Università di Genova

Jet substructure at lepton colliders

FCC-ee and Lepton Colliders, Laboratori Nazionali di Frascati, 23th of January 2025

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Lepton Colliders and Jet Substructure Studies

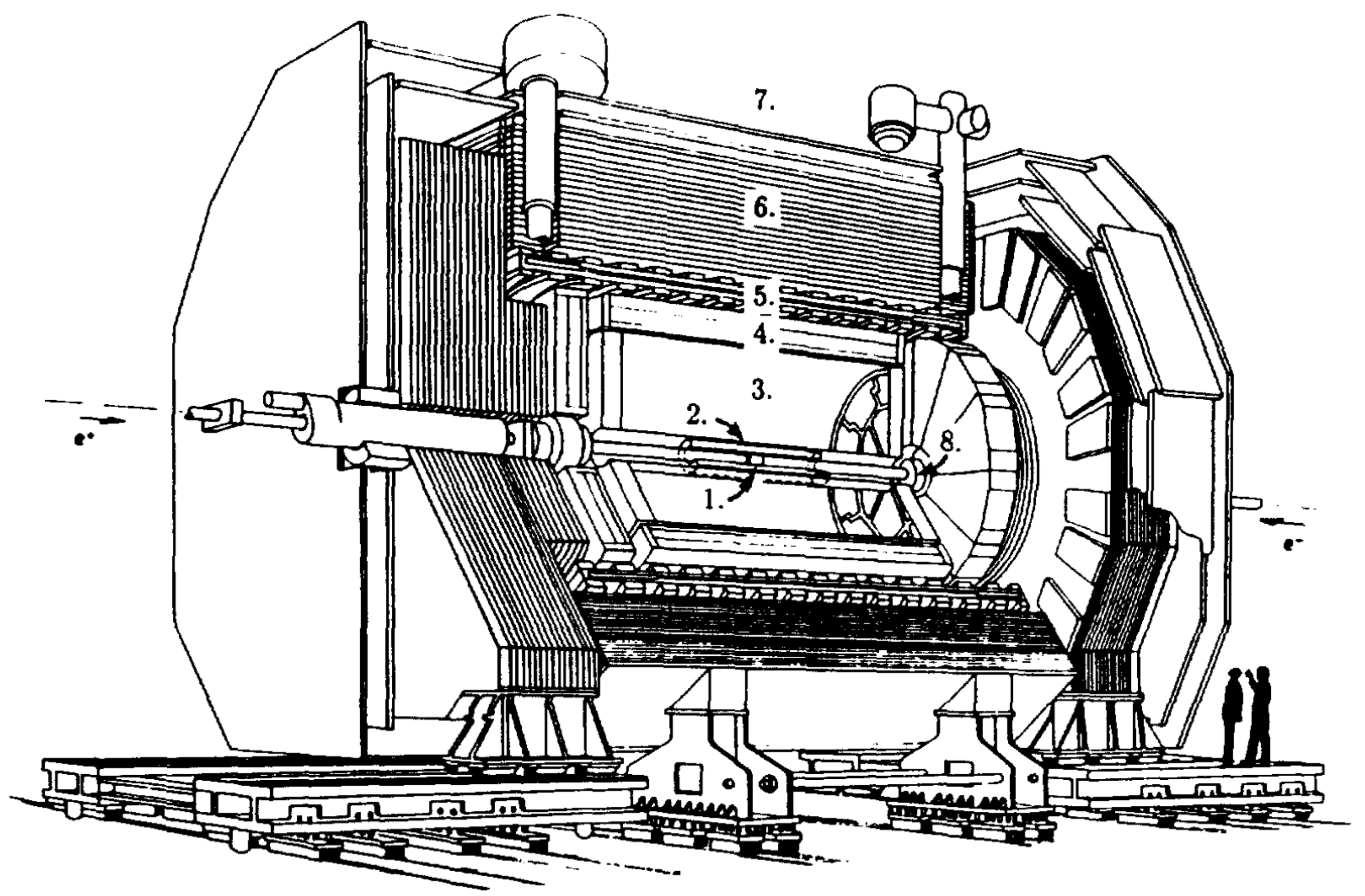
Key Features of Lepton Colliders

- QCD Dynamics Confined to Final State:
  - No Pile-Up or Underlying Event (UE) effects,
  - Free from PDFs complexities,
- Precisely Controlled Initial State:
  - Collision energy directly measurable,
  - All energy is used efficiently (up to QED ISR),

Current Focus in Jet Studies

- Active developments on jets and jet algorithms focus more on:
  - Hadron Collisions (e.g., LHC),
  - Heavy Ion Collisions,
- Yet, Monte Carlo event generators are primarily tuned with:
  - LEP data.

Hadrons vs Leptons



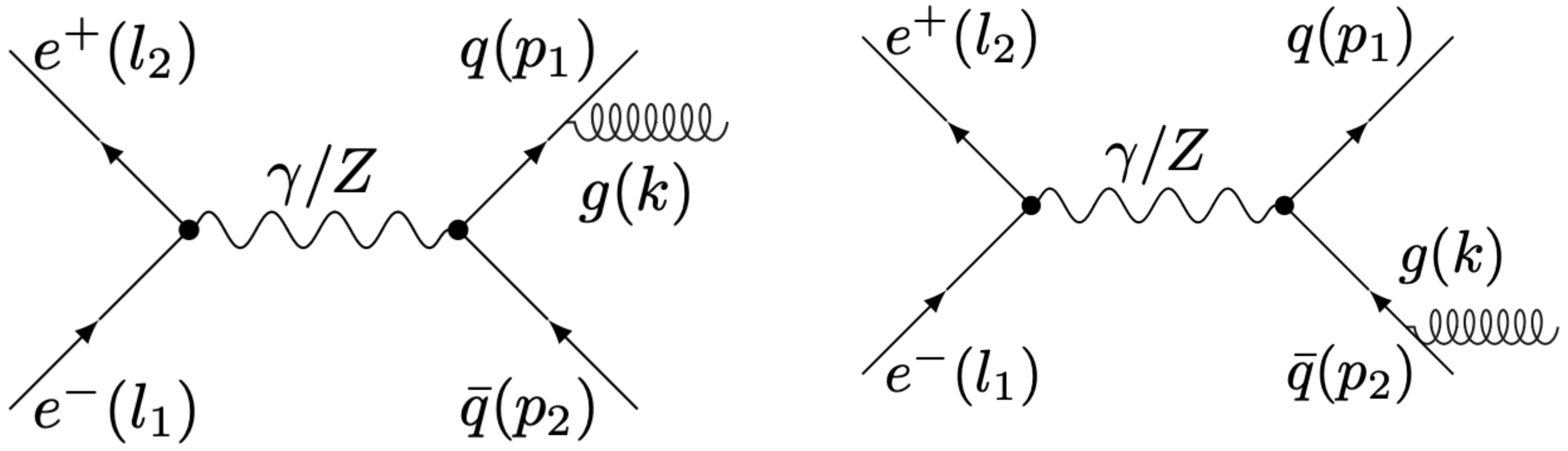
Why Lepton Colliders for Jet Substructure?

- Cleanest Environment for studying:
  - Final state jet substructure,
  - Testing perturbative QCD.
- Impact of JSS studies:
  - Jet flavour identification,
  - Electroweak boson tagging (e.g. $W^{\pm}, Z$),
  - Top quark tagging.

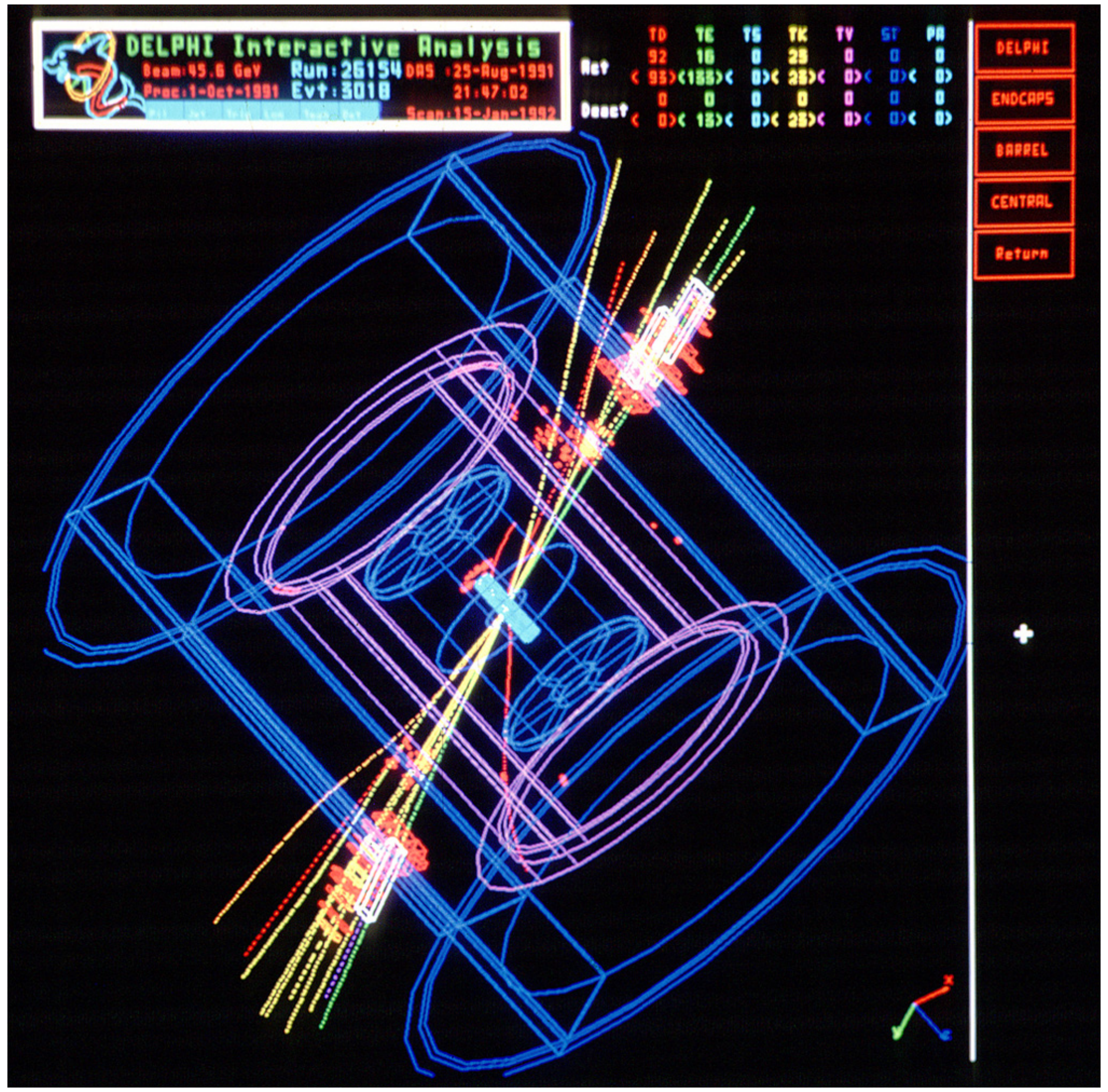
A. Test of light and heavy flavor fragmentation

B. Determination of α_S at per mille accuracy

Electron-positron collisions



- We consider $q\bar{q}$ production in e^+e^- collisions
- At lowest order in perturbation theory, the energy is divided between the two produced quarks

