Spinor fields in κ-Minkowski noncommutative spacetime

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June 25, 2025

Noncommutative spacetime - What? Why?

- We are interested in possible residual effects of quantum gravity in the flat spacetime limit

$$\text{General Relativity} \xrightarrow[\text{flat limit}]{} \text{Special Relativity}$$

$$\downarrow \kappa?$$

$$\text{Quantum Gravity?} \xrightarrow[\text{flat limit}]{?} \textbf{Deformed SR?}$$

- Certain models (e.g. topological QG in 2+1 dimensions, some spin foam models) predict effective noncommutativity of spacetime:

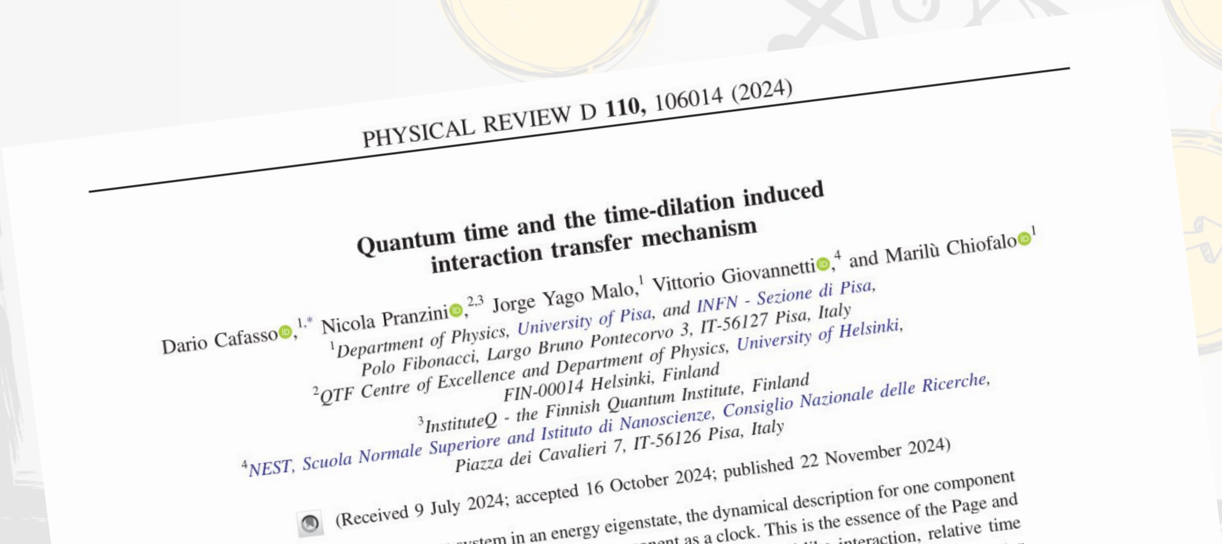
$$[x^\mu, x^\nu] \neq 0$$

- There are many different models of noncommutative spacetime - e.g. Snyder spacetime, Moyal-Weyl spacetime, κ-Minkowski, ρ-Minkowski...
- The attractive feature of κ-Minkowski is that it admits a relativistically invariant length/energy/mass scale, which is a recurring theme across many approaches to QG

κ-field theory

- What kind of phenomenological implications can we extract?
- We investigate the interplay between κ-Poincaré and CPT
- Long-standing research program:
  - L. Freidel, J. Kowalski-Glikman and S. Nowak, *Field theory on kappa-Minkowski space revisited: Noether charges and breaking of Lorentz symmetry*, (2008)
  - M. Arzano, A. Bevilacqua, J. Kowalski-Glikman, G. Rosati, J. Unger, *κ-deformed complex fields and discrete symmetries* (2021)
  - A. Bevilacqua, J. Kowalski-Glikman, W. Wiślicki, *κ -deformed complex scalar field: Conserved charges, symmetries, and their impact on physical observables*, (2022)
- Recent advancements: new perspective on CPT and Dirac fields

Gravitational time-dilation from quantum interactions?



PHYSICAL REVIEW D **110**, 106014 (2024)

Quantum time and the time-dilation induced interaction transfer mechanism

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...system in an energy eigenstate, the dynamical description for one component ... as a clock. This is the essence of the Page and ... interaction, relative time

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University of Naples, Federico II

Time in quantum theory

[Isham, 1993, Page and Wootters, 1983]

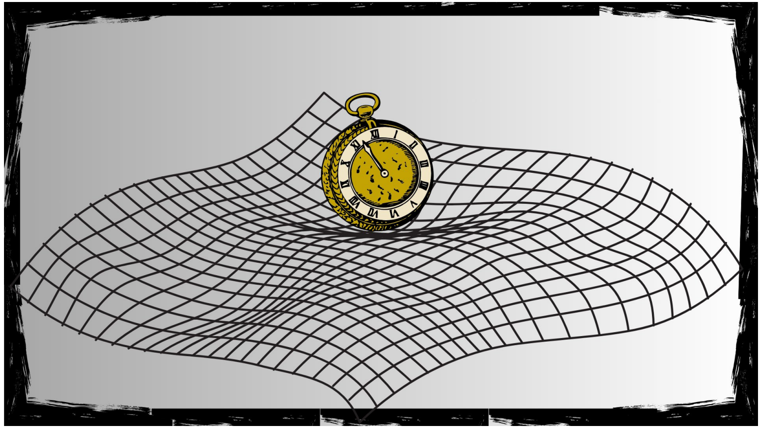
Page and Wootters (PaW)

“If the Universe is truly isolated, there is no place in the theory for an external time parameter”

“Time via a physical clock that is part of the system itself”

Reasonable hypothesis, if we want to compute/simulate...

Engeneered environment as a quantum reference frame



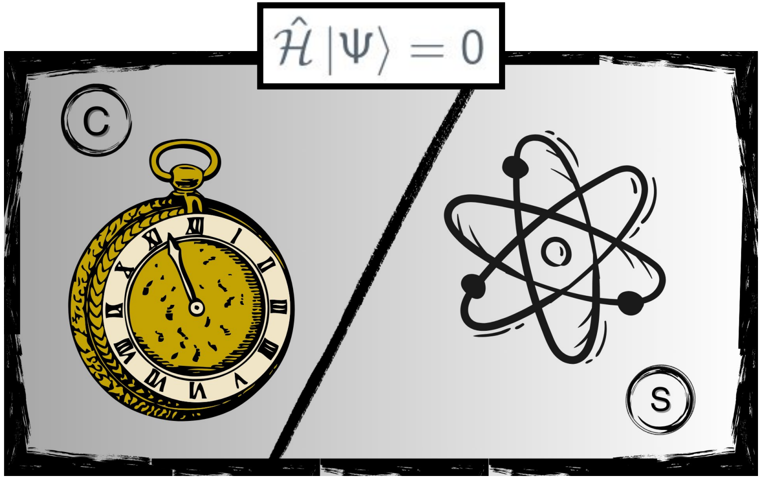
“Whatever can be said about ‘time development’ has to be extracted from this equation.”

$$\hat{\mathcal{H}}\,|\Psi\rangle = 0$$

Evolution without evolution

Bipartite quantum system

$U = C + S$ with $\mathcal{H} = \mathcal{H}_C \otimes \mathcal{H}_S$.



