

Global Fits for Lepton Colliders

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- Introduction
- Current status of projections for future colliders
- New: preliminary results for LEP3
- Roadmap for the future
- Summary

Many thanks to J. de Blas and L. Reina



INTRODUCTION

- $SU(2)_L \times U(1)_Y$ gauge symmetry hidden at low energies, but restored in the UV
 - the renormalizable SM Lagrangian is completely determined by G_F , M_Z , α (or M_W), $\alpha_s(M_Z)$, m_H , 9 fermion masses, 3 angles and 1 phase in the CKM matrix.
 - tree-level relations among weak couplings and masses corrected by finite and calculable loop corrections
 - flavour changing neutral currents absent at the tree level, finite and calculable at loop level
 - precision measurements of calculable observables
 - test the quantum structure of the SM
 - probe NP through its virtual effects

INTRODUCTION

- The effects of heavy NP in the decoupling regime can be described by higher dimensional gauge-invariant operators built with SM fields and suppressed by inverse powers of the NP mass scale
- consider the SM as an effective theory (SMEFT) valid up to the NP scale Λ :

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_5/\Lambda + \mathcal{L}_6/\Lambda^2 + \dots$$

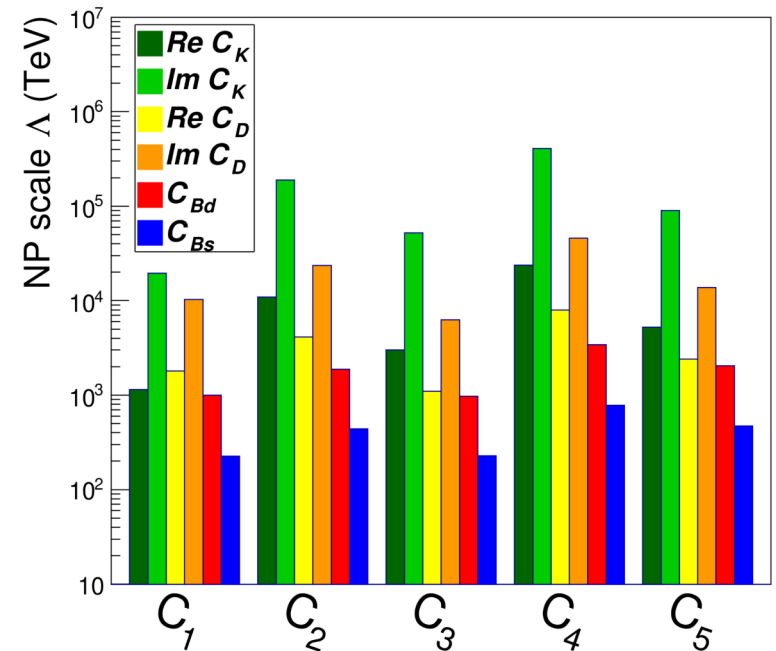
- higher dimensional operators violate accidental symmetries of the SM (B and L_i conservation) and introduce new sources of flavour and CP violation

INTRODUCTION

- Constraining SMEFT coefficients gives information on C/Λ^2 . To interpret this information assume that:
 - $\Lambda \gg$ energy scale of the process
 - $C = \text{NP coupling (} \times \text{ flavour coupling for fermions)}$
 - maximal NP coupling 4π
 - minimal NP coupling α_w
- Maximal reach on Λ for $C=4\pi \times O(1)$
- Lower bound on Λ for $C=\alpha_w \times \text{MFCPV}$

INTRODUCTION

- **B violation** probes scales $\sim 10^{15}$ GeV, ϵ_K up to 10^5 GeV, so any NP we might hope to observe directly within this century must have tiny or zero new sources of B, L, CP and flavour violation.
- For B and L conserving MFCPV NP, we are currently probing scales of $O(1-10)$ TeV.
- The lack of direct NP signals is fully consistent with the absence of indirect evidence for NP: we are efficiently probing any NP coupled to SM interactions with $O(\text{TeV})$ mass.



INTRODUCTION

- The threefold role of global fits:
 - provide the best **unbiased** probe of NP **beyond** the reach of direct detection (for specific models, model-dependent fits are always more efficient)
 - **guarantee** that we do not miss any NP **within** the reach of direct detection
 - allow for a precise determination of the couplings of any directly detected NP

INTRODUCTION

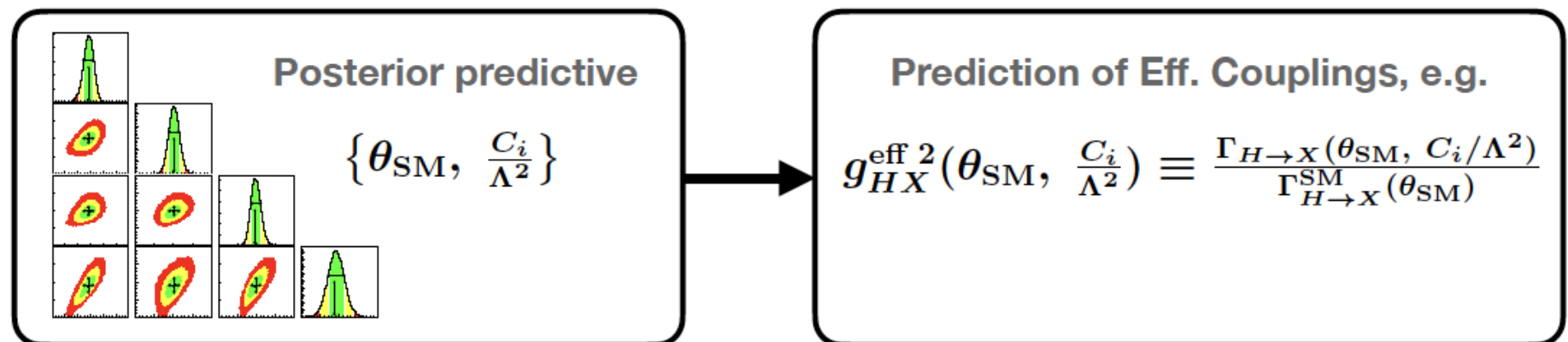
- To match one order of magnitude increase in the direct reach of future colliders, we need:
 - one order of magnitude improvement in the reach of global fits
 - two orders of magnitude improvement on the precision on SMEFT coefficients based on
 - HL-LHC
 - Future colliders

