

# Solid Plasma-based Wakefield Accelerators

EuPRAXIA Camp I: Technologies

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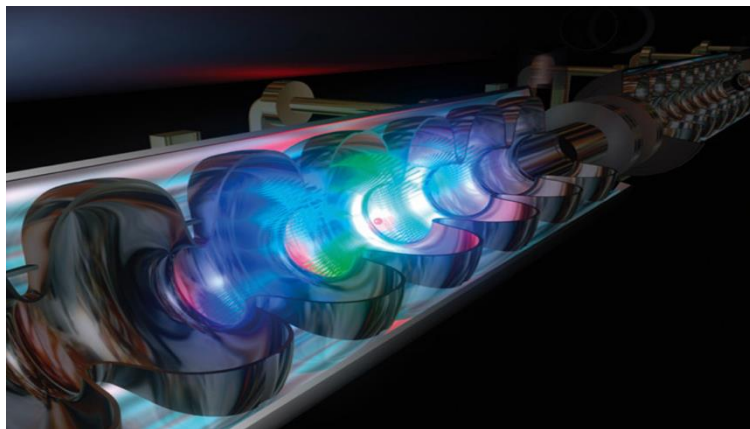
University of Liverpool, The Cockcroft Institute

8<sup>st</sup> April.2025, PISA, Italy

# Introductions & Motivation

Conventional accelerator

100 MV/ m

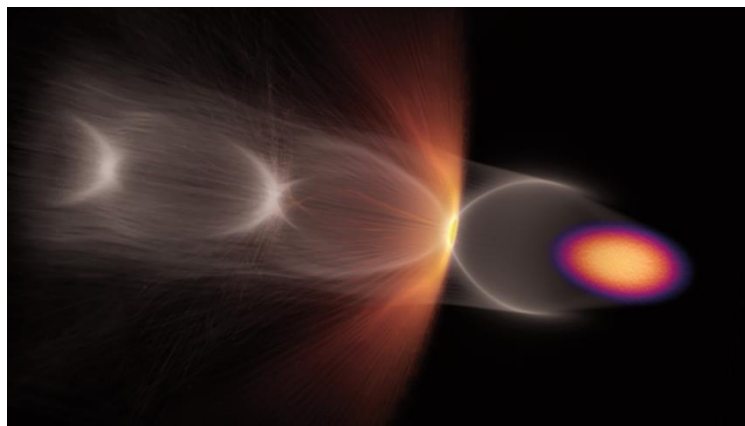


LHC@CERN

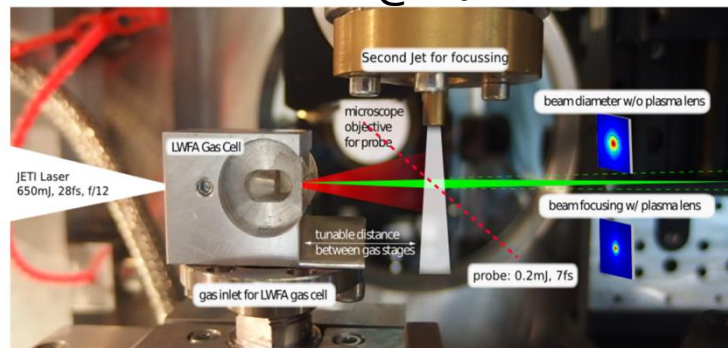


Gaseous plasma accelerator

100 GV/m

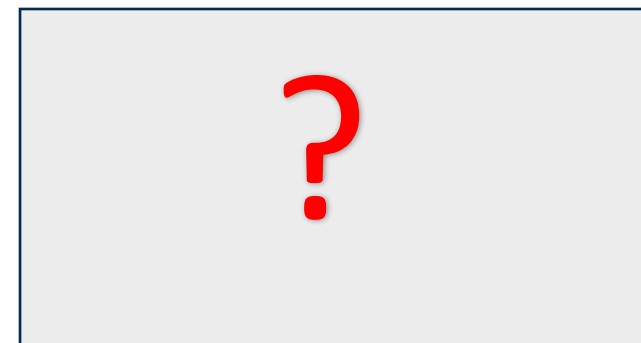
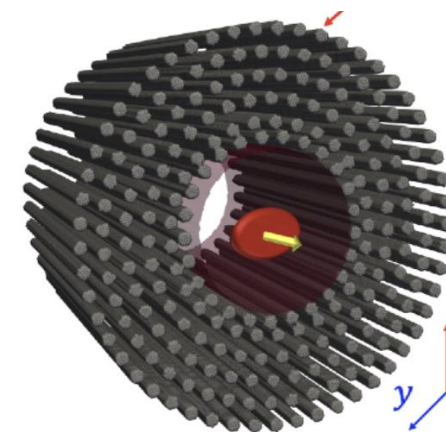


LWFA@HIJ

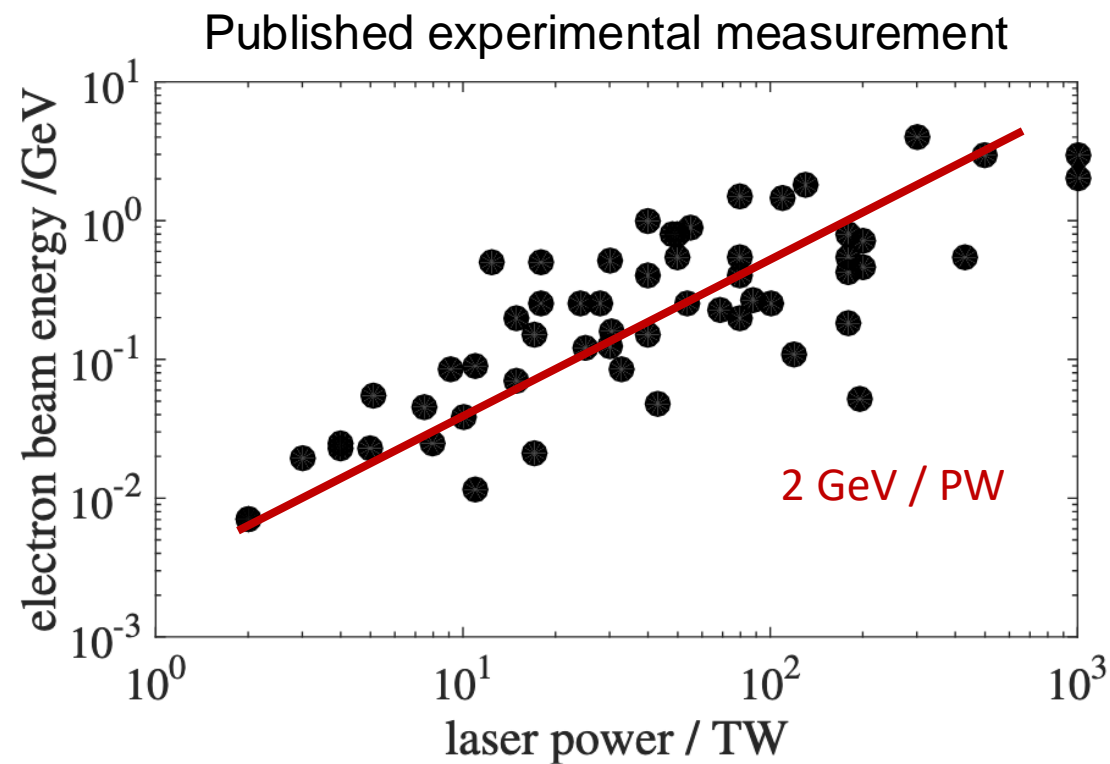
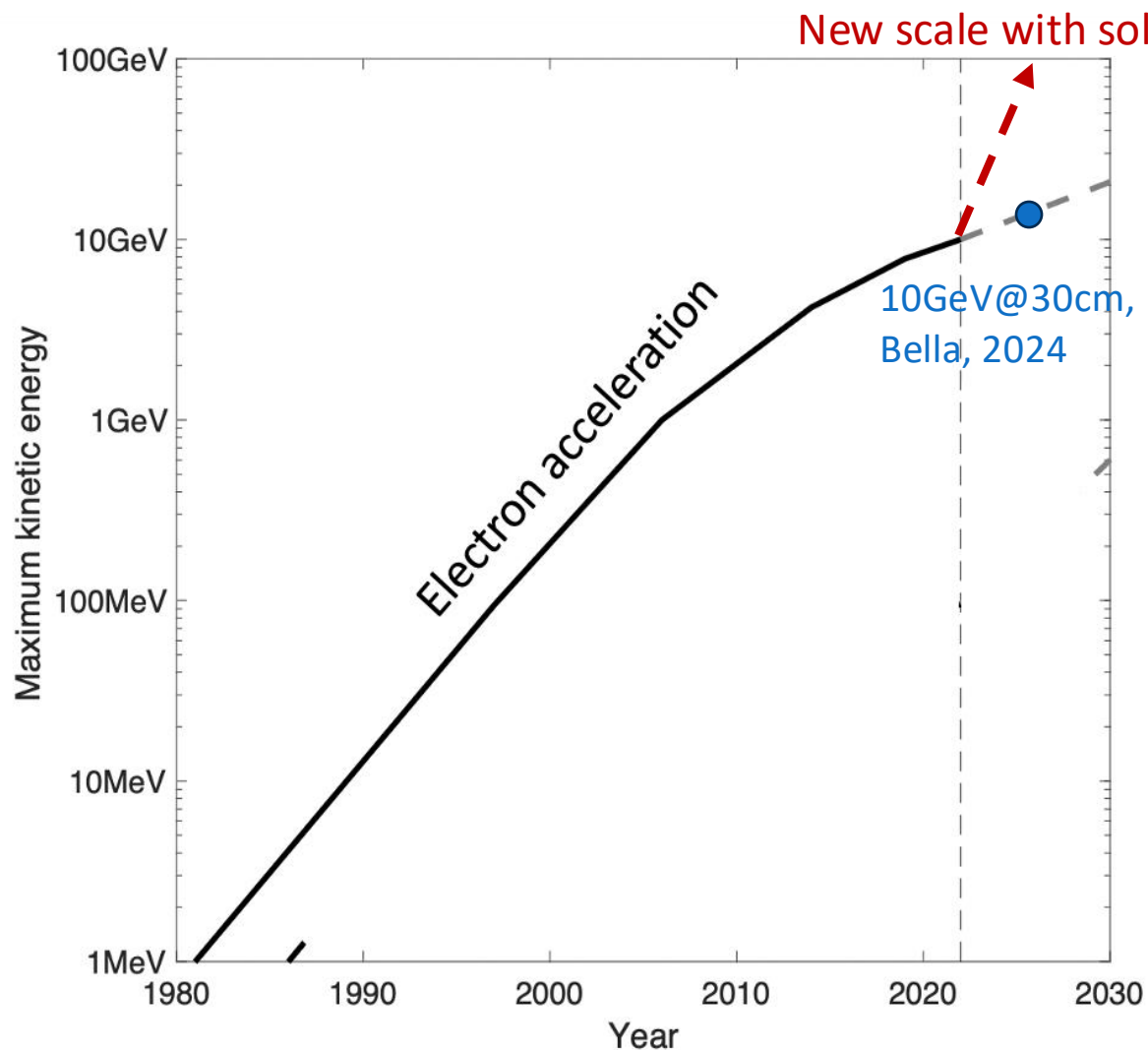


