# Graviton quantum noise on geodesic congruences and gravitational decoherence

Hing-Tong Cho

Department of Physics, Tamkang University, Taiwan

(Collaboration with Bei-Lok Hu)

Relativistic Quantum Information (North) 2025

# Reference


1. HTC and B.-L. Hu, "Quantum noise of gravitons and stochastic force on geodesic separation", Phys. Rev. D 105, 086004 (2022).
2. HTC and B.-L. Hu, "Graviton noise on tidal forces and geodesic congruences", Phys. Rev. D107, 084005 (2023).
3. HTC and B.-L. Hu, "Non-Markovian quantum master and Fokker-Planck equation for gravitational systems and gravitational decoherence", arXiv:2504.11991.

# Outline

# I. Motivation

1. Interferometric observations of gravitational waves since 2015.
2. Is it possible to explore the quantum nature of gravity by interferometric observations?
3. Proposal by Parikh, Wilczek, and Zahariade to study the quantum noise due to gravitons.
4. Quantum noise can be studied by the open quantum system approach of Feynman and Vernon.
5. This approach was first applied to the case of graviton by Galley, Hu, and Lin.

# II. Quantum Brownian motion paradigm

A particle (system) coupled linearly to a set of harmonic oscillators (environment):

$$
\begin{aligned}
S[x] &= \int_0^t ds \left[\frac{1}{2}M\dot{x}^2 - V(x)\right] \\
S_e[q_n] &= \int_0^t ds \sum_n \left[\frac{1}{2}m_n\dot{q}_n^2 - \frac{1}{2}m_n\omega_n^2 q_n^2\right] \\
S_{int}[x,\{q_n\}] &= \int_0^t ds \sum_n (-C_n x\, q_n)
\end{aligned}
$$

(Schwinger, Feynman-Vernon, Caldeira-Leggett, Hu-Paz-Zhang, ...)

The dynamics of the particle is governed by the CTP effective action

$$
\begin{aligned}
e^{i\Gamma[x_+,x_-]} &= e^{iS[x_+]-iS[x_-]} \times \\
&\int_{CTP} \prod_n Dq_{n+} Dq_{n-} \left( e^{iS_e[\{q_{n+}\}]-iS_e[\{q_{n-}\}]} \right. \\
&\qquad \left. e^{iS_{int}[x_+,\{q_{n+}\}]-iS_{int}[x_-,\{q_{n-}\}]} \right) \\
&= e^{iS[x_+]-iS[x_-]+iS_{IF}[x_+,x_-]}
\end{aligned}
$$

where $S_{IF}$ is the influence action due to the quantum harmonic oscillators.

The influence action $S_{IF}$ can be expressed in terms of the Schwinger-Keldysh propagators

$$
\begin{aligned}
& S_{IF}[x_+, x_-] \\
= \; & \sum_n \frac{1}{2} \int ds\, ds' \\
& \quad \big[ x_+(s) G_{n++}(s,s') x_+(s') - x_+(s) G_{n+-}(s,s') x_-(s') \\
& \quad\; - x_-(s) G_{n-+}(s,s') x_+(s') + x_-(s) G_{n--}(s,s') x_-(s') \big]
\end{aligned}
$$

due to the corresponding boundary conditions.

The Schwinger-Keldysh propagators are

$$\begin{aligned} G_{n++}(s,s') &= -\eta_n(s-s')\,\mathrm{sgn}(s-s') + i\nu_n(s-s') \\ G_{n+-}(s,s') &= \eta_n(s-s') + i\nu_n(s-s') \\ G_{n-+}(s,s') &= -\eta_n(s-s') + i\nu_n(s-s') \\ G_{n--}(s,s') &= \eta_n(s-s')\,\mathrm{sgn}(s-s') + i\nu_n(s-s') \end{aligned}$$

where

$$\begin{aligned} \eta_n(s-s') &= -\frac{C_n^2}{2m_n\omega_n}\sin\omega_n(s-s') \\ \nu_n(s-s') &= \frac{C_n^2}{2m_n\omega_n}\cos\omega_n(s-s') \end{aligned}$$