Global SMEFT fit results in FCC feasibility report

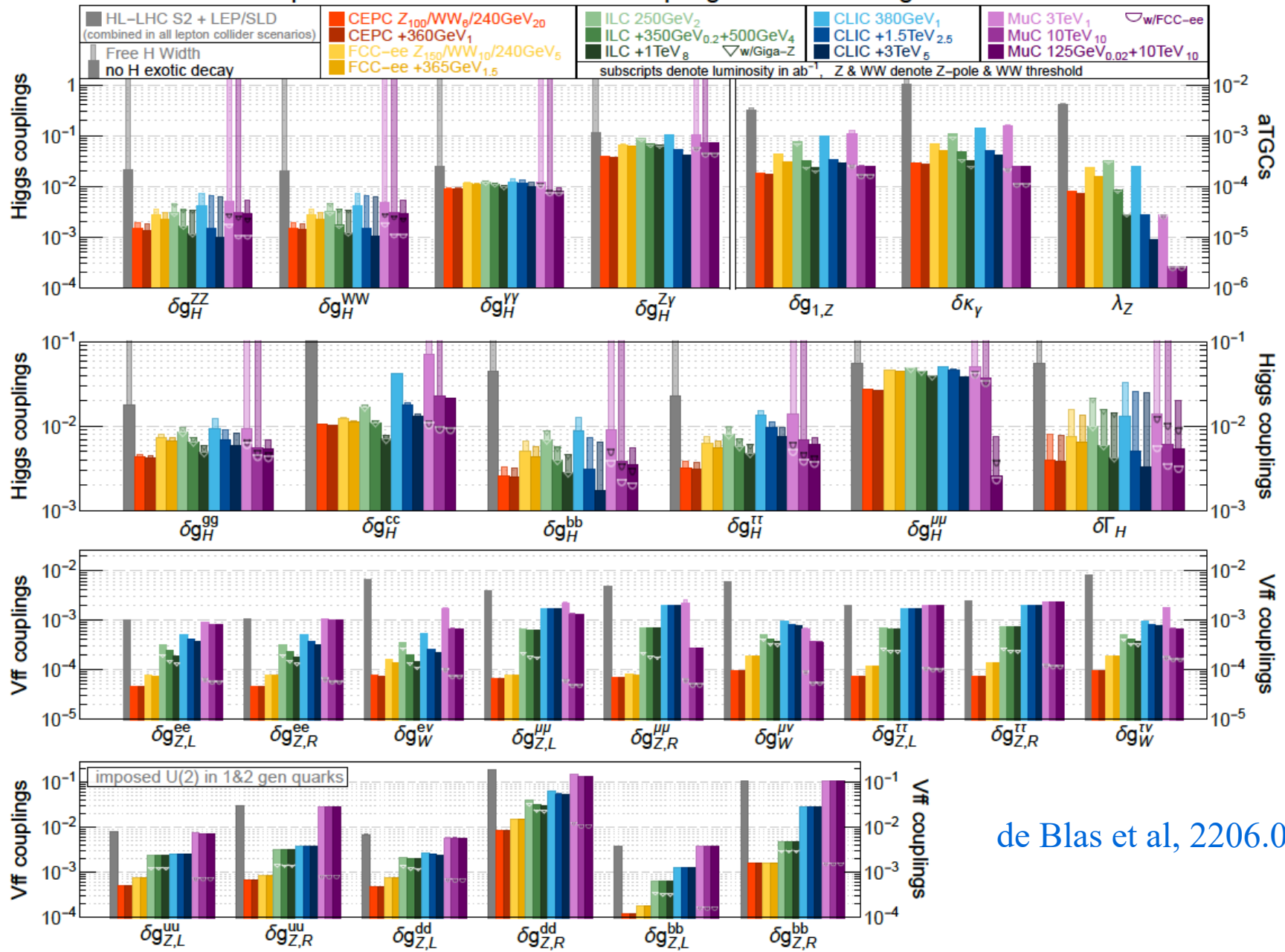
- **Starting point:** setup prepared for the FCC CDR → previous European Strategy Update → later updated in the Snowmass 2021

Designed with focus on the characterization of Higgs boson & role of EW

- ▶ LO dimension-6 SMEFT fit to EW + Higgs + (very minimal) Top
 - ▶ Limited by input available at the time of CDR/2020 ESU. Improved during Snowmass (WW) and afterwards (Top)
- ▶ Flavor assumptions: maximize exploration of deformations in Higgs and EW observables w/o FCNC
 - ▶ Non-universal Diagonal NC → SMEFT_{ND} (Cumbersome from BSM point of view)
- ▶ Bayesian fit including 5 SM + 30 SMEFT free physics parameters using **HEPfit**
- ▶ Performed in Warsaw basis ⇒ projected in terms of sensitivity to NP in “effective” SM couplings



precision reach on effective couplings from SMEFT global fit



de Blas et al, 2206.08326

VF

Effective couplings

$$g_{HX}^{\text{eff} 2} \equiv \frac{\Gamma_{H \rightarrow X}}{\Gamma_{H \rightarrow X}^{\text{SM}}}$$

$$\Gamma_{Z \rightarrow e^+e^-} = \frac{\alpha M_Z}{6 \sin^2 \theta_w \cos^2 \theta_w} (|g_{Zee,L}^{\text{eff}}|^2 + |g_{Zee,R}^{\text{eff}}|^2),$$

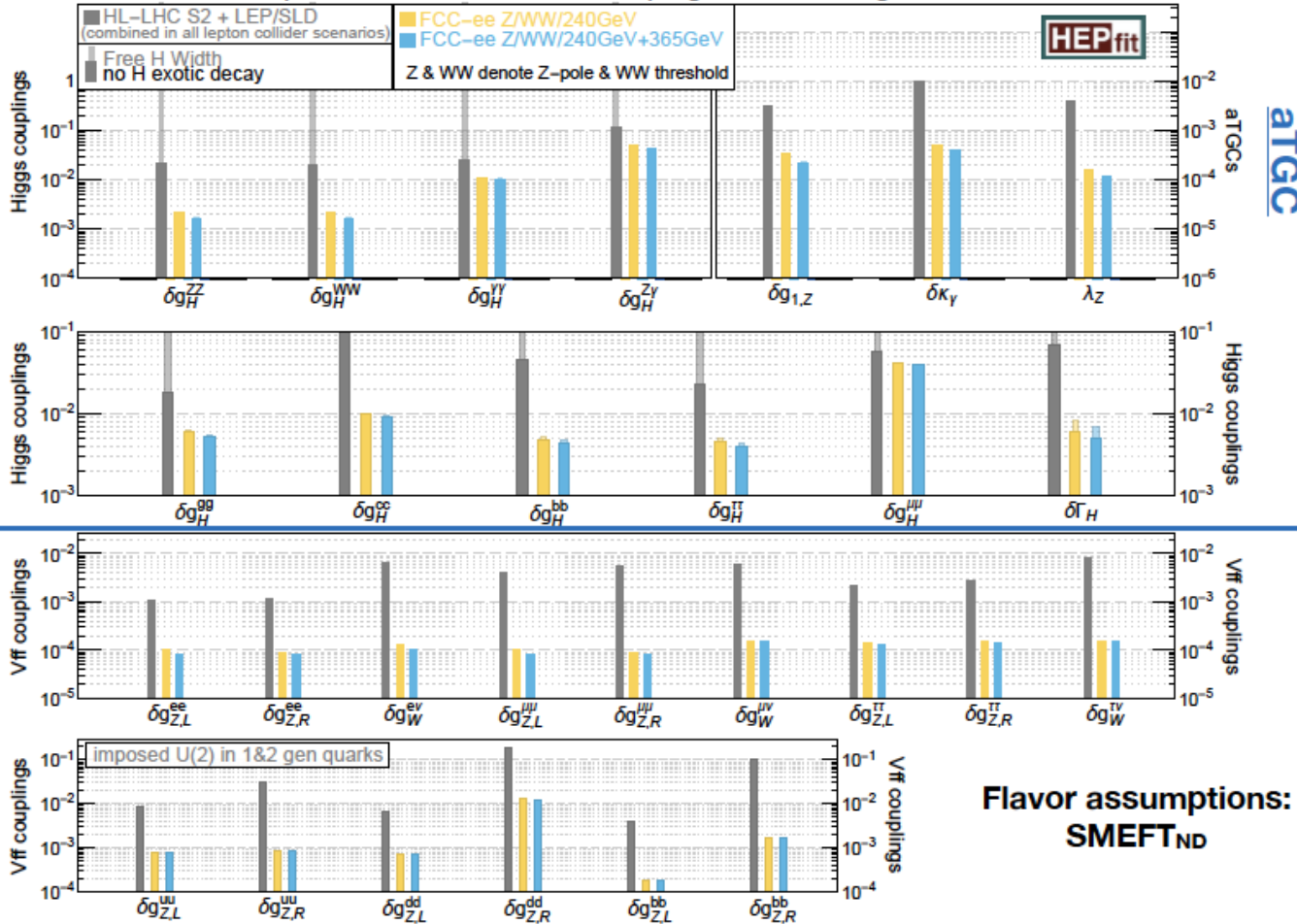
$$A_e = \frac{|g_{Zee,L}^{\text{eff}}|^2 - |g_{Zee,R}^{\text{eff}}|^2}{|g_{Zee,L}^{\text{eff}}|^2 + |g_{Zee,R}^{\text{eff}}|^2}$$

