Evanescent particles

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In collaboration with
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Contents

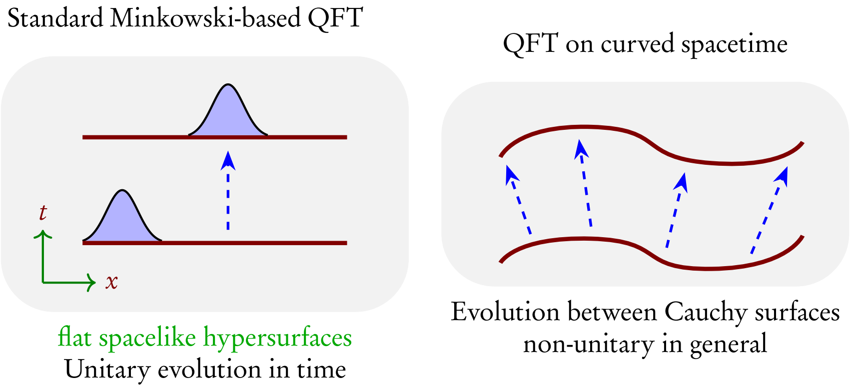
Motivation

Obstacles

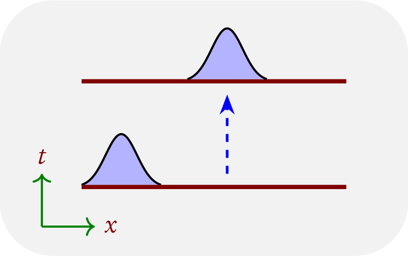
- Quantum Gravity
  - existence of both technical and conceptual difficulties in implementing a gravitational field quantization process
  - locality: cluster decomposition relies on a background metric
- Limits of standard formulation of QFT
  - perturbative techniques $\leftrightarrow$ eternal interactions (bound states, static black holes...)
  - relies on spacetime isometries $\rightarrow$ vacuum state in curved QFT ?
  - quantization of evanescent modes

Wishlist

- Locality (not just in time) $\rightarrow$ role of the boundary, composition
- Solution of QFT difficulties (selection of vacuum state in curved space, S matrix in AdS,...)
- Appropriate formulation for QG


Standard Minkowski-based QFT
t
x
flat spacelike hypersurfaces
Unitary evolution in time
QFT on curved spacetime
Evolution between Cauchy surfaces
non-unitary in general

Standard Minkowski-based QFT



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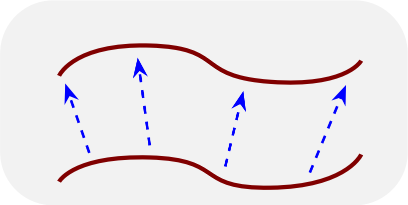
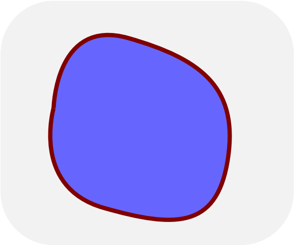


Evolution between Cauchy surfaces
non-unitary in general

GBQFT



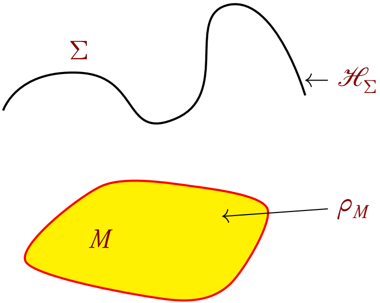
- States and space states defined on general spacetime boundary hypersurfaces
- Evolution defined *inside* the region enclosed by the boundary
- Generalization of the notion of transition amplitude

GBF: basic structures and core axioms

In the GBF algebraic structures are associated to geometric ones.