The influence action $S_{IF}$ can be written as

$$e^{iS_{IF}} = e^{-2i\int_0^t ds \int_0^s ds' [\Delta x(s)\eta(s-s')\Sigma x(s')]} \\ e^{-\frac{1}{2}\int_0^t ds \int_0^t ds' \, [\Delta x(s)\nu(s-s')\Delta x(s')]}$$

where $\Delta x(s) = x_+(s) - x_-(s)$ and $\Sigma x(s) = \frac{1}{2}[x_+(s) + x_-(s)]$, and

$$\eta(s-s') = \sum_n \eta_n(s-s') = -\sum_n \frac{C_n^2}{2m_n\omega_n} \sin\omega_n(s-s')$$

$$\nu(s-s') = \sum_n \nu_n(s-s') = \sum_n \frac{C_n^2}{2m_n\omega_n} \cos\omega_n(s-s')$$

$S_{IF}$ is basically separated into its real and imaginary parts.

Rewriting the imaginary part of $S_{IF}$ as

$$
\begin{aligned}
& e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi e^{-\frac{1}{2}\int \xi \nu^{-1} \xi} e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi e^{-\frac{1}{2}\int (\xi - i\nu\Delta x)\nu^{-1}(\xi - i\nu\Delta x)} e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi P[\xi] e^{i\int \xi \Delta x}
\end{aligned}
$$

where $P[\xi] = e^{-\frac{1}{2}\int \xi \nu^{-1} \xi}$ is the Gaussian probability density of the stochastic force $\xi$.

Due to this probability density one has the stochastic average $\langle \xi(s)\xi(s')\rangle_s = \nu(s - s')$ which is called the noise kernel.

After this procedure the effective action

$$\begin{aligned}\Gamma[x_+, x_-] &= S[x_+] - S[x_-] \\ &\quad -2\int_0^t ds \int_0^s ds' \Delta x(s)\eta(s-s')\Sigma x(s') \\ &\quad + \int_0^t ds \Delta x(s)\xi(s)\end{aligned}$$

The equation of motion for the particle is then given by

$$\left.\frac{\delta\Gamma[x_+, x_-]}{\delta x_+}\right|_{x_+=x_-=x} = 0$$

The equation of motion is a Langevin equation with the stochastic force $\xi(t)$,

$$M\ddot{x} + V'(x) + \int_0^t ds\, \eta(t-s)x(s) = \xi(t)$$

The integral term is related to dissipation as one can write

$$\eta(t) = \frac{d}{dt}\gamma(t) \Rightarrow \gamma(t) = \sum_n \frac{C_n^2}{2m_n\omega_n^2}\; cos\omega_n t$$

and we have

$$M\ddot{x} + V'(x) + \int_0^t ds\, \gamma(t-s)\dot{x}(s) = \xi(t)$$

$\eta(s-s')$ is called the dissipation kernel.

Quantum mechanically the system is described by the density matrix $\hat{\rho}(t)$ with the evolution

$$\hat{\rho}(t) = J(t,0)\,\hat{\rho}(0)$$

where the evolution operator can be expressed in terms of path integrals

$$\begin{aligned} & J(x_+, q_+, x_-, q_-, t | x_{+i}, q_{+i}, x_{-i}, q_{-i}, 0) \\ = & \int_{x_{+i}}^{x_+} Dx_+ \int_{q_{+i}}^{q_+} Dq_+ \int_{x_{-i}}^{x_-} Dx_- \int_{q_{-i}}^{q_-} Dq_- \, e^{iS[x_+,q_+]-iS[x_-,q_-]} \end{aligned}$$

Integrating over the degrees of freedom $q$, we have the reduced density matrix

$$\rho_r(x_+, x_-) = \int_{-\infty}^{\infty} dq_+ \int_{-\infty}^{\infty} dq_- \, \rho(x_+, q_+, x_-, q_-) \, \delta(q_+ - q_-)$$

with the evolution

$$\rho_r(x_+, x_-, t) = \int_{-\infty}^{\infty} dx_{+i} \int_{-\infty}^{\infty} dx_{-i} \, J_r(x_+, x_-, t | x_{+i}, x_{-i}, 0) \, \rho_r(x_{+i}, x_{-i}, 0)$$

Suppose the initial state of the system and the environment is uncorrelated.

