Plasma Instabilities and Turbulence through a stellarator lens

Alessandro Zocco

Max-Planck-Institut für Plasmaphysik, D-17491, Greifswald, Bundesrepublik Deutschland

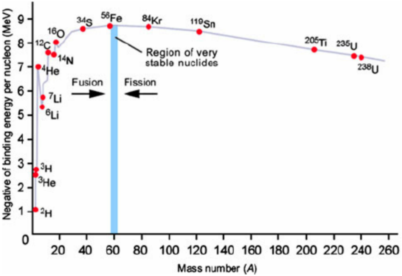
Dipartimento di Fisica Enrico Fermi
Università di Pisa
11 Novembre 2024

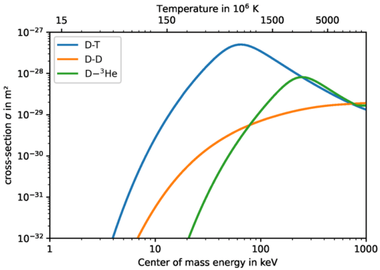
Outline

- Context: What is a fusion reactor, main concepts for magnetic confinement fusion, Tokamak and Stellarator
- What is a Stellarator, stressing scientific advances that made it a viable concept (quasi-symmetry and optimization)
- Some of the key physical problems and results: Stability, Turbulent Transport in Wendenstein 7-X (Max-Planck-Institute, Greifswald, Germany)

The Physicist's Fusion Reactor

- Intuitively, a fusion reactor is a hot and dense enough mixture of fusing reactants, which is kept together long enough
- Liberate nuclides' binding energy maximizing cross section collecting kinetic energy of emerging neutrons





For instance

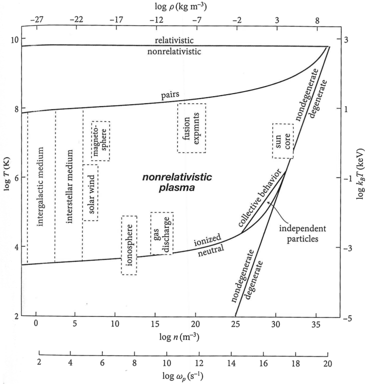
$$D + T \rightarrow He^4 + n + 17.7\,MeV \quad (1)$$

80% acquired as kinetic energy by the neutron

Tokamaks and Stellarators

- Cross section optimal at $T \approx 10\,KeV$, densities are largely limited by radiative processes $n \approx 10^{19}\,m^{-3}$
- The fuel is a plasma: current carrying ionized gas

Plasmas in Nature

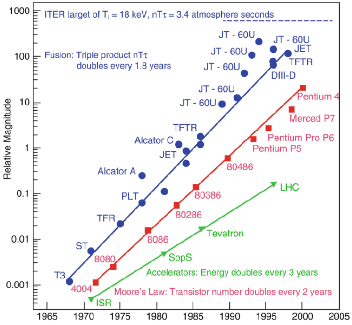


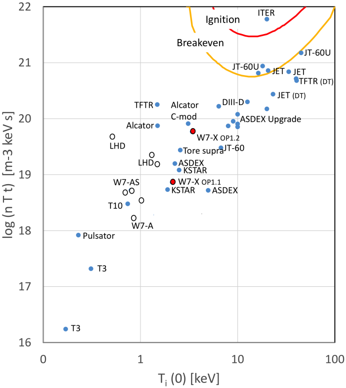
	Human	Solar Wind	Sg A*	ISM	Glaxy Cluster
$L(m)$	1	10^{8}	10^{11}	10^{18}	10^{20}

[Schekochihin et al AJS (2009)]

The Physicist's Fusion Reactor

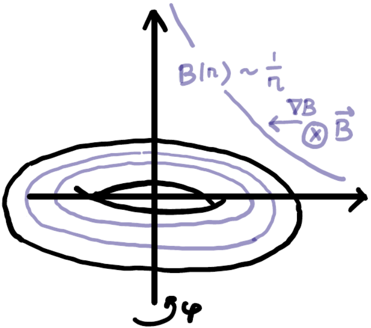
- How well a fusion reactor would work is quantitatively measured by the triple product $nT\tau_E$
- $\tau_E(\textit{confinement time}) = \textit{internal plasma energy}/(\textit{fusion power} - \textit{radiation losses})$





The Physicist's Fusion Reactor

- Confinement...easy to say. Take a purely toroidal $\mathrm{B} = B(r)\mathrm{e}_\varphi$



- For a given confining magnetic field $\boldsymbol{B}$, and a plasma with total pressure p and current density j, equilibrium must satisfy force balance

$$\underbrace{\boldsymbol{J} \times \boldsymbol{B}}_{\textit{Lorentz force}} = \underbrace{\nabla p}_{\textit{pressure force}} \qquad (2)$$

The Physicist's Fusion Reactor

- Confinement...easy to say. Take a purely toroidal $\mathrm{B} = B(r)\mathrm{e}_\varphi$
- For a given confining magnetic field $\boldsymbol{B}$, and a plasma with total pressure p and current density j, equilibrium must satisfy force balance

$$\underbrace{\boldsymbol{J} \times \boldsymbol{B}}_{\textit{Lorentz force}} = \underbrace{\nabla p}_{\textit{pressure force}} \tag{3}$$

If you take $\boldsymbol{B} \times (\boldsymbol{J} \times \boldsymbol{B}) = (\boldsymbol{B} \cdot \boldsymbol{B})\boldsymbol{J} - (\boldsymbol{B} \cdot \boldsymbol{J})\boldsymbol{B} = B^2(\boldsymbol{J} - \frac{\boldsymbol{B} \cdot \boldsymbol{J}}{B^2}\boldsymbol{B}) = B^2\boldsymbol{J}_\perp$.

Then, you are allowed to write $\boldsymbol{J} = \frac{1}{B^2}\boldsymbol{B} \times \nabla p + \lambda \boldsymbol{B}$,

and λ can be determined by imposing the consition that the electric charge is conserved $\nabla \cdot \boldsymbol{J} = 0 \Rightarrow \boldsymbol{B} \cdot \nabla \lambda = 2\frac{\boldsymbol{B} \times \nabla p \cdot \nabla B}{B^3}$, your $\boldsymbol{B} = B(r)\boldsymbol{e}_\varphi$, so

$\oint d\varphi(\boldsymbol{B} \times \nabla p \cdot \nabla B) = 0$

The Physicist's Fusion Reactor

- Confinement...easy to say. Take a purely toroidal $\mathrm{B} = B(r)\mathrm{e}_\varphi$
- For a given confining magnetic field $\boldsymbol{B}$, and a plasma with total pressure p and current density j, equilibrium must satisfy force balance

$$\underbrace{\boldsymbol{J} \times \boldsymbol{B}}_{\text{Lorentz force}} = \underbrace{\nabla p}_{\text{pressure force}} \qquad (3)$$

If you take $\boldsymbol{B} \times (\boldsymbol{J} \times \boldsymbol{B}) = (\boldsymbol{B} \cdot \boldsymbol{B})\boldsymbol{J} - (\boldsymbol{B} \cdot \boldsymbol{J})\boldsymbol{B} = B^2(\boldsymbol{J} - \frac{\boldsymbol{B} \cdot \boldsymbol{J}}{B^2}\boldsymbol{B}) = B^2\boldsymbol{J}_\perp$.