Geometric structures (representing pieces of spacetime):

- **hypersurfaces**: oriented manifolds of dimension $d-1$
- **regions**: oriented manifolds of dimension d with boundary



Algebraic structures:

- To Σ a Hilbert space $\mathcal{H}_\Sigma$
- To M a linear amplitude map $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$

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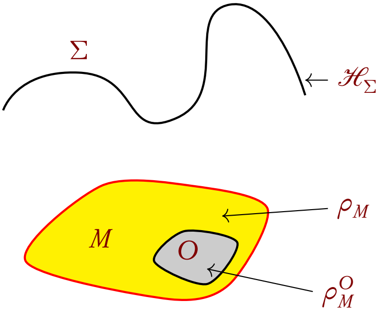

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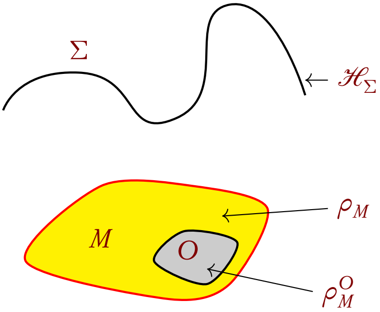

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- Set of axioms + generalization of the Born rule

Advantages of GBF

- The GBF provides a manifestly local description of quantum theory [Oeckl, 2007,2008]
- No background metric structure enters in the algebraic structures of the GBF
- The GBF offers a new perspective on quantum theory, underlying geometric aspects (holography) [DC, Oeckl, 2008; Oeckl, 2013; DC 2015; DC, Raetzel, 2013; DC, Oeckl, 2021]
- It can treat situations where quantum theory fails:
  - vacuum state in curved space [DC, Oeckl, 2019]
  - S-matrix in AdS [DC, Dohse, Oeckl, 2012; DC, Dohse, 2017]
  - QFT in the presence of static black holes
  - evanescent modes [DC, Oeckl, 2021; DC, Oeckl, Zampeli, 2024]
  - time operator in QFT [DC, Oeckl, 2025]

Vacuum state

Lagrangian subspace and quantization

- The image of the map $L_M \to L_{\partial M}$ is a **real Lagrangian subspace**: $L_M \subseteq L_{\partial M}$, meaning that it is

$$\textit{isotropic}, \quad \omega_{\partial M}(\phi,\eta)=0,\ \forall\, \phi,\eta \in L_M$$

and

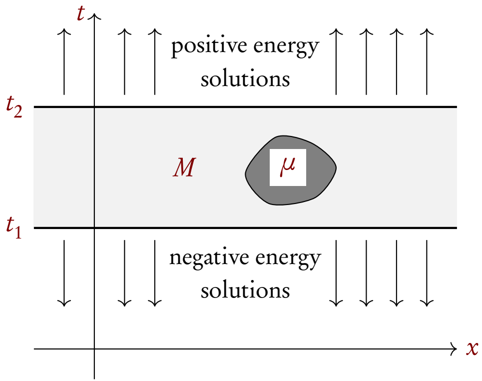
$$\textit{coisotropic}, \quad \omega_{\partial M}(\phi,\eta)=0,\ \forall\, \phi \in L_M \;\Rightarrow\; \eta \in L_M.$$

- To quantize one considers L_Σ^+, a **Lagrangian subspace** of the space of complex solutions in a neighbourhood of Σ **positive definite** w.r.t. the inner product

$$(\phi,\eta)_\Sigma = 4\mathrm{i}\,\omega_\Sigma(\overline{\phi},\eta)$$

Standard quantization

positive energy modes	$\longleftrightarrow$	positive definite Lagrangian subspace L^+
negative energy modes	$\longleftrightarrow$	negative definite Lagrangian subspace L^-



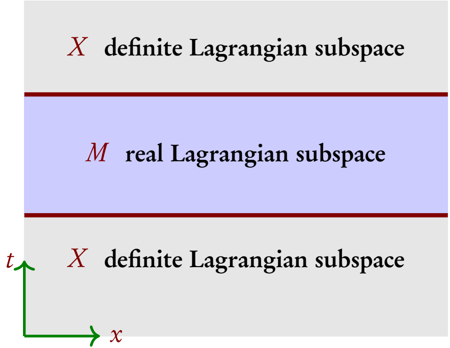
The region $M = [t_1, t_2] \times \mathbb{R}^3$ is a time-interval in Minkowski space. A source μ is located in M.

