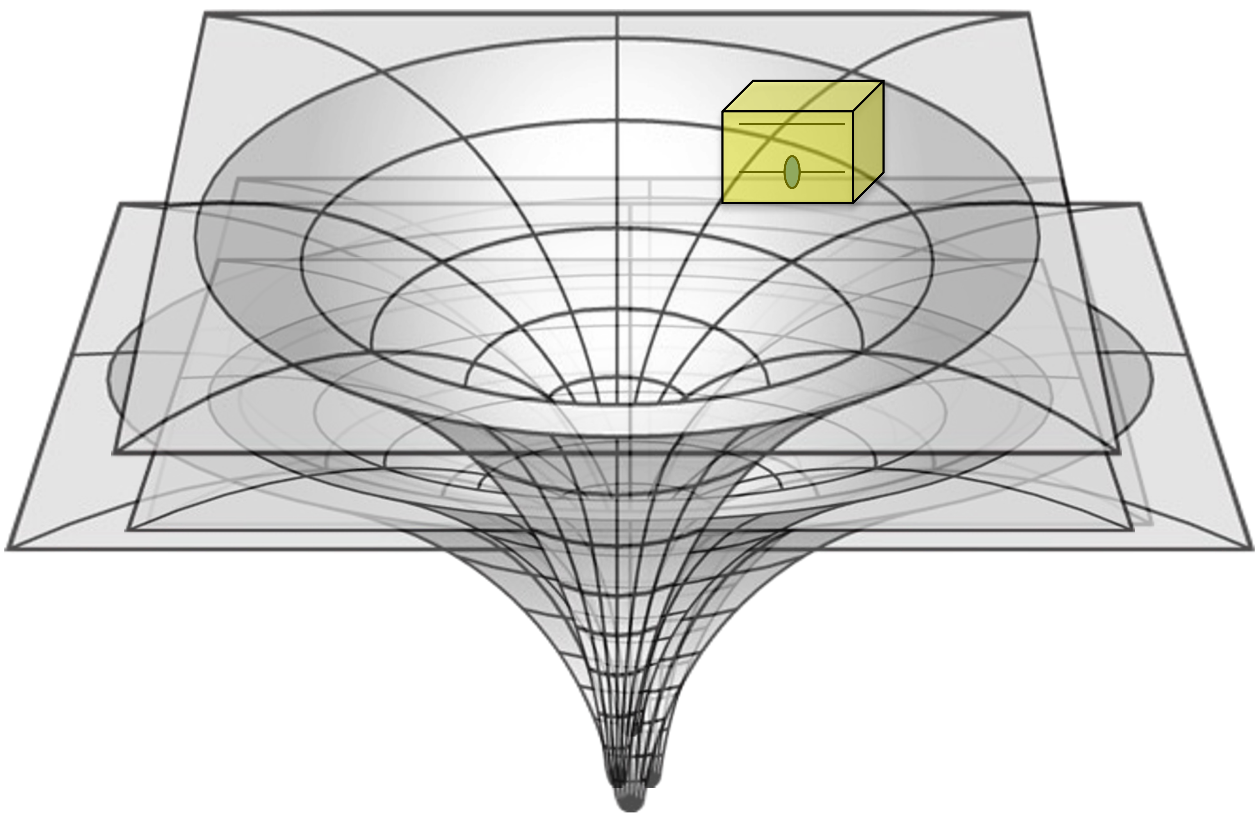
Relativistic Quantum Information
COST Action 23115

UNIVERSITY OF WATERLOO | IQC Institute for Quantum Computing

Sensing Superposed Spacetime



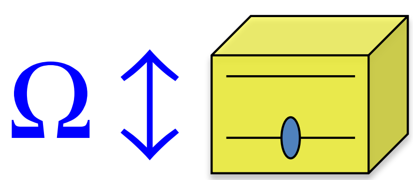
N. Afshordi
C. Arabaci
J. Foo
L. Goel
S. Ono
E. Patterson

Robert B. Mann

M. Preciado Rivas
A. Sajeendran
C. Suryaatmadja
S. Wang
M.Zych

Spacetime in Superposition

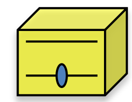
- Still lack a quantum theory of gravity
- General Expectation: Spacetime superposition
- How can this be done without a theory of quantum gravity?
- What would a spacetime superposition "look like"? How would we detect it?



UdW detector

Quantum Detector Formalism

$$S = -\int d^4x\sqrt{-g}\left[R + \frac{1}{2}\partial_\mu \Phi(x)\partial^\mu \Phi(x) - \xi R \Phi^2(x)\right]$$

$\Omega \updownarrow$ 

UdW detector
→ quantum dot

$$+\sum_D \int d\tau_D \frac{m_0}{2}\left[\dot{Q}_D^2(\tau_D) - \Omega_D^2 Q^2\right] + \sum_D \lambda_D \int d^4x Q_D(\tau)\Phi(x)\delta^4\left(x^\mu - z_D^\mu(\tau)\right)\Big\}$$

$$H_{ID}(\tau) = \chi_D(\tau)\left[e^{i\Omega_D\tau}\sigma_D^+ + e^{-i\Omega_D\tau}\sigma_D^-\right]\Phi[z_D(\tau)]$$

switcher

monopole operator

$$\chi_D = \exp\left(-\frac{(\tau-\tau_D)^2}{2\sigma_D^2}\right)$$

$$\sigma_D^+ := |1_D\rangle\langle 0_D| = |e_D\rangle\langle g_D|$$

$$\sigma_D^+ := |0_D\rangle\langle 1_D| = |g_D\rangle\langle e_D|$$

Initial state

$$\rho_I = \rho_{D_I}\rho_{\Phi_I}$$

Unitarily evolve

$$U = \mathcal{T}e^{-i\int dt\left[\sum_D \frac{d\tau_D}{dt} H_{ID}(\tau_D)\right]}$$

Final state

$$\rho_F = U\rho_I U^\dagger$$

Trace over field

$$\rho_{D_F} = \mathrm{Tr}_\Phi\left[\rho_F\right]$$

single detector

$$\rho_D = \begin{pmatrix} 1 - P_D & 0 \\ 0 & P_D \end{pmatrix}$$

response

two detectors

$$\rho = \begin{pmatrix} 1 - P_A - P_B & 0 & 0 & X \\ 0 & P_B & C & 0 \\ 0 & C^* & P_A & 0 \\ X^* & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$

entanglement, mutual information

$$P_D = \lambda^2 \int_{-\infty}^{\infty} d\tau_D \int_{-\infty}^{\infty} d\tau_{D'}\, \chi_D(\tau_D)\chi_{D'}(\tau_{D'}) e^{-i\Omega_D(\tau_D - \tau_{D'})} W\left(x_D(\tau_D), x_D(\tau_{D'})\right) \qquad D = A, B$$

$$C = \lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{d\tau_A}{dt}\frac{d\tau_B}{dt'} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_A\tau_A - \Omega_B\tau_B)} W\left(x_A(t), x_B(t')\right)$$

$$X = -\lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \Big[\frac{d\tau_A}{dt'}\frac{d\tau_B}{dt} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_B\tau_B + \Omega_A\tau_A)} W\left(x_A(t'), x_B(t)\right) + \frac{d\tau_A}{dt}\frac{d\tau_B}{dt'} \chi_A(\tau_A)\chi_B(\tau_B) e^{-i(\Omega_A\tau_A + \Omega_B\tau_B)} W\left(x_B(t'), x_A(t)\right) \Big]$$

Wightman Function

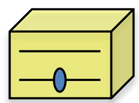
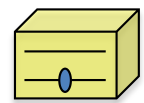
$$W(x, x') := \langle 0 | \phi(x)\phi(x') | 0 \rangle \qquad \tau_D = \gamma_D t$$

- detector motion
- spacetime curvature
- initial field state (vacuum, thermal, coherent, etc)
- choice of quantum field (scalar, vector, …)

Quantum Controlled Detectors

Foo/Onoe/Zych
PRD **102** (2020) 085013
Foo/Onoe/RBM/Zych
PRR **3** (2021) 043056

Superposed Detector → use Quantum Control

$\alpha|$  $c_1\rangle + \beta|$  $c_2\rangle$

control qbits

$$\hat{H}_I(\tau) = \lambda\left[e^{i\Omega\tau}\sigma^+ + e^{-i\Omega\tau}\sigma^-\right]\sum_{i=1}^{N} \chi_i(\tau)\Phi(\mathbf{z}_i(\tau))\otimes|c_i\rangle\langle c_i|$$

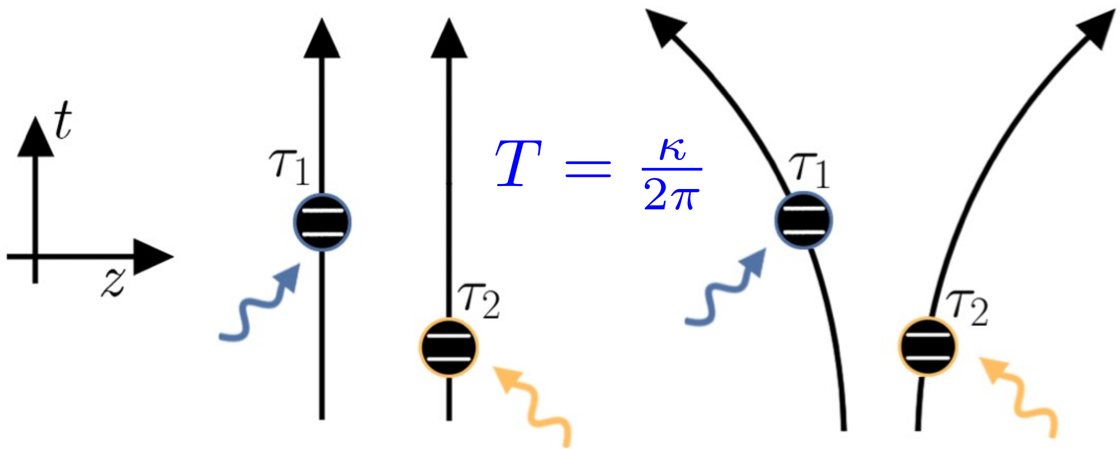
Measure in control state $|c\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|c_i\rangle$

$$P_D = \frac{\lambda^2}{N^2}\sum_{i,j=1}^{N}\int_{-\infty}^{\infty} d\tau' \chi_i(\tau') e^{-i\Omega\tau'}\int_{-\infty}^{\infty} d\tau'' \overline{\chi_j}(\tau'') e^{-i\Omega\tau''}\mathcal{W}_+^{ji}(\tau',\tau'')$$

