Montecarlo and tools for lepton colliders

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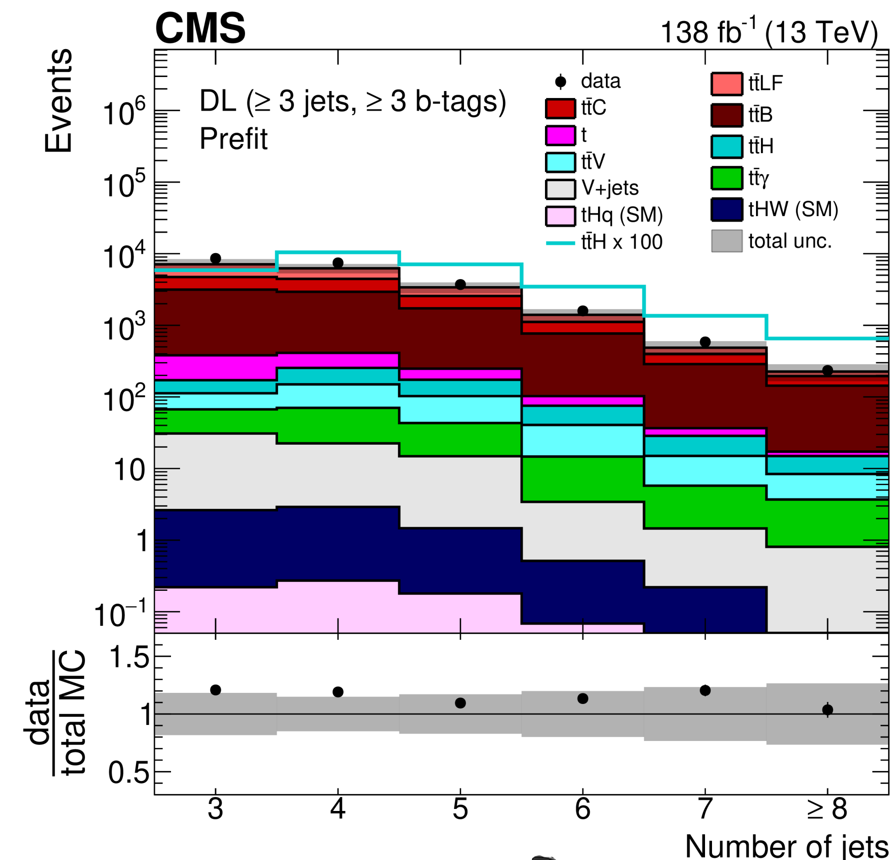
From the LHC to a future collider

- MonteCarlo generators are fundamental workhorses for collider physics
  - Assess the reach of new machines
  - Extract results and compare theory vs data
- Lepton colliders demand <0.1% precision
  - Prominent role of EW corrections
  - NLO mandatory, often not enough
- In general, two kind of tools:
  - Process specific, with the highest achievable accuracy
  - General-purpose (mostly relying on automatic techniques), with a satisfactory accuracy



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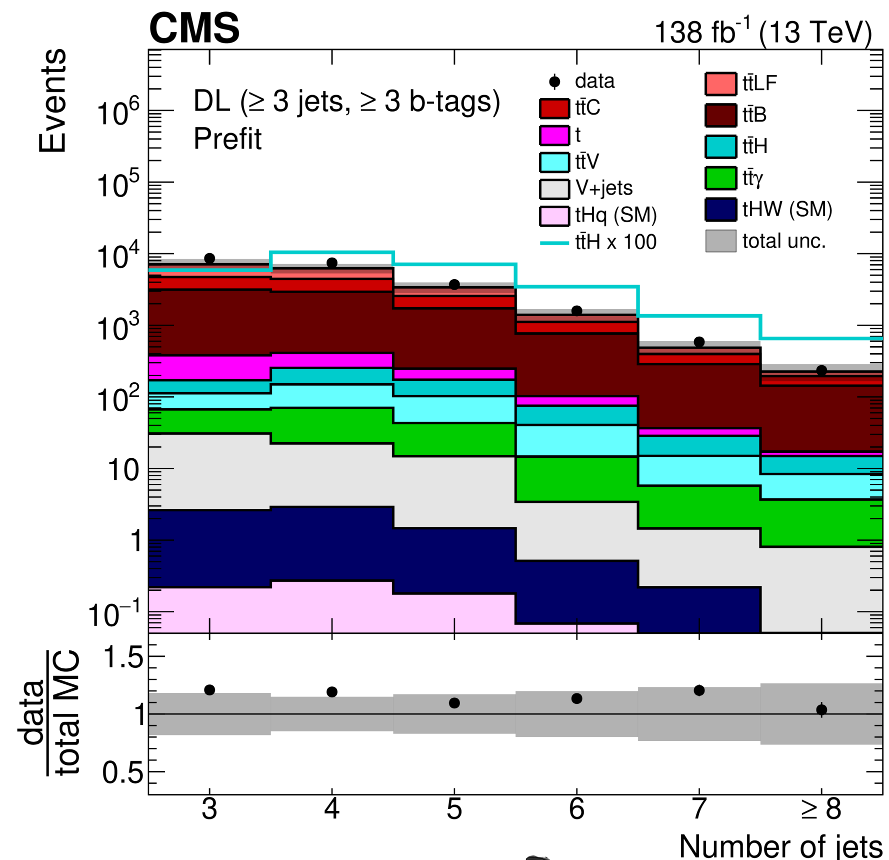


What I'll present focuses on the latter, but may be relevant also for the former

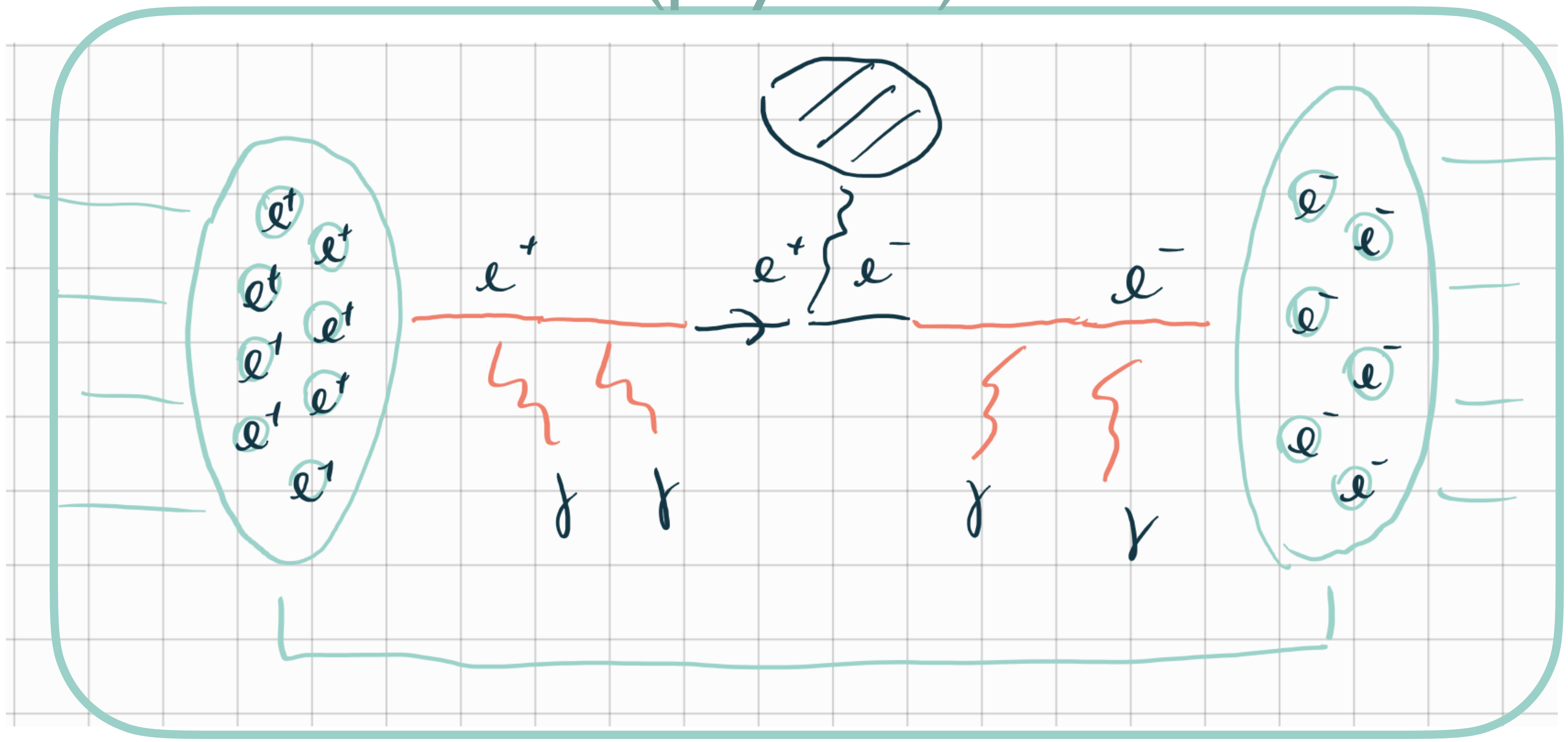
Lepton collisions

(In a collinear-factiorisation inspired picture)

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{kl} \int dy_+ dy_- \, \mathcal{B}_{kl}(y_+, y_-) \, d\sigma_{kl}(y_+ P_{e^+}, y_- P_{e^-})$$

$$d\sigma_{kl}(p_k, p_l) = \sum_{ij} \int dz_+ dz_+ \, \Gamma_{i/k}(z_+, \mu, m) \, \Gamma_{j/l}(z_-, \mu, m) \, d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu)$$

Beam-level (physical) cross section



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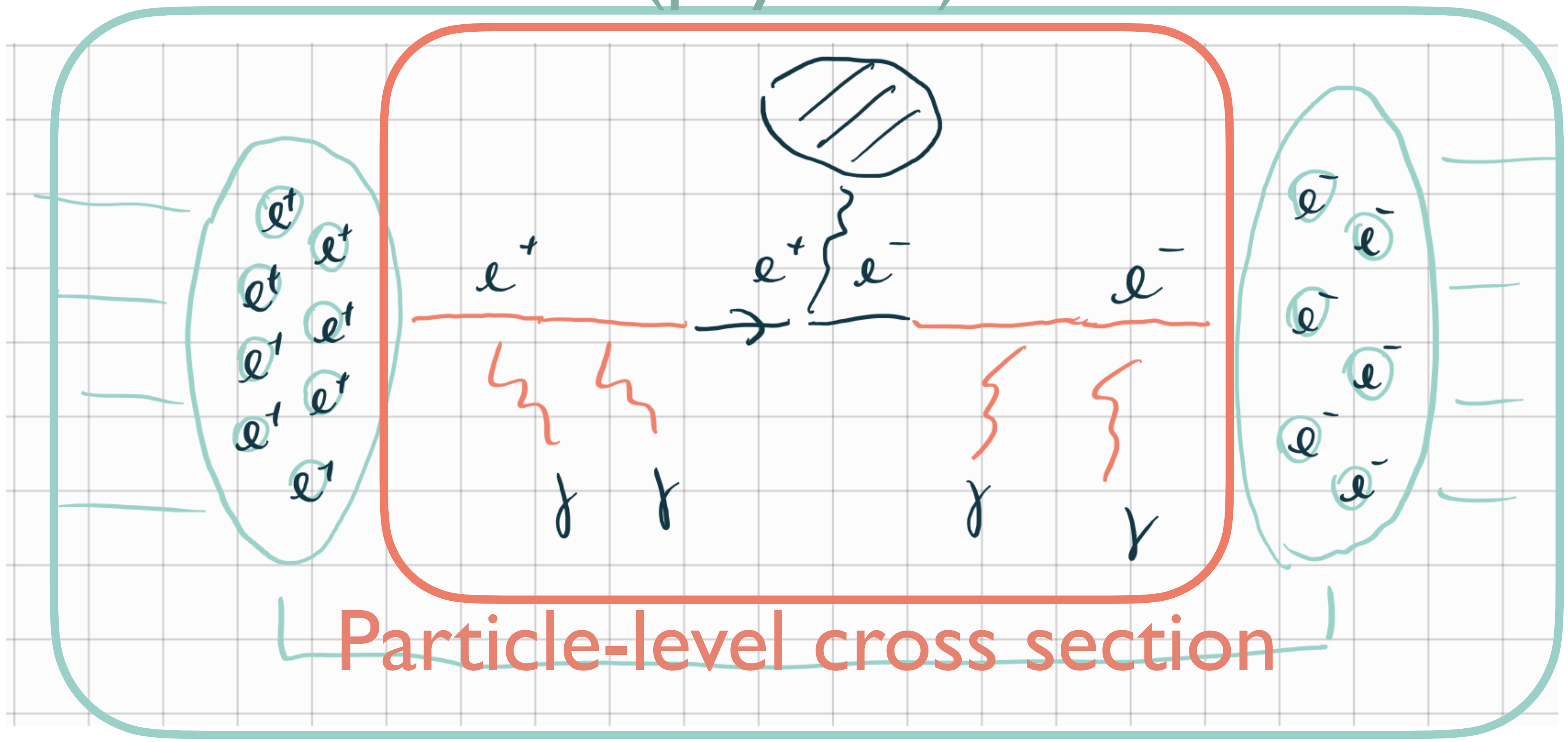