Solid-density plasma accelerator

100 TV/m

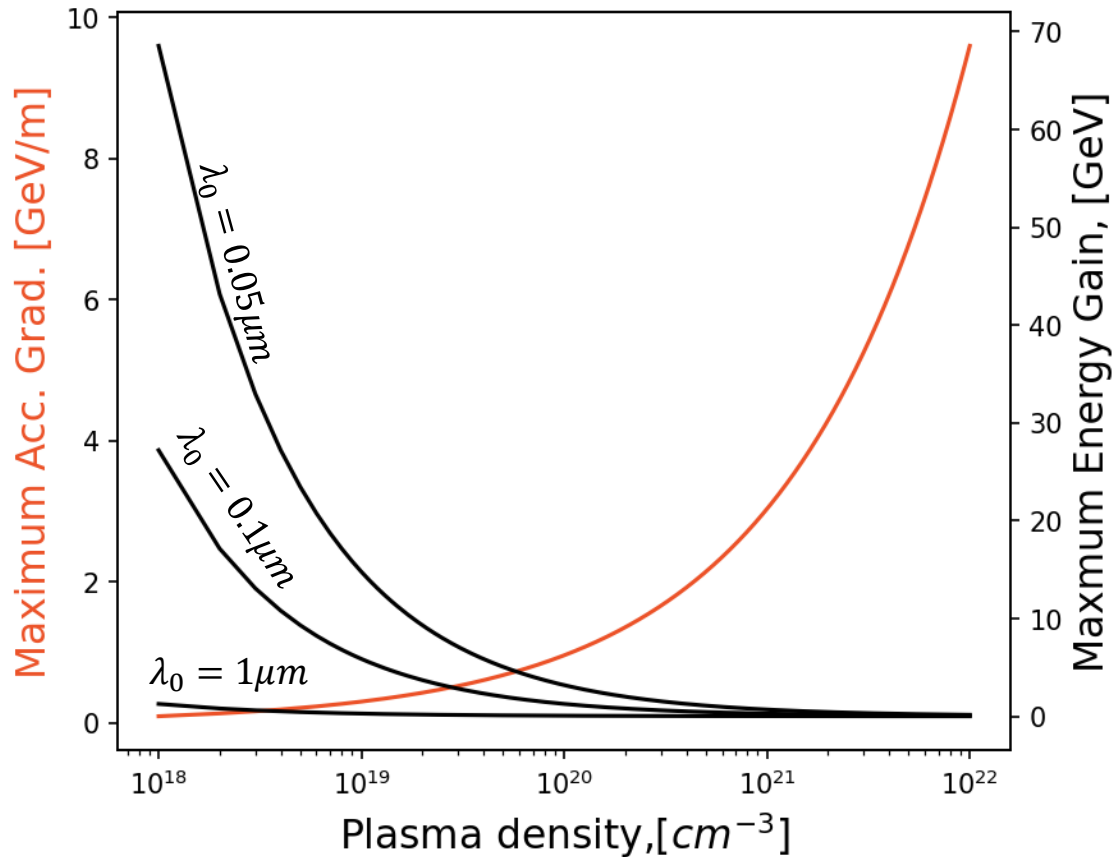


# Energy Frontier of Plasma Accelerator



Scales with infinite laser power? -- > **NO**, due to bubble break!!

# Role of Plasma Density



$$E_{wb} [GeV/m] = \frac{cm_e \omega_p}{e} \simeq 9.6 \sqrt{n_p (cm^{-3})}$$



$$L_d [cm] \sim \frac{3.9}{\lambda_0^2 [\mu m] n_p [10^{18} cm^{-3}]}$$

$$\Delta E [GeV] \simeq 1.7 \left( \frac{P [TW]}{100} \right)^{1/3} \left( \frac{10^{18}}{n_p [cm^{-3}]} \right)^{2/3} \left( \frac{0.8}{\lambda_0 [\mu m]} \right)^{4/3}$$

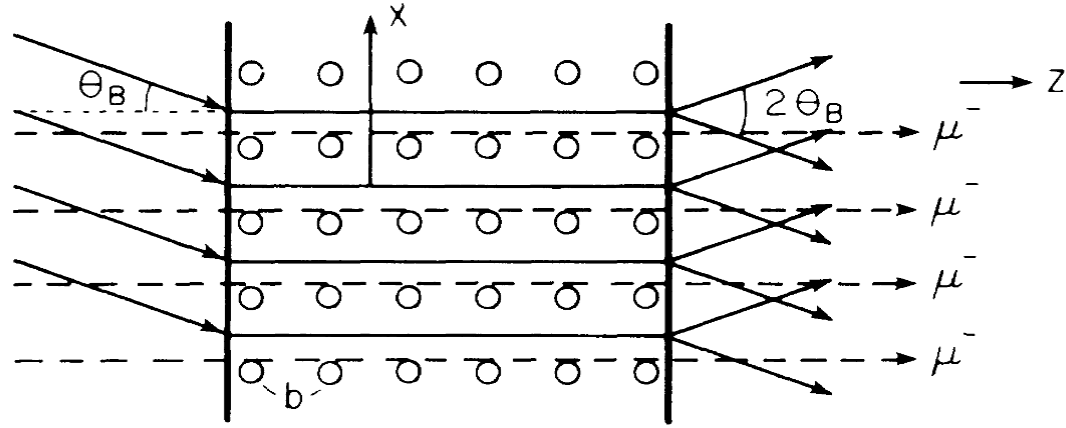


$$n_c = \frac{m_e c^2}{4\pi e^2 \lambda_0^2}$$

How wakefield can be generated in solid plasma  $n_e > n_c$ ?  
 -- > short wavelength of laser -- > X-ray

# X-ray-driven Crystal Accelerator

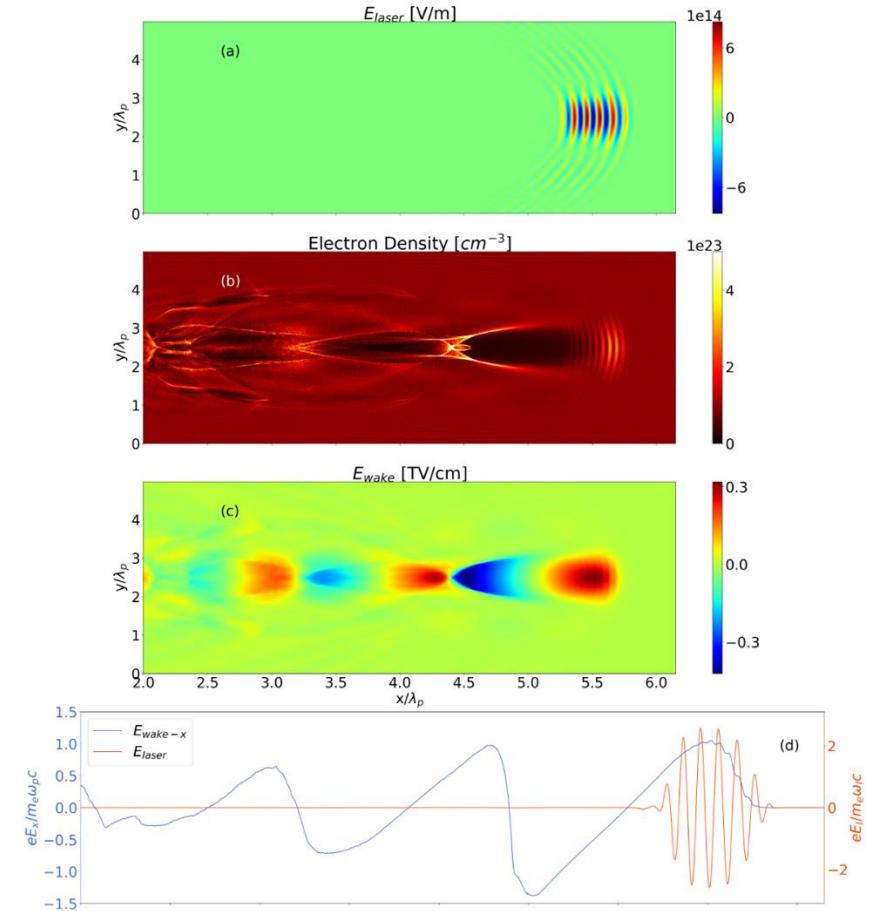
- Plasma density in Crystal: up to  $10^{27} \text{ cm}^{-3}$
- X-ray:  $\lambda_0 = 1 \text{ nm}$  --  $\rightarrow n_c = 1.11 \times 10^{27} \text{ cm}^{-3}$  and  $E_{WB} = 1 \text{ PeV/m}$