$\hat{\mathcal{H}} = \hat{\mathcal{H}}_C + \hat{\mathcal{H}}_S$ we have

$$\langle t|_C\, \hat{\mathcal{H}}\,|\Psi\rangle = \left(-i\hbar\frac{\mathrm{d}}{\mathrm{d}t} + \hat{\mathcal{H}}_S\right)\langle t|_C\,|\Psi\rangle = 0$$

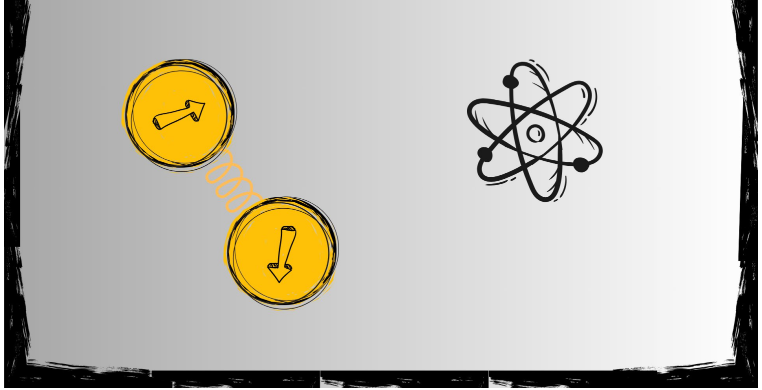
$$|\Psi\rangle = \int \mathrm{d}\mu(t)\; a(t)\,|t\rangle_C\,|\psi(t)\rangle_S$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,|\psi(t)\rangle_S = -\frac{i}{\hbar}\hat{\mathcal{H}}_S\,|\psi(t)\rangle_S \quad \text{with} \quad a(t) = const$$

$$|\psi(t)\rangle_S = \frac{1}{a(t)}\,\langle t|_C\,|\Psi\rangle \quad \text{with} \quad a(t) = \|\langle t|_C\,|\Psi\rangle\|$$

The TiDIT mechanism

$$\hat{\mathcal{H}} = \hbar w\left(\hat{\sigma}_A^x + \alpha\hat{\sigma}_B^x - g\hat{\sigma}_A^x\hat{\sigma}_B^x\right) + \hat{\mathcal{H}}_S$$



The corresponding Schrödinger equation is

$$i\hbar\hat{R}\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} = \left(\alpha\hbar w\hat{\sigma}_B^x + \hat{\mathcal{H}}_S\right)\left|\psi(\tau)\right\rangle_{U|A}$$

$$\hat{R} = \mathbb{1} - g\hat{\sigma}_B^x$$

[Castro-Ruiz et al., 2020]

Quantum time dilation $\xrightarrow{TiDIT}$ new interaction terms!

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}\tau}\left|\psi(\tau)\right\rangle_{U|A} = \hat{\mathcal{H}}_{eff}^{(A)}\left|\psi(\tau)\right\rangle_{U|A}$$

$$\hat{\mathcal{H}}_{eff}^{(A)} = \frac{1}{1-g^2}\left(\alpha\hbar w g + \alpha\hbar w\hat{\sigma}_B^x + \hat{\mathcal{H}}_S + g\hat{\sigma}_B^x\hat{\mathcal{H}}_S\right)$$

Time-Dilation induced Interaction Transfer (TiDIT) mechanism

Thank you for your (classical) time!