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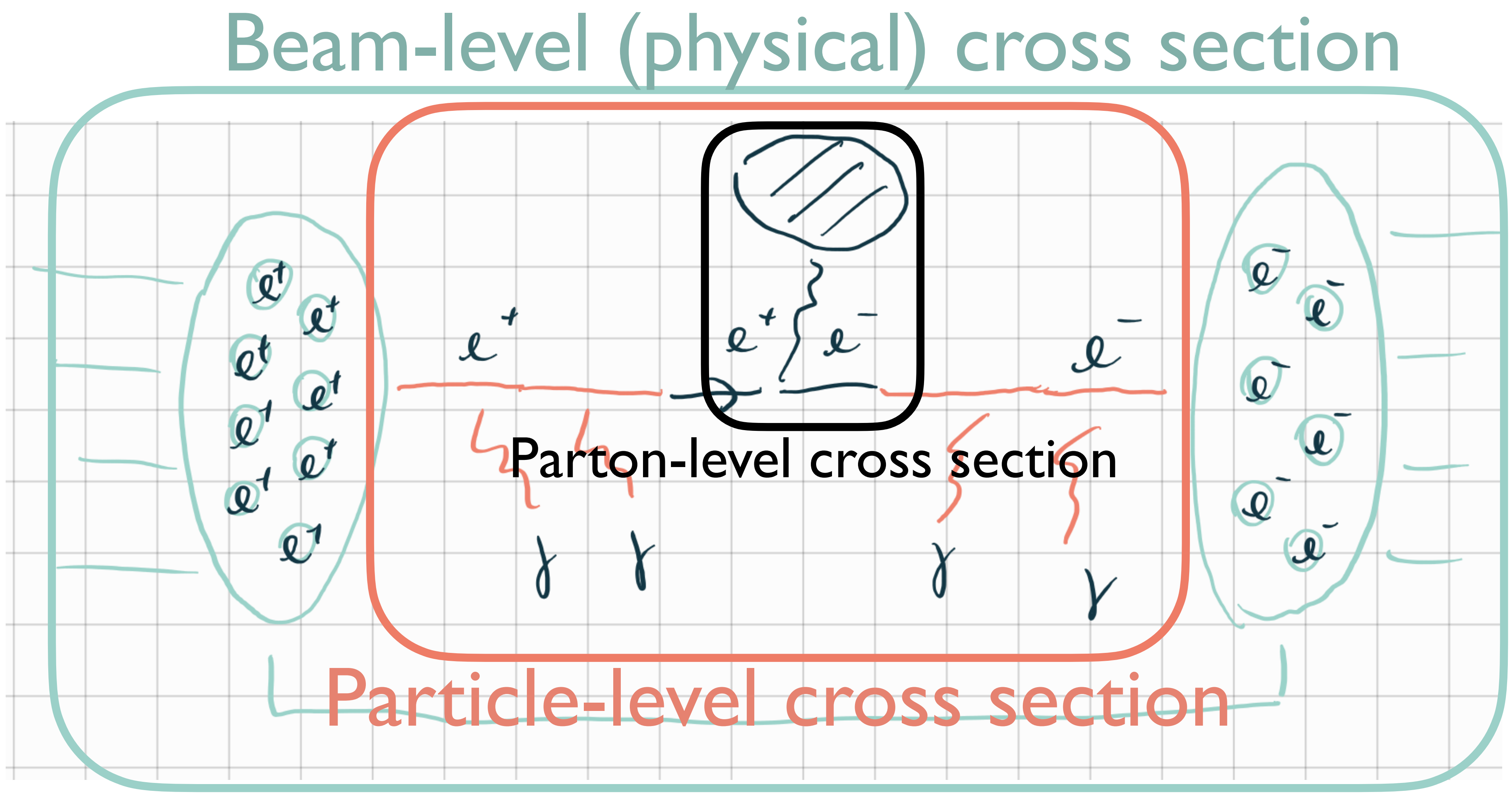


Lepton collisions

- **Beam-beam interactions** (aka beamstrahlung) are machine-dependent collective effects. Can be fitted with ad-hoc tools (e.g. GuineaPig, Circe, …). Less important for circular colliders than for linear ones
- **ISR** is universal (like hadronic PDFs), and can be computed perturbatively (*un*like hadronic PDFs)
- The **partonic cross section** is the usual one. Because of the form of PDFs, needs new phase-space and momentum mappings

Beamstrahlung

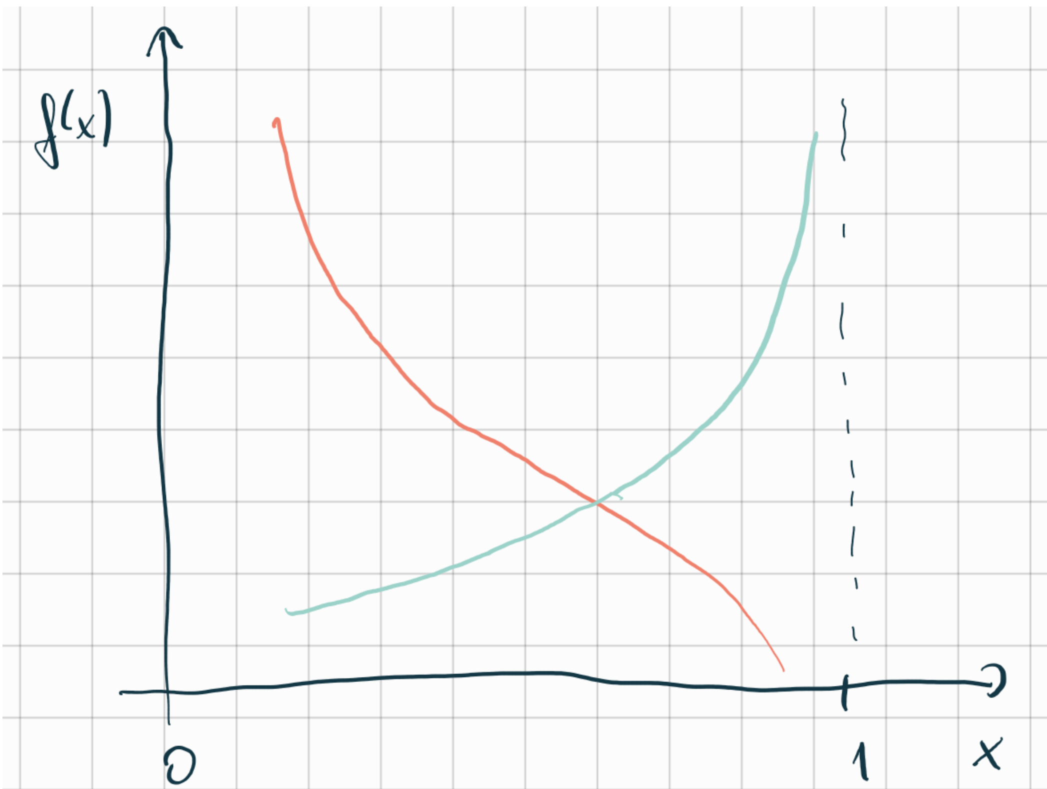
- Description of beam-beam interactions outside the scope of MC generators. Specific tools exist
- E.g.: simulated beamstrahlung with GuineaPig, and fit the spectra (energy spread functions) with Circe
  Circe: Ohl, hep-ph/9607454; GuineaPig: Schulte, 1998
- Specific to collider parameters
- Effects are usually tabulated as grids, pre-convoluted with PDFs
- Need special care when matching with parton-shower:
  - The PS enters between ISR and beamstrahlung

ISR

at LL: GRIBOV V. N. and LIPATOV L. N. Sow. J . Nucl. Phys., 15 (1972) 438

$$D_{\rm GL}(x,Q^2) = \frac{\exp\left[(1/2)\eta(3/4-\gamma_{\rm E})\right]}{\Gamma(1+(1/2)\eta)}\frac{1}{2}\eta(1-x)^{(1/2)\eta-1} \simeq \frac{1}{(1-x)^{1-\eta/2}} = \delta(1-x)+\mathcal{O}(\eta)$$

$$\left(\eta = \frac{2\alpha}{\pi}\log\frac{Q^2}{m^2}\right)$$ ~0.05 for Q=100 GeV



- **Hadronic PDFs** vanish at large x (divergence at small-x avoided by cuts)
- **Leptonic PDFs** diverge (but are integrable) at large x
- While leptonic PDFs have been substantially improved since 1972, the asymptotic behaviour is unchanged
- A different phase space mapping is required wrt pp collisions