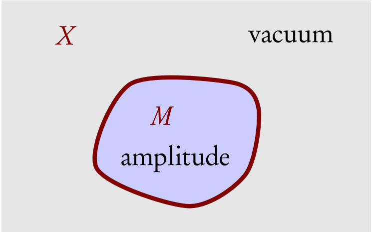
The solution η of the inhom. e.o.m., $(\Box + m^2)\eta(x) = \mu(x)$, is

- a negative energy sol. for $t \le t_1$, $\eta|_{t \le t_1} = \eta^- \in L^-$,
- a positive energy sol. for $t \ge t_2$, $\eta|_{t \ge t_2} = \eta^+ \in L^+$.

These *boundary conditions* for η are precisely tied to the choice of vacuum.

Different Lagrangian subspaces





Field propagator associated to a generic region M, in the Schrödinger rep.

$$Z_M(\varphi) = \int_{\phi|_{\partial M}=\varphi} \mathscr{D}\phi \, e^{iS_M[\phi]}$$

the integral is over the spacetime configurations ϕ that reduce to φ at ∂M.

The propagator Z_X associated to the exterior region X is interpreted as an amplitude which coincides with the vacuum wave function

$$Z_X(\varphi) = \overline{\rho_X(\varphi)} = \psi_0(\varphi)$$

Summary

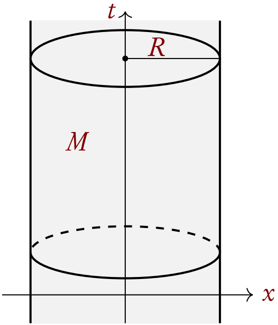
Important property

L^+ and L^- are Lagrangian subspaces of $L^{\mathbb{C}}$

vacuum state $\longleftrightarrow$ postive-definite Lagrangian subspace $\longleftrightarrow$ boundary conditions at infinity (temporal or spatial)

[DC, Oeckl, 2019]

A case study: The timelike hypercylinder



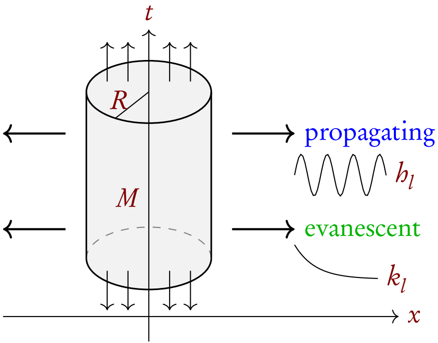
Boundary: *timelike hypercylinder*, i.e. a 3-ball B_R^3 of radius R extended over all of time

$\Longrightarrow$ **connected** and **timelike**

Complex solution space decomposes into propagating and evanescent sectors:

$$L^{\mathbb{C}} = L^{\mathrm{p},\mathbb{C}} \oplus L^{\mathrm{e},\mathbb{C}}$$

- Propagating elements:
  spherical Hankel functions h_l and $\overline{h_l}$
- Evanescent elements:
  modified spherical Hankel functions
  $k_l(z) = -\mathrm{i}^l \pi h_l(\mathrm{i}z)/2$ and $\tilde{k}_l(z) = k_l(-z)$



Propagating sector

The positive-definite and negative-definite Lagrangian subspaces

$$L^{p,+} = \{\phi \in L^{p,\mathbb{C}} \quad \text{defined by } h_l \quad \forall E \geq m\}$$
$$L^{p,-} = \{\phi \in L^{p,\mathbb{C}} \quad \text{defined by } \overline{h_l} \quad \forall E \geq m\}$$

define a Kähler polarization of $L^{\mathbb{C}}$

[DC, R. Oeckl, 2008]

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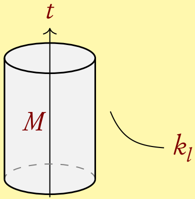
Evanescent sector

Outside the hypercylinder region, the Lagrangian subspace that reproduces the standard Minkowski vacuum state is the Lagrangian subspace

$$L^{e,+} = \{\phi \in L^{e,\mathbb{C}} \quad \text{defined by } k_l\}$$



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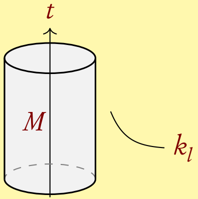


Problem

The *inner product* $(\phi, \xi)^e_{\partial M}$ for the evanescent modes is not positive definite on $L^{e,+}$!
$\Longrightarrow$ The quantum theory cannot be constructed with the traditional technique

α-Kähler quantization

It is possible to construct a Hilbert space for evanescent modes by introduction a real structure that "generalizes" the action of the complex structure.