Then, you are allowed to write $\boldsymbol{J} = \frac{1}{B^2}\boldsymbol{B} \times \nabla p + \lambda \boldsymbol{B}$,

and λ can be determined by imposing the consition that the electric charge is conserved $\nabla \cdot \boldsymbol{J} = 0 \Rightarrow \boldsymbol{B} \cdot \nabla\lambda = 2\frac{\boldsymbol{B} \times \nabla p \cdot \nabla B}{B^3}$, your $\boldsymbol{B} = B(r)\boldsymbol{e}_\varphi$, so

$$\oint d\varphi(\boldsymbol{B} \times \nabla p \cdot \nabla B) = 0$$

The Physicist's Fusion Reactor

- Confinement...easy to say. Take a purely toroidal $\mathrm{B} = B(r)\mathrm{e}_\varphi$
- For a given confining magnetic field $\boldsymbol{B}$, and a plasma with total pressure p and current density j, equilibrium must satisfy force balance

$$\underbrace{\boldsymbol{J}\times\boldsymbol{B}}_{\textit{Lorentz force}} = \underbrace{\nabla p}_{\textit{pressure force}} \tag{3}$$

If you take $\boldsymbol{B}\times(\boldsymbol{J}\times\boldsymbol{B}) = (\boldsymbol{B}\cdot\boldsymbol{B})\boldsymbol{J} - (\boldsymbol{B}\cdot\boldsymbol{J})\boldsymbol{B} = B^2(\boldsymbol{J} - \frac{\boldsymbol{B}\cdot\boldsymbol{J}}{B^2}\boldsymbol{B}) = B^2\boldsymbol{J}_\perp$.

Then, you are allowed to write $\boldsymbol{J} = \frac{1}{B^2}\boldsymbol{B}\times\nabla p + \lambda\boldsymbol{B}$,

and λ can be determined by imposing the consition that the electric charge is conserved $\nabla\cdot\boldsymbol{J} = 0 \Rightarrow \boldsymbol{B}\cdot\nabla\lambda = 2\frac{\boldsymbol{B}\times\nabla p\cdot\nabla B}{B^3}$, your $\boldsymbol{B} = B(r)\boldsymbol{e}_\varphi$, so

$\oint d\varphi(\boldsymbol{B}\times\nabla p\cdot\nabla B) = 0$

The Physicist's Fusion Reactor

- Confinement...easy to say. Take a purely toroidal $\mathrm{B} = B(r)\mathrm{e}_\varphi$
- For a given confining magnetic field $\boldsymbol{B}$, and a plasma with total pressure p and current density j, equilibrium must satisfy force balance

$$\underbrace{\boldsymbol{J} \times \boldsymbol{B}}_{\textit{Lorentz force}} = \underbrace{\nabla p}_{\textit{pressure force}} \tag{3}$$

If you take $\boldsymbol{B} \times (\boldsymbol{J} \times \boldsymbol{B}) = (\boldsymbol{B} \cdot \boldsymbol{B})\boldsymbol{J} - (\boldsymbol{B} \cdot \boldsymbol{J})\boldsymbol{B} = B^2(\boldsymbol{J} - \frac{\boldsymbol{B} \cdot \boldsymbol{J}}{B^2}\boldsymbol{B}) = B^2\boldsymbol{J}_\perp$.

Then, you are allowed to write $\boldsymbol{J} = \frac{1}{B^2}\boldsymbol{B} \times \nabla p + \lambda \boldsymbol{B}$,

and λ can be determined by imposing the consition that the electric charge is conserved $\nabla \cdot \boldsymbol{J} = 0 \Rightarrow \boldsymbol{B} \cdot \nabla\lambda = 2\frac{\boldsymbol{B} \times \nabla p \cdot \nabla B}{B^3}$, your $\boldsymbol{B} = B(r)\boldsymbol{e}_\varphi$, so

$\oint d\varphi(\boldsymbol{B} \times \nabla p \cdot \nabla B) = 0$

The Physicist's Fusion Reactor

- Confinement...easy to say.
- For a given confining magnetic field $\boldsymbol{B}$, and a plasma with total pressure p and current density j, equilibrium must satisfy force balance

$$\underbrace{\boldsymbol{J} \times \boldsymbol{B}}_{\textit{Lorentz force}} = \underbrace{\nabla p}_{\textit{pressure force}} \tag{4}$$

If you take $\boldsymbol{B} \times (\boldsymbol{J} \times \boldsymbol{B}) = (\boldsymbol{B} \cdot \boldsymbol{B})\boldsymbol{J} - (\boldsymbol{B} \cdot \boldsymbol{J})\boldsymbol{B} = B^2(\boldsymbol{J} - \frac{\boldsymbol{B} \cdot \boldsymbol{J}}{B^2}\boldsymbol{B}) = B^2 \boldsymbol{J}_\perp$.

In axisymmetry none of the quantities in the integrand depend on the symmetry angle, your $\boldsymbol{B} = B(r)\boldsymbol{e}_\varphi$, $\oint d\varphi(\mathrm{B} \times \nabla p \cdot \nabla B) = 0$

In axisymmetry none of the quantities in the integrand depend on the symmetry angle, then

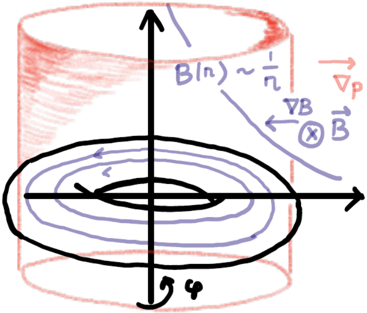
$$\mathrm{B} \times \nabla p \cdot \nabla B = 0 \tag{5}$$

The Physicist's Fusion Reactor

- Confinement...easy to say. Take a purely toroidal $\boldsymbol{B} = B(r)\boldsymbol{e}_\varphi$

$$\boldsymbol{B} \times \nabla p \cdot \nabla B = 0 \Rightarrow \tag{6}$$

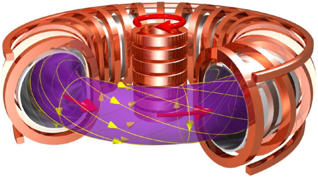
pressure is constant along lines parallel to the symmetry axis, NO PLASMA CONFINEMENT! [JB Taylor (JPP lecture notes)]



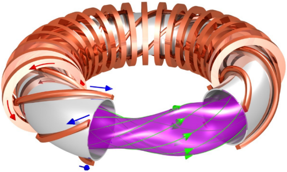
But a helical field would be enough. How can we generate that?

