Recent Advancements in Cavity controlled QFT effects

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15th annual conference on Relativistic Quantum Information (North)

Università degli Studi Federico II, Napoli

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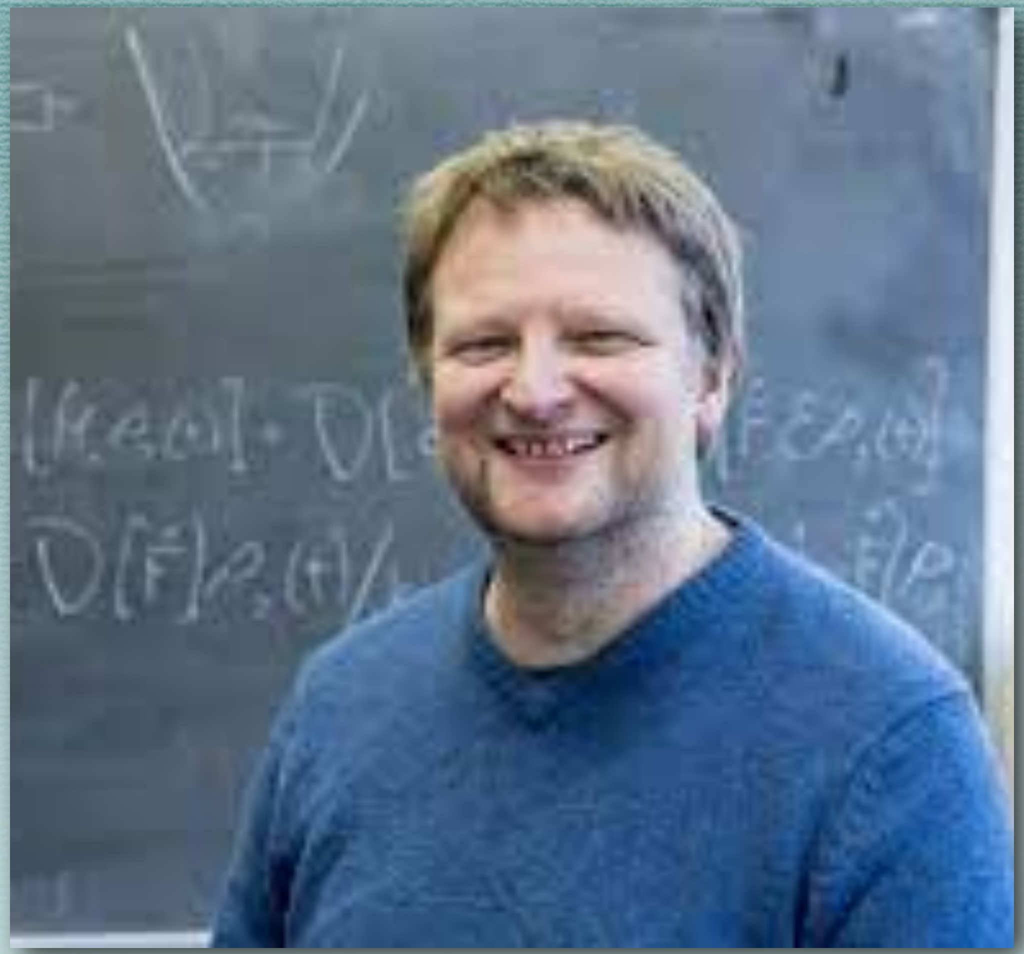
IISER Mohali (INDIA)

Venue for 16th RQI- North meet

Jaffino Stargen @ Joy Univ



Hendrik Ulbricht @ U Southampton

Andrea Vinante @ IFN, Trento

Sandeep Goyal @ IISER M

Pritam Nanda@ IISER M

Akhil Deswal @ IISER M

Harkirat Singh Sahota @IIT Delhi

Navdeep Arya @ U Stockholm

- **Cavity enhanced Unruh effect**

  D. Jaffino Stargen, KL   Phys. Rev. Lett. 129, 111303 (2022)

- **Radiative correction to states of atoms**

  Navdeep Arya, D. Jaffino Stargen, KL, Goyal ; Phys. Rev. D (2024)

- **Superradiance in cavity environment**

  Akhil Deswal, Navdeep Arya, KL, Goyal ; arxiv 2501.16219

- **Interacting quantum field theory in cavity**

  Pritam Nanda, KL; To appear ...

Probes of Quantum fields

- **At not so UV scale the interaction between fields/atoms are typically limited by energy scales of probes**
- **Interesting predictions need extreme conditions for verifications.**
- Lab based test : Unruh Effect, non-inertial QFT, Inertial QFT effects : Hierarchy suppressed interaction
- **Modification from the boundary conditions : Dilution of extreme conditions**

Atomic and Field probes

- Perturbative approach : correlators of field play central role

Probe Response $\propto \int_0^\infty d\omega_k\, \rho(\omega_k)\mathcal{I}(\Delta E, \omega_k)\mathcal{J}(\omega_k, \eta^i)$

$\rho(\omega_k)$ is the density of field modes

$\mathcal{I}(\Delta E, \omega)$ determines the participation of modes in a process

$$(\delta E_a)_{\mathrm{rf}} = -\frac{g^2}{2}\sum_{i,j}\int_0^\infty \mathrm{d}u \sum_d |\langle b|\mu(0)|d\rangle|^2 (e^{i\omega_{bd}u} - e^{-i\omega_{bd}u})\frac{1}{2}\langle 0|\{\Phi^{\mathrm{f}}(\tau_0), \Phi^{\mathrm{f}}(\tau_0 - u)\}|0\rangle$$