The evolution operator for the reduced density matrix can be represented by

$$
\begin{aligned}
& J_r(x_+, x_-, t | x_{+i}, x_{-i}, 0) \\
= \; & \int_{x_{+i}}^{x_+} Dx_+ \int_{x_{-i}}^{x_-} Dx_- \; e^{iS[x_+] - iS[x_-] + iS_{IF}[x_+, x_-]}
\end{aligned}
$$

Hu-Paz-Zhang master equation ($x_+ \to x$ and $x_- \to x'$)

$$\begin{aligned}\frac{\partial}{\partial t}\rho_r(x,x',t) &= \left[\frac{i}{2M}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x'^2}\right)-\frac{i}{2}M\Omega^2_{\rm ren}(t)\left(x^2-x'^2\right)\right.\\ &\quad -\Gamma(t)(x-x')\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)-D_{pp}(t)(x-x')^2\\ &\quad \left.-iD_{px}(t)(x-x')\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial x'}\right)\right]\rho_r(x,x',t)\end{aligned}$$

$\Gamma(t)$ is the dissipation coefficient, whereas $D_{pp}(t)$ and $D_{px}(t)$ are the diffusion coefficients responsible for decoherence.

# III.Influence action and stochastic force from gravitons

Einstein action

$$S_g = \frac{1}{\kappa^2} \int d^4x \sqrt{-g}\, R$$

where $\kappa^2 = 16\pi G$.

With $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, the graviton action

$$S_{grav} = -\frac{1}{2} \int d^4x \sum_s \partial_\alpha h^{(s)}(x) \partial^\alpha h^{(s)}(x)$$

Fermi normal coordinate system $(t, \vec{z})$ along the geodesic of the first mass with the metric

$$
\begin{aligned}
g_{00}(t, \vec{z}) &= -1 - R_{i0j0}(t, 0) z^i z^j + \cdots \\
g_{0i}(t, \vec{z}) &= -\frac{2}{3} R_{0ijk}(t, 0) z^j z^k + \cdots \\
g_{ij}(t, \vec{z}) &= \delta_{ij} - \frac{1}{3} R_{ikjl}(t, 0) z^k z^l + \cdots
\end{aligned}
$$

Action of the second mass,

$$S_m = -m \int \sqrt{-ds^2}$$

In terms of the gravitational perturbation $\kappa h_{\mu\nu}$, the Riemann tensor component

$$R_{i0j0} = -\frac{\kappa}{2}\,\ddot{h}_{ij}$$

$$S_m = \int dt\,\left[\frac{m}{2}\,\delta_{ij}\,\dot{z}^i\dot{z}^j + \frac{m\kappa}{4}\,\ddot{h}_{ij}\,z^i z^j\right] + \cdots$$

With

$$J^{(s)}(x) = \frac{m\kappa}{2\sqrt{2}} \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}} \frac{d^2}{dt^2}\left(\epsilon_i^{(s)*}(\vec{k}) z^i(t)\right)^2$$

Closed-time-path formalism

$$\begin{aligned} e^{iS_{IF}} &= \int_{CTP} Dh_+\, Dh_-\, e^{i(S_g[h_+]-S_g[h_-]+\int J_+h_+-\int J_-h_-)} \\ &= e^{-\frac{i}{2}\int(J_+G_{++}J_+-J_+G_{+-}J_--J_-G_{-+}J_++J_-G_{--}J_-)} \end{aligned}$$

