SM phenomenology at a muon collider

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SEZIONE DI BOLOGNA

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μC and the SM calculations

μC looks like as an "EW collider" as much as the LHC is a "QCD collider".

Partons of the proton at the LHC: $q, \bar{q}, g, \gamma$

Partons of the muon at a μC: μ, γ, W, Z, ν

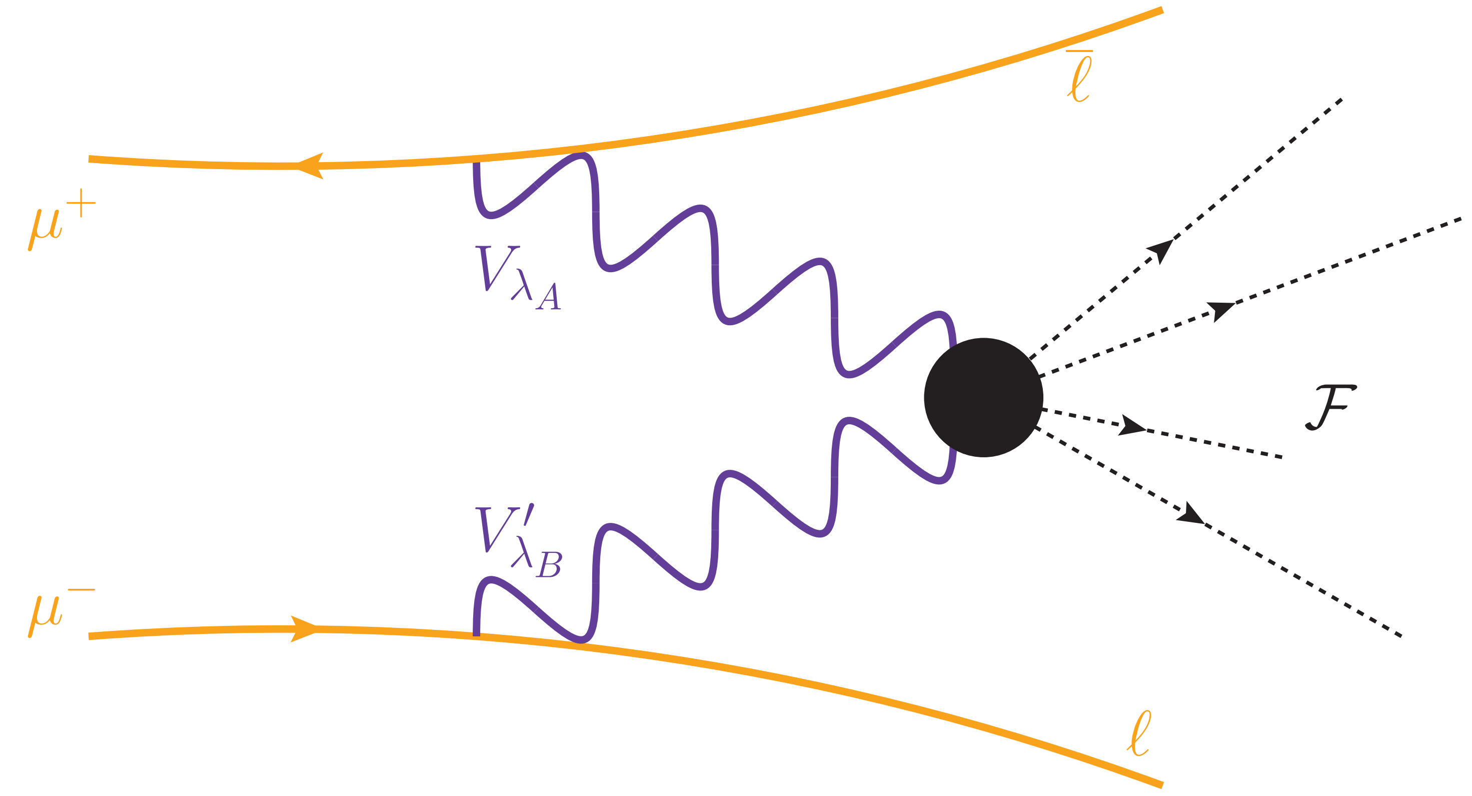
Size of NLO QCD corrections at LHC: ~ 10-100 %

Size of NLO EW corrections at a μC: ~ -(10-100) %

In other words, it looks like that **EW** is the new **QCD.** Is it?

In this talk I will try to discuss at which level this picture is helpful/correct or misleading/imprecise.

μC as VV collider



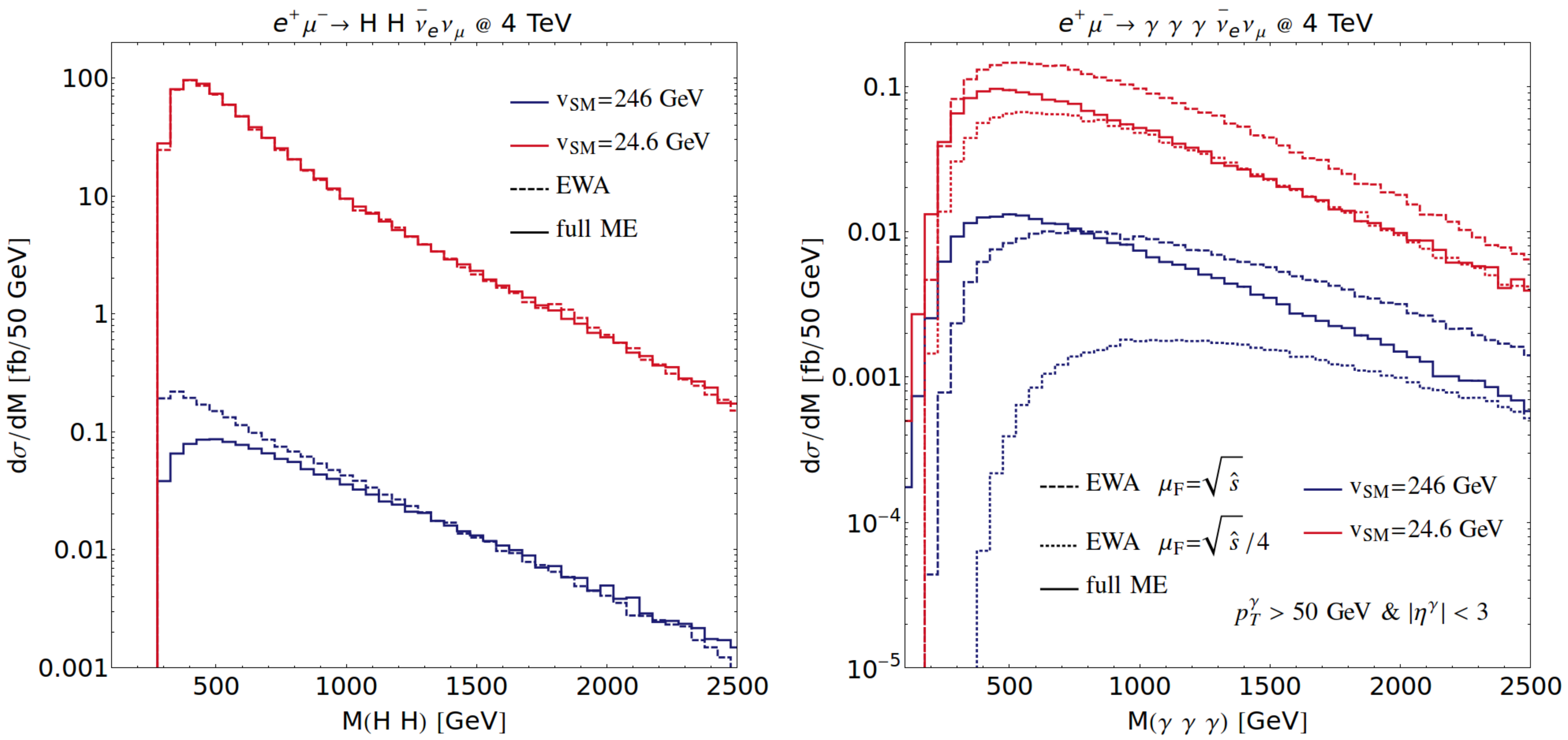
$2 \to n+2$ processes can be described as a $2 \to n$ convoluted with **EVA** (effective vector boson approx.)

Logarithmically enhanced contributions are **captured**. Additional **power-corrections** are **not** included.

Automated in MadGraph5_aMC@NLO and leads to simpler computations (scale dependent).

$$
\begin{aligned}
\sigma(\mu^+\mu^- \to \mathcal{F}+X) =\; & \tilde{f} \otimes \tilde{f} \otimes \hat{\sigma} + \text{Power and Logarithmic Corrections} \\
=\; & \sum_{V_{\lambda_A}, V'_{\lambda_B}} \int_{\tau_0}^{1} d\xi_1 \int_{\tau_0/\xi_1}^{1} d\xi_2 \int dPS_n \\
& \times\; \tilde{f}_{V_{\lambda_A}/\mu^+}(\xi_1, \mu_f)\, \tilde{f}_{V'_{\lambda_B}/\mu^-}(\xi_2, \mu_f) \\
& \times\; \frac{d\hat{\sigma}(V_{\lambda_A} V'_{\lambda_B} \to \mathcal{F})}{dPS_n} \\
& +\; \mathcal{O}\left(\frac{p^2_{T,l_k}}{M^2_{VV'}}\right) + \mathcal{O}\left(\frac{M^2_{V_k}}{M^2_{VV'}}\right) + \mathcal{O}\left(\log \frac{\mu_f^2}{M^2_{V_k}}\right).
\end{aligned}
$$

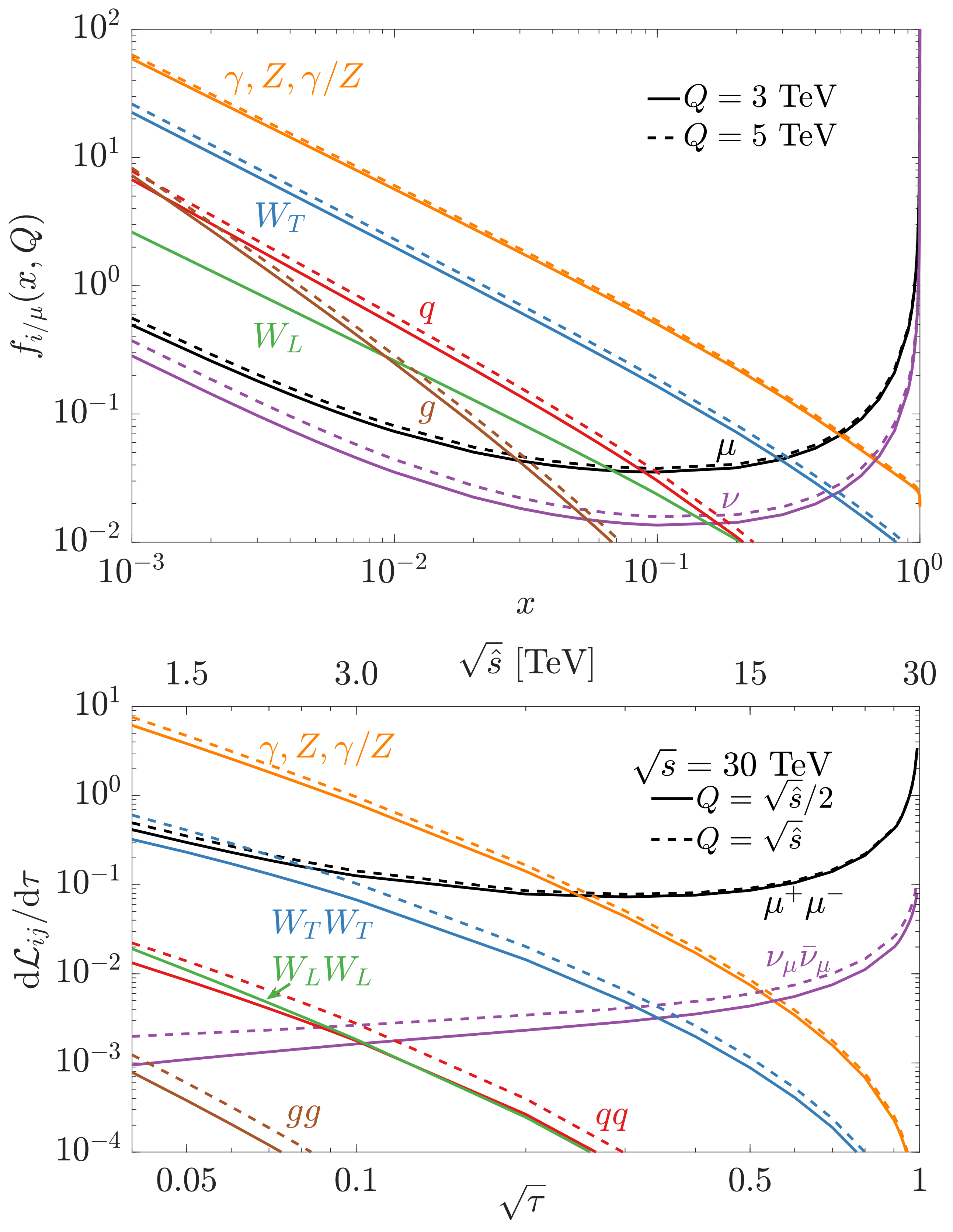
Ruiz, Costantini, Maltoni, Mattelaer '21



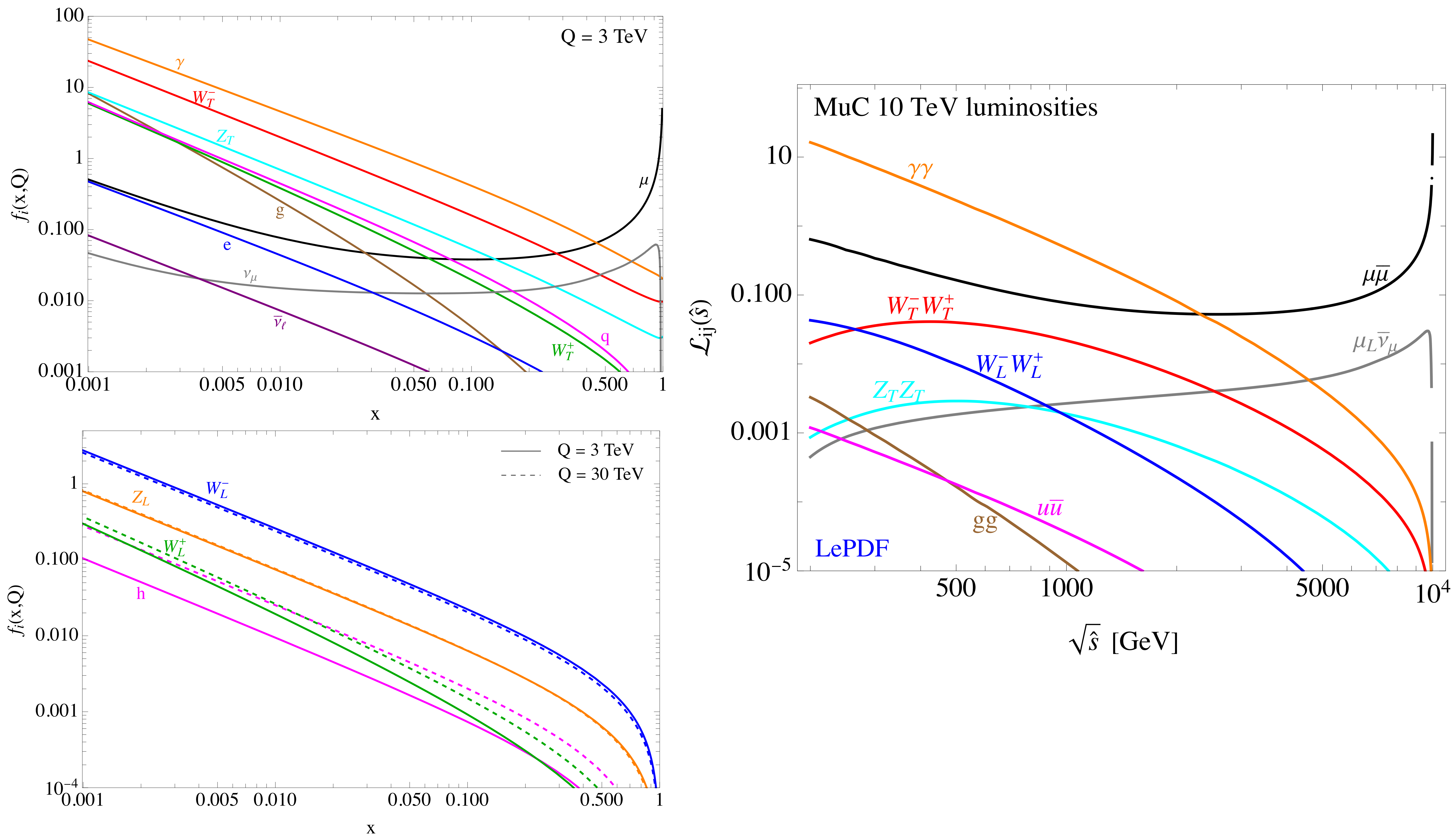
EW: from EVA to PDFs of the muon

Having logs from EW splittings, it is natural to think about EW PDFs and resum these logs.

EW Leading Log PDFs of the muon are available.