Superposed trajectories exhibit interference

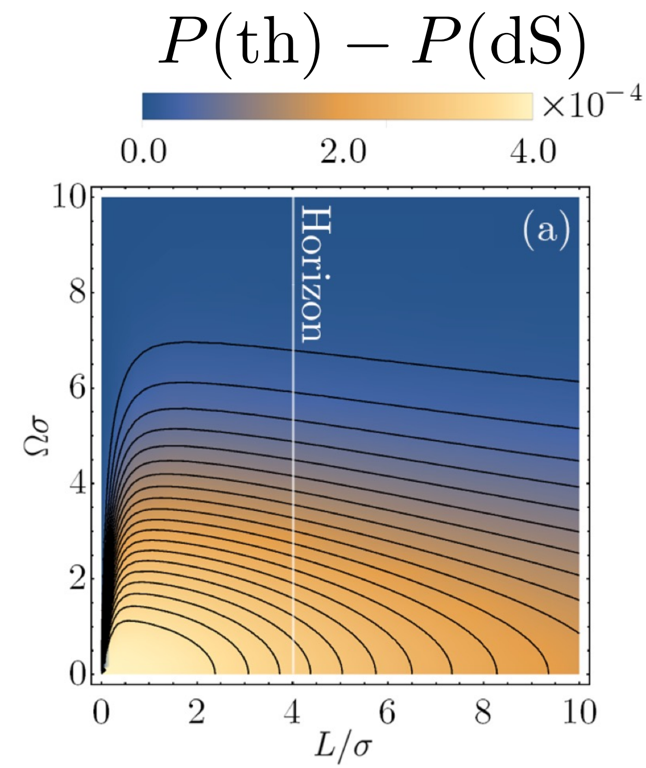
A Superposed Detector gains information about the field!

$$\Gamma \propto \sum_{i,j=1}^{N} \int_{-\infty}^{\tau} d\tau' \chi_j(\tau') e^{-i\Omega\tau'} \int_{-\infty}^{\tau} d\tau'' \overline{\chi}_i(\tau'') e^{-i\Omega\tau''} \mathcal{W}_{+}^{ji}(\tau', \tau'')$$



Superposed Thermal

Superposed de Sitter



- Superposed detector separated by a distance *L*
- Single detector cannot distinguish between a thermal bath and conformal vacuum of expanding universe at same temperature
- Two detectors CAN distinguish these two settings
- Single superposed detector CAN ALSO distinguish these two settings

Gibbons/Hawking PRD (1979)

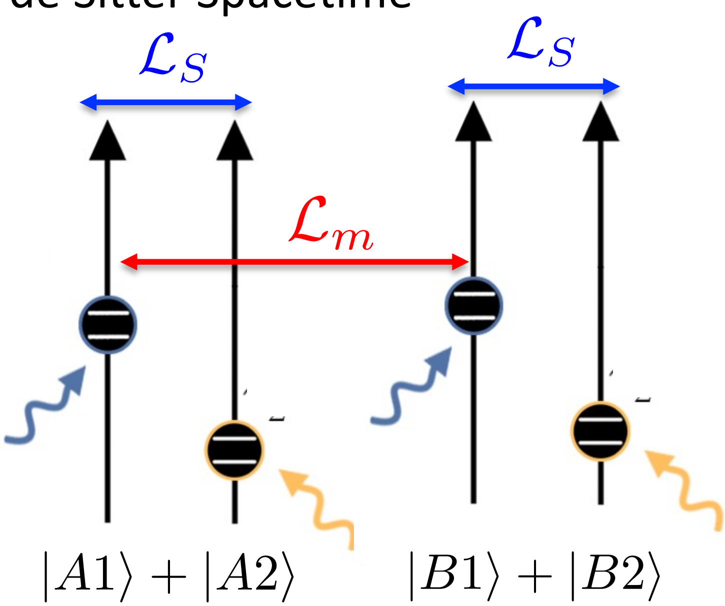
Ver Steeg/Menicucci PRD **79** (2009) 0440276

Foo/Onoe/RBM/Zych PRR **3** (2021) 043056

Superposed Detectors amplify Harvested Entanglment

Foo/Mann/Zych
PRD **103** (2021) 065013

de Sitter Spacetime



$$\mathcal{P}_D = \frac{\lambda^2\sqrt{\pi\sigma^2}}{2}\left\{\int_{-\infty}^{\infty} \mathrm{d}s\, \frac{e^{-s^2/4\sigma^2}e^{-i\Omega s}}{\sinh^2(\beta s/2 - i\varepsilon)} + \int_{-\infty}^{\infty} \mathrm{d}s\, \frac{e^{-s^2/4\sigma^2}e^{-i\Omega s}}{\sinh^2(\beta s/2 - i\varepsilon) - (\beta\mathcal{L}_s/2)^2}\right\}$$

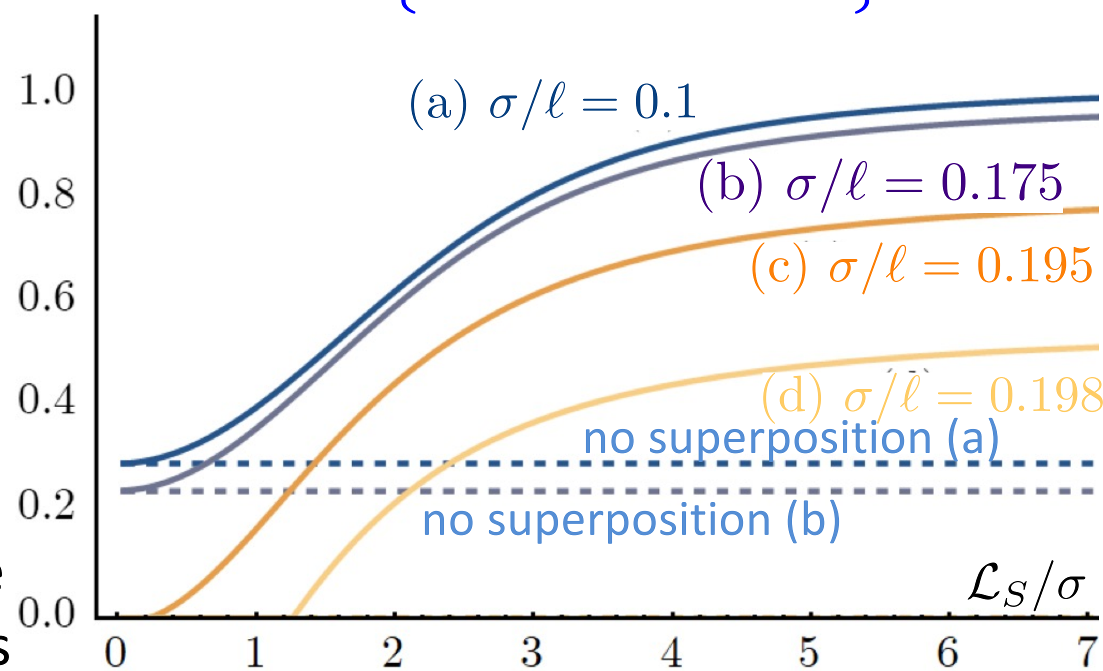
$$\beta = \sqrt{\ell^2 - R_D^2}$$

$$\mathcal{M} = -2\lambda^2\sqrt{\pi\sigma^2}e^{-\sigma^2\Omega^2}\int_0^{\infty} \frac{\mathrm{d}s\, e^{-s^2/4\sigma^2}}{\sinh^2(\beta s/2 - i\varepsilon) - (\beta\mathcal{L}_m/2)^2}$$

$$\mathcal{C}(\hat{\rho}_D) = 2\max\left\{0, |\mathcal{M}| - \sqrt{\mathcal{P}_A\mathcal{P}_B}\right\}$$

- Concurrence → monotonically increases with superposition distance
- Entangling term identical for both superposed and non-superposed trajectories

→ Amplification due to interference terms in local transition probabilities



Schroedinger's Cat in de Sitter Space

Superpositions of stationary detector trajectories in a single spacetime ⟷ A single detector in a superpositions of diffeomorphic spacetimes

$$|\Psi\rangle_{\mathrm{iFD}} = \frac{1}{\sqrt{2}}\big(|\xi\rangle + |\xi + \mathcal{L}\rangle\big)|g\rangle|0_{\mathrm{dS}}\rangle$$

$$= \frac{1}{\sqrt{2}}\underbrace{\big(\mathbb{I} + \hat{\mathcal{T}}(\mathcal{L})\big)|\xi\rangle|g\rangle}_{\text{Detector in superposed trajectory}}|0_{\mathrm{dS}}\rangle$$

Giacomini/Brukner Quantum Sci **4** (2022) 015601

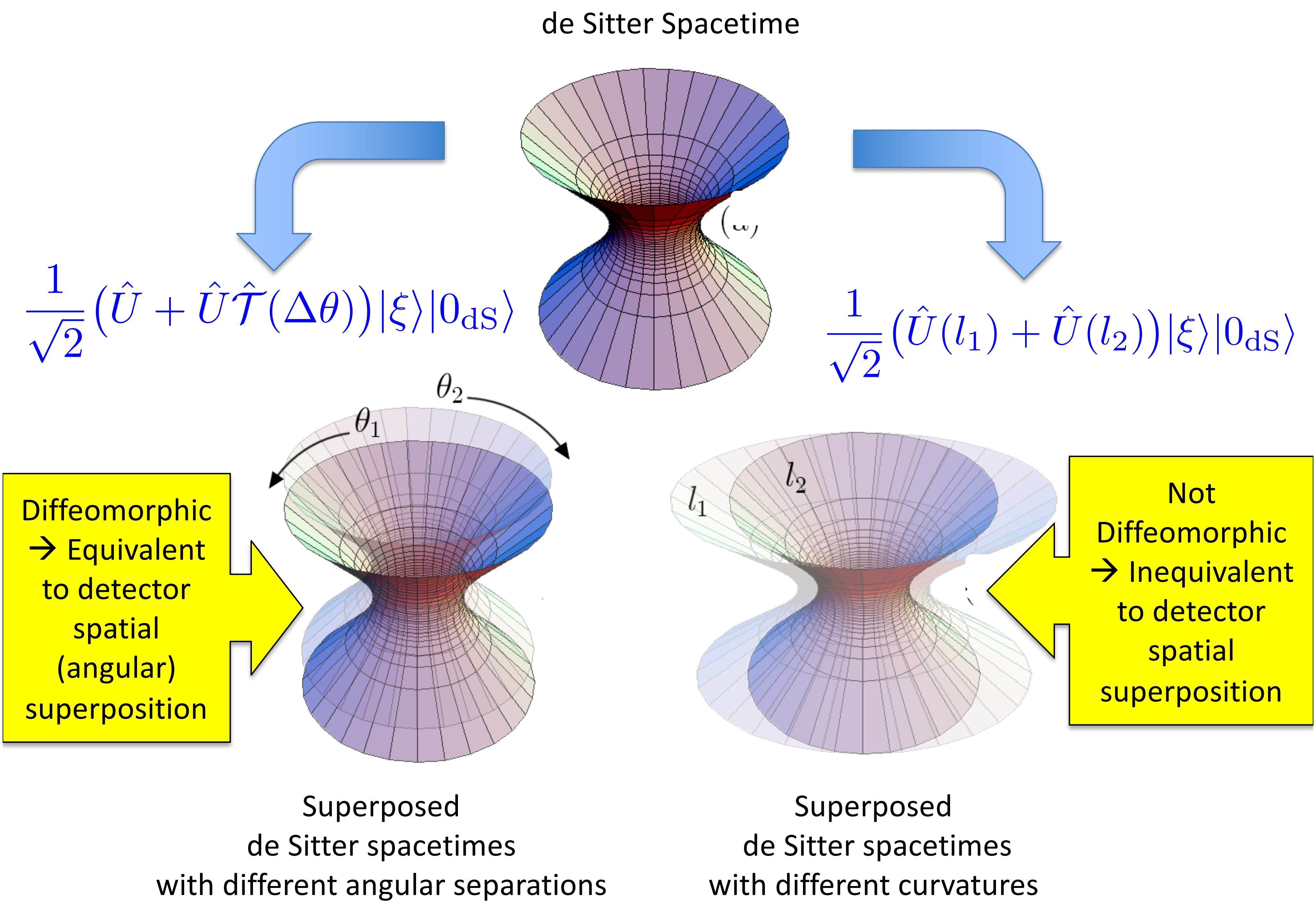
Time evolution

$$\hat{U}|\Psi\rangle_{\mathrm{iFD}} = \frac{1}{\sqrt{2}}\big(\hat{U} + \hat{U}\hat{\mathcal{T}}(\mathcal{L})\big)|\xi\rangle|0_{\mathrm{dS}}\rangle|g\rangle$$