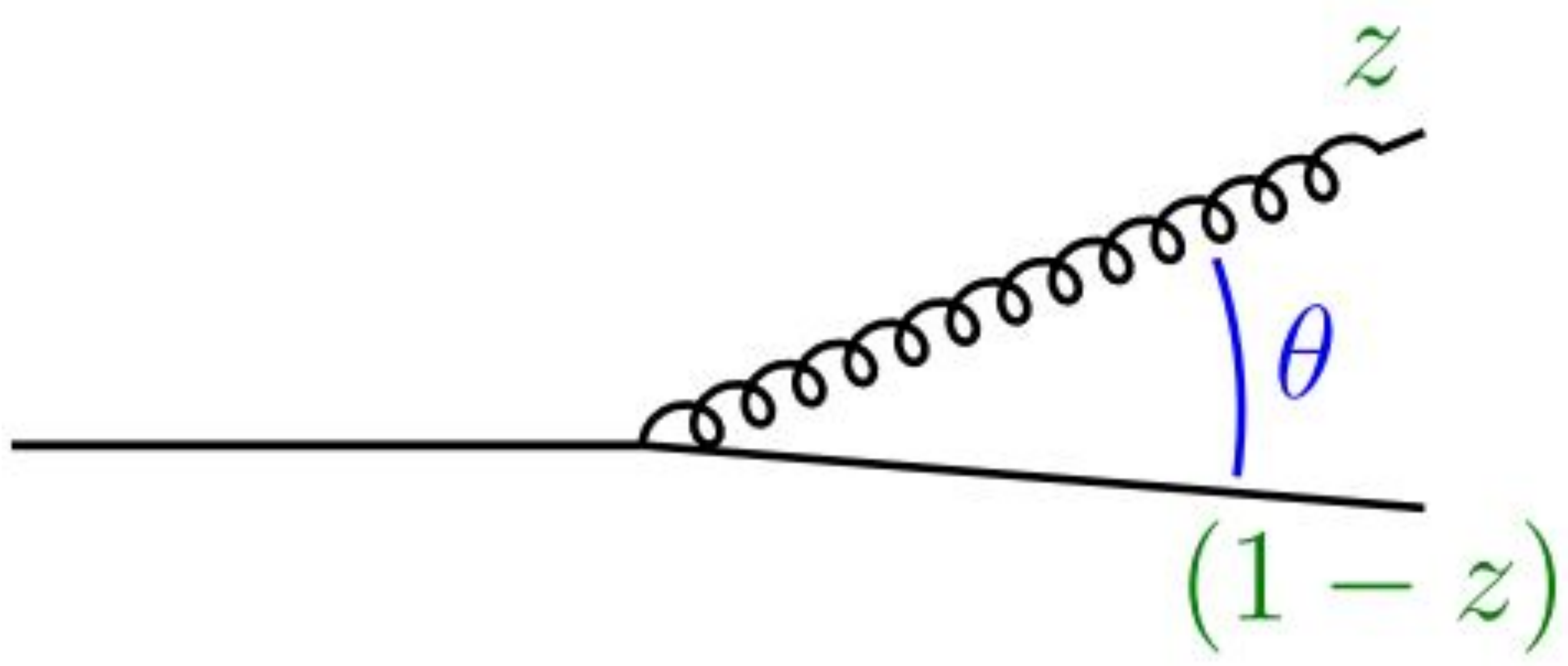


- Gluon soft and collinear radiation does not alter this picture

Jets at e^+e^- can be clustered into two hemispheres

Jet formation at parton level

At high energies, soft ($z \to 0$) and collinear ($\theta^2 \to 0$) emissions are favored in QCD



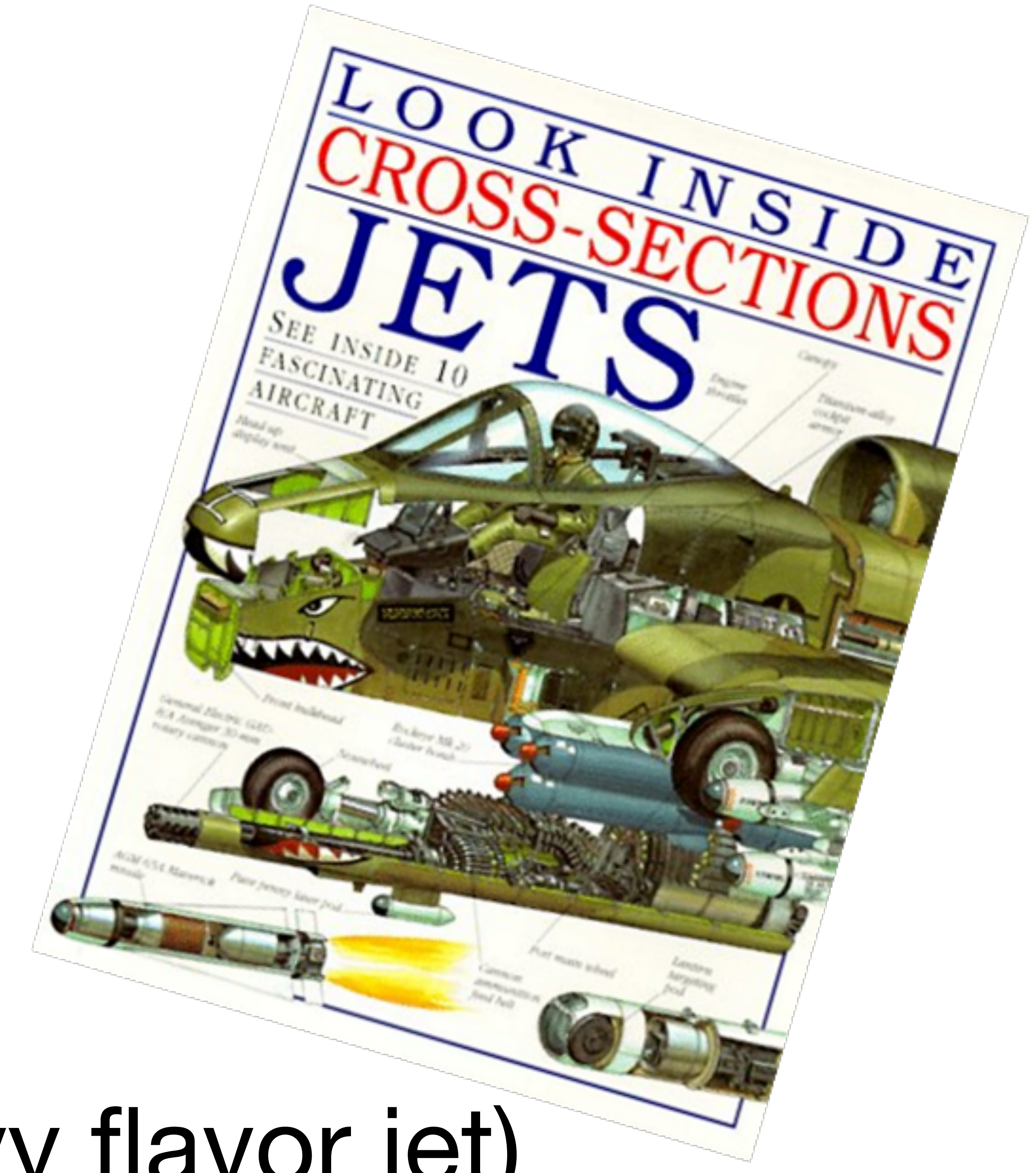
$$d\sigma \simeq \frac{\alpha_S C_F}{2\pi} \frac{dz}{z} \frac{d\theta^2}{\theta^2}$$

z= gluon energy fraction

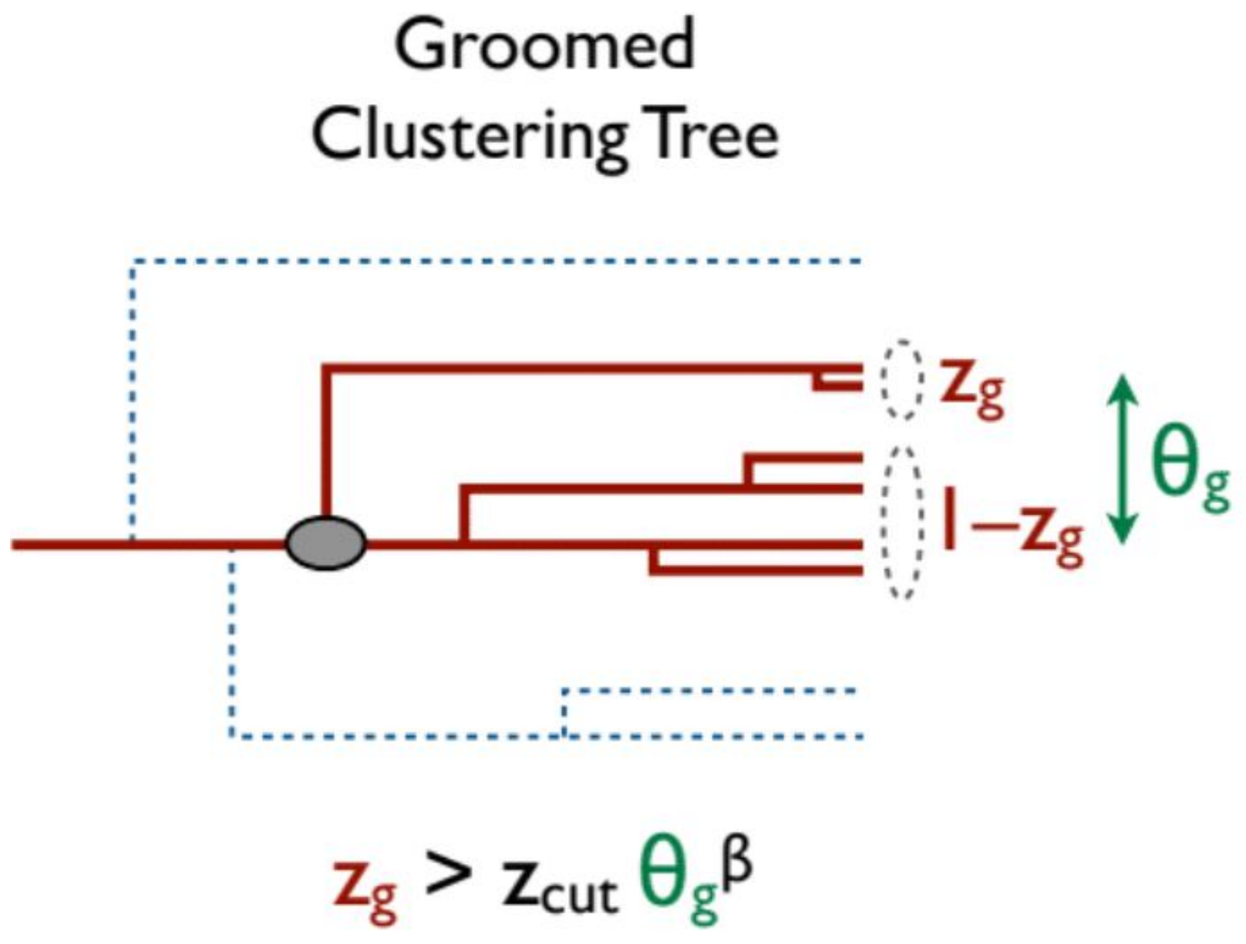
θ^2= gluon splitting angle

This relation reflects the fact that at high energy, massless QCD is a scale invariant QFT

Jet substructure in a nutshell



- Importance of knowing the hard process that originates the jets
- Distinguish different kind of jets (light flavor vs heavy flavor jet)

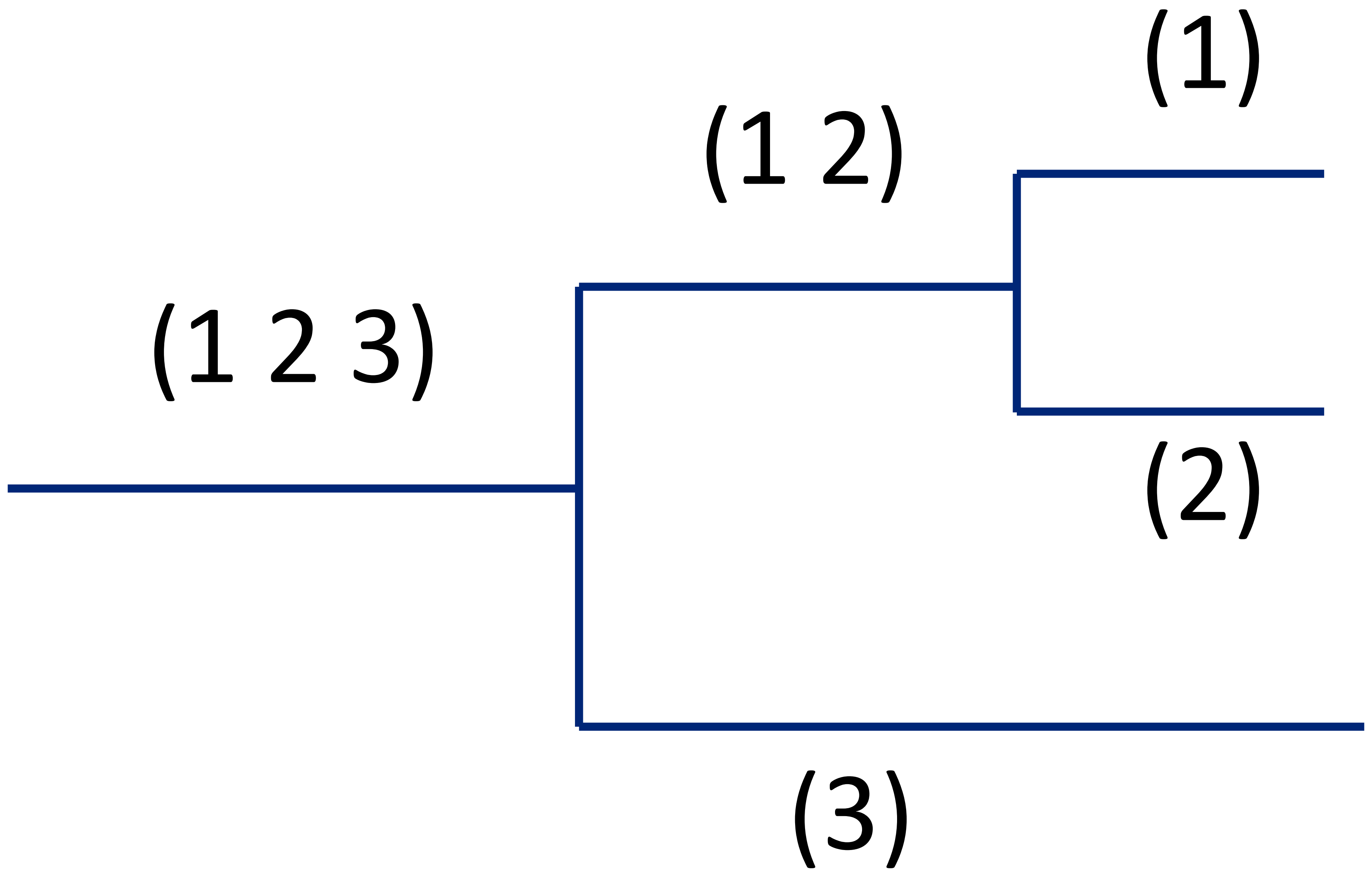


- Prong finders: find hard cores within jets
- Radiation constraints: examine gluon radiation pattern
- Groomers: removes large angle soft radiation