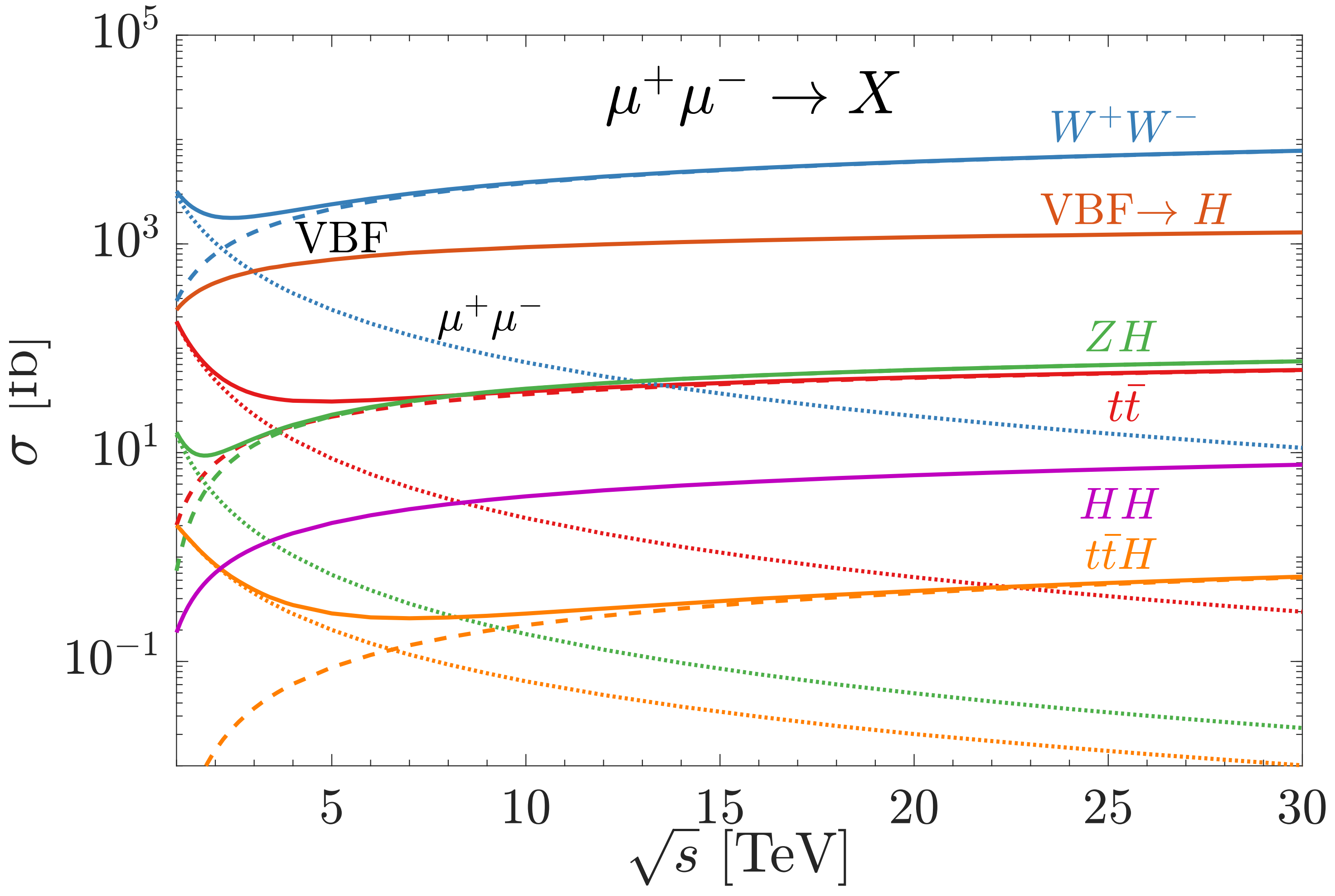


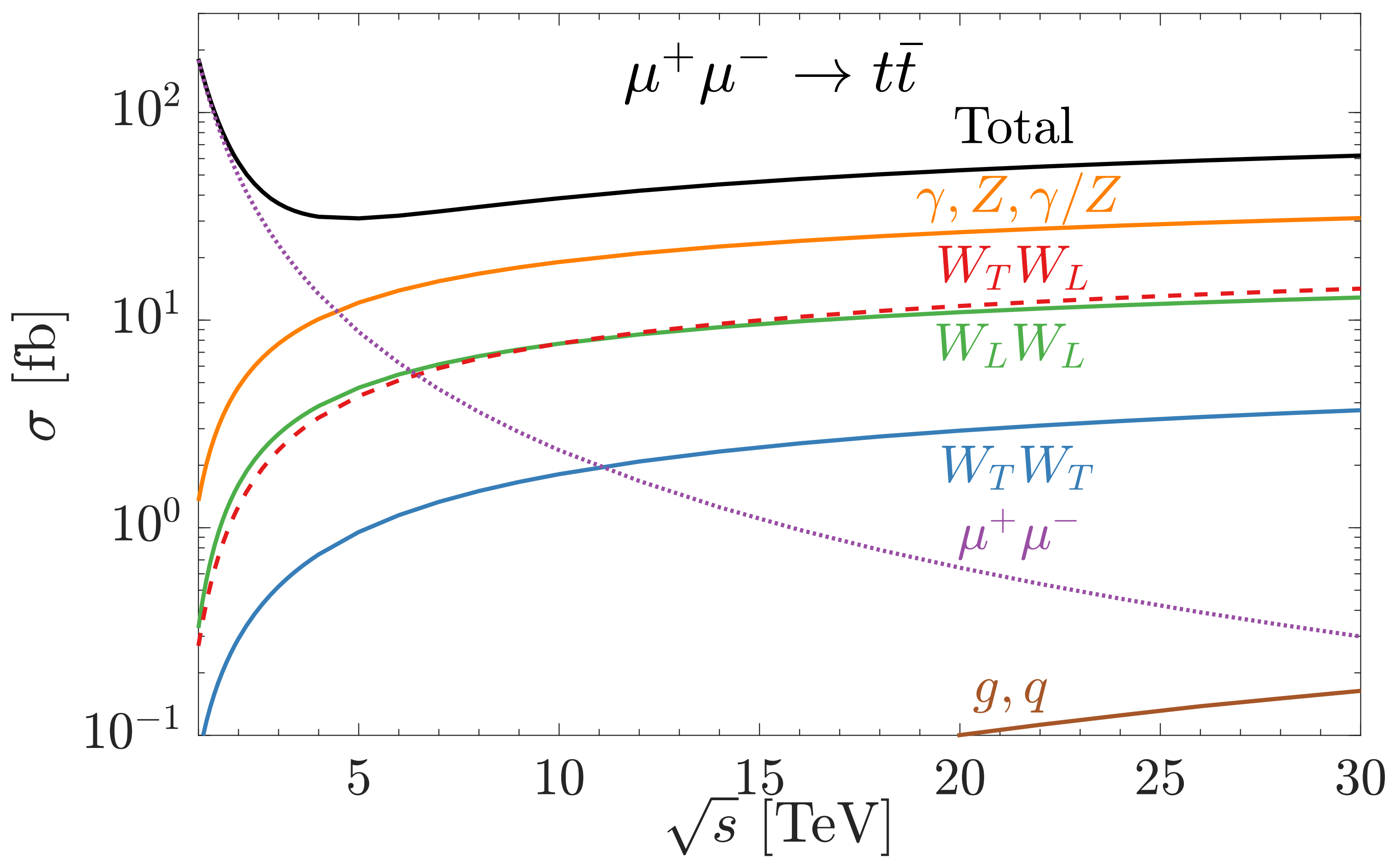
Han, Ma, Xie '20

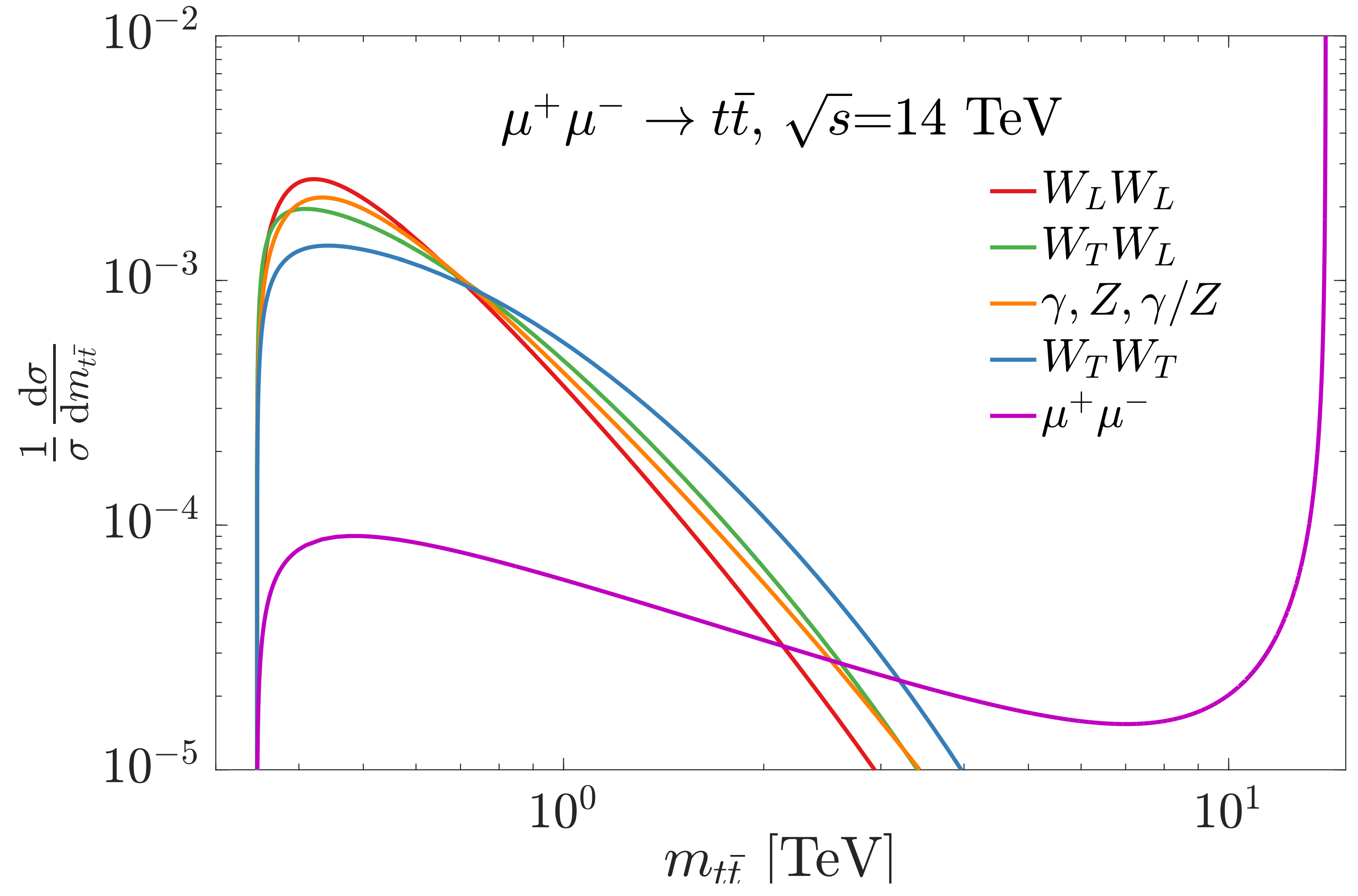


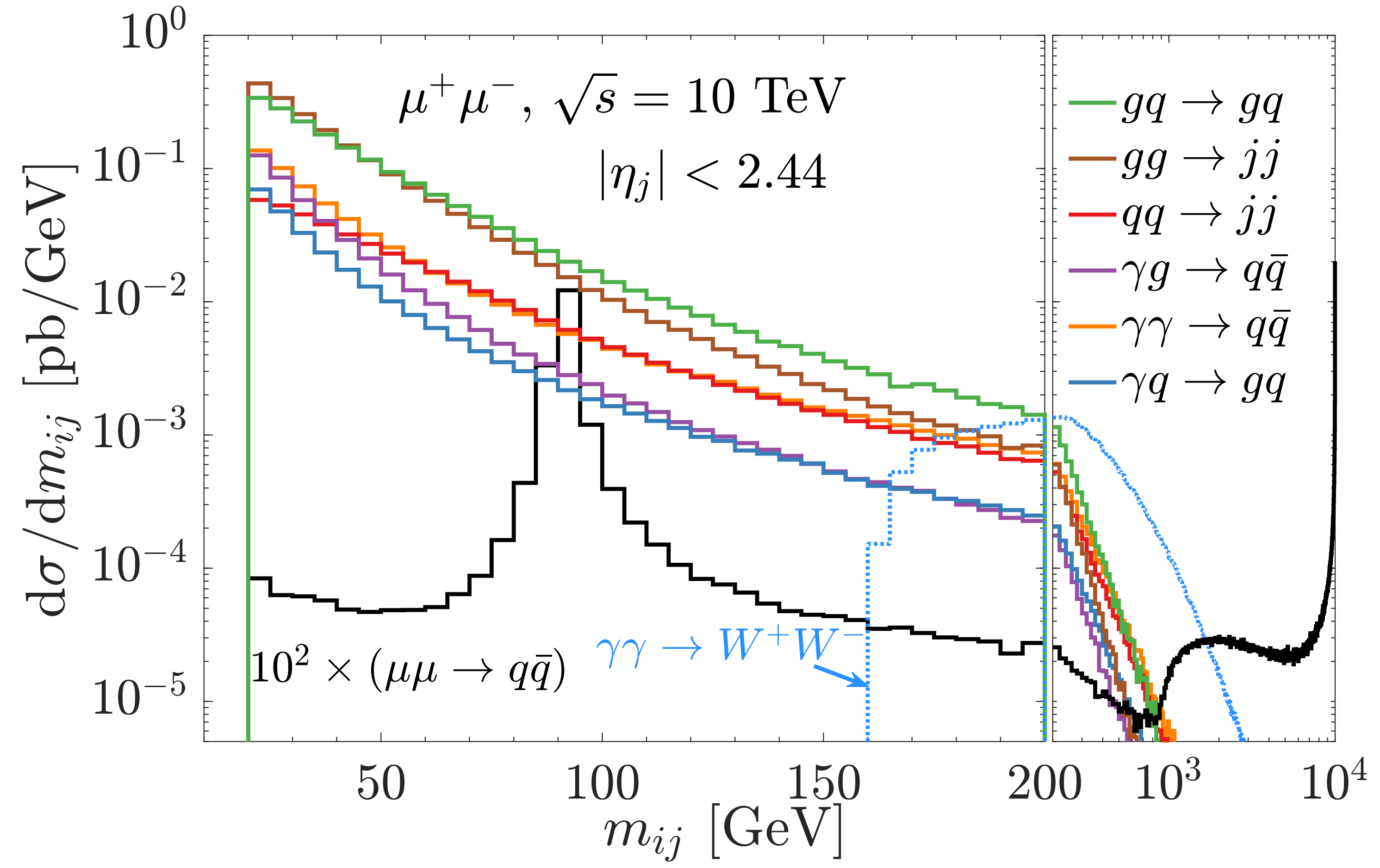
Garosi, Marzocca, Trifinopoulos '23

Phenomenology with PDFs









Han, Ma, Xie '20, '21

At large energies, VBF configurations become the dominant production mode.

Given S, for $m(X) \ll S$ VBF is dominant, while for $m(X) \simeq S$ the direct production is dominant.

Phenomenology with EW PDFs: open questions

How do we calculate NLO EW corrections with EW PDFs?
We need EW factorisation counter-terms in order not to double-count the logs. Anyway, also with them, current PDFs are LL-accurate. Therefore an NLO EW calculation would lead to an artificially small scale-dependence.

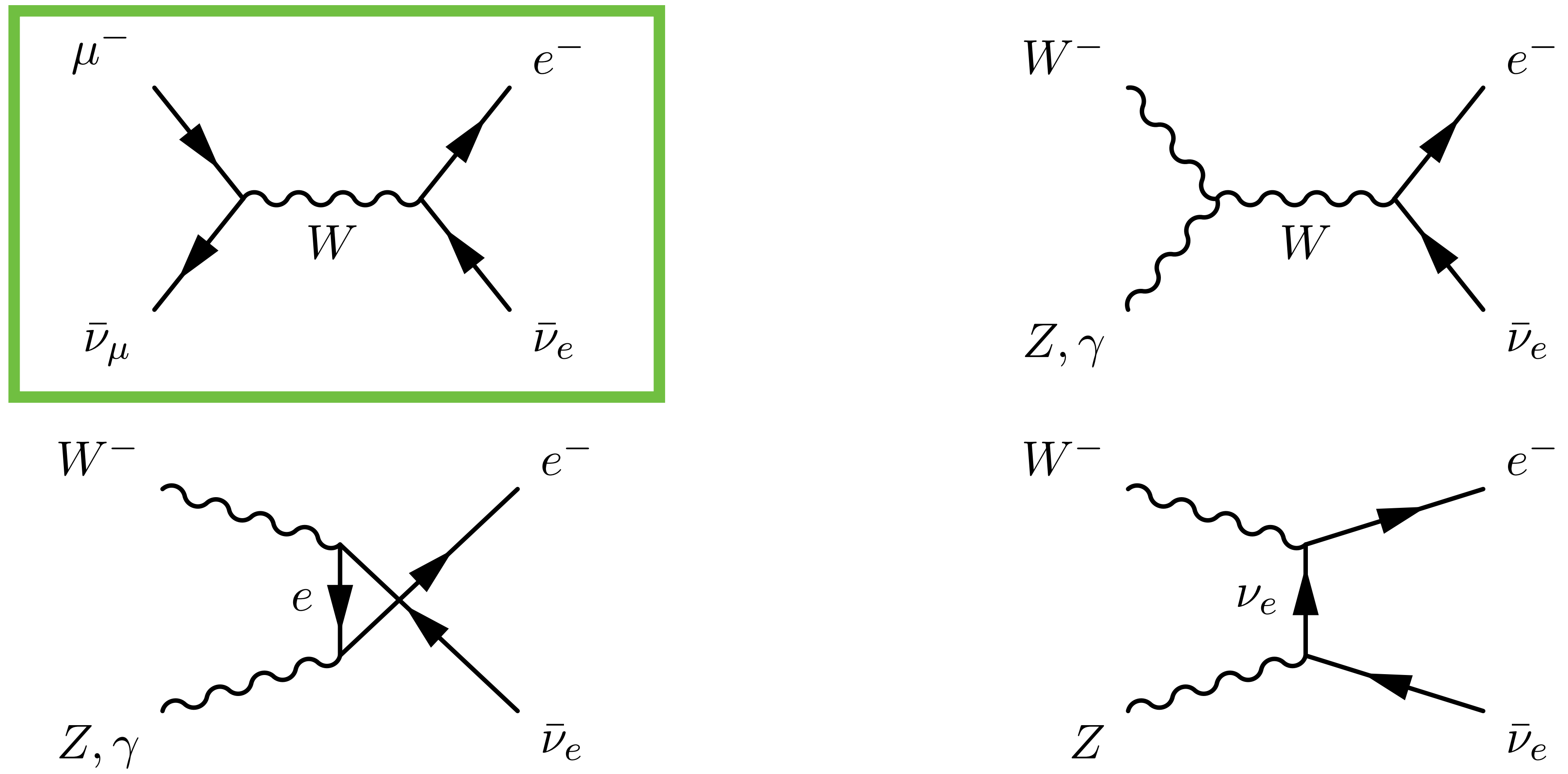
Is more important resumming logs or taking into account power corrections?
Besides the case of photon initial state, it is not obvious that VBF calculated with PDFs is superior w.r.t. exact matrix-element calculations.

How do we improve the accuracy?
For $m(X) \simeq S$, the muon PDF in the muon is the dominant one. We see it later in the talk.
For $m(X) \ll S$ answer to previous questions are crucial: estimate scale unc. and EW corrections.

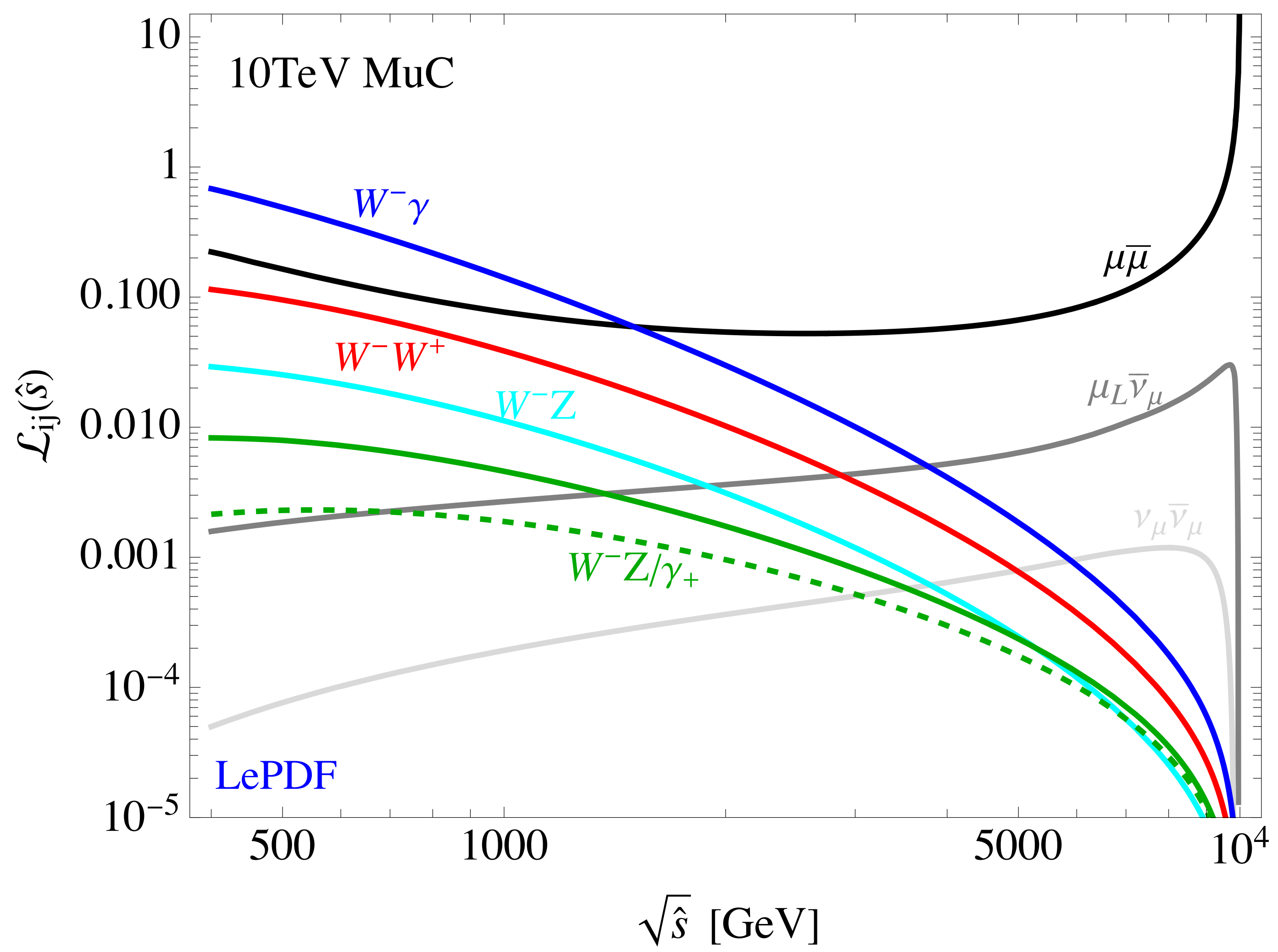
Are new channels appearing without PDFs?

Should be only QCD and QED involved in the PDFs but not the Weak component?

An example: $e^-\bar{\nu}_e$ production



With EW PDFs



Without EW PDFs and only ME.

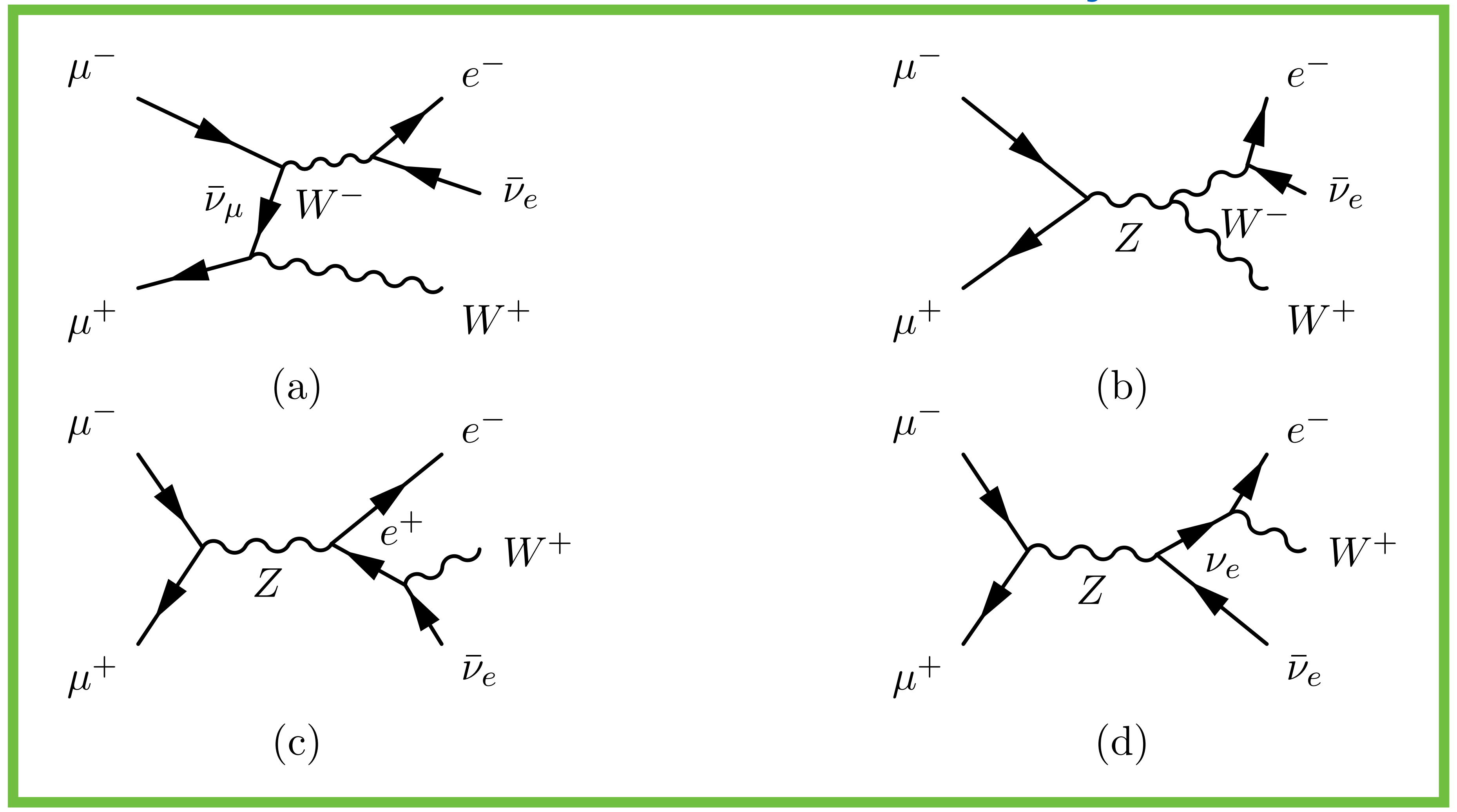


Figure 7. Leading-order Feynman diagrams for $\mu^-\mu^+ \to e^-\bar{\nu}_e W^+$.

