Conceptual differences between mechanics and field theory

Marko Vojinović

Group for Gravitation, Particles and Fields, Institute of Physics Belgrade

CA23115 RQI COST Action & CA23130 BridgeQG COST Action

INTRODUCTION

The idea of this lecture is to distinguish the ontology of field theory from the ontology of mechanics, and emphasize differences between them.

I will discuss three statements, lessons we (should) have learned from field theory:

Statement 1: There are no particles!

Statement 2: There is no Schrödinger equation!

Statement 3: There are no initial conditions!

These sound deliberately provocative, in order stimulate a discussion. . .

And remember — gravity is a field!

NO PARTICLES

Experimental confusion, due to the Unruh effect — put an ideal particle detector in an ideal vacuum:

⇒ Keep the detector at rest — it does not click.

⇒ Shake the detector around — it starts clicking!

Are particles being detected? Is the real world made out of such particles?

Theoretical confusion — two very different concepts described as a “particle”:

⇒ Electron and photon — described as delocalized plane wave solutions of field equations.

⇒ A hydrogen atom — described as a well localized kink solution of field equations.

A plane wave is not the same as a point-mass!

NO SCHRÖDINGER EQUATION

Stone-von Neumann theorem: for a system of *countably* many dofs, the Heisenberg and the Scrödinger pictures are unitarily equivalent.

Haag theorem: for a system of *uncountably* many dofs, the Heisenberg and the Scrödinger pictures are *not* unitarily equivalent.

$\Rightarrow$ Harmonic oscillator equations of motion in mechanics, in Heisenberg picture:

$$\left[\frac{d^2}{dt^2} + \omega^2\right] \hat{q}(t) = 0\,, \qquad \text{can correspond to} \qquad |\Psi(t)\rangle = e^{-i\hat{H}(t-t_0)}\,|\psi(t_0)\rangle\,,$$

$\Rightarrow$ Harmonic oscillator equations of motion in field theory, in Heisenberg picture:

$$\left[\eta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} + m^2\right] \hat{\phi}(x) = 0\,, \qquad \text{corresponds only to} \qquad \hat{\phi}(x) = \int d^4y\, G(x-y)\,\hat{\phi}(y)\,.$$

Hamiltonian $\hat{H}$ describes evolution in *time*,

while

propagator G describes evolution in *time and in space*!

NO BOUNDARY CONDITIONS

Start from an action for N identical harmonic oscillators:

$$S[q] = \int dt \sum_{k=1}^{N} q_k(t) \left[\frac{d^2}{dt^2} \qquad + \omega^2 \right] q_k(t) \,.$$

Generalize to uncountably many identical harmonic oscillators, one at each point in space:

$$S[q] = \int dt \int d^3\vec{x} \;\; q(t,\vec{x}) \left[\frac{d^2}{dt^2} \qquad + \omega^2 \right] q(t,\vec{x}) \,.$$

Compare to the action of a Klein-Gordon field:

$$S[\phi] = \int dt \int d^3\vec{x} \;\; \phi(t,\vec{x}) \left[\frac{d^2}{dt^2} - \nabla^2 + m^2 \right] \phi(t,\vec{x}) \,.$$

A purely spatial derivative (the red term) is *alien to mechanics*!

To avoid confusion, recall that ∇^2 term in the Schrödinger equation is

$$\sum_{k=1}^{n} \frac{\partial^2}{\partial q_k^2} \,, \qquad \text{which is not the same as} \qquad \sum_{k=1}^{3} \frac{\partial^2}{\partial x_k^2} \qquad (\; q \neq x \text{ !!! })$$

Initial conditions not enough, one must also deal with *spatial boundary conditions*!

THANK YOU!