$$|\Psi\rangle_{FD} = \frac{1}{2}\underbrace{\Big(\hat{U}(\xi) + \hat{U}(\xi + \mathcal{L})\Big)|0_{\mathrm{dS}}\rangle}_{\text{superposed spacetime/field}}|g\rangle$$

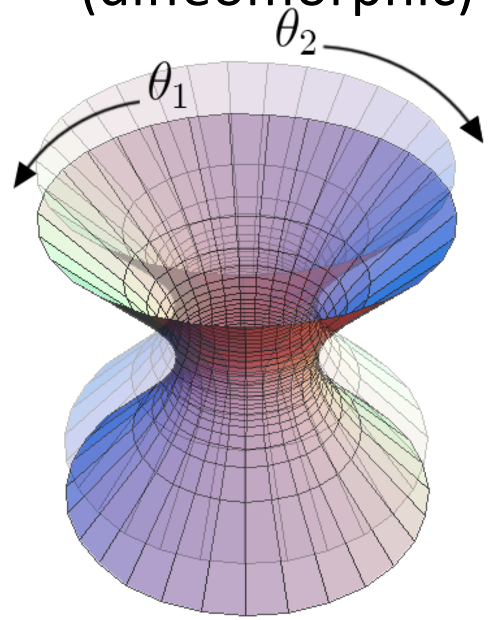
Measure in control basis

Foo/RBM/Zych CQG **38** (2021) 115010

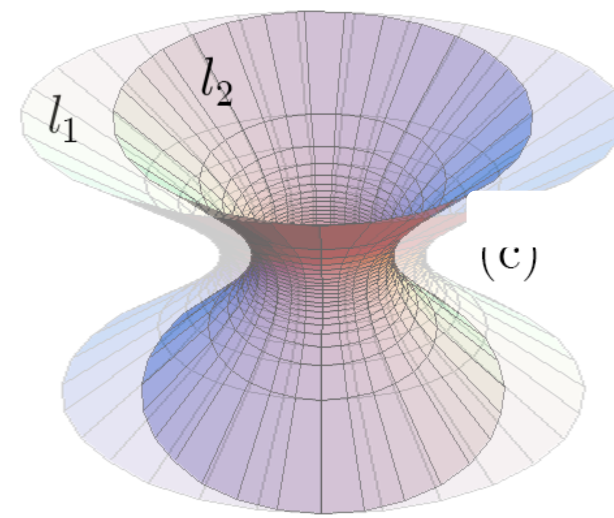


de Sitter Spacetime
$\frac{1}{\sqrt{2}}(\hat{U} + \hat{U}\hat{\mathcal{T}}(\Delta\theta))|\xi\rangle|0_{\rm dS}\rangle$
$\frac{1}{\sqrt{2}}(\hat{U}(l_1) + \hat{U}(l_2))|\xi\rangle|0_{\rm dS}\rangle$
θ_1
θ_2
l_1
l_2
Diffeomorphic → Equivalent to detector spatial (angular) superposition
Not Diffeomorphic → Inequivalent to detector spatial superposition
Superposed de Sitter spacetimes with different angular separations
Superposed de Sitter spacetimes with different curvatures

Angular Superposition (diffeomorphic)

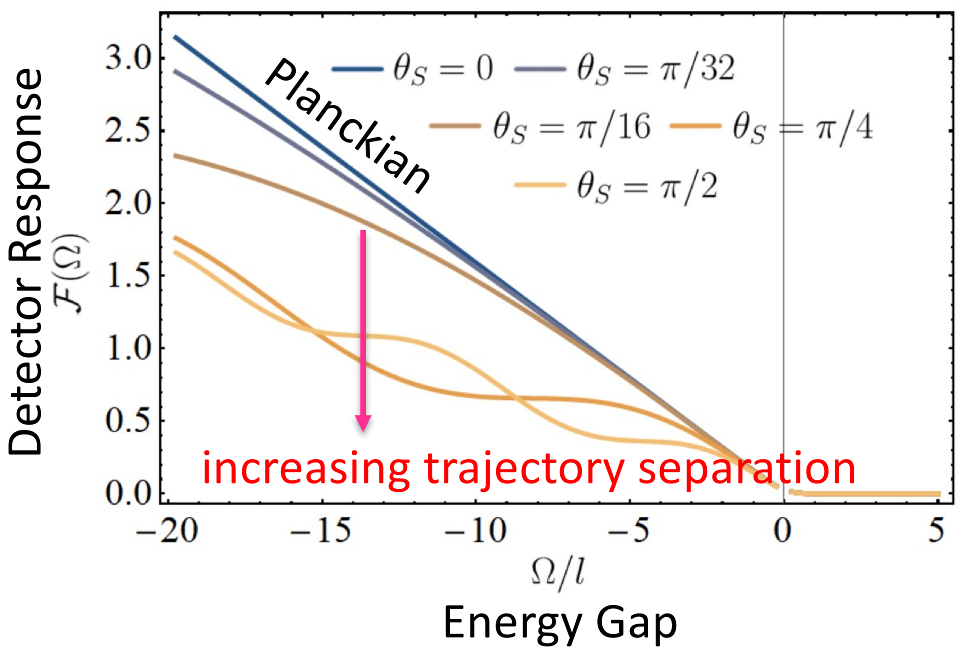


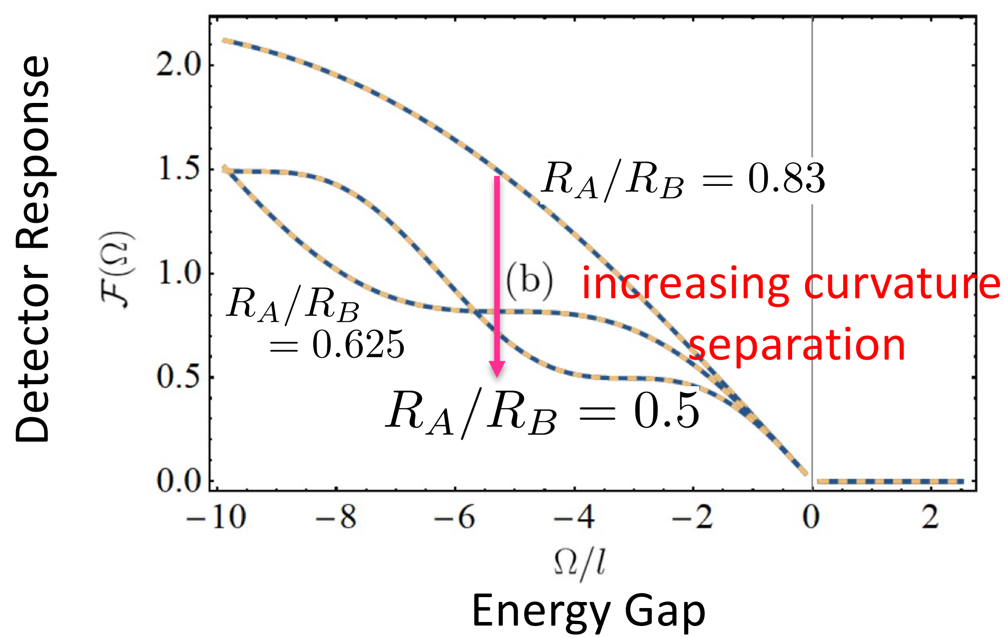
Curvature Superposition (not diffeomorphic)