Global SMEFT fit results in FCC feasibility report

Characterization of Higgs boson & role of EW

- Updated to the current baseline (4IP) and luminosities

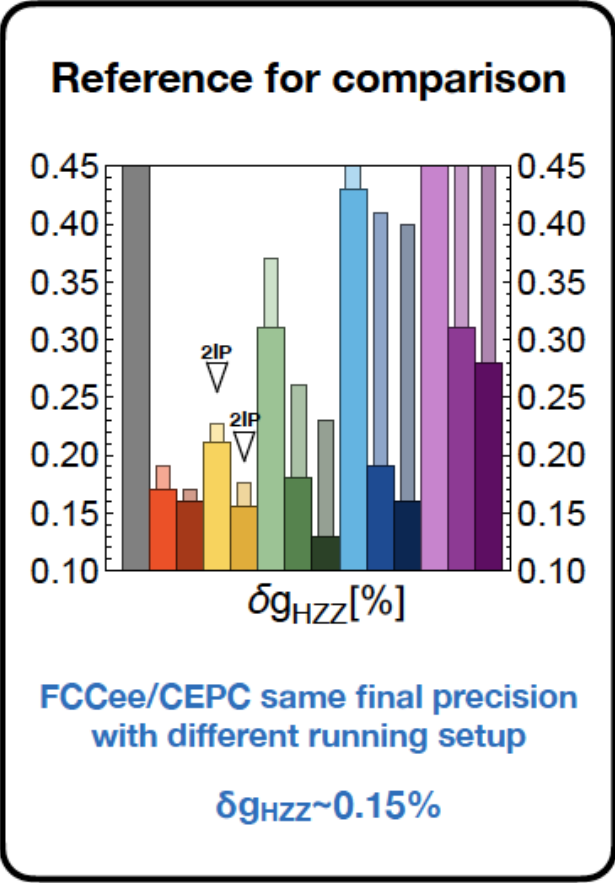
precision reach on effective couplings from SMEFT global fit



Higgs interactions

EW Vff interactions

atGC



FCCee/CEPC same final precision with different running setup

$\delta g_{HZZ} \sim 0.15\%$

Effective couplings

$$g_{HX}^{\text{eff} 2} \equiv \frac{\Gamma_{H \rightarrow X}}{\Gamma_{H \rightarrow X}^{\text{SM}}}$$

$$\Gamma_{Z \rightarrow e^+e^-} = \frac{\alpha M_Z}{6 \sin^2 \theta_w \cos^2 \theta_w} (|g_{Zee,L}^{\text{eff}}|^2 + |g_{Zee,R}^{\text{eff}}|^2),$$

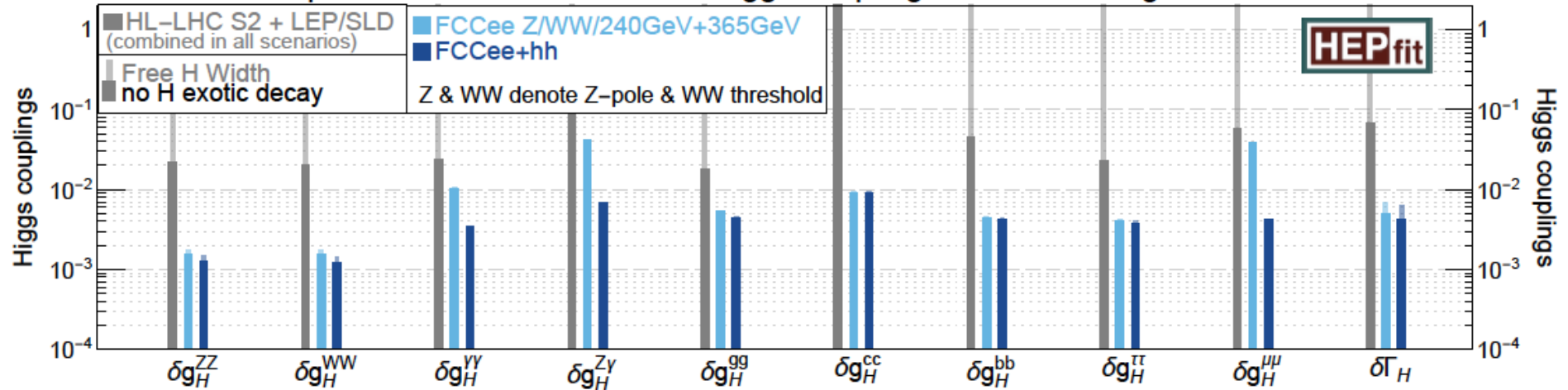
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Global SMEFT fit results in FCC feasibility report

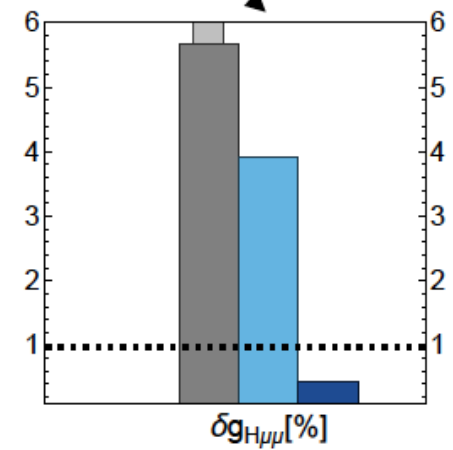
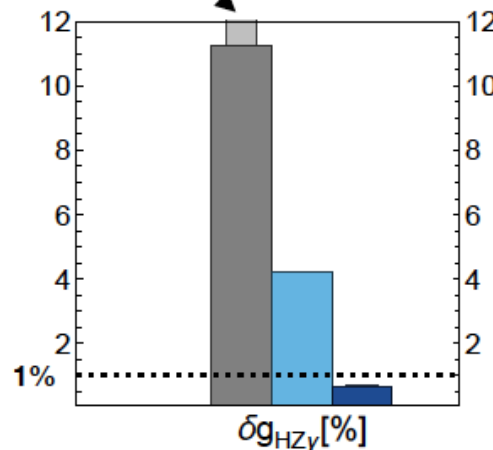
Characterization of Higgs boson & role of EW

- Updated to the current baseline (4IP) and luminosities and in combination with FCC-hh (Higgs)

precision reach on effective Higgs couplings from SMEFT global fit



Sensitivity to any BSM generating 1% corrections in any Higgs coupling



Effective couplings

$$g_{HX}^{\text{eff} 2} \equiv \frac{\Gamma_{H \rightarrow X}}{\Gamma_{H \rightarrow X}^{\text{SM}}}$$