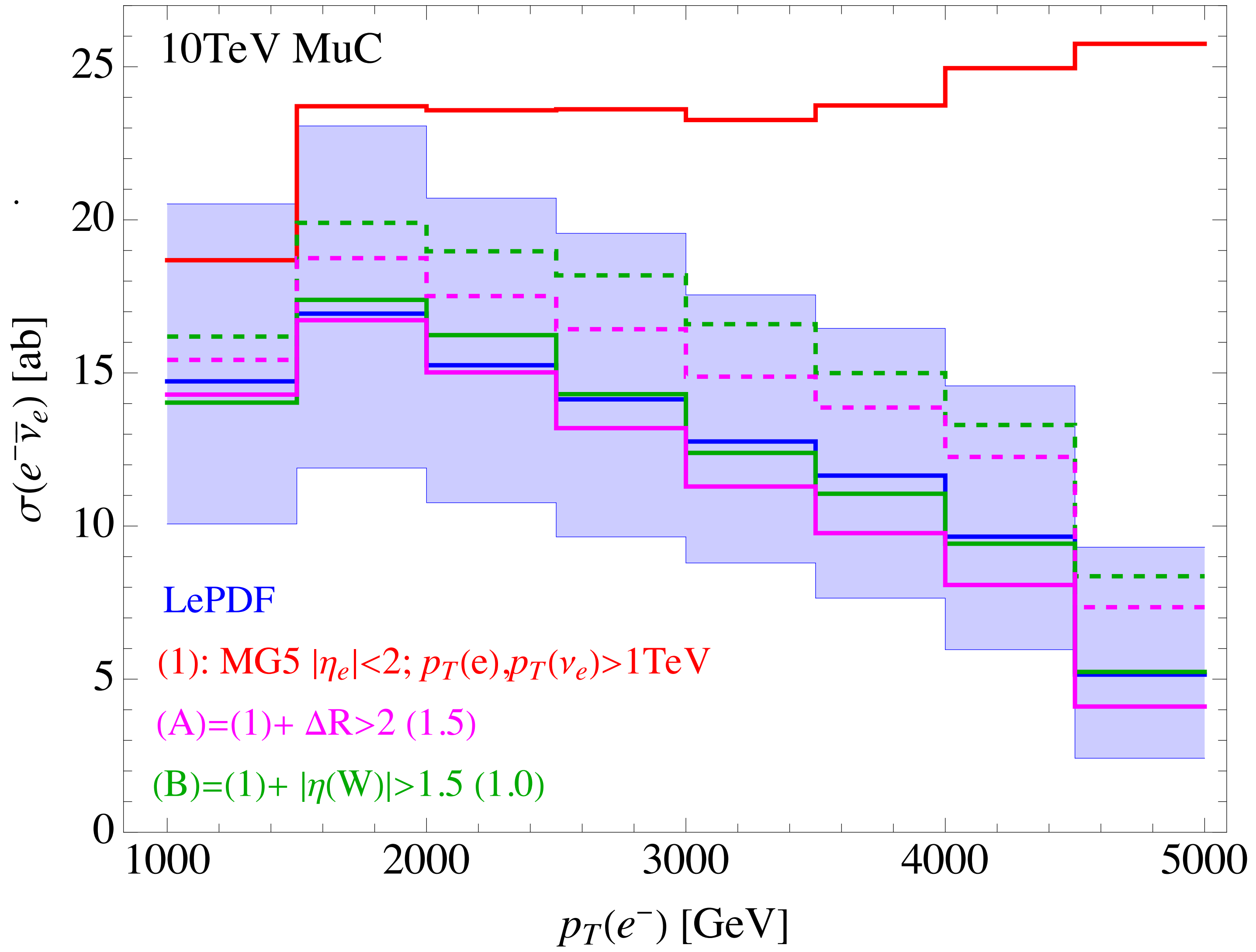
COMPARISON:

$|y_e| < 2\,, \quad p_T^e > 1\,\text{TeV}\,, \quad p_T^\nu > 1\,\text{TeV}\,, \quad M(e,\nu_e) > 500\,\text{GeV}\,.$

(A) : $\Delta R(i,j) > 2$ or 1.5

(B) : $|\eta_W| > 1.5$ or 1.0

Agreement within (large scale unc.) only for specific cuts.



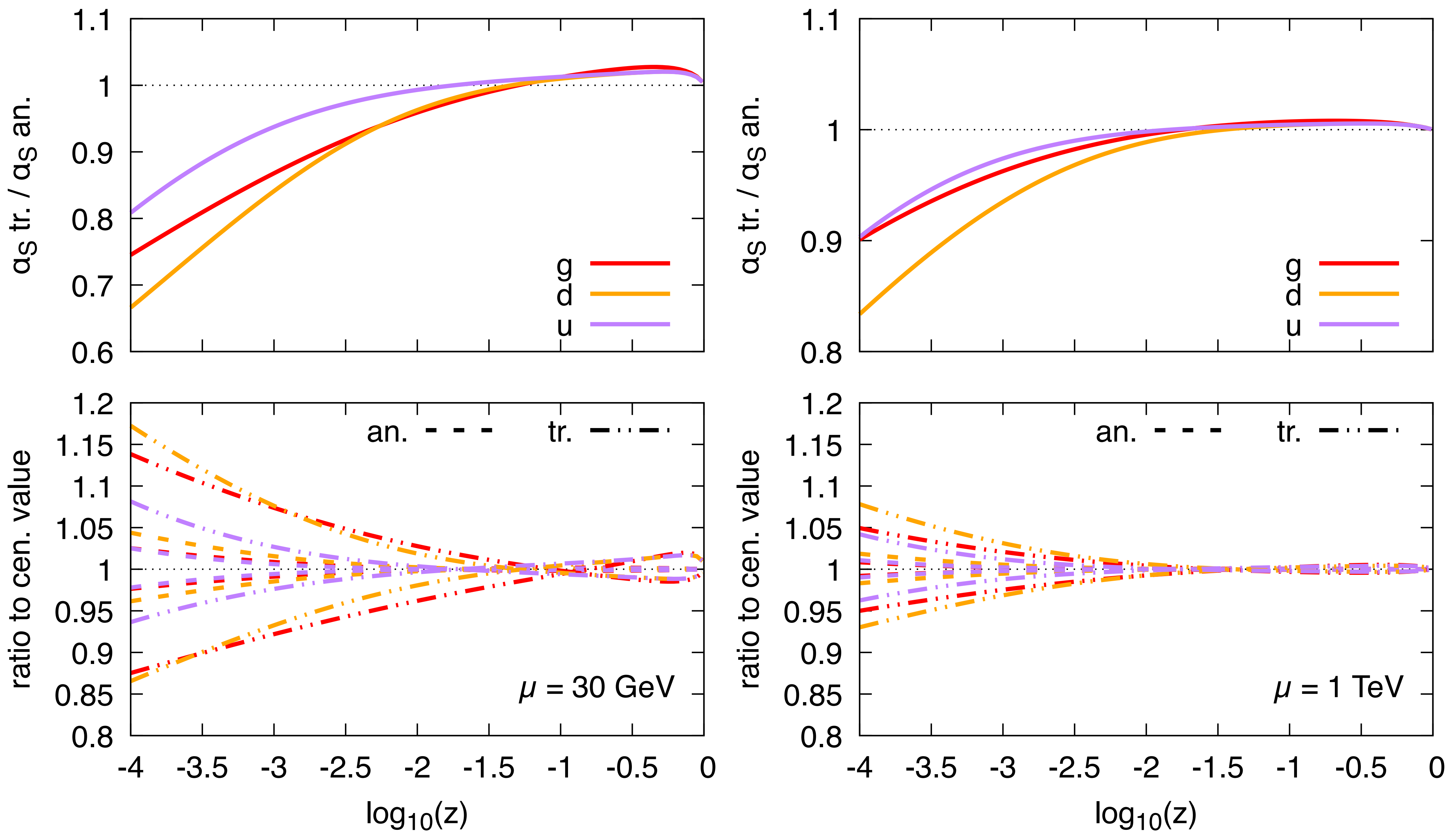
Capdevilla, Garosi, Marzocca, Stechauner '23

PDFs of the muon: QCD and QED only

- NLO initial condition, NLL evolution.
- QCD $\neq$ 0 for $Q^2 \lesssim 1$ GeV, unlike the cases discussed before.
- No W, Z, ν PDFs.

Total dijet rates for $p_T^{cut} = 10$ GeV, in pb.

$\sigma(p_T^{cut} = 10 \text{ GeV})$ [pb]	an.	tr.
$\mathcal{O}(\alpha_S^2)$	$18.33^{+1.30\%}_{-1.25\%}$	$15.00^{+10.23\%}_{-10.99\%}$
γ-ind.	$8.24^{+0.68\%}_{-0.91\%}$	$7.56^{+3.71\%}_{-3.75\%}$
total	$26.58^{+1.11\%}_{-1.15\%}$	$22.57^{+8.04\%}_{-8.56\%}$



Uncertainties and central values at small x for this method (**an.**) are very different w.r.t. setting QCD $\neq$ 0 for $Q^2 \lesssim Q_0^2$ and varying $0.5 \text{ GeV} \lesssim Q_0^2 \lesssim 1 \text{ GeV}$ (**tr.**).

NLO EW corrections do not involve Weak subtraction counter terms when are calculated with such PDFs and NLO accuracy for the QED component can be already achieved.

Frixione, Stagnitto '23

NLO EW corrections at high energies

NLO EW corrections for energies of the order of few TeVs are as large as (or even more than) NLO QCD corrections at the LHC. Origin: **EW Sudakov logarithms.**

EW corrections should be considered not only for precision physics, since they give $\mathcal{O}(10-100\%)$ effects. This includes also BSM scenarios.

$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma^{\rm incl}_{\rm LO}$ [fb]	$\sigma^{\rm incl}_{\rm NLO}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-Z	$3.330(2) \cdot 10^1$	$2.568(8) \cdot 10^1$	$-22.9(2)$
W^+W^-H	$1.1253(5) \cdot 10^0$	$0.895(2) \cdot 10^0$	$-20.5(2)$
ZZZ	$3.598(2) \cdot 10^{-1}$	$2.68(1) \cdot 10^{-1}$	$-25.5(3)$
HZZ	$8.199(4) \cdot 10^{-2}$	$6.60(3) \cdot 10^{-2}$	$-19.6(3)$
HHZ	$3.277(1) \cdot 10^{-2}$	$2.451(5) \cdot 10^{-2}$	$-25.2(1)$
HHH	$2.9699(6) \cdot 10^{-8}$	$0.86(7) \cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1) \cdot 10^0$	$0.993(6) \cdot 10^0$	$-33.1(4)$
W^+W^-ZZ	$1.209(1) \cdot 10^0$	$0.699(7) \cdot 10^0$	$-42.2(6)$
W^+W^-HZ	$8.754(8) \cdot 10^{-2}$	$6.05(4) \cdot 10^{-2}$	$-30.9(5)$
W^+W^-HH	$1.058(1) \cdot 10^{-2}$	$0.655(5) \cdot 10^{-2}$	$-38.1(4)$
$ZZZZ$	$3.114(2) \cdot 10^{-3}$	$1.799(7) \cdot 10^{-3}$	$-42.2(2)$
$HZZZ$	$2.693(2) \cdot 10^{-3}$	$1.766(6) \cdot 10^{-3}$	$-34.4(2)$
$HHZZ$	$9.828(7) \cdot 10^{-4}$	$6.24(2) \cdot 10^{-4}$	$-36.5(2)$
$HHHZ$	$1.568(1) \cdot 10^{-4}$	$1.165(4) \cdot 10^{-4}$	$-25.7(2)$

3 TeV Muon Collider

WHIZARD

Bredt, Kilian, Reuter, Steinemeier '22

What are EW Sudakov logarithms?

QCD: virtual and real terms are separately IR divergent ($1/\epsilon$ poles). In physical cross sections the contributions are combined and poles cancel.

QED: same story, but I can also regularise IR divergencies via a photon-mass λ. So $1/\epsilon$ poles $\rightarrow \log(Q^2/\lambda^2)$, where Q is a generic scale.

EW: with weak interactions $\lambda \rightarrow m_W, m_Z$ and W and Z radiation are typically not taken into account, which is anyway IR-safe.

Therefore, at high energies EW loops induce corrections of order

$$-\alpha^k \log^n(s/m_W^2)$$

where k is the number of loops and $n \leq 2k$. These logs are physical. Even including the real radiation of W and Z, there is not the full cancellation of this kind of logarithms.

Future Colliders: are EW Sudakov logarithms a good and robust approximation for EW corrections at high energies?

Currently: exact NLO EW automated for SM but not for BSM.

Since EW corrections are expected to be relevant also for BSM, can we safely use the high-energy Sudakov approximation?

MadGraph5_aMC@NLO: EW corrections for FC

NLO EW hadron colliders: *Frederix, Frixione, Hirshi, DP, Shao, Zaro '18*

NLO EW e^+e^- colliders: *Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao '22*

One-loop EW Sudakov alone: *DP, Zaro '21*

one-loop EW virtual corrections $\mathcal{O}(\alpha)$

=

α [Sudakov Logs $\mathcal{O}(-\log^k(s/m_W^2), k = 1,2)$ +

constant term $\mathcal{O}(1)$ +

mass-suppressed terms $\mathcal{O}(m_W^2/s)$]

Having separately exact NLO EW and EW Sudakov logarithms it is possible to study the goodness of the high-energy approximation(s). **SM as a test case!**

Master formula (Denner&Pozzorini)

Born amplitude: $\mathcal{M}_0^{i_1\dots i_n}(p_1,\dots,p_n)$

Denner Pozzorini '01

One-loop EW Sudakov corrections: $\delta\mathcal{M}^{i_1\dots i_n}(p_1,\dots,p_n) = \mathcal{M}_0^{i'_1\dots i'_n}(p_1,\dots,p_n)\delta_{i'_1 i_1\dots i'_n i_n}$

other tree-level amplitudes — the logs

$$\delta = \delta^{\mathrm{LSC}} + \delta^{\mathrm{SSC}} + \delta^{\mathrm{C}} + \delta^{\mathrm{PR}}$$

eikonal approximation of soft EW boson exchange ($\delta^{\mathrm{LSC}} + \delta^{\mathrm{SSC}}$)

- **Leading Soft-Collinear**: It depends only on s and it is the only term involving double logarithms.
- **Subleading Soft-Collinear**: The only one involving ratios of s with other invariants and also angular dependences.
- **Collinear**
- **Parameter renormalis.**: In an on-shell scheme, the dependence on the UV regularisation scale cancels. No μ_r dependence is left.

The logs inside the δ^i have always the form:

$$L(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi}\log^2\frac{|r_{kl}|}{M^2}$$

$$l(|r_{kl}|, M^2) \equiv \frac{\alpha}{4\pi}\log\frac{|r_{kl}|}{M^2}$$

$$M = M_W, M_Z, m_f, \lambda, \dots$$
$$r_{kl} \equiv (p_k + p_l)^2$$

ASSUMPTIONS:

$$r_{kl} \equiv (p_k + p_l)^2 \simeq 2p_k p_l \gg M_W^2 \simeq M_H^2, m_t^2, M_W^2, M_Z^2$$

the high-energy limit

$$r_{kl}/r_{k'l'} \simeq 1$$

All invariants $\simeq s$. Reasonable, but $r_{kl} = s$ is impossible.

Our revisitation:

DP, Zaro '21

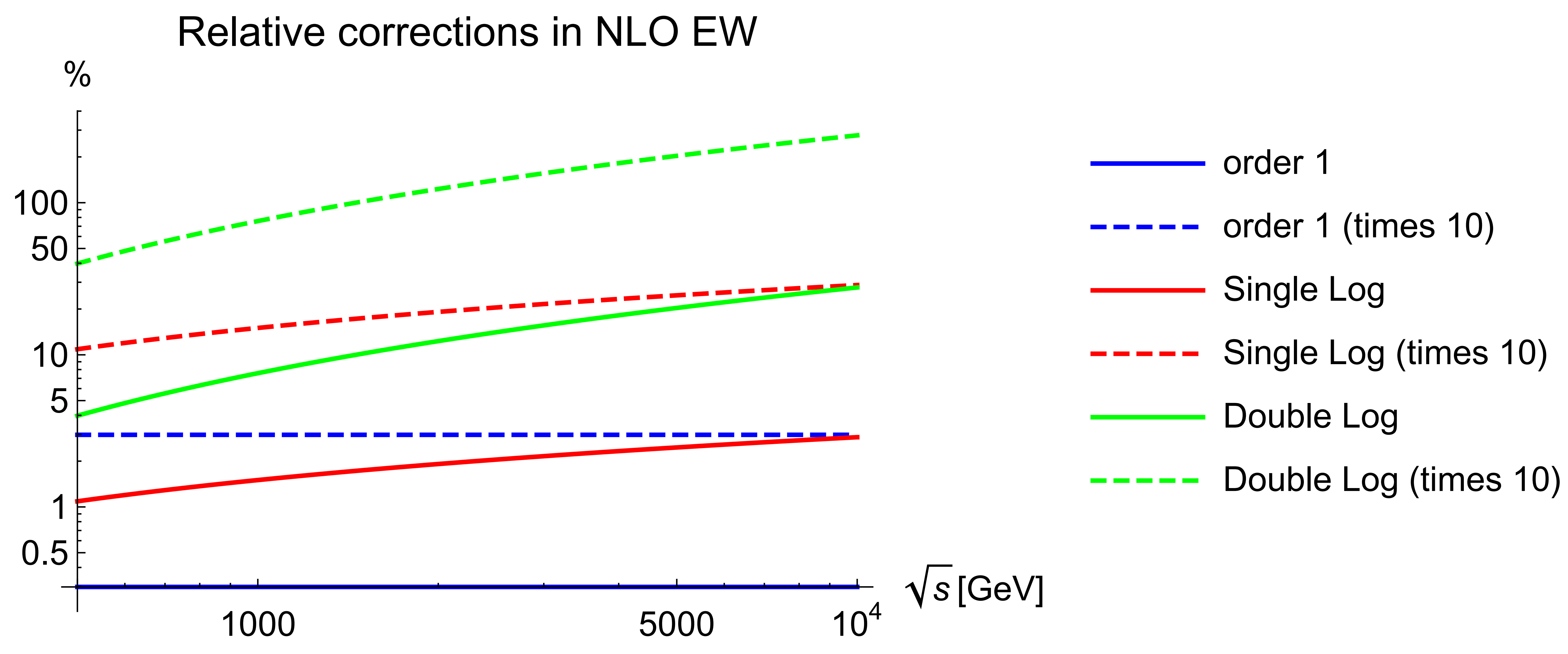
Logs among invariants: Logs like $\log(t/s)$ taken into account.

$\mathrm{SDK}_{\mathrm{Weak}}$ scheme: A purely Weak (no QED) scheme for improving approximation of IR-finite physical observables. Different to the more common SDK_0 scheme that has been used in the literature.

How large are expected to be the EW Sudakov?

$$\mathcal{O}(1) \to \frac{\alpha}{4\pi s_w^2} \sim 0.3\,\%\,,\quad \textbf{Single Log} \to \frac{\alpha}{4\pi s_w^2}\log(s/m_W^2),$$

$$\textbf{Double Log} \to \frac{\alpha}{4\pi s_w^2}\log^2(s/m_W^2)$$



Taking into account only DL, and not SL, is not safe for partonic energies up to 10 TeV.

Just a representative example of a process

The estimate done via the variation of a factor of 10 is actually conservative.

$$\delta^{\mathrm{RR,ew}}_{\mathrm{e^+e^-}\to\mu^+\mu^-} = -2.58\,L(s) - 5.15\left(\log\frac{t}{u}\right) l(s) + 0.29\,l_{\mathrm{Z}} + 7.73\,l_{\mathrm{C}} + 8.80\,l_{\mathrm{PR}},$$

$$\delta^{\mathrm{RL,ew}}_{\mathrm{e^+e^-}\to\mu^+\mu^-} = -4.96\,L(s) - 2.58\left(\log\frac{t}{u}\right) l(s) + 0.37\,l_{\mathrm{Z}} + 14.9\,l_{\mathrm{C}} + 8.80\,l_{\mathrm{PR}},$$

Denner Pozzorini '01

NLO EW: some open questions/issues

Resummation?

When is it necessary to resum EW (Sudakov) corrections?

BSM?

What features of NLO EW corrections are universal and can be extended to the BSM case?

Heavy Boson Radiation (HBR)?

What should one do with Z,W radiation? Experimental set-up may impact the calculation result.

PDFs or VBF with matrix elements?