The Soft Drop algorithm

<u>A. Larkoski et al</u>

The SD algorithm removes consistently soft emission at large angle



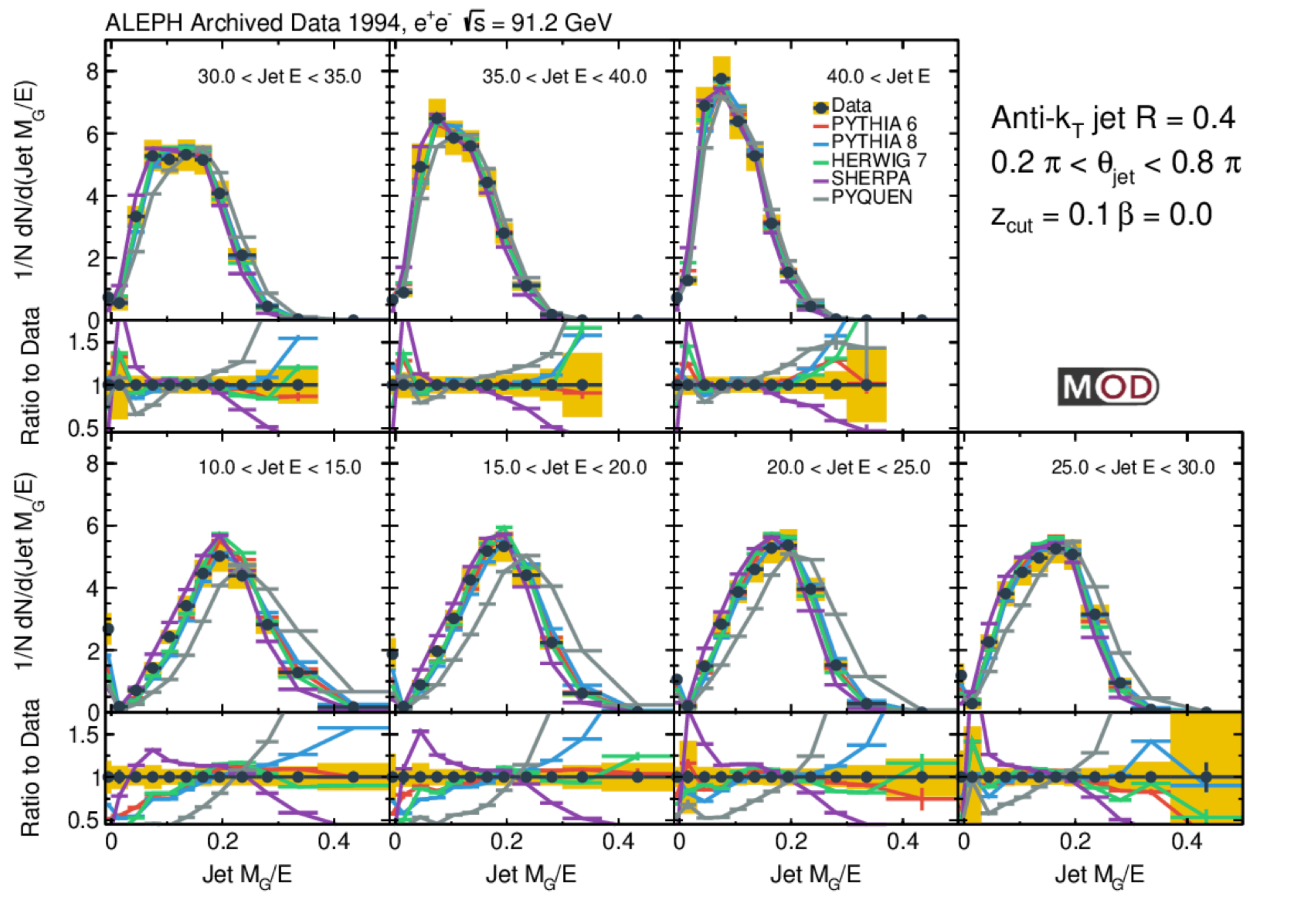
$$\frac{\min(E_i, E_j)}{E_i + E_j} > z_{\text{cut}} \left(2(1 - \cos\theta_{ij})\right)^{\frac{\beta}{2}}$$

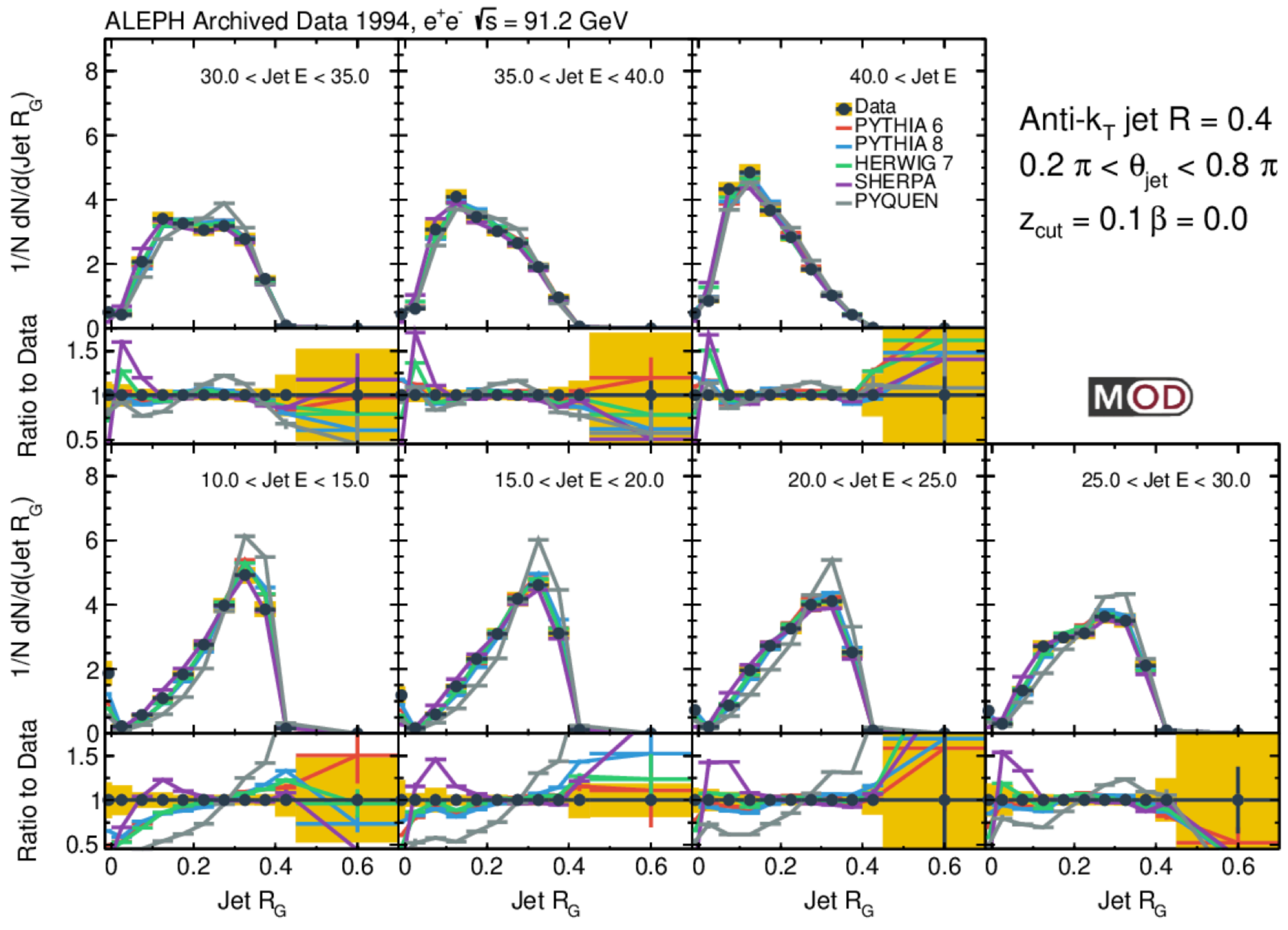
θ_{ij}: angle between branches i, j

The jet constituents are re-clustered to form an angular ordered tree. The declustering is then applied.

Jet substructure at LEP

Recent comparison with ALEPH data for z_g, R_g and for the jet mass distributions

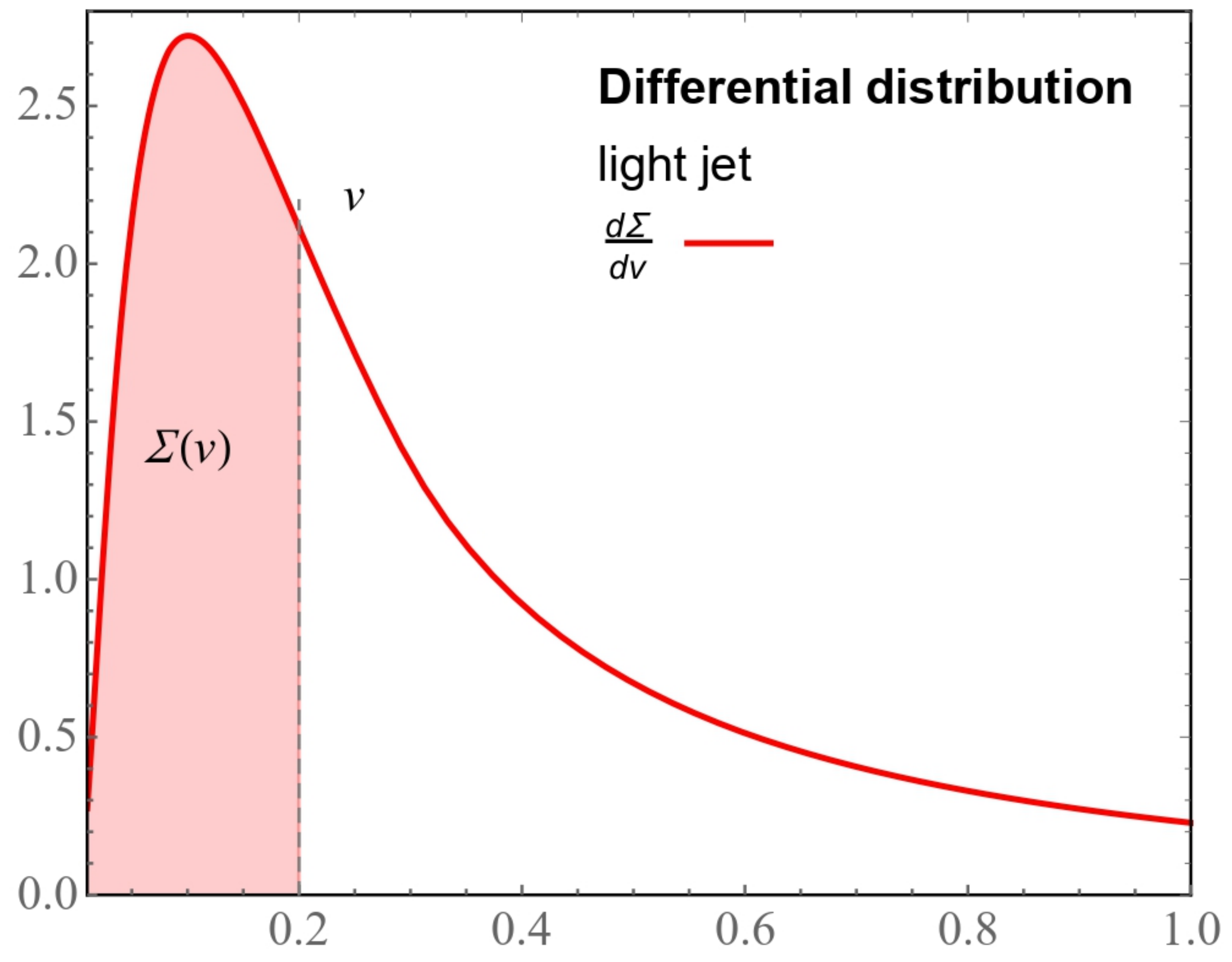




See Yang-Ting Chien et al.