- Scattering amplitudes

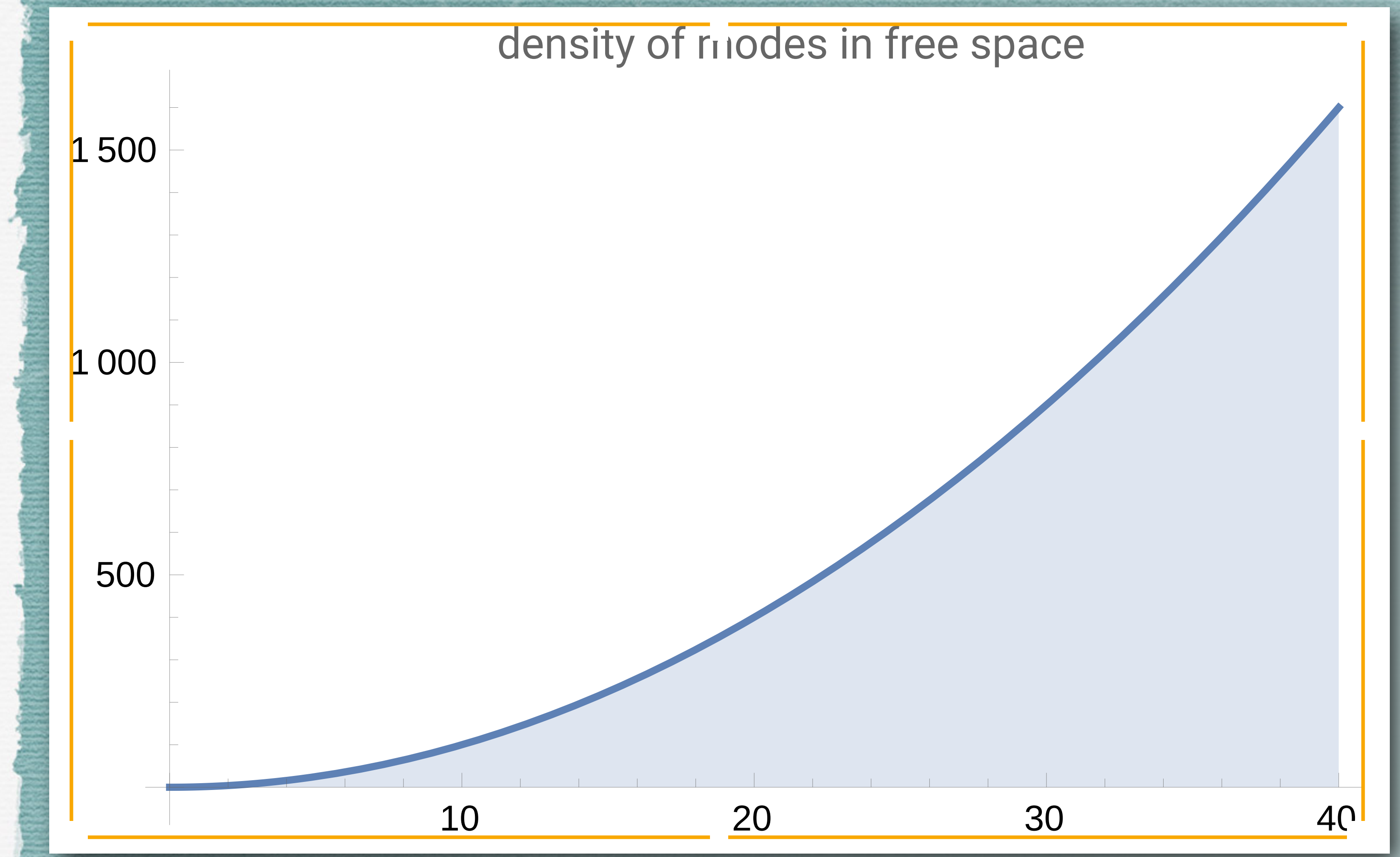
$$\langle K_1, K_2, ..K_\ell; k_1, k_2, ..k_p|\,\Phi^n(x)\phi^m(x)|K'_1, K'_2, ..K'_{n-\ell}; k'_1, k'_2, ..k_{m-p}\rangle$$