When the x rays are injected at the Bragg angle, the Bormann effect takes place. Particle beams injected along the crystal axis can be accelerated.

@T. Tajima et al 1987

## X-ray driven plasma wakefield in crystal



@Sahel, et al 2019

# Challenges and Future Research

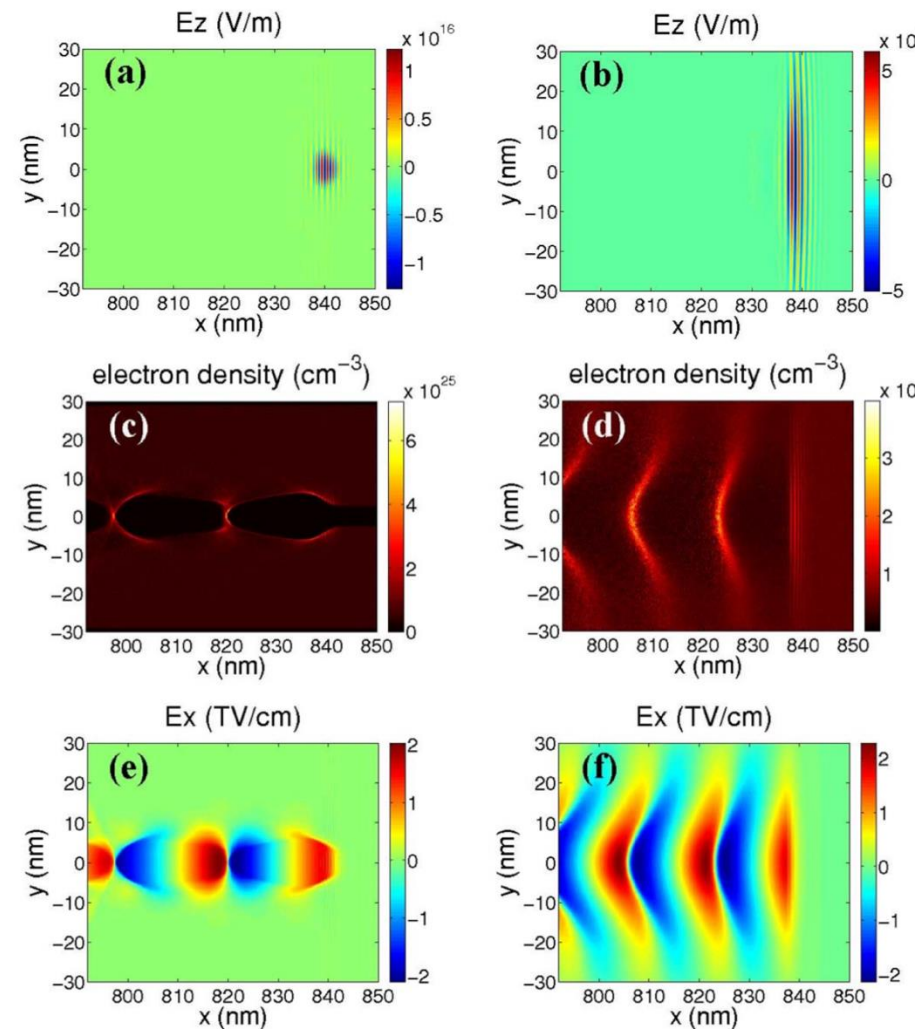
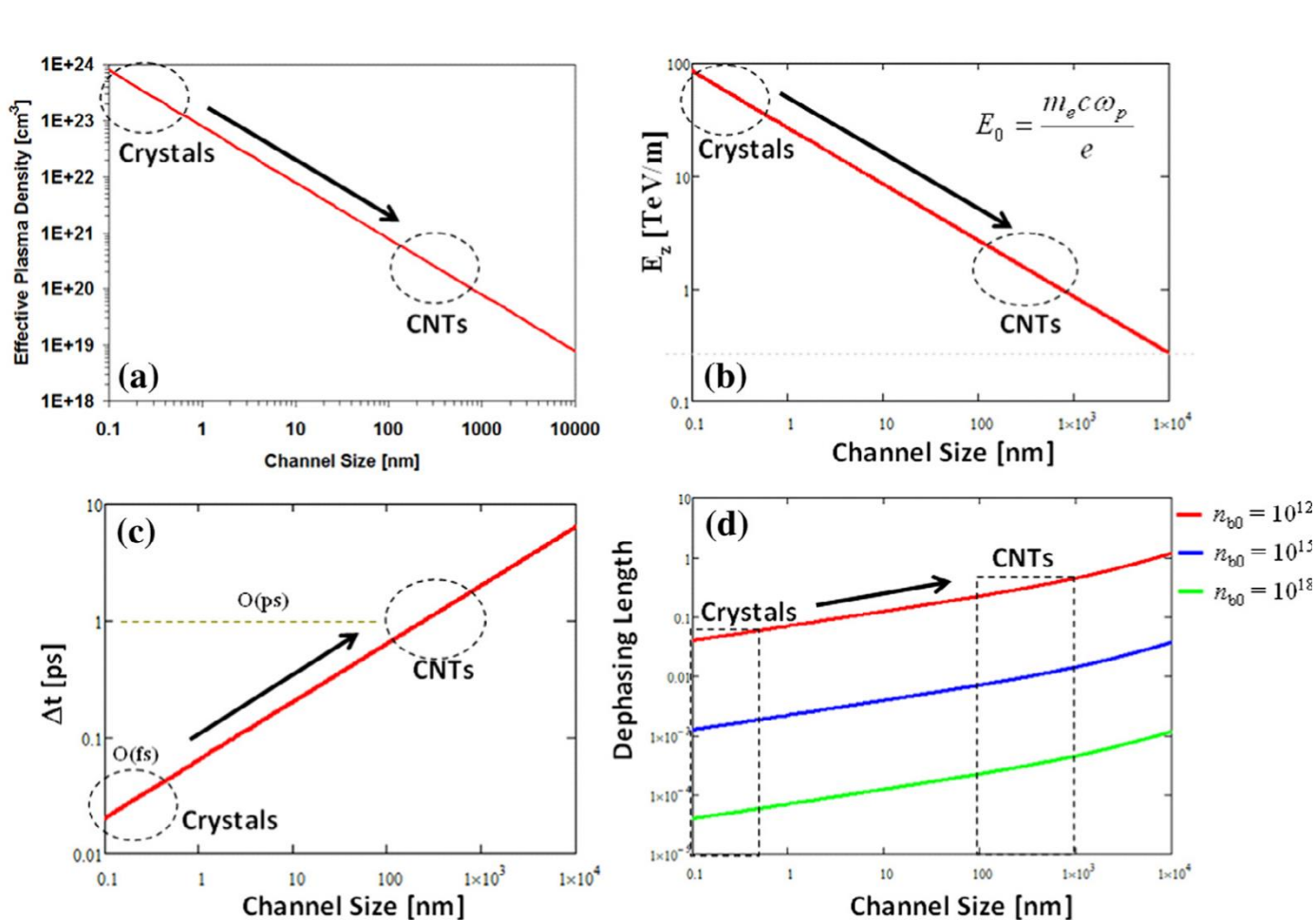
## Open questions:

1. Material study: crystal damage and durability, ångströms-size limiting the acceleration volume and leads to fast dissociation
2. Availability of high intensity X-ray .
3. Complex particle and beam dynamics in crystal acceleration channels; Instabilities in crystal acceleration channels, such as filamentation/Weibel instability
4. Complex interaction of intense X-rays and crystalline structures
5. Practical Implementation: Transitioning from theoretical concepts to practical accelerator designs
6. Engineering challenges in crystal fabrication, beam handling, and detector systems need to be addressed.
7. ...