Schwinger-Keldysh Green functions

$$\begin{aligned} G_{++}(x,x') &= -i\langle Th_+(x)h_+(x')\rangle \\ G_{+-}(x,x') &= -i\langle h_-(x')h_+(x)\rangle \\ G_{-+}(x,x') &= -i\langle h_-(x)h_+(x')\rangle = G_{+-}(x',x) \\ G_{--}(x,x') &= -i\langle \bar{T}h_-(x)h_-(x')\rangle = -G^*_{++}(x,x') \end{aligned}$$

Influence action

$$
\begin{aligned}
S_{IF} &= \int dt\, dt' \left( \frac{d^2}{dt^2} \Delta^{ij}(t) \right) D_{ijkl}(t,t') \left( \frac{d^2}{dt'^2} \Sigma^{kl}(t') \right) \\
&\quad + \frac{i}{2} \int dt\, dt' \left( \frac{d^2}{dt^2} \Delta^{ij}(t) \right) N_{ijkl}(t,t') \left( \frac{d^2}{dt'^2} \Delta^{kl}(t') \right)
\end{aligned}
$$

where

$$
\begin{aligned}
\Sigma^{ij}(t) &= \frac{1}{2} \left[ z_+^i(t) z_+^j(t) + z_-^i(t) z_-^j(t) \right] \\
\Delta^{ij}(t) &= z_+^i(t) z_+^j(t) - z_-^i(t) z_-^j(t)
\end{aligned}
$$

The dissipation and the noise kernels,

$$
\begin{aligned}
D_{ijkl}(t,t') &= \left(\frac{m^2\kappa^2}{512\pi^6}\right)\int d^3k\, d^3k' \int d^3x\, d^3x' \\
&\quad e^{-i\vec{k}\cdot\vec{x}} e^{-i\vec{k}'\cdot\vec{x}'} \sum_s \epsilon_{ij}^{(s)}(\vec{k})\epsilon_{kl}^{(s)}(\vec{k}') G_{ret}(x,x') \\
N_{ijkl}(t,t') &= \left(\frac{m^2\kappa^2}{1024\pi^6}\right)\int d^3k\, d^3k' \int d^3x\, d^3x' \\
&\quad e^{-i\vec{k}\cdot\vec{x}} e^{-i\vec{k}'\cdot\vec{x}'} \sum_s \epsilon_{ij}^{(s)}(\vec{k})\epsilon_{kl}^{(s)}(\vec{k}') G^{(1)}(x,x')
\end{aligned}
$$

$G_{ret}(x,x') = i\theta(t-t')\langle[h(x),h(x')]\rangle$ is the retarded Green function, and $G^{(1)}(x,x') = \langle\{h(x),h(x')\}\rangle$ is the Hadamard function.

Feynman-Vernon formalism

$$
\begin{aligned}
& e^{-\frac{1}{2}\int \Delta^{ij}\tilde{N}_{ijkl}\Delta^{kl}} \\
= \; & \mathcal{N}\int D\xi \; e^{-\frac{1}{2}\int (\xi_{ij}+i\Delta^{mn}\tilde{N}_{ijmn})(\tilde{N}^{-1})^{ijkl}(\xi_{kl}+i\tilde{N}_{klpq}\Delta^{pq})} \; e^{-\frac{1}{2}\int \Delta^{ij}\tilde{N}_{ijkl}\Delta^{kl}} \\
= \; & \int D\xi \; P[\xi] \; e^{-i\int \xi_{ij}\Delta^{ij}}
\end{aligned}
$$

where $\tilde{N}_{ijkl} = (d^2/dt^2)(d^2/dt'^2)N_{ijkl}$ and with the gaussian probability density

$$P[\xi] = \mathcal{N}\, e^{-\frac{1}{2}\int \xi_{ij}(\tilde{N}^{-1})^{ijkl}\xi_{kl}}$$

where $\xi_{ij}$ is a stochastic force and $\mathcal{N}$ is a normalization constant.