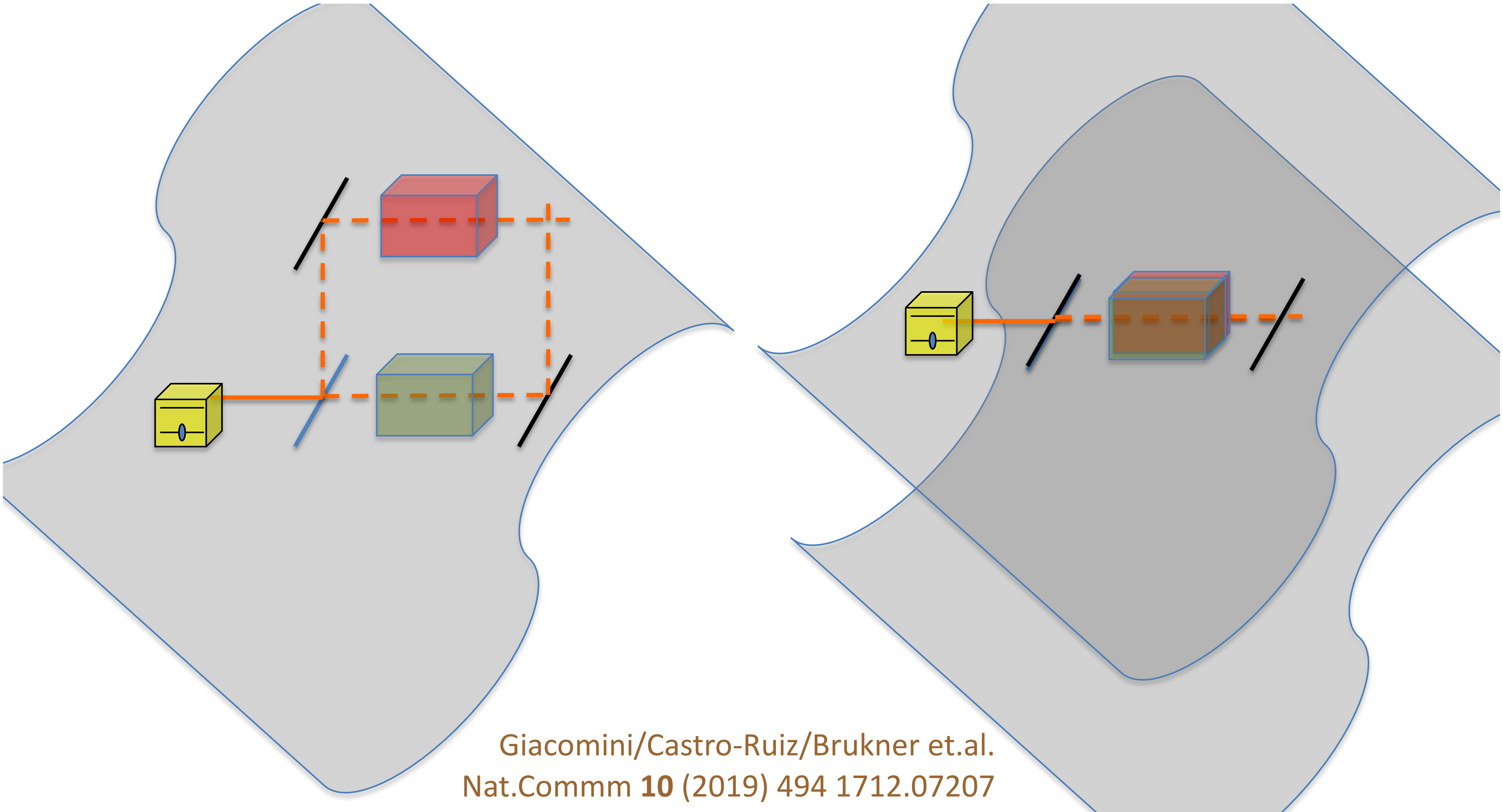


$$T = \frac{\kappa}{2\pi}$$

calibrate temperatures







Giacomini/Castro-Ruiz/Brukner et.al.
Nat.Commm **10** (2019) 494 1712.07207

Superposed detector paths in a single spacetime ⟺ (equivalent) Single detector paths in superposed diffeomorphic spacetimes

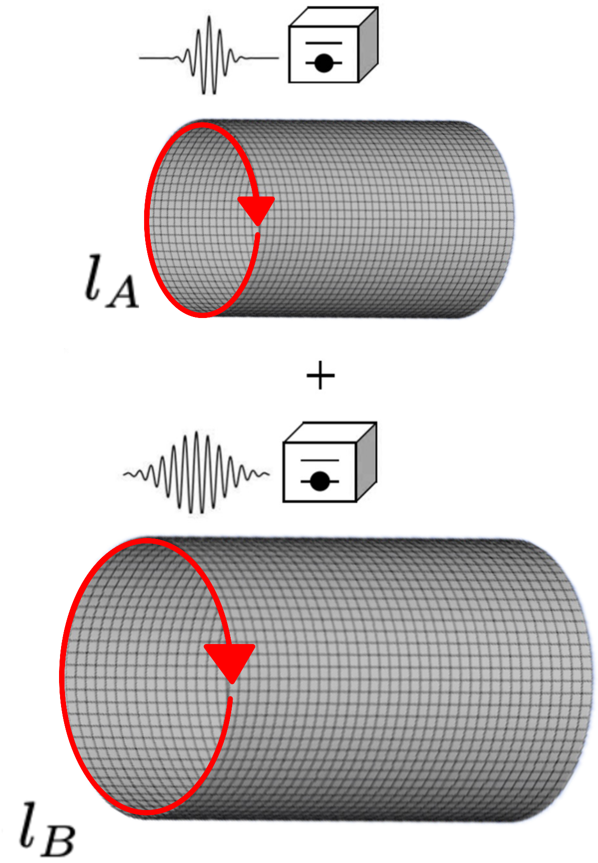
Hypothesis: Assume spacetime superposition holds for non-diffeomorphic spacetimes

Superposed Flat Spacetime

Superpose two cylindrically identified flat spacetimes ➔ circumferences l_A and l_B

Not globally diffeomorphic: they differ in their topological scale

$$J_{0_D} : (\tau, r, y, z) \mapsto (\tau, r, y, z + l_D)$$

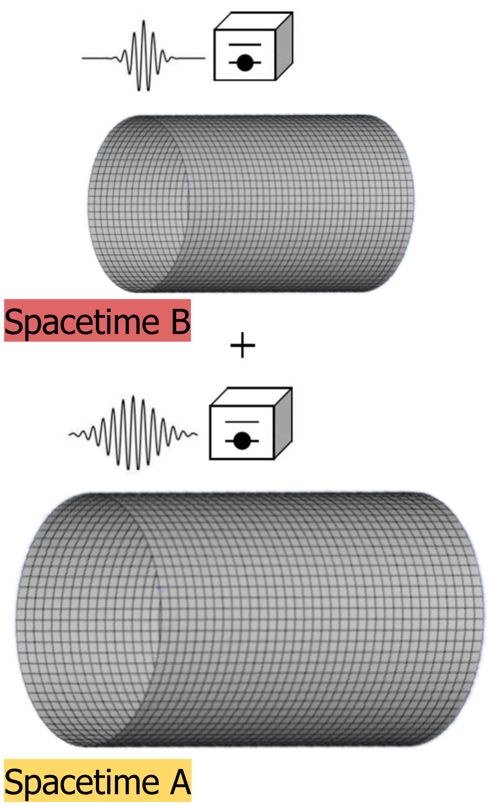


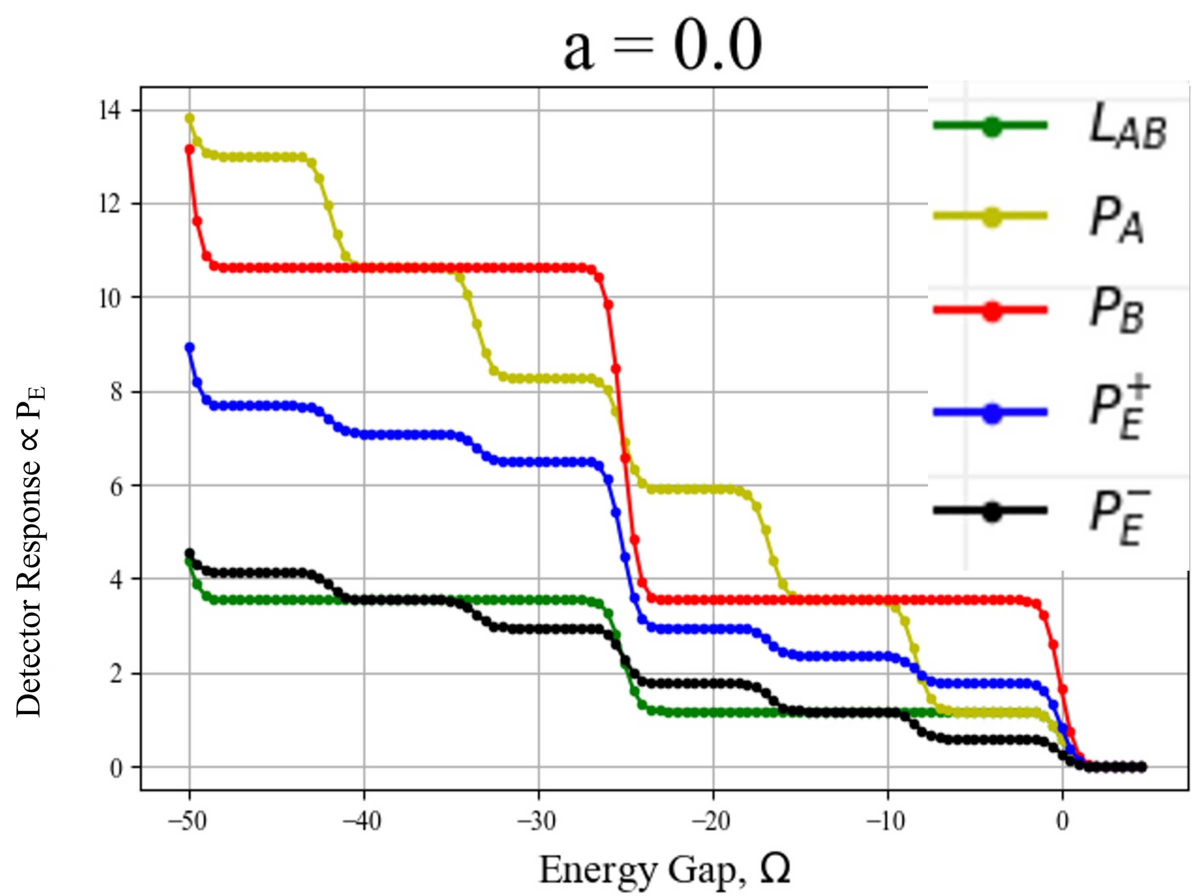
Static detector

Quantum superpositions of Minkowski spacetime

Joshua Foo,[1,*] Cemile Senem Arabaci,[2] Magdalena Zych,[3] and Robert B. Mann[2,4]
[1]*Centre for Quantum Computation & Communication Technology, School of Mathematics & Physics, The University of Queensland, St. Lucia, Queensland, 4072, Australia*
[2]*Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, Canada, N2L 3G1*
[3]*Centre for Engineered Quantum Systems, School of Mathematics and Physics, The University of Queensland, St. Lucia, Queensland, 4072, Australia*
[4]*Perimeter Institute, 31 Caroline St., Waterloo, Ontario, N2L 2Y5, Canada*
(Dated: October 26, 2022)

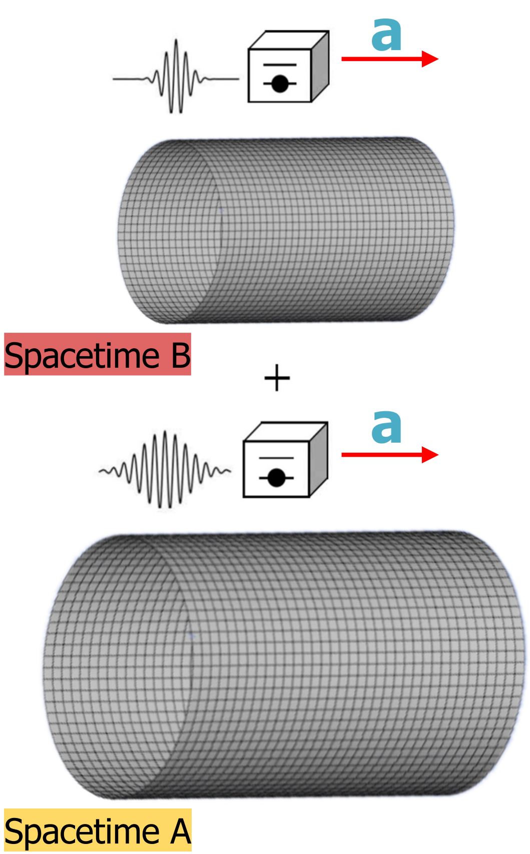
Within any anticipated unifying theory of quantum gravity, it should be meaningful to combine the fundamental notions of quantum superposition and spacetime to obtain so-called "spacetime superpositions": that is, quantum superpositions of different spacetimes not related by a global coordinate transformation. Here we consider the quantum-gravitational effects produced by superpositions of periodically identified Minkowski spacetime (i.e. Minkowski spacetime with a periodic boundary condition) with different characteristic lengths. By coupling relativistic quantum matter to fields on such a spacetime background (which we model using the Unruh-deWitt particle detector model), we are able to show how one can in-principle "measure" the field-theoretic effects produced by such a spacetime. We show that the detector's response exhibits discontinuous resonances at rational ratios of the superposed periodic length scale.