## Important topics:

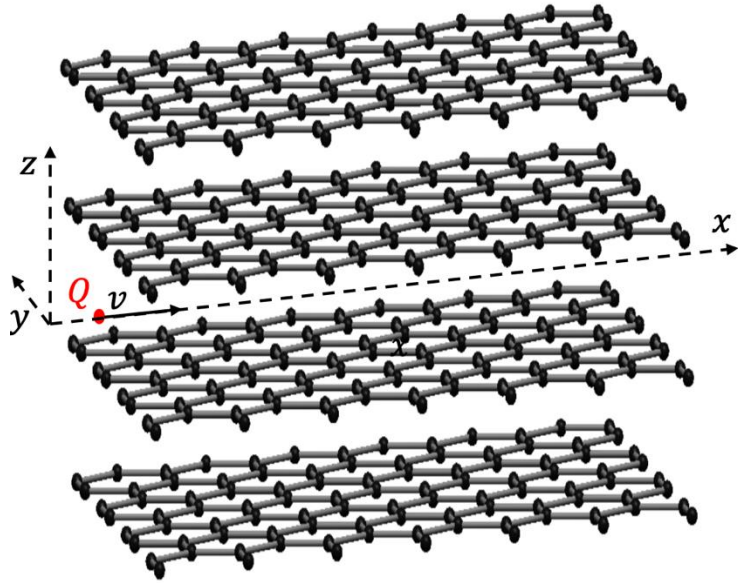
1. Exploring different crystal materials and geometries.
2. Developing advanced X-ray optics and beam delivery systems.
3. Simulating and modelling the interaction processes, e.g. particle and beam dynamics in crystal acceleration channels instabilities in crystal acceleration channels, such as filamentation/Weibel instability ,
4. Investigating novel techniques for beam diagnostics and control.
5. ...

# Wakefield Excitation in Nano Structures

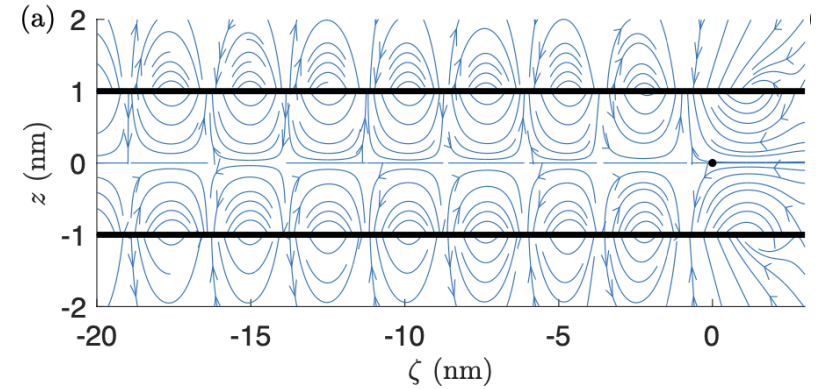
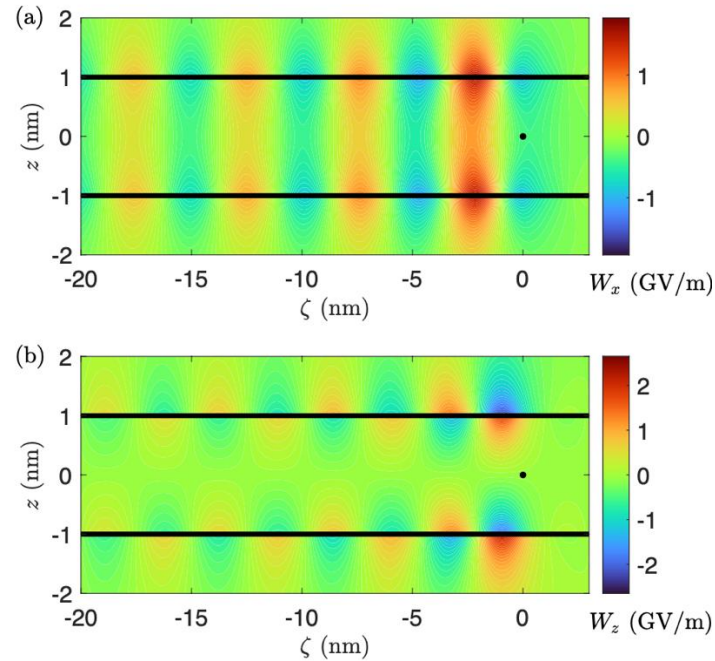


@Shin et al 2019

# Wakefield in Graphene Layers: one particle model



@Martín et al. 2025



Continuity equation

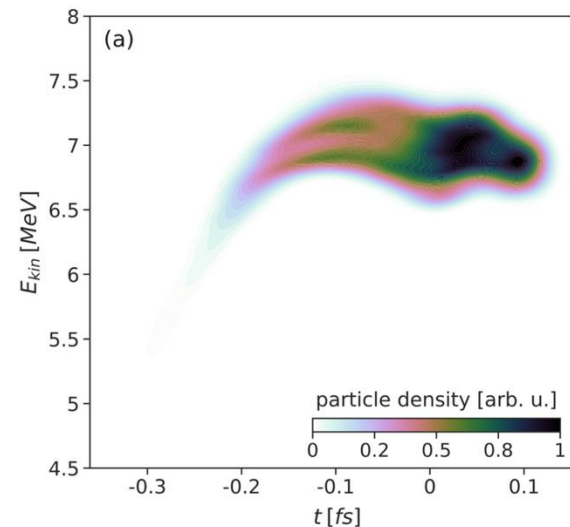
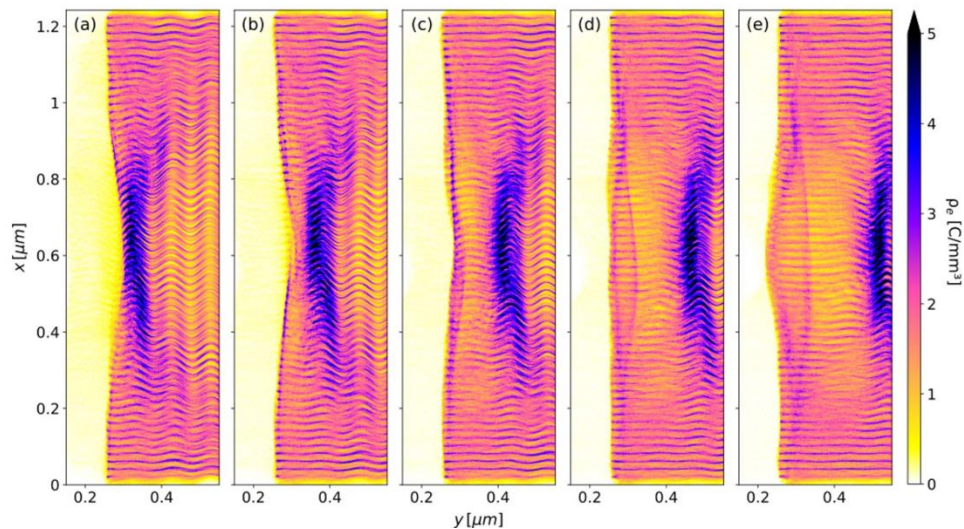
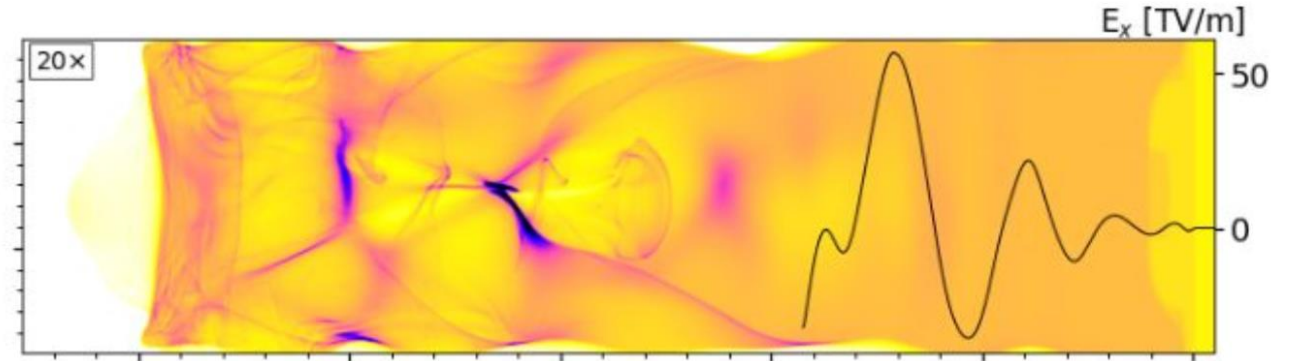
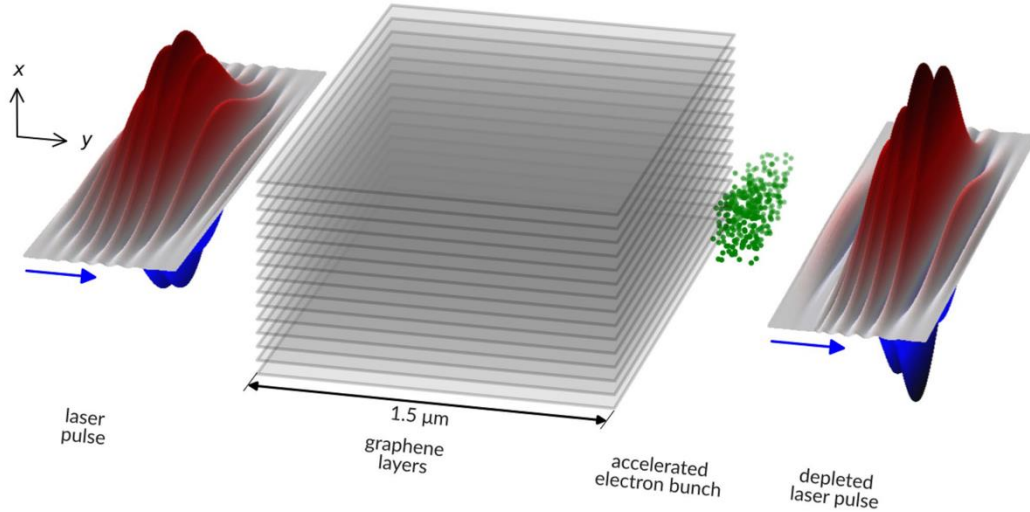
$$\frac{\partial n_j(\mathbf{r}_j, t)}{\partial t} + n_{0j} \nabla_j \cdot \mathbf{u}_j(\mathbf{r}_j, t) = 0,$$