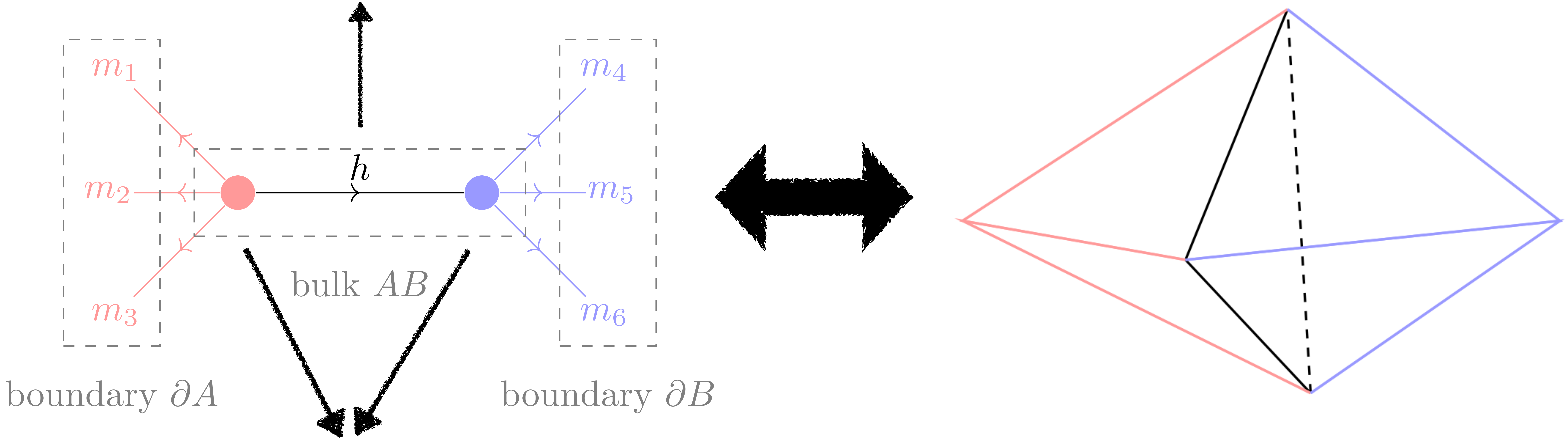
Spin networks on quantum computers

Grzegorz Czelusta

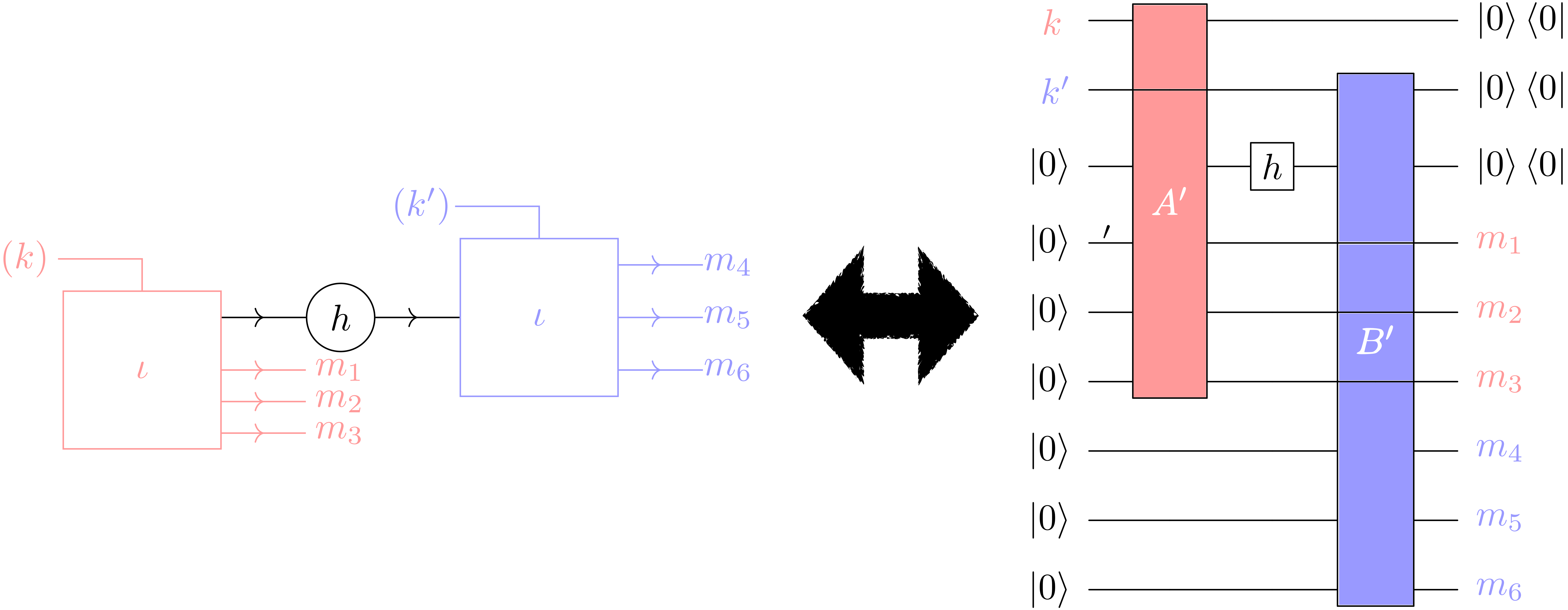
Jagiellonian University, Kraków, Poland

Quantum Cosmos Lab

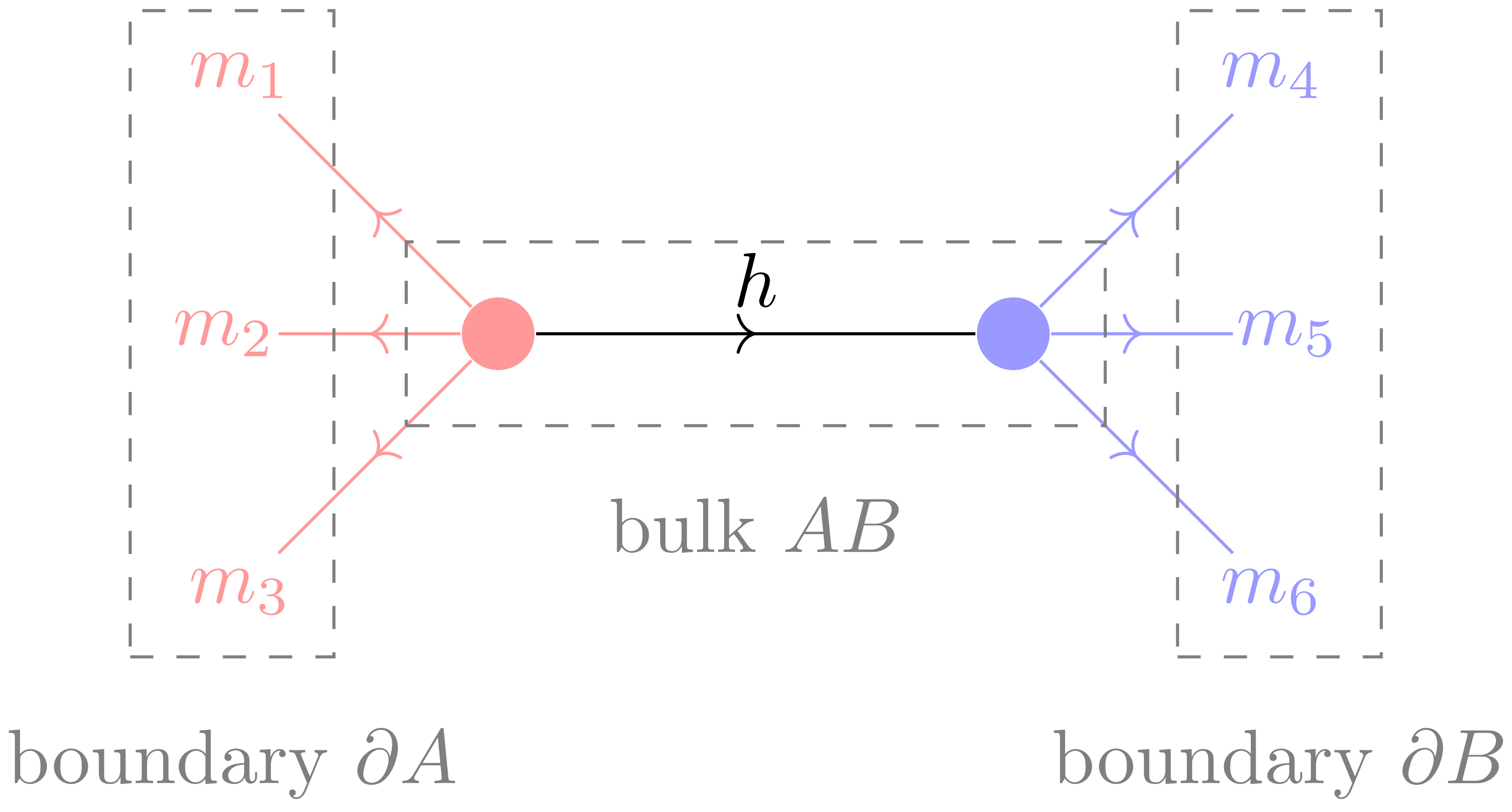
$h \in SU(2)$



$$|\mathcal{I}\rangle \in \mathrm{Inv}_{SU(2)}\left(\mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}}\right)$$

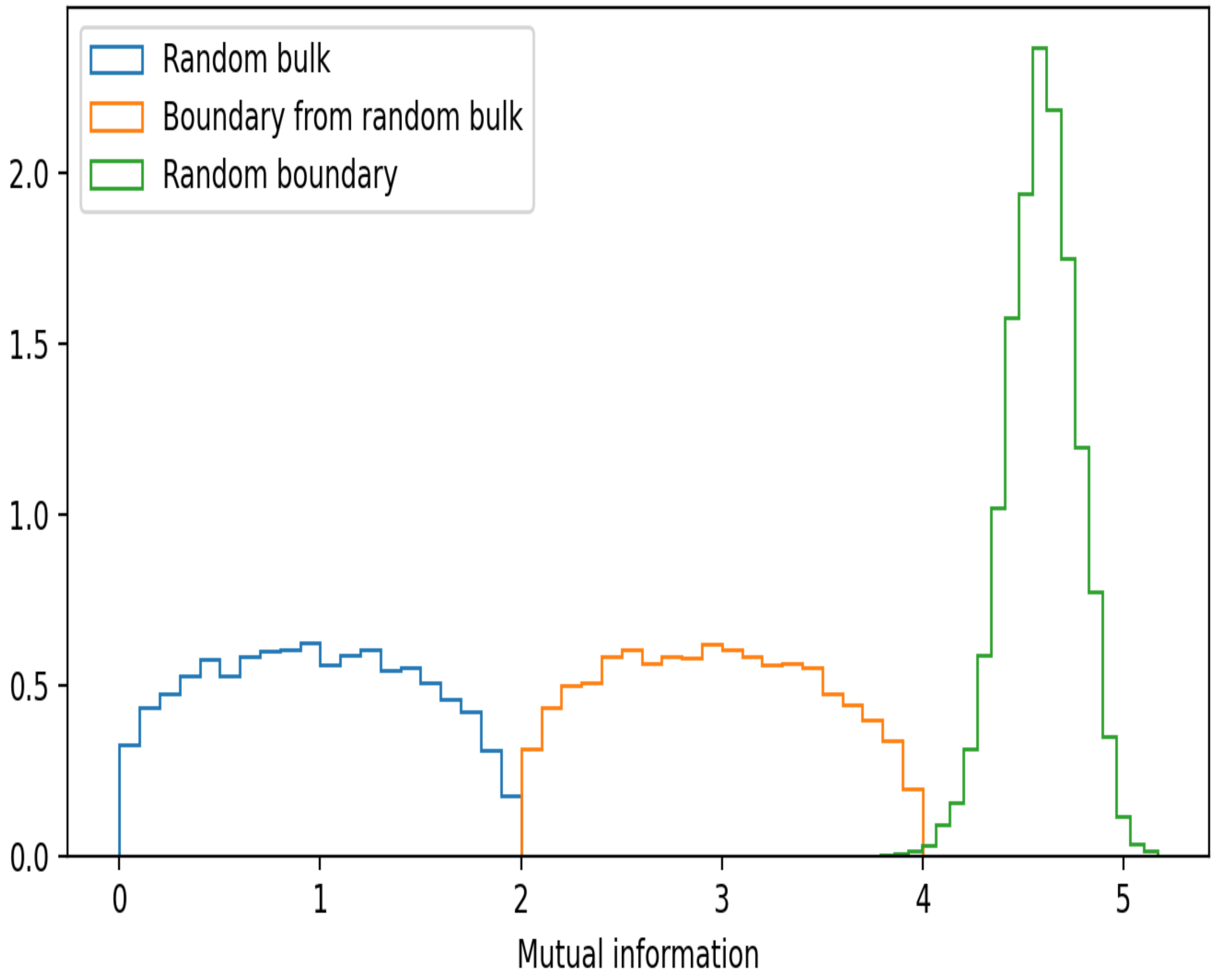


$$\iota_{(k)}^{\;m_1 m_2 m_3 m} \, D\left(h\right)^{m'}{}_{m} \, \iota_{(k')m'}{}^{m_4 m_5 m_6}$$