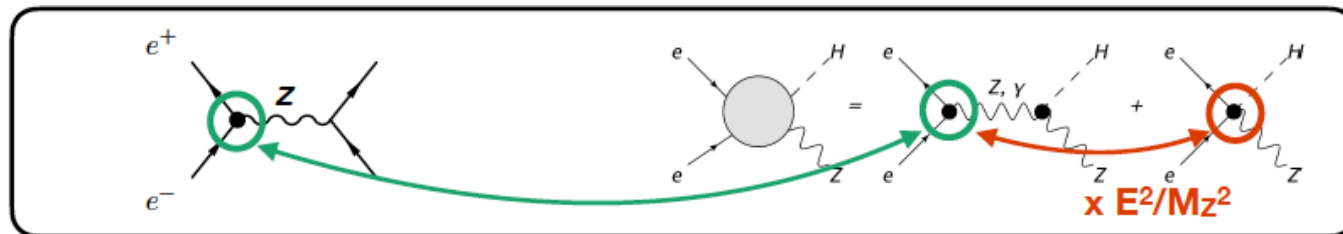
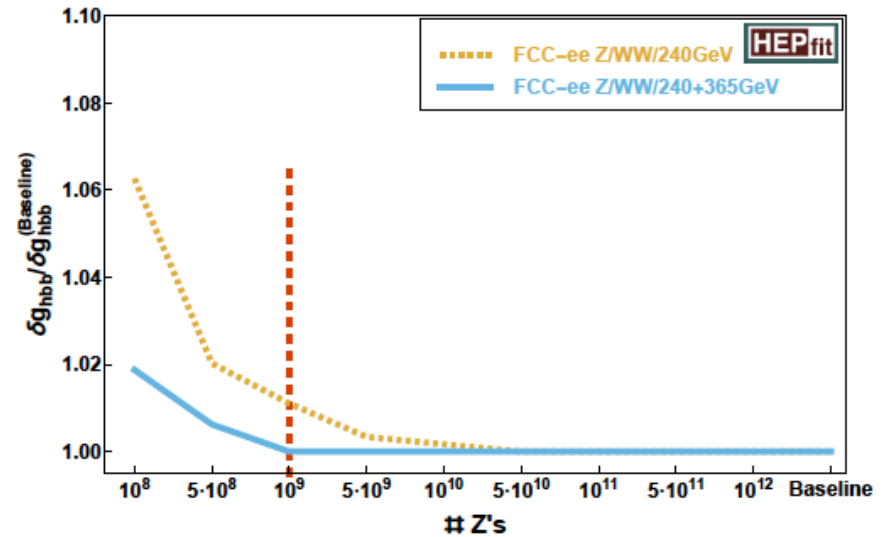
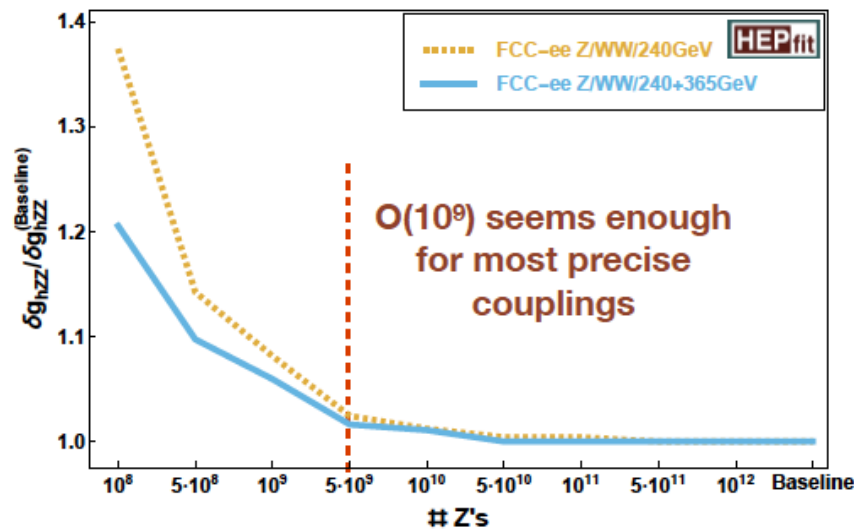
Global SMEFT fit results in FCC feasibility report

Characterization of Higgs boson & role of EW

- Made more precise the interplay between Z-pole and Higgs measurements

Influence of # Z's in Higgs precision

Degradation in Higgs precision wrt Tera Z



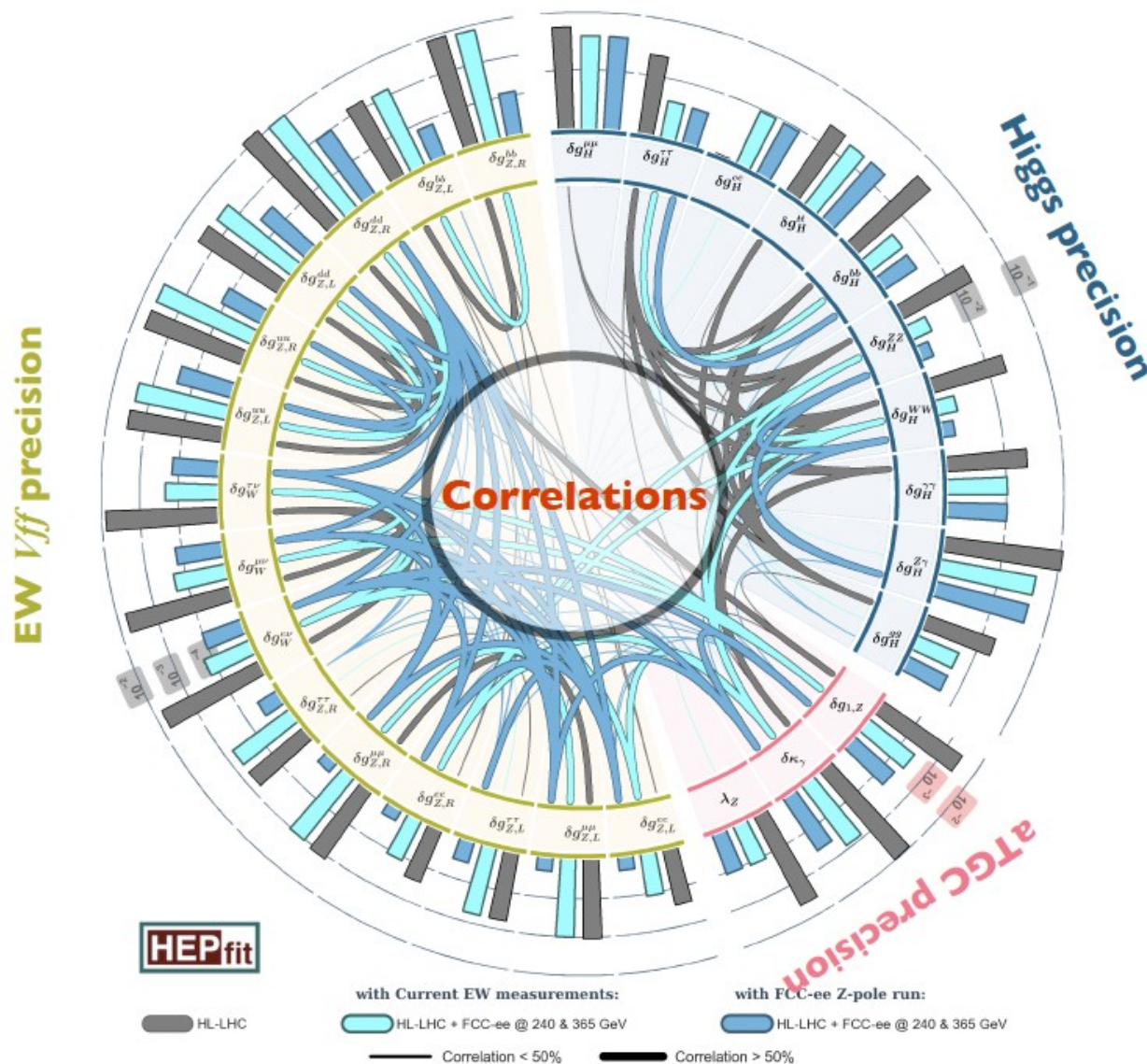
Effective couplings

$$g_{HX}^{\text{eff} 2} \equiv \frac{\Gamma_{H \rightarrow X}}{\Gamma_{H \rightarrow X}^{\text{SM}}}$$