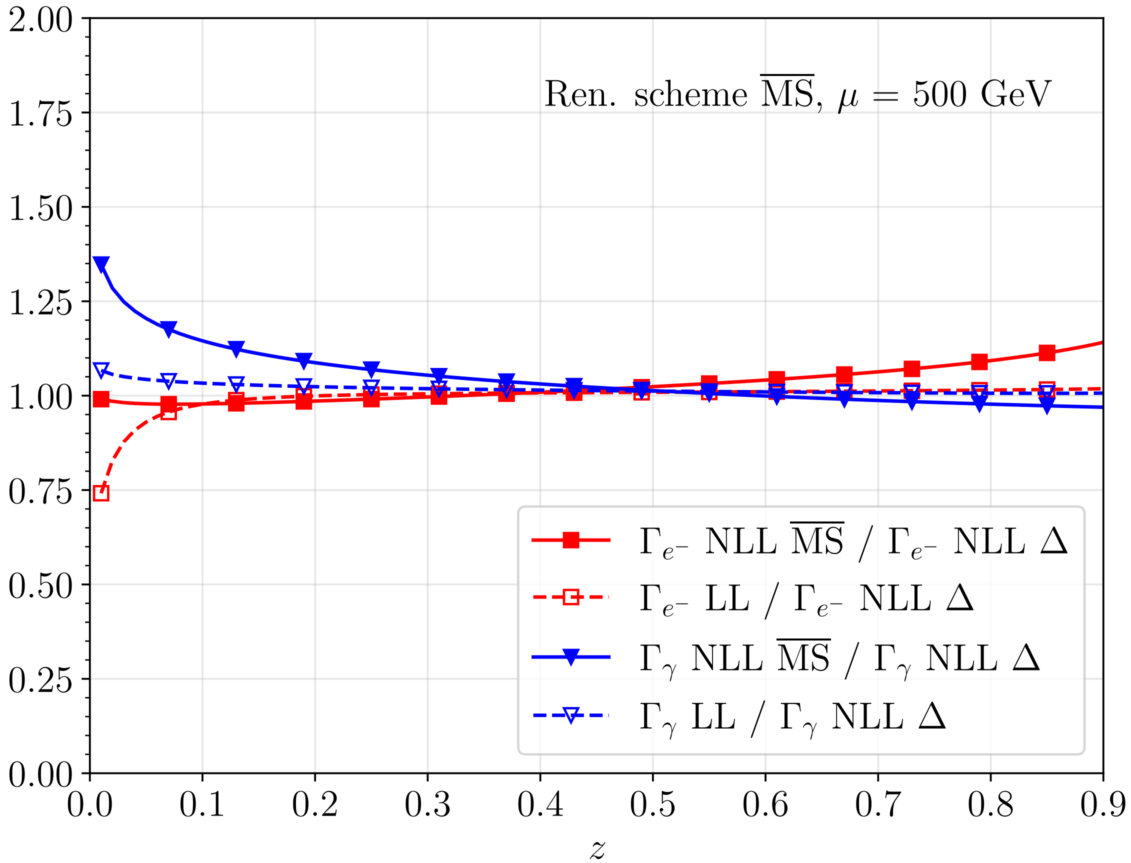
ISR for e^+e^-: State of the art

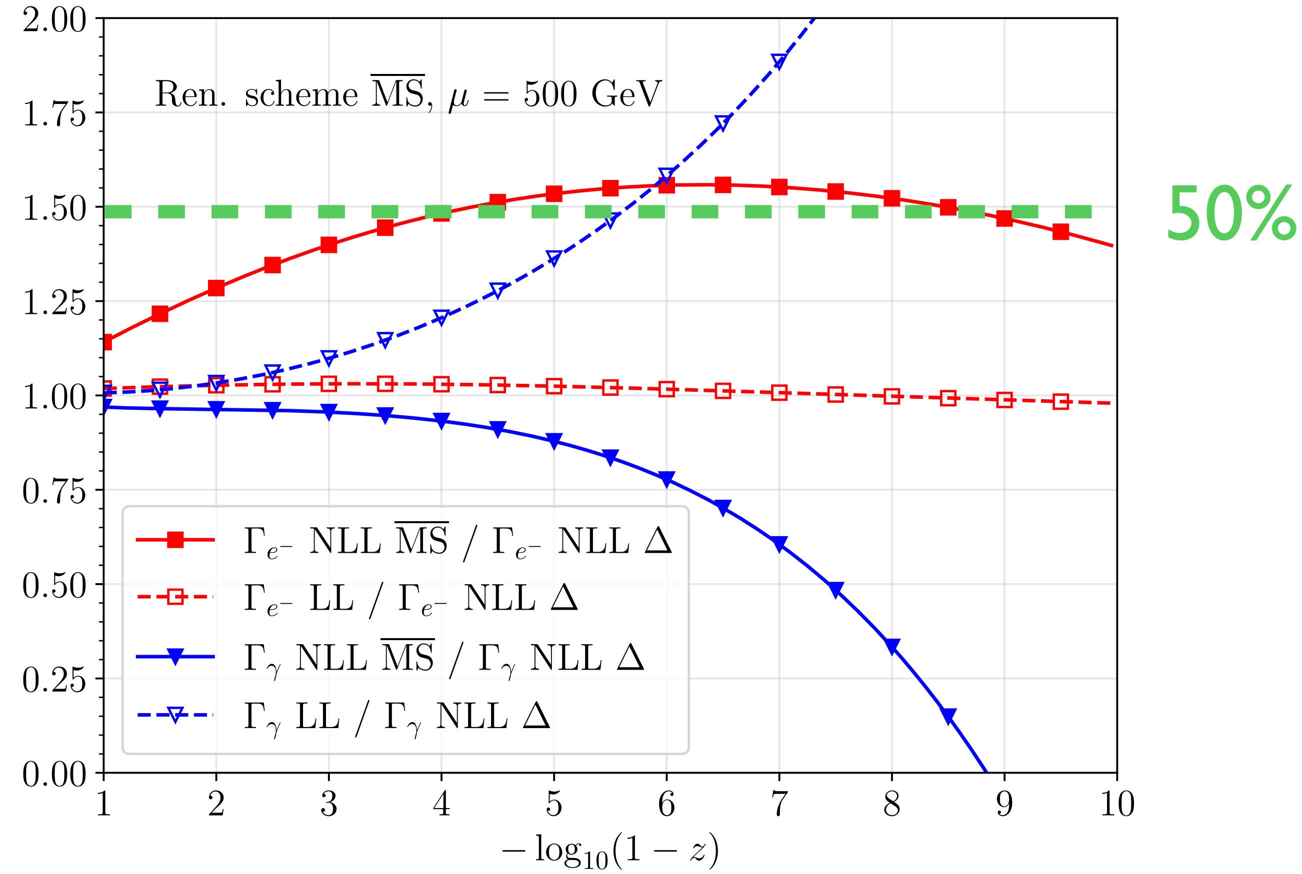
- Recently, the ISR structure function was obtained at NLL accuracy[1]. This required the NLO initial conditions[2]
- A new factorisation scheme (alternative to MSbar) has been proposed (Δ-scheme)[3], which improves the behaviour of the evolved PDF at large x
- PDFs available with α in three ren. scheme: G_μ, $\alpha(m_Z)$, MSbar (with proper treatment of all thresholds)[4]. All available within eMELA[5]
- Photon and e^+-in-e^- densities available as well

1 Bertone, Cacciari, Frixione, Stagnitto, 1911.12040
2 Frixione 1909.03886
3 Frixione 2105.06688
4 Bertone, Cacciari, Frixione, Stagnitto, MZ, Zhao 2207.03265
5 https://github.com/gstagnit/eMELA

ISR at NLL

Bertone, Cacciari, Frixione, Stagnitto, MZ, Zhao 2207.03265

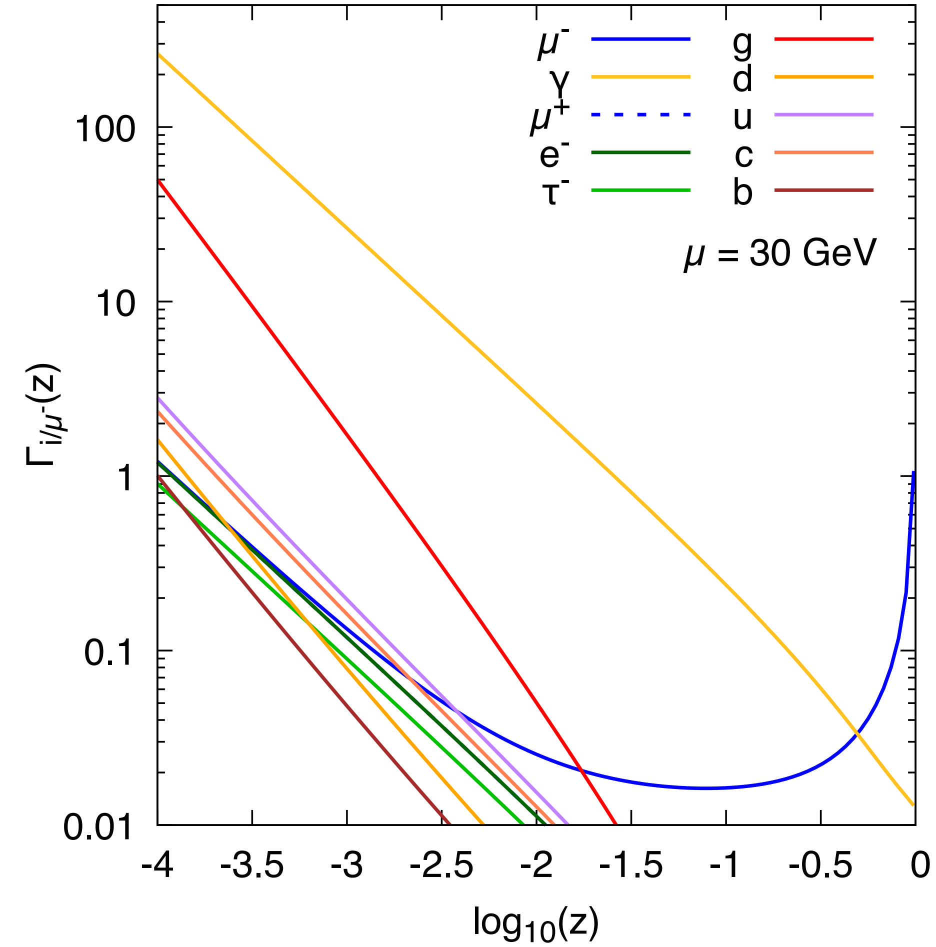


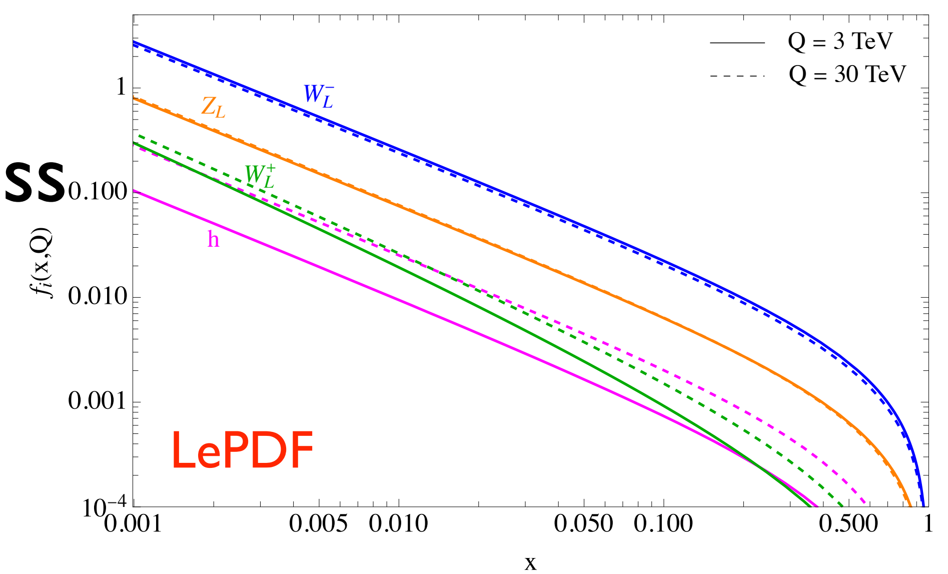


- NLL-MSbar seems very different from LL, while NLL-Δ is closer
- Differences between NLL-MSbar and NLL-Δ are 10-50% at large x
- Physical cross sections (NLO-accurate) will display much smaller discrepancies

ISR for $\mu^+\mu^-$

- In principle, a muon is just a heavier electron, so most of what has been said so far should apply
- However, muon colliders can reach far higher energy, lower values of x are probed
  - The photon, and even quarks and gluons are important in this region
- Muon density including all massless/light particles available in eMELA at LL (QED+QCD), WIP at NLL (+small x)
  Frixione, Stagnitto, 2309.07516, +Bonvini WIP
- In principle, one can also treat W/Z as massless parsons in those regions where $m_W \ll \hat{s}$
  Han, Ma, Xe, 2007.14300,
  Garosi, Marzocca, Trifinopoulos, 2303.16964 (LePDF)
  See Davide Pagani's talk