Stochastic effective action for the geodesic separation

$$
\begin{aligned}
S_{SEA} &= S_m[z_+] - S_m[z_-] + S_{IF} \\
&= \frac{m}{2}\int dt\, \delta_{ij}\dot{z}^i_+(t)\dot{z}^j_+(t) - \frac{m}{2}\int dt\, \delta_{ij}\dot{z}^i_-(t)\dot{z}^j_-(t) \\
&\quad + \int dt\, dt'\, \Delta^{ij}(t)\tilde{D}_{ijkl}(t,t')\Sigma^{kl}(t') - \int dt\, \xi_{ij}(t)\Delta^{ij}(t)
\end{aligned}
$$

Langevin equation

$$\left.\frac{\delta S_{SEA}}{\delta z_+^i}\right|_{z_+=z_-=z} = 0$$

$$\Rightarrow \quad m\ddot{z}^i(t) + 2\delta^{im}\int dt'\, \tilde{D}_{mnkl}(t,t')\, z^n(t) z^k(t') z^l(t') \\ -2\delta^{ik}\xi_{kl}(t)\, z^l(t) = 0$$

# IV. Langevin equation and geodesic congruences

We shall concentrate on the noise effect due to the stochastic force

$$m\ddot{z}^i(t) = 2\delta^{ik}\xi_{kl}(t)\, z^l(t)$$

with the correlation function

$$\begin{aligned}\langle \xi_{ij}(t)\xi_{kl}(t')\rangle_s &= \int D\xi \; P[\xi]\, \xi_{ij}(t)\xi_{kl}(t') \\ &= \tilde{N}_{ijkl}(t,t').\end{aligned}$$

Geodesic separation

$$z^i = z_0^i + \delta z^i$$

where $z_0^i(t)$ corresponds to the geodesic motion without the graviton effects,

$$m\ddot{z}_0^i(t) = 0$$

Imposing the initial conditions: $\delta z^i(0) = \delta \dot{z}^i(0) = 0$,

$$\delta z^i(t) = \frac{2}{m} \int_0^t dt' \, (t - t') \delta^{ik} \xi_{kl}(t') z_0^l(t')$$

Geodesic separation correlation

$$\langle \delta z^i(t) \delta z^j(t') \rangle_s = \frac{4}{m^2} \delta^{ik} \delta^{jl} \int_0^t dt'' \int_0^{t'} dt''' (t - t'')(t' - t''') z_0^m(t'') z_0^n(t''') \tilde{N}_{kmln}(t'', t''')$$

In the Minkowski vacuum, take the cutoff scale $\Lambda \sim 1/z_0$ and $t \sim z_0$, the fluctuation can be estimated to be

$$\sqrt{\langle (\delta z^i(t))^2 \rangle^{(0)}} \sim \kappa \sim 10^{-35}\mathrm{m}$$

which is the order of Planck length.

Thermal state at low temperature,

$$\sqrt{\langle(\delta z^i(t))^2\rangle^{(\beta)}} \sim T^3$$

and at high temperature,

$$\sqrt{\langle(\delta z^i(t))^2\rangle^{(\beta)}} \sim T^{5/2}$$

Enhancement only at very high temperature

Squeezed state

$$|\zeta\rangle = \prod_{\vec{k}} e^{\frac{1}{2}\zeta(\hat{a}_{\vec{k}}^2 - \hat{a}_{\vec{k}}^{\dagger 2})}|0\rangle$$

Enhancement by the squeeze parameter,

$$\sqrt{\langle(\delta z^3(t))^2\rangle^{(\zeta)}} \sim e^{\zeta}\sqrt{\langle(\delta z^3(t))^2\rangle^{(0)}}$$

Primordial gravitons from the inflationary era are in a squeezed quantum state.

With the grand unified inflation, for example,

$$e^{\zeta} \sim 10^{18}$$

it will bring the noise effect to a detectable level.

Geodesic congruence

Deformation tensor $\Omega_{ij}$ defined by

$$\frac{d(\Delta z)^i}{dt} = \Omega^i{}_j (\Delta z)^j$$

$\Omega_{ij}$ can be decomposed into

$$\Omega_{ij} = \frac{1}{3}\theta\,\delta_{ij} + \sigma_{ij} + \omega_{ij}.$$

with $\theta$ being the expansion scalar, $\sigma_{ij}$ the shear tensor, and $\omega_{ij}$ the rotation tensor.