Global SMEFT fit results in FCC feasibility report

Characterization of Higgs boson & role of EW

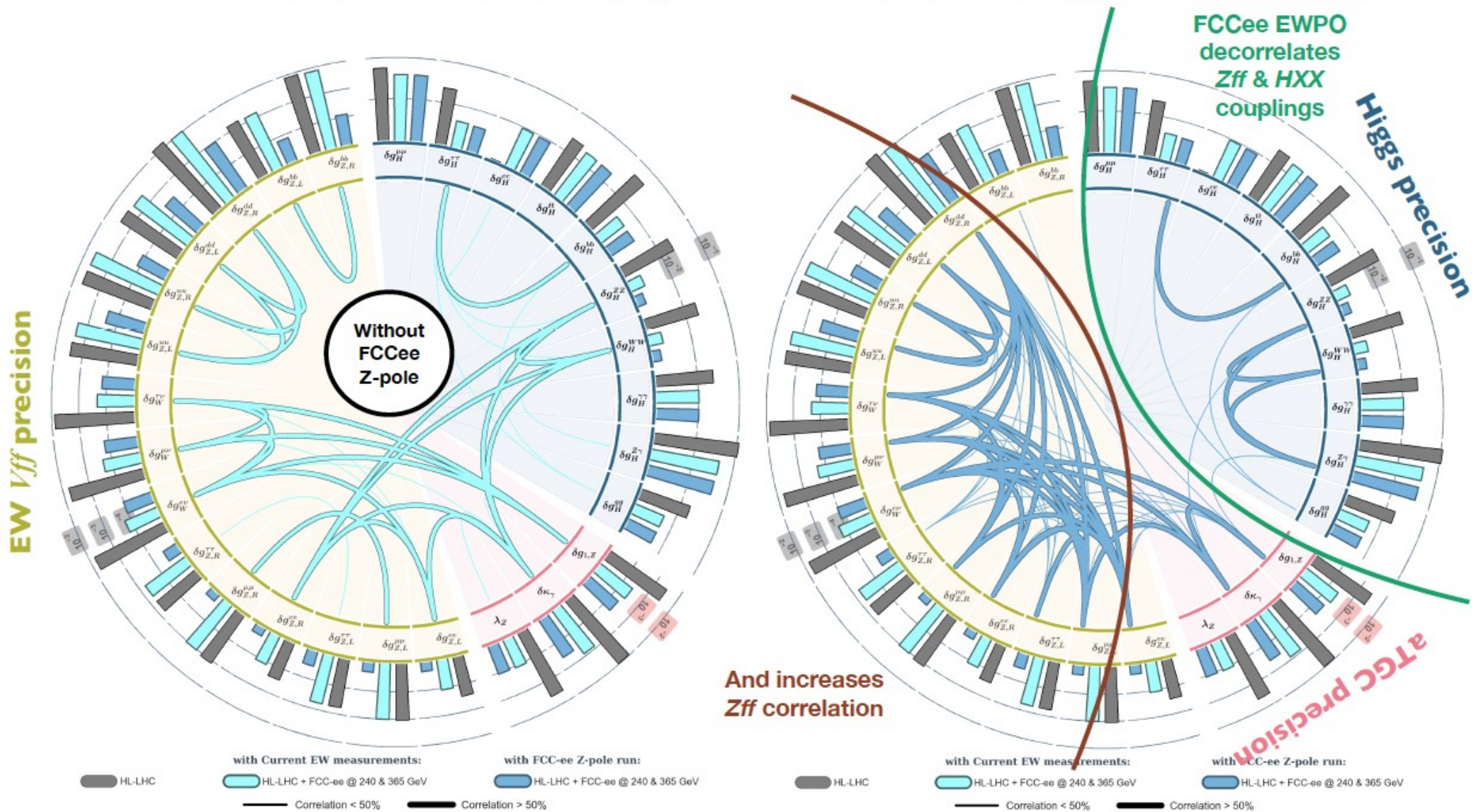
- Made more precise the interplay between Z-pole and Higgs measurements



Global SMEFT fit results in FCC feasibility report

Characterization of Higgs boson & role of EW

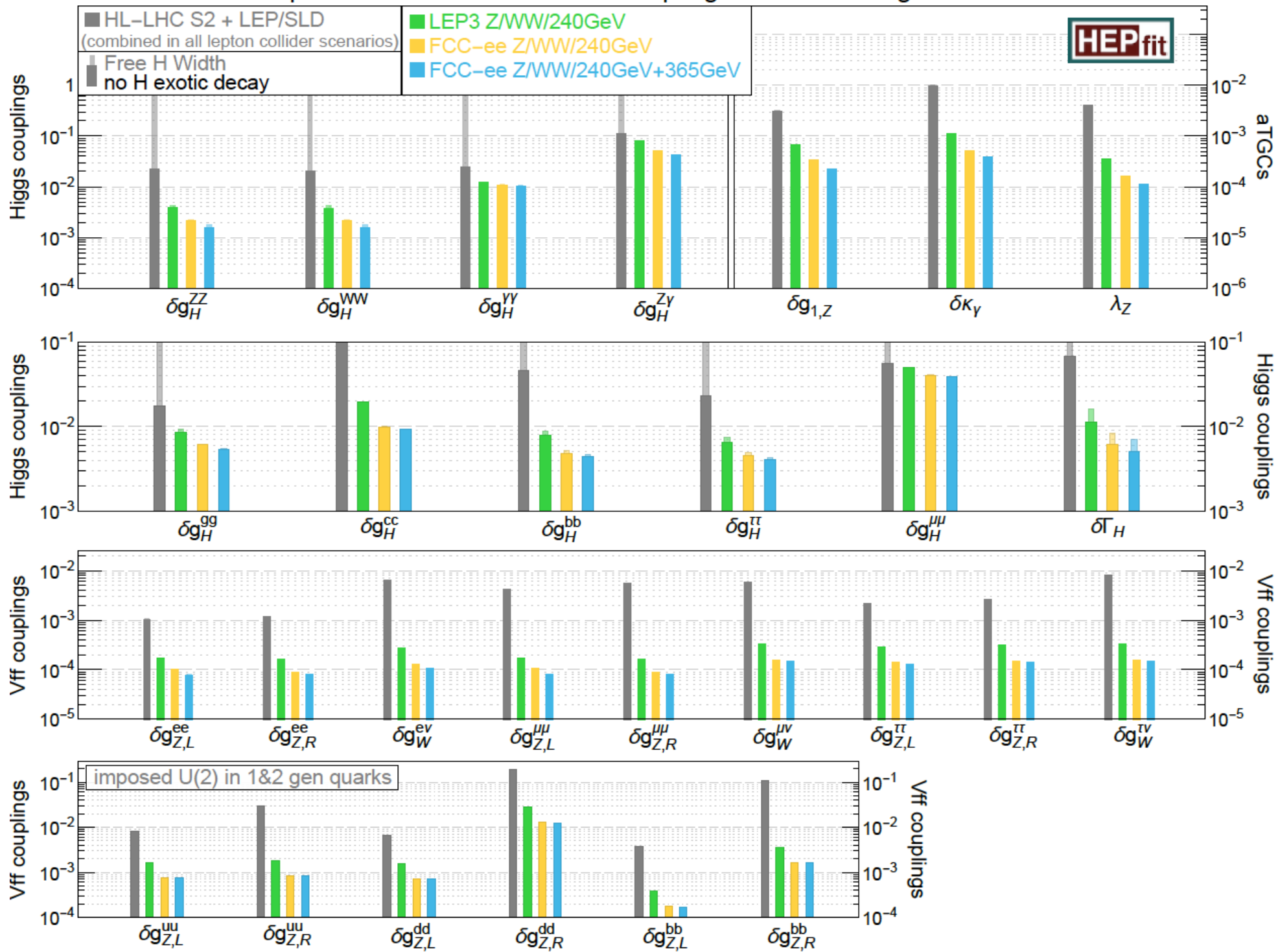
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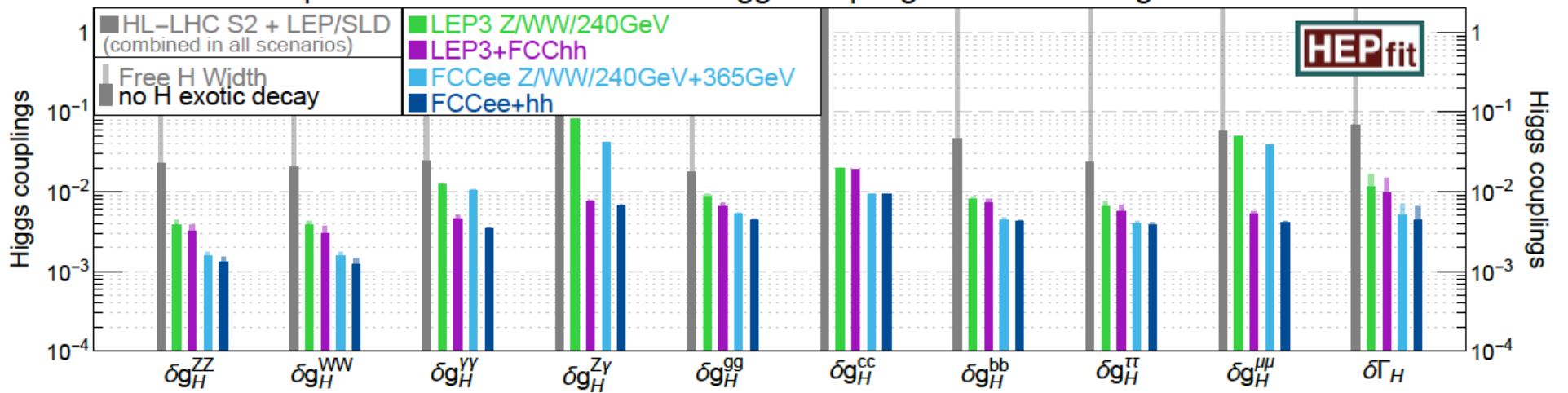
IMPACT OF LEP3