General-purpose MCs for lepton colliders

MG5_aMC

Whizard

SHERPA

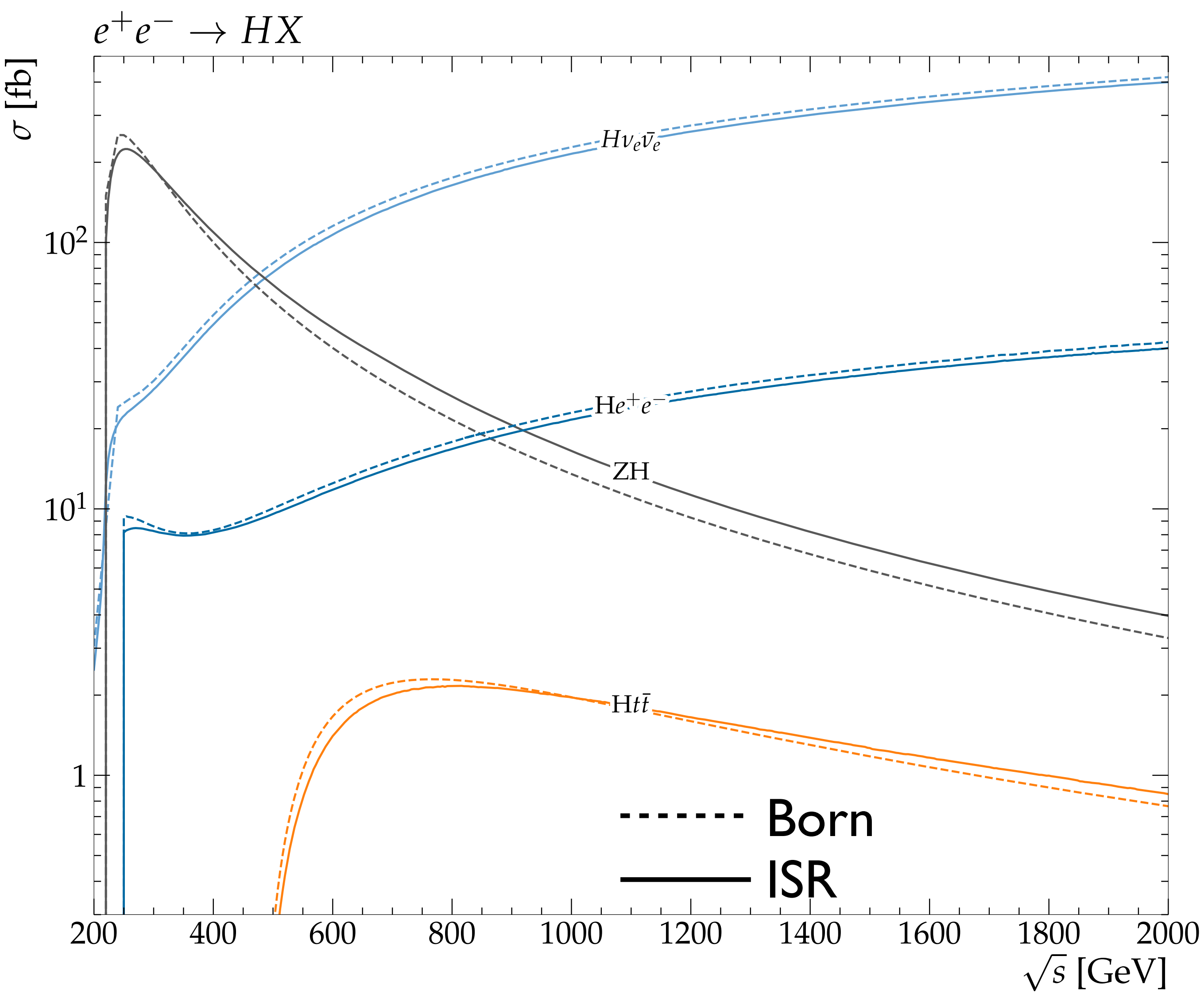
Sherpa3 at lepton colliders

Bothmann et al, 2410.22148; Krauss et al, 2203.10948

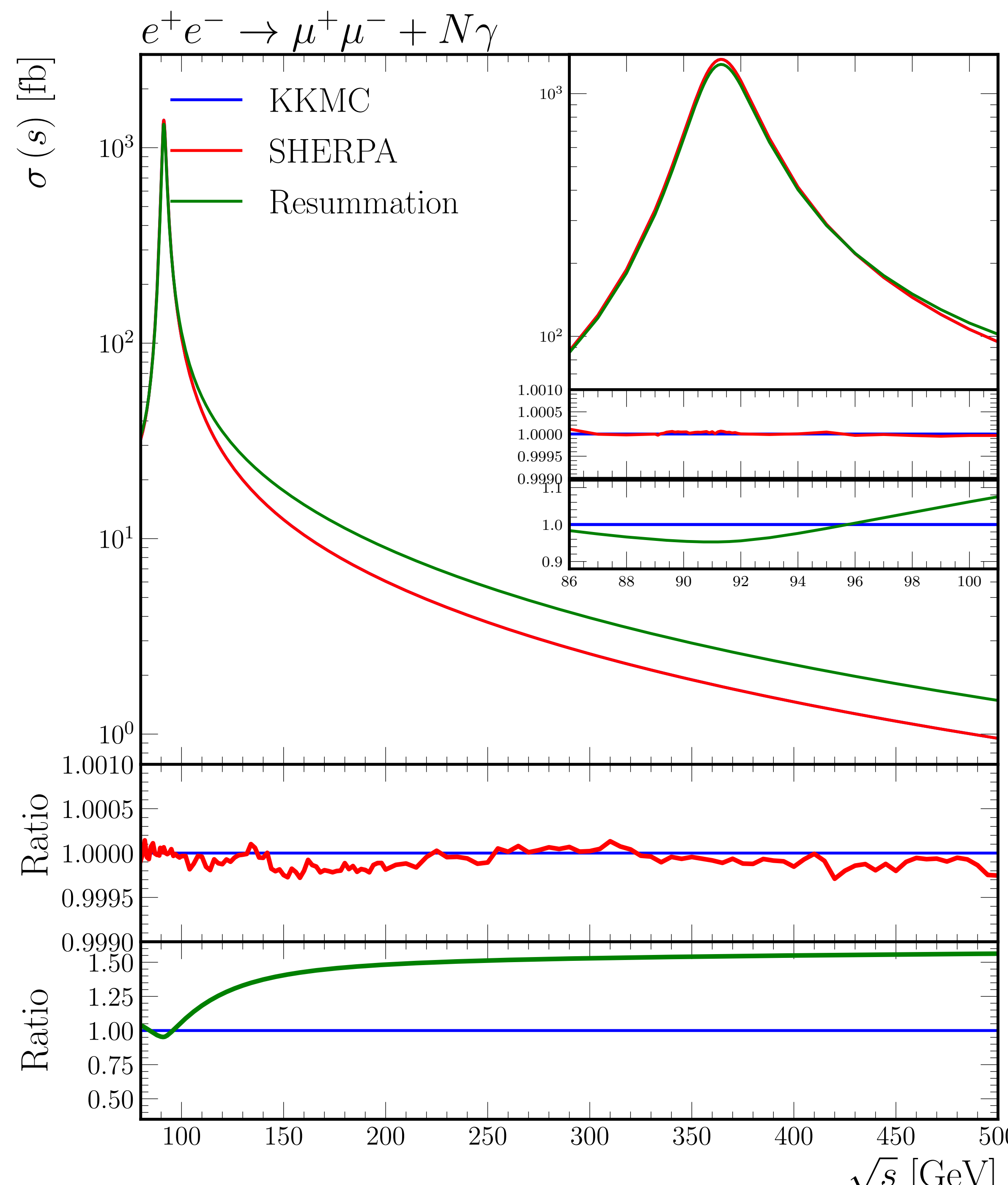
- Treatment of polarised beams
- Photon resummation available both with PDFs (LL) and with YSF
- EW corrections for lepton collider in the future
- Photoproduction with the EPA Hoche et al, 2310.18674
- QED effects in parton shower, including $\gamma \to f\bar{f}$ Flower et al, 2210.07007

Results with Sherpa

Krauss et al, 2203.10948



$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \tilde{S}(k_i)\, \Theta(k_i,\Omega)\right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{S}(k_j)} + \sum_{\substack{j,k=1\\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j,k_k)}{\tilde{S}(k_j)\,\tilde{S}(k_k)} + \cdots\right)$$