Momentum-balance equation of the electron fluid at each tube surface

$$\frac{\partial \mathbf{u}_j(\mathbf{r}_j, t)}{\partial t} = \nabla_j \Phi(\mathbf{r}_j, t) - \frac{\alpha_j}{n_{0j}} \nabla_j n_j(\mathbf{r}_j, t) + \frac{\beta}{n_{0j}} \nabla_j [\nabla_j^2 n_j(\mathbf{r}_j, t)] - \gamma_j \mathbf{u}_j(\mathbf{r}_j, t),$$



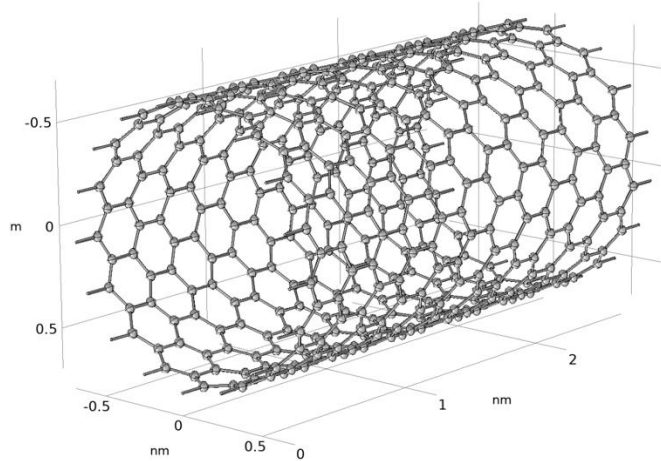
# Wakefield in Graphene Layers: Laser-driven



- Acceleration gradient: 4.79 TeV/m
- Self-injection at edge
- 0.4 fs-long bunch

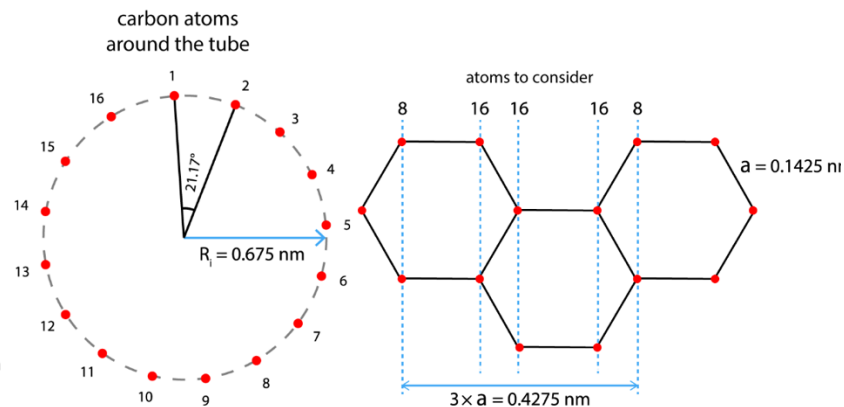
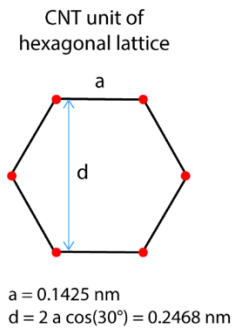
@ Cristian, et al, 2024

# Metallic Carbon Nanotubes (CNTs): Armchair Type

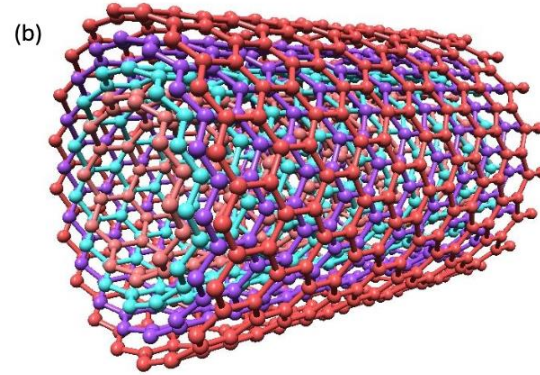
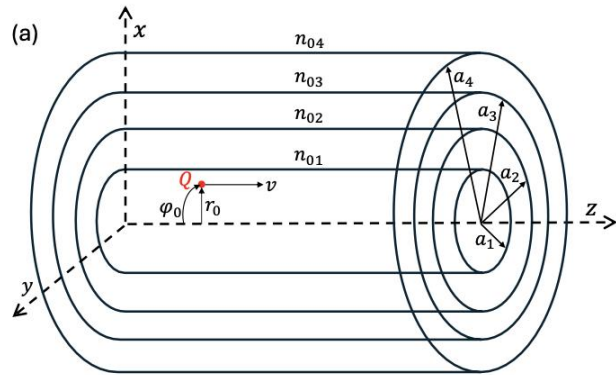


$$\rho = \frac{64 \text{ atoms}}{0.181 \text{ cm}^3} = 3.53 \times 10^{23} \text{ atoms/cm}^3$$

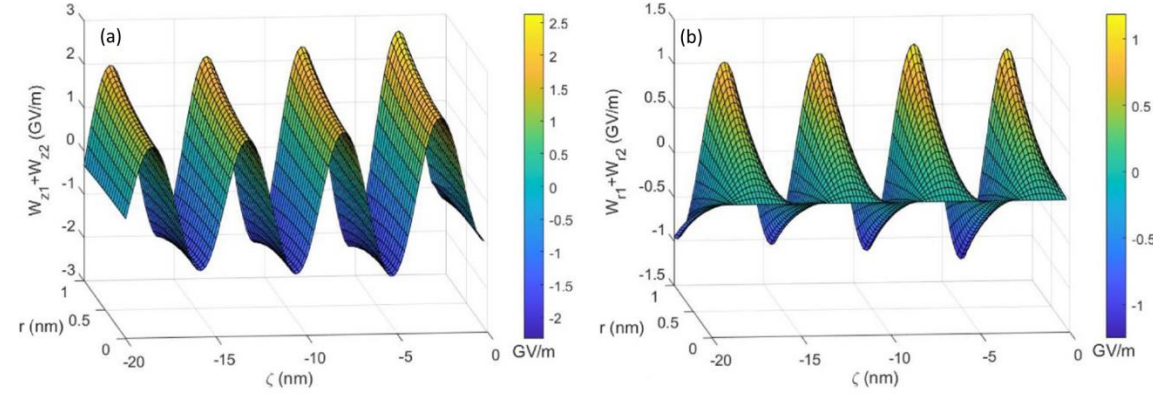
Number	Layer				Tube		
	Radius [nm]	Volume [nm <sup>3</sup> ]	Atoms	Density [cm <sup>-3</sup> ]	Volume [nm <sup>3</sup> ]	Atoms	Density [cm <sup>-3</sup> ]
1	0.675	0.181	64	$3.530 \times 10^{23}$	N/A	64	N/A
2	1.115	0.299	108	$3.606 \times 10^{23}$	1.058	172	$1.626 \times 10^{23}$
3	1.555	0.418	152	$3.639 \times 10^{23}$	2.636	324	$1.229 \times 10^{23}$
4	1.995	0.536	196	$3.658 \times 10^{23}$	4.733	520	$1.099 \times 10^{23}$
5	2.435	0.654	240	$3.669 \times 10^{23}$	7.351	760	$1.034 \times 10^{23}$
6	2.875	0.772	288	$3.729 \times 10^{23}$	10.489	1048	$9.991 \times 10^{22}$
7	3.315	0.890	332	$3.729 \times 10^{23}$	14.147	1380	$9.755 \times 10^{22}$
8	3.755	1.009	376	$3.728 \times 10^{23}$	18.325	1756	$9.583 \times 10^{22}$
9	4.195	1.127	420	$3.727 \times 10^{23}$	23.023	2176	$9.452 \times 10^{22}$
10	4.635	1.245	464	$3.727 \times 10^{23}$	28.241	2640	$9.348 \times 10^{22}$
...	...	...	...	...	...	...	...
45	20.035	5.382	2036	$3.783 \times 10^{23}$	538.482	47184	$8.762 \times 10^{22}$