- What would be the impact of an e^+e^- machine in the LEP tunnel running at the Z pole and at 240 GeV with ~ 5 times less luminosity than FCC-ee?
- In the lack of an updated public document, very roughly scale all uncertainties by luminosity (see Marco's talk)
- Super-preliminary results (runs ended a few hours ago) - not to be taken seriously!

precision reach on effective couplings from SMEFT global fit



precision reach on effective Higgs couplings from SMEFT global fit



ROADMAP FOR FUTURE DEVELOPMENTS

- Several crucial improvements and extensions needed. On the exp side:
 - HL-LHC inputs only include signal strengths, need more projections; also need updated flavour projections including e.g. CMS B-parking, etc.
 - inputs from energy frontier very limited so far (only a few signal strengths for FCC-hh, very limited input from μ coll; impact definitely underestimated, see e.g. trilinear vs quartic h coupling)

ROADMAP FOR FUTURE DEVELOPMENTS

- Several crucial improvements and extensions needed. On the th/pheno side:
 - Add LO RG evolution from the NP scale ✓
 - LO SMEFT anomalous dimension available
 - At linear order in SMEFT, the running of SMEFT coefficients can be computed neglecting SMEFT effects in the running of SM parameters, so neglecting uncertainties on SM parameters it's a fixed 2599-dimensional matrix $U(\mu,\Lambda)$

D. Marzocca

ROADMAP FOR FUTURE DEVELOPMENTS

- Several crucial improvements and extensions needed. On the th/pheno side:
 - Work towards NLO. Full NLO requires two-loop ADM and one-loop matching and matrix elements:
 - Full NLO only possible in explicit UV completions
 - Partial NLO is scheme-dependent. Need NLO ADM to make sense of NLO matrix elements at the EW scale or below.
 - Waiting for the heroic effort of computing the NLO ADM, partial NLO is being widely considered. However, partial NLO results should be taken cum grano salis. Not implemented in HEPfit for this reason.
 - See Javi Fuentes' talk at FCC Workshop for the current status of NLO calculations

ROADMAP FOR FUTURE DEVELOPMENTS

- Several crucial improvements and extensions needed. On the th/pheno side:
 - Add flavour observables:
 - Keeping track of indirect SMEFT contributions much more difficult than in EW/Higgs/top, both from exp and th point of view. E.g. several lattice collaborations use F_π as fundamental input to fix the lattice spacing
 - In the flavour sector, power counting complicated by the presence of tiny Yukawa couplings of light fermions: need flavour symmetries, but even then indirect effects must be evaluated carefully
 - In many cases, only rough estimates of matrix elements of new operators available. Several analyses, e.g. V_{cb} , must be completely reinterpreted in terms of the SMEFT, with additional Ffs
 - Currently main line of development in HEPfit.

CONCLUSIONS

- Global fits are a crucial ingredient in our quest for NP, even more so in the absence of a preferred NP model
- First steps made for the previous strategy need to be generalized and updated, both from the exp and th point of view, in particular with the inclusion of flavour observables
- Not quite close yet to a full-fledged global analysis, even with current data and even assuming constrained flavour structure (MFV, $U(2)_5$,...)
- Intense but exciting work still needed to fully assess the impact of future machines (including HL-LHC)

BACKUP

Machine	Pol. (e^-, e^+)	Energy	Luminosity	Reference
HL-LHC	Unpolarised	14 TeV	3 ab ⁻¹	[17]
ILC	(\mp 80%, \pm 30%)	250 GeV	2 ab ⁻¹	[18]
		350 GeV	0.2 ab ⁻¹	
	(\mp 80%, \pm 20%)	500 GeV	4 ab ⁻¹	
		1 TeV	8 ab ⁻¹	
CLIC	(\pm 80%, 0%)	380 GeV	1 ab ⁻¹	[19]
		1.5 TeV	2.5 ab ⁻¹	
		3 TeV	5 ab ⁻¹	
FCC- <i>ee</i>	Unpolarised	Z-pole	150 ab ⁻¹	[20]
		$2m_W$	10 ab ⁻¹	
		240 GeV	5 ab ⁻¹	
		350 GeV	0.2 ab ⁻¹	
		365 GeV	1.5 ab ⁻¹	
CEPC	Unpolarised	Z-pole	100 ab ⁻¹	[21]
		$2m_W$	6 ab ⁻¹	
		240 GeV	20 ab ⁻¹	
		350 GeV	0.2 ab ⁻¹	
		360 GeV	1 ab ⁻¹	
MuC	Unpolarised	125 GeV	0.02 ab ⁻¹	[22, 23]
		3 TeV	3 ab ⁻¹	
		10 TeV	10 ab ⁻¹	

Table 2: Future collider scenarios considered in this work.