Manipulating correlations

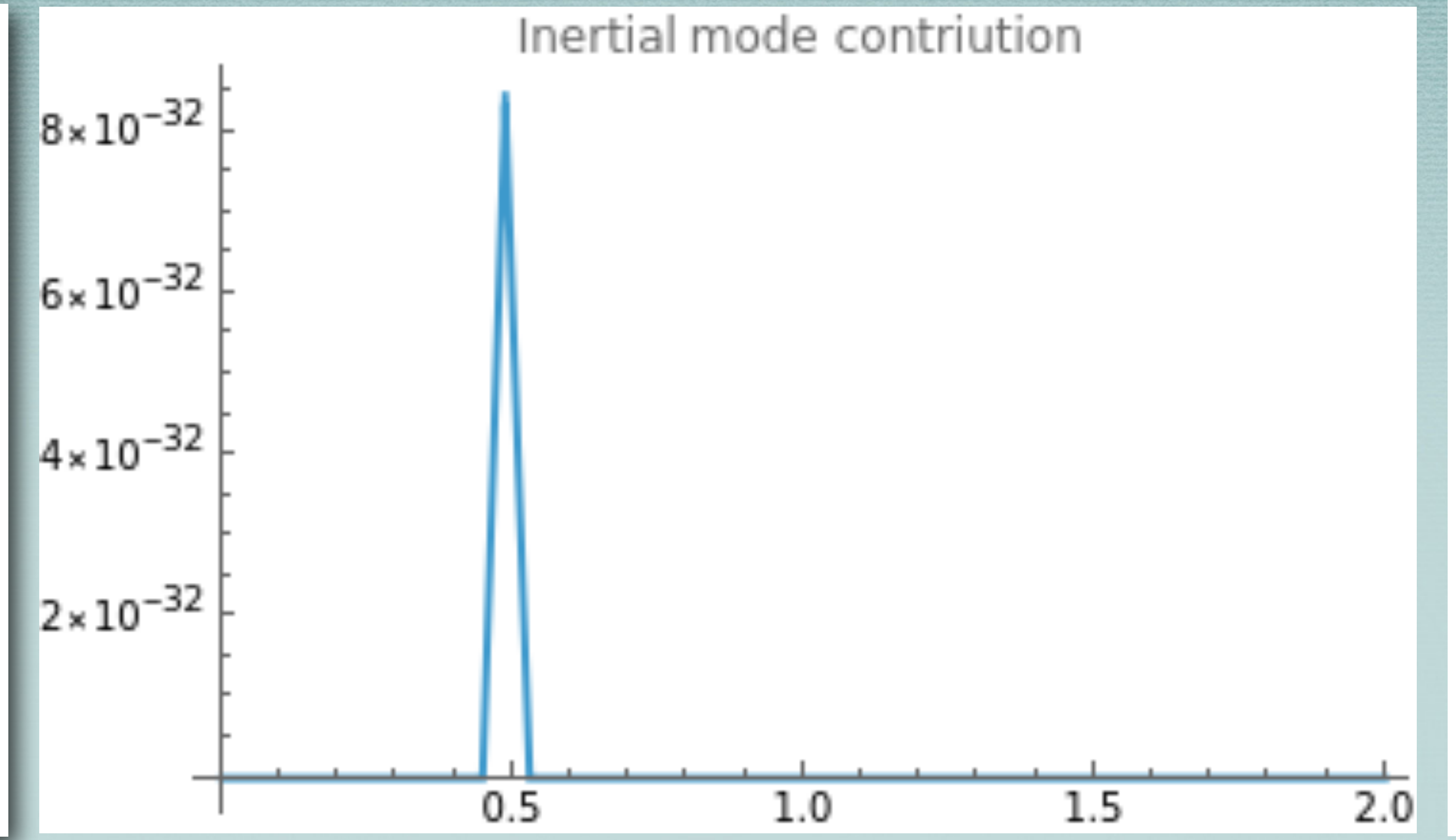
Bridging the gap of extreme conditions

Exclusive Regimes

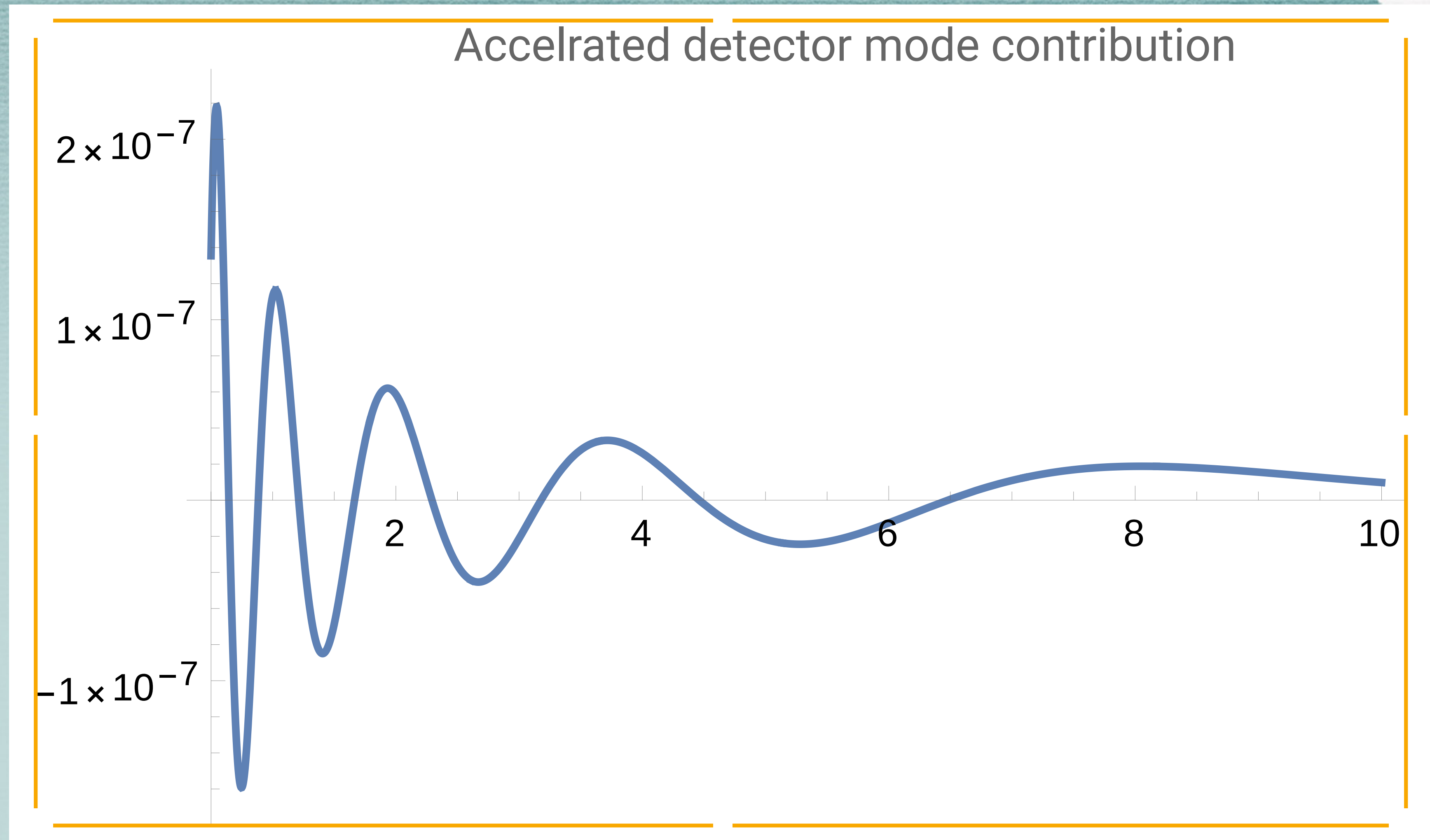
- The density of modes and participating modes live mostly in exclusively separated regime.



$\rho(\omega_k)$



$\delta(-\Delta E + \omega_k)$



$\frac{K_{2i\Delta E/a}(2\omega/a)}{2\omega/a}$

$$\mathcal{I}(\Delta E, \omega)$$

- **Typically would want to excite large energy modes** : Go to large acceleration : Too taxing
- Can we arrest the IR end of mode distribution ?
- Culminate the need of extreme accelerations !

Midi*superspace quantization of field

Putting boundaries starts building up resonance structure

$$\int \frac{d^3k}{\omega_k} u_k(x) u_k^*(y)$$

$$\omega_k^2 = k^2$$

$$\longrightarrow$$

$$\sum_n \int \frac{d^2k}{\omega_{k,n}} u_{k,n}(x) u_{k,n}^*(y)$$

$$\omega_{k,n}^2 = k^2 + \frac{f(n)^2}{L^2}$$

$$\downarrow$$

- Strongly enhances field correlations at resonance

$$\sum_n \sum_m \int \frac{dk_z}{\omega_{k_z,n,m}} u_{k_z,n,m}(x) u_{k_z,n,m}^*(y)$$

$$\omega_{k_z,n,m}^2 = k_z^2 + \frac{f(n,m)^2}{L^2}$$

$$\longleftarrow$$

$$\sum_{nml} \delta\left(\omega_{nml - f(nml)/L}\right)$$

- Scully et. al. Phys. Rev. Lett. (2003), Lopp, Martin-Martinez, Page CQG (2018)

Measure of modes

$$\omega_k^2 = k^2$$

$$\frac{d^3k}{\omega_k} = \omega^2 d\omega d\Omega$$

- Low energy suppression

$$\omega_{k,n}^2 = k^2 + \frac{f(n)^2}{L^2}$$

$$\frac{d^2k}{\omega_k} = d\omega_k d\theta \;\; \Theta(\omega_k - f(n)/L)$$

- Energy dilution arrested, coupling dictated

- Introduces a cutoff in the system

Zheng, Zhou, Gou, Zhou; Phys. Rev. Res. (2025)

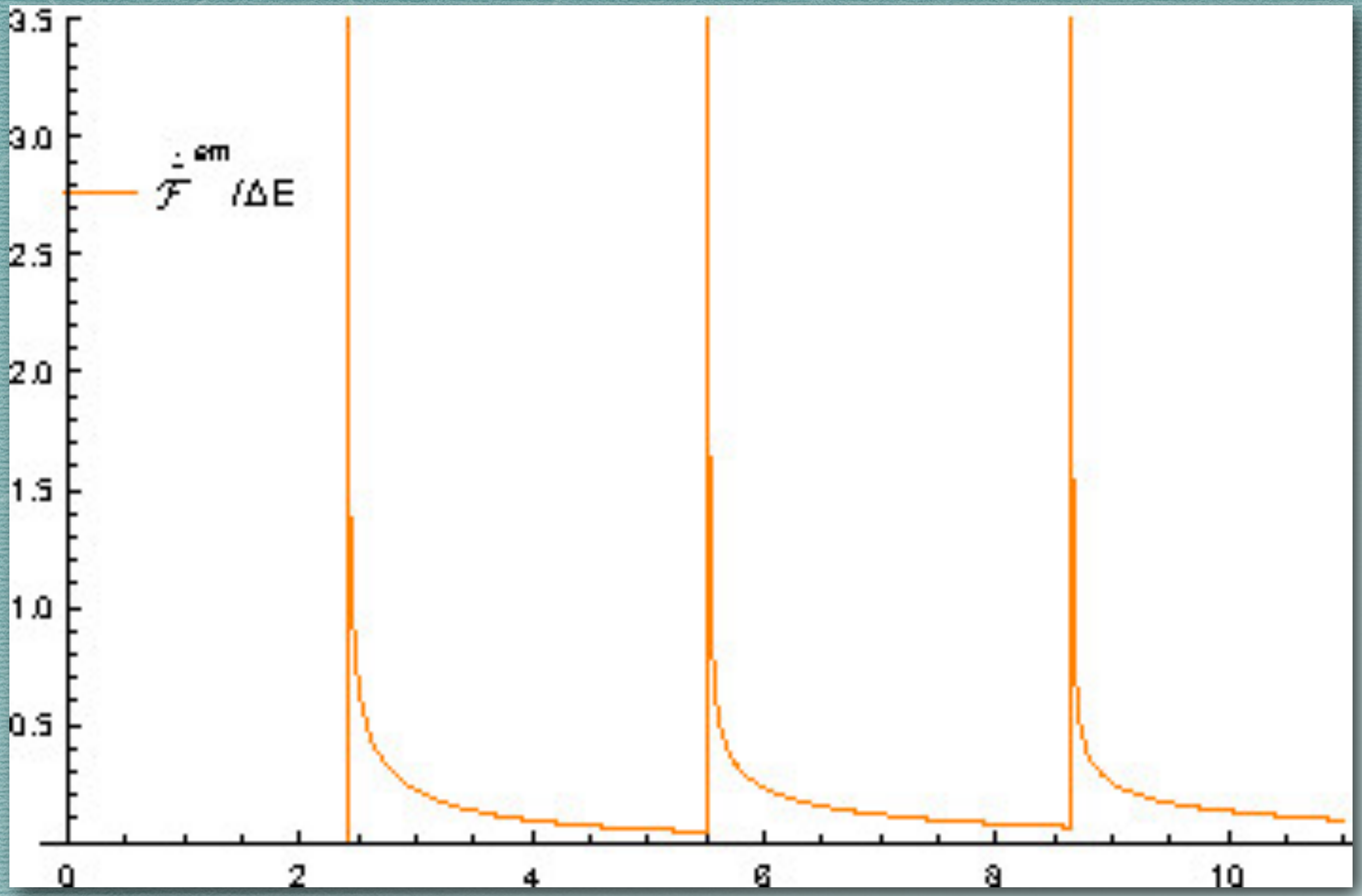
Akhil Deswal, Navdeep Arya, KL, Goyal ; arxiv 2501.16219