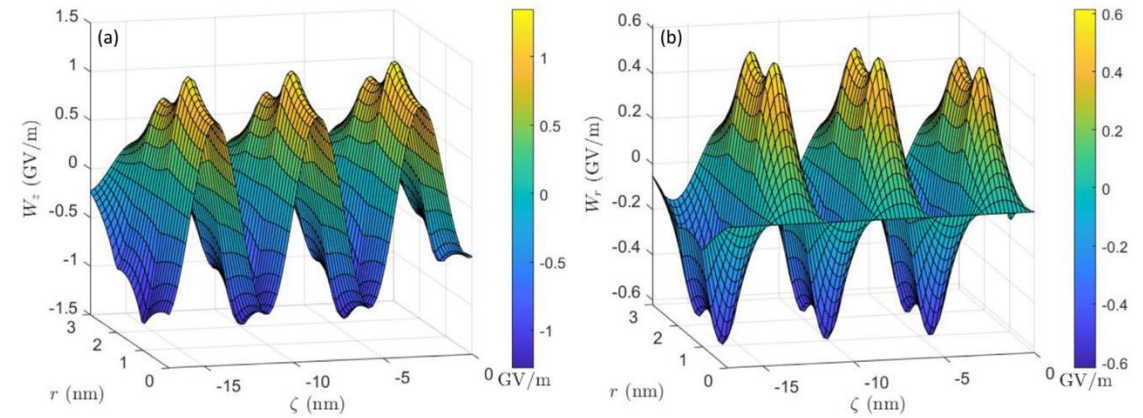
# Plasma Wakefield in CNT Tube



SWCNT



DWCNT



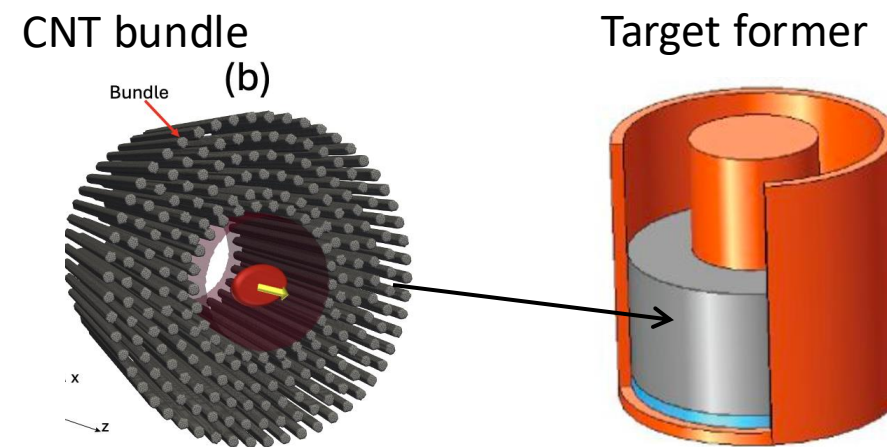
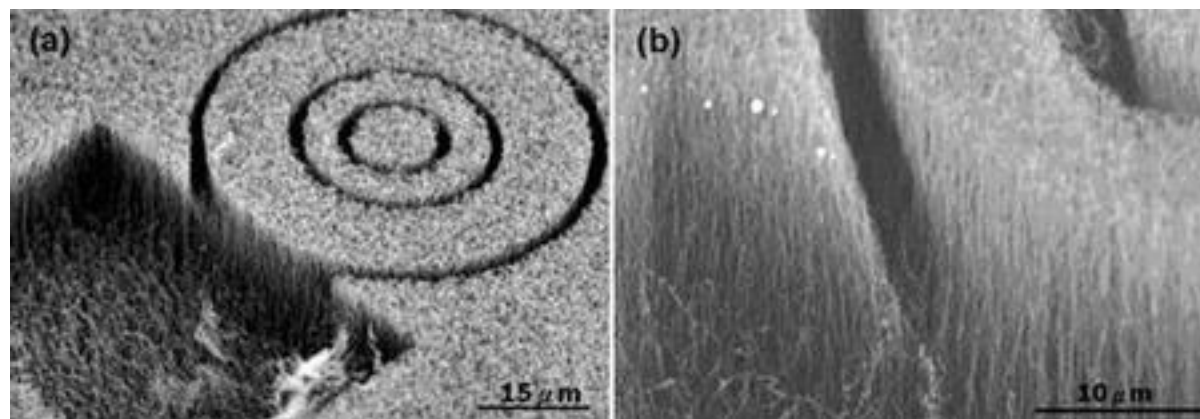
$$\frac{\partial n_j(\mathbf{r}_j, t)}{\partial t} + n_{0j} \nabla_j \cdot \mathbf{u}_j(\mathbf{r}_j, t) = 0.$$

$$\frac{\partial \mathbf{u}_j(\mathbf{r}_j, t)}{\partial t} = \nabla_j \Phi(\mathbf{r}_j, t) - \frac{\alpha_j}{n_{0j}} \nabla_j n_j(\mathbf{r}_j, t) + \frac{\beta}{n_{0j}} \nabla_j [\nabla_j^2 n_j(\mathbf{r}_j, t)] - \gamma_j \mathbf{u}_j(\mathbf{r}_j, t),$$

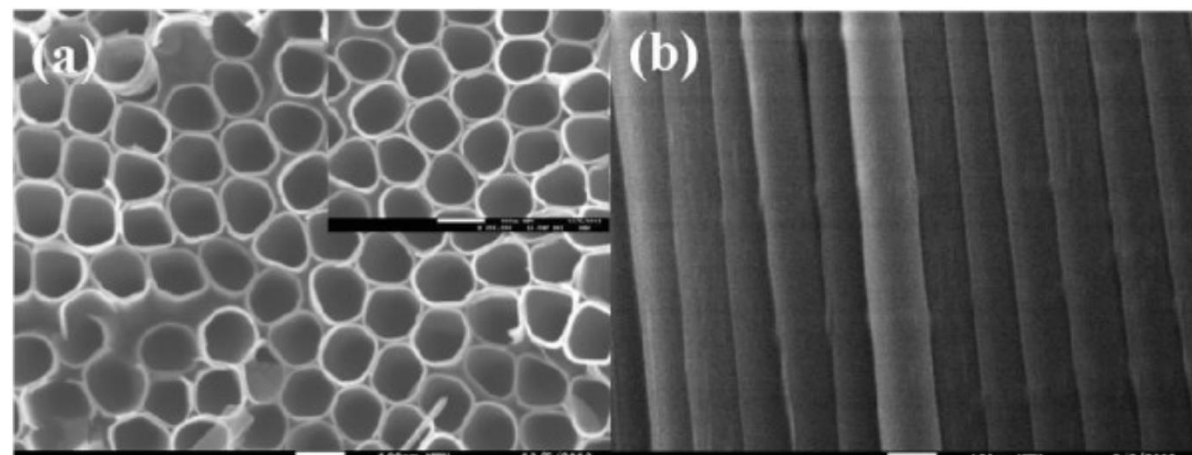
@ Palo et al 2025

# Nano-structured CNTs: CNTs in dense forest form

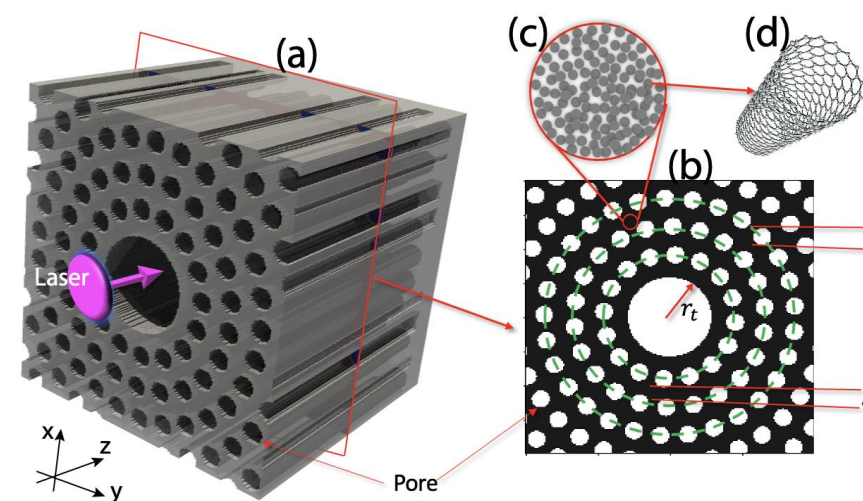
@Hung et al APL 91, 093121 (2017)



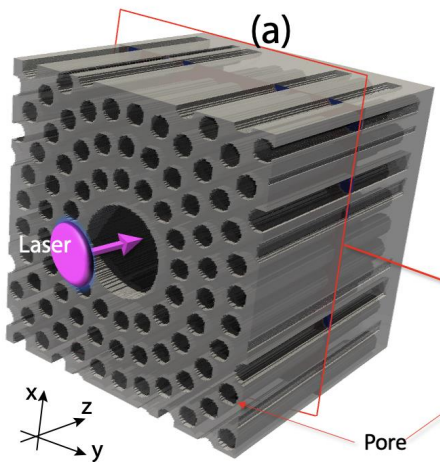
@Teen et al nano express, 579 (2012)