Cumulative distributions

Given a substructure observable v, from a theoretical point of view, it is natural to compute the resummation of the cumulative distribution



Probability of measuring a value of the observable less than v

$$\Sigma(v) = \int_0^v dv' \frac{d\sigma}{dv}$$

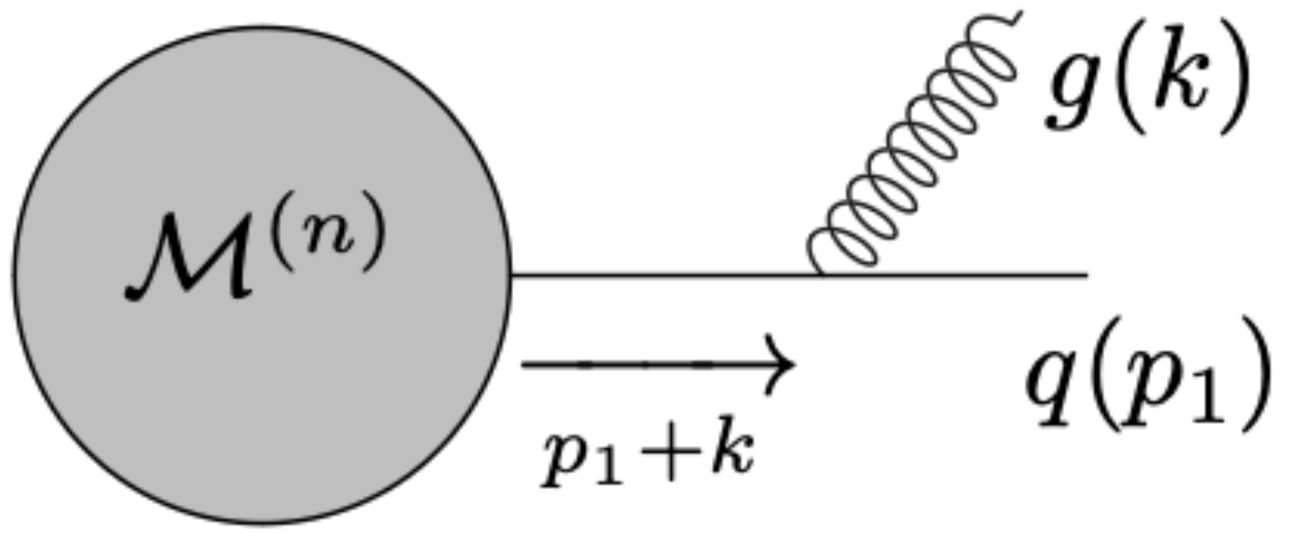
- v is a function of momenta that vanish when no emissions occur (Born level)
- v must be IRC safe
- $\Sigma(v)$ is computed to all orders exploiting QCD factorization theorems

Soft-Collinear factorization

We begin studying the case of the single emission off a quark.

The matrix element factorizes in the soft and collinear limit:

$$\Sigma(v) = 1 - \frac{\alpha_S C_F}{2\pi} \sum_{\ell} \int_0^1 \frac{dz}{z} \int_0^1 \frac{d\theta^2}{\theta^2} \Theta\left(\mathcal{V}_{\ell}(z,\theta^2) - v\right)$$



Corresponds to the LO radiator $R(v)$

$\mathcal{V}$ represents the soft and collinear limit of the observable and in general can be written

$$\mathcal{V} = z^a \theta^{a+b}$$

All order calculation

$\Sigma(v)$ is computed order by order in perturbation theory:

$$\Sigma(v) = \sum_k \left(\frac{\alpha_S}{\pi}\right)^k c_k(v)$$

However, if we are interested in the regime where $v \ll 1$, the convergence of the perturbative series is spoiled:

$$\Sigma(v) \simeq 1 + \underbrace{\alpha_S L^2 + \alpha_S^2 L^4}_{\mathcal{O}(1)} + \dots \qquad L = -\log v$$

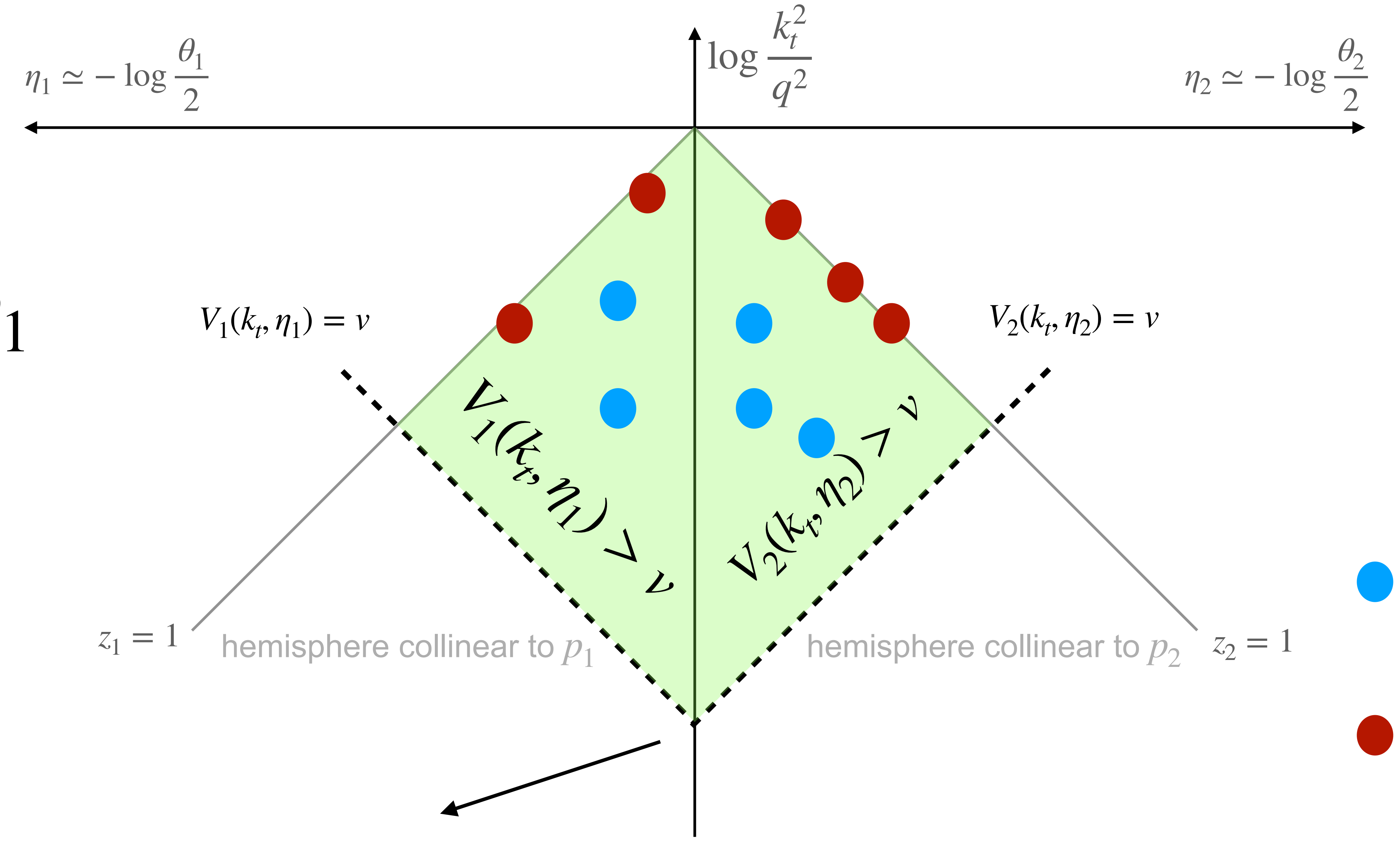
Need to rearrange the perturbative series: $\Sigma(v) = \exp\left[\frac{1}{\alpha_S} \underbrace{g_1(\alpha_S L)}_{\text{LL}} + \underbrace{g_2(\alpha_S L)}_{\text{NLL}} + \dots\right]$

Lund Plane geography

The all order calculation of the cumulative distribution can be performed exploiting Lund diagrams

Lund diagrams: representation of the phase space available by emissions

Emission collinear to p_1



Emission collinear to p_2

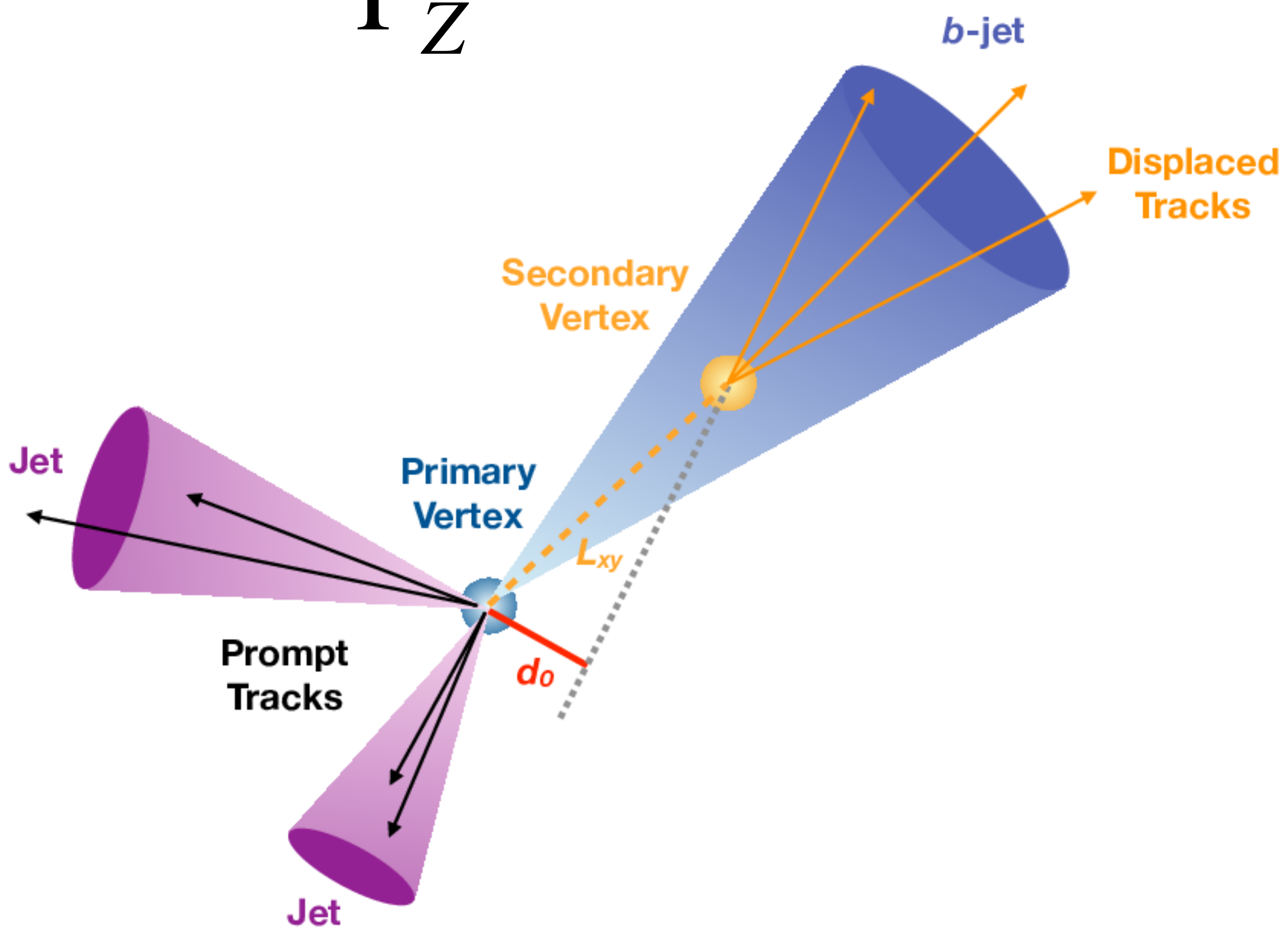
Soft and collinear emissions

Hard collinear emissions at $z_i = 1$

Small k_t region: Non perturbative

Heavy flavor jets

- $m_{\mathcal{Q}} > \Lambda_{\text{QCD}}, \quad \mathcal{Q} = c, b, t$
- Linked to Higgs physics and to EW symmetry breaking
- $\dfrac{\Gamma_{Z\to b\bar{b}}}{\Gamma_Z} \simeq 15\,\%\,, \dfrac{\Gamma_{Z\to c\bar{c}}}{\Gamma_Z} \simeq 12\,\%$



- Heavy flavor processes offer a more robust test of pQCD
- Long-enough lifetime of B hadrons and D mesons: easily detected in collider experiments identifying their displaced vertices
- Top quark decays before hadronizing