$$S(\hat{\rho}_{\partial A}) = S(\hat{\rho}_A) + \log(2j+1)$$

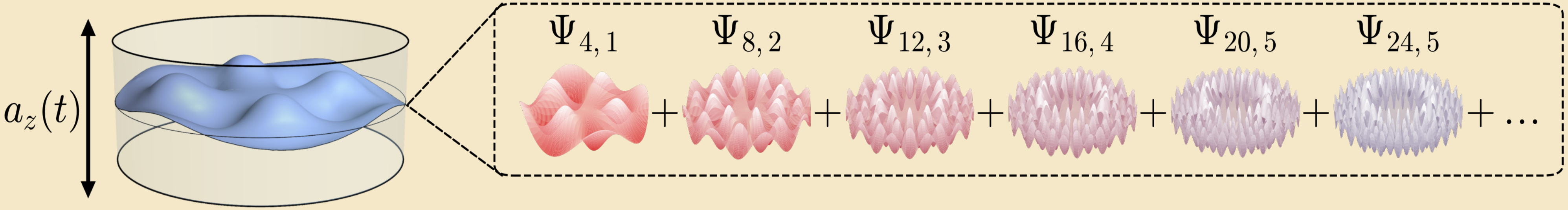
Livine, E. R. (2018). Physical Review D, 97(2), 026009.



Czelusta, G., Mielczarek, J. (2025). Physical Review D, 111(6), 066012.

Formation of an interfacial wave spectral cascade: from *one* to *few* to *many*

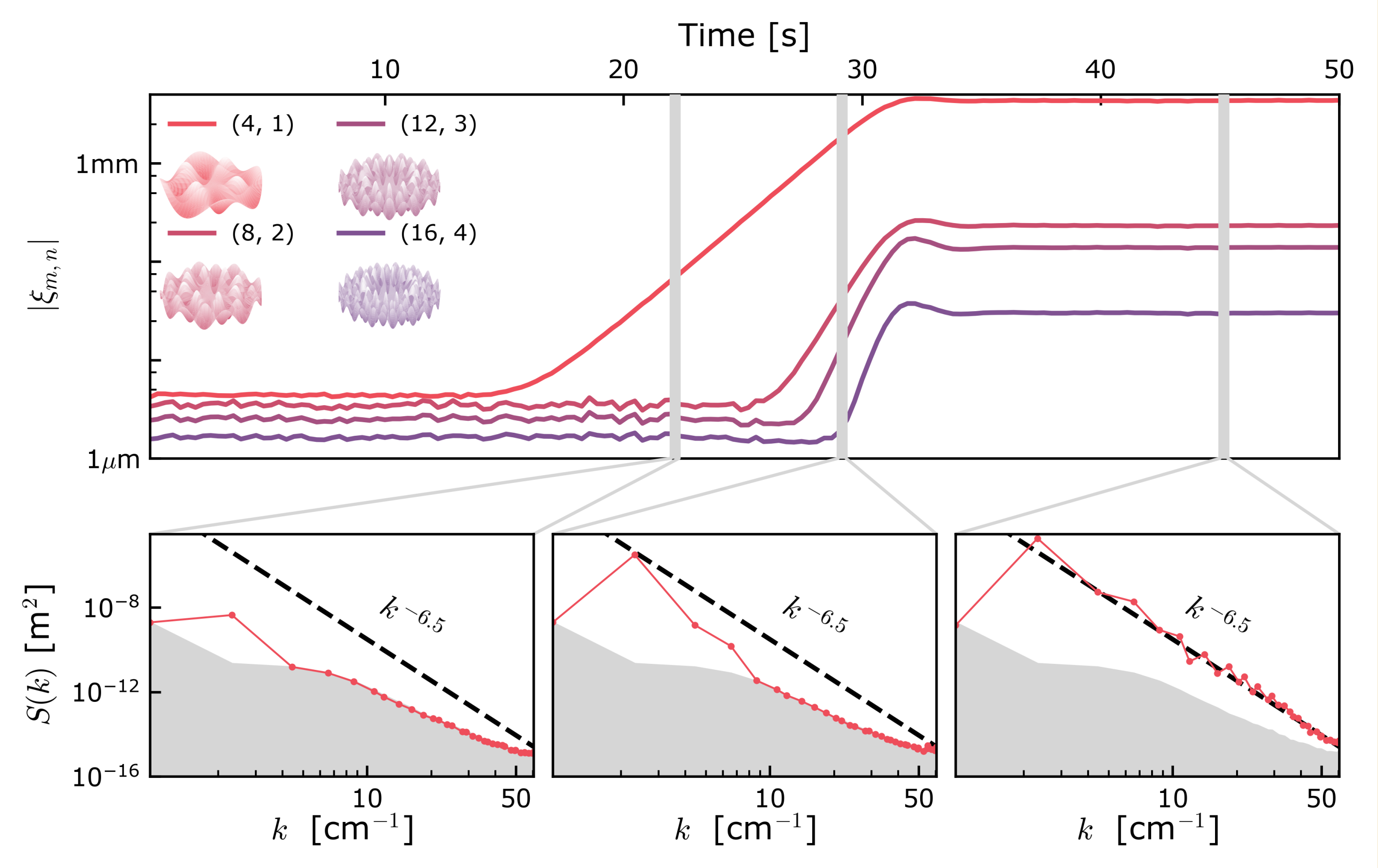
Seán Gregory, Silvia Schiattarella, Vitor Barroso, David Kaiser, Anastasios Avgoustidis, & Silke Weinfurtner



The interface between two fluids, $\xi(t, \vec{x})$, is oscillated with a shaking platform

This excites one low wavenumber k mode through parametric resonance

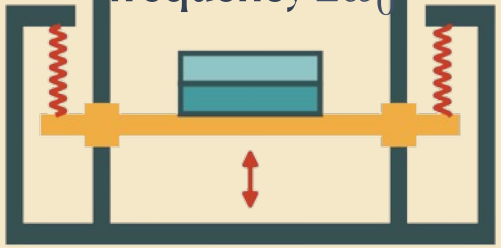
Through nonlinear interaction, this *one* mode excites a *few*, which in turn excite *many* more



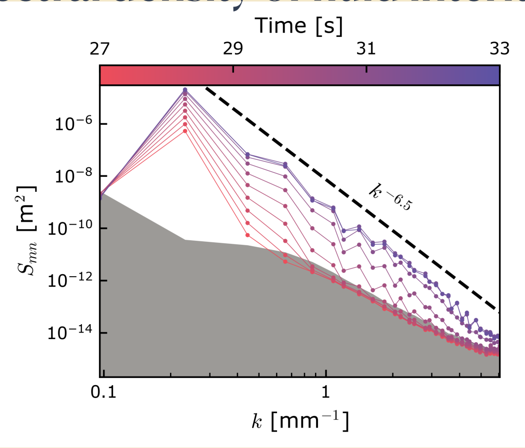
Analogue cosmology

Fluid-fluid interface $\xi(t, \vec{x})$

Periodic forcing from shaking platform at amplitude A, frequency $2\omega_0$



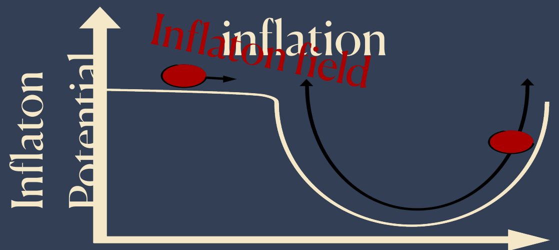
Power spectral density of fluid interface S_{mn}