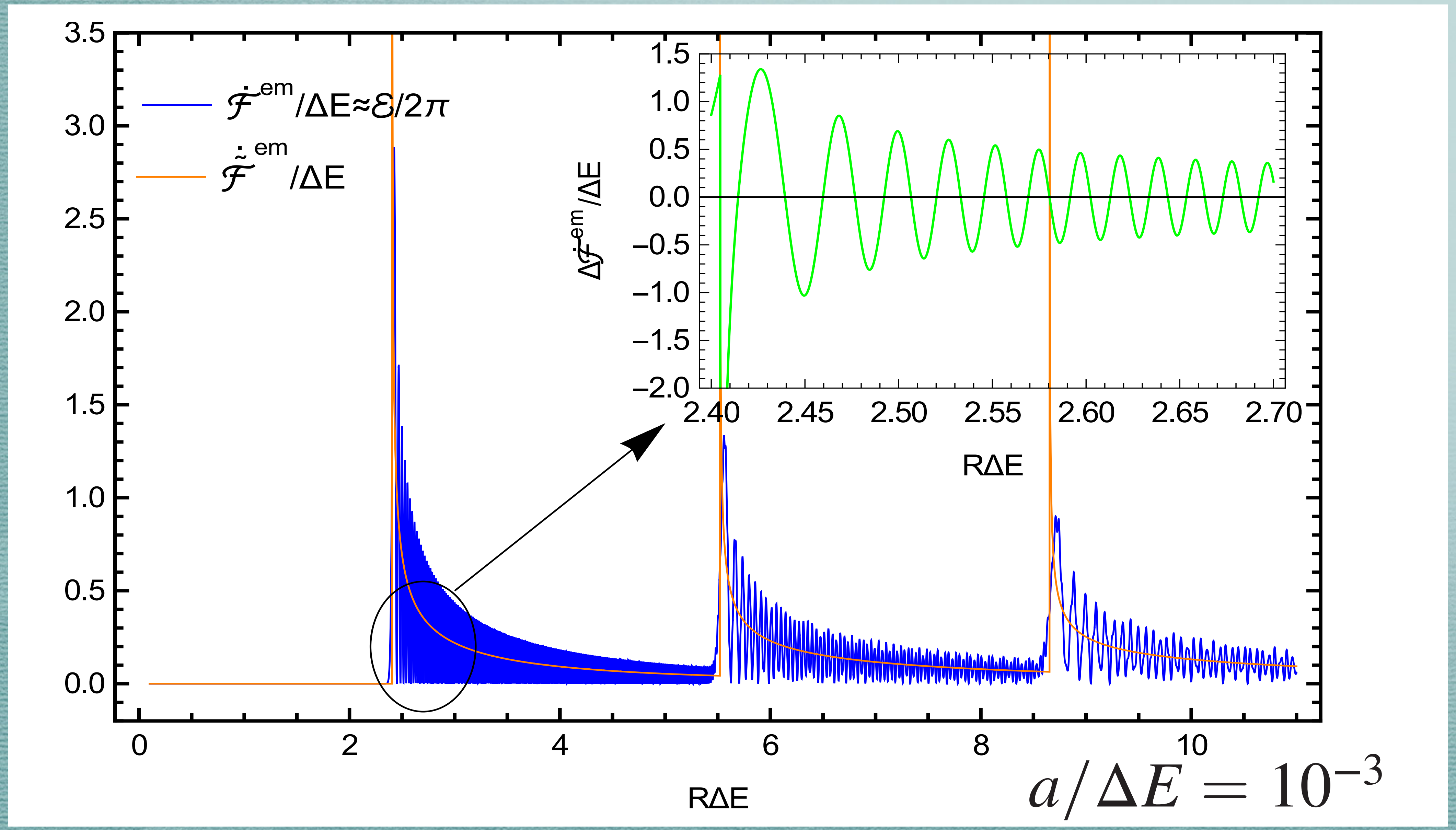
$$\omega_{k_z,n,m}^2 = k_z^2 + \frac{f(n,m)^2}{L^2}$$

$$\frac{dk_z}{\omega_k} = \frac{d\omega_k}{k_z} = \frac{d\omega_k}{\sqrt{\omega_k^2 - f(n,m)^2/L^2}}\Theta(\omega_k - f(n,m)/L)$$

- Strongly enhances field correlations at resonance

SPONTANEOUS EMISSION IN CYLINDRICAL CAVITY





$$\dot{\mathcal{F}}(\Delta E) = \frac{1}{2\pi}\int_0^\infty d\omega_k \underbrace{\frac{8}{a^2 e^{-\pi\Delta E/a}} \frac{K_{2i\Delta E/a}(2\omega_k/a)}{(2\omega_k/a)}}_{\mathcal{I}(\Delta E,\omega_k)}$$

$$\times \sum_{m=-\infty}^{\infty} \underbrace{\sum_{n=1}^{\infty} \frac{(\omega_k/\pi R^2)}{J^2_{|m|+1}(\xi_{mn})} \frac{\Theta l(\omega_k - \xi_{mn}/R)}{\sqrt{\omega_k^2 - (\xi_{mn}/R)^2}}}_{\rho(\omega_k)} \times \underbrace{J_m^2(\xi_{mn}\rho_0/R)}_{\mathcal{J}(\rho_0/R)}$$

$$= \frac{e^{\pi\Delta E/a}}{\pi^2 R(Ra)} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} \frac{J_m^2(\xi_{mn}\rho_0/R)}{J^2_{|m|+1}(\xi_{mn})} \times K^2_{i\Delta E/a}(\xi_{mn}/Ra)$$

- Fall off is less steep and non monotonic !
- Possible to catch the radius where it is above inertial and not too small
- Stargen, KL Phys. Rev. Lett. (2022)]

Radiative energy corrections

Navdeep Arya, D. Jaffino Stargen, KL, Goyal ; Phys. Rev. D (2024)

If atoms see more pronounced quantum fluctuations, their atomic lines must respond