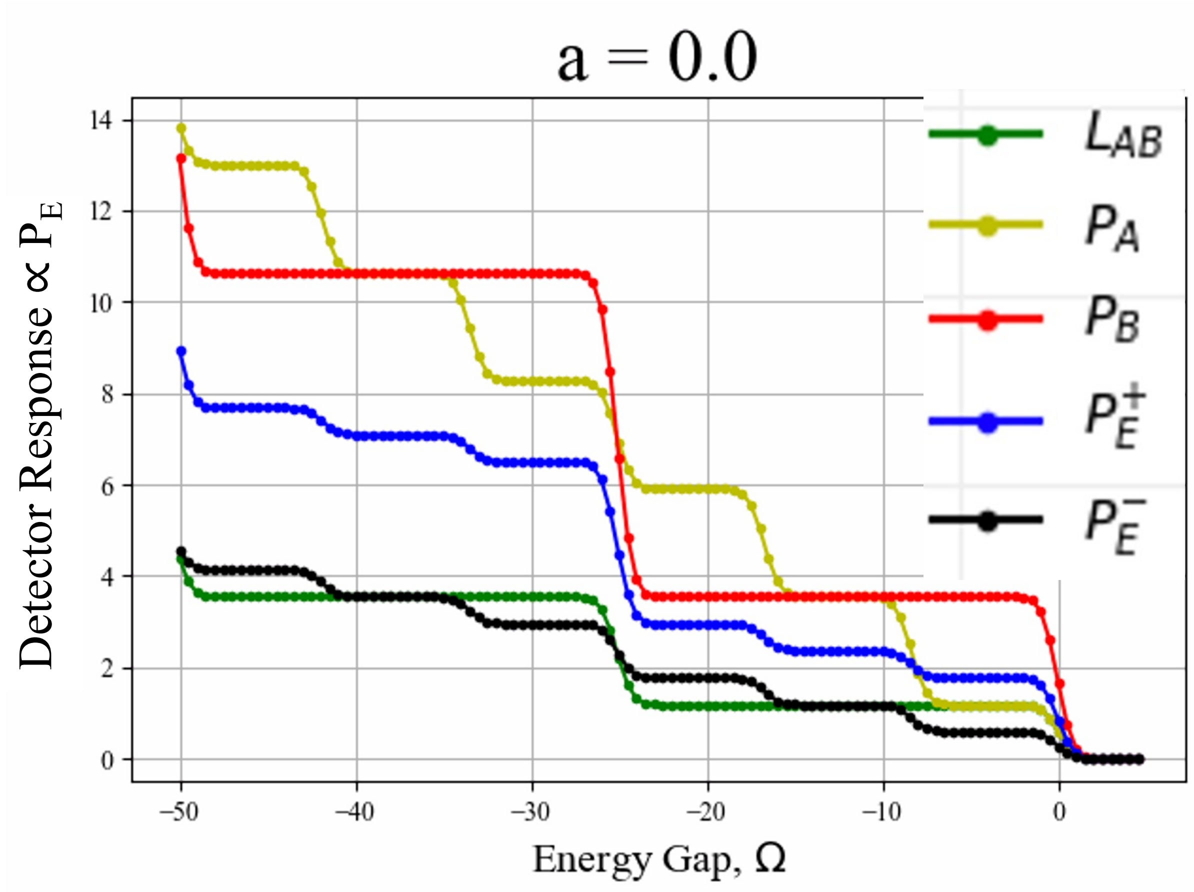




Foo/Arabaci/Zych/RBM
PRD *107* (2023), 045014.

Accelerating Detector





Goel/Patterson/Preciado-Rivas/
Torbian/RBM/Afshordi
PRD **111** (2025) 025015 2409.06818

Black Hole Superposition

- Can we detect a black hole in superposition?
  - Complicated in general: curvature not constant
  - Wightman functions are mode superpositions
- Test lab: BTZ black hole
  - Constant curvature black hole
  - Superpose using methods from de Sitter space

Identified AdS = BTZ Black Hole

$$ds^2 = -\left(\frac{\tilde{r}^2}{l^2} - M\right) d\tilde{t}^2 + \left(\frac{\tilde{r}^2}{l^2} - M\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\tilde{\phi}^2$$