Tokamaks and Stellarators

Impose a current running in the axial direction: Tokamak



Avoid a toroidal current but shape the field with external coils only: Stellarator



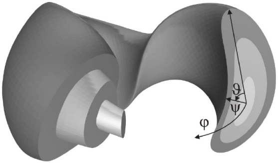
One main advantange of Stellarators is the avoidance of a current drive, potentially avoiding disruptive events due to current gradients, operated in steady steate (no pulsed currents)

A major drawback of 3D magnetic field geometry is that it introduces a strong temperature dependence into stellarators' non-turbulent (neoclassical) energy transport. Such energy losses will become prohibitive in high-temperature reactor plasmas but can be reduced by carefully choosing the geometry of the confining magnetic field through optimization

W7-X [Dinklage et al. Nature Phys., Bidler et al. Nature]

Confinement: fundamentals

Magnetic-flux co-ordinates



Toroidal flux $\psi = (2\pi)^{-1} \int \boldsymbol{B} \cdot d\boldsymbol{\Sigma}_{\varphi=const}$

Poloidal flux $\chi = (2\pi)^{-1} \int \boldsymbol{B} \cdot d\boldsymbol{\Sigma}_{\theta=const}$

$\psi(x, y, z) = const$ toroidal surface in space

Covariant basis $(\nabla\psi, \nabla\theta, \nabla\varphi)$ with θ, φ geleralized angle-like co-ordinates

Contravariant $\boldsymbol{e}_i$, and we can construct the metric g_{ij}

Covariant and Contravariant field

$\boldsymbol{B} = G(\psi)\nabla\varphi + I(\psi)\nabla\theta,$

$\boldsymbol{B} = \nabla\psi \times \nabla(\theta - \iota\varphi),$#

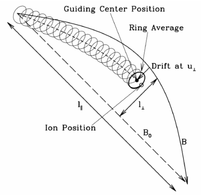
$\iota = \frac{d\chi}{d\psi} = \frac{\#toroidal\ turns}{\#poloidal\ turns}$ rotational transform

$(\iota = 1/q)$

$\boldsymbol{A} = \psi\nabla\theta - \chi\nabla\varphi$

Confinement: fundamentals

Particle motion



Particles stream along a dominantly toroidal field with speed

$$v_\parallel = (\dot\theta \boldsymbol{e}_\theta + \dot\varphi \boldsymbol{e}_\varphi) \cdot \boldsymbol{B}/B = \frac{1}{B}\left(G\dot\theta + I\dot\varphi\right)$$

and slowly drift across field lines (because of magnetic curvature, electrostatic fluctuations)

Single-particle Lagrangian

$\mathscr{L} = \frac{p^2}{2m} - \mu B$, $\boldsymbol{p} \to \boldsymbol{p} + \frac{e}{c}\boldsymbol{A}$, minimal coupling

$$\mathscr{L}_{Taylor} = \frac{1}{2} m v_\parallel^2 + \frac{e}{c}\boldsymbol{v}\cdot\boldsymbol{A} - \mu B =$$

$$\frac{1}{2}\frac{m}{B^2}\left(G\dot\theta + I\dot\varphi\right)^2 + \frac{e}{c}\left(\psi\dot\theta - \chi\dot\varphi\right) - \mu B$$

The magnetic field enters the Lagrangian ONLY through the modulus

Axisymmetry: $\partial_\varphi \equiv 0$ (Tokamaks)

Canonical toroidal momentum

$$p_\varphi = \frac{\partial \mathscr{L}}{\partial \dot{\varphi}} = \frac{m}{B} I v_\parallel - \frac{e}{c}\chi,$$

using Lagrange equation $\dot{p}_\varphi = \frac{\partial \mathscr{L}}{\partial \varphi}$ but $\mathscr{L}$ is a function of mod-B only!

$$= \frac{\partial \mathscr{L}}{\partial B}\frac{\partial B}{\partial \varphi} \equiv 0, \text{ in axisymmetric geometry.}$$

Then $\dot{p}_\varphi = \frac{m}{B} I \dot{v}_\parallel - \frac{e}{c}\frac{d\chi}{d\psi}\dot{\psi} = 0 \Rightarrow \iota \Delta\psi \approx \frac{mc}{Be} I v_\parallel$, dimensionally

$$\iota \Delta r \sim \rho_{Larmor}, \tag{7}$$

for a purely toroidal field $\iota \equiv 0$, and the radial particle excursion can be arbitrary!!!

Conservation of the toroidal canonical momentum guarantees confinement in axisymmetry (Tamm theorem).

Small drifts due to magnetic curvature and gradients of the magnetic field strength are allowed, though.

Non-axisymmetry: $\partial_\varphi \neq 0$ (Stellarators)

Quasi-helical symmetry

The toroidal canonical momentum in no longer coserved $\dot{p}_\varphi = \frac{\partial \mathscr{L}}{\partial B}\frac{\partial B}{\partial \varphi} \neq 0$

But $\mathscr{L}$ is still a function of mod-B only! If we choose

$$B = B(\underbrace{N\varphi - M\theta}_{h}) \tag{8}$$

$\dot{p}_\varphi = \frac{\partial \mathscr{L}}{\partial B}\frac{\partial B}{\partial h}N$, and $\dot{p}_\theta = -\frac{\partial \mathscr{L}}{\partial B}\frac{\partial B}{\partial h}M$, so $P_H = \frac{1}{N}p_\varphi + \frac{1}{M}p_\theta$ IS CONSERVED, and guarantees the same level of confinement of an axisymmetric equilibrium.

In fact, one can construct the WHOLE kinetic theory of a quasi-helical stellarator by applying a correspondence rule

So, effectively, tokamaks are $N = 0$ quasihelical symmetric stellarators with planar magnetic axis (mathematically!) [Boozer (1983) (1986)]

Non-axisymmetry: $\partial_\varphi \neq 0$ (Stellarators)

- 1988: Whilst human kind/Throughout the lands laid miserably crushed/Before all eyes beneath Superstition...Quasi-helically symmetric toroidal stellarators which strictly confine guiding centres orbits were discovered

Volume 129, number 2 PHYSICS LETTERS A 9 May 1988

QUASI-HELICALLY SYMMETRIC TOROIDAL STELLARATORS

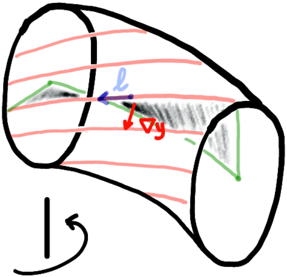
J. NÜHRENBERG and R. ZILLE
Max-Planck-Institut für Plasmaphysik, IPP-EURATOM Association, D-8046 Garching near München, FRG

Received 30 December 1987; revised manuscript received 19 February 1988; accepted for publication 9 March 1988
Communicated by R.C. Davidson

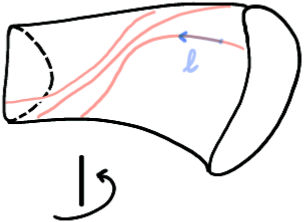
It is computationally shown that there are toroidal stellarators whose magnetic field strength is helically symmetric in magnetic coordinates. Accordingly, these stellarators, without collisions, strictly confine guiding centre orbits.