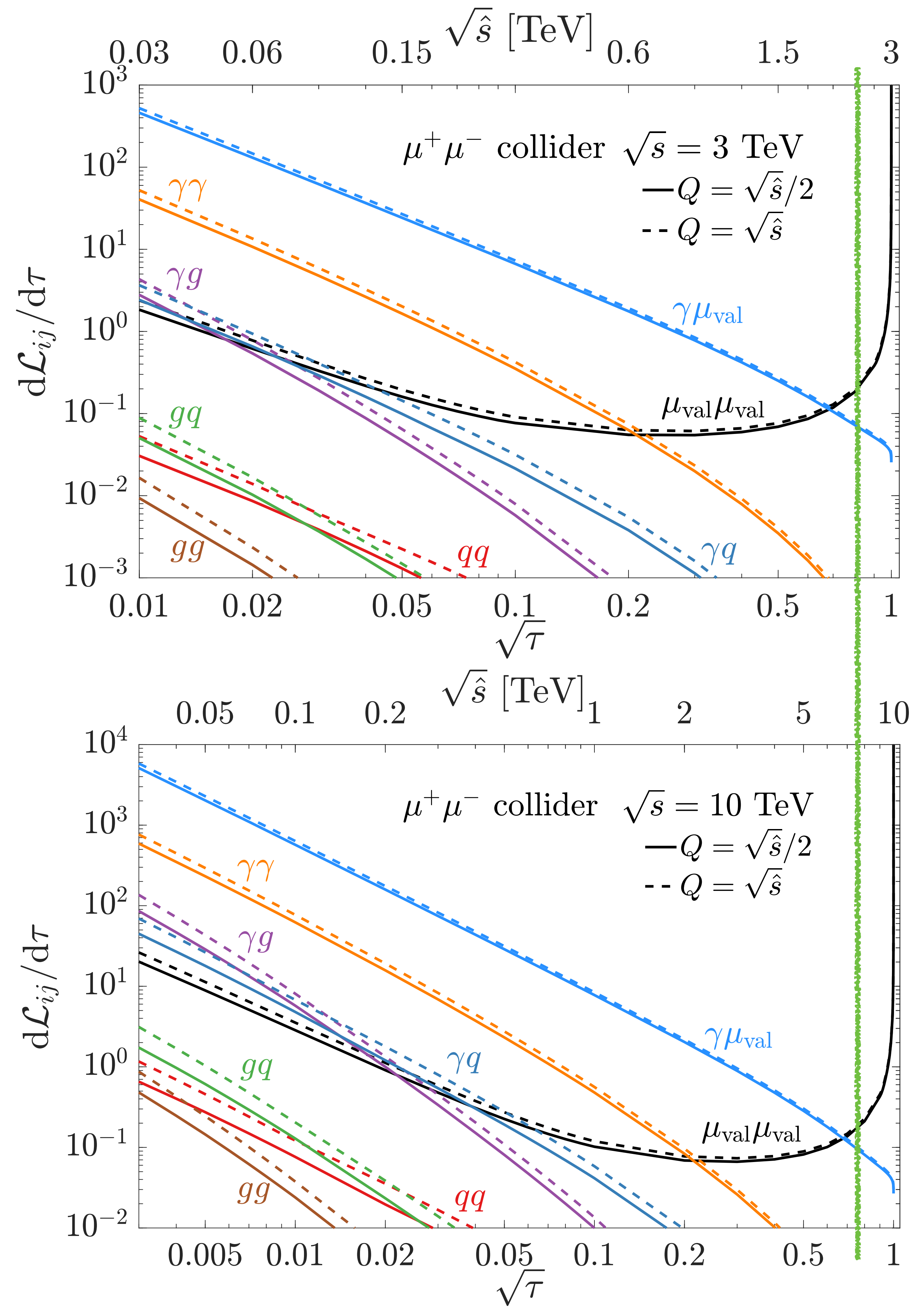
If PDFs involve weak effects, weak counter terms in NLO EW corrections should be included. Resum logs or keep power corrections? Both?

Direct production at high energy

$\mu^+\mu^- \longrightarrow F$, where F is a generic final state involving W, Z, t, H.

Ma, DP, Zaro ' 24

We select direct production, with no VBF contributions.



We require $m(F) > 0.8\sqrt{S}$, so that neither VBF nor PDFs other than μ are relevant.

We apply further experimentally motivated cuts for each X, Y particle in F:

$$p_T(X) > 100 \text{ GeV}, \ |\eta(X)| < 2.44, \ \Delta R(X, Y) > 0.4$$

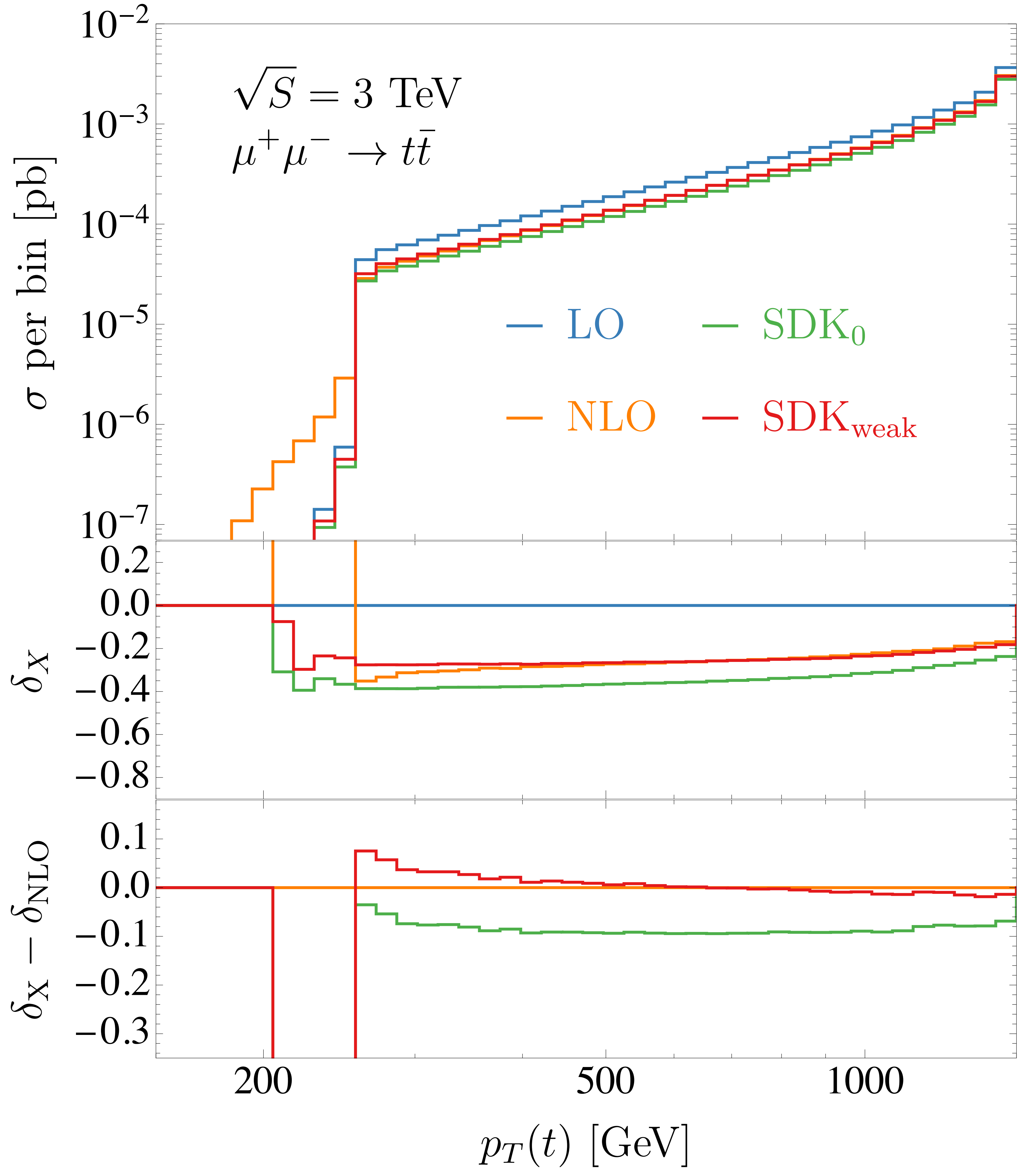
And we recombine photons with charged (also massive) particles.

Han, Ma, Xie '20, '21

The μ PDF in the μ is peaked at Bjorken-x=1, therefore: Collider $S \simeq$ partonic s

$t\bar{t}$

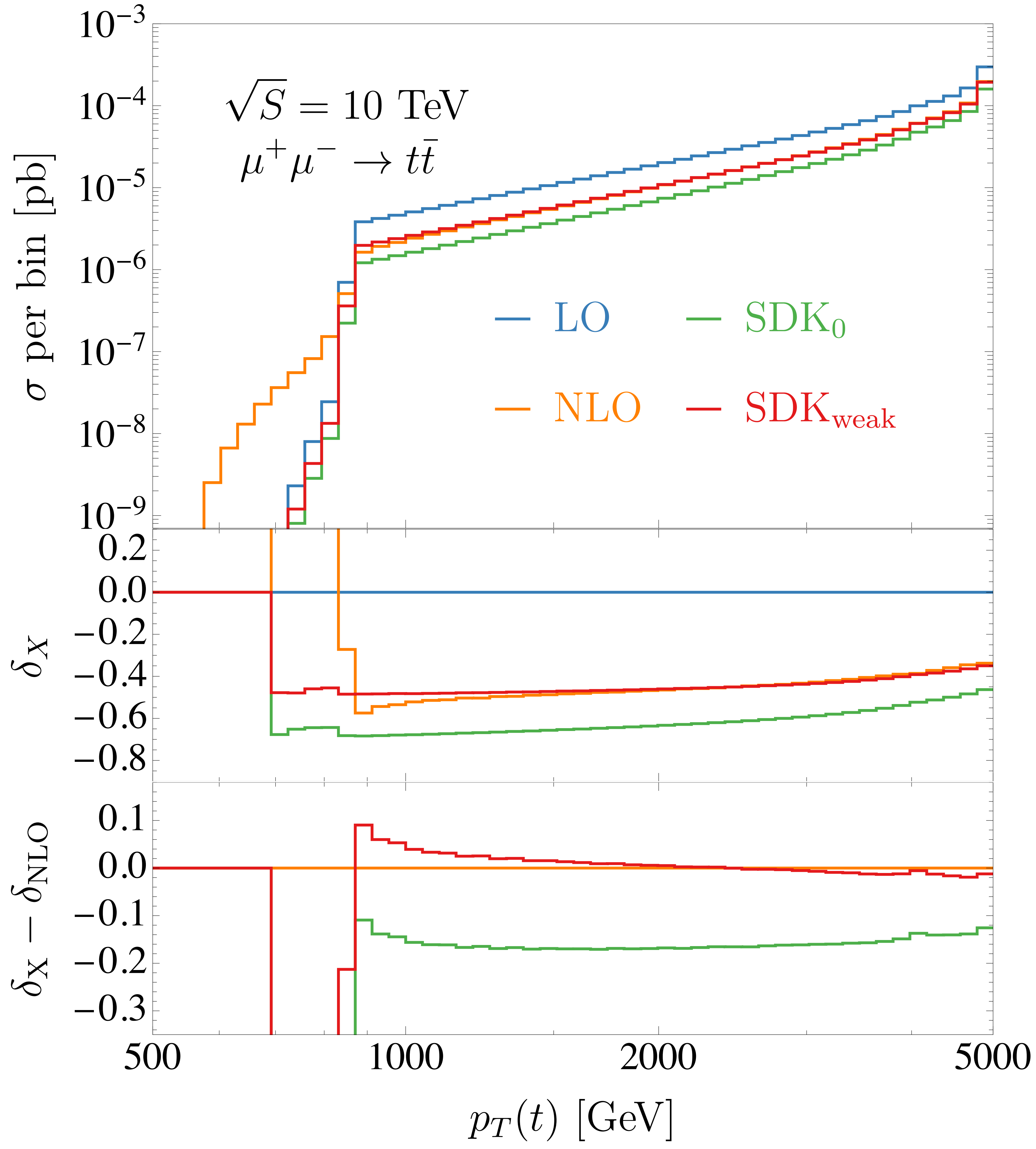
Ma, DP, Zaro ' 24



For smaller p_T, larger corrections.

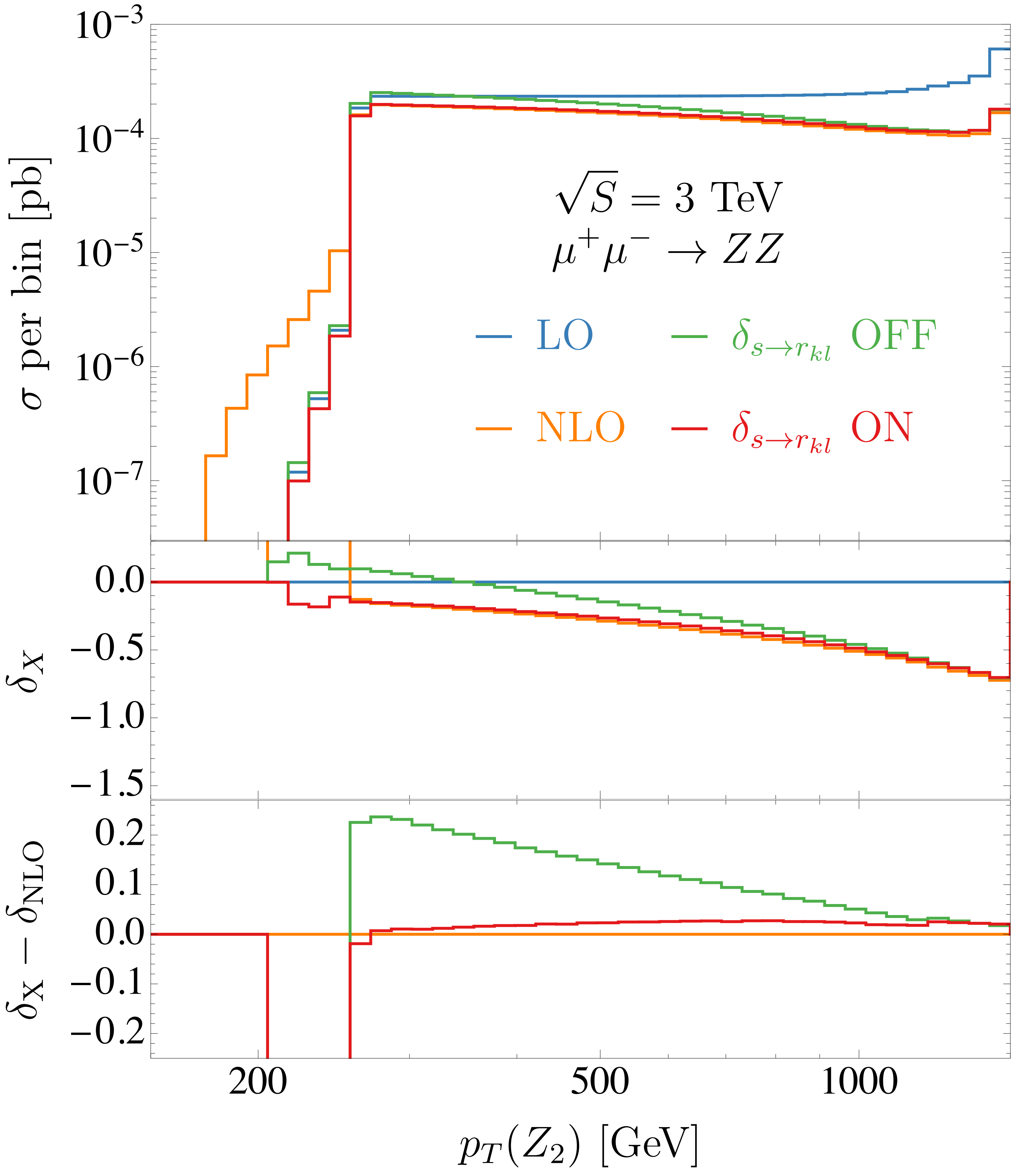
Sudakov (in the $\mathrm{SDK_{weak}}$ scheme) **capture NLO EW corrections** up to the % level.

If double logs are written in the form $\log^2(s/m_W^2)$, the shapes observed here are all arising from **single logs**.



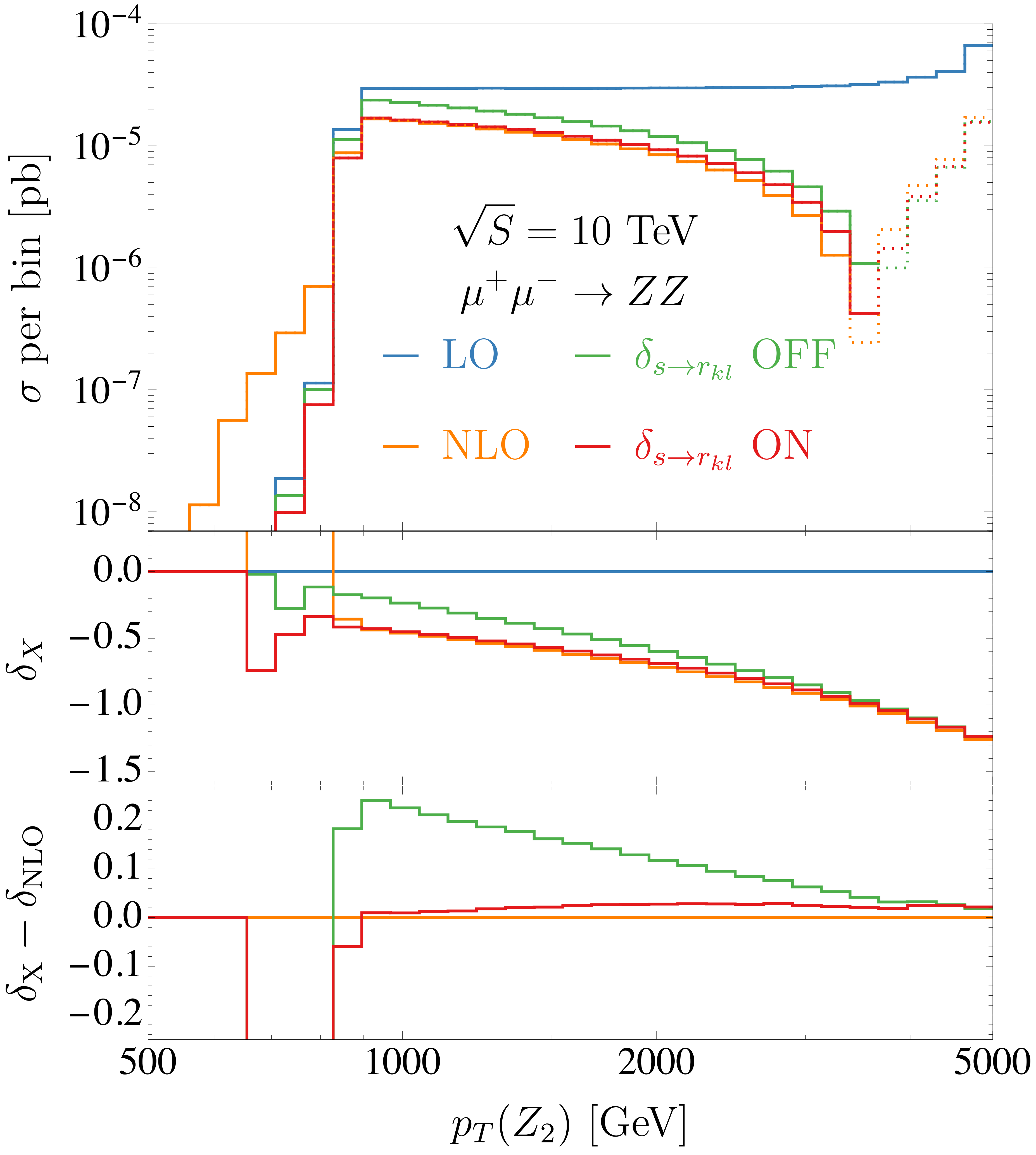
ZZ

Ma, DP, Zaro ' 24



Sudakov logs **capture NLO EW corrections** up to the % level, but only if all the logs of the form $\log(t/s)$ are taken into account.