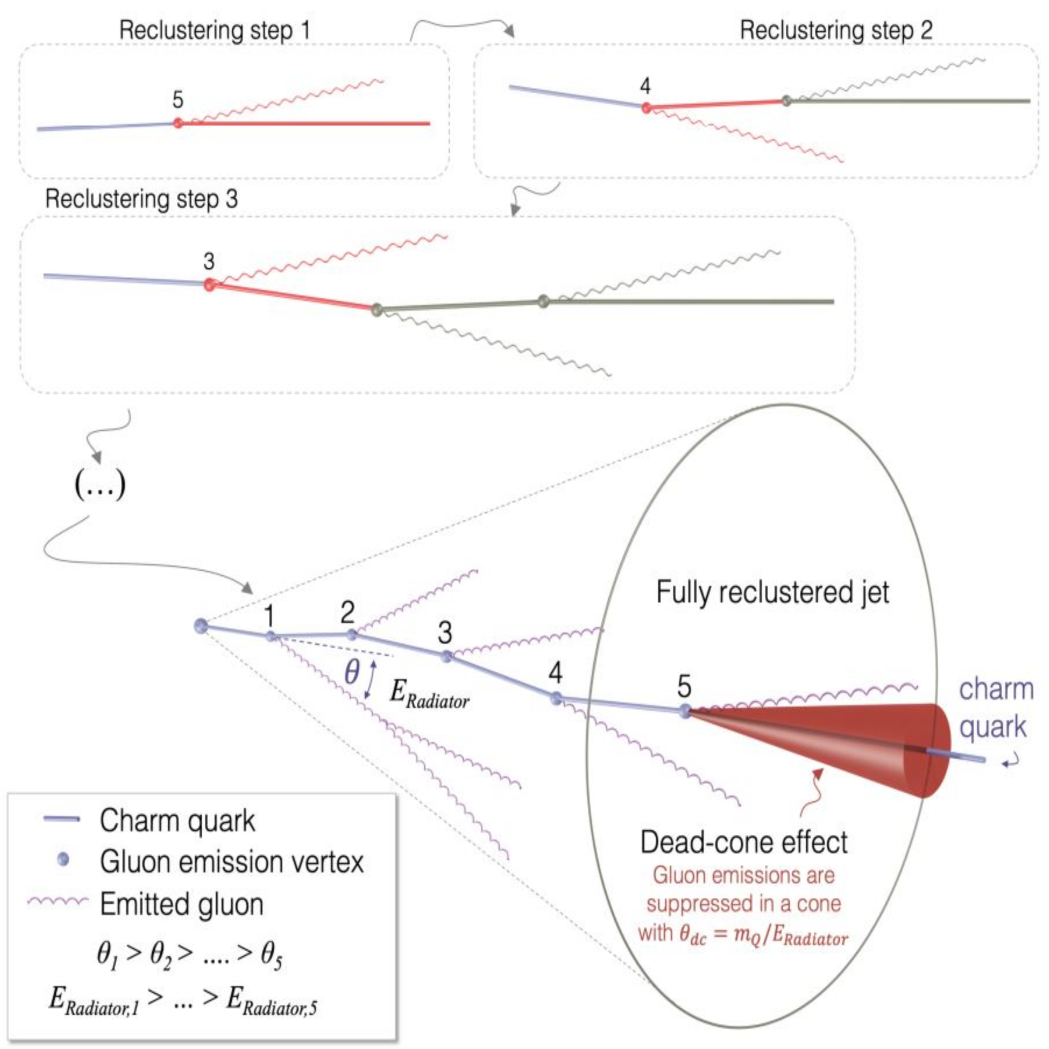
Dead cone effect

When jets are initiated by a heavy flavor, the quark mass shields the collinear singularity

$$d\sigma \simeq \frac{\alpha_S C_F}{2\pi}\frac{dz}{z}\frac{d\theta^2}{\theta^2 + \frac{m^2}{E^2}}$$

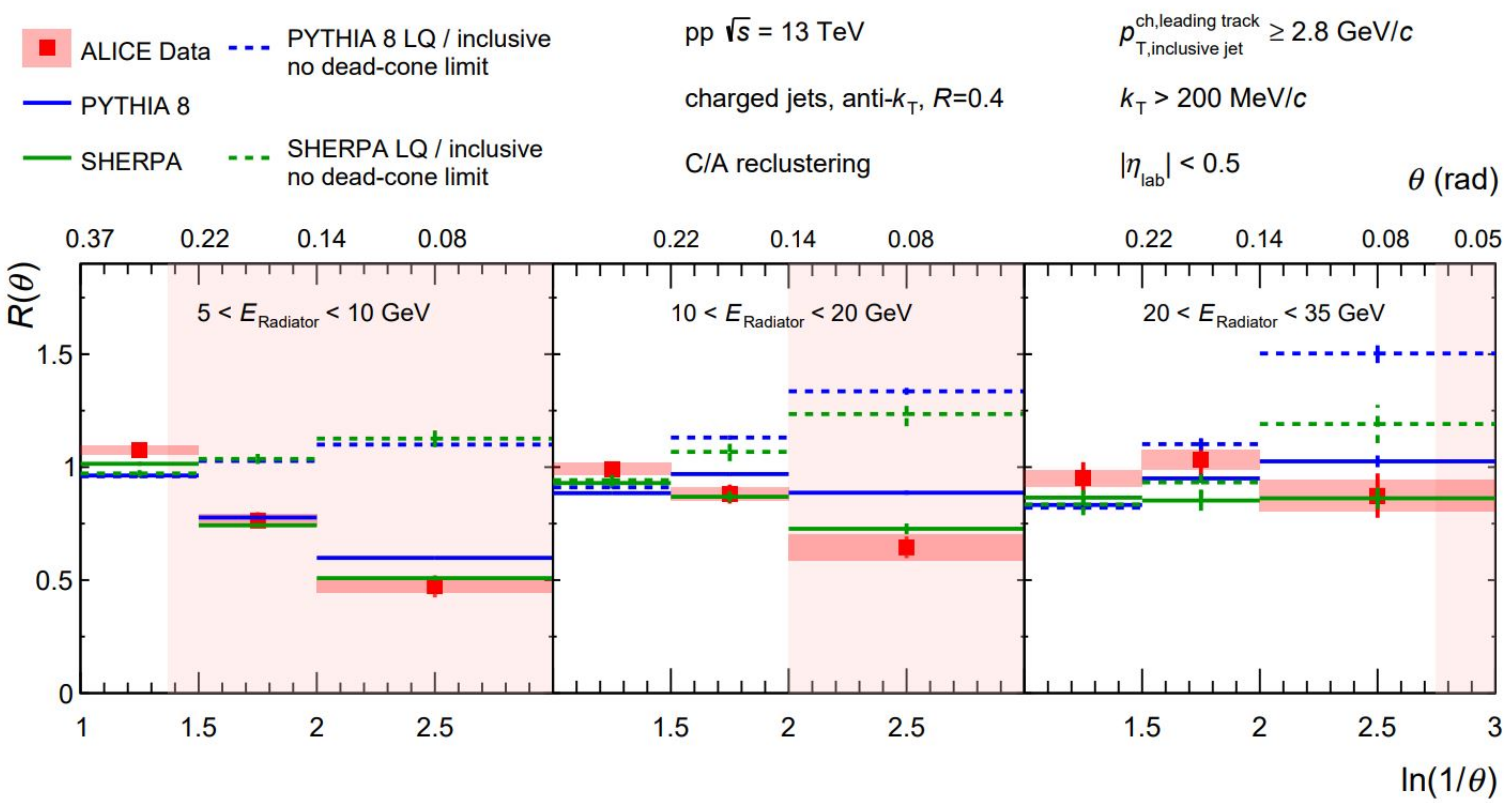
$m =$ heavy quark mass

$E =$ heavy quark energy



Dead cone effect

the radiation emitted off a heavy flavor is suppressed inside a cone of opening angle $\theta_D \sim m/E$ (ALICE)



All order calculation with heavy flavours

- How do we model calculations we heavy flavours?
- Many scales involved:

1. Mass of the heavy quark m, larger than $\Lambda_{\mathrm{QCD}} \sim 1\ \mathrm{GeV}$
2. Hard scale of the process $\sqrt{s}$ (center of mass energy)
3. Substructure variable v we want to probe

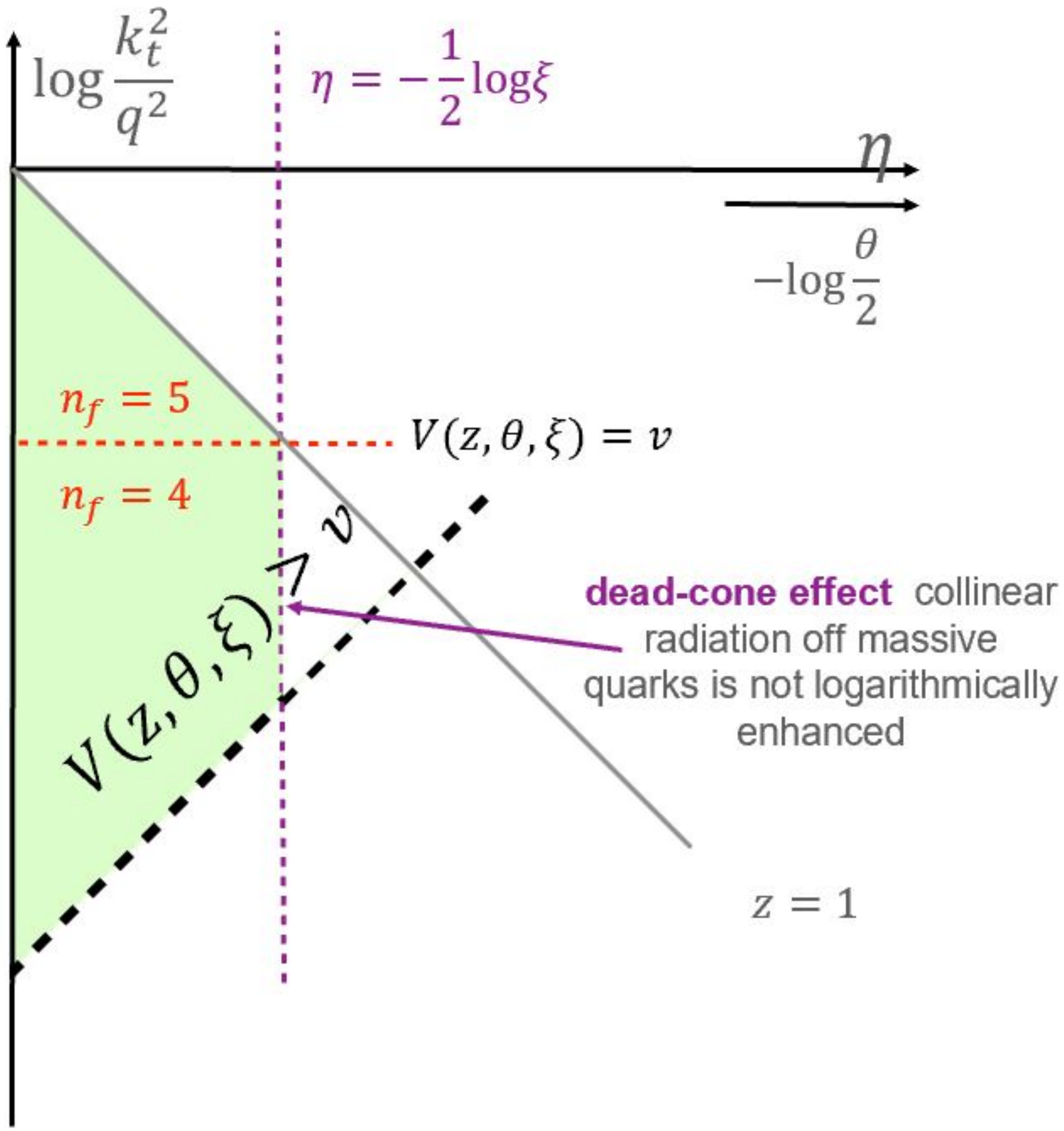
We need to understand the hierarchy between the various scales and to perform multiple resummations $(\log \frac{m^2}{s}, \log v)$

Lund Plane with heavy flavors

In the case of emissions off heavy flavour Lund plane diagrams receives substantial corrections:

(A.G, S. Marzani, G. Ridolfi)

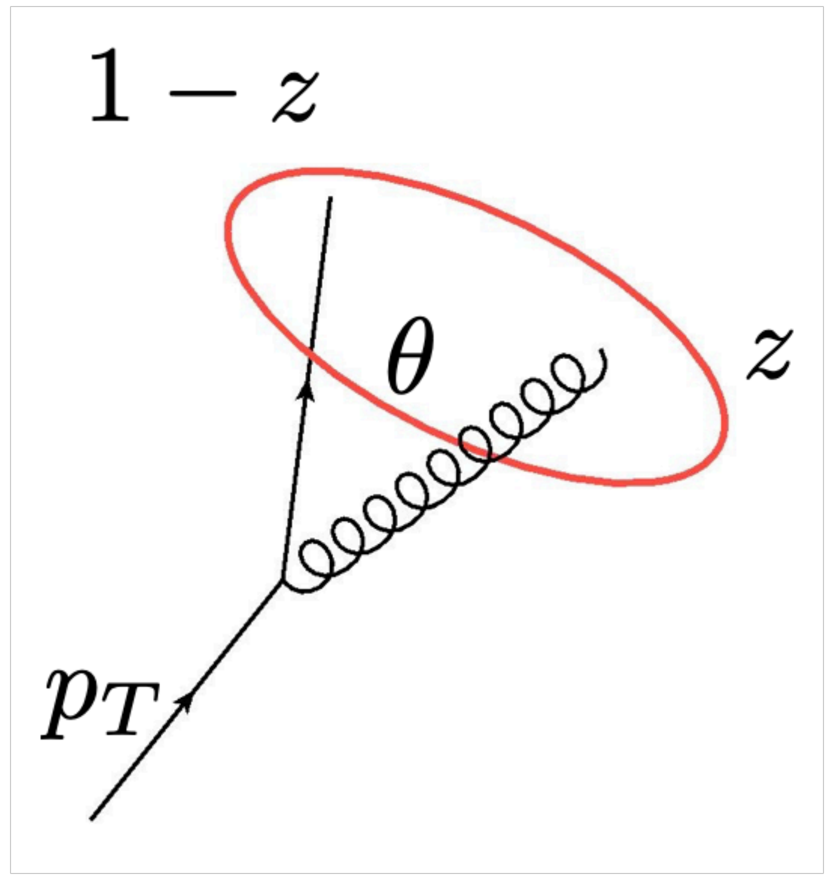
1. Collinear factorization is replaced by quasi-collinear factorization ($k_t \sim m \ll \sqrt{s}$)
2. The mass of the heavy flavour imposes a boundary on the emission rapidity.
3. Horizontal line at constant k_t divides the five flavour region from the four flavour one.