Evolution of the deformation tensor

$$\frac{d\Omega^i{}_j}{dt} = -\Omega^i{}_k\Omega^k{}_j + K^i{}_j$$

For example, for the expansion scalar

$$\dot{\theta} = -\frac{1}{3}\theta^2 - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} + K^i{}_i$$

Raychaudhuri equation with an external tidal force

However, for the shear tensor

$$\langle \sigma_{ij}(t)\sigma_{kl}(t')\rangle = (\sigma_0)_{ij}(\sigma_0)_{kl} + \frac{4}{m^2}\int_0^t dt'' \int_0^{t'} dt'''\, N_{ijkl}(t'', t''')$$

For example, taking $(\sigma_0)_{ij} = 0$, the stochastic average of $(\sigma_{33})^2$ in the Minkowski vacuum

$$\sqrt{\langle \sigma_{33}(t)\sigma_{33}(t)\rangle^{(0)}} \sim \sqrt{\left(\frac{\hbar}{c}\right)\frac{\kappa^2}{30\pi^2}} \approx 1.94\times 10^{-27}\,\mathrm{s}^{-1}$$

which is again too small to be detectable.

Again, the magnitude of the shear tensor fluctuation due to graviton quantum noise can be enhanced by the squeezing parameter in the squeezed state.

$$\sqrt{\langle \sigma_{33}(t)\sigma_{33}(t)\rangle^{(\zeta)}} \;\sim\; e^{\zeta}\sqrt{\langle \sigma_{33}(t)\sigma_{33}(t)\rangle^{(0)}}$$

# V. Quantum noise of gravitons and gravitational decoherence

Reduced density matrix of the particle

$$\begin{aligned} & \rho_r(\vec{z}_+, \vec{z}_-, t) \\ = & \int_{-\infty}^{\infty} d^3 z_{+i} \int_{-\infty}^{\infty} d^3 z_{-i} \; J_r(\vec{z}_+, \vec{z}_-, t | \vec{z}_{+i}, \vec{z}_{-i}, 0) \rho_r(\vec{z}_{+i}, \vec{z}_{-i}, 0) \end{aligned}$$

with the evolution operator

$$J_r(\vec{z}_+, \vec{z}_-, t | \vec{z}_{+i}, \vec{z}_{-i}, 0) = \int_{\vec{z}_{+i}}^{\vec{z}_+} D\vec{z}_+ \int_{\vec{z}_{-i}}^{\vec{z}_-} D\vec{z}_- \; e^{i\, S_{\mathrm{CG}}[\vec{z}^+, \vec{z}^-]}$$

where $S_{\mathrm{CG}} = S_m[z_+] - S_m[z_-] + S_{IF}$ is called the coarse-grained action.

Influence action

$$S_{IF} = \int dt\, dt' \left(\frac{d^2}{dt^2}\Delta^{ij}(t)\right) D_{ijkl}(t,t') \left(\frac{d^2}{dt'^2}\Sigma^{kl}(t')\right) + \frac{i}{2}\int dt\, dt' \left(\frac{d^2}{dt^2}\Delta^{ij}(t)\right) N_{ijkl}(t,t') \left(\frac{d^2}{dt'^2}\Delta^{kl}(t')\right)$$

where

$$\Sigma^{ij}(t) = \frac{1}{2}\left[z_+^i(t)z_+^j(t) + z_-^i(t)z_-^j(t)\right]$$

$$\Delta^{ij}(t) = z_+^i(t)z_+^j(t) - z_-^i(t)z_-^j(t)$$

As the graviton degrees of freedom have been integrated over, $S_{\rm CG}$ can be expressed as