Quantity	current	ILC250	ILC-GigaZ	FCC-ee	CEPC	CLIC380
$\Delta\alpha(m_Z)^{-1} (\times 10^3)$	17.8*	17.8*		3.8 (1.2)	17.8*	
Δm_W (MeV)	12*	0.5 (2.4)		0.25 (0.3)	0.35 (0.3)	
Δm_Z (MeV)	2.1*	0.7 (0.2)	0.2	0.004 (0.1)	0.005 (0.1)	2.1*
Δm_H (MeV)	170*	14		2.5 (2)	5.9	78
$\Delta\Gamma_W$ (MeV)	42*	2		1.2 (0.3)	1.8 (0.9)	
$\Delta\Gamma_Z$ (MeV)	2.3*	1.5 (0.2)	0.12	0.004 (0.025)	0.005 (0.025)	2.3*
$\Delta A_e (\times 10^5)$	190*	14 (4.5)	1.5 (8)	0.7 (2)	1.5	64
$\Delta A_\mu (\times 10^5)$	1500*	82 (4.5)	3 (8)	2.3 (2.2)	3.0 (1.8)	400
$\Delta A_\tau (\times 10^5)$	400*	86 (4.5)	3 (8)	0.5 (20)	1.2 (6.9)	570
$\Delta A_b (\times 10^5)$	2000*	53 (35)	9 (50)	2.4 (21)	3 (21)	380
$\Delta A_c (\times 10^5)$	2700*	140 (25)	20 (37)	20 (15)	6 (30)	200
$\Delta\sigma_{\text{had}}^0$ (pb)	37*			0.035 (4)	0.05 (2)	37*
$\delta R_e (\times 10^3)$	2.4*	0.5 (1.0)	0.2 (0.5)	0.004 (0.3)	0.003 (0.2)	2.7
$\delta R_\mu (\times 10^3)$	1.6*	0.5 (1.0)	0.2 (0.2)	0.003 (0.05)	0.003 (0.1)	2.7
$\delta R_\tau (\times 10^3)$	2.2*	0.6 (1.0)	0.2 (0.4)	0.003 (0.1)	0.003 (0.1)	6
$\delta R_b (\times 10^3)$	3.0*	0.4 (1.0)	0.04 (0.7)	0.0014 (< 0.3)	0.005 (0.2)	1.8
$\delta R_c (\times 10^3)$	17*	0.6 (5.0)	0.2 (3.0)	0.015 (1.5)	0.02 (1)	5.6

Table 3: EWPOs at future e^+e^- : statistical error (experimental systematic error). Δ (δ) stands for absolute (relative) uncertainty, while * indicates inputs taken from current data [24]. See Refs. [9, 18, 21, 25–27].

EWPO uncertainties	Current theory error	Projected theory error	Current param. error	Projected param. error	
				Scenario 1	Scenario 2
Δm_W (MeV)	4	1	5	2.8	0.6
$\Delta \Gamma_Z$ (MeV)	0.4	0.1	0.5	0.3	0.1
$\Delta \sin^2 \theta_{\text{eff}}^\ell$ ($\times 10^5$)	4.5	1.5	4.2	3.7	1.1
ΔA_ℓ ($\times 10^5$)	32	11	30	25	7.5
δR_ℓ ($\times 10^3$)	6	1.5	6	3.2	1.3

Table 4: Impact of theory and parametric uncertainties on the prediction of a few selected EWPOs (see Ref. [66]). For the theory errors, the uncertainty estimates from currently available calculations are compared to the projected improvement when assuming the availability of N³LO corrections and leading N⁴LO corrections. For the parametric errors, current uncertainties are compared to two future scenarios, see eq. (12).

	Δm_t [GeV]	Δm_H [GeV]	Δm_Z [MeV]	$\Delta(\Delta\alpha)$	$\Delta\alpha_s$	(12)
Current	0.6	0.17	2.1	10^{-4}	9×10^{-4}	
Scenario 1	0.3	0.02	0.8	10^{-4}	5×10^{-4}	
Scenario 2	0.05	0.01	0.1	3×10^{-5}	2×10^{-4}	