- DDC formalism for radiative correction

Dalibard, Dupont-Roc, and C. Cohen-Tannoudji, J. Physique 43, 1617 (1982).
Audretsch and Muller Phys. Rev. A. 50, 1755 (1994)

$$H_I(\tau) = -g\mu(\tau)\Phi(\tilde{x}(\tau))$$

- Due to interaction, both field and atomic states change

$$\frac{d}{d\tau}\hat{\mathcal{O}}_A = \frac{i}{\hbar}\left[\hat{H}_A, \hat{\mathcal{O}}_A\right] + \frac{i}{\hbar}\left[\hat{H}_I, \hat{\mathcal{O}}_A\right]$$

$$\frac{d}{d\tau}\hat{\mathcal{O}}_\phi = \frac{i}{\hbar}\left[\hat{H}_\phi, \hat{\mathcal{O}}_\phi\right] + \frac{i}{\hbar}\left[\hat{H}_I, \hat{\mathcal{O}}_\phi\right]$$

- For leading order shift in eigenstates of the atomic Hamiltonian

$$(\delta E_a)_{\mathrm{vf}} = -\frac{g^2}{2}\int_0^\infty du C^{(\mathrm{R})}(u)\chi^{(\mathrm{S,a})}(u)$$

$$C^{(\mathrm{R})}(u) = \frac{1}{2}\langle 0|\{\Phi^{\mathrm{f}}(\tau_0), \Phi^{\mathrm{f}}(\tau_0 - u)\}|0\rangle,$$

$$\chi^{(\mathrm{S,a})}(u) = i\sum_d |\langle a|\mu(0)|d\rangle|^2 \left(e^{i\omega_{ad}u} - e^{-i\omega_{ad}u}\right)$$

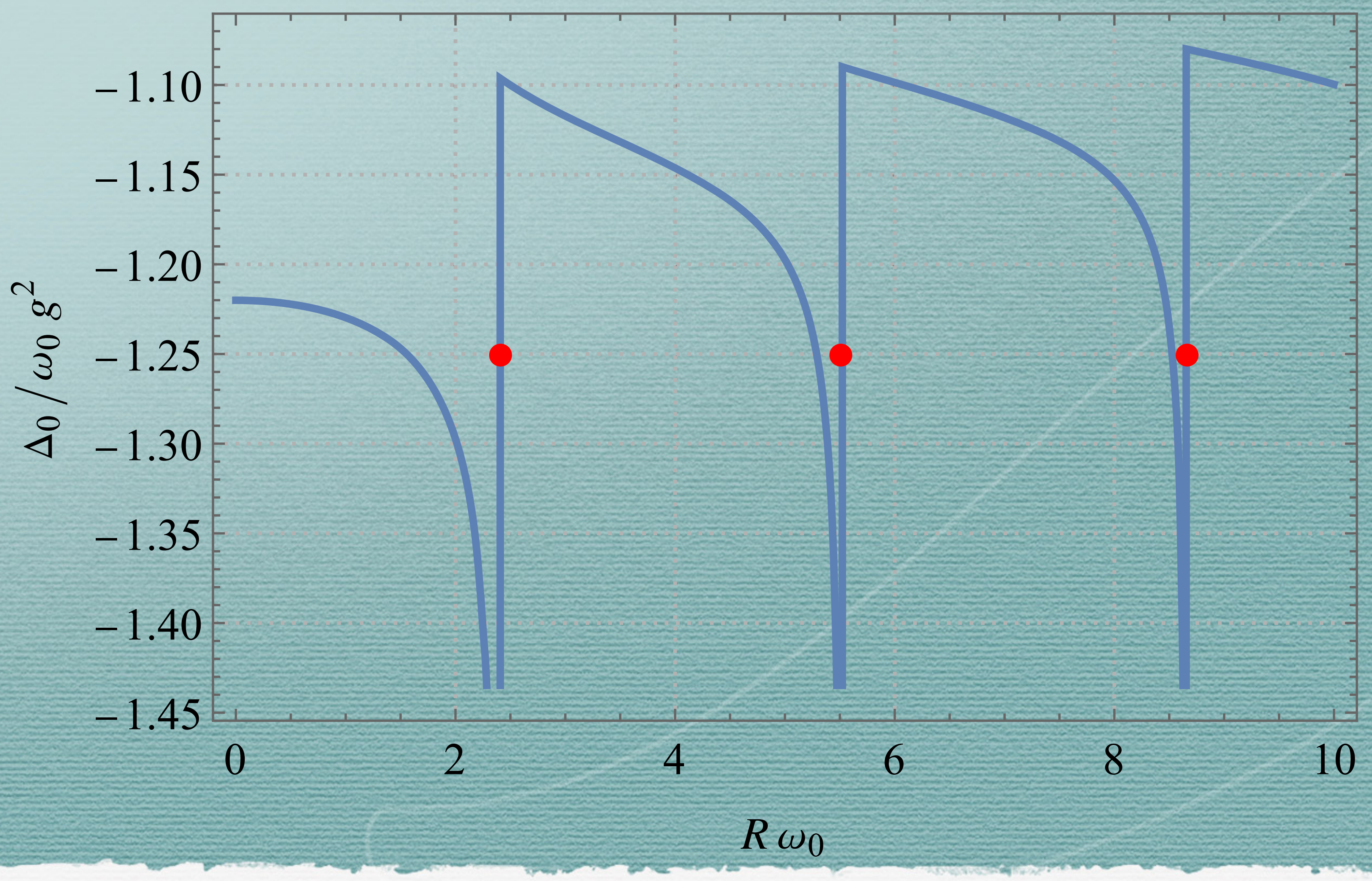
- On Inertial trajectories $\tilde{x}^{\nu}(\tau) = (\gamma\tau, 0, 0, \gamma v_0 \tau)$

$$\langle 0|\Phi(\tilde{x}(\tau_2))\Phi(\tilde{x}(\tau_1))|0\rangle = \frac{1}{(2\pi R)^2}\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\frac{J_m^2(\xi_{mn}\rho_0/R)}{J_{|m|+1}^2(\xi_{mn})}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_z'}{\omega_k'}e^{-i\omega_k'(\tau_2-\tau_1)}$$

$$\Delta_0 = \frac{g^2\omega_0}{\pi^2(R\omega_0)}\sum_{m=-\tilde{m}}^{\tilde{m}}\sum_{n=1}^{\tilde{n}}\frac{J_m^2(\xi_{mn}\rho_0/R)}{J_{|m|+1}^2(\xi_{mn})}\frac{1}{\xi_{mn}}\begin{cases}\frac{1}{\sqrt{1-\left(\frac{R\omega_0}{\xi_{mn}}\right)^2}}\left[\tan^{-1}\left(\frac{\sqrt{1-\left(\frac{R\omega_0}{\xi_{mn}}\right)^2}}{\left(\frac{R\omega_0}{\xi_{mn}}\right)}\right)-\frac{\pi}{2}\right], & R\omega_0 < \xi_{mn}, \\ \frac{1}{\sqrt{\left(\frac{R\omega_0}{\xi_{mn}}\right)^2-1}}\log\left[\frac{R\omega_0}{\xi_{mn}}+\sqrt{\left(\frac{R\omega_0}{\xi_{mn}}\right)^2-1}\right], & R\omega_0 \geq \xi_{mn}.\end{cases}$$