constant t

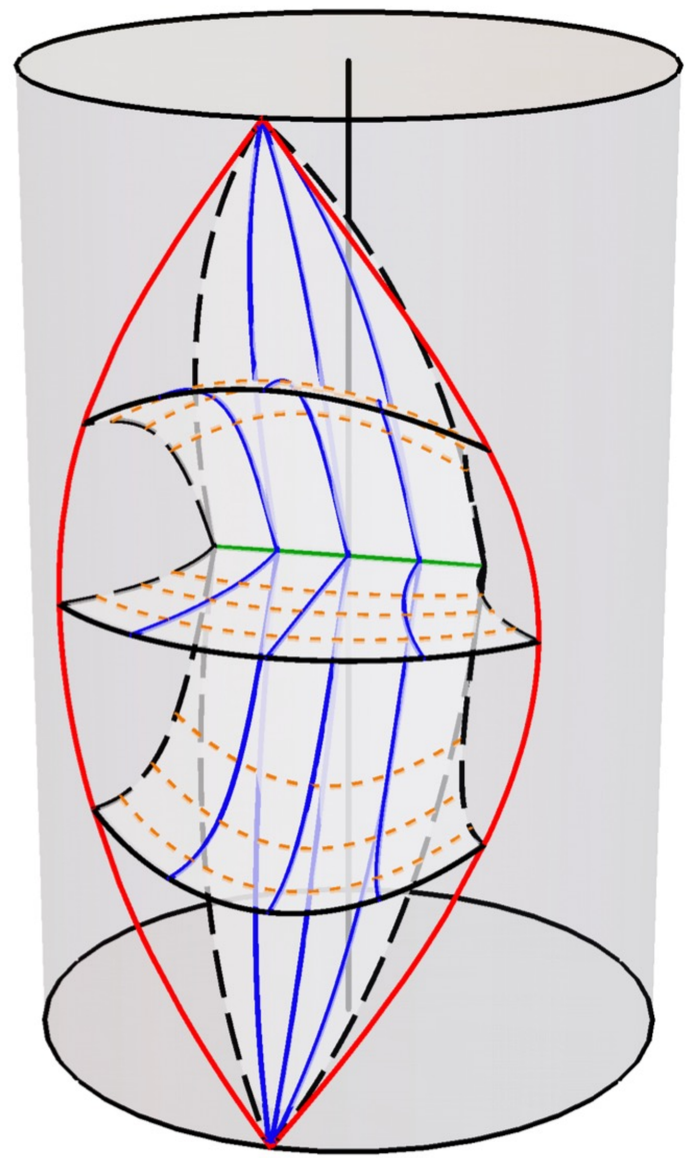
constant ϕ

constant r

identification

$$\phi \to \phi + 2\pi\sqrt{M}$$

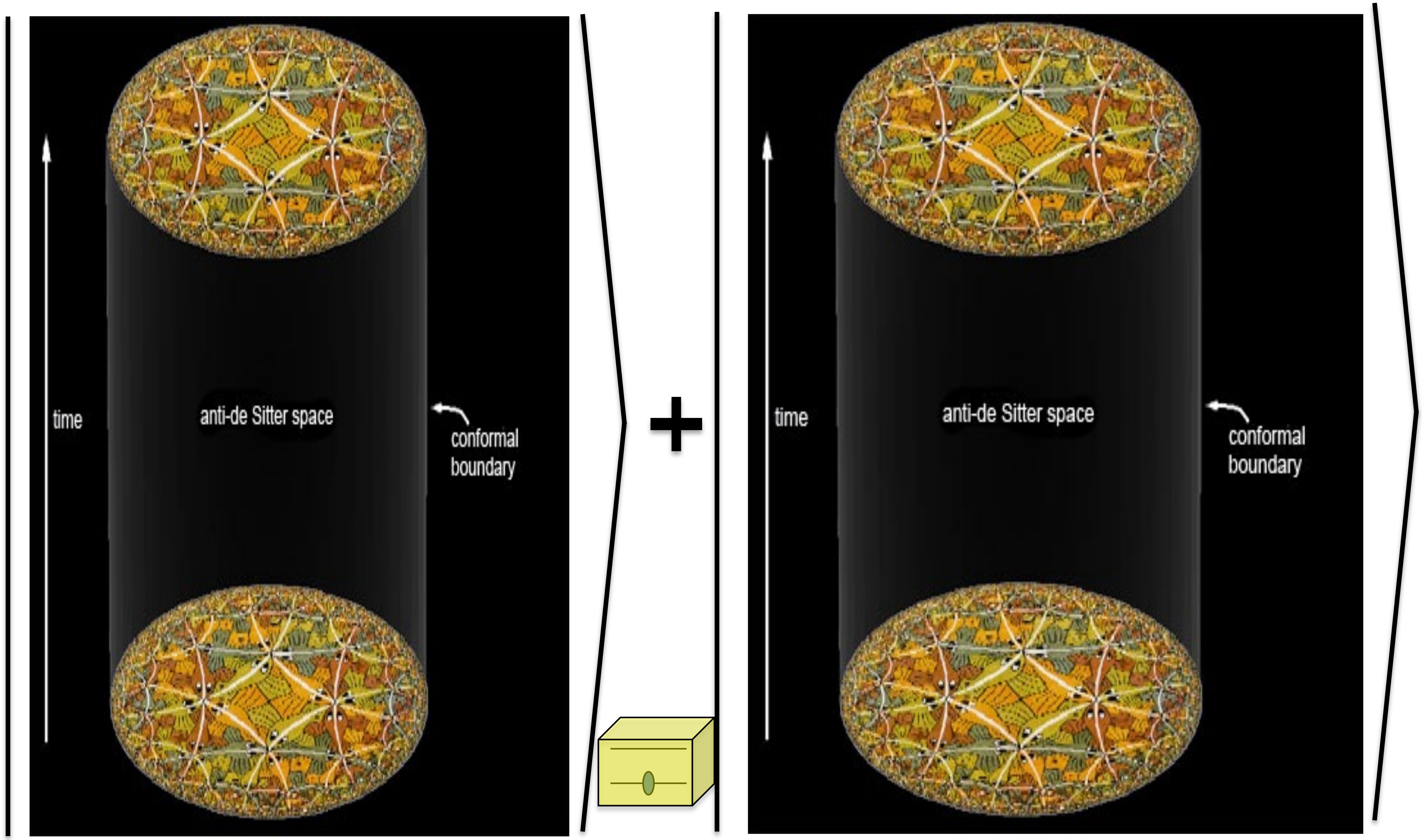
horizon



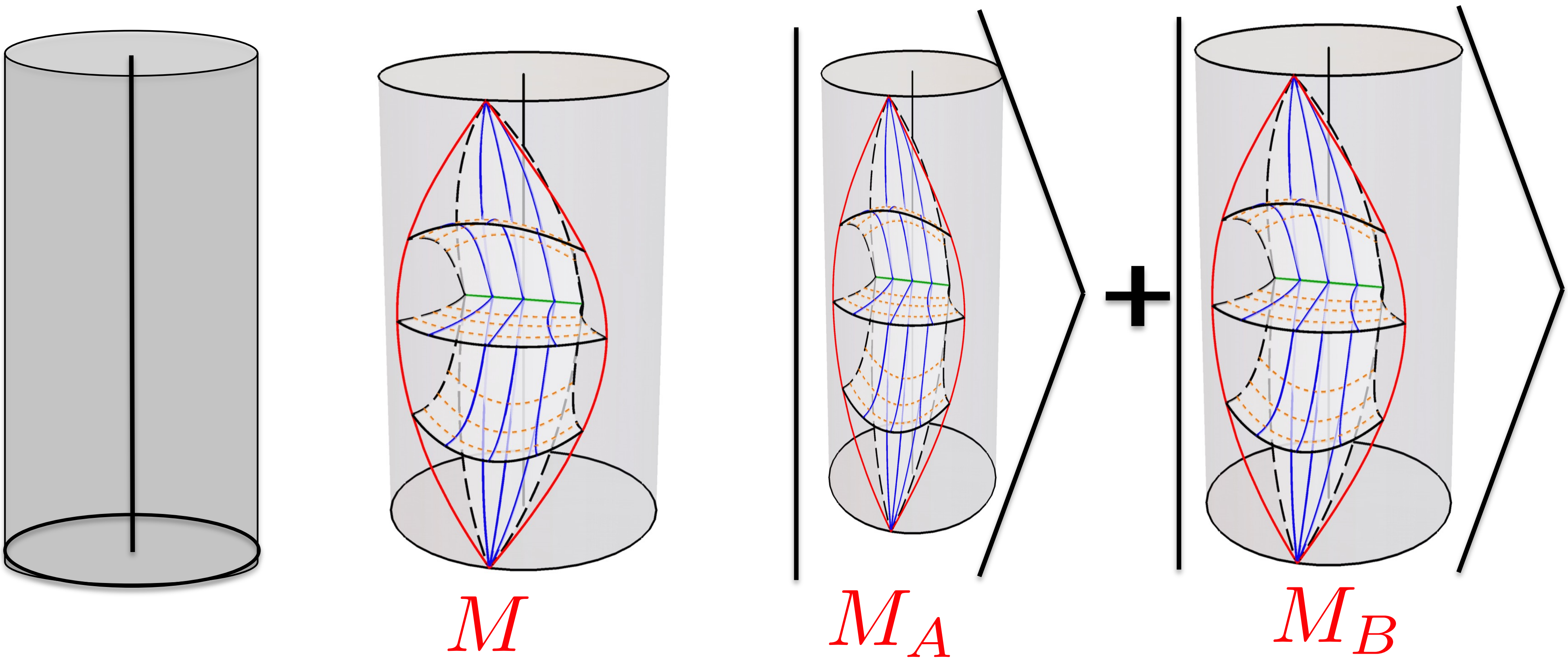
Arias-Henriques/
Gregory/Scoins
JHEP 05 (2022) 063

One Detector in Superposed Anti de Sitter Space

Arabaci/Foo/RBM/Zych
PRL **129** (2022) 181301



The Superposed BTZ Black Hole



AdS → BTZ = Identified Rindler Ads → Superposed BTZ

$$\phi \to \phi + 2\pi\sqrt{M}$$

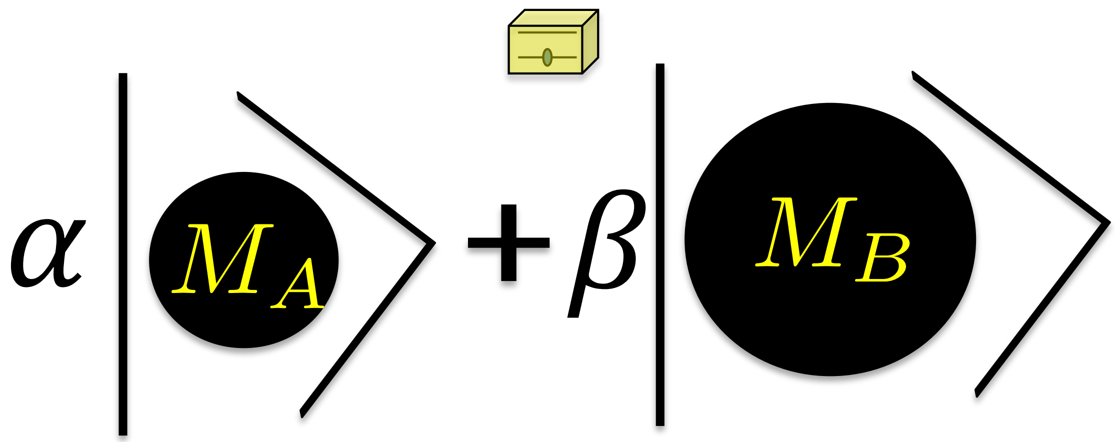
$$\phi \to \phi + 2\pi\sqrt{M_A}$$

$$\phi \to \phi + 2\pi\sqrt{M_B}$$

Probing a Superposed BTZ Black Hole

Arabaci/Foo/RBM/Zych
PRL **129** (2022) 181301

$$ds^2 = -\left(\frac{r^2}{l^2} - 1\right) dt^2 + \left(\frac{r^2}{l^2} - 1\right)^{-1} dr^2 + r^2 d\phi^2$$



$$\Gamma_A : \phi \to \phi + 2\pi\sqrt{M_A} \qquad \Gamma_B : \phi \to \phi + 2\pi\sqrt{M_B}$$

$$W(x, x') = \langle 0|\phi(\mathsf{x})\phi(\mathsf{x}')|0\rangle$$

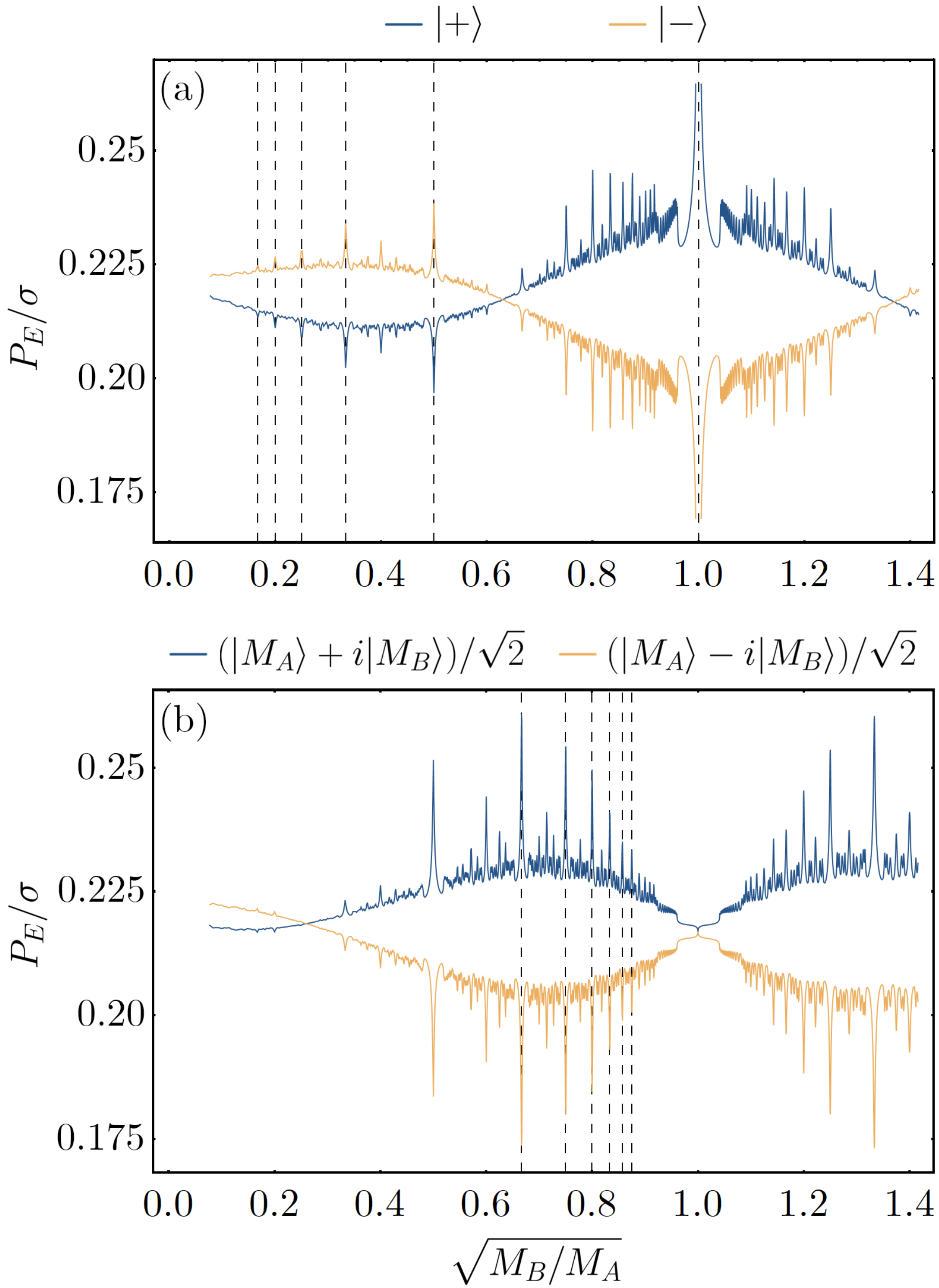
$$\Rightarrow \quad W_{\rm BTZ}^{(AB)}(\mathsf{x}, \mathsf{x}') = \frac{1}{\sum_k \eta^{2k}} \sum_{n,m} \eta^n \eta^m W_{\rm AdS}(\Gamma_A^n \mathsf{x}, \Gamma_B^m \mathsf{x}')$$