Any real structure $\alpha : L^{\mathbb{C}} \to L^{\mathbb{C}}$ must satisfies the following properties:

- polarization interchange
- identity property: $(\alpha)^2 = \mathrm{id}$
- compatibility with the symplectic structure: $\omega(\alpha(\eta), \alpha(\zeta)) = \overline{\omega(\eta, \zeta)}$
- positive definiteness fo the inner product: $(\eta, \zeta) = 4\mathrm{i}\omega(\alpha(\eta), \zeta)$

Using the positive definite inner product a Hilbert state can now be constructed: This is called the α**-Kähler quantization**.

- Hypercyinder symmetries $\Longrightarrow$ $\boxed{\alpha \text{ is unique}}$

[DC, R. Oeckl, SIGMA, 2021]

[DC, R. Oeckl, Int.J.Mod.Phys., 2021]

Dynamics

$$G_F^{\text{propagating}} + G_F^{\text{evanescent}} = \text{ standard Feynman propagator in Minkowski}$$

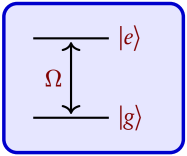
Analogous results have been obtained in Euclidean, Rindler, Milne, dS, AdS spaces.

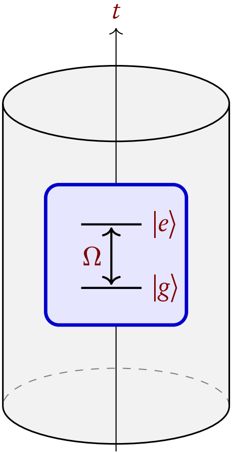
Evanescent particles

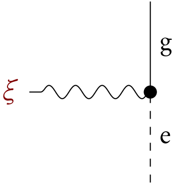
UDW detector inside the hypercylinder region

Fee Hamiltonian: $H_0 = \frac{\Omega}{2}\left(\sigma^+\sigma^- - \sigma^-\sigma^+\right)$

Interaction: $H_I(\tau) = \lambda\chi(\tau)\left(\sigma^- + \sigma^+\right)\hat{\phi}(x(\tau))$



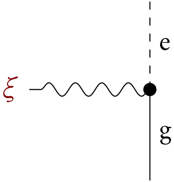




emission

outgoing particle on ∂M

$\rho^{\mathrm{UDW}_{e\to g}}(\Psi^{\mathrm{out},E})$



absorption

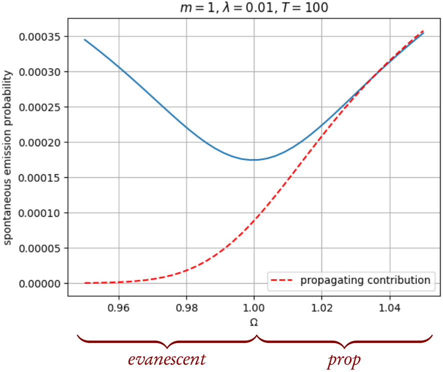
incoming particle on ∂M

$\rho^{\mathrm{UDW}_{g\to e}}(\Psi^{\mathrm{in},E})$

Comparison radial picture *vs.* temporal picture

Emission spectrum for the temporal picture

$=$ Emission spectrum for the radial picture without evanescent sector



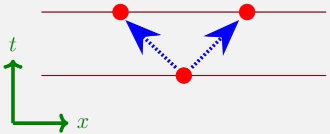
[DC, Oeckl, Zampeli, 2024]

Time
operator

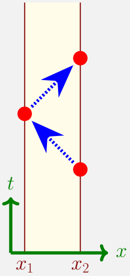
As is well known there are obstacles in the construction of a time operator in quantum theory.

The main idea is to construct a time operator analogous to the Newton-Wigner position operator.