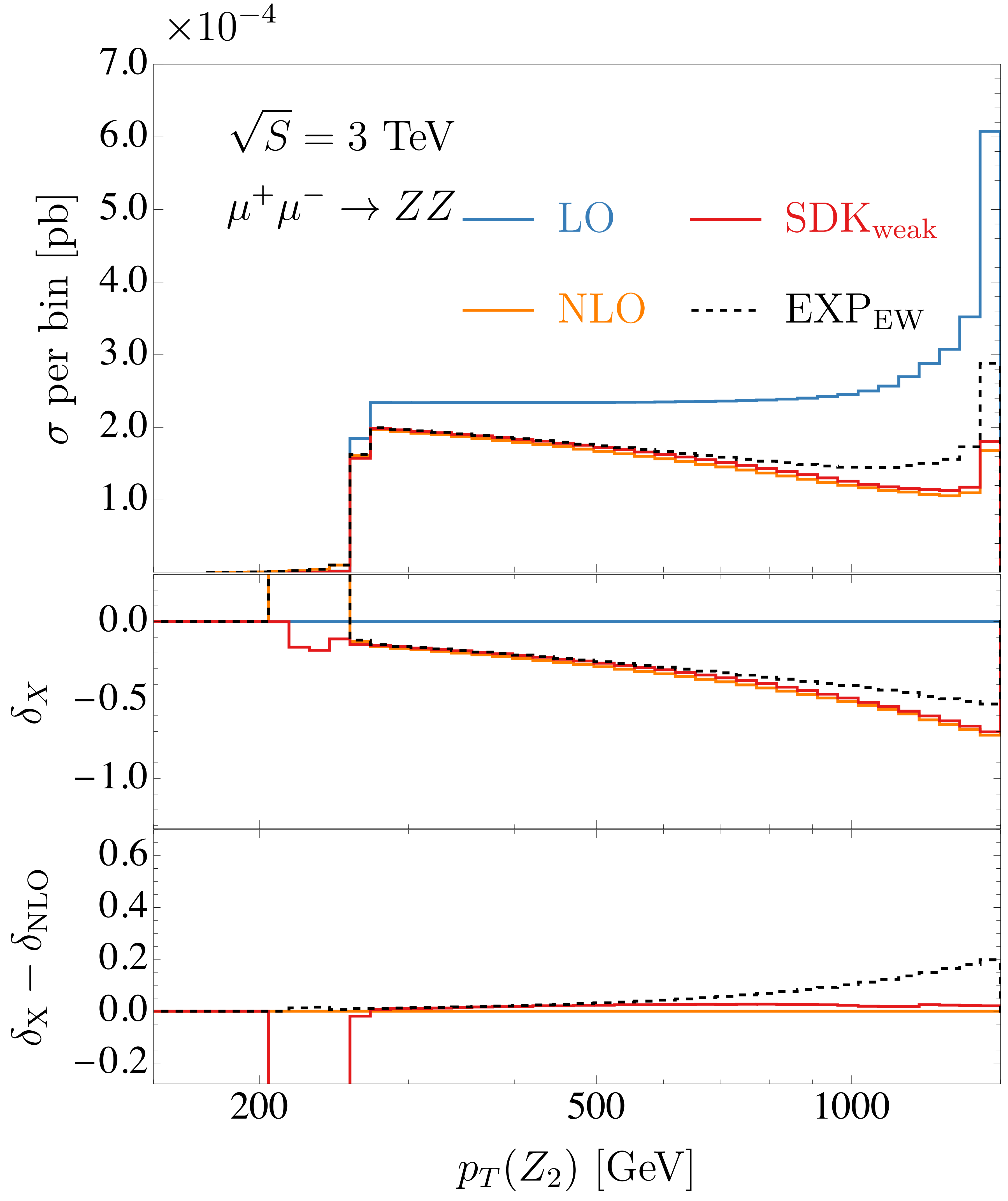
Green: logs of the form $\log^2(t/s)$ or $\log(t/s)$ ignored.



ZZ

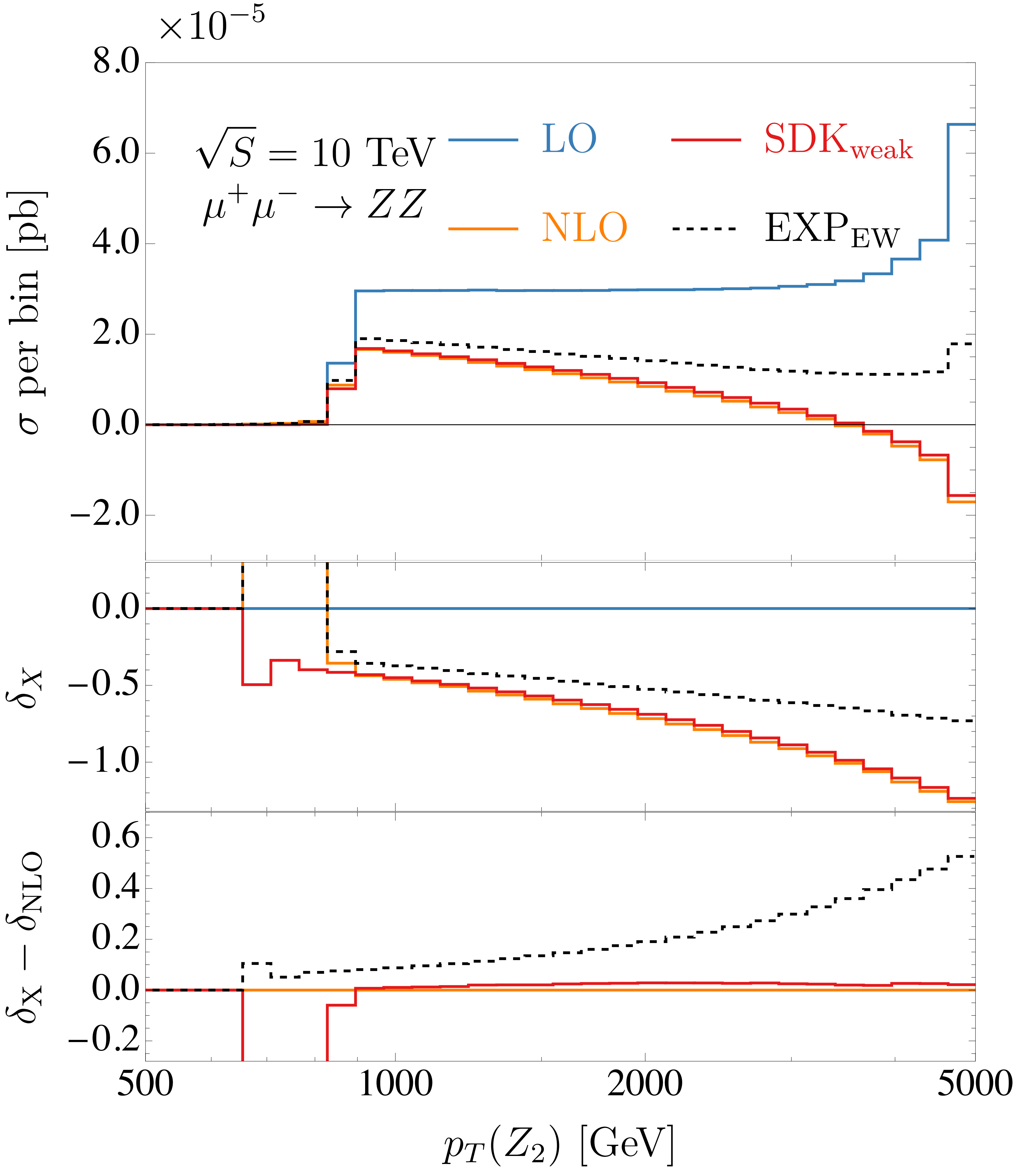
Ma, DP, Zaro ' 24

$$\sigma_{\mathrm{EXP_{EW}}} \equiv \left(\sigma_{\mathrm{LO}}\, e^{\delta_{\mathrm{SDK_{weak}}}}\right) + \left(\sigma_{\mathrm{NLO_{EW}}} - \sigma_{\mathrm{SDK_{weak}}}\right) = \sigma_{\mathrm{NLO_{EW}}} + \mathcal{O}(\alpha^2) \times \sigma_{\mathrm{LO}}.$$



Exponentiation as an approximation of proper resummation.

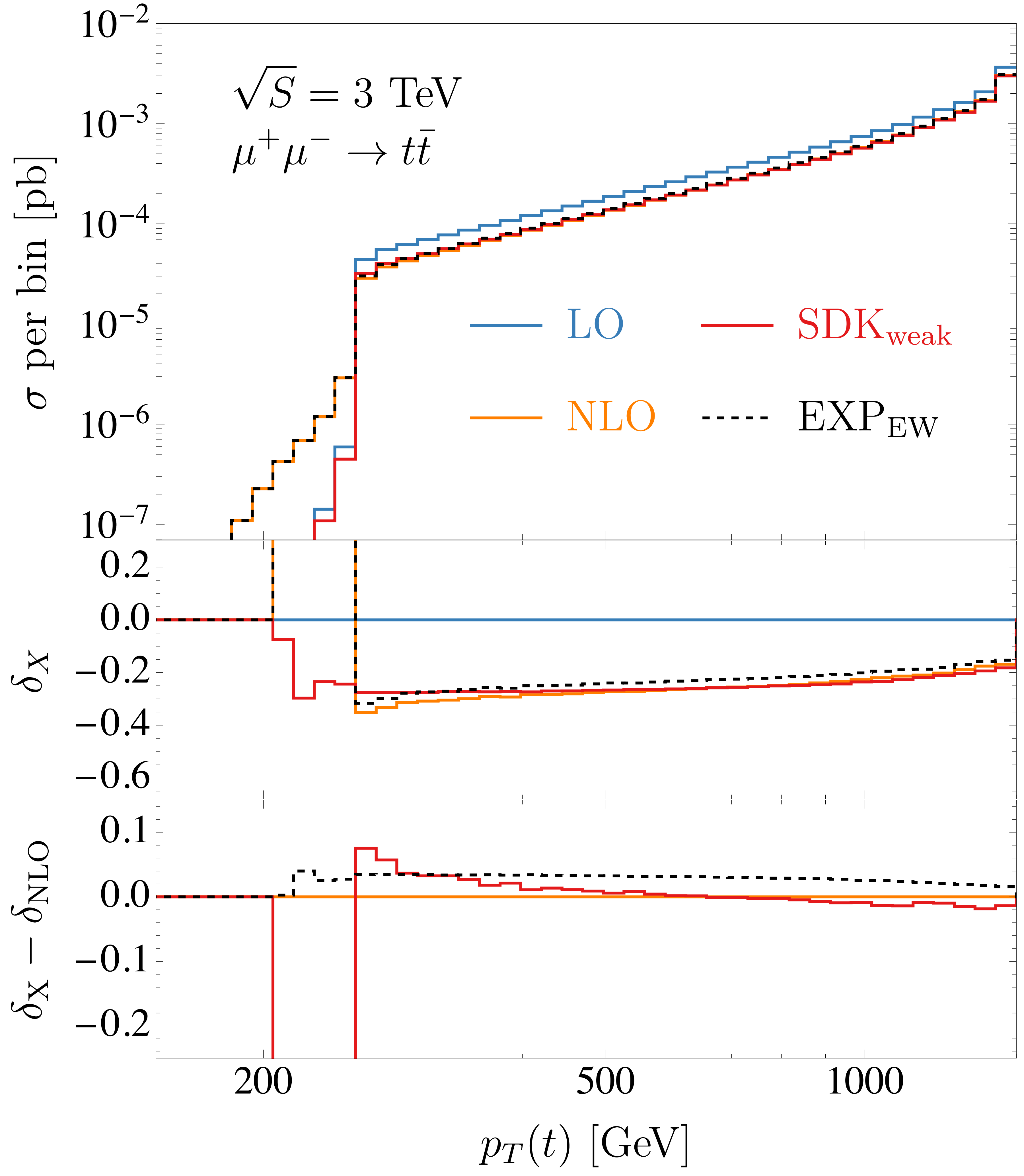
At 10 TeV resummation is unavoidable for sensible predictions, and it is necessary for precision at 3 TeV.



$t\bar{t}$

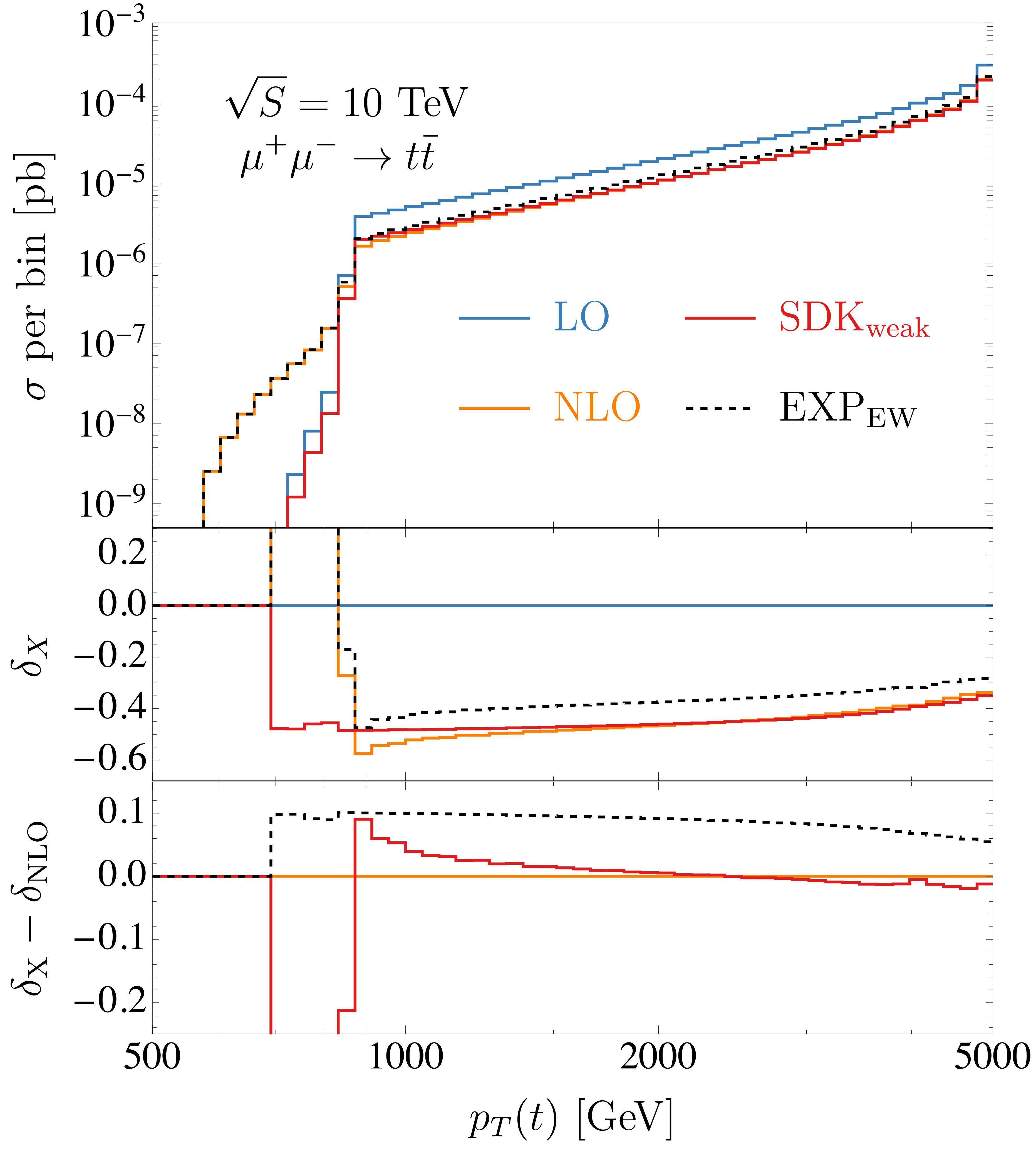
Ma, DP, Zaro ' 24

$$\sigma_{\mathrm{EXP_{EW}}} \equiv \left(\sigma_{\mathrm{LO}}\, e^{\delta_{\mathrm{SDK_{weak}}}}\right) + \left(\sigma_{\mathrm{NLO_{EW}}} - \sigma_{\mathrm{SDK_{weak}}}\right) = \sigma_{\mathrm{NLO_{EW}}} + \mathcal{O}(\alpha^2) \times \sigma_{\mathrm{LO}}.$$



Exponentiation as an approximation of proper resummation.

Unlike ZZ, for $t\bar{t}$ **also at 10 TeV resummation is** necessary only for precision.



What about extra radiation of Z (and H)?

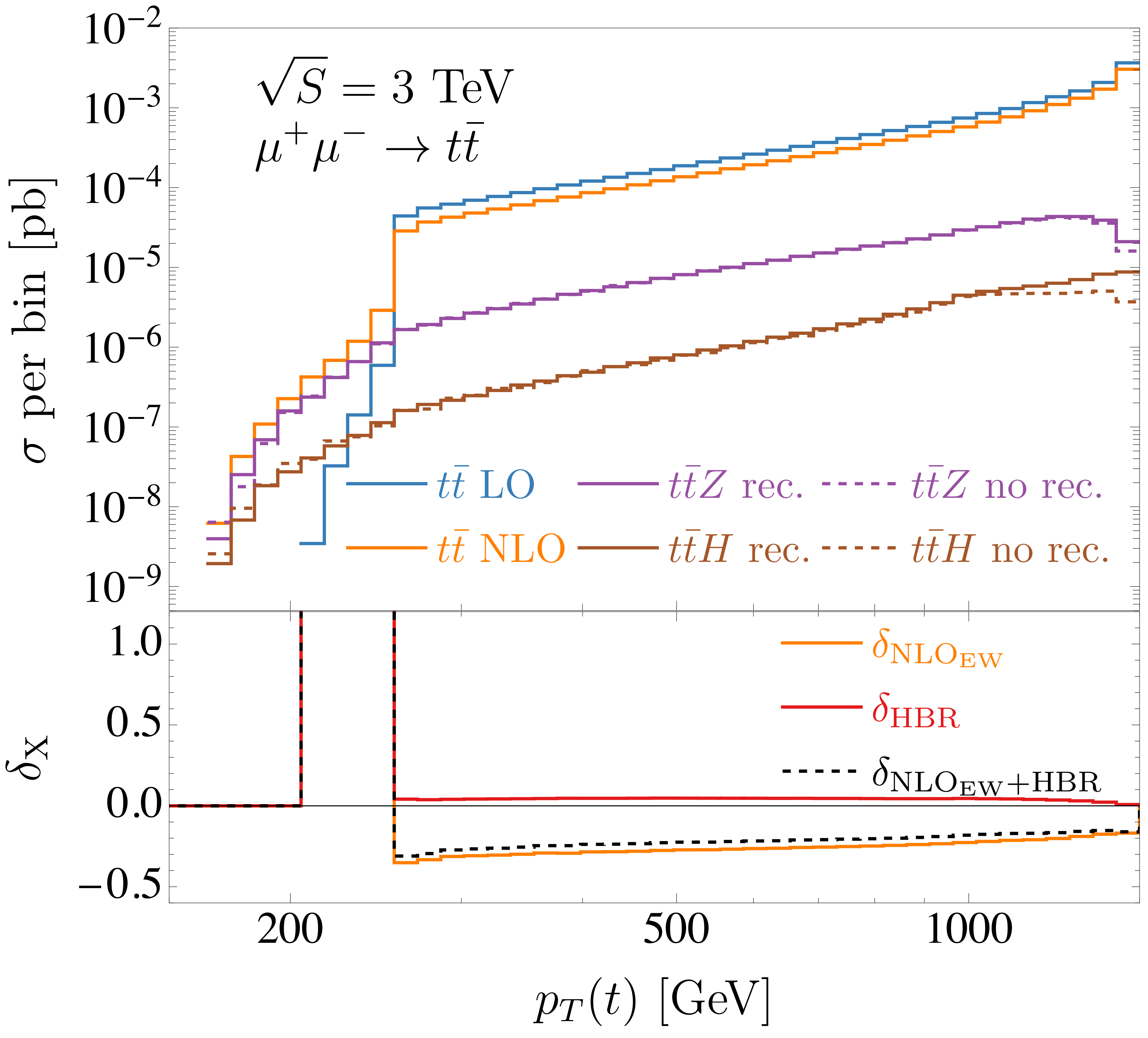
We know that unlike QCD in virtual+real there is not the exact cancellation of logarithms.

But a cancellation is still present, how much large?

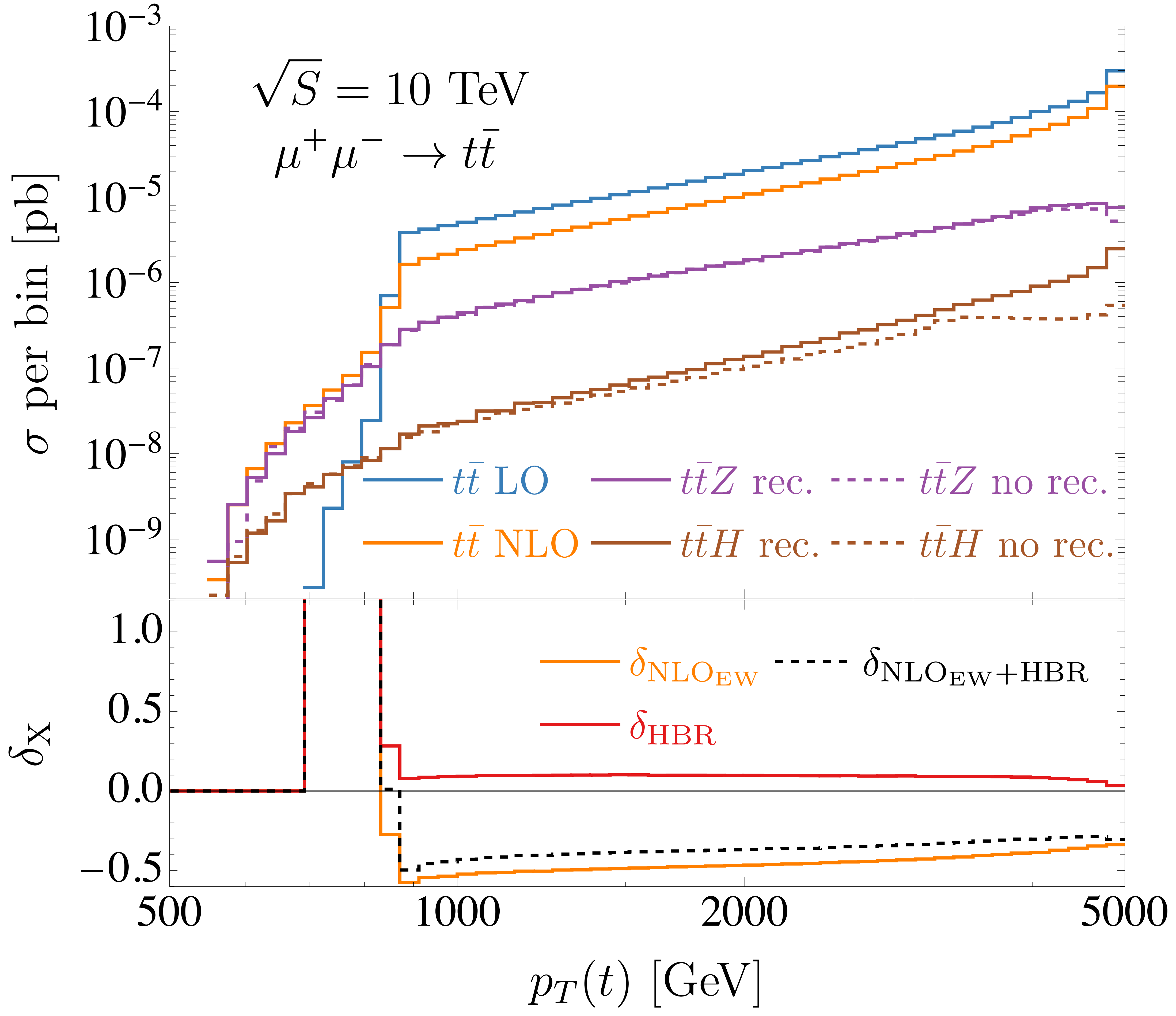
Is it really Heavy-Boson-Radiation (HBR) leading to $\mathcal{O}(1)$ corrections?