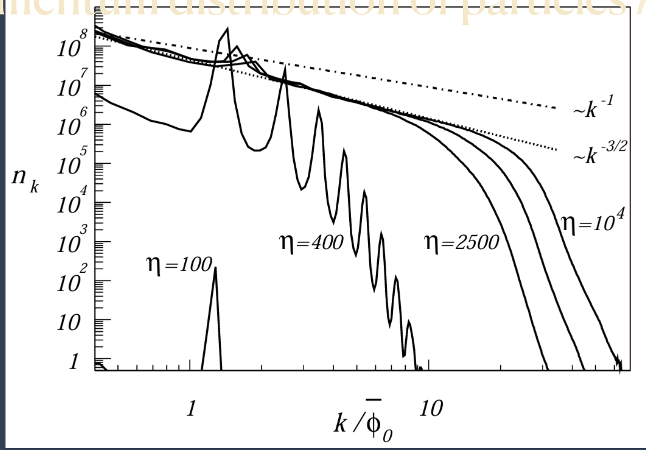
[1]

(2+1) Scalar field in the early universe $\phi(t, \vec{x})$

Periodic forcing from inflaton oscillating around potential minima at amplitude A, frequency $2\omega_0$, immediately after inflation



Momentum distribution of particles n_k



[2]

[1] Gregory, S., Schiattarella, S., Barroso, V. S., Kaiser, D. I., Avgoustidis, A., & Weinfurtner, S. (2024). Tracing the nonlinear formation of an interfacial wave spectral cascade from one to few to many. arXiv preprint arXiv:2410.08842

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Shannon wavelets and ‘holographic-like’ lattices in QFT

Speaker: Dominic G. Lewis

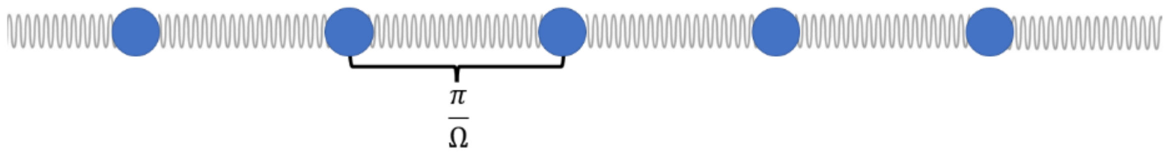
In collaboration with: Nicholas Funai, Simon Vedl, Dan George, Achim Kempf, Gavin Brennan, Nicolas C. Menicucci

June 2025

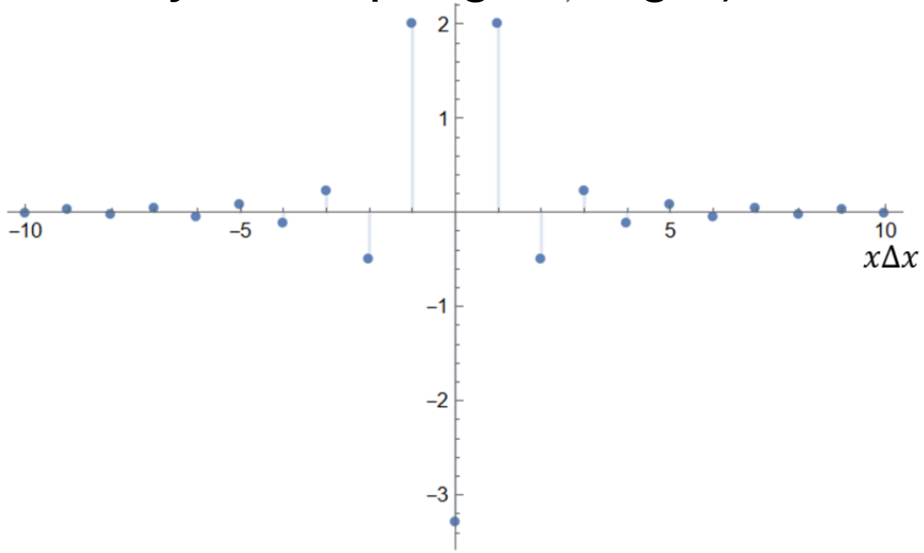
RMIT University, Macquarie University, University of Waterloo

Some history

An ultraviolet cut-off to QFT → Permits an equivalent lattice model of coupled harmonic oscillators

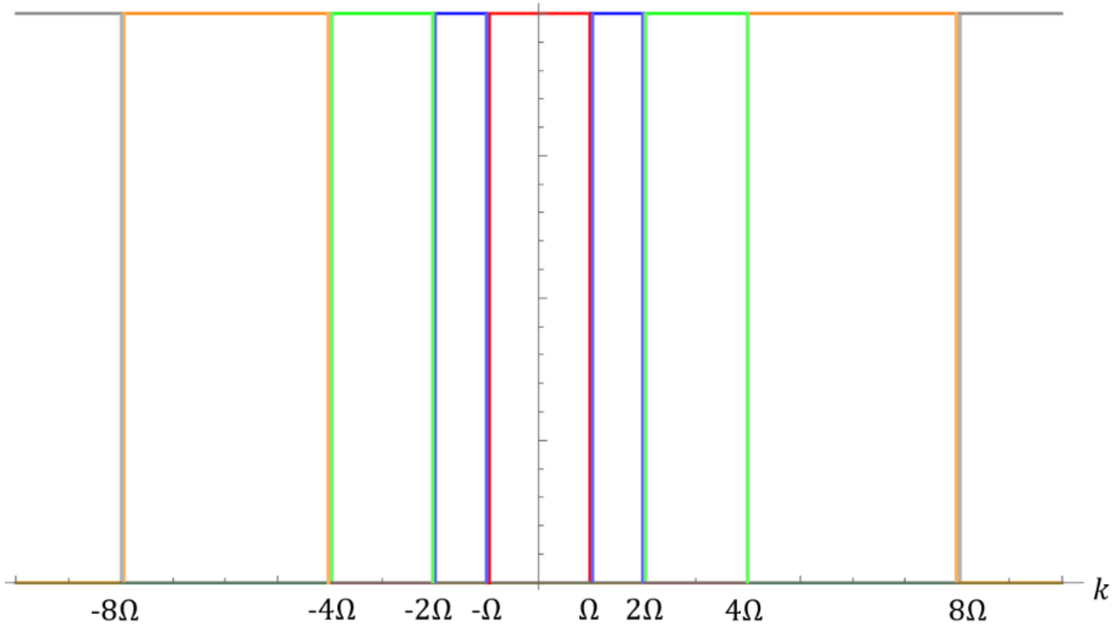


Local derivative operators on the continuous field are non-local on the lattice (polynomial decay in coupling strength)



What about without a cut-off?

Using a Shannon wavelet basis, we can use sampling theory on fields without a cut-off!