$$
\begin{aligned}
& S_{\rm CG}[\vec{\Sigma}, \vec{\Delta}] \\
= \; & m \int_{t_a}^{t} dt_1 \, \delta^{ij} \left[ \dot{\Sigma}_i(t_1) \dot{\Delta}_j(t_1) \right] \\
& + 2 \int_{t_a}^{t} dt_1 \int_{t_a}^{t} dt_2 \left[ \frac{d^2}{dt_1^2} \Delta_i(t_1) \Sigma_j(t_1) \right] D^{ijkl}(t_1, t_2) \\
& \qquad \left[ \frac{d^2}{dt_2^2} \left( \Sigma_k(t_2) \Sigma_l(t_2) + \frac{1}{4} \Delta_k(t_2) \Delta_l(t_2) \right) \right] \\
& + 2i \int_{t_a}^{t} dt_1 \int_{t_a}^{t} dt_2 \left[ \frac{d^2}{dt_1^2} \Delta_i(t_1) \Sigma_j(t_1) \right] N^{ijkl}(t_1, t_2) \\
& \qquad \left[ \frac{d^2}{dt_2^2} \Delta_k(t_2) \Sigma_l(t_2) \right]
\end{aligned}
$$

with $\vec{\Sigma}(t) = [\vec{z}_+(t) + \vec{z}_-(t)]/2$ and $\vec{\Delta} = \vec{z}_+(t) - \vec{z}_-(t)$.

Master equation of the reduced density matrix

$$
\begin{aligned}
&\frac{\partial}{\partial t}\rho_r(\vec{\Sigma},\vec{\Delta};t)\\
=\;&\left[\frac{i}{m_{\rm ren}(t)}\frac{\partial}{\partial\Sigma^i}\frac{\partial}{\partial\Delta_i}+N_1(t)+iN_2(t)\Sigma^i\Delta_i\right.\\
&+D_1(t)\left(\Sigma^i\Sigma_i+\frac{1}{4}\Delta^i\Delta_i\right)+iD_2(t)\left(\Sigma^i\frac{\partial}{\partial\Delta^i}+\frac{1}{4}\Delta^i\frac{\partial}{\partial\Sigma^i}\right)\\
&+N_3(t)\left(\Sigma^i\frac{\partial}{\partial\Sigma^i}+\Delta^i\frac{\partial}{\partial\Delta^i}\right)+D_3(t)\left(\frac{\partial}{\partial\Delta^i}\frac{\partial}{\partial\Delta_i}+\frac{1}{4}\frac{\partial}{\partial\Sigma^i}\frac{\partial}{\partial\Sigma_i}\right)\\
&+N_4(t)\left(3\frac{\partial}{\partial\Delta^i}\frac{\partial}{\partial\Delta_i}\frac{\partial}{\partial\Sigma^j}\frac{\partial}{\partial\Sigma_j}+\frac{\partial}{\partial\Delta^i}\frac{\partial}{\partial\Sigma_i}\frac{\partial}{\partial\Delta^j}\frac{\partial}{\partial\Sigma_j}\right)\\
&\left.+iD_4(t)\frac{\partial}{\partial\Sigma^i}\frac{\partial}{\partial\Delta_i}\left(\frac{\partial}{\partial\Delta^j}\frac{\partial}{\partial\Delta_j}+\frac{1}{4}\frac{\partial}{\partial\Sigma^j}\frac{\partial}{\partial\Sigma_j}\right)\right]\rho_r(\vec{\Sigma},\vec{\Delta};t)
\end{aligned}
$$

with $\vec{\Sigma}=[\vec{z}_+ + \vec{z}_-]/2$ and $\vec{\Delta}=\vec{z}_+ - \vec{z}_-$.

Terms quadratic in $\Sigma^i$, $\Delta^i$ and their derivatives can be considered as renormalization contributions.

The term with $N_4(t)$ is related to the noise kernel and is associated with decoherence phenomena.

The term with $D_4(t)$ is related to the dissipation kernel and is associated with processes with loss of energy.