Superposed BTZ Wightman fn

untwisted

$\eta = \pm 1$

twisted



$$|\pm\rangle = (|M_A\rangle \pm |M_B\rangle)/\sqrt{2}$$

Dashed lines:
$\sqrt{M_B/M_A} = (n-1)/n$
where $n = \{3, \ldots 8\}$

Arabaci/Foo/RBM/Zych
PRL **129** (2022) 181301
2111.13315

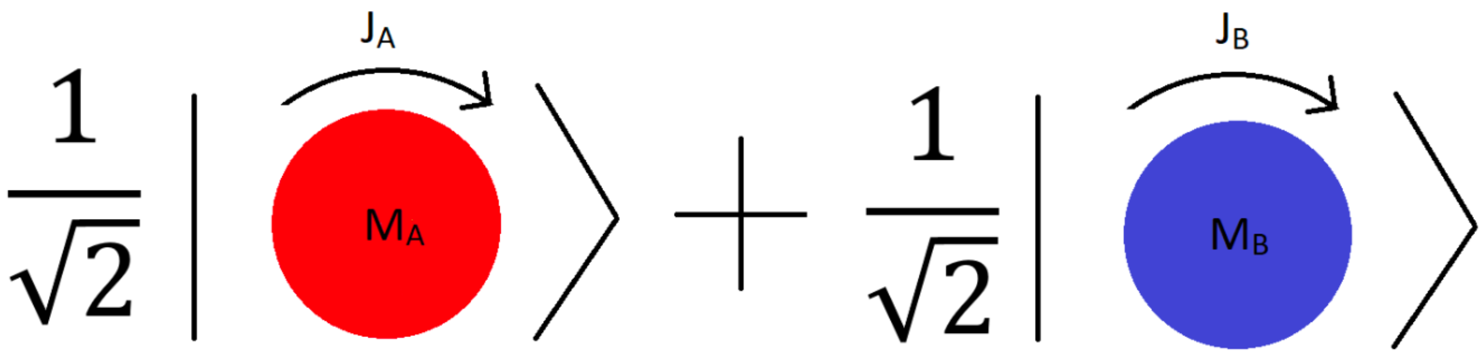
Resonant peaks at integer values of (mass ratios)$^{1/2}$!

Consistent with Bekenstein's black hole mass quantization conjecture

Rotating Superposed Black Holes

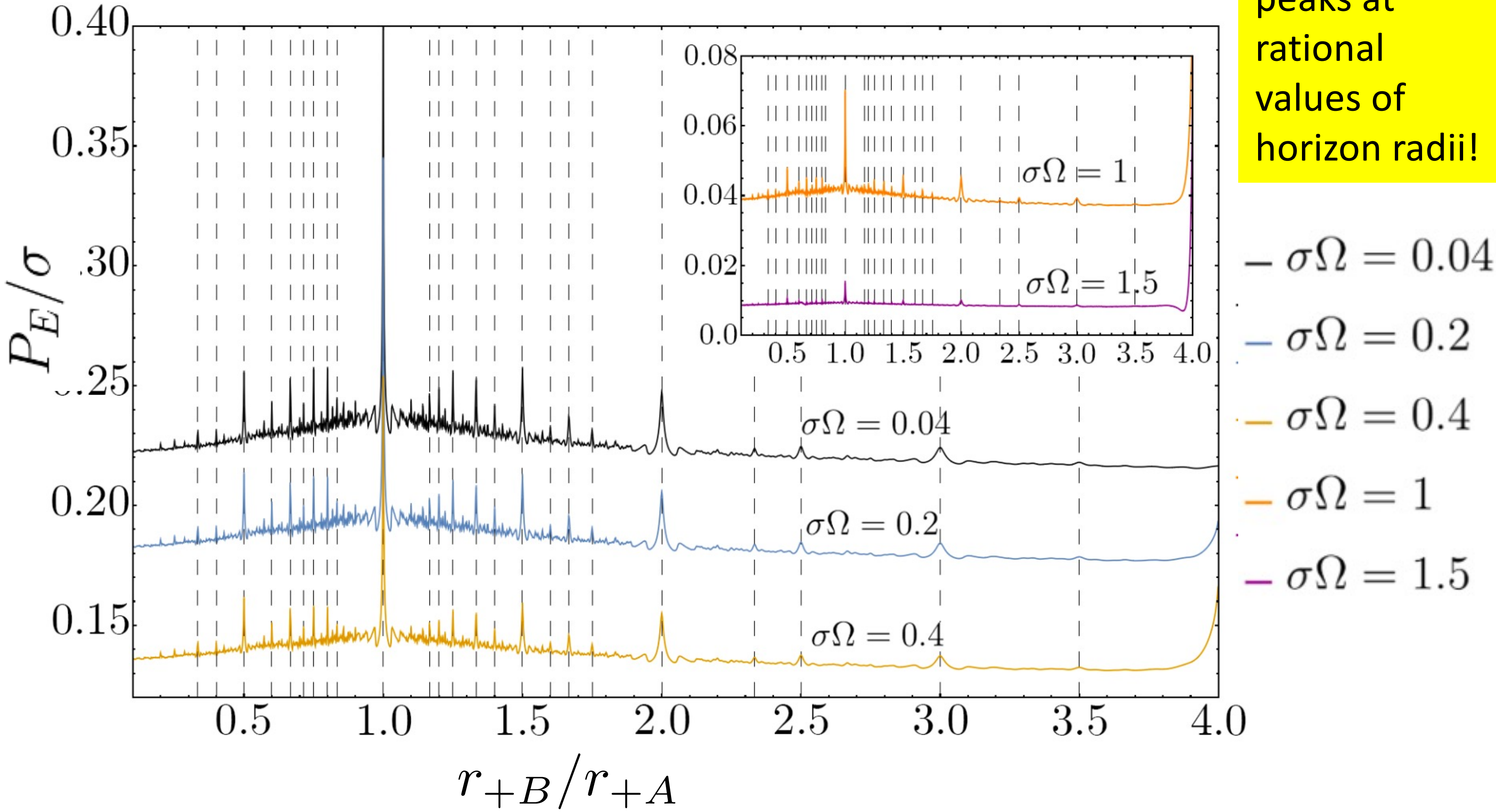
$$f = -M + \frac{r^2}{\ell^2} + \frac{J^2}{4r^2}$$

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 (d\phi - \frac{J}{2r^2} dt)^2$$



Suryaatmadja/Arabaci/
Robbins/Foo/RBM/Zych
PRD **110** (2024) 066018

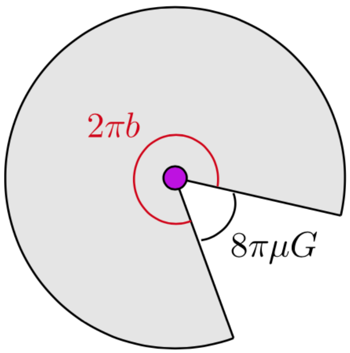
Response Parametrized by Energy Gap

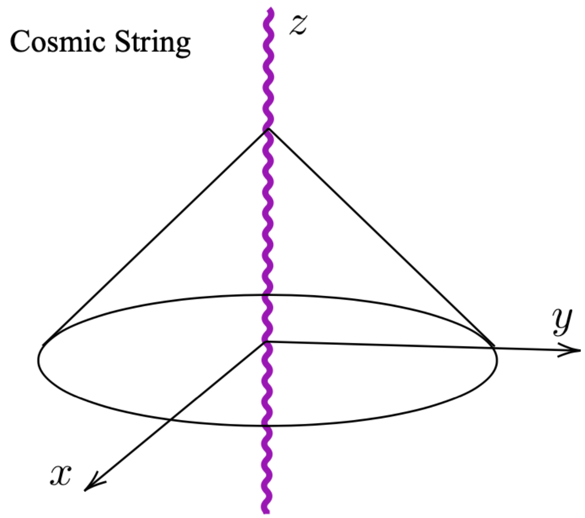


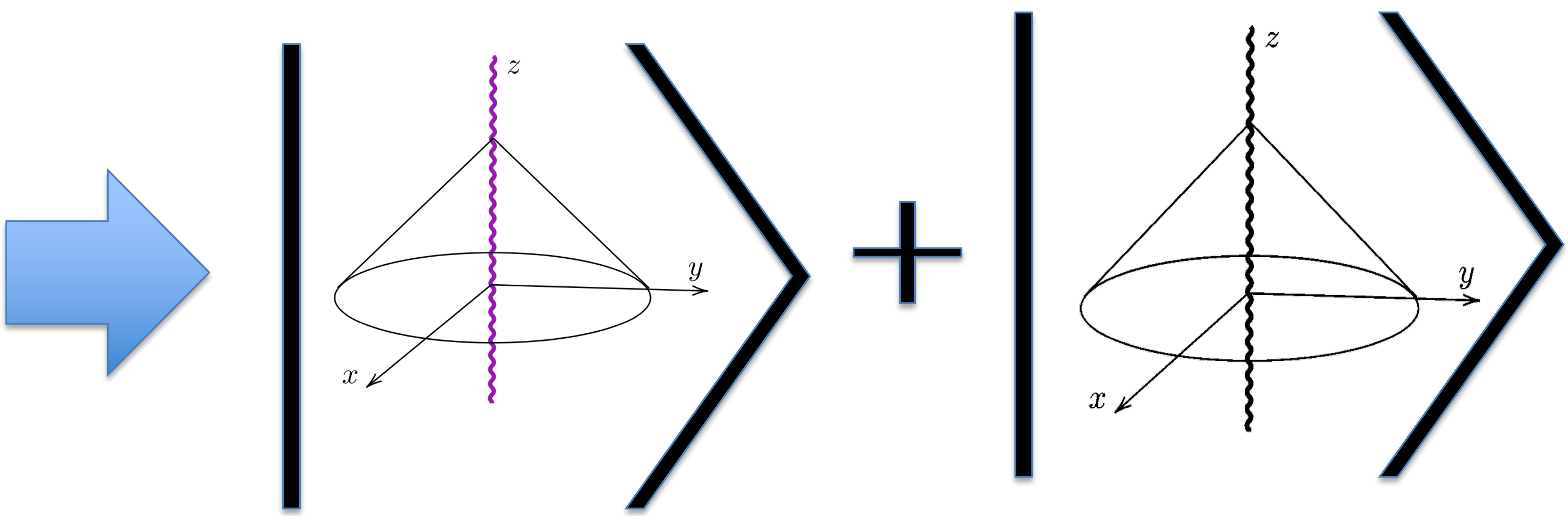
Resonant peaks at rational values of horizon radii!