- Radiative energy shifts in cavity

D. J. Heinzen and M. S. Feld; Phys. Rev. Lett. 59, 2623 (1987)



- On a Rindler trajectory on the other hand

$$t(\tau) = a^{-1}\sinh(a\tau),$$
$$z(\tau) = a^{-1}\cosh(a\tau),$$

$$\begin{aligned}\Delta_{\rm rf} &= (\delta E_e)_{\rm rf} - (\delta E_g)_{\rm rf}\\ &= \frac{g^2}{(2\pi R)^2}\sum_{m,n}\frac{J_m^2(\xi_{mn}\rho_0/R)}{J_{|m|+1}^2(\xi_{mn})}\int_0^\infty \mathrm{d}u\int_{-\infty}^{\infty}\frac{\mathrm{d}k_z'}{\omega_k'}\,\mathrm{Im}\Big(e^{-i(2\omega_k'/a)\sinh(au/2)}e^{i\omega_0 u} + e^{i(2\omega_k'/a)\sinh(au/2)}e^{i\omega_0 u}\Big).\end{aligned}$$

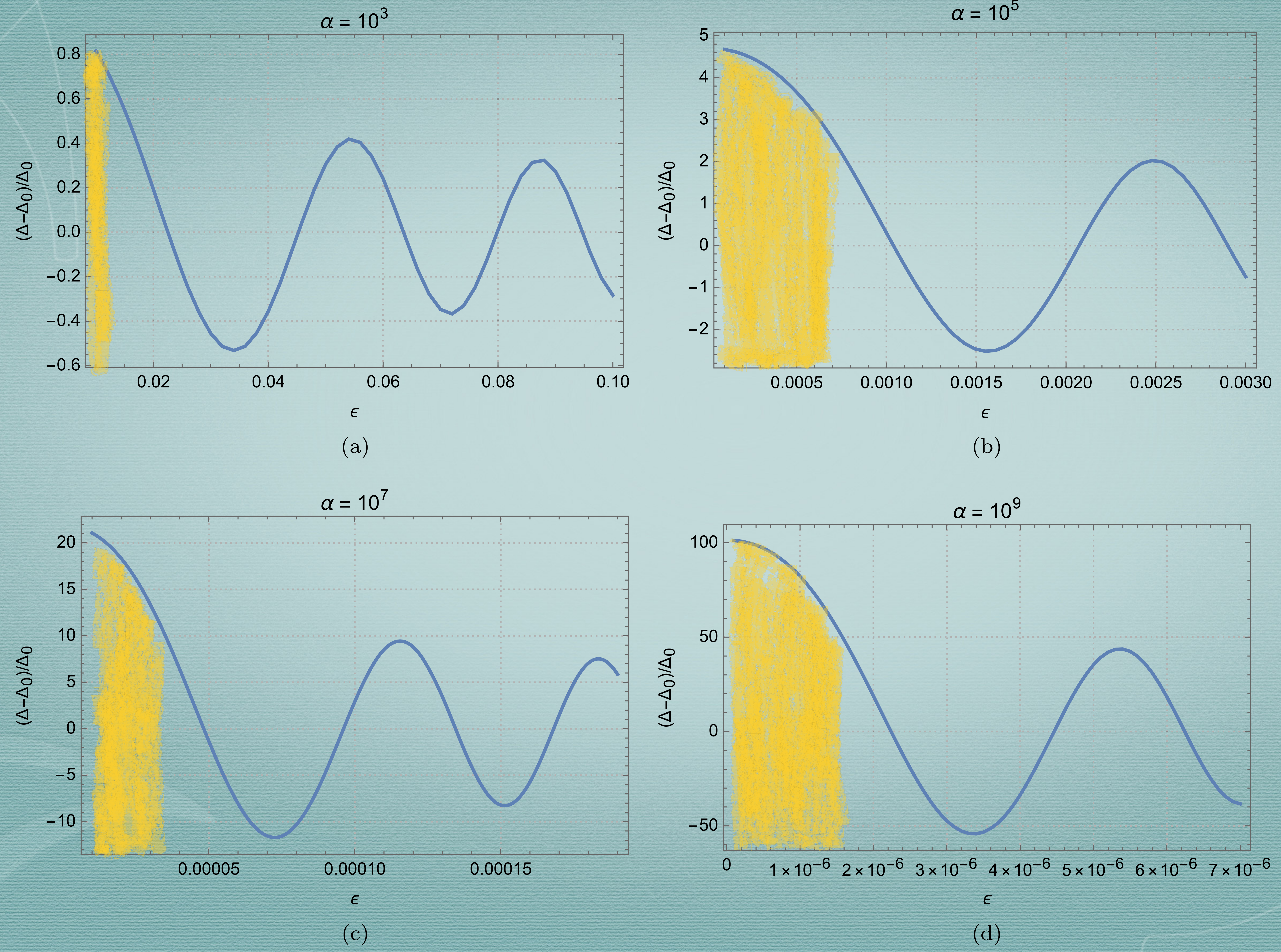
- Again, slightly off resonant cavity helps enhance the thermal fluctuations

$$\Delta - \Delta_0 = \frac{g^2\omega_0}{(2\pi)^2(R\omega_0)^2}\sum_{m,n}\frac{J_m^2(\xi_{mn}\rho_0/R)}{J_{|m|+1}^2(\xi_{mn})}\times\int_0^\infty \mathrm{d}\varpi\left(\frac{4\alpha}{\pi}\cosh(\pi\varpi\alpha)K_{i\varpi\alpha}^2(\xi_{mn}\alpha/R\omega_0) - \frac{\Theta(\varpi-(\xi_{mn}/R\omega_0))}{\sqrt{\varpi^2-(\xi_{mn}/R\omega_0)^2}}\right)$$

$$\mathrm{P.V.}\left(\frac{1}{\varpi+1} - \frac{1}{\varpi-1}\right)$$

$$\varpi \equiv \omega_k/\omega_0 \qquad \alpha \equiv \omega_0/a$$

- **More pronounced shifts in the neighbourhood of resonance points**



$$\alpha \equiv \omega_0 / a$$

- As. Before : The fall in radiative shift away from resonance is less steep and non monotonous

- This gives a more room for tuning cavity geometry : A more sensitive measure

- Generalisable for other accelerating setting as well

Navdeep Arya, Sandeep Goyal ; Phys. Rev. D (2024)

Field - Field interaction

Pritam Nanda, KL; To appear ...

- Field interaction : How fields talk to each other

$$\mathcal{L} = \frac{1}{2}\left[-\partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2 - \partial_\mu\phi_1\partial^\mu\phi - m^2\phi^2\right] - \lambda\Phi^n(x)\phi^m(x)$$

What they end up doing to each other is captured in

$$-i\lambda\langle K_1, K_2, ..K_\ell; k_1, k_2, ..k_p| \int d^4x\Phi^n(x)\phi^m(x)|K_1', K_2', ..K_{n-\ell}'; k_1', k_2', ..k_{m-p}\rangle$$

- Illustrative example : Particle decay

$$\mathcal{L} = \frac{1}{2}\partial^\mu\Phi\partial_\mu\Phi - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda\Phi\phi\phi$$

$$\langle p_1 p_2| - i\lambda\int d^4x\Phi(x)\phi(x)\phi(x)\,|P\rangle$$

- Coupling hierarchy sets up the dominant decay channels

$$\Gamma = \frac{1}{2M}\int\frac{d^3\vec{p_1}d^3\vec{p_2}}{(2\pi)^6}\frac{1}{4E_{p_1}E_{p_2}}|\mathcal{M}(\Phi\to\phi\phi)|^2(2\pi)^4\delta^4(P_\Phi - p_{1\phi} - p_{2\phi})$$

Decay rate and branching ratio

- The decay rate is dependent on the "effective coupling"

$$\Gamma = \quad \gamma \quad \frac{\lambda^2}{M}\int \frac{d^3\vec{p_1}}{(2\pi)^2}\frac{1}{4(\vec{p_1}^{\,2}+m^2)}\delta\left(M-2\sqrt{\vec{p_1}^{\,2}+m^2}\right) \quad = \frac{\lambda^2 E_P}{M^2}\sqrt{1-\frac{4m^2}{M^2}}$$

- Weak coupling : Longer decay
- Migrate to high energy to appreciate this process !