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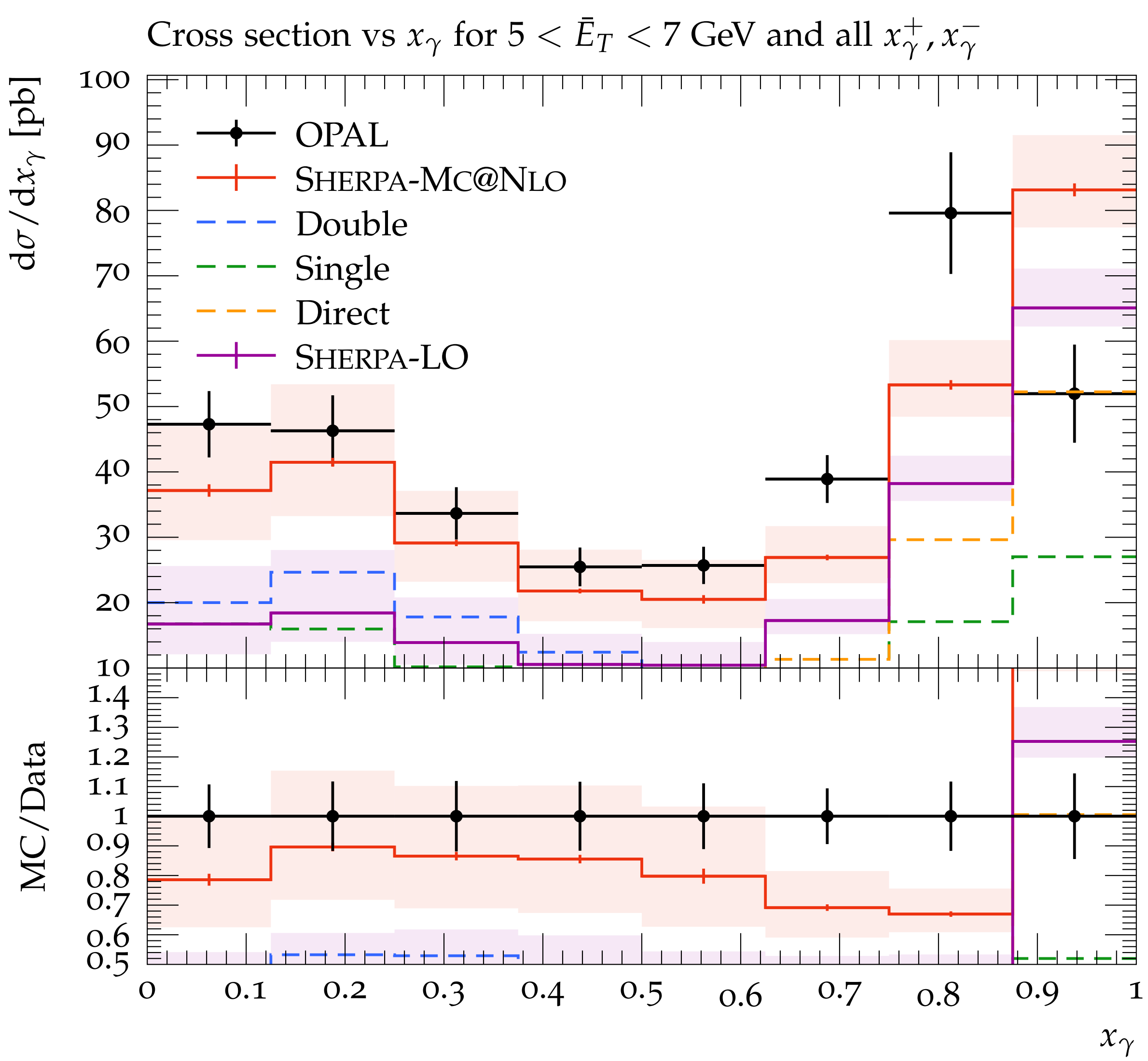
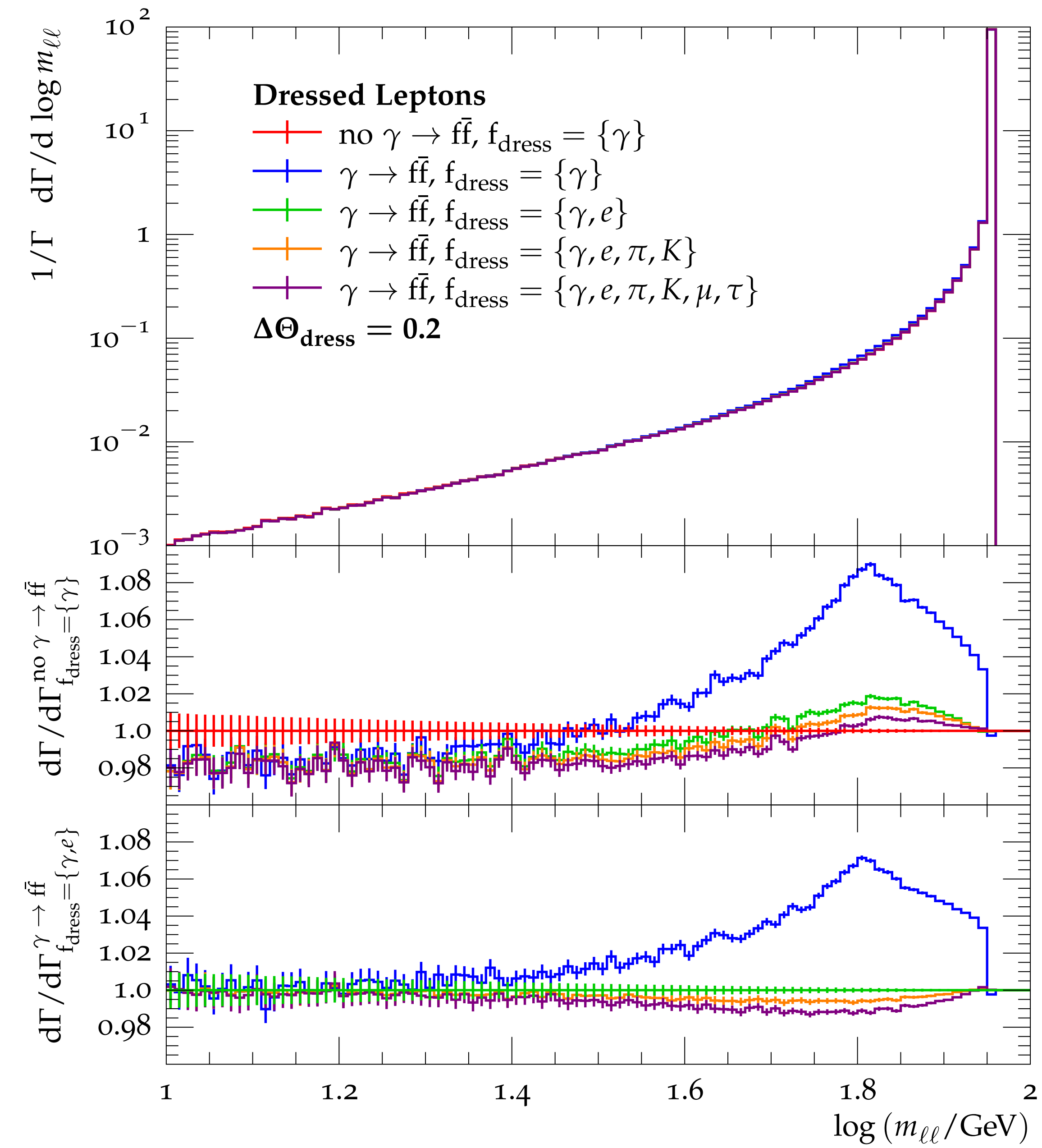


Photoproduction at LEP:

Hoche et al, 2310.18674



$\gamma \to ff$ splittings:

Flower et al, 2210.07007

Whizard at lepton colliders

- NLO QCD/EW accuracy for arbitrary processes
- NLO QCD matched à la Powheg
- Beamstrahlung included via Circe
- ISR at LL, wip for NLL (independent from eMELA)
- Full support of polarised beams
- WIP for EW PDFs

Results with Whizard

ee @ 1 TeV, NLO QCD

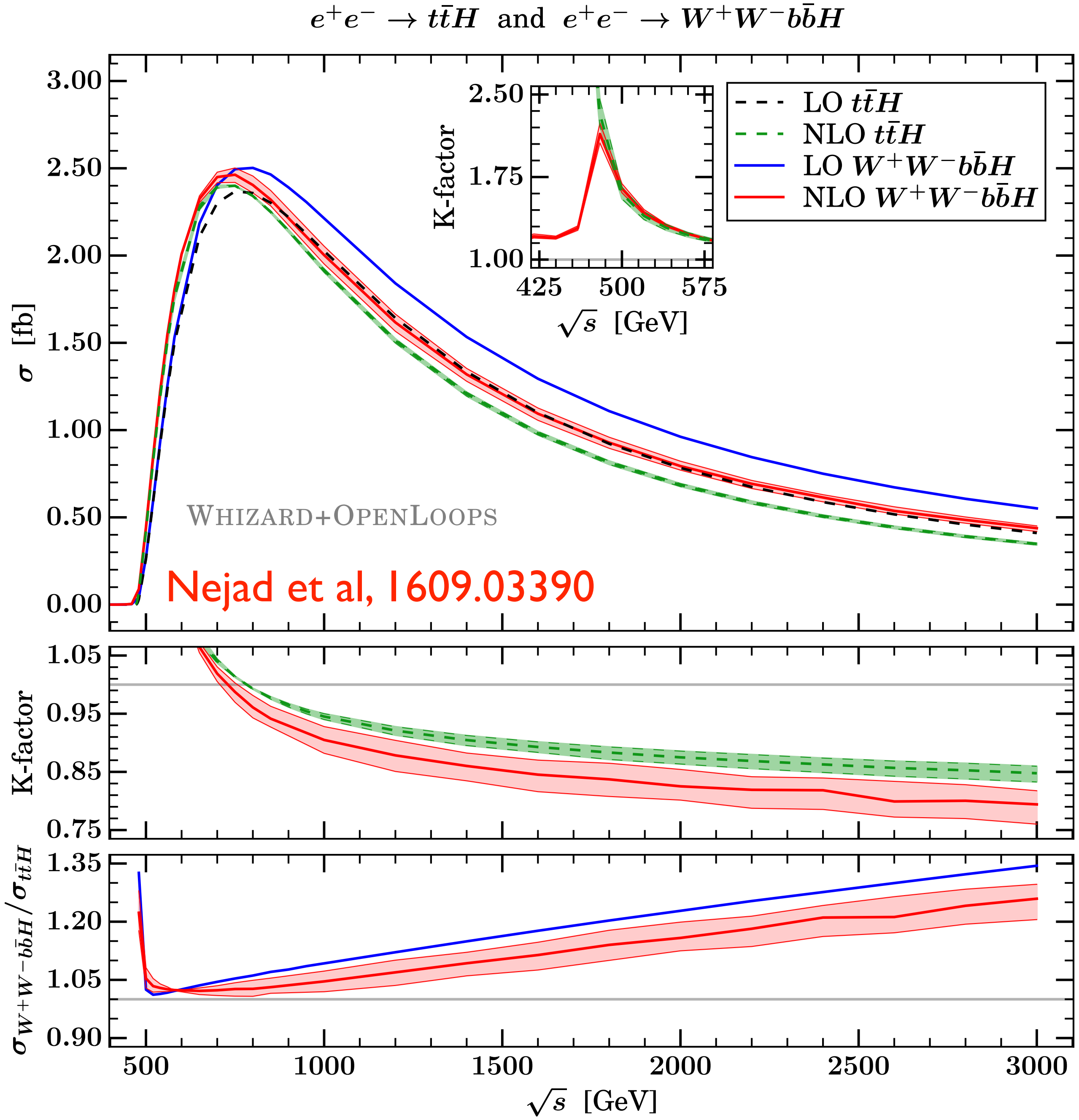
Process	WHIZARD+OpenLoops $\sigma_{\rm LO}$ [fb]	$\sigma_{\rm NLO}$ [fb]
$e^+e^- \to jj$	622.737(8)	639.39(5)
$e^+e^- \to jjj$	340.6(5)	317.8(5)
$e^+e^- \to jjjj$	105.0(3)	104.2(4)
$e^+e^- \to jjjjj$	22.33(5)	24.57(7)
$e^+e^- \to jjjjjj$	3.583(17)	4.46(4)
$e^+e^- \to t\bar{t}$	166.37(12)	174.55(20)
$e^+e^- \to t\bar{t}j$	48.12(5)	53.41(7)
$e^+e^- \to t\bar{t}jj$	8.592(19)	10.526(21)
$e^+e^- \to t\bar{t}jjj$	1.035(4)	1.405(5)
$e^+e^- \to t\bar{t}t\bar{t}$	$0.6388(8) \cdot 10^{-3}$	$1.1922(11) \cdot 10^{-3}$
$e^+e^- \to t\bar{t}t\bar{t}j$	$2.673(7) \cdot 10^{-5}$	$5.251(11) \cdot 10^{-5}$
$e^+e^- \to t\bar{t}H$	2.020(3)	1.912(3)
$e^+e^- \to t\bar{t}Hj$	$2.536(4) \cdot 10^{-1}$	$2.657(4) \cdot 10^{-1}$
$e^+e^- \to t\bar{t}Hjj$	$2.646(8) \cdot 10^{-2}$	$3.123(9) \cdot 10^{-2}$
$e^+e^- \to t\bar{t}Z$	4.638(3)	4.937(3)
$e^+e^- \to t\bar{t}Zj$	$6.027(9) \cdot 10^{-1}$	$6.921(11) \cdot 10^{-1}$
$e^+e^- \to t\bar{t}Zjj$	$6.436(21) \cdot 10^{-2}$	$8.241(29) \cdot 10^{-2}$
$e^+e^- \to t\bar{t}W^\pm jj$	$2.387(8) \cdot 10^{-4}$	$3.716(10) \cdot 10^{-4}$
$e^+e^- \to t\bar{t}HZ$	$3.623(19) \cdot 10^{-2}$	$3.584(19) \cdot 10^{-2}$
$e^+e^- \to t\bar{t}ZZ$	$3.788(6) \cdot 10^{-2}$	$4.032(7) \cdot 10^{-2}$
$e^+e^- \to t\bar{t}HH$	$1.3650(15) \cdot 10^{-2}$	$1.2168(16) \cdot 10^{-2}$
$e^+e^- \to t\bar{t}W^+W^-$	$1.3672(21) \cdot 10^{-1}$	$1.5385(22) \cdot 10^{-1}$