EW is the new **QCD**,
but it is not exactly as the QCD!

Small effects from Z and H radiation, especially in the bulk: $p_T(t) \simeq \sqrt{S}/2$



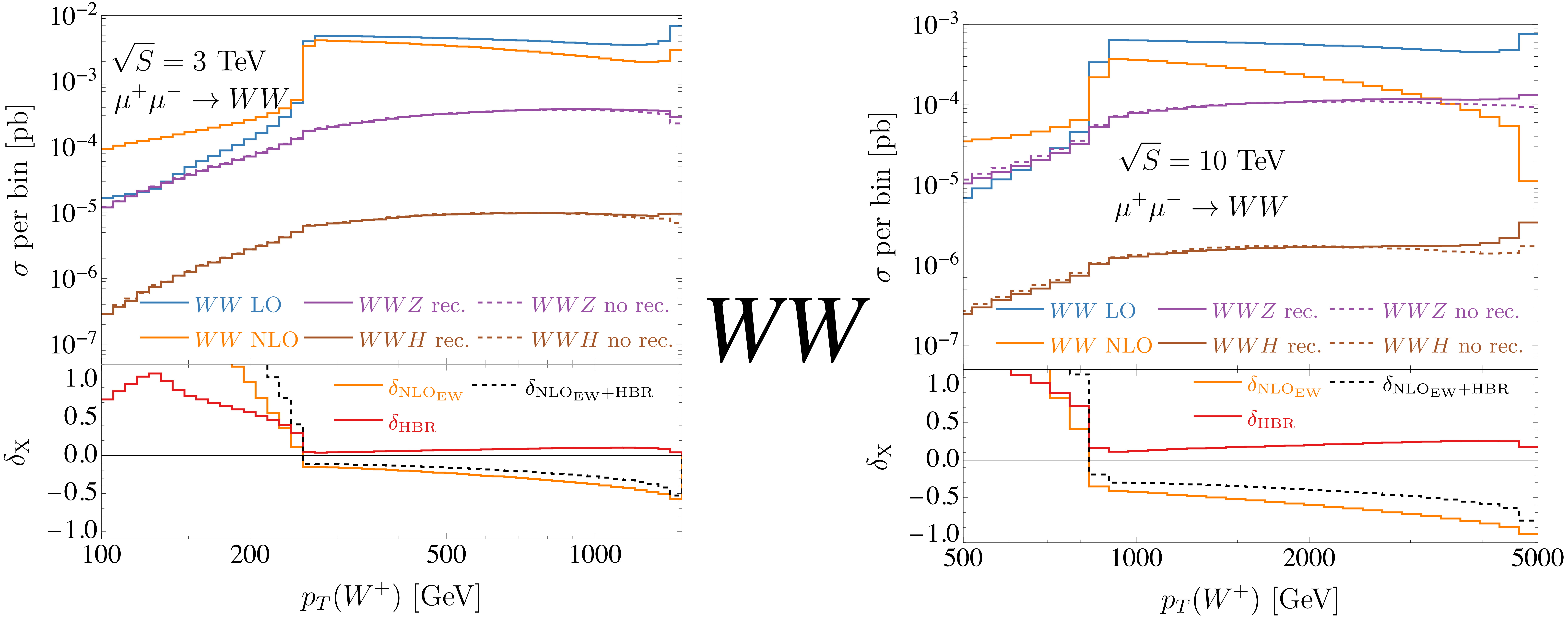
$t\bar{t}$



Notice that in order to allow more phase space we required just $m(F) > 0.8\sqrt{S}$.
Still HBR << NLO EW in absolute value.

Ma, DP, Zaro ‘ 24

Small effects from Z and H radiation, especially in the bulk: $p_T(W) \simeq \sqrt{S}/2$

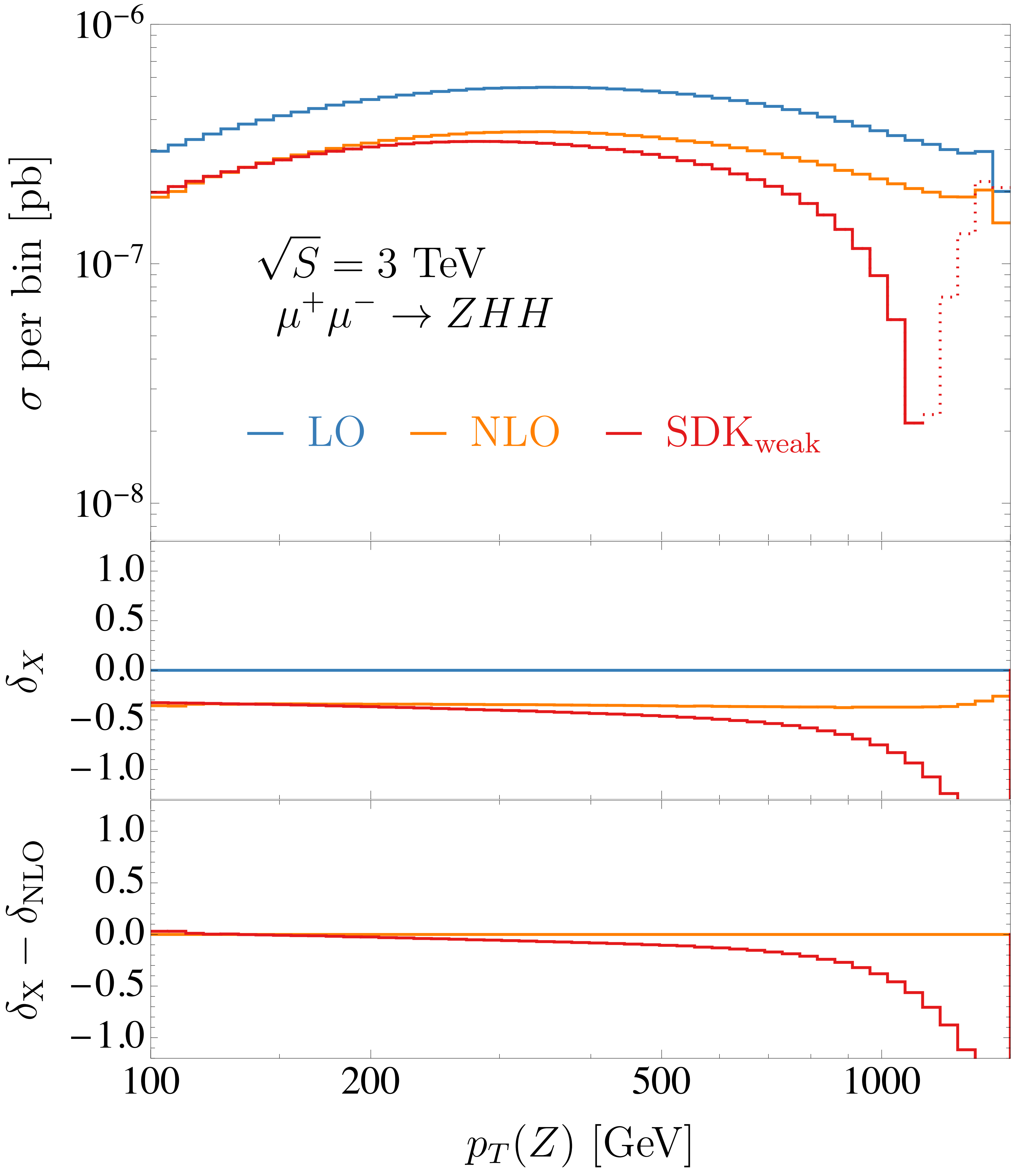


It is a general pattern: radiation of heavy bosons is less important than loops!

Ma, DP, Zaro ' 24

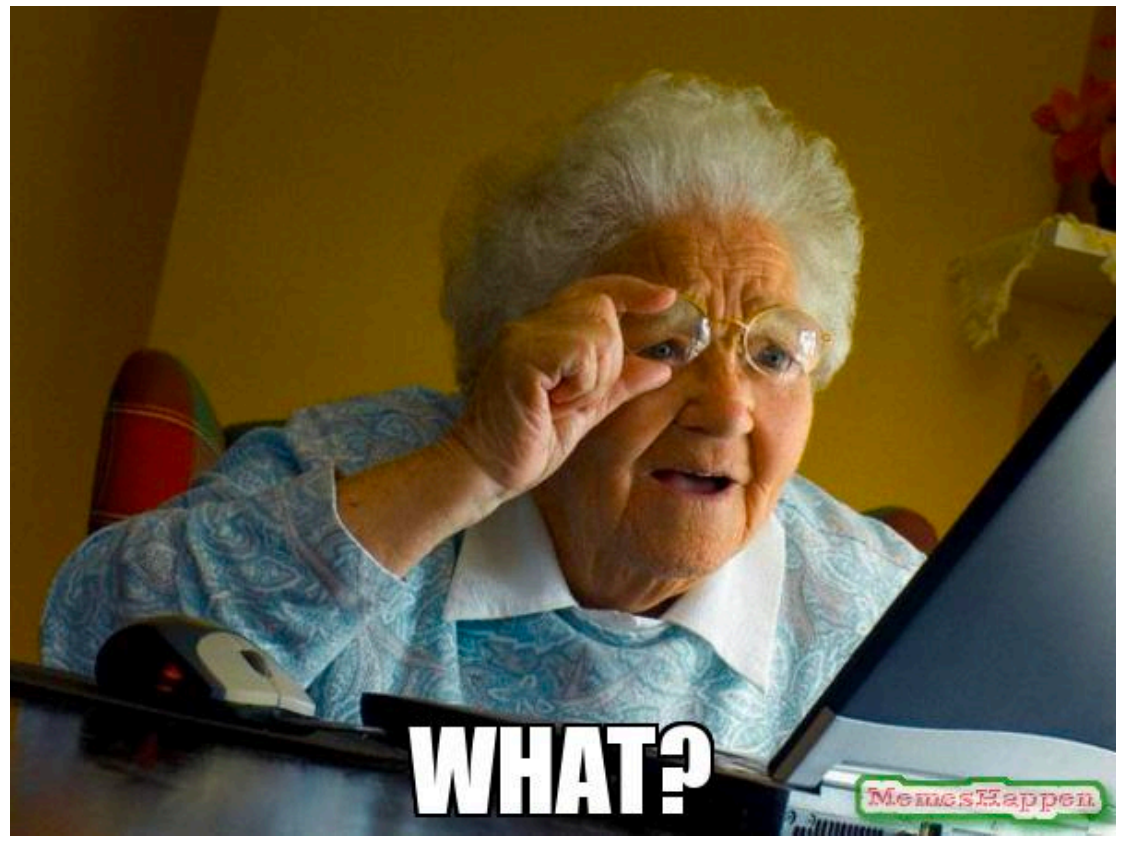
Sudakov may completely fail: ZHH

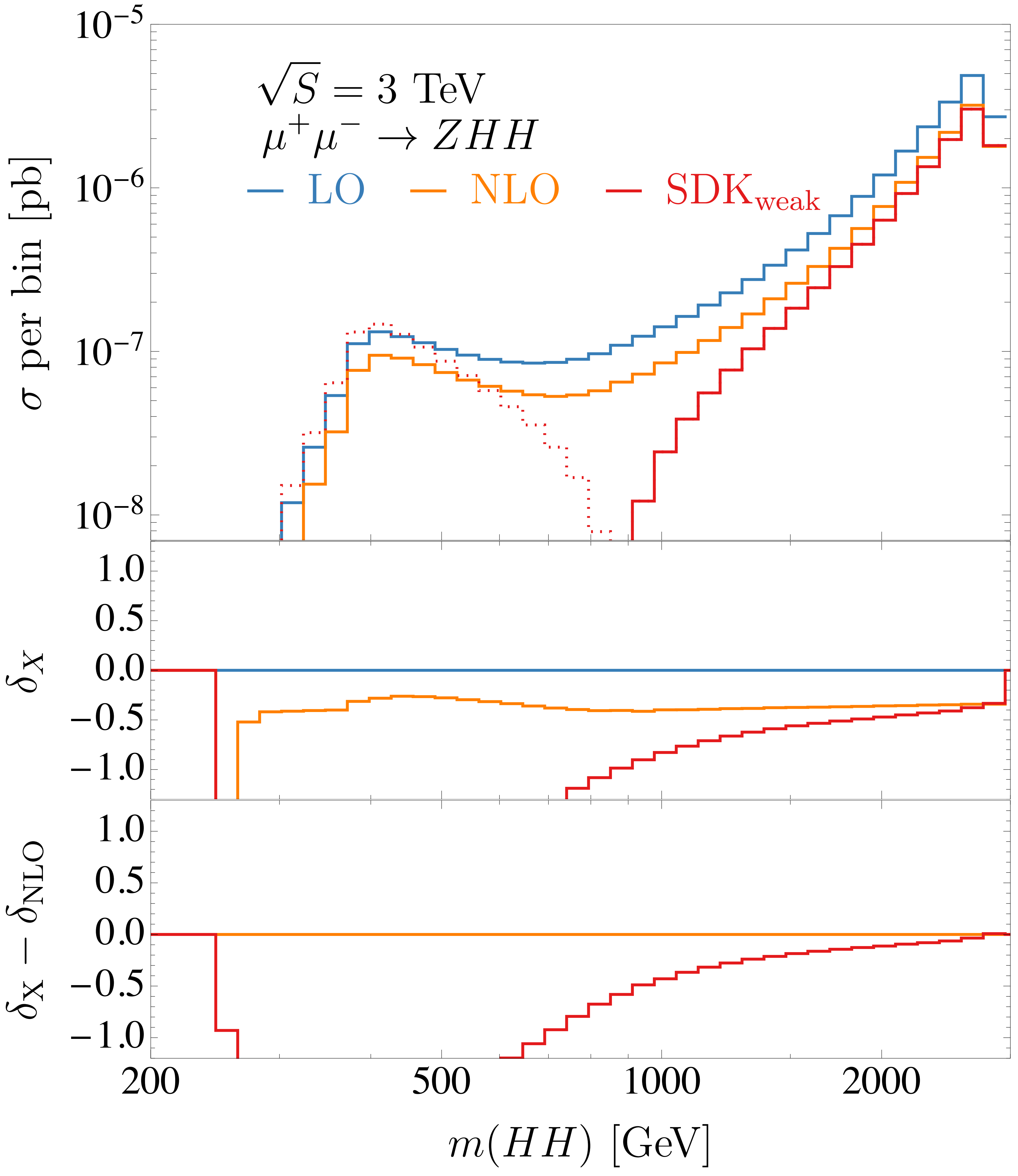
Ma, DP, Zaro ‘ 24



NLO EW corrections are flat.

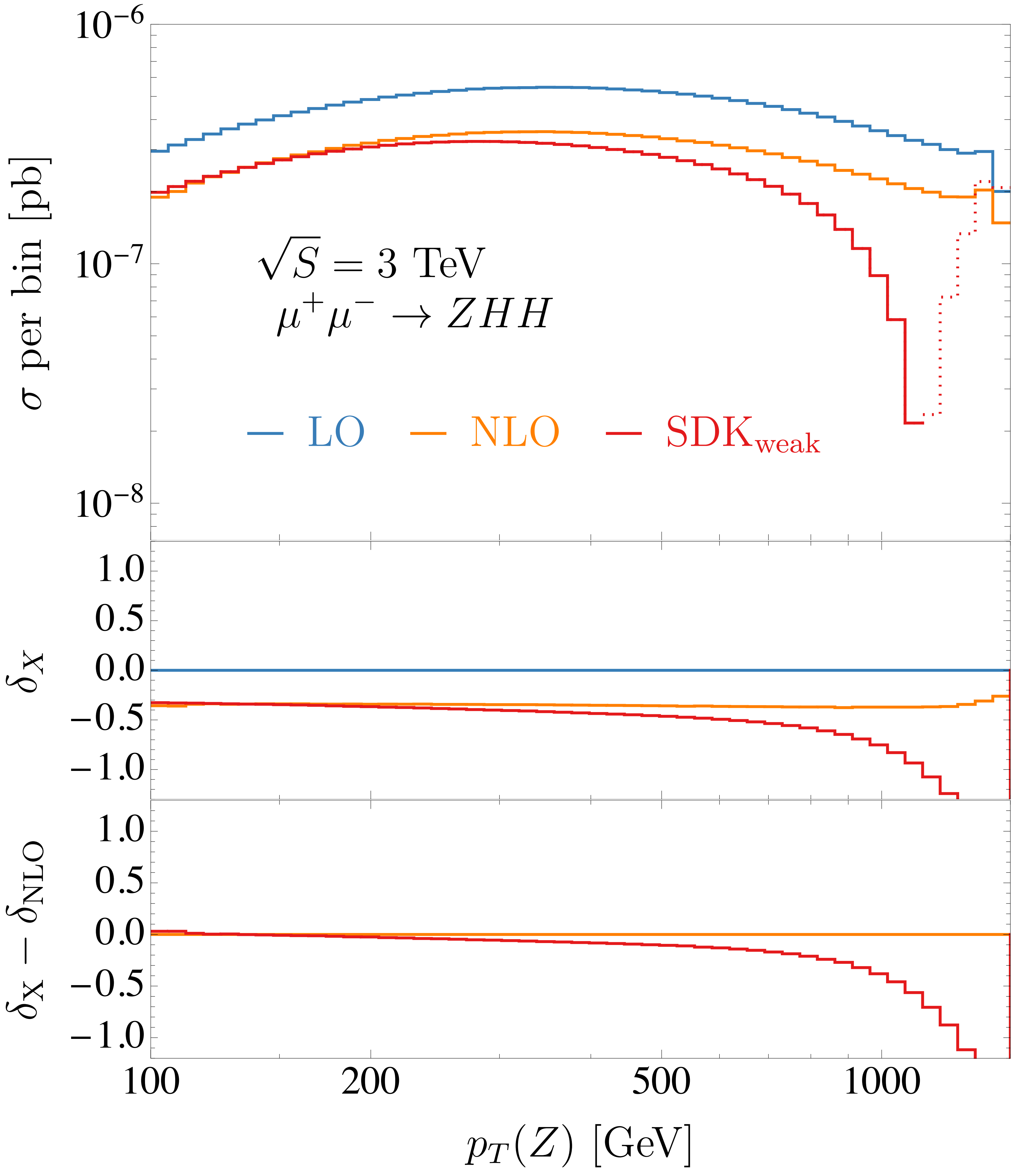
Sudakov logarithms work **very well at low pt and very bad at high pt.**





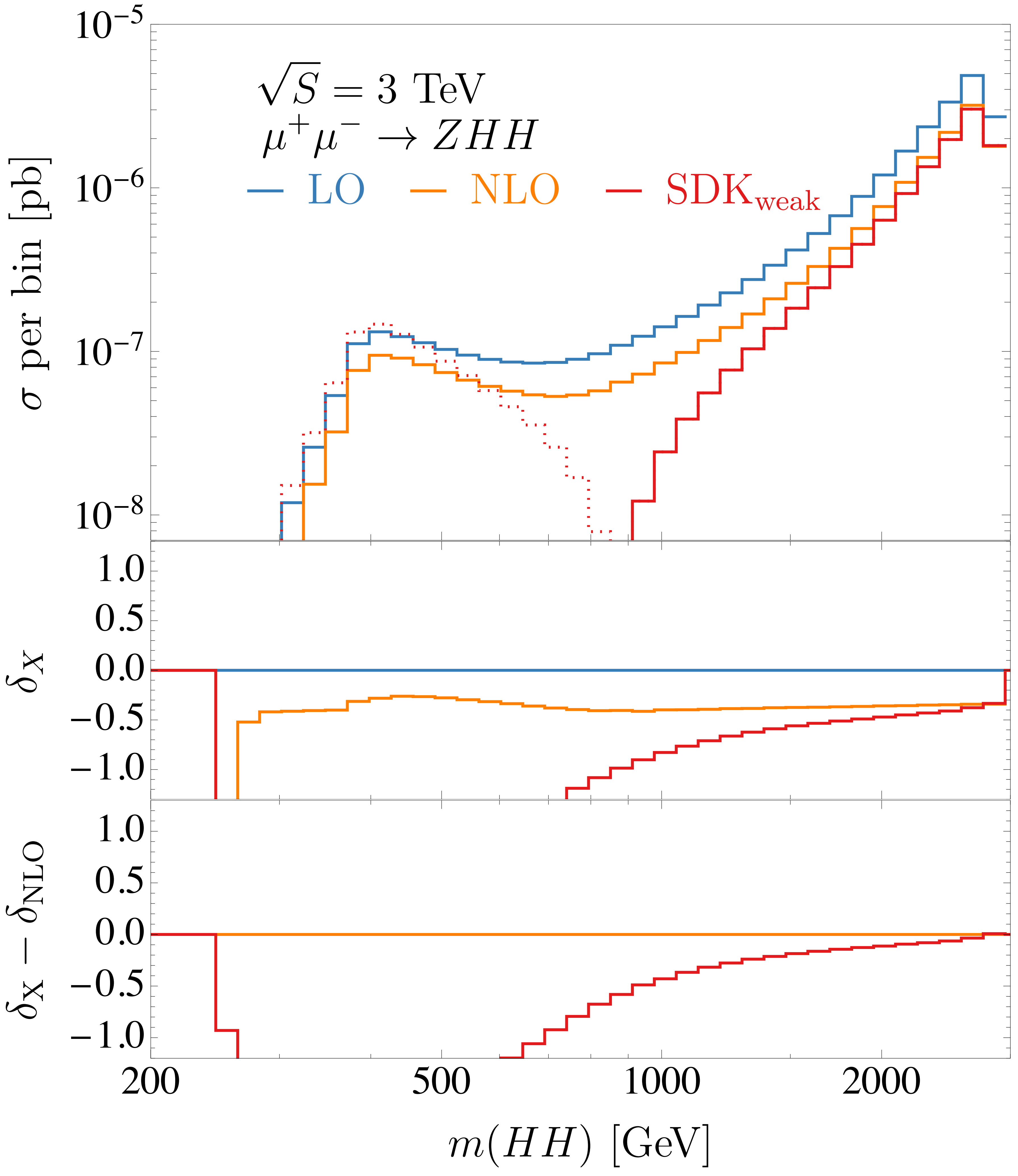
Sudakov may completely fail: ZHH

Ma, DP, Zaro ' 24



For High pt of the Z boson, the two Higgs can have very small ΔR and so small $m(HH)$, recoiling against the Z.