Superposed Cosmic String

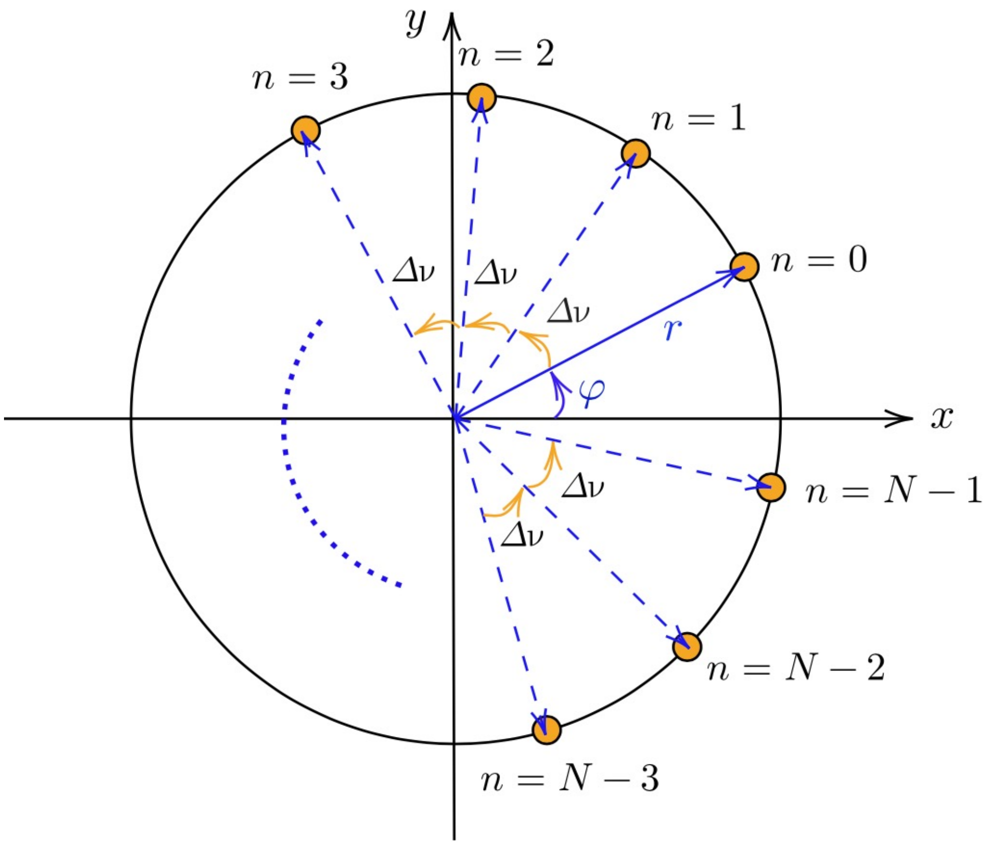
Arias/Chinga/Gauvin/RBM
(in progress)



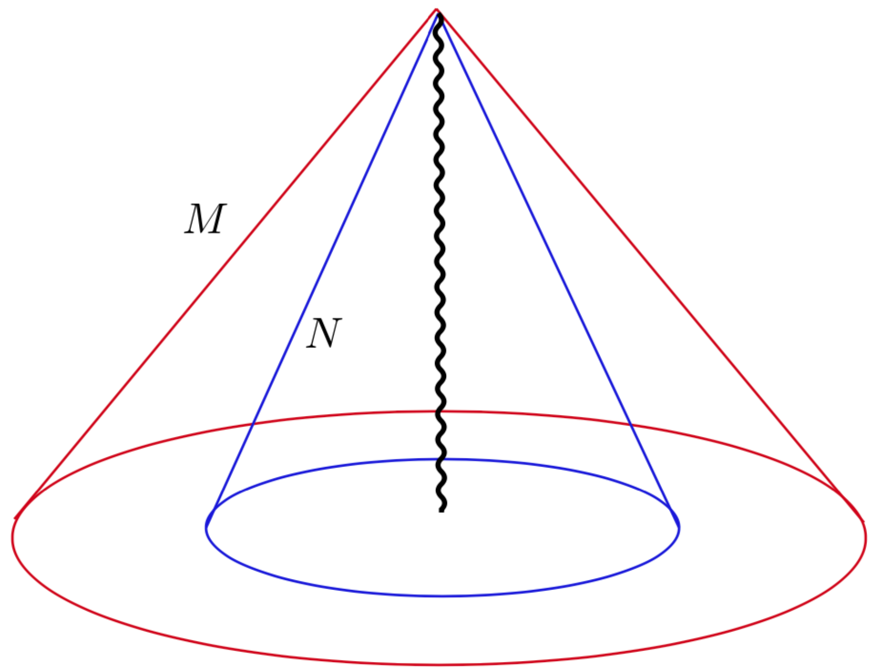




Topological Cosmic String



$$\Delta\nu = \frac{2\pi}{N} = 2\pi(1 - 4\mu G)$$

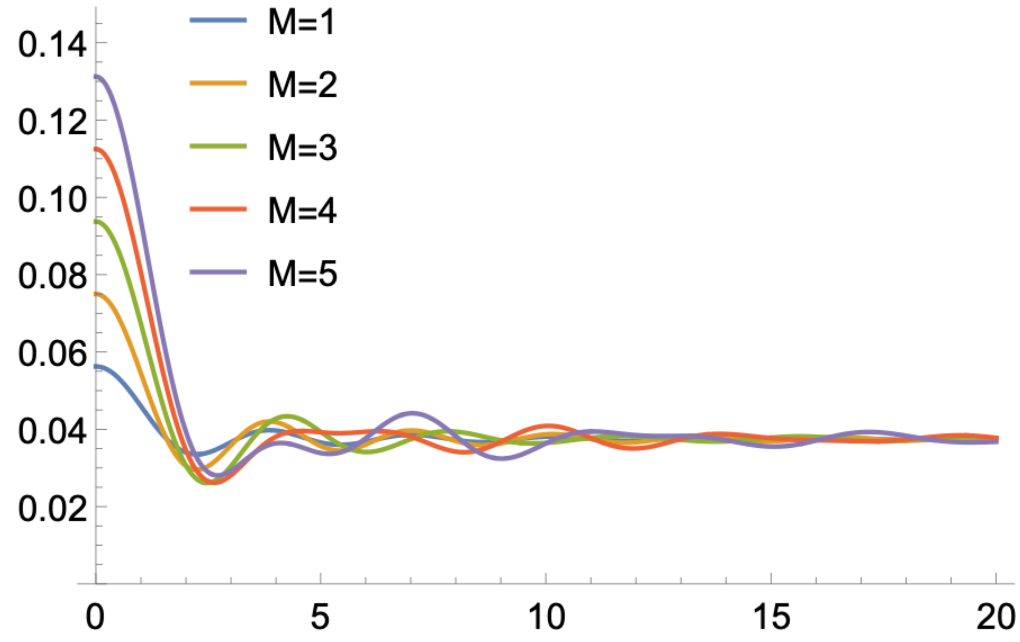


$$\Delta\nu = \frac{2\pi}{N} \qquad J_N : (r, \phi, z) \to (r, \phi + \Delta\nu, z).$$

$$\Delta\mu = \frac{2\pi}{M} \qquad J_M : (r, \phi, z) \to (r, \phi + \Delta\mu, z).$$

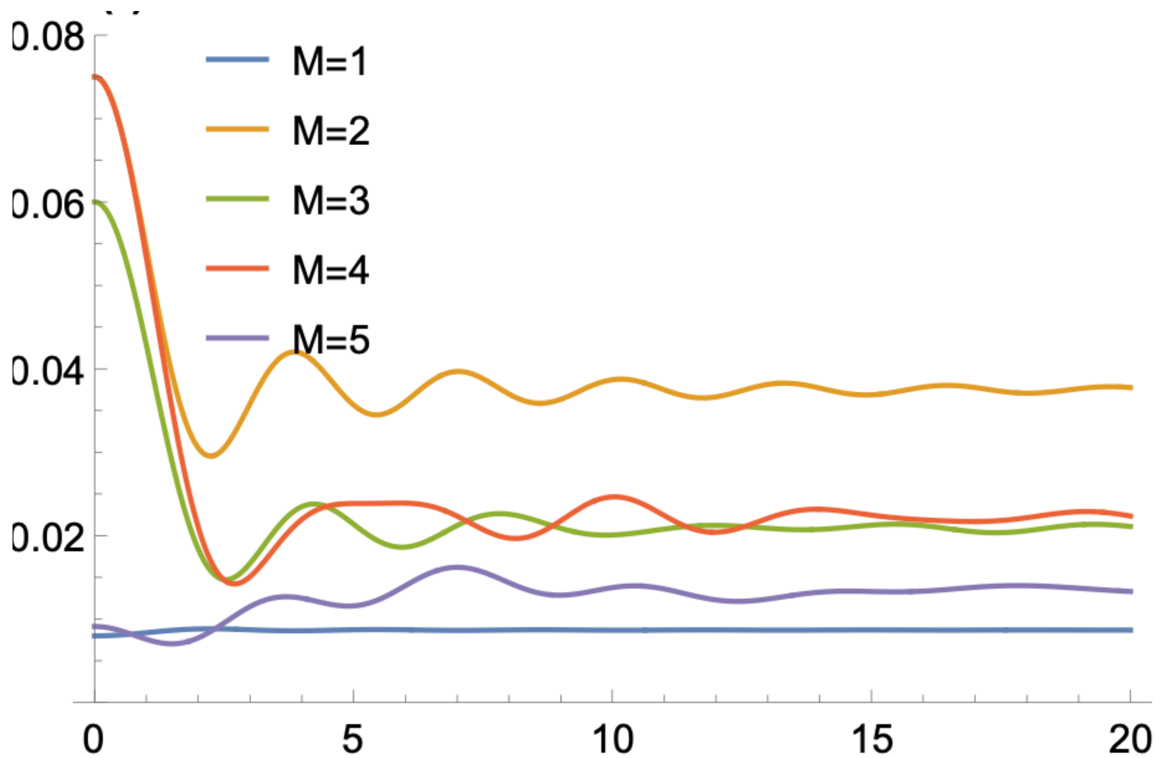
Superposition of two Topological Strings

Not-Superposed



$$N = 2$$

Superposed



Open Issues

- Other Examples
  - Non-constant curvature
  - Dynamics
- Hilbert Space?
  - Basis of spacetime states

Sajeendran/Foo/RBM/Wang
In progress

- Role of the Vacuum?
  - Different vacua in different branches?
- Entanglement? Mutual Information?
- Connections with other quantum gravity approaches?
- Experiments?

Paczos/Foo/Zych
2406.19037

Summary

- Construction of superposed spacetimes
  - superposed (identified) flat space
  - curvature-superposed de Sitter
  - Generalizable to other spacetimes (cosmic string)
- Mass-superposed black hole
  - Response peaks at rational values of horizon ratio
  - Holds for rotating black holes as well
  - Consistent with Bekenstein's Conjecture
- Provides a pathway for understanding effects of quantum gravitational phenomena even without a quantum theory of gravity!
- Can we witness this in the lab?

Foo/Arabaci/Zych/RBM
PRD *107* (2023), 045014.

Goel/Patterson/Preciado-Rivas/
Torbian/RBM/Afshordi
PRD **111** (2025) 025015

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