Waiting around for Unruh

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RQI North 2025, Naples, 23-27 June 2025

C. J. Fewster, J. Louko, L. J. A. Parry and D. Vidal-Cruzprieto (to appear)

Plan

1. **Unruh effect**
   - **Relativistic** spacetime and **analogue** spacetime
   - Circular motion in $2+1$ dimensions: **low energy** regime
2. **Quantum dot**
   - **Stationary** motion response
3. **Circular motion in 2+1 dimensions**
   - Coupling of **uniform** and **non-uniform** sign
4. **Outlook**

1. Unruh effect

Well established

- Uniformly **linearly** accelerated observer sees Minkowski vacuum as thermal, $T = \frac{a}{2\pi}$ Unruh 1976
- Weak coupling, long time, negligible switching effects
- Thermal: Observer/detector records detailed balance:

$$\boxed{\frac{P_{\downarrow}}{P_{\uparrow}} = e^{E_{\mathrm{gap}}/T}}$$

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Can cook a steak!

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Uniform **circular** motion?

- Long time in finite size lab!
- Accelerator storage rings Bell and Leinaas 1983,...
- Analogue spacetime: BEC, ^{4}He,...
  Gooding *et al.* 2020, Bunney *et al.* 2023

Aims

Why now

2+1 dimensions: effective temperature

Today

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Why now

- Analogue spacetime experiment proposals
  - Finite size lab
  - Time dilation $\leftrightarrow$ time-independent energy scaling $\rightarrow$ data analysis

Gooding *et al.* 2020
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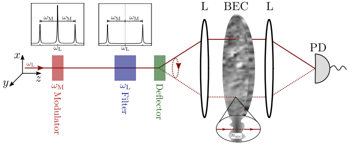

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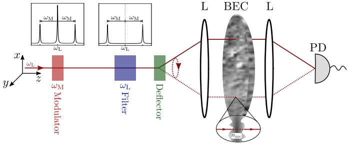
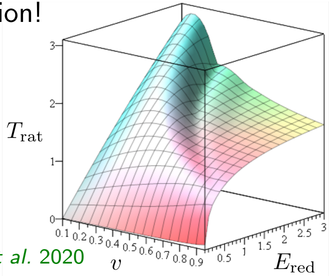


2+1 dimensions: effective temperature

Linear acceleration $\neq$ circular acceleration!
Long time limit:

- $T_{\text{rat}} := T_{\text{circ}}/T_{\text{lin}}$
- $E_{\text{red}} := E/a$ energy gap
- v orbital velocity



Biermann *et al.* 2020

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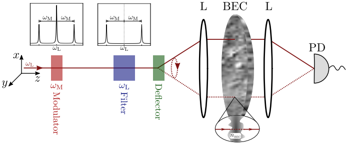

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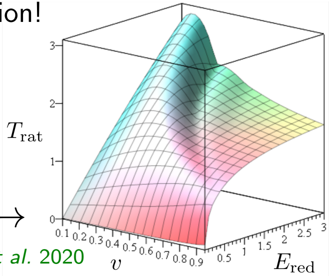

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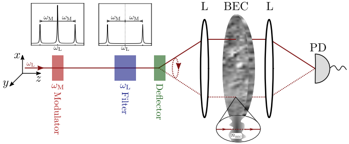

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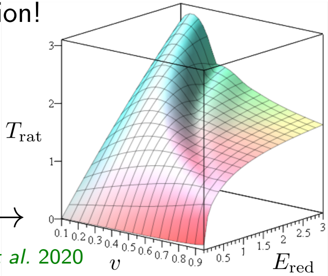

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Today

Cure: $T_{\text{rat}} \simeq 1$ **at long time** $\longleftrightarrow$ **small gap double limit**

Inspiration

Fewster, Juárez-Aubry and JL 2016

Linear acceleration Unruh effect in $3+1$:
long time $\longleftrightarrow$ **large gap** double limit

- $|E| \to \infty$ at **fixed** interaction duration: no thermality

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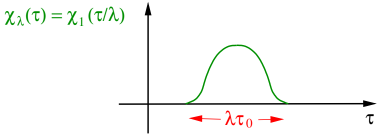
- $|E| \to \infty$ with interaction duration **polynomial** in $|E|$:

Thermality for **adiabatic** scaling
with sufficient decay of $\widehat{\chi}_1$



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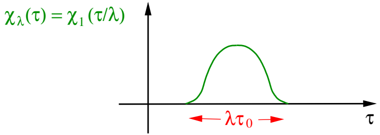
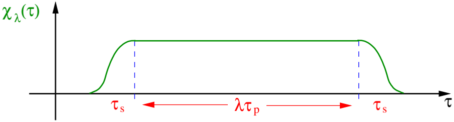


but not for **plateau** scaling
non-polynomially long time needed



2. **Quantum dot** (relativistic) Unruh(1976)-DeWitt(1979)

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Unruh(1976)-DeWitt(1979)

Quantum field

Two-state detector (atom)

Interaction

2. Quantum dot (relativistic)   Unruh(1976)-DeWitt(1979)

Quantum field

D	spacetime dimension
ϕ	real scalar field
$\lvert 0\rangle$	Minkowski vacuum

Two-state detector (atom)

$\lVert 0\rangle\rangle$	state with energy 0
$\lVert 1\rangle\rangle$	state with energy E
$\mathsf{x}(\tau)$	detector worldline, τ proper time

Interaction

$$H_{\text{int}}(\tau) = c\chi(\tau)\mu(\tau)\phi\big(\mathsf{x}(\tau)\big)$$

c	coupling constant
χ	switching function, C_0^∞, real-valued
μ	detector's monopole moment operator

Probability of transition

$$\|0\rangle\rangle \otimes |0\rangle \longrightarrow \|1\rangle\rangle \otimes |\text{anything}\rangle$$

in first-order perturbation theory:

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in first-order perturbation theory:

$$P(E) = c^2 \underbrace{\left|\langle\!\langle 0\|\mu(0)\|1\rangle\!\rangle\right|^2}_{\substack{\text{detector internals only:}\\ \text{drop!}}} \times \underbrace{F_\chi(E)}_{\substack{\text{trajectory and } |0\rangle\text{:}\\ \text{response function}}}$$

$$F_\chi(E) = \int \mathrm{d}\tau'\mathrm{d}\tau''\, \mathrm{e}^{-iE(\tau'-\tau'')}\, \chi(\tau')\chi(\tau'')\, W(\tau',\tau'')$$

$$W(\tau',\tau'') = \langle 0|\phi\big(\mathsf{x}(\tau')\big)\phi\big(\mathsf{x}(\tau'')\big)|0\rangle$$

Wightman function
(distribution)

Stationary motion response

Effective temperature

Long interaction duration

Long interaction duration as $E \to 0$?

Stationary motion response

$W(\tau', \tau'') = W(\tau' - \tau'', 0) =: W(\tau' - \tau'')$ time translation invariance

$$\Rightarrow \quad F_\chi(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\omega \, |\widehat{\chi}(\omega)|^2 \, \widehat{W}(\omega + E)$$

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$$\frac{|\widehat{\chi}(\omega)|^2}{\Delta\tau} \xrightarrow[\Delta\tau \to \infty]{} 2\pi\delta(\omega) \quad \Rightarrow \quad \frac{F_\chi(E)}{\Delta\tau} \xrightarrow[\Delta\tau \to \infty]{} \widehat{W}(E)$$

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Long interaction duration as $E \to 0$?

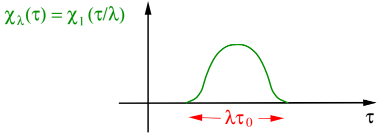
$\widehat{W}(E)$ **discontinuous** at $E = 0$ for $2 + 1$ massless field! **Ouch.**

3. $2+1$ circular motion: $\chi \geq 0$

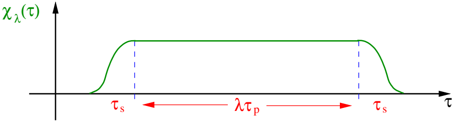
Duration **inverse power-law** long at small gap:

$$\Delta\tau \;\propto\; \lambda \;\propto\; \frac{1}{|E|^{\alpha}} \qquad\qquad \alpha > 0$$

Adiabatic



Plateau



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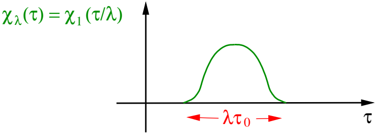

