

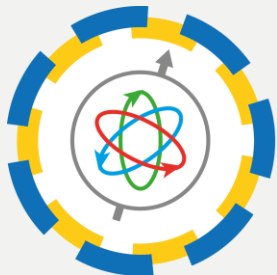
Studies of the absorption parameter $3\gamma/2\gamma$ in positronium decays

Kamila Kasperska on behalf of the J-PET collaboration

Under the supervision of dr hab. Magdalena Skurzok

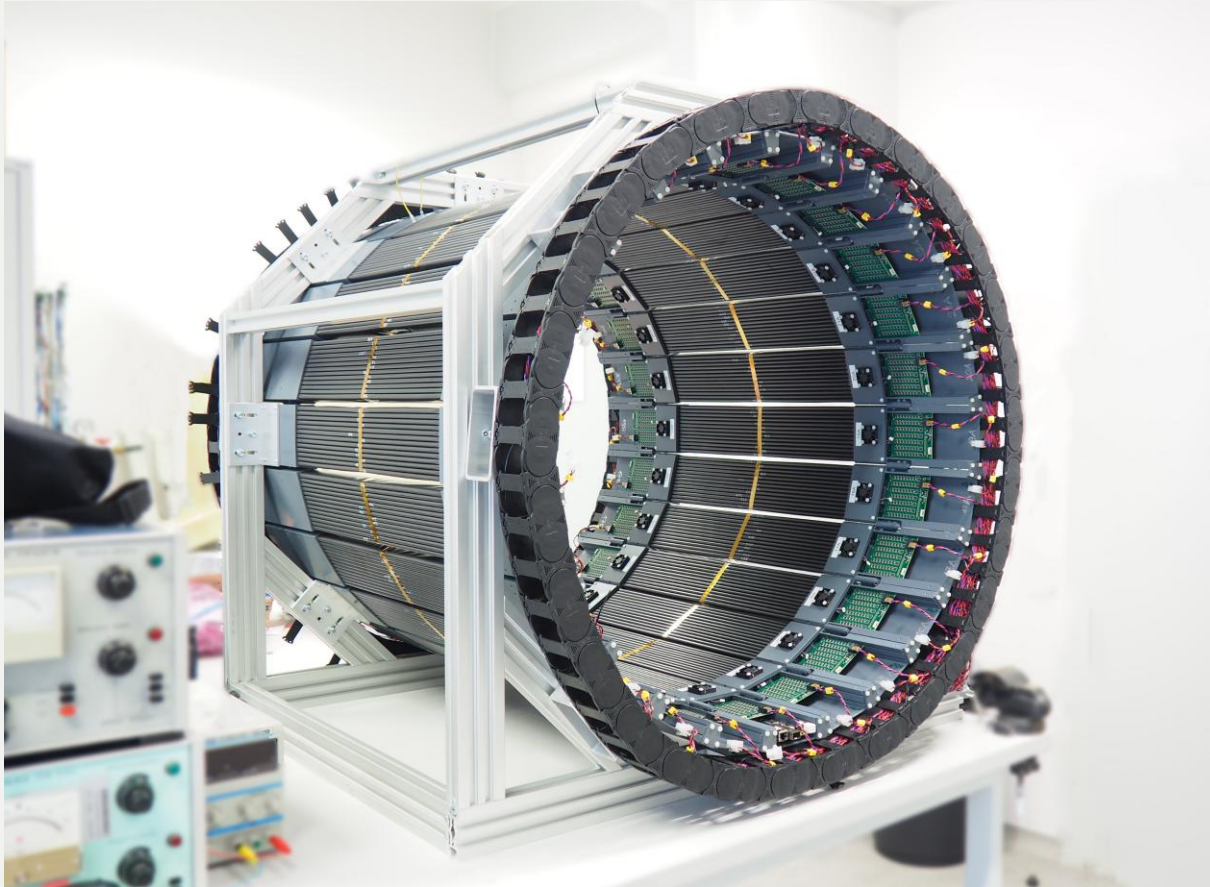
HPXM Workshop

Frascati, 19.06.2025



J-PET

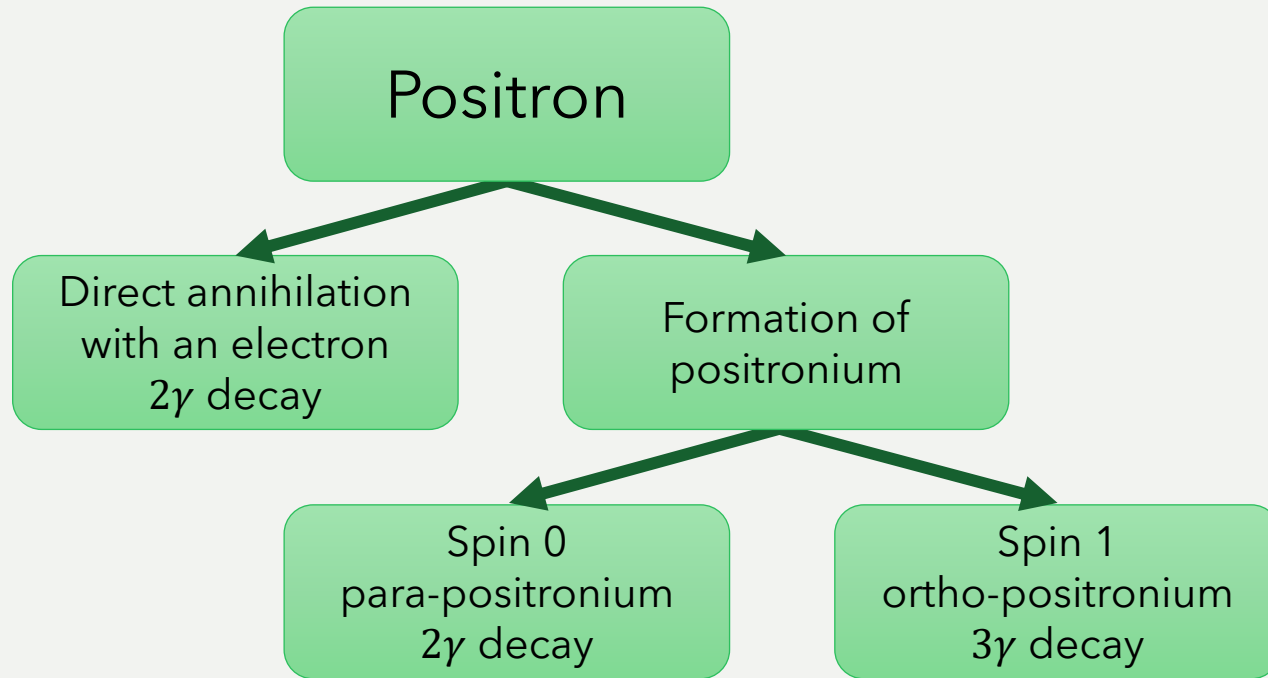