- In multiple field interaction, the coupling hierarchy sets the branching ratio

$$\mathcal{L} = \frac{1}{2}\left[-\partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2 - \partial_\mu\phi_1\partial^\mu\phi_1 - m_1^2\phi_1^2 - \partial_\mu\phi_2\partial^\mu\phi_2 - m_2^2\phi_2^2\right] - \lambda_1\Phi\phi_1\phi_1 - \lambda_2\Phi\phi_2\phi_2\phi_2$$

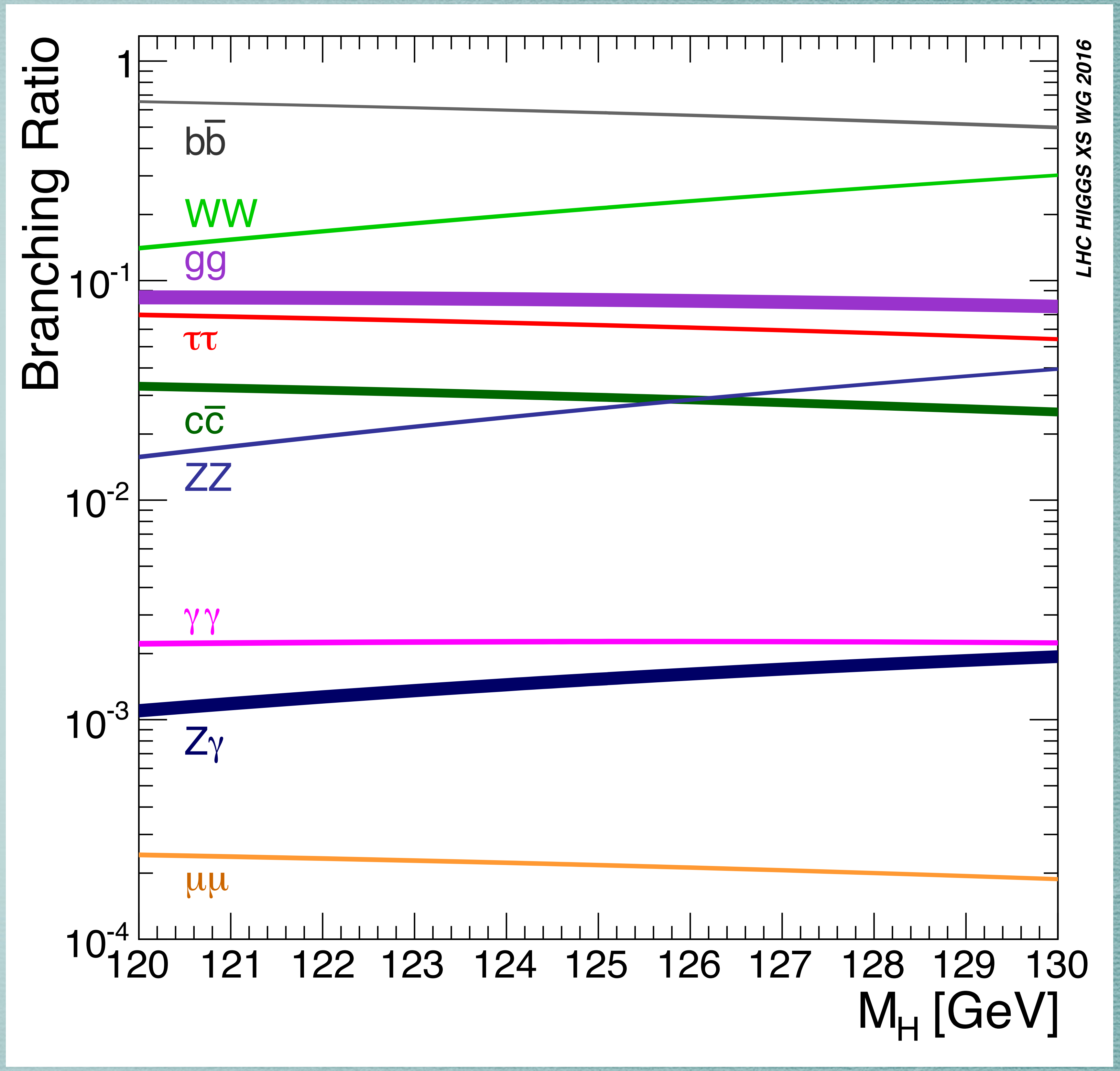
Higgsplosion : Khoze, Spannowsky; Nuclear Physics B (2018)

Higgs decay

Decay channel	Branching ratio	Rel. uncertainty
$H \to \gamma\gamma$	2.27×10^{-3}	2.1%
$H \to ZZ$	2.62×10^{-2}	$\pm 1.5\%$
$H \to W^+W^-$	2.14×10^{-1}	$\pm 1.5\%$
$H \to \tau^+\tau^-$	6.27×10^{-2}	$\pm 1.6\%$
$H \to b\bar{b}$	5.82×10^{-1}	$^{+1.2\%}_{-1.3\%}$
$H \to c\bar{c}$	2.89×10^{-2}	$^{+5.5\%}_{-2.0\%}$
$H \to Z\gamma$	1.53×10^{-3}	$\pm 5.8\%$
$H \to \mu^+\mu^-$	2.18×10^{-4}	$\pm 1.7\%$

$m_H = 125\,\text{GeV}$

Status of Higgs boson : Particle data group (2023)



LHC Higgs cross section working group report (2016)

Can we enhance $\langle p_1 p_2| - i\lambda \int d^4x \Phi(x)\phi(x)\phi(x) |P\rangle$?

Selective Enhancement in 2-D square cavity

$$\Phi = \frac{2}{L}\sum_{n_3=1}^{\infty}\sum_{m_3=1}^{\infty}\sin\frac{n_3\pi x}{L}\sin\frac{m_3\pi y}{L}\int\frac{dk_z^{(3)}}{\sqrt{\omega_{k_z^{(3)}}}}\left(a_{k_z^{(3)}}e^{ik_z^{(3)}z}+a^{\dagger}_{k_z^{(3)}}e^{-ik_z^{(3)}z}\right)$$

$$\phi = \frac{2}{L}\sum_{n_1=1}^{\infty}\sum_{m_1=1}^{\infty}\sin\frac{n_1\pi x}{L}\sin\frac{m_1\pi y}{L}\int\frac{dk_z^{(1)}}{\sqrt{\omega_{k_z^{(1)}}}}\left(a_{k_z^{(1)}}e^{ik_z^{(1)}z}+a^{\dagger}_{k_z^{(1)}}e^{-ik_z^{(1)}z}\right)$$

$$\omega^2_{k_z^{(3)}} = {k_z^{(3)}}^2 + \frac{\pi^2}{L^2}(n_3^2+m_3^2) + M^2$$

$$\omega^2_{k_z^{(1)}} = {k_z^{(1)}}^2 + \frac{\pi^2}{L^2}(n_1^2+m_1^2)$$

- **Transition amplitude : Reduced symmetry**

$$(-i\lambda)\int < p_1p_2|\Phi\phi\phi|P > d^4x = \frac{1}{L}\frac{2n_1n_2n_3(\cos n_1\pi\cos n_2\pi\cos n_3\pi - 1)}{(n_1+n_2+n_3)(n_1-n_2+n_3)(n_1+n_2-n_3)(n_1-n_2-n_3)} \times \frac{2m_1m_2m_3(\cos m_1\pi\cos m_2\pi\cos m_3\pi-1)}{(m_1+m_2+m_3)(m_1-m_2+m_3)(m_1+m_2-m_3)(m_1-m_2-m_3)}$$

$$\times \quad \delta(k_z^{(3)}-k_z^{(2)}-k_z^{(1)})\delta(\omega^{(3)}-\omega^{(2)}-\omega^{(1)})$$