In that configuration, formally **mass suppressed** terms $\sim \dfrac{v}{m(H_1H_2)}$ can become numerically sizeable, and the **DP algorithm fails**.



EW Sudakov and SMEFT

NLO EW for SMEFT is challenging, Sudakov approximation would simplify the calculation and allow for dominant effects.

$$\mathcal{M} \propto \frac{s^{(4-n)/2}}{\Lambda^2}$$

The Denner-Pozzorini algorithm work only for non-mass suppressed amplitude at LO = no powers of $M_W/\sqrt{S} \sim v/\sqrt{S}$

Often in the SMEFT a vev is appearing in the Feynman rules leading at dim=6 to

$$\mathcal{M} \propto \frac{v\, s^{(3-n)/2}}{\Lambda^2} \propto \frac{M}{\sqrt{s}} \times \frac{s^{(4-n)/2}}{\Lambda^2} \qquad \mathcal{M} \propto \frac{v^2 s^{(2-n)/2}}{\Lambda^2} \propto \frac{M^2}{s} \times \frac{s^{(4-n)/2}}{\Lambda^2}$$

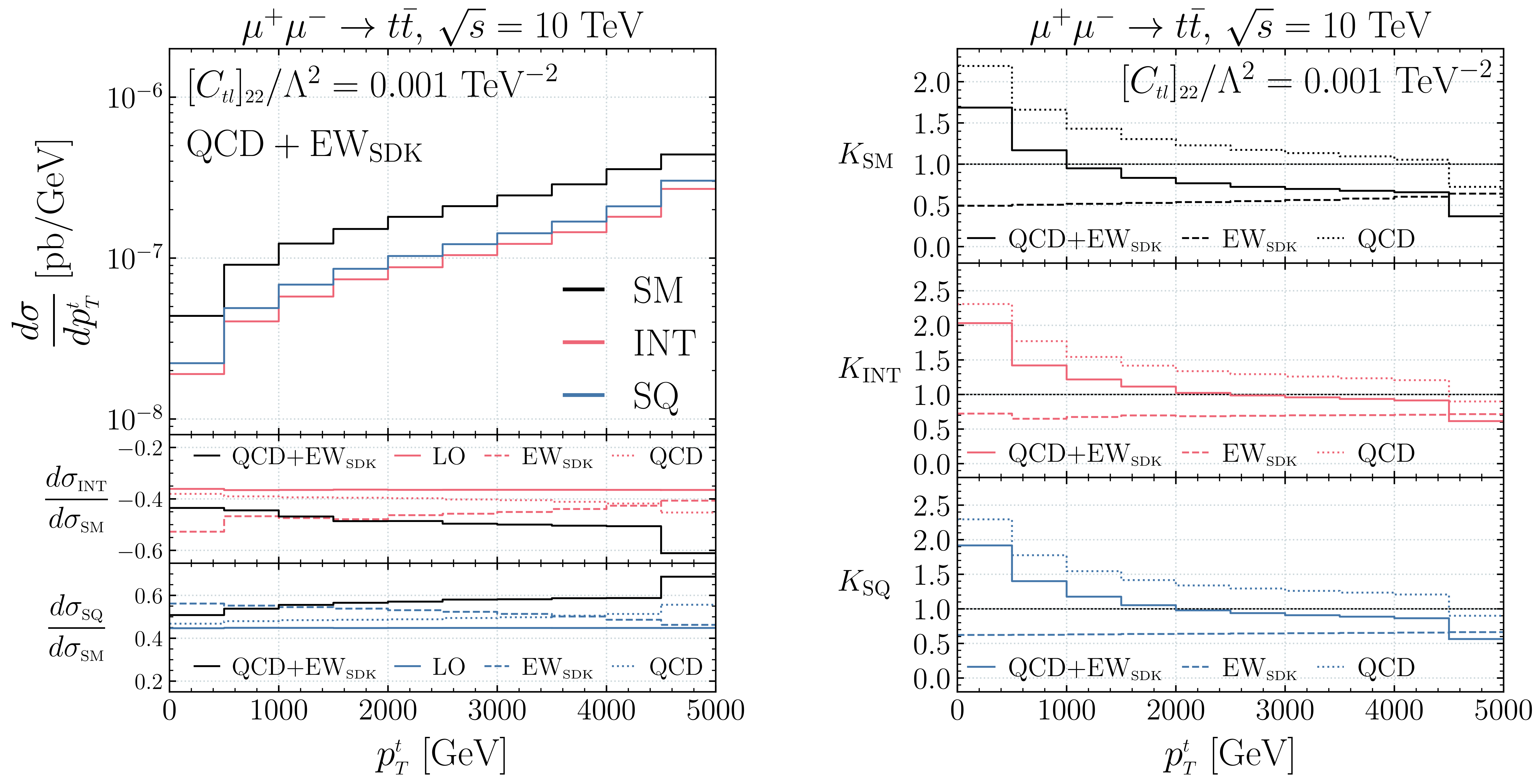
This is clearly a limitation and indicates that the exact NLO EW is necessary also for SMEFT. However this limitation applies to those processes which are not maximally growing with the energy and so have less sensitivity on possible BSM dynamics.

El Faham, Mimasu, DP, Severi, Vryonidou, Zaro '24

EW Sudakov and SMEFT: $t\bar{t}$

Only Four-Fermion operators are considered in the study.

$$\mathcal{O}_{tl} = (\bar{t}\gamma^{\mu}t)(\bar{l}_i\gamma_{\mu}l_i)$$



10 Tev μ-coll

Both QCD (exact) and EW (Sudakov)corrections are different for SM, SM-SMEFT interference, and SMEFT^2 contributions of dim-6.

QCD and EW cancel each other: both are important.

K-factors can be different in SM and BSM!

El Faham, Mimasu, DP, Severi, Vryonidou, Zaro '24

CONCLUSION

- The high-energy **Muon Collider** can be seen as a **VV collider**.
- **PDFs** of the muon are available both including **Weak** effects at **LL** or **without them at NLL**. Dedicated studies with comparisons with ME are necessary.
- EW corrections are mandatory for phenomenology at a muon collider colliders, especially for high energies. **Not only for the SM also for BSM!**
- **Sudakov logs** are the dominant contribution of EW corrections at high energy and they are a **good approximation** of them, but only under certain conditions.
- **Heavy-Boson Radiation** has an impact, but not always so large and typically **smaller than** the **virtual** contributions.
- **Resummation may be mandatory for sensible results** in many configurations and in general for precision.
- **Effects** observed **in** the **SM may be different with BSM** (see SMEFT example). Still, EW corrections are important and dedicated studies are necessary,

EXTRA SLIDES

Calculation set up for showcasing some results

$\mu^+\mu^- \longrightarrow X$, where X is a generic final state involving W, Z, t, H, ℓ. Thus direct production, no VBF considered.

ISR Treatment: we use the LL PDF for the muon only

$$\Gamma_{\rm LO}(z) = \frac{\exp\left(3\beta_S/4 - \gamma_E \beta_E\right)}{\Gamma\left(1+\beta_E\right)}\, \beta_E (1-z)^{\beta_E - 1} - \frac{1}{2}\beta_H (1+z) + \mathcal{O}(\alpha^2)$$

Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao '22

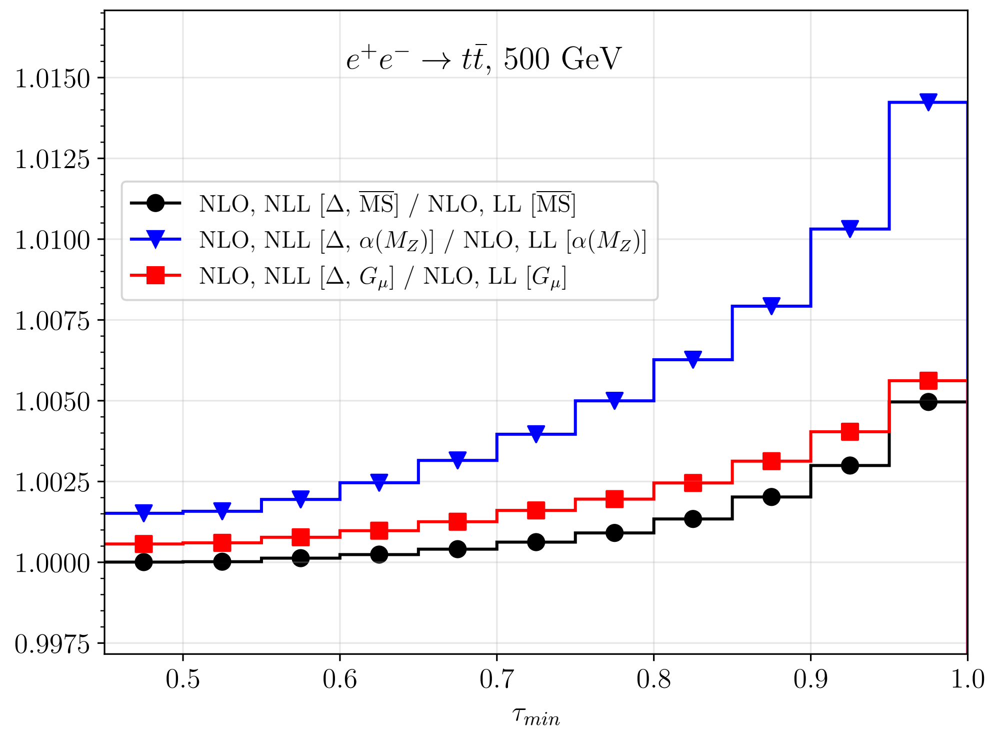
- Beta scheme:

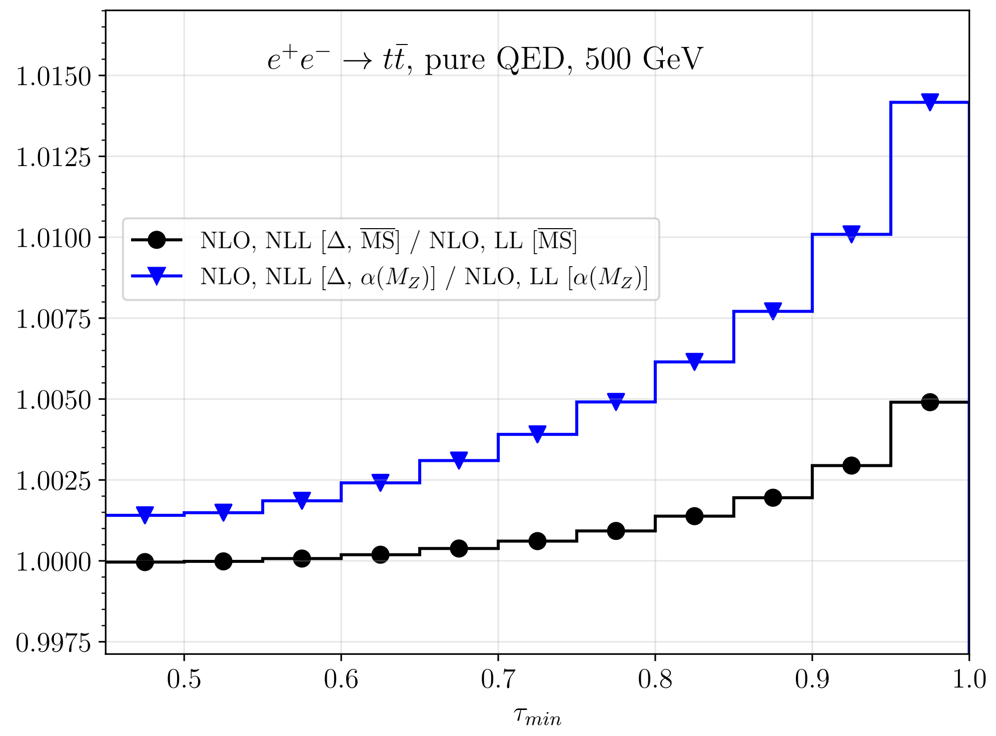
$$\beta_E = \beta_S = \beta_H = e_e^2 \beta\,.$$

- Eta scheme:

$$\beta_E = \beta_S = e_e^2 \beta\,, \qquad \beta_H = e_e^2 \eta\,.$$

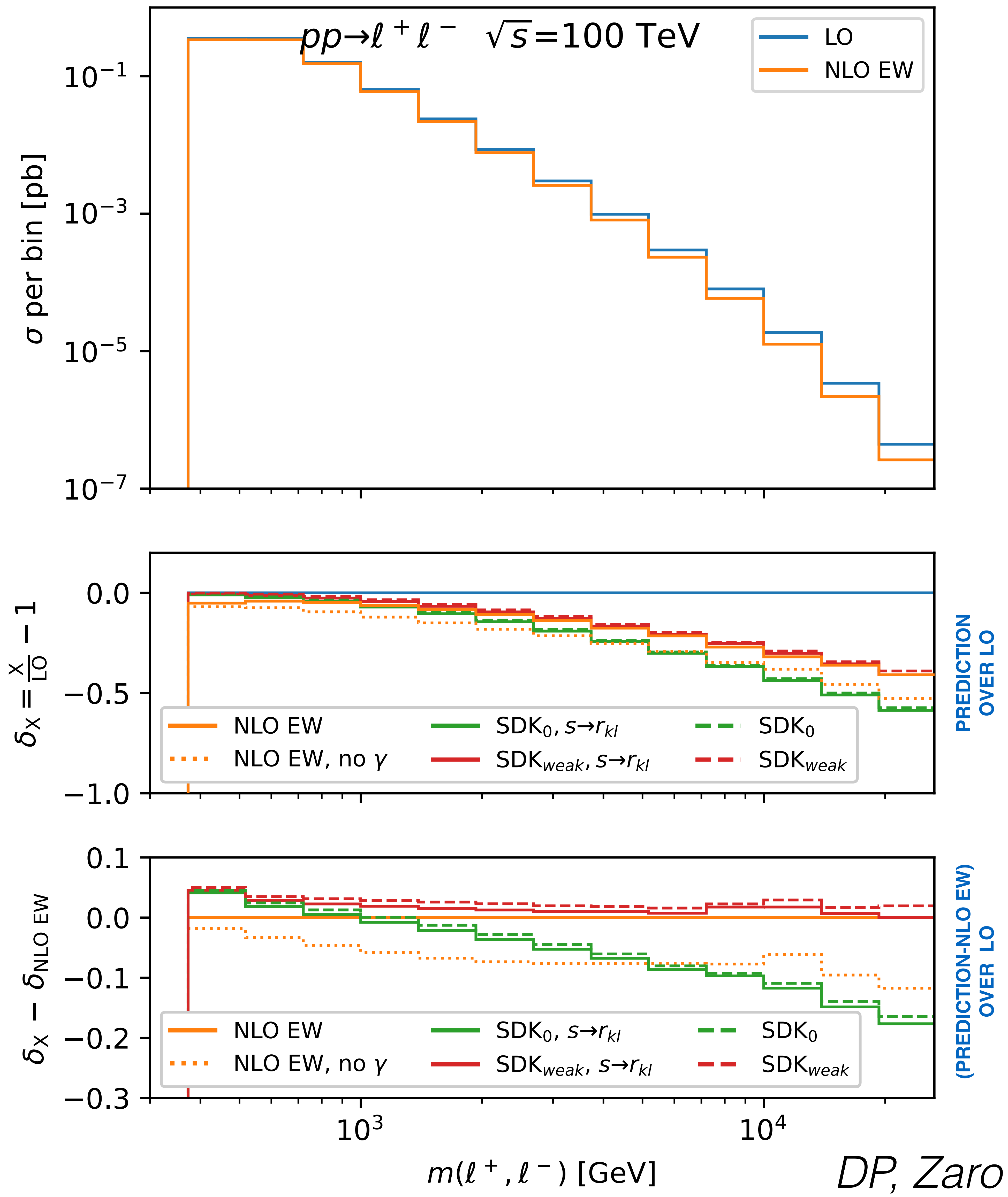
$$\eta = \frac{\alpha}{\pi} \log \frac{\mu^2}{m^2}\,, \qquad \beta = \frac{\alpha}{\pi}\left(\log\frac{\mu^2}{m^2} - 1\right)$$





For precision physics the scheme adopted and the NLL accuracy (*Frixione, Stagnitto '23*) are mandatory.

e^+e^- production at 100 TeV **FCC-hh**



DP, Zaro '21

$$p_T(\ell^{\pm}) > 200\ \text{GeV}, \quad |\eta(\ell^{\pm})| < 2.5, \quad m(\ell^+,\ell^-) > 400\ \text{GeV}, \quad \Delta R(\ell^+,\ell^-) > 0.5.$$

Orange: NLO EW, (**dotted**: NLO EW no γ PDF)
Green = SDK_0, **Red** = $\mathrm{SDK}_{\mathrm{weak}}$
Dashed: standard approach for amplitudes.
Solid: our formulation (more angular information)

Reference Prediction: Red-solid line

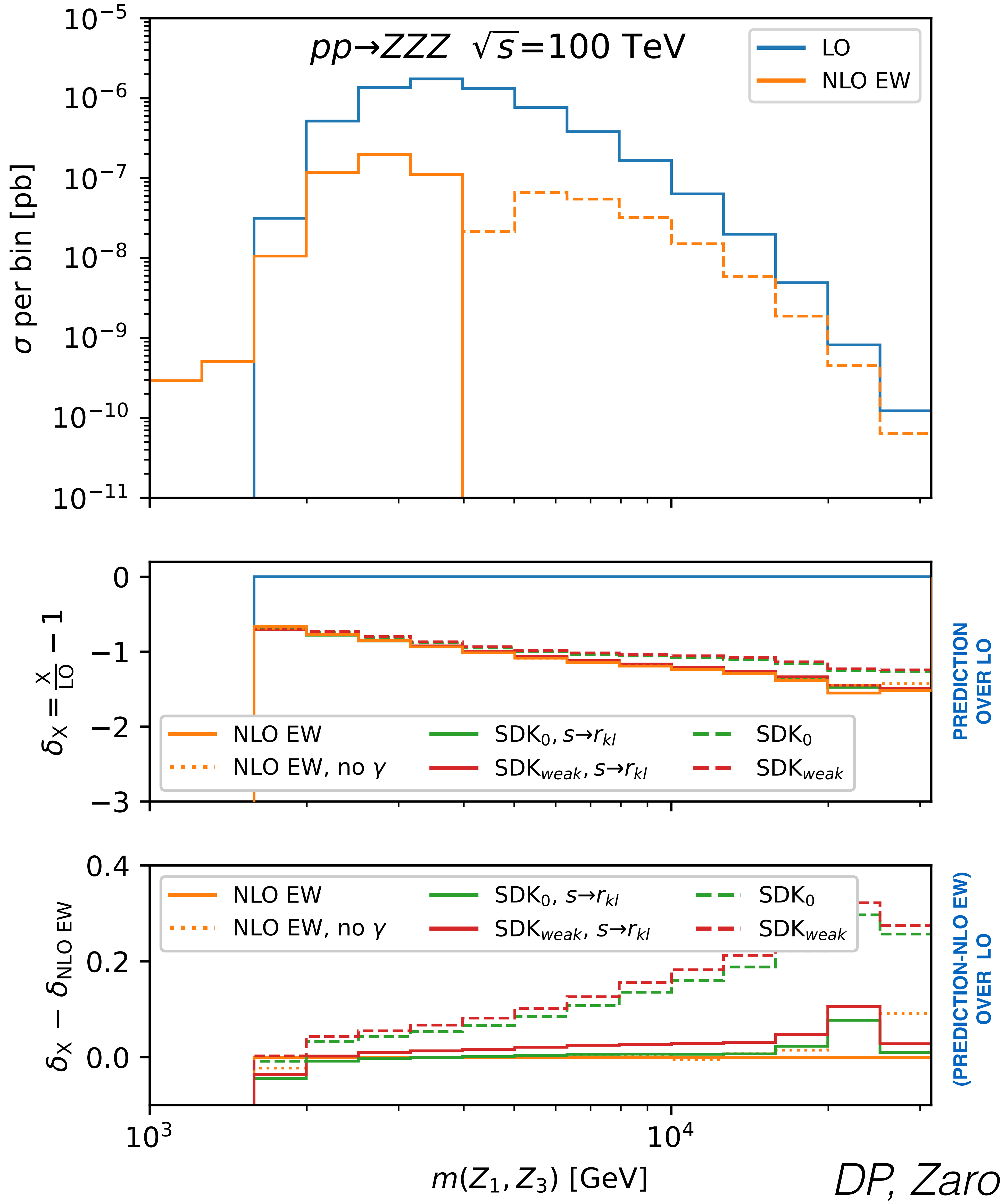
Only the $\mathrm{SDK}_{\mathrm{weak}}$ approach correctly captures the NLO EW prediction.

Solid and dashed very similar.

Photon PDF cannot be ignored.