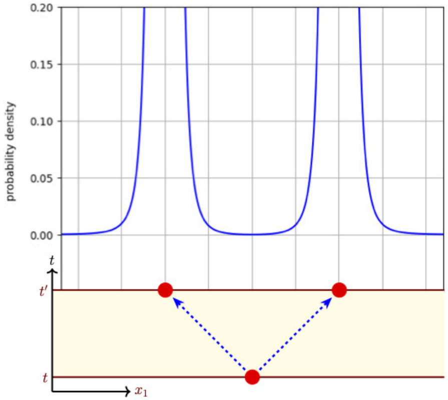
$$\hat{x}_{\mathrm{NW}} = \int \mathrm{d}^3x \, x \, a_x^\dagger a_x$$



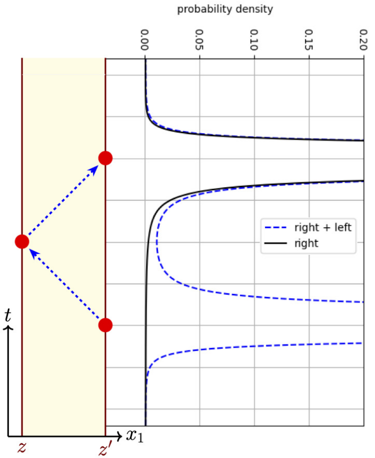
$$\hat{t}_{\mathrm{NW}} = \int \mathrm{d}^2y \, \mathrm{d}t \, t \, a_{y,t}^\dagger a_{y,t}$$



Results

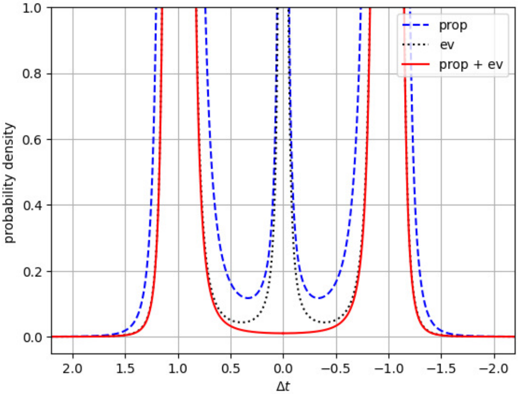


One-particle state localized on each spacelike hyperplane



One-particle state localized on each timelike hyperplane

Propagating and evanescent contribution

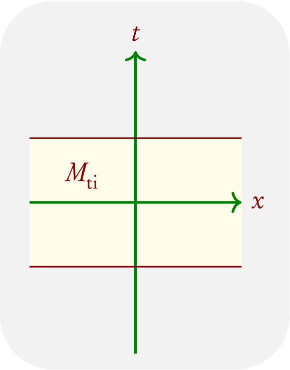


Evanescent contribution resolves the infinite probability density at $\Delta t = 0$!

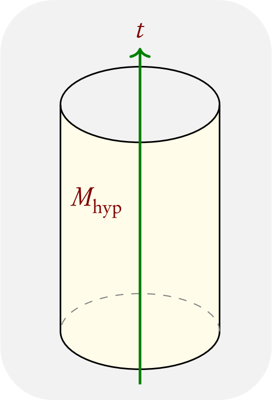
[DC, Oeckl, to appear in Found. of Physics]

Electromagnetic field

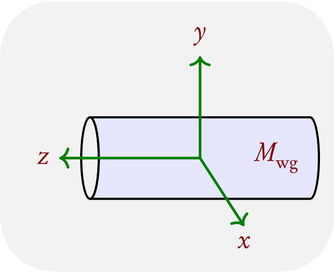
Different spacetime regions in Minkowski (Coulomb gauge)



no evanescent modes
plane waves



no evanescent modes
vector spherical harmonics



evanescent modes ✓
cylindrical spherical harmonics

Dynamics

$$\boxed{G_F^{M_{\mathrm{ti}}} \;=\; G_F^{M_{\mathrm{hyp}}} \;=\; G_F^{M_{\mathrm{wg}},\mathrm{propagating}} + G_F^{M_{\mathrm{wg}},\mathrm{evanescent}}}$$

Conclusion and outlook

- The GBF is a viable formulation for QFT
- Can handle situation where the standard formulation fails
- vacuum, evanescent particle, time operator
- Maybe an appropriate arena for QG

- To explore the physics of evanescent particles (colab. Zampeli)
- Localization of state in more general hypersurfaces
- Quantum cosmological model; linearized gravity