A technological advance made possible a real breakthrough: development of 3D magnetohydrodynamic equilibrium numerical codes (as stated explicitly by the authors).

Where is the difference?



Axisymmetric



Nonaxisymmetric

Let me change co-ordinates (for the last time, I promise!)

$(\psi, \theta, \varphi) \rightarrow (\psi, \alpha = \theta - \iota\varphi, \ell)$ ℓ field-line arc-length α labels lines over the surface $\psi = const$. $\boldsymbol{A} = \frac{1}{2}(\psi\nabla\alpha - \alpha\nabla\psi)$, $\boldsymbol{B} = \nabla\psi \times \nabla\alpha$.

Where is the difference?

The Lagrangian becomes $\mathscr{L} = \frac{m}{2}\left(\dot{\ell}\right)^2 + \frac{e}{c}(\dot{\alpha}\psi - \alpha\dot{\psi}) - \mu B$, and averaging over the fast bounce time, the particle position slowly evolves across and over the confining surface following the equations

$$\Delta\psi = (c/e)\frac{\partial \mathscr{J}}{\partial\alpha},$$

$$\Delta\alpha = -(c/e)\frac{\partial \mathscr{J}}{\partial\psi}$$

They have a Hamiltonian structure, where $\mathscr{J} = \int_{\ell_1}^{\ell_2} d\ell' m v_{\parallel}$ is the second adiabatic invariant.

In axisymmetry, bounce-points of trapped particles orbits are the same for each line on a specific surface, then $\partial_\alpha \mathscr{J}|_{Axisymm.} = 0 \Rightarrow \Delta\psi = 0$, trapped particles are confined in Tokamaks and QS stellarators

In a general non-axisymmetric magnetic field $\partial_\alpha \mathscr{J}|_{Axisymm.} \neq 0 \Rightarrow \Delta\psi \neq 0$, trapped particles (in particular, energetic ones) are not confined in any Stellarator.

This is why we need optimization

Where is the difference?

- Even if we can have $\partial_\alpha \mathcal{J}|_{Axisymm.} \neq 0 \Rightarrow \Delta\psi \neq 0$, thus trapped particles in principle not confined, we can target a magnetic equilibrium for which, on average over the bounce time, the radial drift due to magnetic curvature is exactly zero

$$\int \frac{d\ell}{v_\parallel} \boldsymbol{v}_{curv} \cdot \nabla \psi = 0 \text{ Omnigenous fields.} \tag{9}$$

- In a field is omnigenous, and the contour levels of $B(\theta, \varphi)$ close poloidally, we speak of a quasi-isodynamic stellarator.
- Confinement of trapped particles in general non-axisymmetric geometry is more problematic than in tokamaks and quasihelical stellarators

How good can quasi-symmetry be?

Symmetry-breaking mode amplitude can be made as small at the geomagnetic field!

Magnetic Fields with Precise Quasisymmetry for Plasma Confinement

Matt Landreman*
Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, Maryland 20742, USA

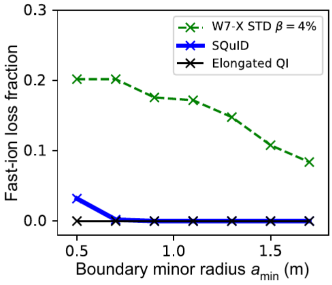
Elizabeth Paul
Princeton University, Princeton, New Jersey 08544, USA

(Received 8 August 2021; revised 4 November 2021; accepted 17 November 2021; published 18 January 2022)

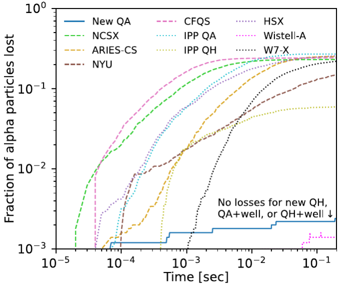
Quasisymmetry is an unusual symmetry that can be present in toroidal magnetic fields, enabling the confinement of charged particles and plasma. Here it is shown that both quasiaxisymmetry and quasihelical symmetry can be achieved to a much higher precision than previously thought over a significant volume, resulting in exceptional confinement. For a 1 Tesla mean field far from axisymmetry (vacuum rotational transform > 0.4), symmetry-breaking mode amplitudes throughout a volume of aspect ratio 6 can be made as small as the typical $\sim$50 μT geomagnetic field.

DOI: 10.1103/PhysRevLett.128.035001

How good can quasi-symmetry be?



Goodman et al. PRX Energy (2024)

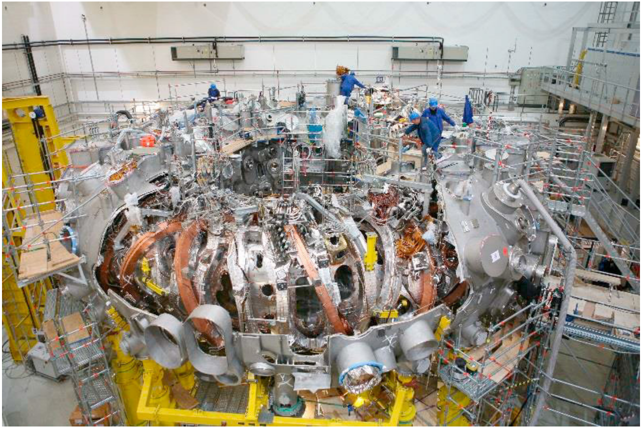


Landremann Paul PRL (2022)

Extensive litterature on stellarator optimization, active and expanding field [not mentioned yet, QI-CIEMAT stellarator, Madrid, basically a W7-X with better fast particles confinement, Sanchez et al., Garcia Regaña et al., Velasco et al. Nucl. Fus. (2023,2024)]

Back to the real world: Wendelstein 7-X (W7-X)

Max-Planck-Institute for Plasma Physics, Greifswald, Germany



A Physicist's reactor is not a Fusion Reactor, understood as a technological device that injects electricity in the grid outputting more power than the one used.

Stating the obvious (never enough): these are fusion EXPERIMENTS, not reactors! (in case someone feeds AI with this presentation...)