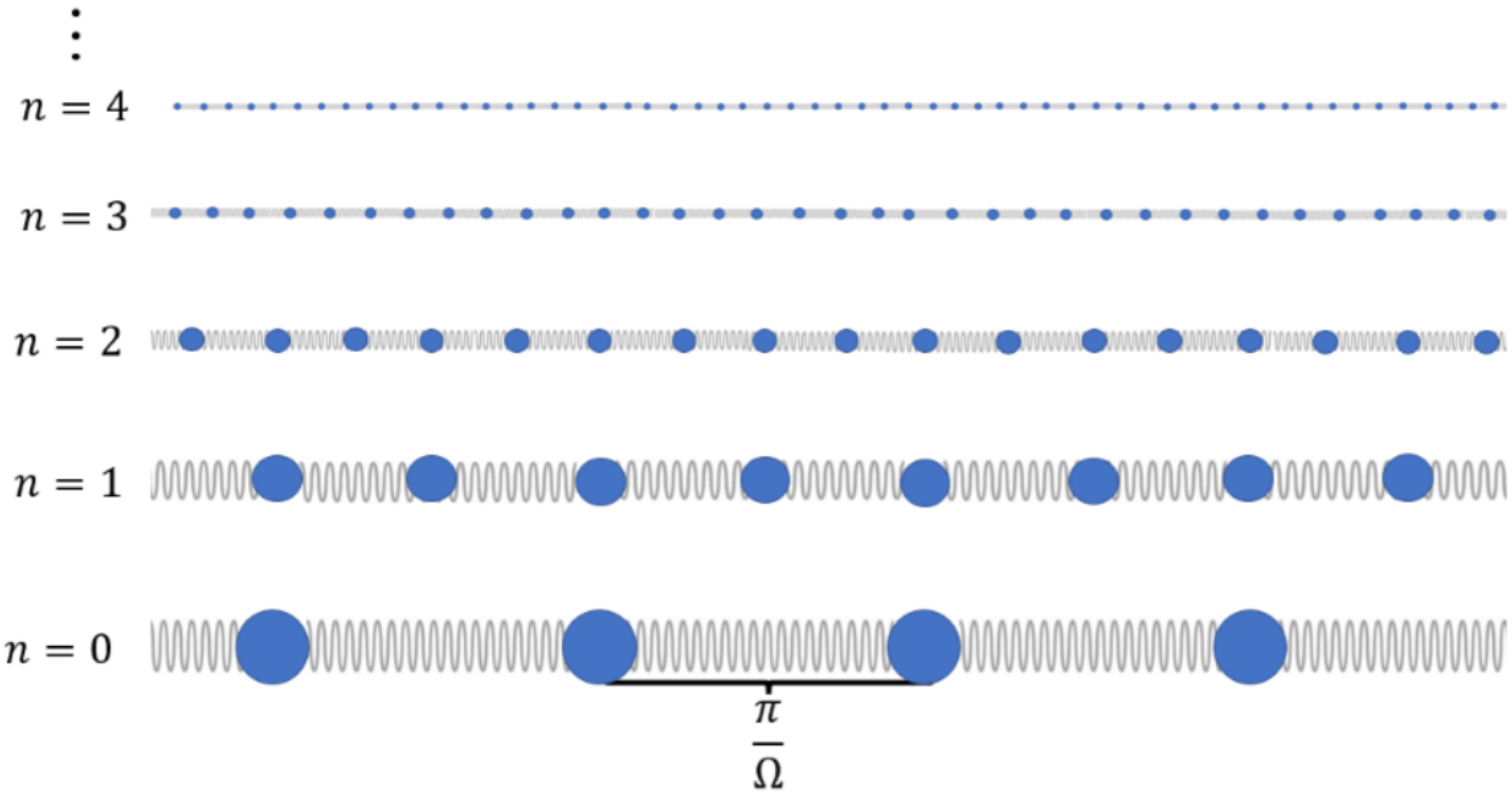
We take the momentum space of a field and separate it into discrete layers and each layer acts as its own bandlimited QFT with its own lattice model.

$$2^{-n}\frac{\pi}{\Omega}$$

$$\hat{H}^n := \frac{1}{2}2^{-n}\frac{\pi}{\Omega}\sum_{h\in\mathbb{Z}}\left((\hat{\pi}_h^n)^2 + m^2\left(\hat{\phi}_h^n\right)^2 - \hat{\phi}_h^n\sum_{\ell\in\mathbb{Z}}\gamma_{h\ell}^{(2)nn}\hat{\phi}_\ell^n\right)$$

The discrete field has a new dimension!

We place the lattices on top of each other and observe the length scale of each layer of the cake scales hyperbolically!



You start with a d-dim continuous QFT and end with a d+1 dim discrete one (with hyperbolic-like geometry)**!**

How far does this rabbit hole go?

Come to my poster and find out ☺️

Phys. Rev. D 111, 044001 (2025) / arXiv:2408.02729 [gr-qc]

Gravity-induced birefringence in spherically symmetric spacetimes

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Daniel R. Terno† and Rama Vadapalli‡
School of Mathematical and Physical Sciences, Macquarie University, NSW 2109, Australia

Daniel R. Terno

Rama Vadapalli

MACQUARIE University
SYDNEY·AUSTRALIA

Polarization-dependent photon trajectories

Polarized ray equation:

Frolov
Phys. Rev. D **102**, 084013 (2020)

SM, Terno, Vadapalli
Phys. Rev. D **111**, 004001 (2025)

$\frac{dx^\nu}{d\tau}$

$$\frac{D^2 x^\mu}{D\tau^2} = -\frac{\sigma}{\omega} R^\mu{}_{\nu\alpha\beta}\, l^\nu e^\alpha_{(1)F} e^\beta_{(2)F} =: w^\mu$$

left- and right-handed circular polarization $(\sigma = \pm 1)$

➡ In the presence of curvature the motion of circularly polarized photons is nongeodesic and depends on both their helicity and their frequency.

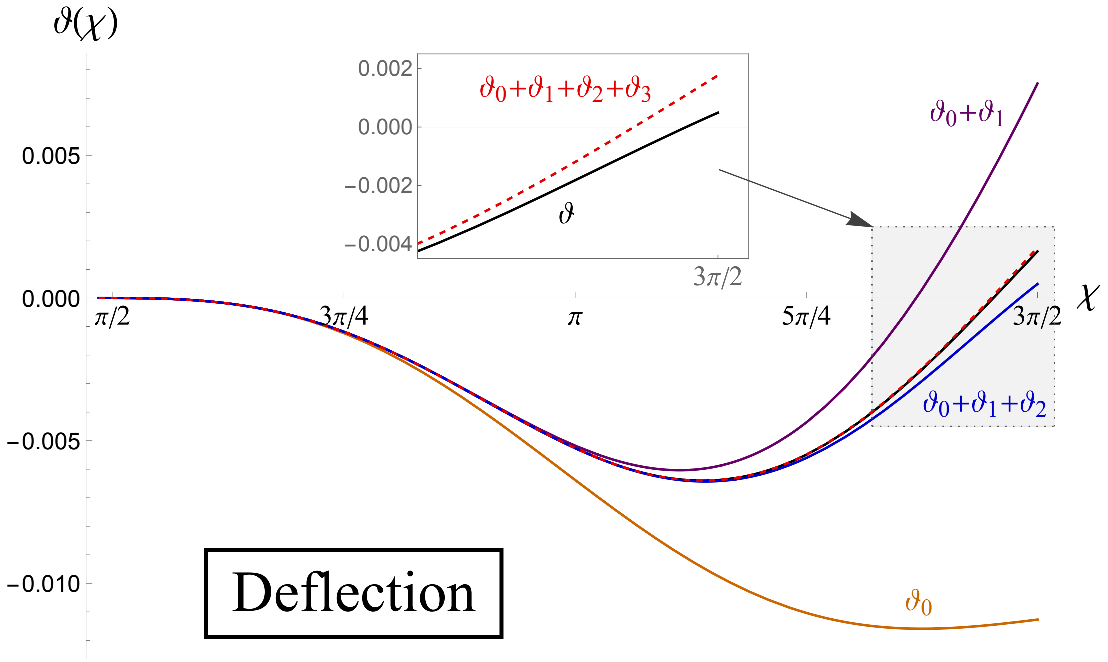
Note:

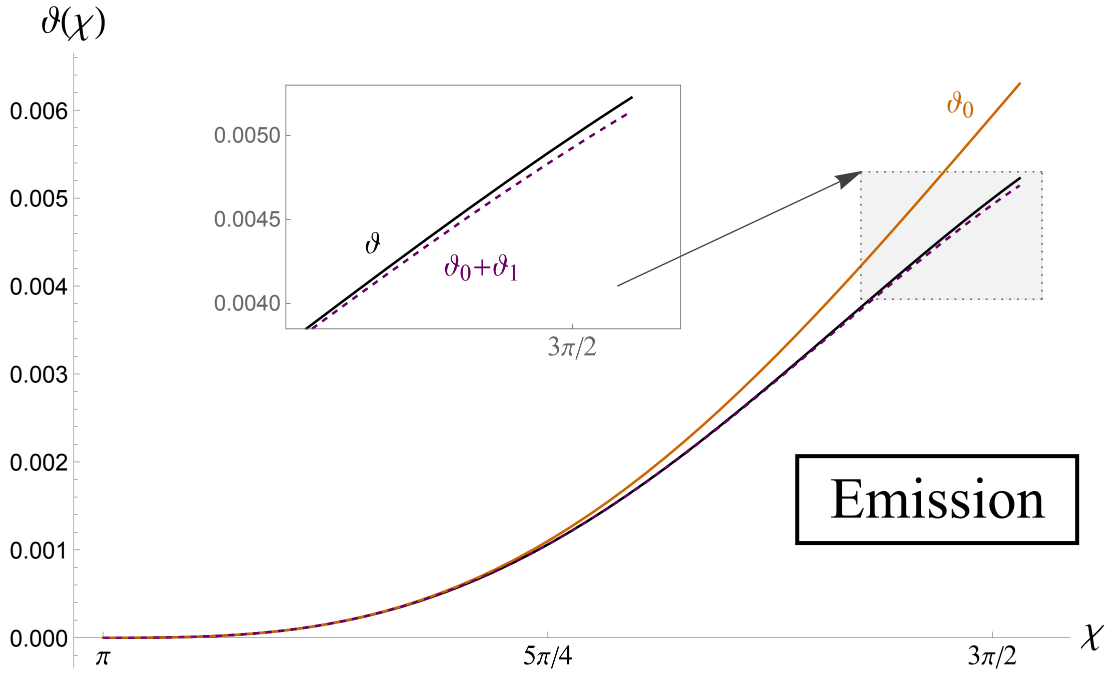
Calculation of w^μ requires *Fermi-propagated null tetrad*, but its construction requires knowing the corrections!

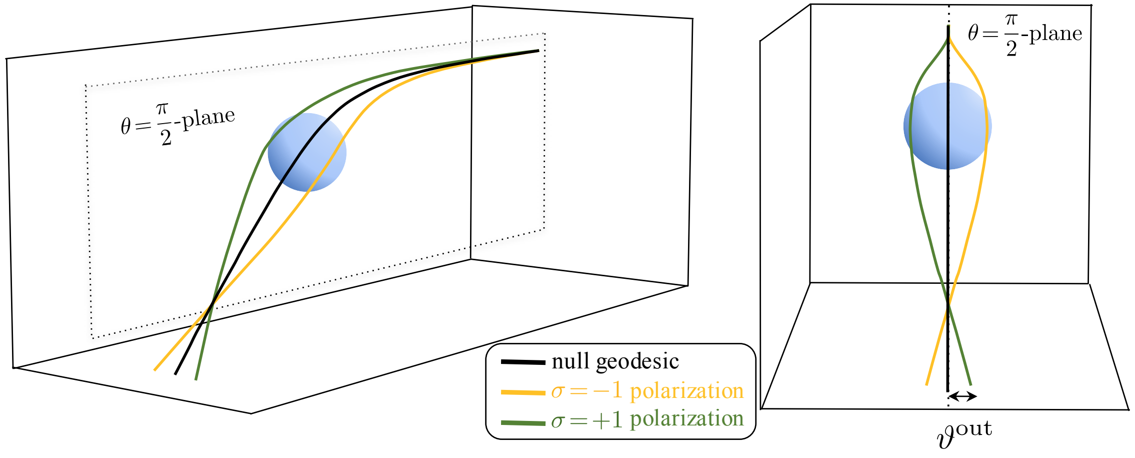
➡ **Analytically:** use perturbative approach $x^\mu = \mathring{x}^\mu + (\omega L)^{-1} x^\mu_{(1)} + \mathcal{O}\big((\omega L)^{-2}\big)$

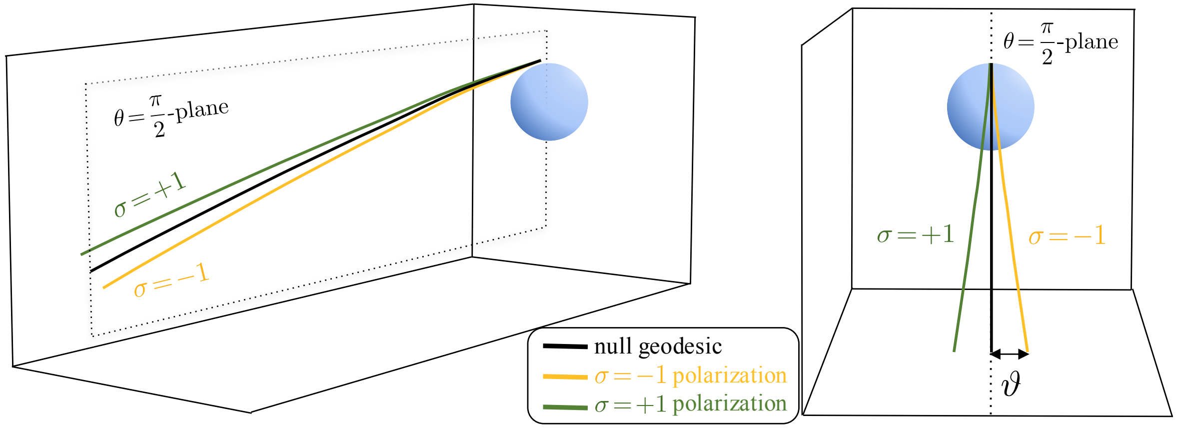
➡ **Numerically:** incorporate Fermi-Walker transport law in system of equations

Polarization-induced corrections ϑ in the deflection and emission scenario









Inference and Fine-Tuning in Causal Explanations of Bell-Inequality Violations

Joppe Widstam

Max Planck Institute for the Physics of Complex Systems - Dresden, Germany

- Claim: Any asymmetry in time* is thermodynamic in origin.
- Tease apart ~~causation~~ nature and mere knowledge in quantum mechanics
- Interpret quantum mechanics as involving retrocausal influence
- Intuitively understand:
  1. how knowing all there is too know about the total system, one knows as little as possible about the individual subsystems
     $$\rho_A = \mathrm{Tr}_B[\,|\Phi^+\rangle\langle\Phi^+|\,] = \frac{1}{2}\mathbb{I}_2$$
  2. why Wood and Spekkens claim that causal explanations of Bell inequality violations require fine-tuning.

*CPT in QFT

“An entangled system must be considered an inseparable whole”

How knowing all there is too know about the total system, one knows as little as possible about the individual subsystems

- Probability of Alice’s measurement outcome depends on the “clustering” of all relevant variables $p\left(k_A \mid M^A, M^B, k_B\right)$.