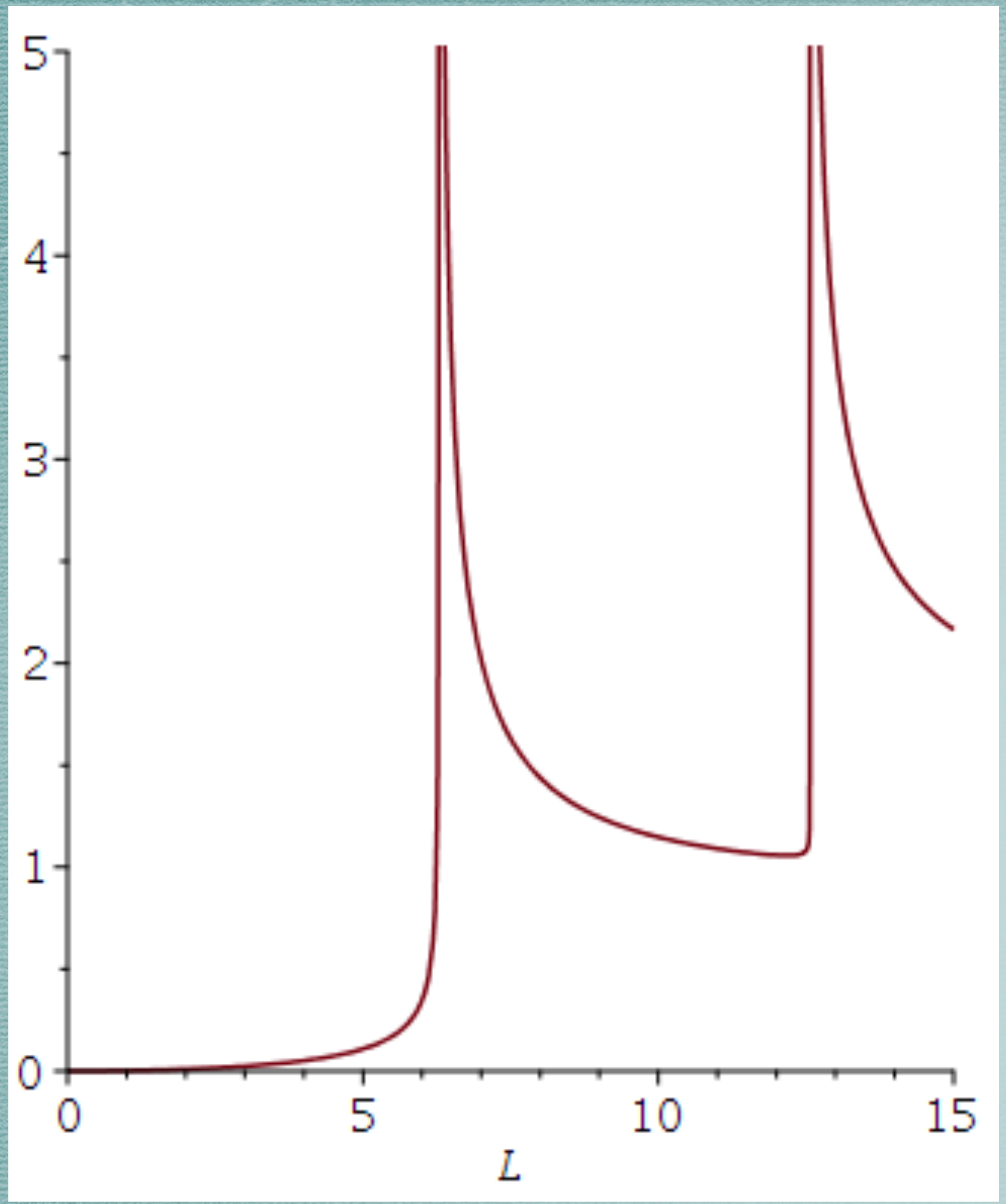
$$\Gamma = \quad \gamma \; \frac{\lambda^2}{ML^2} \sum_{n_1,n_2} \sum_{m_1,m_2} \left(A_{n_1,n_2,1}\right)^2 \left(B_{m_1,m_2,1}\right)^2 \int \frac{dk_z^{(1)} dk_z^{(2)}}{\omega_{k_z^{(1)}} \omega_{k_z^{(2)}}} \delta(M - \omega_{k_z^{(1)}} - \omega_{k_z^{(2)}}) \; \delta(k_z^{(2)} + k_z^{(1)})$$

- **Transition amplitude : Decay into scalar photons**

$$\Gamma = 2 \frac{\lambda^2}{ML^2} \sum_{n_1,n_2} \sum_{m_1,m_2} \frac{\left(A_{n_1,n_2,1}\right)^2 \left(B_{m_1,m_2,1}\right)^2}{\left(\frac{M^2 + \frac{\pi^2}{L^2}(n_1^2 + m_1^2 - n_2^2 - m_2^2)}{2M} + \sqrt{\frac{\left(M^2 + \frac{\pi^2}{L^2}(n_1^2 + m_1^2 - n_2^2 - m_2^2)\right)^2}{4M^2} + \frac{\pi^2}{L^2}(n_2^2 + m_2^2 - n_1^2 - m_1^2)} \right)}$$

$$\frac{E_P}{M} \quad \boxed{\times \frac{1}{\sqrt{\frac{\left(M^2 + \frac{\pi^2}{L^2}(n_1^2 + m_1^2 - n_2^2 - m_2^2)\right)^2}{4M^2} - \frac{\pi^2}{L^2}(n_1^2 + m_1^2)}}}$$

- Transition amplitude : Each process has its own characteristic resonance



$$\frac{1}{\sqrt{\frac{\left(M^2+\frac{\pi^2}{L^2}(n_1^2+m_1^2-n_2^2-m_2^2)\right)^2}{4M^2}-\frac{\pi^2}{L^2}(n_1^2+m_1^2)}}$$

- n-particle decay will have different amplitude and has a different resonance location

- Thus a particular decay channel can be selectively enhanced by identifying the proper resonance condition.

Summary

- Atom/field or field-field interaction at low energy is typically made uninteresting due to dilution of participating modes.

- Controlling the density fall off can be achieved by selecting Dirichilet boundary conditions on specific surface.

- Leads to surge in correlations and cross section of processes around these resonances.

- Selective enhancement of interesting processes can alternatively be achieved at low energies

Thank you !