in %	HL-LHC	CEPC		FCC-ee		ILC						CLIC			muon-collider		
		+Z/WW	+360	+Z/WW	+365	250		+500		+1TeV		380	+1.5TeV	+3TeV	3TeV	10TeV	10TeV +125
						Giga-Z	Giga-Z	Giga-Z	Giga-Z								
$\delta g_{H\gamma}^Z$	2.2	0.17	0.16	0.28	0.22	0.31	0.29	0.18	0.18	0.13	0.13	0.43	0.19	0.16	0.48	0.31	0.28
	-	0.19	0.17	0.31	0.25	0.37	0.35	0.26	0.25	0.23	0.23	0.56	0.41	0.4	-	-	0.39
$\delta g_{H^*W}^Z$	2.	0.17	0.15	0.28	0.22	0.32	0.31	0.19	0.18	0.14	0.14	0.44	0.21	0.17	0.49	0.31	0.28
	-	0.18	0.17	0.31	0.25	0.37	0.36	0.26	0.26	0.24	0.23	0.56	0.42	0.41	-	-	0.39
$\delta g_{H^*H}^Z$	2.5	0.91	0.89	1.2	1.1	1.2	1.2	1.1	1.1	0.98	0.97	1.2	1.1	1.	1.2	0.7	0.69
	-	0.91	0.9	1.2	1.1	1.2	1.2	1.1	1.1	1.	1.	1.3	1.2	1.1	-	-	0.74
$\delta g_{H^*H}^{\gamma\gamma}$	11.	4.	3.8	6.7	6.1	9.3	9.1	7.	6.8	6.7	6.6	10.	8.3	5.8	9.7	5.2	5.2
	-	4.	3.8	6.7	6.1	9.3	9.1	7.	6.8	6.7	6.6	10.	8.3	5.8	-	-	5.2
$\delta g_{1,Z}$	0.31	0.025	0.023	0.044	0.03	0.069	0.067	0.031	0.025	0.025	0.022	0.1	0.06	0.052	0.1	0.025	0.025
	0.31	0.025	0.023	0.043	0.03	0.069	0.067	0.031	0.025	0.025	0.022	0.1	0.06	0.052	0.1	0.025	0.025
$\delta \kappa_\gamma$	0.97	0.046	0.042	0.069	0.05	0.1	0.092	0.047	0.036	0.031	0.026	0.15	0.071	0.06	0.16	0.025	0.024
	0.97	0.046	0.043	0.069	0.05	0.1	0.092	0.047	0.036	0.031	0.026	0.15	0.071	0.061	0.16	0.025	0.025
λ_α	0.4	0.012	0.011	0.023	0.016	0.031	0.031	0.0082	0.0082	0.0028	0.0028	0.025	0.0028	0.00092	0.0027	0.00026	0.00025
	0.4	0.012	0.011	0.023	0.016	0.031	0.031	0.0083	0.0082	0.0028	0.0028	0.025	0.0028	0.00092	0.0027	0.00026	0.00026
$\delta g_{H^*H}^{\gamma Z}$	1.8	0.44	0.43	0.74	0.68	0.85	0.85	0.66	0.66	0.49	0.49	0.94	0.71	0.59	0.87	0.46	0.43
	-	0.45	0.44	0.77	0.69	0.9	0.89	0.69	0.69	0.53	0.53	1.1	0.79	0.69	-	-	0.51
$\delta g_{H^*H}^{\gamma\gamma}$	-	1.2	1.1	1.3	1.2	1.8	1.8	1.2	1.2	0.87	0.87	4.3	1.9	1.4	6.2	1.9	1.8
	-	1.2	1.1	1.4	1.3	1.8	1.8	1.2	1.2	0.9	0.9	4.3	1.9	1.5	-	-	1.8
$\delta g_{H^*H}^{\gamma Z}$	4.5	0.41	0.4	0.6	0.53	0.77	0.77	0.5	0.51	0.42	0.42	0.96	0.46	0.37	0.92	0.46	0.44
	-	0.43	0.42	0.66	0.58	0.83	0.83	0.56	0.56	0.48	0.47	1.1	0.6	0.54	-	-	0.53
$\delta g_{H^*H}^{\gamma\gamma}$	2.3	0.34	0.32	0.64	0.56	0.8	0.8	0.58	0.58	0.49	0.48	1.4	0.98	0.76	1.3	0.62	0.58
	-	0.36	0.34	0.68	0.6	0.87	0.86	0.63	0.63	0.53	0.53	1.4	1.	0.84	-	-	0.63
$\delta g_{H^*H}^{\gamma\gamma}$	5.6	2.7	2.7	4.6	4.5	4.9	4.9	4.5	4.5	4.	4.	5.1	4.7	3.8	4.9	2.5	0.24
	-	2.7	2.7	4.6	4.5	4.9	4.9	4.5	4.5	4.	4.	5.1	4.7	3.8	-	-	0.49
$\delta \Gamma_H$	6.7	0.47	0.44	0.82	0.69	1.1	1.	0.62	0.62	0.46	0.46	1.4	0.6	0.45	1.5	0.7	0.63
	-	0.61	0.59	1.1	0.98	1.5	1.5	1.1	1.1	0.94	0.93	2.3	1.6	1.6	-	-	1.3
$\delta g_{Z_L}^{\gamma Z}$	0.11	0.017	0.016	0.01	0.0083	0.036	0.027	0.03	0.023	0.028	0.023	0.061	0.051	0.046	0.095	0.085	0.085
	0.11	0.017	0.016	0.01	0.0083	0.036	0.027	0.03	0.024	0.028	0.023	0.061	0.051	0.046	0.095	0.085	0.086
$\delta g_{Z_R}^{\gamma Z}$	0.12	0.019	0.019	0.0092	0.0085	0.036	0.027	0.028	0.023	0.023	0.02	0.06	0.041	0.037	0.11	0.11	0.11
	0.12	0.02	0.019	0.0092	0.0085	0.036	0.027	0.028	0.023	0.023	0.02	0.06	0.041	0.038	0.11	0.11	0.11
$\delta g_W^{\gamma Z}$	0.65	0.01	0.0097	0.016	0.013	0.031	0.027	0.02	0.015	0.016	0.013	0.058	0.036	0.032	0.17	0.068	0.068
	0.65	0.01	0.0097	0.016	0.013	0.031	0.027	0.02	0.015	0.016	0.013	0.058	0.036	0.032	0.18	0.068	0.068
$\delta g_{Z_L}^{\gamma\gamma}$	0.42	0.019	0.018	0.011	0.0085	0.071	0.028	0.07	0.025	0.07	0.024	0.19	0.19	0.19	0.23	0.12	0.12
	0.42	0.019	0.018	0.011	0.0085	0.071	0.028	0.07	0.025	0.07	0.024	0.19	0.19	0.19	0.23	0.12	0.12
$\delta g_{Z_R}^{\gamma\gamma}$	0.55	0.019	0.019	0.0093	0.0086	0.076	0.028	0.075	0.026	0.075	0.026	0.23	0.23	0.23	0.23	0.027	0.027
	0.55	0.019	0.019	0.0091	0.0086	0.076	0.028	0.075	0.026	0.075	0.026	0.23	0.23	0.23	0.23	0.027	0.027
$\delta g_W^{\gamma\gamma}$	0.6	0.013	0.012	0.019	0.018	0.044	0.039	0.038	0.033	0.035	0.032	0.1	0.087	0.083	0.068	0.035	0.034
	0.6	0.013	0.012	0.019	0.018	0.044	0.039	0.038	0.033	0.035	0.032	0.1	0.087	0.083	0.069	0.035	0.035
$\delta g_{Z_L}^{\gamma\gamma}$	0.22	0.019	0.018	0.015	0.013	0.076	0.032	0.075	0.03	0.074	0.029	0.19	0.19	0.19	0.22	0.22	0.22
	0.22	0.019	0.018	0.014	0.013	0.076	0.033	0.075	0.03	0.075	0.029	0.19	0.19	0.19	0.22	0.22	0.22
$\delta g_{Z_R}^{\gamma\gamma}$	0.27	0.019	0.019	0.015	0.015	0.08	0.032	0.079	0.031	0.079	0.03	0.22	0.22	0.22	0.26	0.26	0.26
	0.27	0.02	0.02	0.015	0.015	0.081	0.032	0.079	0.031	0.079	0.031	0.22	0.22	0.22	0.26	0.26	0.26
$\delta g_W^{\gamma\gamma}$	0.79	0.013	0.012	0.019	0.018	0.044	0.039	0.038	0.033	0.035	0.032	0.1	0.087	0.083	0.18	0.068	0.068
	0.79	0.013	0.013	0.019	0.018	0.044	0.039	0.038	0.033	0.035	0.032	0.1	0.087	0.083	0.18	0.068	0.068
$\delta g_{Z_L}^{\gamma Z}$	0.82	0.052	0.052	0.077	0.076	0.24	0.13	0.24	0.13	0.24	0.13	0.26	0.26	0.26	0.73	0.7	0.7
	0.83	0.052	0.052	0.077	0.076	0.24	0.13	0.24	0.13	0.24	0.13	0.26	0.26	0.26	0.73	0.7	0.7
$\delta g_{Z_R}^{\gamma Z}$	3.	0.071	0.071	0.084	0.084	0.32	0.14	0.31	0.14	0.31	0.14	0.39	0.39	0.39	2.9	2.9	2.9
	3.	0.071	0.071	0.084	0.084	0.32	0.14	0.31	0.14	0.31	0.14	0.39	0.39	0.39	2.9	2.9	2.9
$\delta g_{Z_L}^{\gamma\gamma}$	0.66	0.051	0.051	0.075	0.074	0.21	0.13	0.2	0.12	0.2	0.12	0.28	0.26	0.25	0.56	0.56	0.56
	0.66	0.051	0.051	0.075	0.074	0.21	0.13	0.2	0.12	0.2	0.12	0.28	0.26	0.25	0.56	0.56	0.56
$\delta g_{Z_R}^{\gamma\gamma}$	19.	1.	1.	1.5	1.4	3.6	2.9	3.1	2.3	3.	2.2	6.8	6.	5.8	15.	14.	14.
	19.	1.	1.	1.5	1.4	3.6	2.9	3.1	2.3	3.	2.2	6.8	6.	5.8	15.	14.	14.
$\delta g_{Z_L}^{\gamma Z}$	0.38	0.013	0.013	0.017	0.017	0.063	0.034	0.062	0.033	0.062	0.033	0.13	0.13	0.13	0.38	0.37	0.37
	0.38	0.013	0.013	0.017	0.017	0.063	0.034	0.062	0.033	0.062	0.033	0.13	0.13	0.13	0.38	0.37	0.37
$\delta g_{Z_R}^{\gamma Z}$	11.	0.16	0.16	0.16	0.16	0.49	0.3	0.49	0.3	0.49	0.3	2.8	2.8	2.8	11.	10.	10.
	11.	0.16	0.16	0.16	0.16	0.49	0.3	0.49	0.3	0.49	0.3	2.8	2.8	2.8	11.	10.	10.

Table 29: Precision reach (in percentage) on effective couplings from a SMEFT global analysis of the Higgs and EW measurements at various future colliders listed in Table 2. For each coupling, the first (second) row shows the results from the constrained- Γ_H (free- Γ_H) fit. The results match those in Fig. 3.