Jet angularities and ECFs

We want to study observable sensitive to dead-cone effect

We study jet angularities λ^α and energy correlation functions e_2^α



In a massless theory, considering only one emission:

$$\lambda^\alpha \simeq e_2^\alpha \simeq z\theta^\alpha$$

Many possible choices in the case of massive particles within the jet. ECFs with massive quarks studied in (C. Lee, P. Shrivastava, V. Vaidya) with SCET

Which one is more sensitive to the dead-cone effect?

Possible definitions in e^+e^- collisions

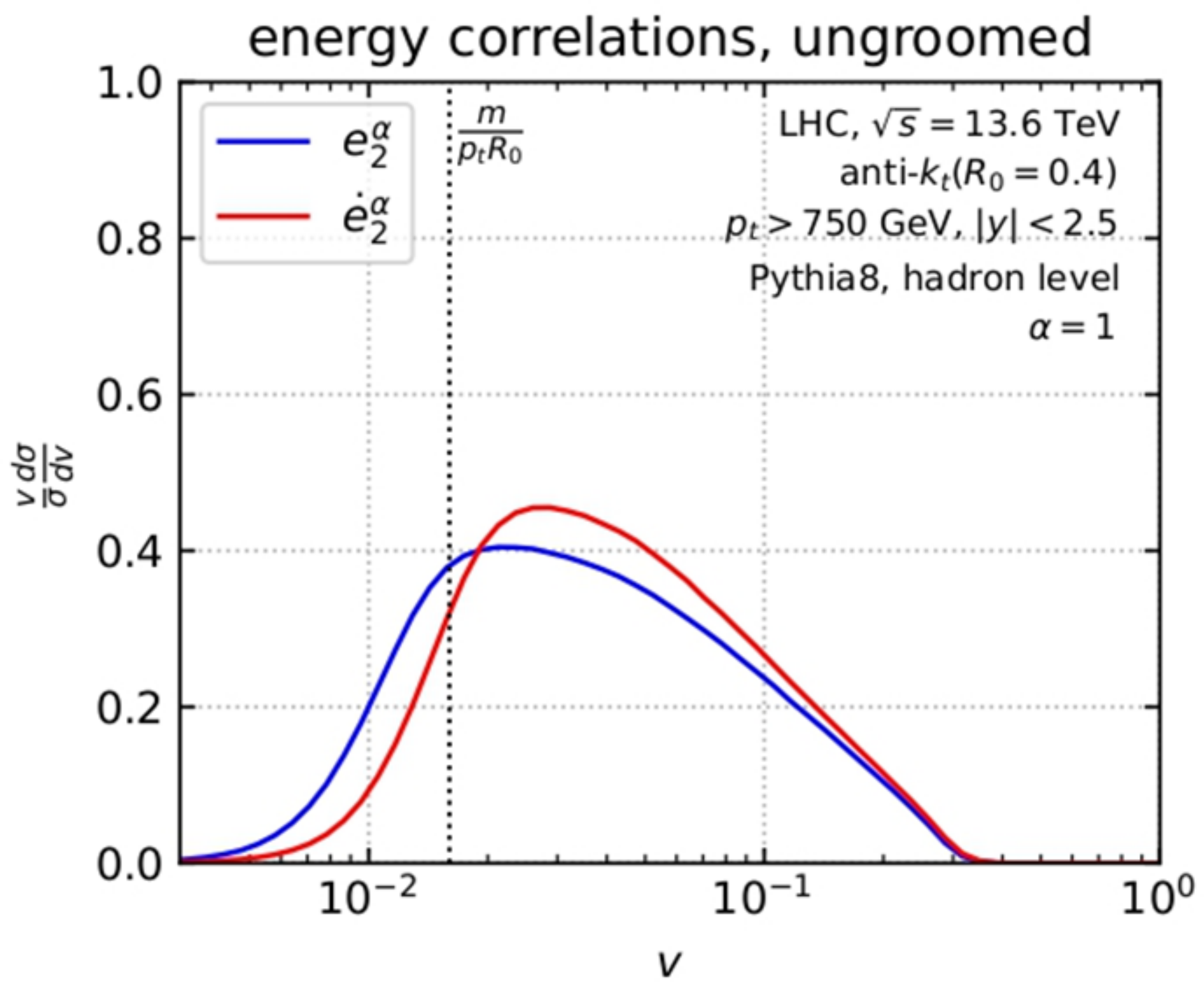
$$e_2^\alpha = \sum_{\mathscr{H}} \sum_{i,j\in\mathscr{H},i<j} z_i z_j \left[2(1-\cos\theta_{ij})\right]^{\frac{\alpha}{2}} \Theta\left((\vec{p}_i\cdot\vec{n})(\vec{p}_j\cdot\vec{n})\right) \simeq \sum_{\mathscr{H}} \sum_{i,j\in\mathscr{H},i<j} z_i z_j\, \theta_{ij}^\alpha$$

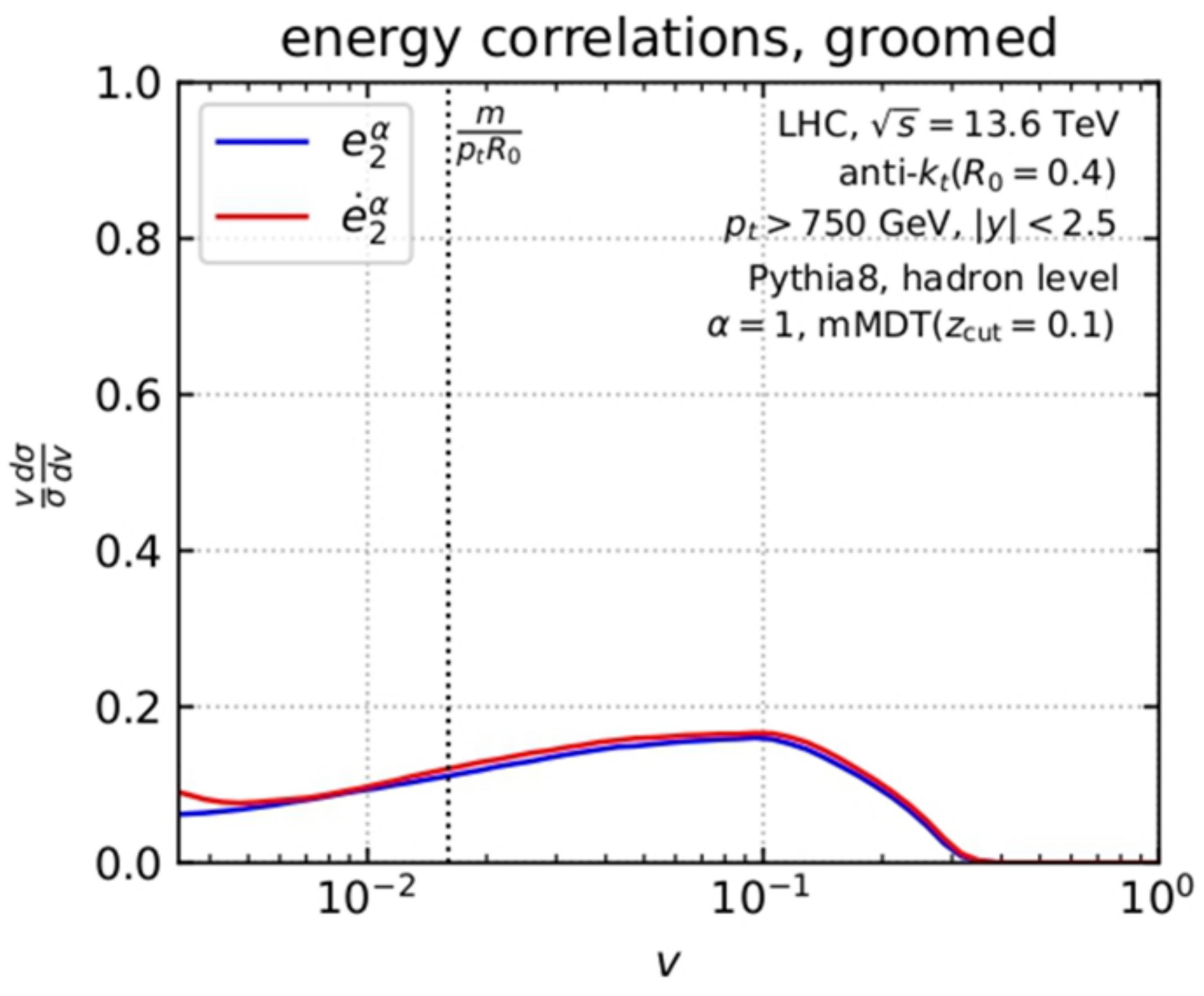
$$\dot{e}_2^\alpha = \sum_{\mathscr{H}} \sum_{i,j\in\mathscr{H},i<j} z_i z_j \left[\frac{2p_i\cdot p_j}{E_iE_j}\right]^{\frac{\alpha}{2}} \Theta\left((\vec{p}_i\cdot\vec{n})(\vec{p}_j\cdot\vec{n})\right) \simeq \sum_{\mathscr{H}} \sum_{i,j\in\mathscr{H},i<j} z_i z_j \left(\theta_{ij}^2 + \frac{m_i^2}{E_i^2} + \frac{m_j^2}{E_j^2}\right)^{\frac{\alpha}{2}}$$

$\vec{n}$: reference vector that defines the hemisphere $\mathscr{H}$

- We focus for simplicity on ECFs (many more possible definitions for λ^α)
- The two observables do not coincide in the quasi collinear limit

Monte Carlo analysis





$$e_2^\alpha \simeq \sum_{\mathscr{H}} \sum_{i,j \in \mathscr{H}, i<j} z_i z_j \, \theta_{ij}^\alpha$$

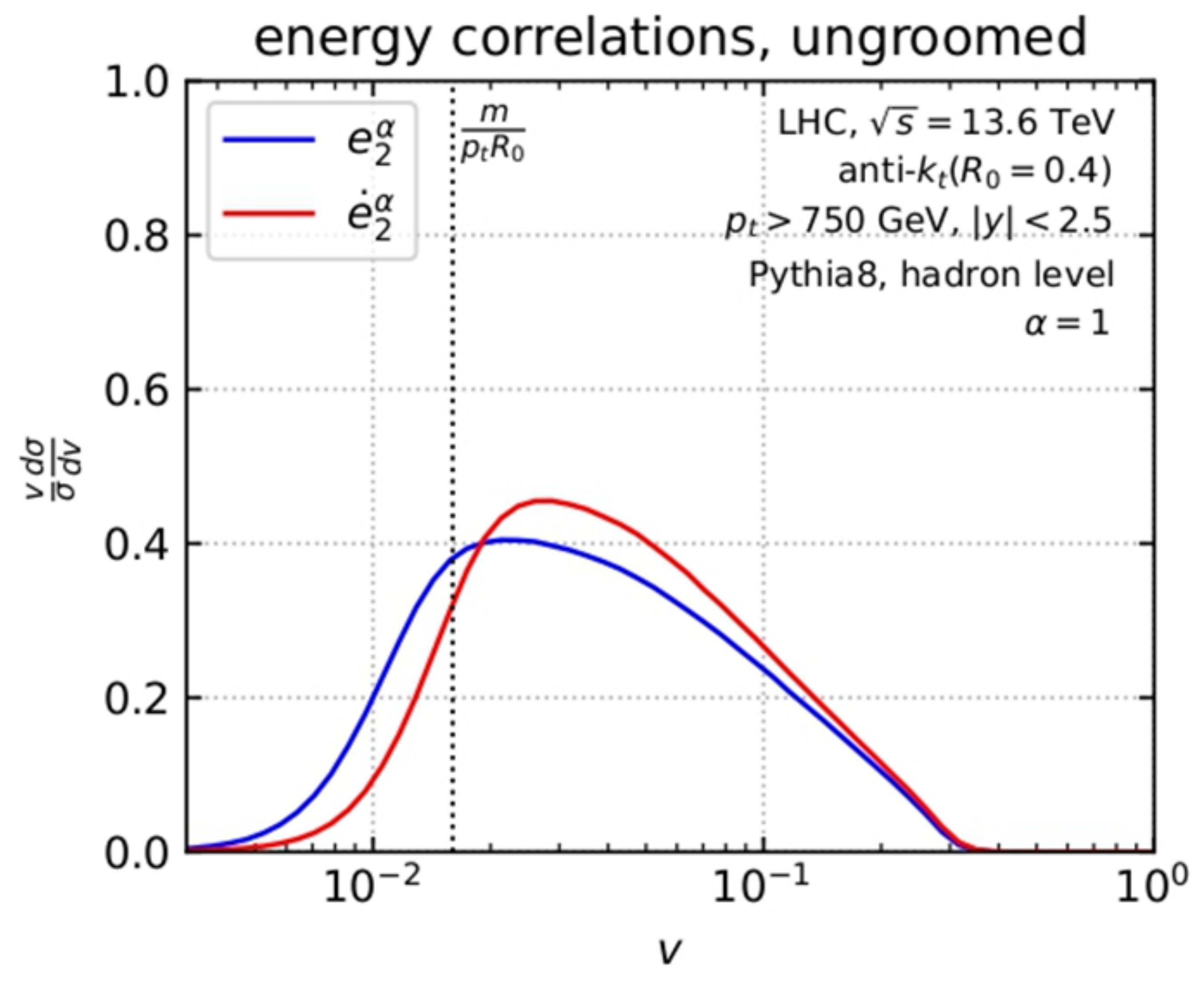
$$\dot{e}_2^\alpha \simeq \sum_{\mathscr{H}} \sum_{i,j \in \mathscr{H}, i<j} z_i z_j \left(\theta_{ij}^2 + \frac{m_i^2}{E_i^2} + \frac{m_j^2}{E_j^2} \right)^{\frac{\alpha}{2}}$$