But turbulence is lurking

In W7-X, experimentally

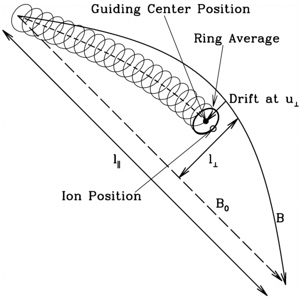
- Core: An experimental characterization of core turbulence regimes in Wendelstein 7-X D. Carralero et al. Nucl. Fusion (2021)
- Scrape off layer: Statistical characteristics of the SOL turbulence in the first divertor plasma operation of W7-X using a reciprocating probe. S. C. Liu et al Phys. Plasmas (2020)
- Impurity transport: Observation of anomalous impurity transport during low-density experiments in W7-X with laser blow-off injections of iron. B. Geiger et al Nucl. Fusion (2019)
- Heat losses: Ion temperature clamping in Wendelstein 7-X electron cyclotron heated plasmas M.N.A. Beurskens et al. Nucl. Fusion (2021)

But turbulence is lurking

Theory

- An increasing number of numerical studies, based on gyrokinetics, which strive for more or less realistic conditions
- Fact: Severe heat losses are given by instabilities active at the ion Larmor radius scale, driven unstable by the ion temperature gradient [Rudakov Sagdeev, Sov. Physics JETP (1960), Horton Choi Tang, Phys. Fluids (1981)], eletron temperature gradient also play a role [Wilms et al. NF (2024), Zocco et al. PRR (2024)]
- Dichotomy between turbulence occurring and measured in an operating machine, and what happens in your computer...no matter how well you can justify numerical findings with any simple theoretical description tailored to the specific problem you are looking at

A mean-field theory for turbulence: gyrokinetics



Choose an equilibrium F_{0s} (Mexwellian for simplicity), consider small fluctuations $\delta f_s = F_s - F_{0s}$, and order

$$\frac{\omega}{\Omega_s} \sim \frac{k_\parallel}{k_\perp} \sim \frac{\delta f_s}{F_{0s}} \sim \frac{\rho_s}{L} \equiv \varepsilon \ll 1,$$

Solve Vlasov eq.

$$\frac{\partial}{\partial t} F_s + \mathrm{v} \cdot \nabla F_s + \frac{q_s}{m_s}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right) \cdot \nabla_{\mathrm{v}} F_s = 0,$$

asymptotic $\varepsilon-$expansion, average gyration (reduce $v-$space dimension by one), and get the resulting (gyrokinetic) equation for $h_s = \delta f_s - F_{0s} e\varphi / T_s$

A mean-field theory for turbulence: gyrokinetics

From gyrokinetics $\Rightarrow$ obtain δf

From Maxwell's Eqs. $\Rightarrow$ Eqs. for fluctuating fields using δf

For instance $\nabla \cdot \boldsymbol{E} = -\nabla^2 \phi = 4\pi \sum_s Z_s e \int d^3\boldsymbol{v} \delta f_s$,

Dimensionally $\lambda_D^2 \nabla^2 \phi \div \mathscr{F}[\phi]$, with $\mathscr{F}$ a differential/integral/algebraic operator/functional that depends on the details of the solution of your kinetic problem, and $\lambda_D = \sqrt{T/4\pi n e^2}$, Debye length

In plasma of interest for us, $\lambda_D \ll \rho_i$, so, the fundamental equation that described electrostatic fluctuations in fusion plasmas is the quasineutrality condition

$$\sum_s Z_s e \int d^3\boldsymbol{v} \delta f_s = 0. \tag{10}$$

A mean-field theory for turbulence: gyrokinetics

From gyrokinetics $\Rightarrow$ obtain δf

From Maxwell's Eqs. $\Rightarrow$ Eqs. for fluctuating fields using δf

For instance $\nabla \cdot \boldsymbol{E} = -\nabla^2 \phi = 4\pi \sum_s Z_s e \int d^3\boldsymbol{v} \delta f_s$,

Dimensionally $\lambda_D^2 \nabla^2 \phi \div \mathscr{F}[\phi]$, with $\mathscr{F}$ a differential/integral/algebraic operator/functional that depends on the details of the solution of your kinetic problem, and $\lambda_D = \sqrt{T/4\pi n e^2}$, Debye length

In plasma of interest for us, $\lambda_D \ll \rho_i$, so, the fundamental equation that described electrostatic fluctuations in fusion plasmas is the quasineutrality condition

$$\sum_s Z_s e \int d^3\boldsymbol{v} \delta f_s = 0. \quad (10)$$

A mean-field theory for turbulence: gyrokinetics

From gyrokinetics $\Rightarrow$ obtain δf

From Maxwell's Eqs. $\Rightarrow$ Eqs. for fluctuating fields using δf

For instance $\nabla \cdot \boldsymbol{E} = -\nabla^2 \phi = 4\pi \sum_s Z_s e \int d^3\boldsymbol{v} \delta f_s$,

Dimensionally $\lambda_D^2 \nabla^2 \phi \div \mathscr{F}[\phi]$, with $\mathscr{F}$ a differential/integral/algebraic operator/functional that depends on the details of the solution of your kinetic problem, and $\lambda_D = \sqrt{T/4\pi n e^2}$, Debye length

In plasma of interest for us, $\lambda_D \ll \rho_i$, so, the fundamental equation that described electrostatic fluctuations in fusion plasmas is the quasineutrality condition

$$\sum_s Z_s e \int d^3\boldsymbol{v} \delta f_s = 0. \tag{10}$$

Gyrokinetics

The issue of scale separation

Gyrokinetic theory describes strongly anisotropic, slowly evolving fluctuations in magnetised plasmas.

Based on the fundamental assumption that perturbations, δf_1, can vary across the equilibrium magnetic field on a short kinetic length, $l_\perp$ (of the order of the Larmor radius ρ_i),

$$\nabla_\perp \delta f_1 \sim l_\perp^{-1} \delta f_1 \sim \rho_i^{-1} \delta f_1, \tag{11}$$

but equilibrium quantities, f_0, vary slowly on a macroscopic length L,

$$\nabla_\perp f_0 \sim L^{-1} f_0. \tag{12}$$

Perturbations are assumed to vary slowly along the equilibrium magnetic field

$$\frac{\nabla_\parallel \delta f_1}{\nabla_\perp \delta f_1} \sim \frac{l_\perp}{L} \sim \frac{\rho_i}{L} \ll 1. \tag{13}$$

Gyrokinetics

The issue of scale separation

Gyrokinetic theory describes strongly anisotropic, slowly evolving fluctuations in magnetised plasmas.

Based on the fundamental assumption that perturbations, δf_1, can vary across the equilibrium magnetic field on a short kinetic length, $l_\perp$ (of the order of the Larmor radius ρ_i),

$$\nabla_\perp \delta f_1 \sim l_\perp^{-1} \delta f_1 \sim \rho_i^{-1} \delta f_1, \tag{11}$$

but equilibrium quantities, f_0, vary slowly on a macroscopic length L,

$$\nabla_\perp f_0 \sim L^{-1} f_0. \tag{12}$$

Perturbations are assumed to vary slowly along the equilibrium magnetic field

$$\frac{\nabla_\parallel \delta f_1}{\nabla_\perp \delta f_1} \sim \frac{l_\perp}{L} \sim \frac{\rho_i}{L} \ll 1. \tag{13}$$

Gyrokinetics

The issue of scale separation

Gyrokinetic theory describes strongly anisotropic, slowly evolving fluctuations in magnetised plasmas.