CNT porous

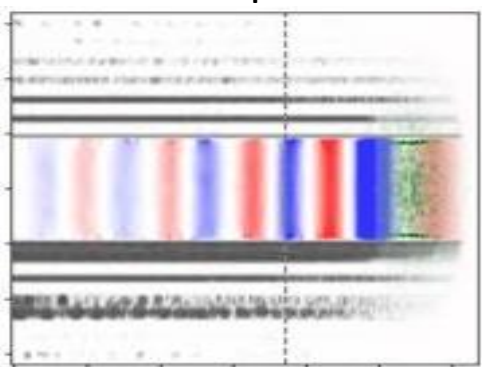


# Plasma Wakefield in Structured CNT Forest

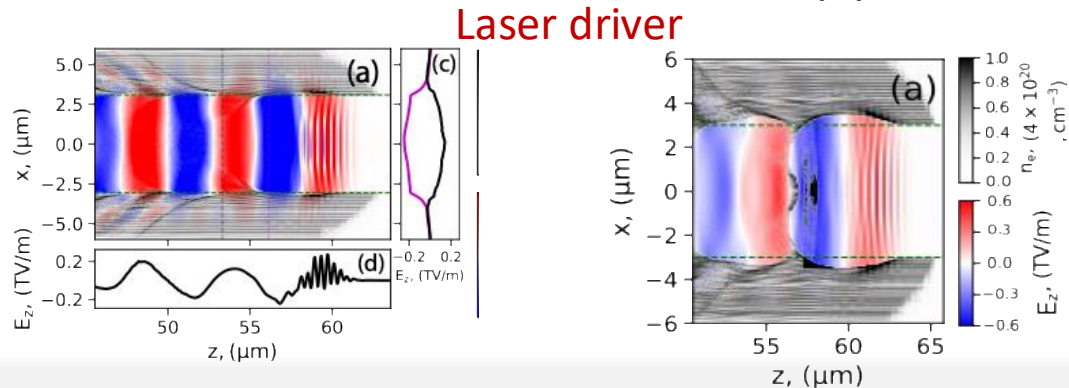
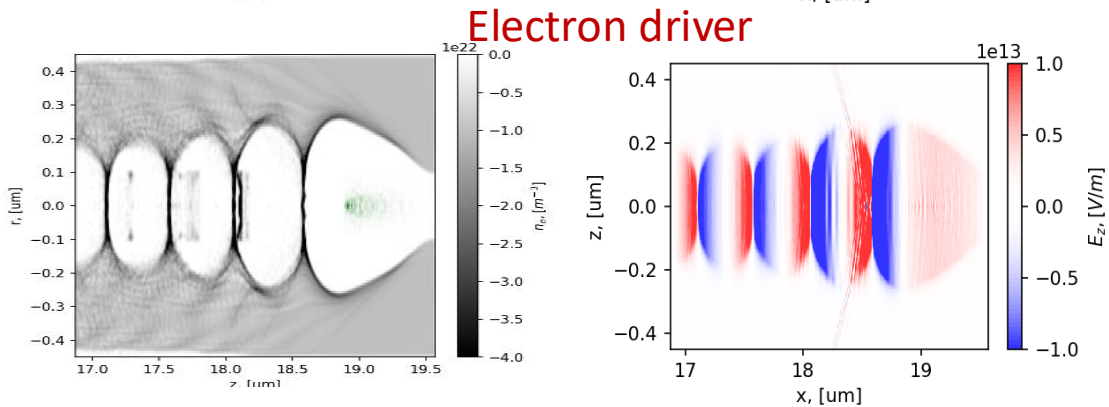
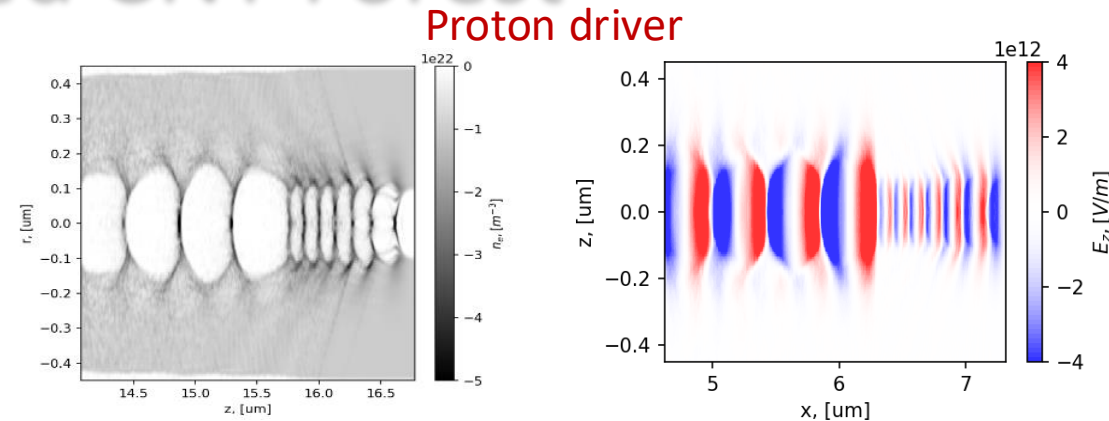
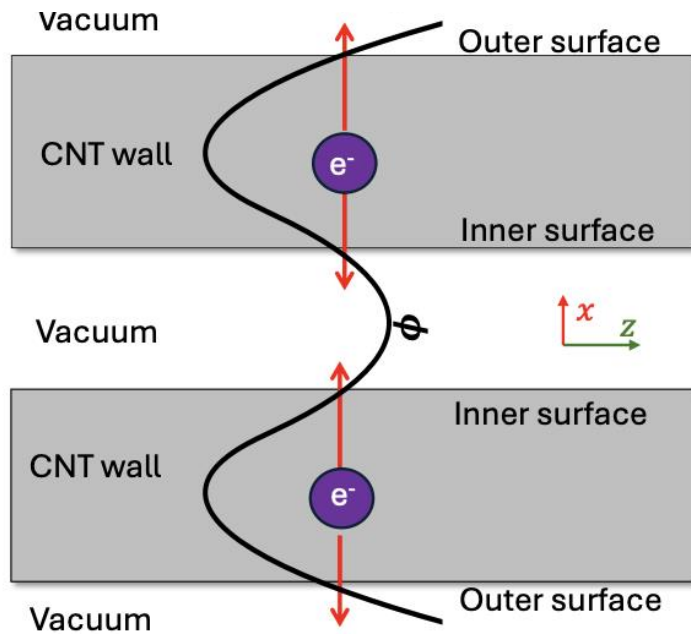
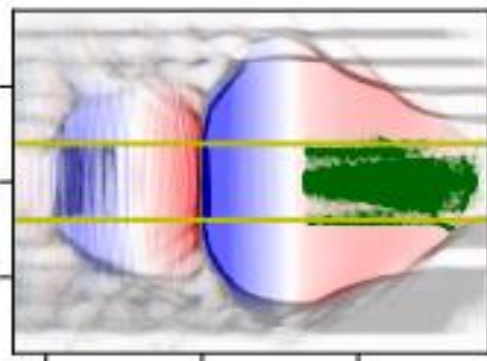


@B.Lei et al 2025

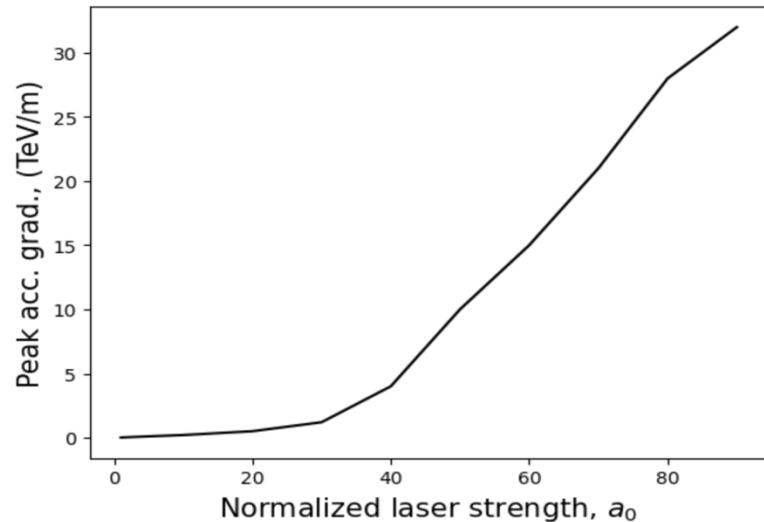
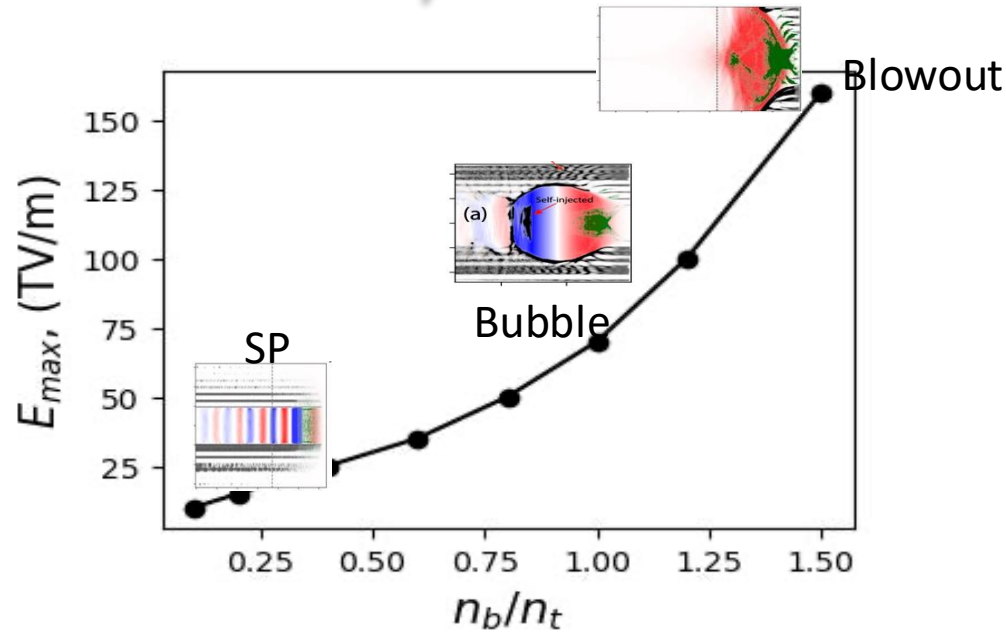
Surface plasmon