μμ @ 3 Tev, NLO EW

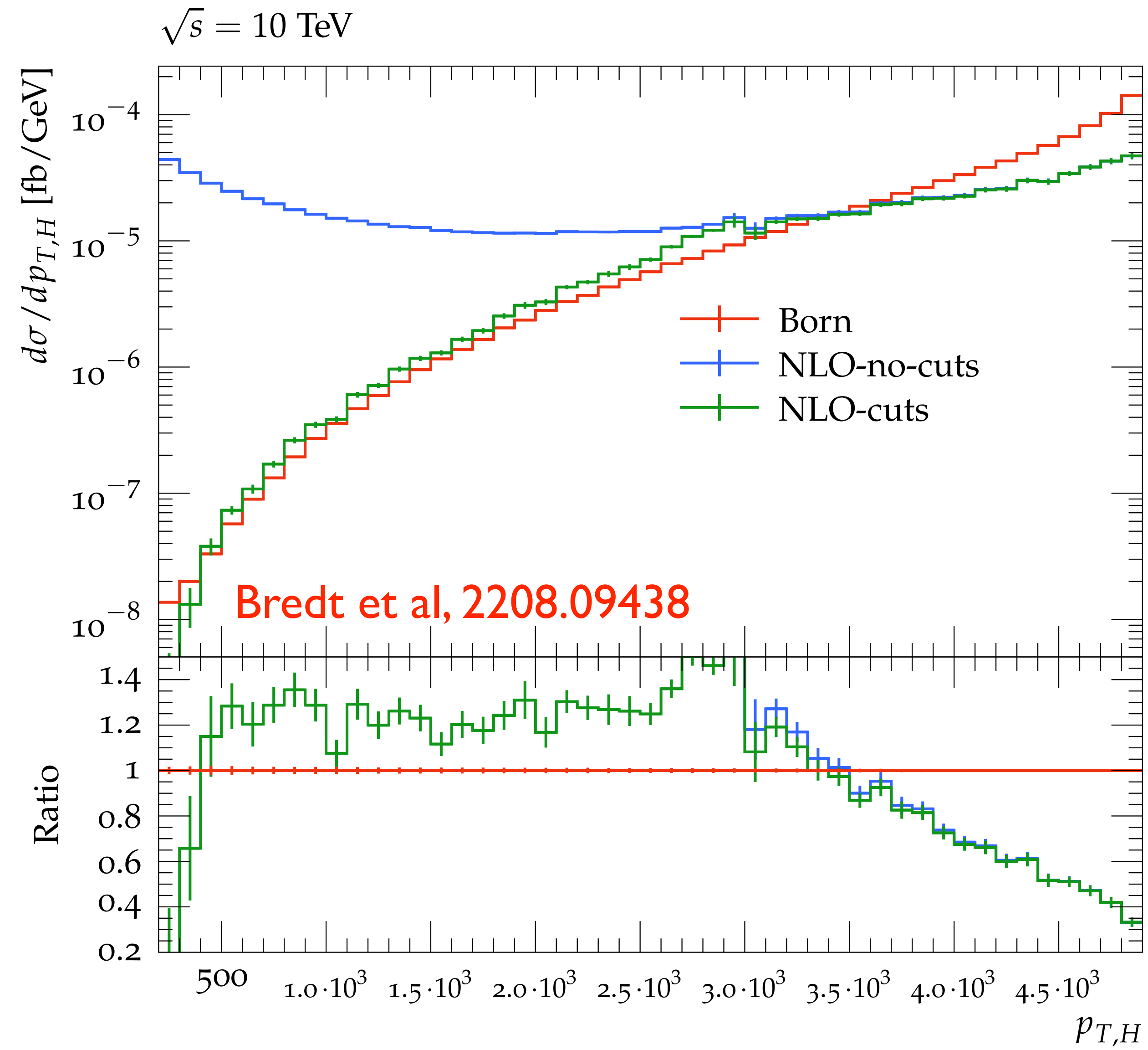
$\mu^+\mu^- \to X, \sqrt{s} =$	$\sigma^{\rm incl}_{\rm LO}$ [fb]	$\sigma^{\rm incl}_{\rm NLO}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-	$4.6591(2) \cdot 10^2$	$4.847(7) \cdot 10^2$	+4.0(2)
ZZ	$2.5988(1) \cdot 10^1$	$2.656(2) \cdot 10^1$	+2.19(6)
HZ	$1.3719(1) \cdot 10^0$	$1.3512(5) \cdot 10^0$	−1.51(4)
HH	$1.60216(7) \cdot 10^{-7}$	$5.66(1) \cdot 10^{-7}$ *	
W^+W^-Z	$3.330(2) \cdot 10^1$	$2.568(8) \cdot 10^1$	−22.9(2)
W^+W^-H	$1.1253(5) \cdot 10^0$	$0.895(2) \cdot 10^0$	−20.5(2)
ZZZ	$3.598(2) \cdot 10^{-1}$	$2.68(1) \cdot 10^{-1}$	−25.5(3)
HZZ	$8.199(4) \cdot 10^{-2}$	$6.60(3) \cdot 10^{-2}$	−19.6(3)
HHZ	$3.277(1) \cdot 10^{-2}$	$2.451(5) \cdot 10^{-2}$	−25.2(1)
HHH	$2.9699(6) \cdot 10^{-8}$	$0.86(7) \cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1) \cdot 10^0$	$0.993(6) \cdot 10^0$	−33.1(4)
W^+W^-ZZ	$1.209(1) \cdot 10^0$	$0.699(7) \cdot 10^0$	−42.2(6)
W^+W^-HZ	$8.754(8) \cdot 10^{-2}$	$6.05(4) \cdot 10^{-2}$	−30.9(5)
W^+W^-HH	$1.058(1) \cdot 10^{-2}$	$0.655(5) \cdot 10^{-2}$	−38.1(4)
$ZZZZ$	$3.114(2) \cdot 10^{-3}$	$1.799(7) \cdot 10^{-3}$	−42.2(2)
$HZZZ$	$2.693(2) \cdot 10^{-3}$	$1.766(6) \cdot 10^{-3}$	−34.4(2)
$HHZZ$	$9.828(7) \cdot 10^{-4}$	$6.24(2) \cdot 10^{-4}$	−36.5(2)
$HHHZ$	$1.568(1) \cdot 10^{-4}$	$1.165(4) \cdot 10^{-4}$	−25.7(2)

Results with Whizard

ee→ttH offshell at NLO QCD



μμ→ZH at NLO EW



MG5_aMC at letpon colliders

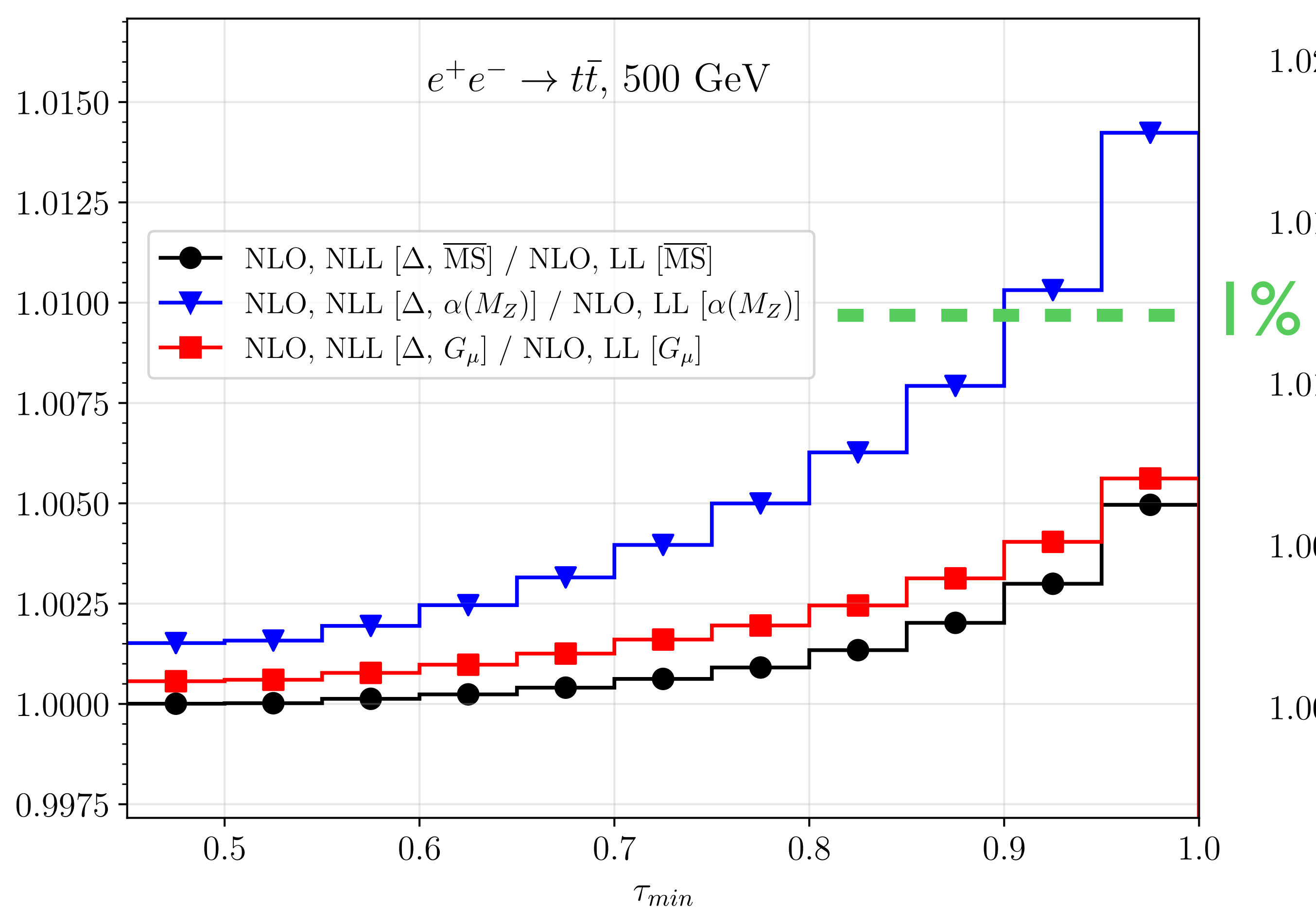
- NLO EW and QCD corrections for (almost) all processes
- Beam polarisation for LO simulations
- Via eMELA, ISR (+beamstrahlung) in different ren/fact schemes
- The code automatically takes care to add to the short-distance xsection those terms necessary for consistency
  - Factorisation-scheme kernels included in the cross-section for Δ scheme and LL PDFs
  - Virtuals are corrected in order to account for different ren. scheme in model and PDFs ($\alpha(m_Z) \rightarrow$ MSbar)
- For details and how-to, see https://answers.launchpad.net/mg5amcnlo/+faq/3324

Some results:

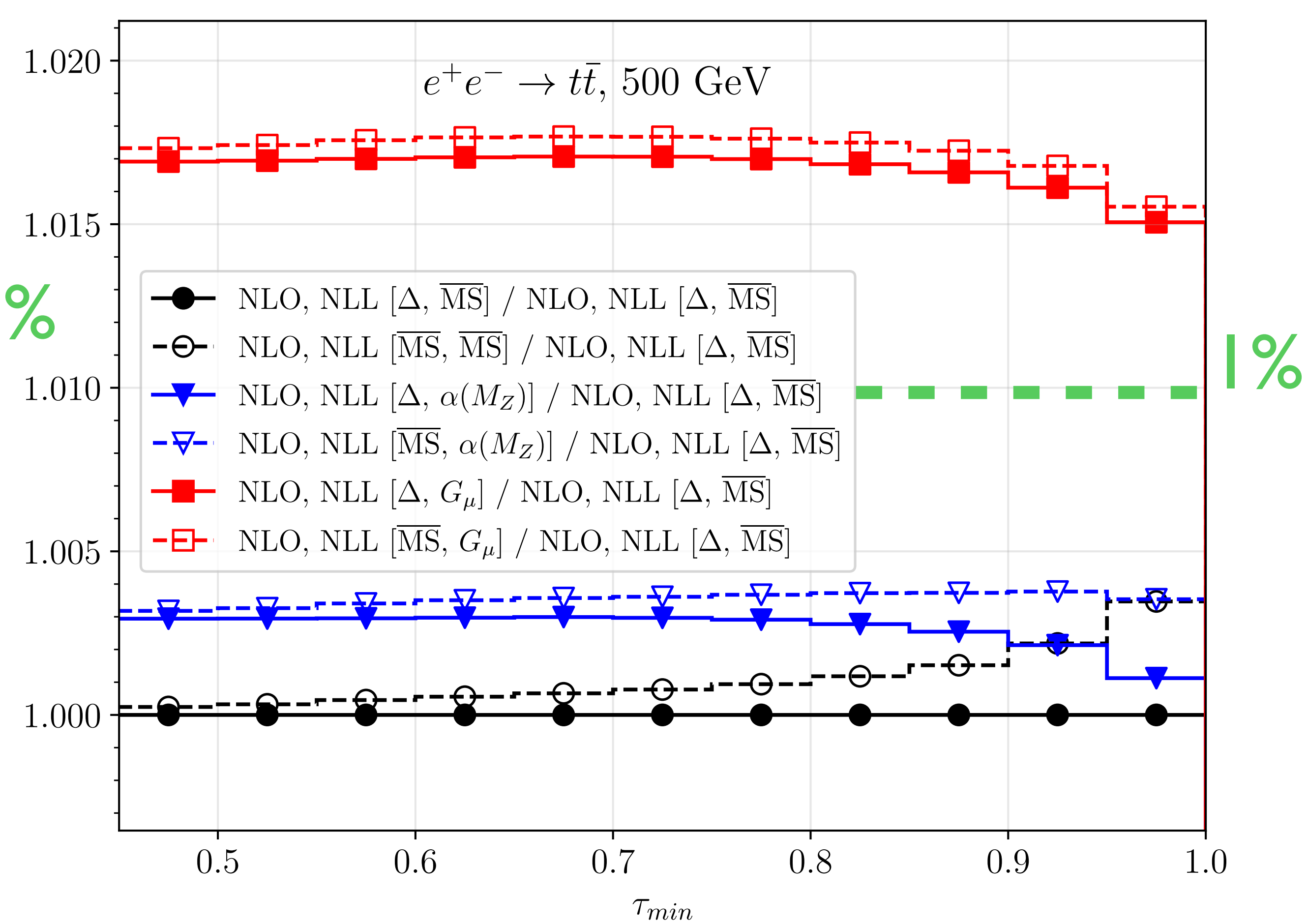
Bertone, Cacciari, Frixione, Stagnitto, MZ, Zhao 2207.03265

- $e^+e^- \to t\bar{t}$ @500 GeV; Observable: $\sigma(\tau_{min}) = \int d\sigma\, \Theta\left(\tau_{min} \leq \frac{M^2_{p\bar{p}}}{s}\right)$

LL vs NLL



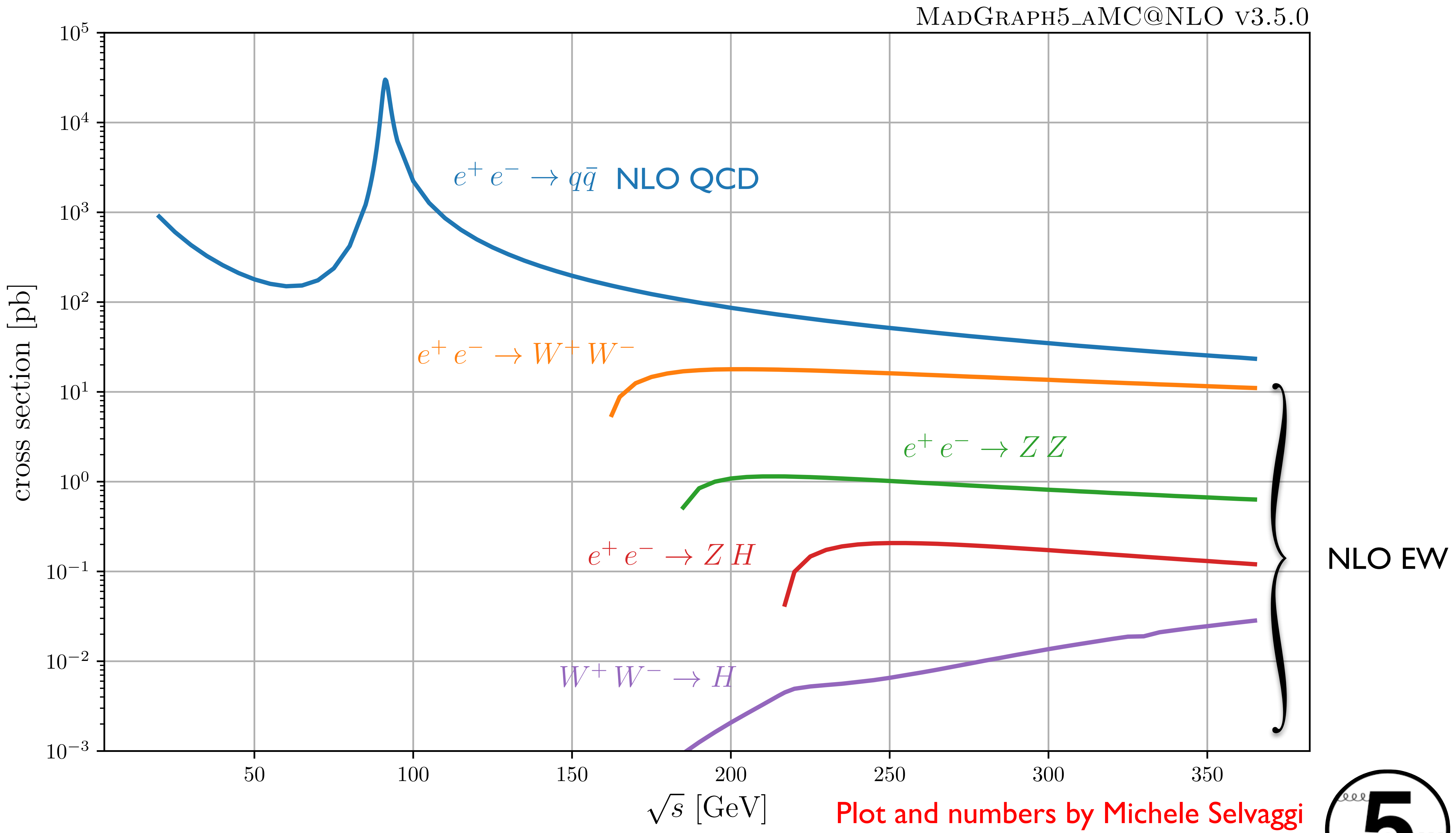
NLL different fact. schemes



- NLL-MSbar vs NLL-Δ is (at most) at the few-per-mil level

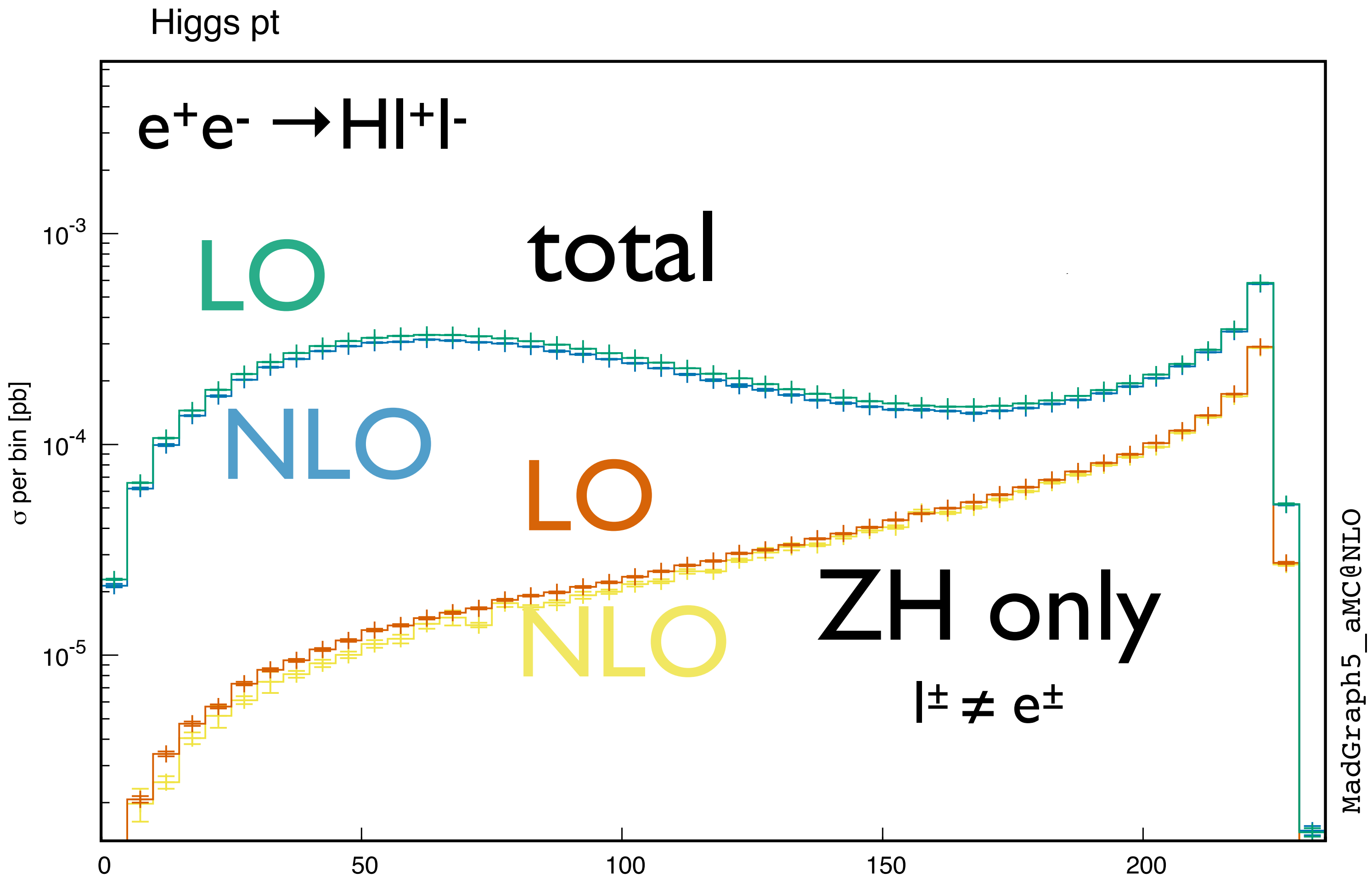
More results:

t



Plot and numbers by Michele Selvaggi

Complex-mass scheme: $e^+e^- \to Hl^+l^-$

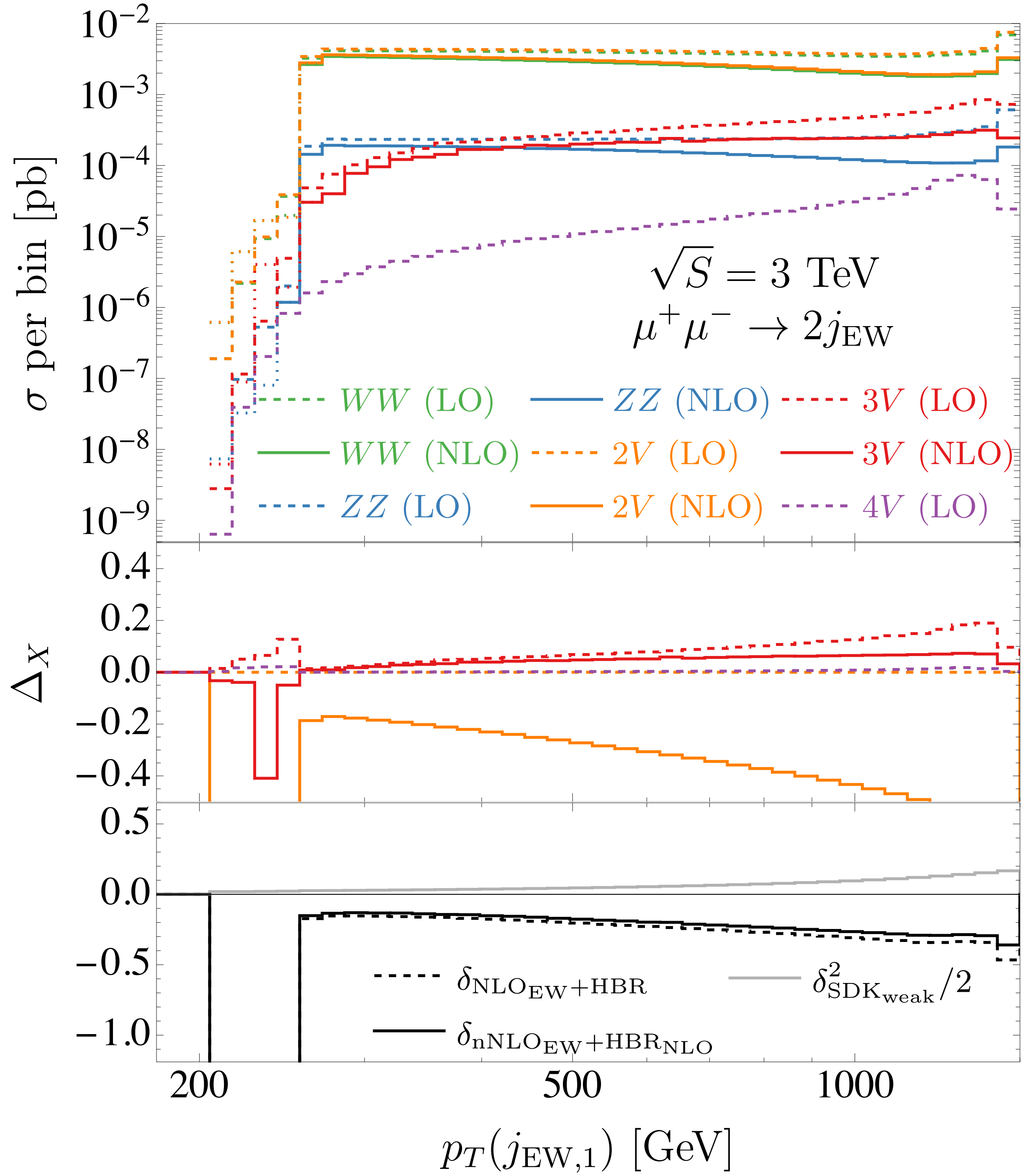


- Qualitatively similar results to Denner, Dittmaier, Roth, Weber, hep-ph/0302198
- Results obtained in 15mins (on a cluster) @ 0.1%

Inclusive timing profile :	
Overall slowest channel	0:06:15
Average channel running time	0:03:42
Aggregated total running time	8:05:57

EW corrections at muon colliders

Pagani, Ma, MZ, 2409.09129, see also Davide's talk



Investigate several aspect of EW corrections, real radiation, etc…

The food drives your appetite

LHC approved
Tevatron running

Automatic tree-lev. ME computation
(no integration/event gen)

Automatic tree-lev ME computation
with event gen

LHC started

NLO revolution

Future (lepton) collider approved

Today:
NLO QCD+EW
NNLO QCD for 2→3,
NNLO subtraction
NNNLO for 2→1
NLL parton showers
…

Outlook

- Not covered in this talk, but still relevant: KKMC-ee for $f\bar{f}$ production, process specific tools, e.g. BabaYaga
- The LHC has thought us many lessons:
  - General-purpose MC tools are fundamental workhorses for all physics analyses
  - The possibility of using different tools is key for validation and proper estimate of uncertainties
- Progress in MCs is driven by the destination: we are in a much better shape than 30 years ago, when LHC started

Thank you!

Matching with PS: ISR at NLL?

QED Parton Shower

see for instance review in 0912.0749

It allows for exclusive photon emission in the context of collinear factorisation.

Photon energies dictated by distribution in z, whereas angles are generated independently according to the YFS formula, valid in the soft limit:

$$\cos\theta_l \propto -\sum_{i,j=1}^{N} \eta_i \eta_j \frac{1-\beta_i\beta_j\cos\theta_{ij}}{(1-\beta_i\cos\theta_{il})(1-\beta_j\cos\theta_{jl})}$$

with η_i a charge factor and β_i the speed of the emitting particle.

Algorithm adopted in BabaYaga $[e^+e^- \to e^+e^-, e^+e^- \to \mu^+\mu^-, e^+e^- \to \gamma\gamma]$

hep-ph/0003268, hep-ph/0103117, hep-ph/0312014, hep-ph/0801.3360, hep-ph/0607181

Balossini, Bignamini, Carloni Calame, Lunardini, Montagna, Nicrosini, Piccinini

BabaYaga also includes a matching to NLO QED in the short distance cross section

slide by G. Stagnitto

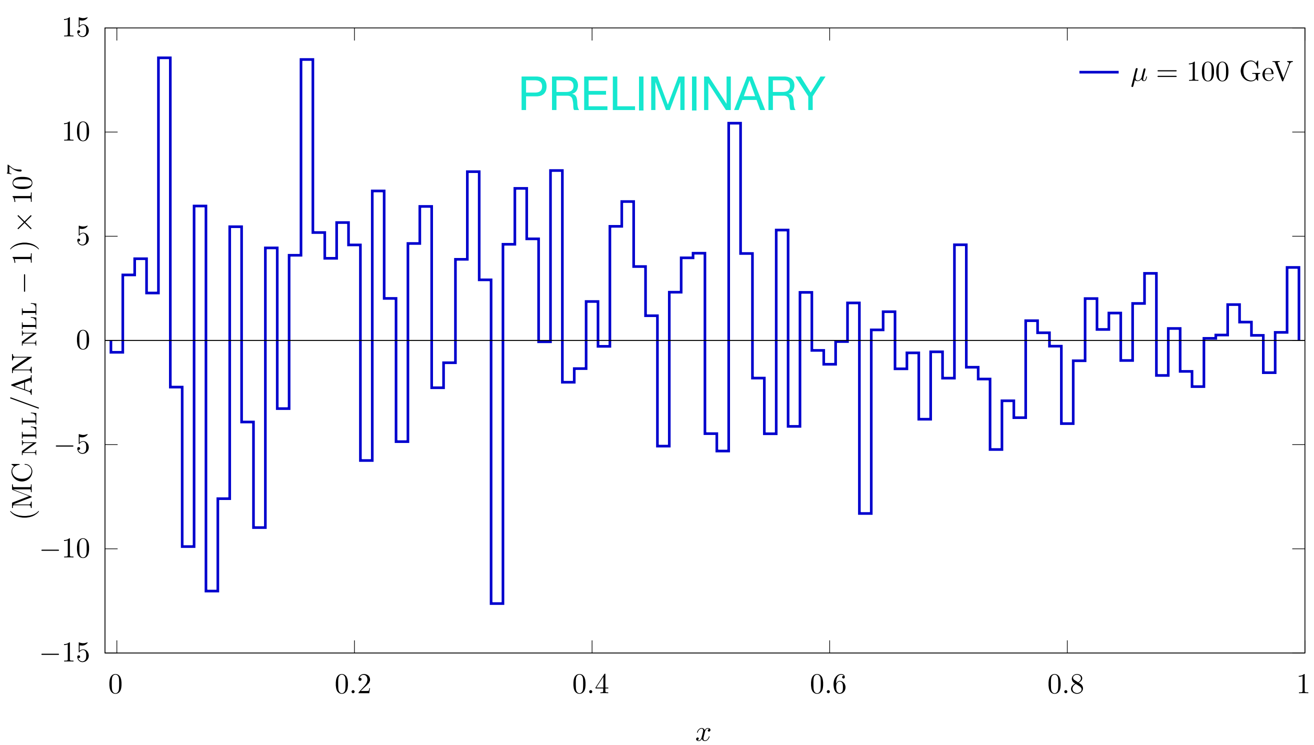
Matching with PS: ISR at NLL?

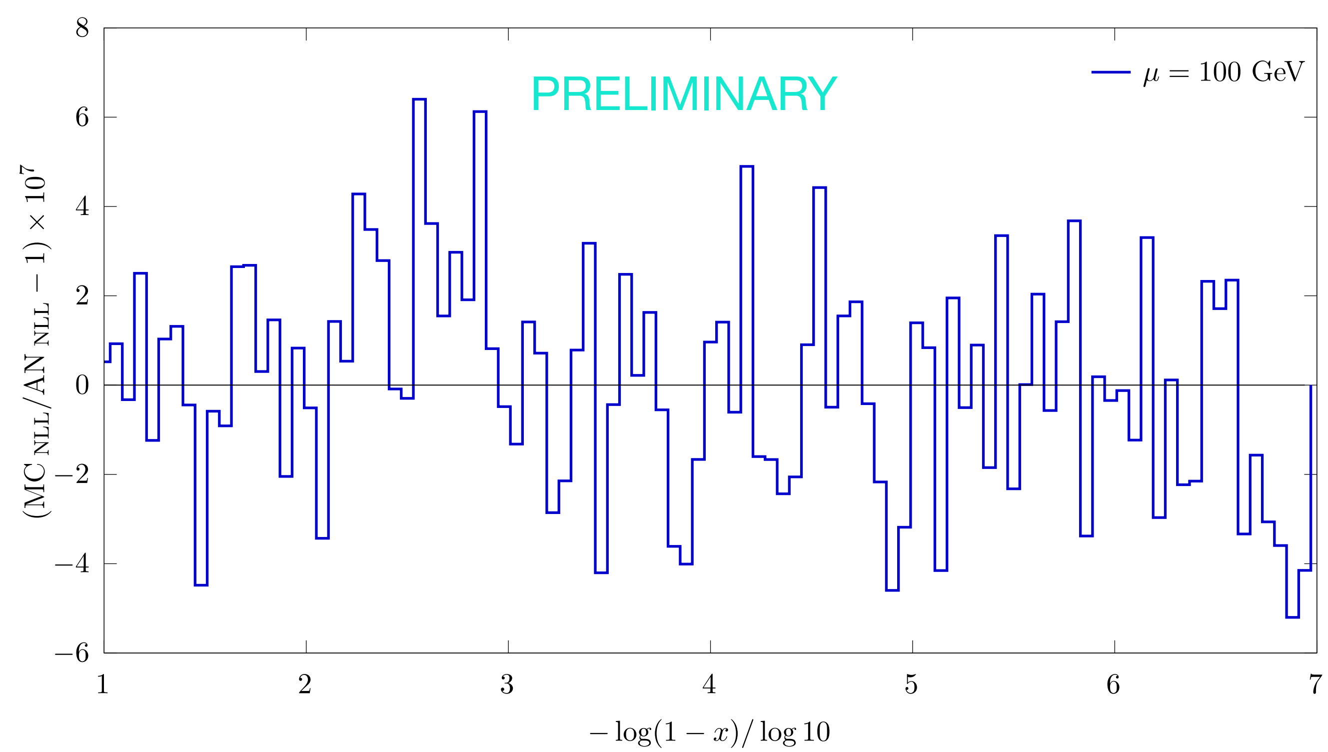
Towards a “NLL” QED Parton Shower

C. M. Carloni Calame, M. Chiesa, S. Frixione, G. Montagna, F. Piccinini, GS

$$D(x,s) = \sum_{n=0}^{\infty} \prod_{i=1}^{n} \left\{ \int_{m_e^2}^{s_{i-1}} \frac{ds_i}{s_i} \Pi(s_{i-1}, s_i) \frac{\alpha}{2\pi} \int_{x/(z_1 \cdots z_{i-1})}^{x_+} \frac{dz_i}{z_i} P(z_i) \right\} \Pi(s_n, m_e^2) D\left(\frac{x}{z_1 \cdots z_n}, m_e^2 \right)$$

With a NLL iterative solution, we recover the known (non-singlet) NLL PDFs





WIP towards exclusive kinematics of final-state photons and singlet components

slide by G. Stagnitto