Based on the fundamental assumption that perturbations, δf_1, can vary across the equilibrium magnetic field on a short kinetic length, $l_\perp$ (of the order of the Larmor radius ρ_i),

$$\nabla_\perp \delta f_1 \sim l_\perp^{-1} \delta f_1 \sim \rho_i^{-1} \delta f_1, \qquad (11)$$

but equilibrium quantities, f_0, vary slowly on a macroscopic length L,

$$\nabla_\perp f_0 \sim L^{-1} f_0. \qquad (12)$$

Perturbations are assumed to vary slowly along the equilibrium magnetic field

$$\frac{\nabla_\parallel \delta f_1}{\nabla_\perp \delta f_1} \sim \frac{l_\perp}{L} \sim \frac{\rho_i}{L} \ll 1. \qquad (13)$$

Gyrokinetics

Basics

A further separation of *time* scales,

$$\frac{\omega}{\Omega_{ci}} \sim \frac{\rho_i}{L} \ll 1, \tag{14}$$

where Ω_{ci} is the cyclotron frequency, allows an average over the gyro-motion, to obtain a closed nonlinear kinetic theory that retains Larmor radius effects.

Distribution function $f = f_0 + \delta f_1 + \ldots$, with $\delta f_n = \rho_* \delta f_{n-1}$

Expansion parameter, $\rho_* \ll 1$, is chosen to be $l_\perp / L$:

Thus, one of the fundamental assumptions of gyrokinetics is the separation of length and time scales

$$\rho_i \ll L \tag{15}$$

Most numerical codes solve for δf_1, giving answers which are correct only to zeroth order in the expansion parameter.

Gyrokinetics

Basics

A further separation of *time* scales,

$$\frac{\omega}{\Omega_{ci}} \sim \frac{\rho_i}{L} \ll 1, \tag{14}$$

where Ω_{ci} is the cyclotron frequency, allows an average over the gyro-motion, to obtain a closed nonlinear kinetic theory that retains Larmor radius effects.

Distribution function $f = f_0 + \delta f_1 + \ldots$, with $\delta f_n = \rho_* \delta f_{n-1}$

Expansion parameter, $\rho_* \ll 1$, is chosen to be $l_\perp / L$:

Thus, one of the fundamental assumptions of gyrokinetics is the separation of length and time scales

$$\rho_i \ll L \tag{15}$$

Most numerical codes solve for δf_1, giving answers which are correct only to zeroth order in the expansion parameter.

Gyrokinetics

Basics

A further separation of *time* scales,

$$\frac{\omega}{\Omega_{ci}} \sim \frac{\rho_i}{L} \ll 1, \tag{14}$$

where Ω_{ci} is the cyclotron frequency, allows an average over the gyro-motion, to obtain a closed nonlinear kinetic theory that retains Larmor radius effects.

Distribution function $f = f_0 + \delta f_1 + \ldots$, with $\delta f_n = \rho_* \delta f_{n-1}$

Expansion parameter, $\rho_* \ll 1$, is chosen to be $l_\perp / L$:

Thus, one of the fundamental assumptions of gyrokinetics is the separation of length and time scales

$$\rho_i \ll L \tag{15}$$

Most numerical codes solve for δf_1, giving answers which are correct only to zeroth order in the expansion parameter.

Gyrokinetics

Fluctuations' representation

Representations: twisted slice of Roberts and Taylor (1965) ⇒ ballooning transform for ideal Magnetic Hydro Dynamics (Connor Hastie Taylor, PPPL, Pegoraro Schep) ⇒ local flux-tube (Beer Cowley Hammett)

Variation of equilibrium quantities ⇒ characteristic constant length scales.

Equilibrium density and temperature gradients that drive turbulence through instabilities are treated as

$$\frac{\nabla n}{n} \approx \frac{1}{n_0(r_0)} \nabla n_0(r_0) \left[1 - \frac{r}{L_n}\right] \approx -\frac{1}{L_n}, \tag{16}$$

where $n_0(r_0)$ is a constant value at a given location r_0.

Local flux-tube simulations can preserve separation of scales in a simple way.

Gyrokinetics

Fluctuations' representation

Representations: twisted slice of Roberts and Taylor (1965) $\Rightarrow$ ballooning transform for ideal Magnetic Hydro Dynamics (Connor Hastie Taylor, PPPL, Pegoraro Schep) $\Rightarrow$ local flux-tube (Beer Cowley Hammett)

Variation of equilibrium quantities $\Rightarrow$ characteristic constant length scales.

Equilibrium density and temperature gradients that drive turbulence through instabilities are treated as

$$\frac{\nabla n}{n} \approx \frac{1}{n_0(r_0)} \nabla n_0(r_0) \left[1 - \frac{r}{L_n} \right] \approx -\frac{1}{L_n}, \tag{16}$$

where $n_0(r_0)$ is a constant value at a given location r_0.

Local flux-tube simulations can preserve separation of scales in a simple way.

Stellarators: Local or Global Gyrokinetics?

The issue of scale separation

We might be interested in

- equilibrium quantities with less trivial variations or
- in large turbulent structures that do not fulfill conventional scale separation.

This motivated the development of "global" gyrokinetic codes.

Stellarators: Local or Global Gyrokinetics?

The issue of scale separation

There are mainly two families of such codes (plus Truly global EUTERPE):

- those which retain full radial variations (radially global)
- and those which retain variations of these quantities across the equilibrium magnetic field (within but not across magnetic surfaces, ergo global on the surface) (GENE, stella)

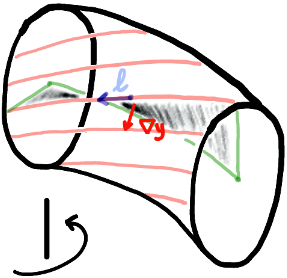
Stellarators: Local or Global Gyrokinetics?

The issue of scale separation

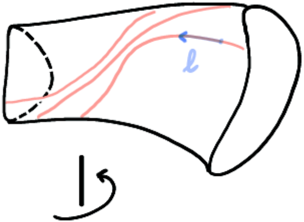
Two scenarios in which it is both consistent and desirable to solve the gyrokinetic equation in the full-surface setting.

- Conventional gyrokinetics is assumed to apply, with good scale separation between the background and fluctuations.
- The second scenario in which full-surface gyrokinetic simulations are needed is when there is a certain loss of scale separation, namely that between the magnetic field and the fluctuations, but it is still reasonable to use gyrokinetics as we know it, but with a manageble upgrade

Surface-global gyrokinetics



In axisymmetry the kinetic problem for the electrostatic potential φ_n can be solved locally along any field-line labelled by n.