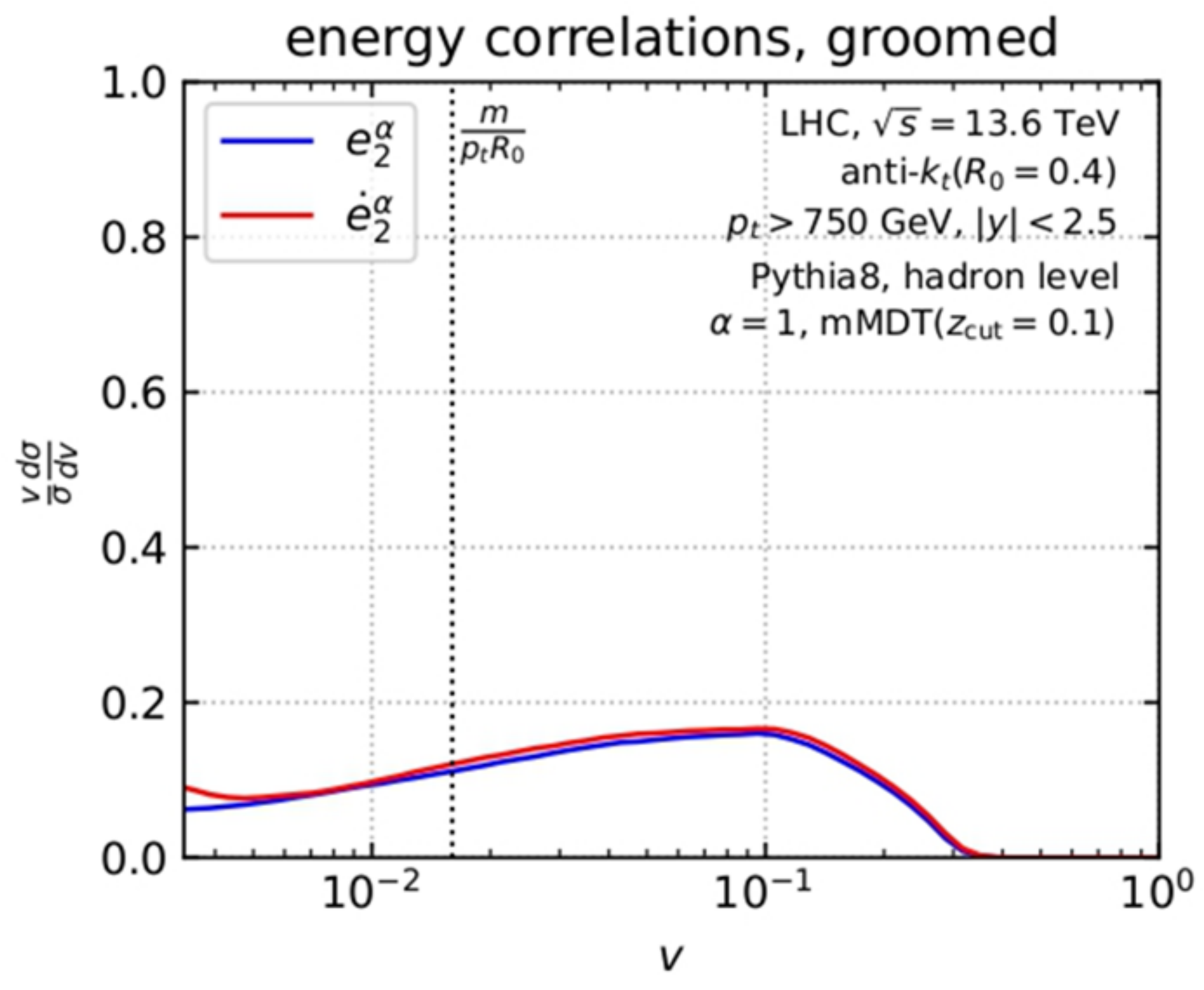
See P. Dhani et al

- The dot observables exhibits a larger peak than e_2^α: more mass sensitive.

- The mass contribution in the e_2^α distribution comes only from the matrix elements (“dynamical”)

Monte Carlo analysis





$$e_2^\alpha \simeq \sum_{\mathscr{H}} \sum_{i,j \in \mathscr{H}, i<j} z_i z_j \, \theta_{ij}^\alpha$$

$$\dot{e}_2^\alpha \simeq \sum_{\mathscr{H}} \sum_{i,j \in \mathscr{H}, i<j} z_i z_j \left(\theta_{ij}^2 + \frac{m_i^2}{E_i^2} + \frac{m_j^2}{E_j^2} \right)^{\frac{\alpha}{2}}$$

See P. Dhani et al

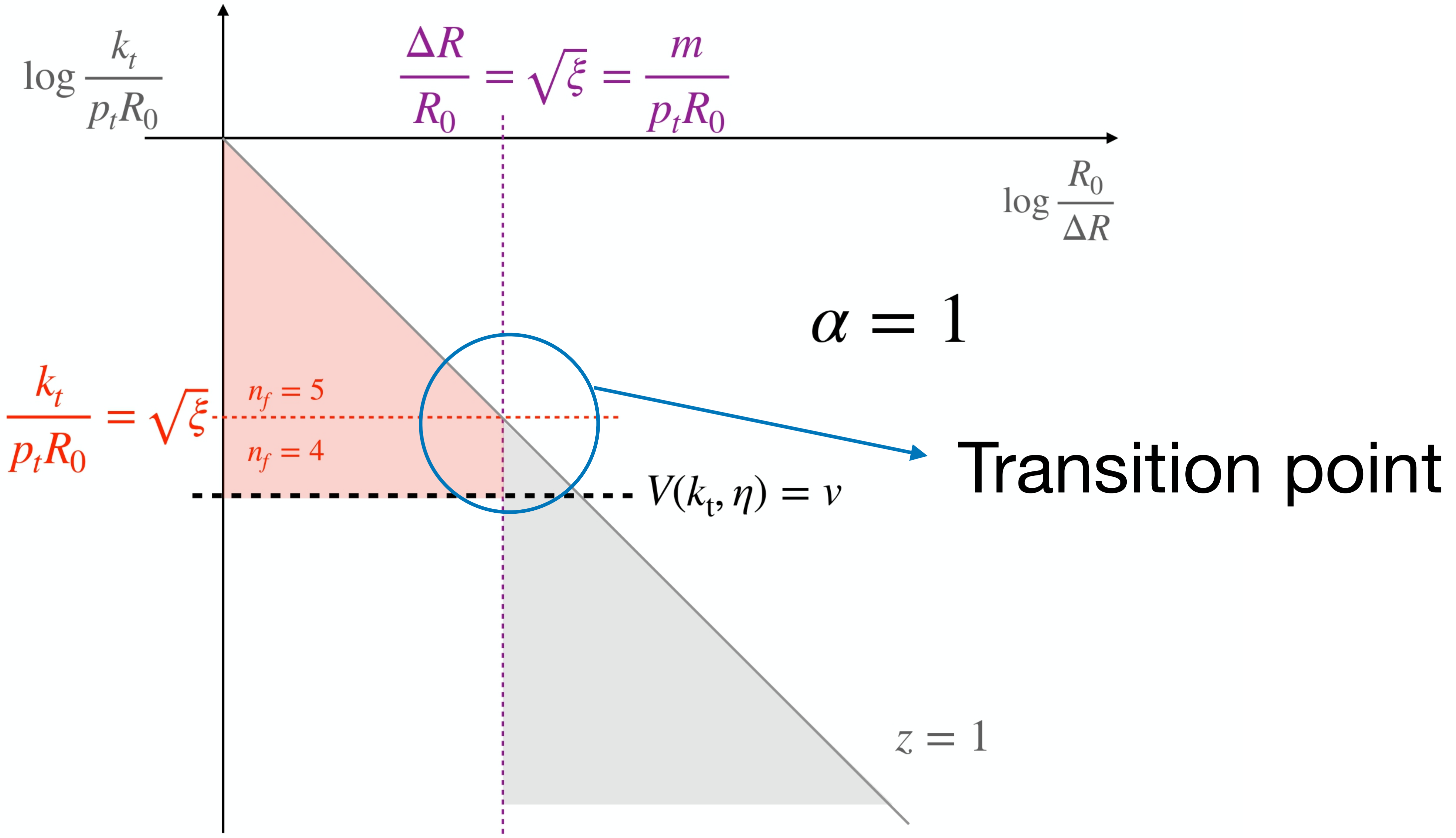
In the groomed case, the solid red curve starts to exhibit a small peak in the tail of the distribution. We cannot have an arbitrarily soft emission: $\min(z_i, z_j) > z_{\rm cut}$

All order calculation

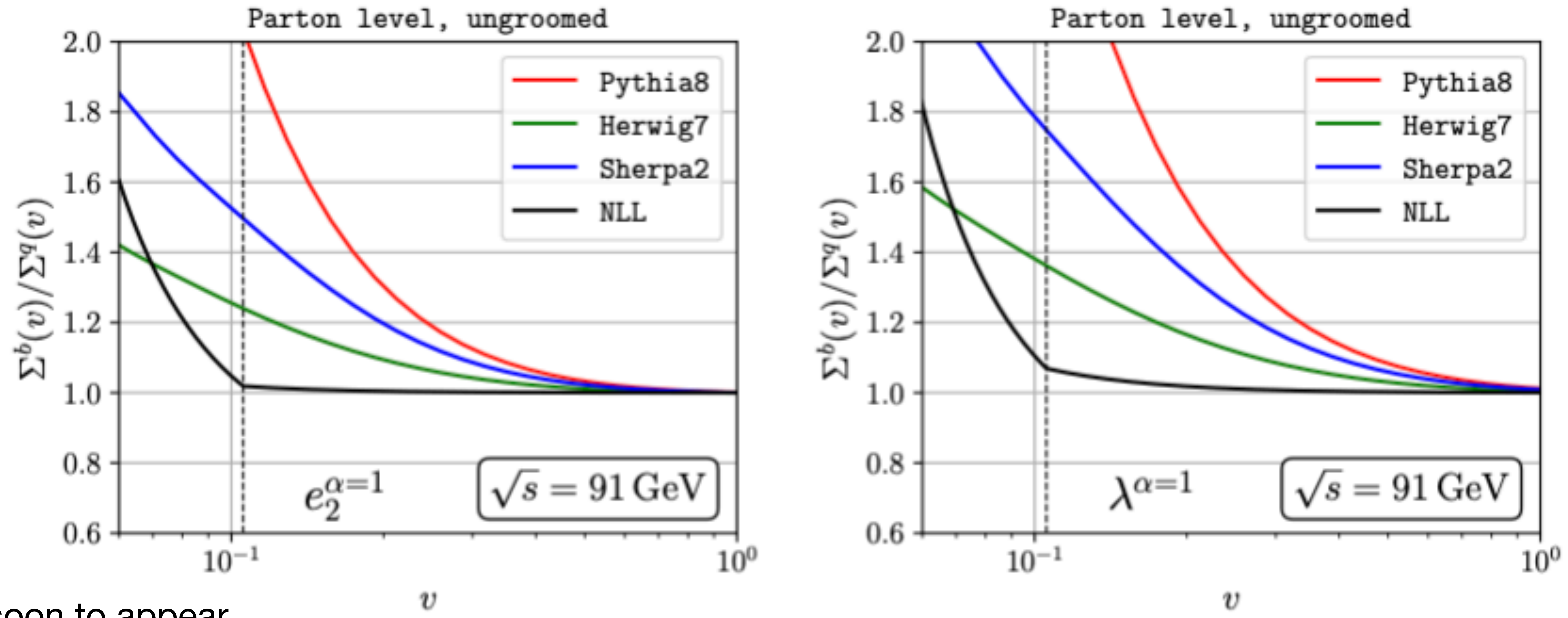
From analytical point of view, all the cumulative distribution resum in the same way at NLL.