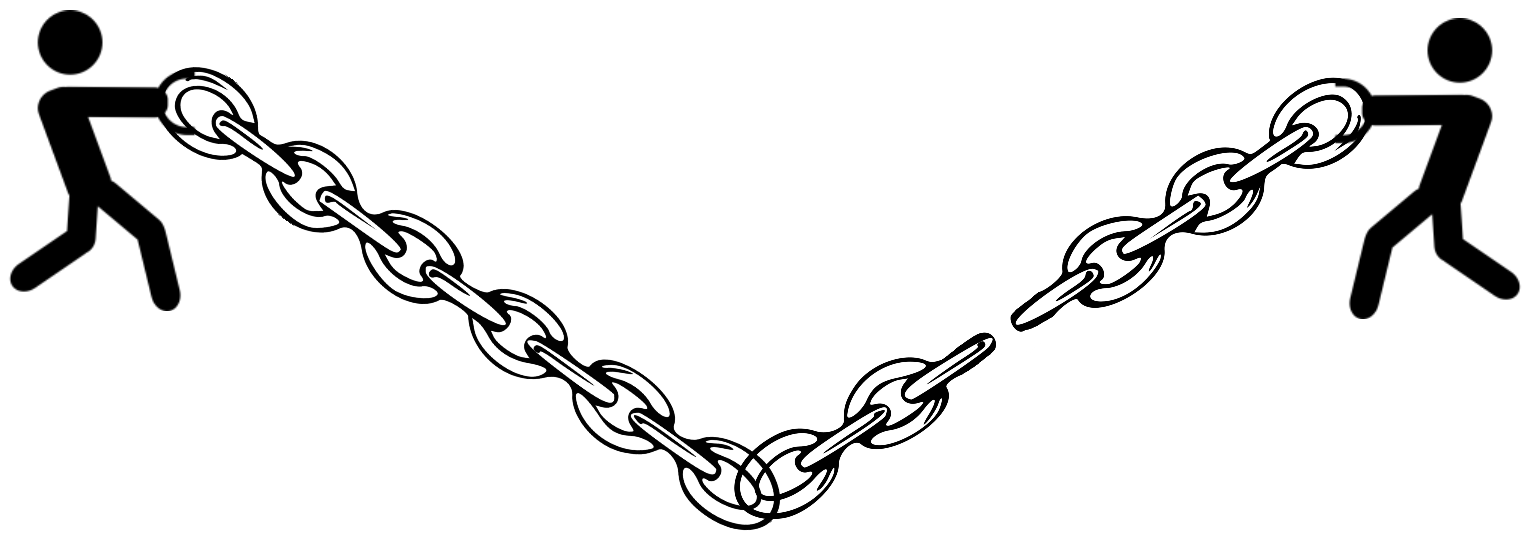
- Maximal ignorance about one of the relevant variables hides the dependence of Alice's outcome on the other relevant variables.

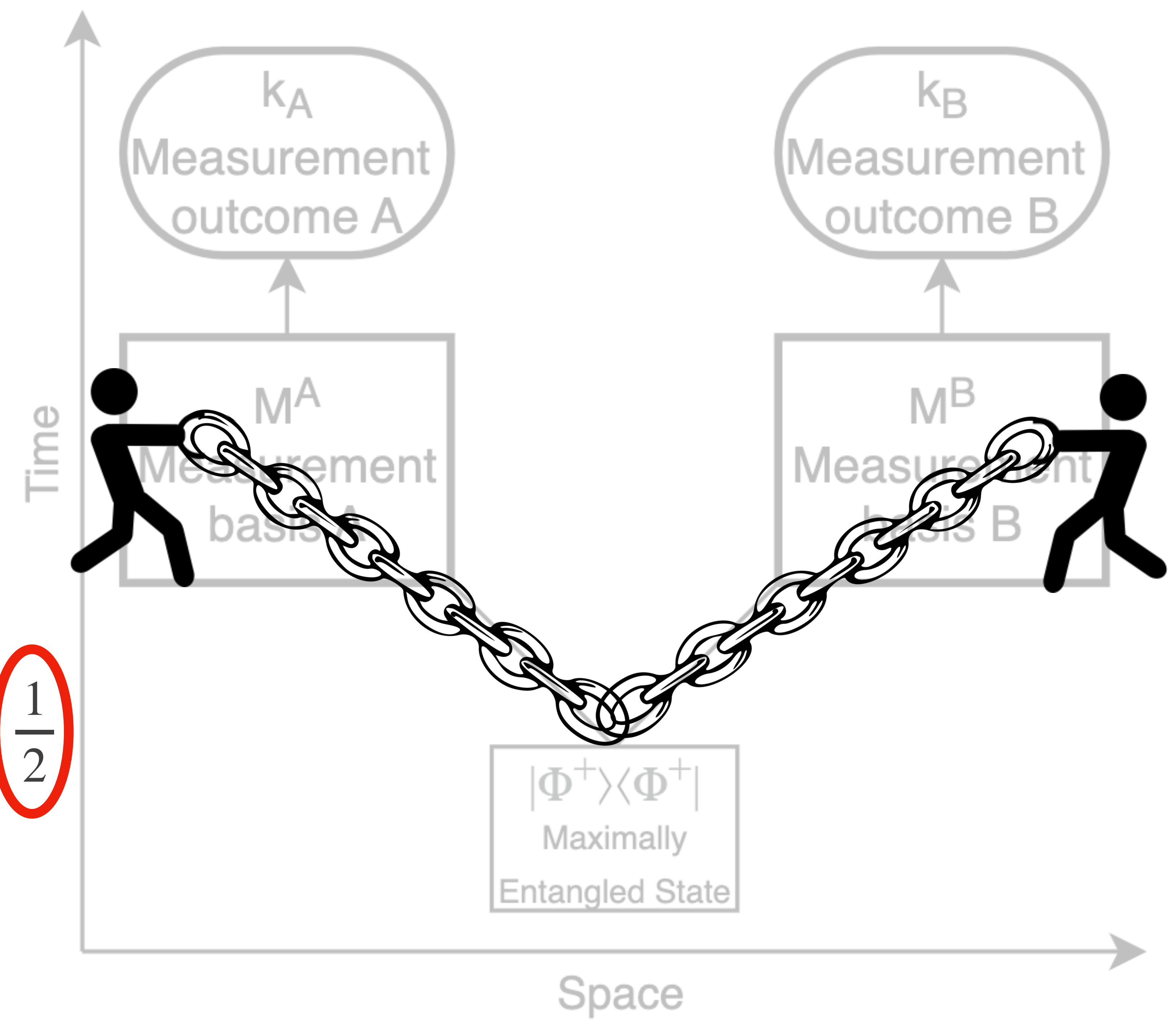
$$p\left(k_A \mid M^A, M^B, k_B\right) \to p\left(k_A \mid M^A, k_B\right) = p\left(k_A \mid M^A, M^B\right) = p\left(k_A \mid M^B, k_B\right) = \frac{1}{2}$$

- Tracing out Bob’s subsystem:

$$\rho_A = \mathrm{Tr}_B[\,|\Phi^+\rangle\langle\Phi^+|\,] = \frac{1}{2}\mathbb{I}_2$$

$$p_{k_A} = \mathrm{Tr}\left(|k_A\rangle\langle k_A|\rho_A\right) = \frac{1}{2}$$





Thank you for listening!

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Price, H., & Wharton, K. (2015). Disentangling the quantum world. *Entropy*, *17*(11), 7752–7767.

Evans, P. W. (2021). A sideways look at faithfulness for quantum correlations. *The Journal of Philosophy*, *118*(1), 28–42.

Grothus, M., & Vilasini, V. (2024). *Characterizing Signalling: Connections between Causal Inference and Space-time Geometry.* arXiv:2403.00916

If you are the author of this paper please visit my poster or talk to me at coffee break, there is a small community of philosophers I think will want to cite your paper.

Vilasini, V., & Colbeck, R. (2022). General framework for cyclic and fine-tuned causal models and their compatibility with space-time. *Physical Review A*, *106*(3), 032204.