Plasma bubble

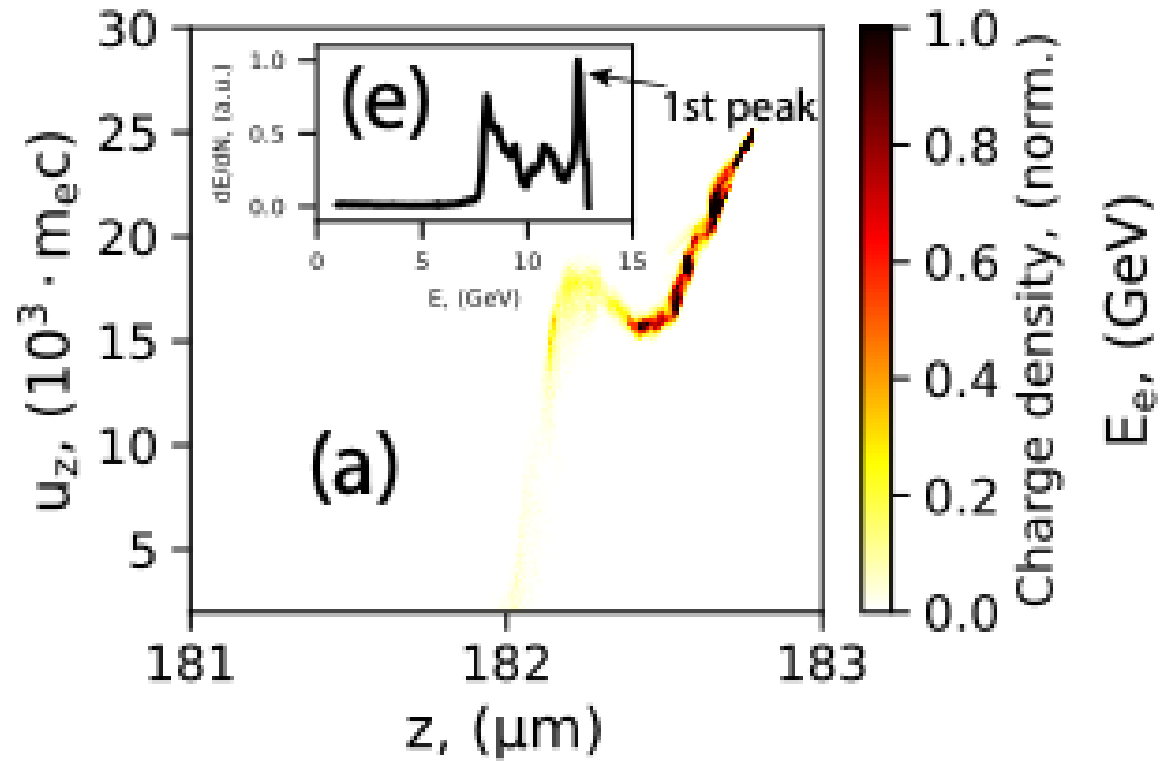


# 100s TeV/m Acceleration Gradient Achievable



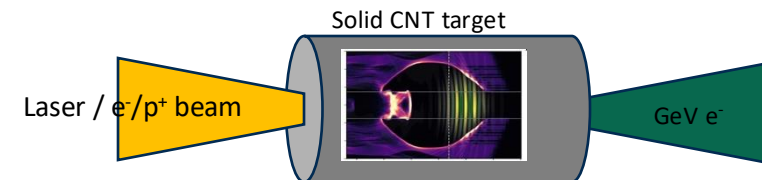
@B.Lei et al 2025

$$n_p = 2 \times 10^{22} \text{ cm}^{-3}, \frac{n_b}{n_t} = 0.5$$



$$G \approx 100 \text{ TeV/m}$$

# Potential Drivers: Particle Beams



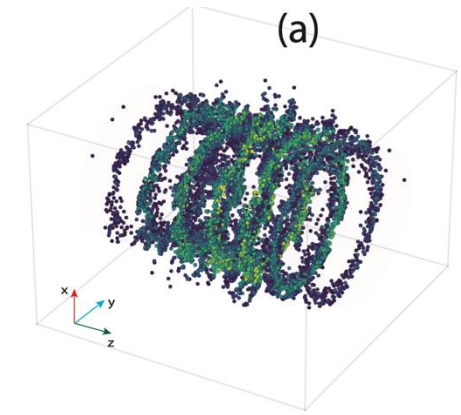
	Unit	FACET-II e <sup>-</sup> beam	FACET-II e <sup>+</sup> beam	CLARA e <sup>-</sup> beam	NPQED Collider e <sup>-</sup> beam	CERN p <sup>+</sup> beam	
Energy	GeV	10 [4.0-13.5]	10 [13GeV]	0.25	125	400	
Charge	nC	2 [0.5-5]	1	0.25 [0.02-0.25]	0.14-1.4	48	
Norm. emitt._x,y	um-rad	4.4, 3.2 [3-6]	3	<1		3.5	
Peak current	kA	300	20	1-3	1700	0.16	
Beam size	x	um	3	3	10	0.01	10
	y	um	2	2	10	0.01	10
	z	um	0.48	0.48	9/30	0.01-0.1	6-12 <b>cm</b>
Max. beam density	Cm-3	1e22	1e22	1e18	1e34	1e14	
Min. energy spread	%	1.4 [0.4-1.6]	0.1	0.01-0.06	0.1	0.01	
Peak electric field	TV/m	3.2			4500		

	Unit	100TW FEBE@STFC	200TW laser@HIJ	VEGE@CLP U	
Intensity	W/cm <sup>2</sup>	$>1 \times 10^{20}$	$>3.5 \times 10^{21}$	$1 \times 10^{20}$	
Peak Power	TW	120	>200	200 [1 PW]	
Wavelength	nm	800	800	800	
Max. strength	/	2	7	10	
On-target energy	J		4	2	
Beam size	Focus, x	um	30	6	8
	Focus, y	um	30	6	8
Duration, z	fs	<40fs	17	25	
Temporal contrast	/	$10^{-13}$	$10^{-12}$	$10^{-13}$	

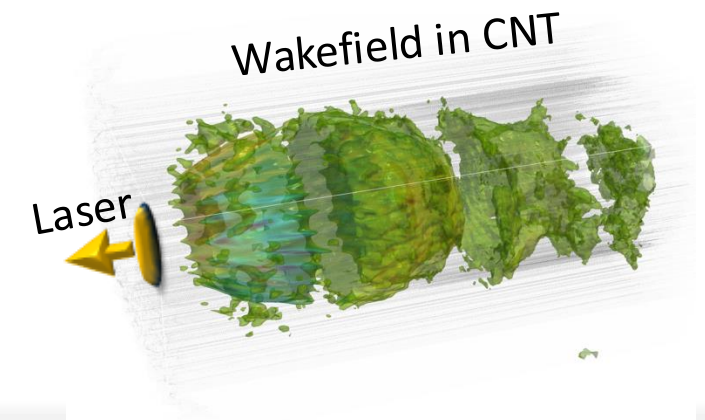
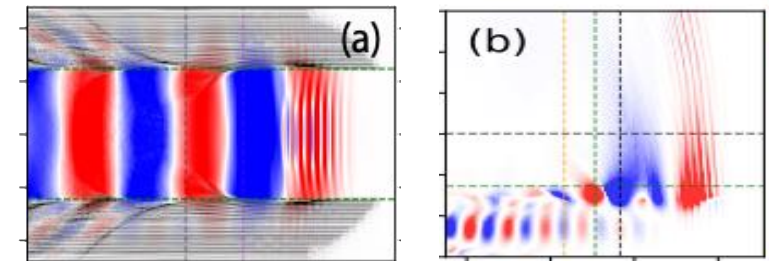
@PRL. 122, 190404 (2019)

# Challenges and Future Research

- Theoretical modelling of the complex dynamics of laser-crystal interaction
- Geometrical study of surface plasmon and wakefield excitation
- Target fabrication, controllability and flexibility.
- New simulation capabilities of multi-physics: plasma, crystal lattice, radiation and QED
- Preparation of high density particle beam as drivers,  $n_b > 10^{20} \text{cm}^{-3}$
- Practical application in particle acceleration, radiation generation, strong field generation, astrophysics, etc.
- New diagnostic technologies required to understand the ultrafast plasma dynamics in crystal
- Experimental demonstration are demanded for proof of principle: CLPU laser, FEBE, ELI-beamlines, Jeti200, FACET-II, X-ray@LCLS-II/XFEL, Proton beam@CERN, etc.



Cylindrical VS. Flat





**Thank you for your attention!**

**Any questions?**