Larger invariant -> larger correction

ZZZ production at 100 TeV **FCC-hh**



DP, Zaro '21

$$p_T(Z_i) > 1 \text{ TeV}, \quad |\eta(Z_i)| < 2.5, \quad m(Z_i, Z_j) > 1 \text{ TeV}, \quad \Delta R(Z_i, Z_j) > 0.5.$$

Orange: NLO EW, (**dotted**: NLO EW no γ PDF)
Green = SDK_0, **Red** = $\mathrm{SDK}_{\mathrm{weak}}$
Dashed: standard approach for amplitudes.
Solid: our formulation (more angular information)

Reference Prediction: Red-solid line

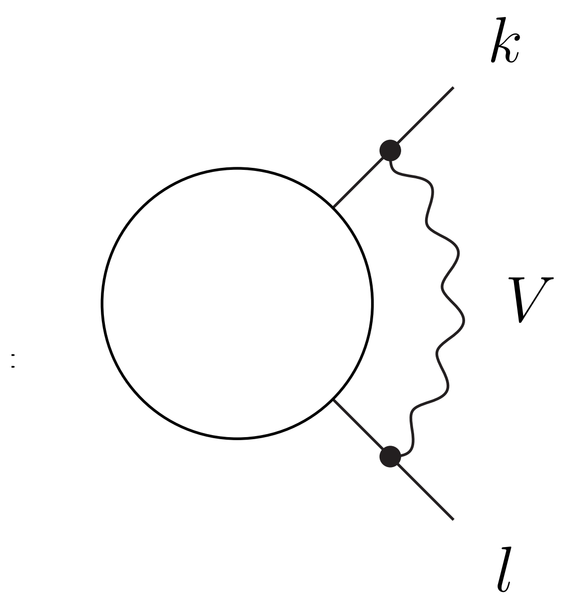
$\mathrm{SDK}_{\mathrm{weak}}$ and SDK_0 not so relevant for neutral final state).

Only the solid lines, having more angular information, correctly capture NLO EW.

One cannot forget terms as $\log^2[m^2(Z_2, Z_3)/s]$

Larger invariant -> larger correction

Derivation of LSC and SSC



Denner&Pozzorini

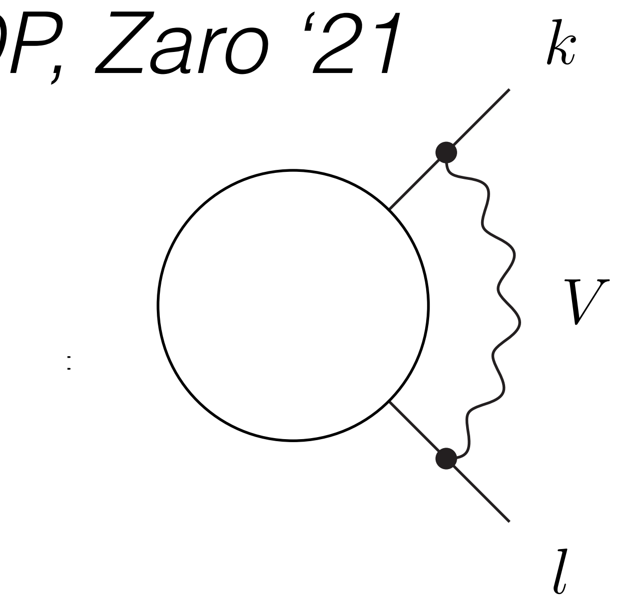
$$L(|r_{kl}|, M^2) = L(s, M^2) + 2l(s, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s)$$

$$= \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\text{LSC}} + \underbrace{2l(s)\log\frac{|r_{kl}|}{s}}_{\text{SSC}} + \cdots$$

$$L(s) \equiv L(s, M_W^2) \quad \text{and} \quad l(s) \equiv l(s, M_W^2)$$

The relation $r_{kl} = r_{k'l'} = s$ is used in all logs, unless they multiply $l(s)$.

DP, Zaro '21



Our approach: ~~$r_{kl} = r_{k'l'} = s$~~ **in the expressions**

$$C_0(p_k, p_l, M, M_k, M_l) \;\Rightarrow\; L(|r_{kl}|, M^2) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2)$$

Previously omitted imaginary term

$$L(|r_{kl}|, M^2) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, M^2) =$$

$$= L(s, M^2) + 2l(s, M^2)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right) + L(|r_{kl}|, s) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, s) =$$

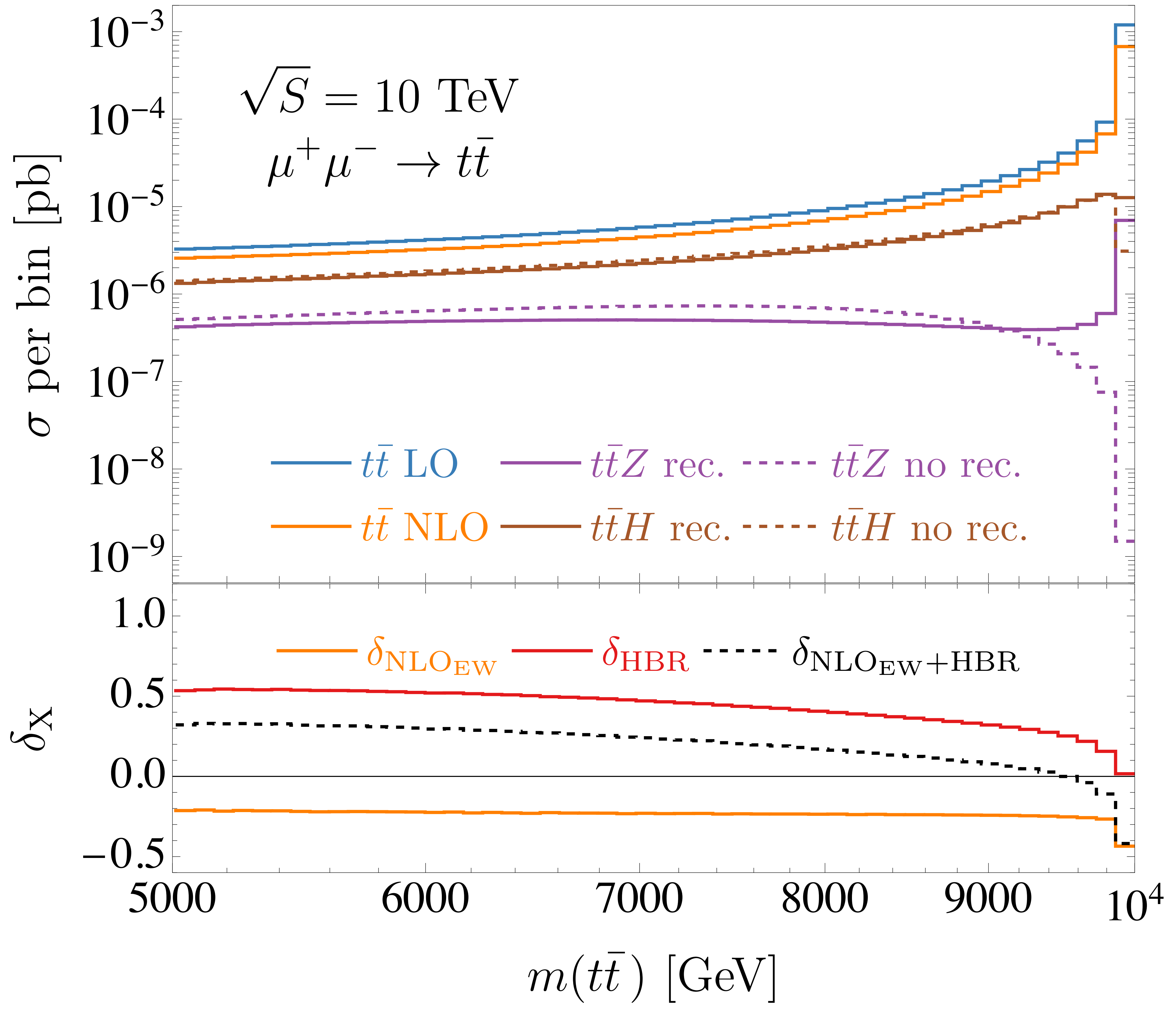
$$= \underbrace{L(s) + 2l(s)\log\frac{M_W^2}{M^2}}_{\text{LSC}} + \underbrace{2l(s)\left(\log\frac{|r_{kl}|}{s} - i\pi\Theta(r_{kl})\right)}_{\text{SSC}} +$$

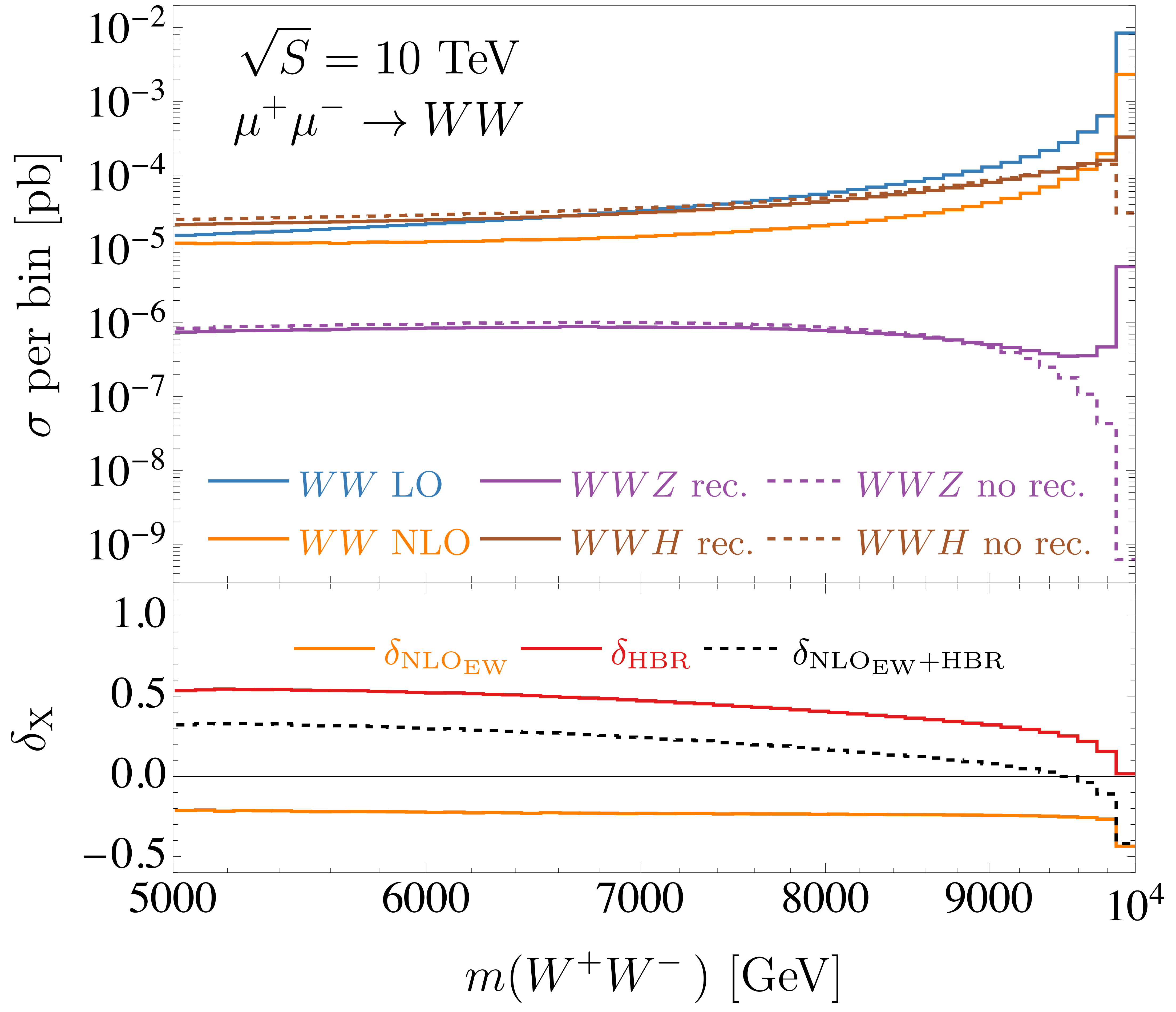
$$\underbrace{2l(M_W^2, M^2)\log\frac{|r_{kl}|}{s} + L(|r_{kl}|, s) - 2i\pi\Theta(r_{kl})l(|r_{kl}|, s)}_{\text{SSC}^{s\to r_{kl}}} + \cdots$$

The conceptual derivation relies on the assumption $s = r_{kl}$, but is not actually used in the expressions. Therefore, further angular dependencies are taken into account.

New angular dependences via ratios among invariants

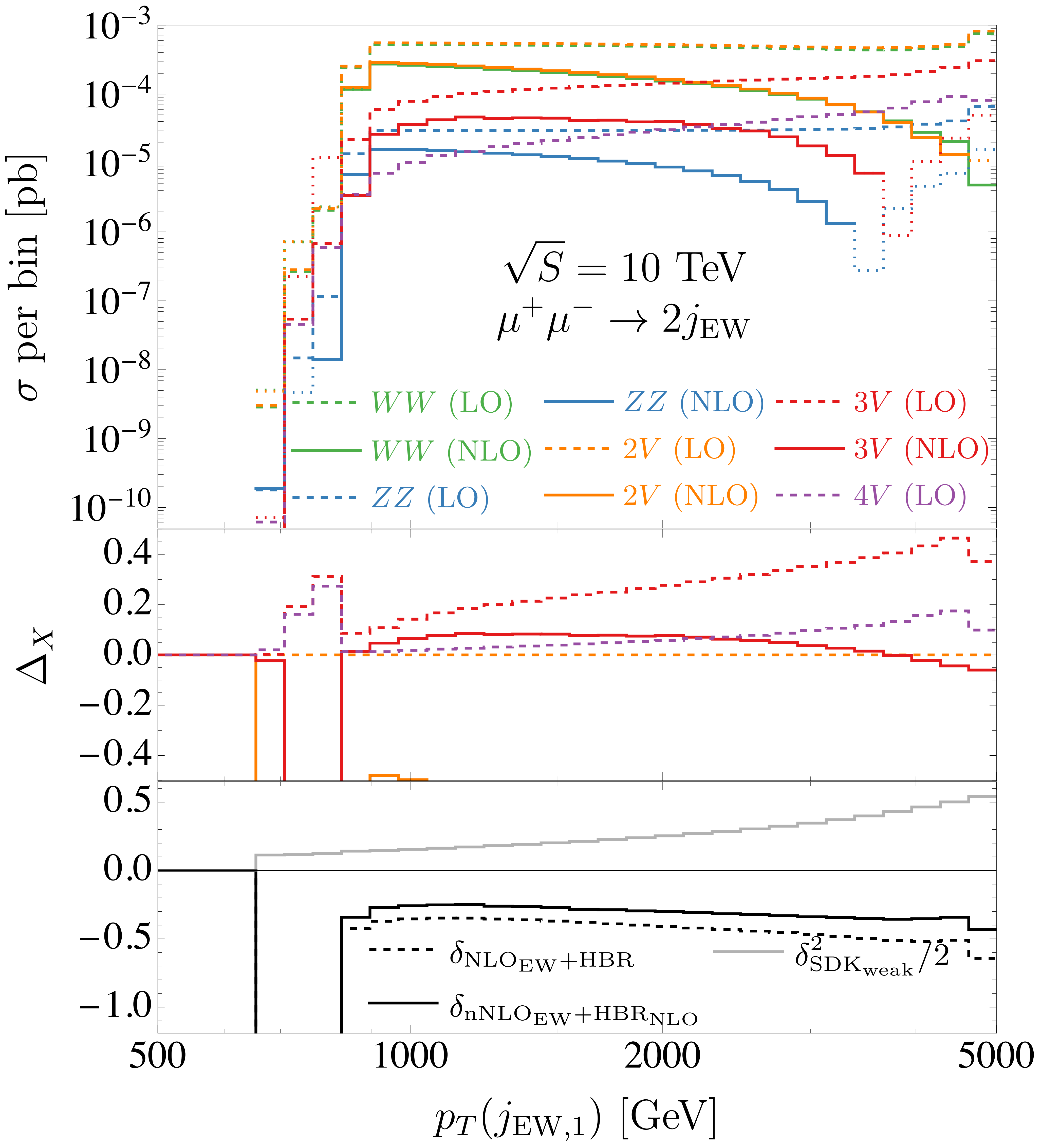
more on Z and H radiation





Ma, DP, Zaro ‘ 24

EW jets



$$\sigma_X(2j_{\rm EW}) \equiv \sigma_X(2V) \qquad \text{for } X = \text{LO, NLO EW, SDK}_{\rm weak}$$

$$\sigma_{\rm HBR}(2j_{\rm EW}) \equiv \sigma_{\rm LO}(3V)\,,$$

$$\sigma_{\rm NLO_{EW}+HBR}(2j_{\rm EW}) \equiv \sigma_{\rm NLO_{EW}}(2V) + \sigma_{\rm LO}(3V)$$

$$\sigma_{\rm nNLO_{EW}+HBR_{NLO}}(2j_{\rm EW}) \equiv \sigma_{\rm LO}(2V)\left(1 + \delta_{\rm NLO_{EW}} + \frac{\delta^2_{\rm SDK_{weak}}}{2}\right) + \sigma_{\rm NLO_{EW}}(3V) + \sigma_{\rm LO}(4V)\,.$$

$$\Delta_X(2V) \equiv \frac{\sigma_X(2V) - \sigma_{\rm LO}(2V)}{\sigma_{\rm LO}(2V)}$$

$$\Delta_X(3V) \equiv \frac{\sigma_X(3V)}{\sigma_{\rm LO}(2V)}$$

$$\Delta_X(4V) \equiv \frac{\sigma_X(4V)}{\sigma_{\rm LO}(2V)}$$

It is a general pattern: radiation of heavy bosons is less important than loops!

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Cross-sections: our approach.

FOR WHAT EW SUDAKOV ARE USEFUL?
For providing a very **good approximation of NLO EW** in the **high-energy** limit.

HOW SHOULD ONE PERFORM THE CALCULATION IN THE HIGH-ENERGY LIMIT?
Photons have to be **always clustered with massless charged particle for IR**-safety reasons. But from an experimental point of view, **at high energy also clustering tops and W bosons with photons** is very reasonable, either if you imagine to tag heavy object directly or via their massless decay products.

The **QED Logs,** involving s and λ^2 (or Q^2), **cancel against their real-emission counterparts and PDF counterterms**. The only one surviving are those from tops in vacuum polarisation for external (not tagged) photons, both in the initial and final state:

$$\mathrm{SDK}_{\mathrm{weak}}$$

Almost all the contributions of QED are removed
(e.g. $C_{\mathrm{EW}}(k) \to C_{\mathrm{EW}}(k) - Q_k^2, Q_k^2 = 0$),
but NOT in the parameter renormalisation δ^{PR}.

DP, Zaro '21

Organisation of the logs in the algorithm

Two examples: LSC and C for fermions

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}_{i'_k i_k}(k) L(s) - 2(I^Z(k))^2_{i'_k i_k} \log\frac{M_Z^2}{M_W^2} l(s) + \delta_{i'_k i_k} Q_k^2 L^{\mathrm{em}}(s, \lambda^2, m_k^2)\right]$$

Casimir for the entire $SU(2)_L \times U(1)_B$

Charge for $U(1)_{QED}$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2}C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8s_{\mathrm{w}}^2}\left((1+\delta_{\kappa\mathrm{R}})\frac{m^2_{f_\sigma}}{M_{\mathrm{W}}^2} + \delta_{\kappa\mathrm{L}}\frac{m^2_{f_{-\sigma}}}{M_{\mathrm{W}}^2}\right)\right] l(s) + Q^2_{f_\sigma} l^{\mathrm{em}}(m^2_{f_\sigma})\right\},$$

$L(s) \equiv L(s, M_W^2)$ and $l(s) \equiv l(s, M_W^2)$

$$l^{\mathrm{em}}(m_f^2) := \frac{1}{2} l(M_{\mathrm{W}}^2, m_f^2) + l(M_{\mathrm{W}}^2, \lambda^2) \qquad L^{\mathrm{em}}(s, \lambda^2, m_k^2) := 2l(s)\log\left(\frac{M_{\mathrm{W}}^2}{\lambda^2}\right) + L(M_{\mathrm{W}}^2, \lambda^2) - L(m_k^2, \lambda^2)$$

The full EW is present between s and M_W^2, while only QED is present between M_W^2 and λ^2.

So the QED contribution is split between the intervals $(s, M_W^2) + (M_W^2, \lambda^2)$. But the division at M_W^2 is simply determined by convenience, in parallel with the weak case. In this case M_W^2 is just a technical parameter and not a physical quantity.

Cross-sections: standard approach in the literature

SDK_0

Two examples: LSC and C for fermions

$$\delta^{\mathrm{LSC}}_{i'_k i_k}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}_{i'_k i_k}(k) L(s) - 2(I^Z(k))^2_{i'_k i_k} \log\frac{M_Z^2}{M_W^2}\, l(s) + \delta_{i'_k i_k} Q_k^2 \ldots (s, \lambda^2, m_k^2) \right]$$

Casimir for the entire $SU(2)_L \times U(1)_B$

$$\delta^{\mathrm{C}}_{f_\sigma f_{\sigma'}}(f^\kappa) = \delta_{\sigma\sigma'}\left\{\left[\frac{3}{2} C^{\mathrm{ew}}_{f^\kappa} - \frac{1}{8 s_W^2}\left((1+\delta_{\kappa \mathrm{R}})\frac{m^2_{f_\sigma}}{M_W^2} + \delta_{\kappa \mathrm{L}}\frac{m^2_{f_{-\sigma}}}{M_W^2}\right)\right] l(s) + Q^2_{f_\sigma} \ldots m^2_{f_\sigma}) \right\}$$

$L(s) \equiv L(s, M_W^2)$ and $l(s) \equiv l(s, M_W^2)$

The logarithms between M_W^2 and the infrared scale are simply removed. Equivalently in the case of DR, logarithms involving M_W^2 and the IR regulator Q^2.

Easy, but not very well motivated.

We will denote in the following this approach as SDK_0.