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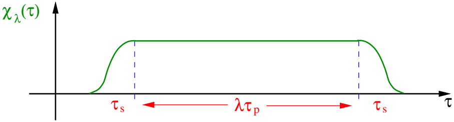

$$\Rightarrow \quad \frac{T_{\mathrm{circ}}(E)}{T_{\mathrm{lin}}(E)} \xrightarrow[|E|\to 0]{} 0$$ as **positive power-law** in $|E|$

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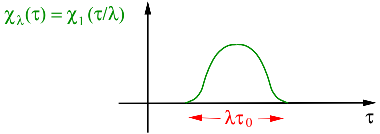

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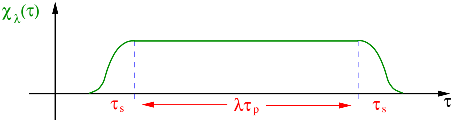

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Epic failure?

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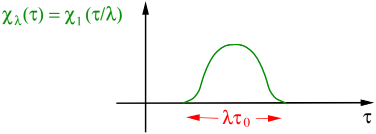

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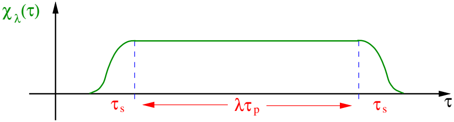

$\Rightarrow \quad \dfrac{T_{\text{circ}}(E)}{T_{\text{lin}}(E)} \xrightarrow[|E|\to 0]{} 0$ as **positive power-law** in $|E|$

Instructive

~~Epic~~ failure: comes from $\int_{-\infty}^{\infty} d\tau\, \chi(\tau) > 0$ **relax this!**

3. $2+1$ circular motion (cont'd): χ changes sign

Wish list

- $\dfrac{|\widehat{\chi}_\lambda(\omega)|^2}{\lambda} \xrightarrow[\lambda\to\infty]{} 2\pi\delta(\omega)$ long time limit

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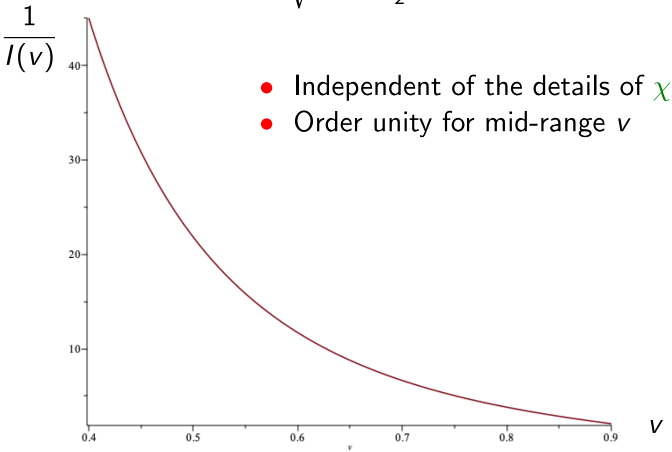
No! Make $\dfrac{|\widehat{\chi}_\lambda(\omega)|^2}{\lambda}$ a "two-bump" nascent delta:

- $0 = \widehat{\chi}_\lambda(0) = \int_{-\infty}^{\infty} \mathrm{d}\tau \, \chi_\lambda(\tau) \quad \Rightarrow \quad \chi_\lambda$ changes sign somewhere
- Scaling adiabatic (or near-adiabatic)
- See (upcoming) paper for example families of χ_λ

$$\Rightarrow \frac{T_{\text{circ}}(E)}{T_{\text{lin}}(E)} = \frac{1}{I(v)} + o(1) \text{ as } E \to 0 \text{ with } \lambda \to \infty$$

$$I(v) = \frac{4v}{\pi^2} \int_0^\infty \mathrm{d}z \left(\frac{1}{\sqrt{1 - \frac{v^2 \sin^2 z}{z^2}}} - 1 \right)$$



- Independent of the details of χ_λ
- Order unity for mid-range v

4. Summary and outlook

- **Circular acceleration Unruh effect**
  - Differs from the linear acceleration effect
  - Strongly so for **2+1 massless field at small energies**
  - Occurs in analogue spacetime
- Small energy effect recoverable in controlled long time limit with **sign-changing coupling**
- Sign change implementable in analogue spacetime