In the high temperature ($\beta \to 0$) or Markovian limit,

$$N_{ijkl}(t_1, t_2) \sim -\frac{64\pi^5\alpha^2}{15\beta} \left[2\delta_{ij}\delta_{kl} - 3(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\right] \delta(t_1 - t_2)$$

and the master equation

$$\begin{aligned} & \frac{\partial}{\partial t}\rho(\vec{z}, \vec{z}\,', t) \\ = \quad & \cdots - \left(\frac{512\pi^5\alpha^2}{15m^4\beta}\right)\left(\frac{1}{4}\right)\left[3\left(\frac{\partial}{\partial \vec{z}}\cdot\frac{\partial}{\partial \vec{z}} + 2\,\frac{\partial}{\partial \vec{z}}\cdot\frac{\partial}{\partial \vec{z}\,'} + \frac{\partial}{\partial \vec{z}\,'}\cdot\frac{\partial}{\partial \vec{z}\,'}\right)\right. \\ & \qquad \left(\frac{\partial}{\partial \vec{z}}\cdot\frac{\partial}{\partial \vec{z}} - 2\,\frac{\partial}{\partial \vec{z}}\cdot\frac{\partial}{\partial \vec{z}\,'} + \frac{\partial}{\partial \vec{z}\,'}\cdot\frac{\partial}{\partial \vec{z}\,'}\right) \\ & \qquad \left. + \left(\frac{\partial}{\partial \vec{z}}\cdot\frac{\partial}{\partial \vec{z}} - \frac{\partial}{\partial \vec{z}\,'}\cdot\frac{\partial}{\partial \vec{z}\,'}\right)^2\right]\rho(\vec{z}, \vec{z}\,', t) \end{aligned}$$

Estimation of the decoherence time scale due to gravitons

$$t_{dec}^{-1} \sim \left(\frac{128\pi^5\alpha^2}{15m^4\beta}\right)\left[3(\vec{p}+\vec{p}\,')^2(\vec{p}-\vec{p}\,')^2+(\vec{p}\,^2-\vec{p}\,'^2)^2\right]$$

in the momentum representation for the off-diagonal density matrix elements.

Similar results in this Markovian case have been obtained by Blencowe, and Anastopoulos and Hu.

In the low temperature (non-Markovian) case, the temperature dependent part of the noise kernel

$$N^{(\beta)}_{ijkl}(t,t') \sim -\frac{32\pi^6\alpha^2}{45\beta^2}[2\delta_{ij}\delta_{kl} - 3(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})]$$

and the master equation

$$\frac{\partial}{\partial t}\,\rho(\vec{p},\vec{p}\,',t) = \cdots - a\left(\frac{1}{t} + \frac{\pi^2\,t}{3\,\beta^2}\right)\,\rho(\vec{p},\vec{p}\,',t)$$

where

$$a = \left(\frac{128\pi^4\alpha^2}{15m^4}\right)\left[3(\vec{p}+\vec{p}\,')^2(\vec{p}-\vec{p}\,')^2 + (\vec{p}\,^2 - \vec{p}\,'^2)^2\right]$$

The perturbative solution with $a$ proportional to $\alpha^2$,

$$\rho(\vec{p}, \vec{p}\,', t) \sim -a \ln t - \frac{a\,\pi^2}{6\,\beta^2} t^2 + \cdots$$

Hence, at low temperature or the non-Markovian case, the decay of the density matrix element is $-\ln t$ for the zero temperature part, and $-t^2$ in the temperature dependent part which is proportional to $1/\beta^2$ or $T^2$.

This is distinctly different from the high temperature (Markovian) decay.

# VI. Conclusions

1. Quantum effect of gravitons could manifest as noise on the particle/detector.
2. Noise effects to separation fluctuations have been analyzed for the Minkowski, thermal, and squeezed vacua. For the squeezed state, the effect may be enhanced to a detectable level.
3. Similar results are obtained for the fluctuations of geodesic congruences due to graviton noise effects.
4. We have derived a master equation for the particle density matrix due to graviton effects. This non-Markovian master equation is valid for all temperatures, as contrast to the previous one (the ABH equation) which is for the high temperature (Markovian) regime.