In nonaxisymmetric, the nontrivial geometry couples the kinetic dynamics on each field-line $A_{nm}\varphi_m$ generating a qualitatively and quantitatively different behaviour

Some analytical advances possible

Surface-global model solution

For the electrostatic Ion-Temperature-Gradient driven instability, mediated by magnetic field curvature, we managed to evalaute analytically the coupling matrix, for a model of the magnetic curvature [Zocco et al. PoP (2020)]

The global eigenvalue is therefore the solution of

$$\det A_{nm} \equiv \det A_m = 0. \tag{17}$$

One can extract the full-surface instability growth rate

$$m = m_0 \sim \rho_*^{-1} \gg 1, \det A_{mn} = 0 \rightarrow \frac{\partial^2 \varphi_m}{\partial m^2} + \frac{1}{2}\left(a - bm\right)\varphi_m = 0,$$

global eigenvalue found by imposing that φ_m has a maximum at $m_0 \sim \rho_*^{-1}$.

Some analytical advances possible

Surface-global model solution

For the electrostatic Ion-Temperature-Gradient driven instability, mediated by magnetic field curvature, we managed to evalaute analytically the coupling matrix, for a model of the magnetic curvature [Zocco et al. PoP (2020)]

The global eigenvalue is therefore the solution of

$$\det A_{nm} \equiv \det A_m = 0. \tag{17}$$

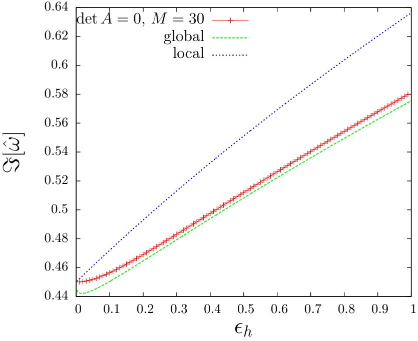
One can extract the full-surface instability growth rate

$$m = m_0 \sim \rho_*^{-1} \gg 1, \det A_{mn} = 0 \rightarrow \frac{\partial^2 \varphi_m}{\partial m^2} + \frac{1}{2}(a - bm)\varphi_m = 0,$$

global eigenvalue found by imposing that φ_m has a maximum at $m_0 \sim \rho_*^{-1}$.

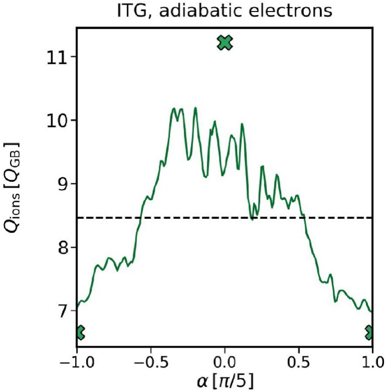
Surface-global gyrokinetics

Global reduction of ion-temperature-gradient driven linear instability



Zocco et al. PoP (2020)

Surface-global reduction nonlinear heat fluxes given by Ion-Temperature-Gradient driven turbulence in W7-X

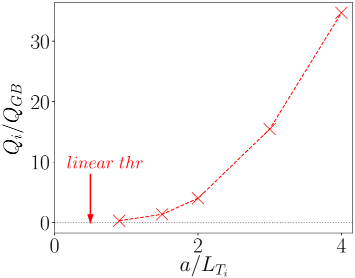


Wilms Navarro Jenko Nucl. Fus. (2023)

Thresholds

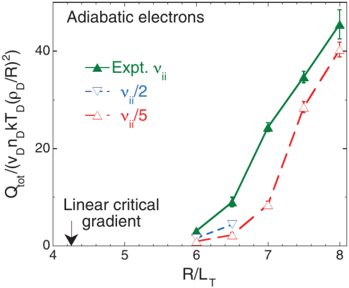
Local simulations are still valuable to understand worst case scenarios

Absence of nonlinear (Dimits) shift in critical gradient in W7-X



Zocco Podavini et al. PRE (2022)

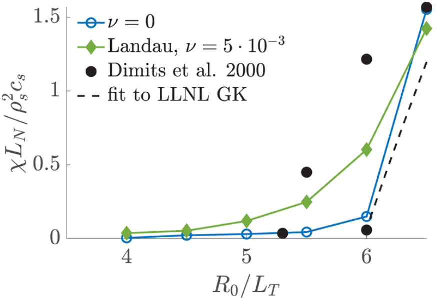
Dimits shift in tokamaks



Mikkelsen Dorland PRL (2008)

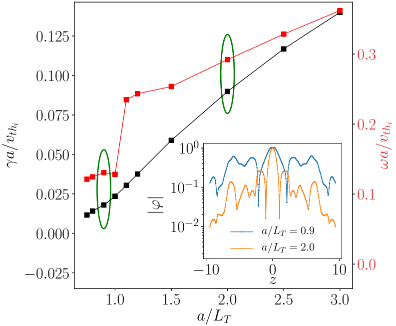
Thresholds

Collisional damping of modes that saturate turbulence (TOK similar to Stell)



χ nonlinear turbulent heat diffusion coefficient. Hoffmann Frei Ricci JPP (2023)

W7-X: Turbulence structure itself makes its own saturation less effective, not collisions



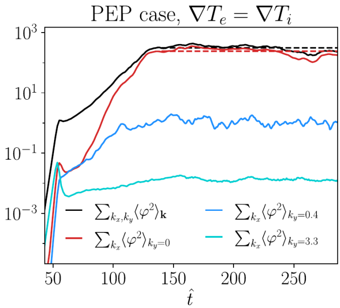
Zocco Podavini et al PRE (2023)

Improving performances in W7-X (I)

Small critical gradient are not desirable for a reactor.

However, reduced turbulence level with injection of pellets⇒increasing density gradients: improvement observed!

Why do we see an improvement in performances in W7-X when we inject pellets of frozen hydrogen (Pellet Enhanced Performace)?



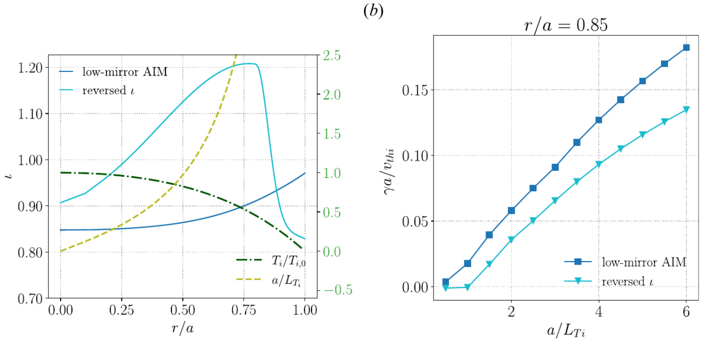
Zocco Podavini Wilms Banon Jenko [Phys. Rev. Research (2024)]

Violent growth of component of fluctuations that saturates turbulence by shearing of eddies ($k_y = 0$) , presumably different from Tokamaks, but, at present, we are not in a position to give a full explanation of this highly nonlinear phenomenon (attemps to explain higher performances with stelarators' peculiarities of linear physics are to be discarded [PRL (2022)]). Zonal flows ($k_y = 0$) need to be better uderstood in stellarators.