However, the differential distribution is discontinuous.

- To smooth the transition we decide to incorporate fixed order calculation
- These are NNLL contributions, which depend on the specific definition of the observable



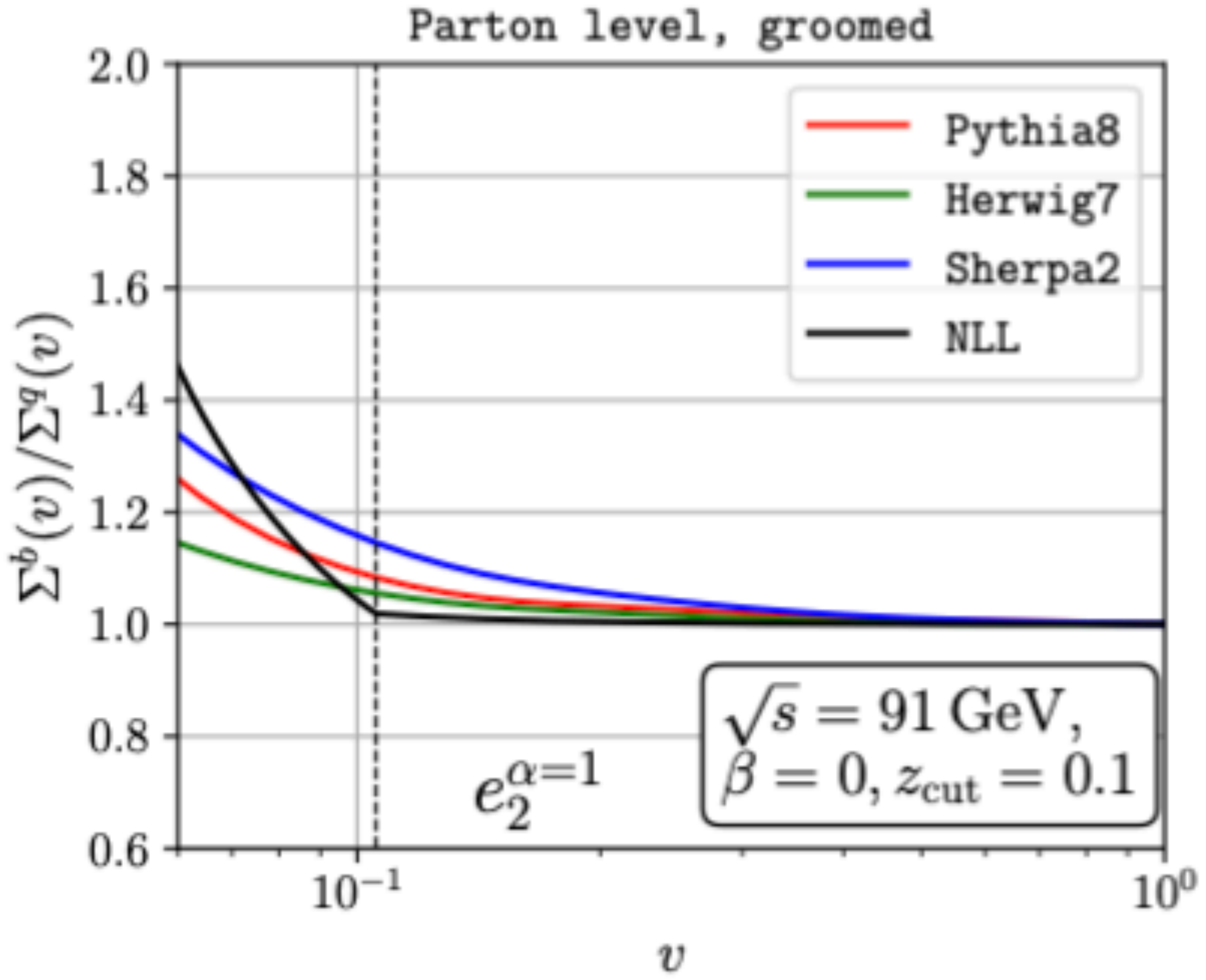
Comparison with MC: ungroomed case

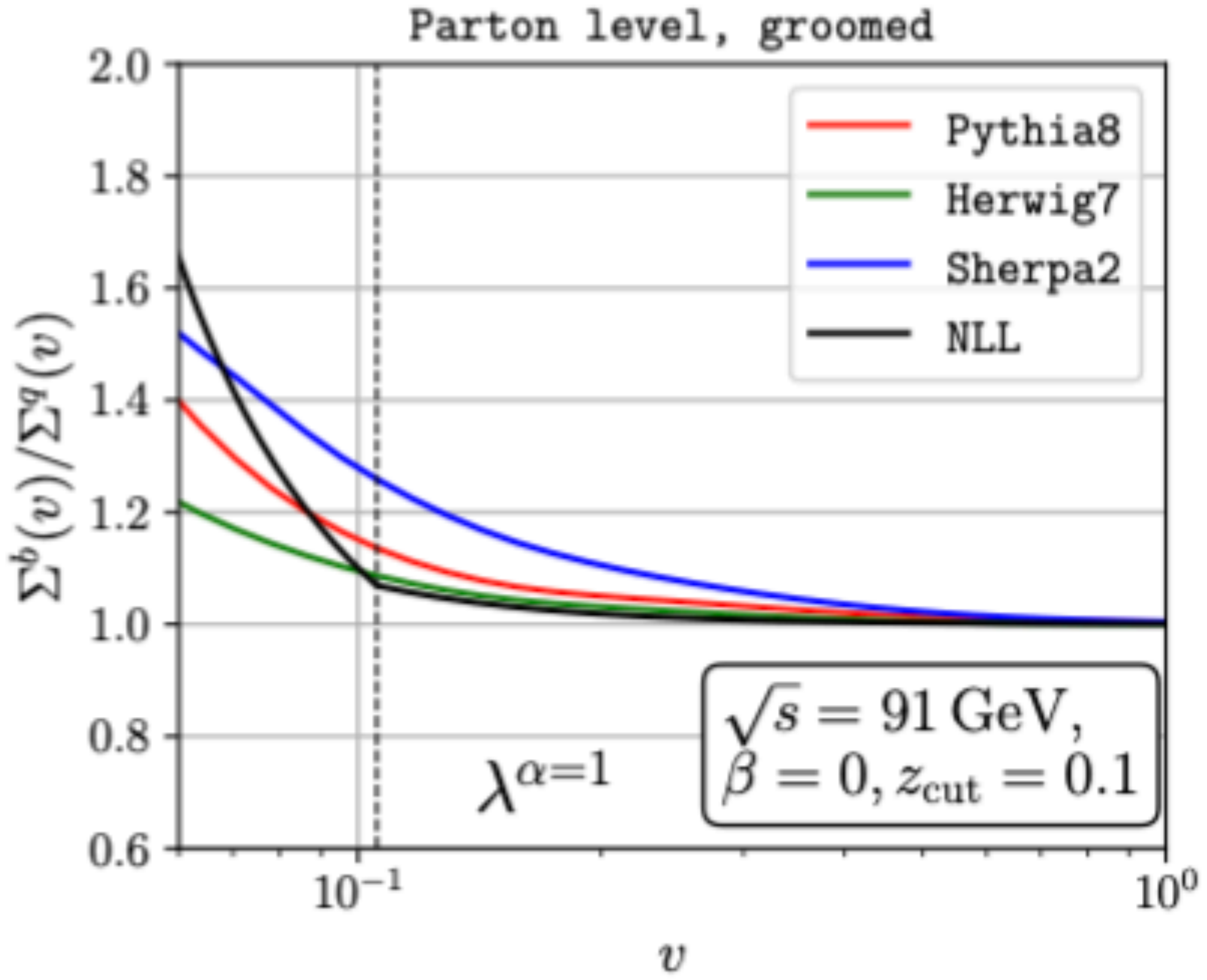


P. Dhani, O. Fedkevych, A. Ghira soon to appear

- Plot of there ratio of the cumulative Σ^b/Σ^q (massive/massless) for $e^+e^- \to 2$ jets
- It appears that the dead cone effect manifests earlier than predicted by theoretical calculations $v \simeq 2m_b/\sqrt{s}$
- Large discrepancy with between analytics and MC in the ungroomed case

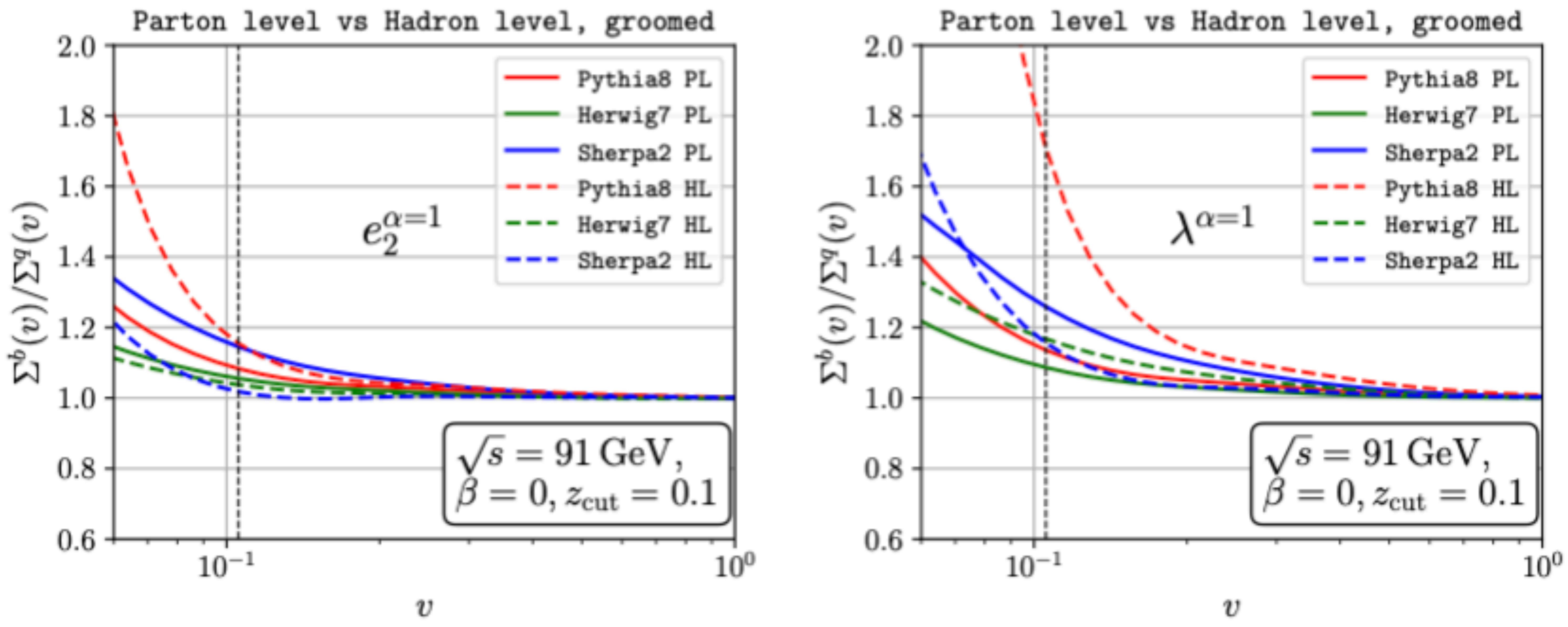
Comparison with MC: groomed case





- Plot of there ratio of the cumulative Σ^b/Σ^q (massive/massless) for $e^+e^- \to 2$ jets
- Very good agreement between MC and analytics
- MC predictions are close to each other

Hadron correction groomed case



- Comparison between parton level and hadron level simulations
- Soft Drop reduces the impact of hadronization corrections
- Both the observables are very robust under NP corrections (e_2^α in particular)

Conclusions & outlook

- Study of JSS observable for HF jets at e^+e^-: the angularities and ECFs defined with the scalar products are more sensitive to mass effects. Mass dependence both in the definition of the observable and at amplitude level.
- The distribution associated to plain observable depends on the mass only through the square matrix element, thus all the mass effects that we see are related to a dynamical suppression of the radiation: $\lambda^{\alpha=1}, e_2^{\alpha=1}$ best way to probe the dead-cone
- Study of top quark dead cone effect at ILC $\sqrt{s} = 2$ TeV (Maltoni et al.)

Thanks for your attention!!!