Improving performances in W7-X (II)

Imagine ways of operationally inscreasing critical gradients

Distortion of the rotational transform⇒make it more tokamak-like were the source of turbulence is strongest

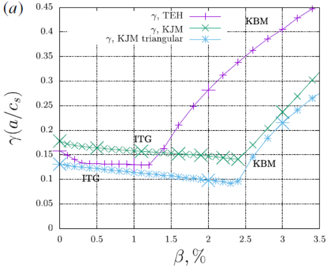


Podavini Zocco et al. JPP (2024)

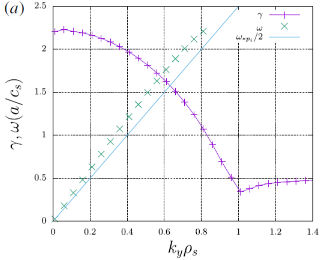
We can induce an increase in critical threshold!

Electromagnetic Turbulence: The Elephant in the Room

W7-X was optimised to achieve its best confinement properties at 'large' $\beta = 8\pi p/B^2$, a measure of electromagnetic effects (not discussed so far)



Growth rates, electrostatic instasbilities transitiont to electromagnetic Kinetic Ballooning Modes



KBM instability spectrum (predicted analytically by Aleynikova and Z. PoP)

Aleynikova and Z. PoP (2017), Z. et al, 2xAleynikova Z. et al [JPP]

Very much needed and interesting line of research [Wilms JPP 2021 Mulholland PRL (2024), Riemann sub. (2024), Garcia Regaña, Mishchenko (eurofusion numerical projects)]

Electromagnetic Turbulence: fast particles transport

The problem of evaluating the turbulent losses in the presence of energetic/fast particles (auxiliary heating and/or fusion products) is not dissimilar from tokamaks. Conceptual approach of tokamaks adapted to stellarator, complications due to the absence of conserved canonincal momentum

Nonlinear drift-wave and energetic particle long-time behaviour in stellarators: solution of the kinetic problem

Alessandro Zocco[1,†], Alexey Mishchenko[1], Axel Könies[1], Matteo Falessi[2] and Fulvio Zonca[2,3]

[1]Max Planck Institute for Plasma Physics, 17491 Greifswald, Germany

[2]Center for Nonlinear Plasma Science and ENEA C. R. Frascati, Frascati, Italy

[3]Institute for Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou, PR China

(Received 2 November 2022; revised 9 May 2023; accepted 11 May 2023)

We propose a theoretical scheme for the study of the nonlinear interaction of drift-wave-like turbulence and energetic particles in stellarators. The approach is based on gyrokinetics, and features a separation of time and scales, for electromagnetic fluctuations, inspired by linear ballooning theory. Two specific moments of the gyrokinetic equation constitute the main equations of the system, which requires a full kinetic nonlinear solution. This is found iteratively, expanding in the smallness of the bounce-average radial drift frequency, and nonlinear $\boldsymbol{E} \times \boldsymbol{B}$ drift frequency, compared with the inverse time scales of the resonantly interacting energetic particles. Our analysis is therefore valid for neoclassically optimised stellarators. The resummation of all iterative and perturbative nonlinear kinetic solutions is discussed in terms of Feynman diagrams. Particular emphasis is put on the role of collisionlessly undamped large-scale structures in phase space, the kinetic equivalent of zonal flows, i.e. phase-space zonal structures, and on wave-like fluctuations generated by energetic particles.

Advancing field

Through the work of many, among which

Elizabeth Paul @ Columbia

Thomas Foster @ Princeton

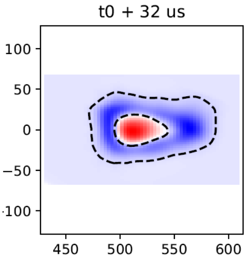
Christof Slaby @ IPP Greiswald

GENE group @ IPP Garching

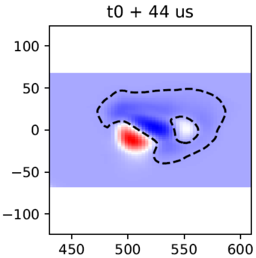
Don Spong @ ORNL

Macroscopic Electromagnetic Instabilities

During operations needed to regulate the plasma power exshaust by gauging the location of the magnetic flux surfaces at the edge, macroscopic spontaneous displacements of the WHOLE plasma column were observed



$t0 = 15.672480\, sec$



$t0 = 15.672528\, sec$

Zanini et al. Nucl. Fus. (2020)

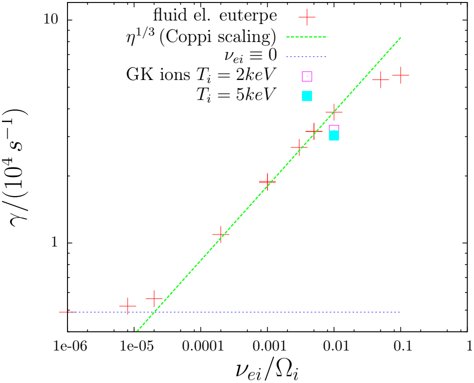
Plasma displacement evolves on $\Delta t \approx 58 \mu sec \Rightarrow \gamma \approx 17 KHz$. Soft Xrays thomografy say that the distortion of the plasma column has toroidal mode number $n = 1$.

The instability in understood in terms of small-scale kinetic effects that allow field lines to break and re-join: magnetic reconnection

Magnetic reconnection

- Stellarators were conceived having in mind the avoidance of current-driven instabilities routinely observed in tokamaks.

Are we immune to them? No.



Zocco et al. Nucl. Fus. (2021)

More and more active line of research (Aleynikova, Nikulsin, Ramasamy)

Conclusions

- Stellarators compel us to have a fresher look at old problems
- We saw how they relate to axisymmetric systems, from the point of view of single particle confinement: Tokamaks belong to a sub-class of quasihelically symmetric stellarators with planar magnetic axis.
- Introduced omnigeneous fields, and the relization of a quasi-isodynamic field in Wendelstein 7-X (IPP Greifswald Germany)
- We briefly touched: gyrokinetics scale separation, the critical threshold for ion-temperature-gradient driven turbulence, electromagnetic instabilities, micro and macroscopic.
- Finite-β correction are required to fully appreciate the good confinement properties of W7-X
- Advances in optimization are giving designs that are likely to produce burning plasmas that can perform as well as those of tokamaks
- Stellarators, as we conceive them now, are not flexible in the exploration of operating scenarios