J-PET detector



Talk by Magdalena Skurzok, Monday 15:50

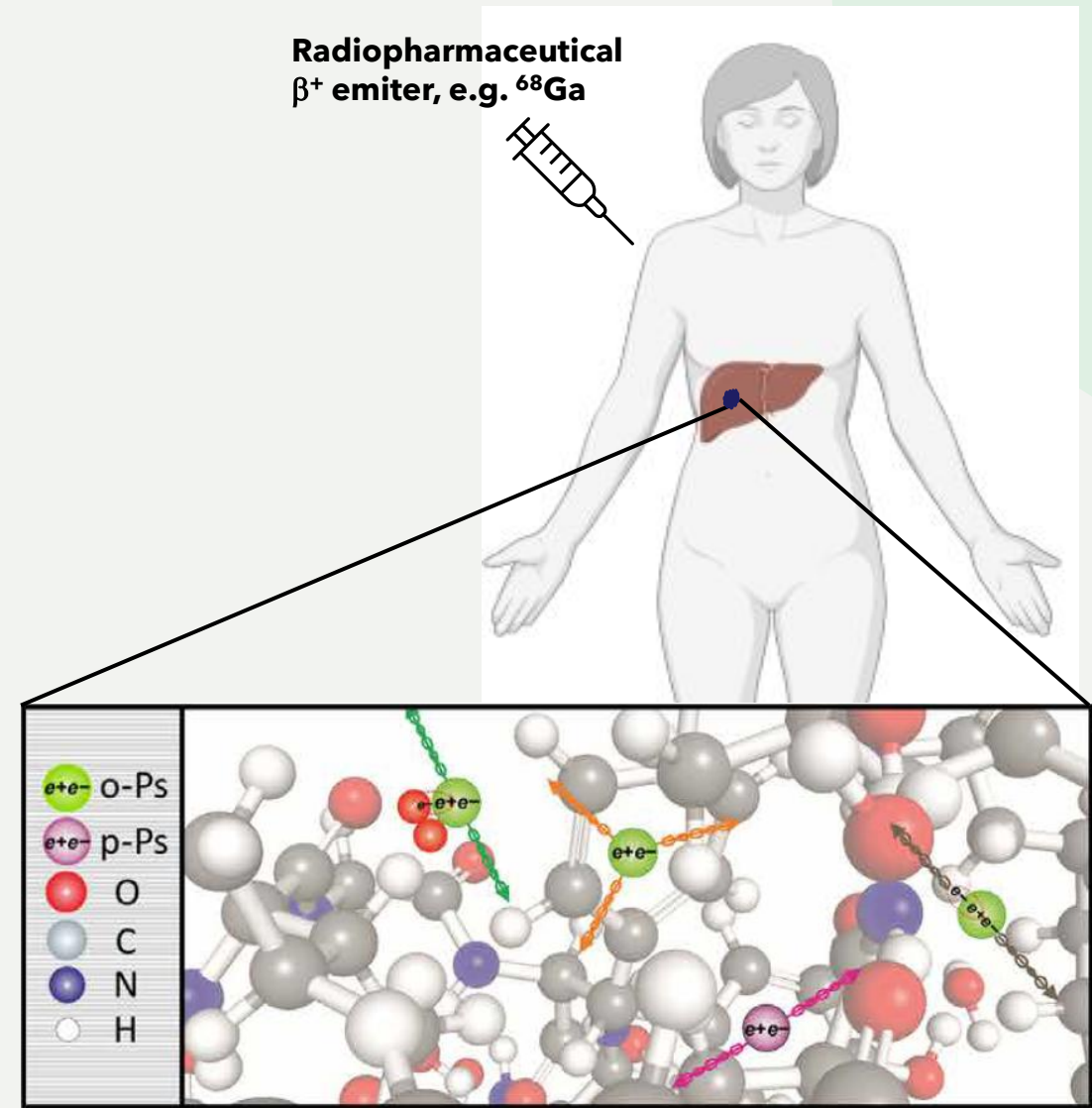
- Positron Emission Tomography detector based on plastic scintillators
- Cost-efficient
- Multi-photon detection
- Medical imaging:
 - Total body PET
 - **Positronium Imaging**
- Fundamental physics:
 - Quantum entanglement
 - Discrete symmetries

Positronium imaging



Volume of free spaces $\searrow$, oxygenation $\nearrow \rightarrow$ oPs lifetime $\searrow$, 3γ fraction $\searrow$

Volume of free spaces $\nearrow$, oxygenation $\searrow \rightarrow$ oPs lifetime $\nearrow$, 3γ fraction $\nearrow$



The **$3\gamma/2\gamma$ ratio** depends on the tissue's molecular structure.

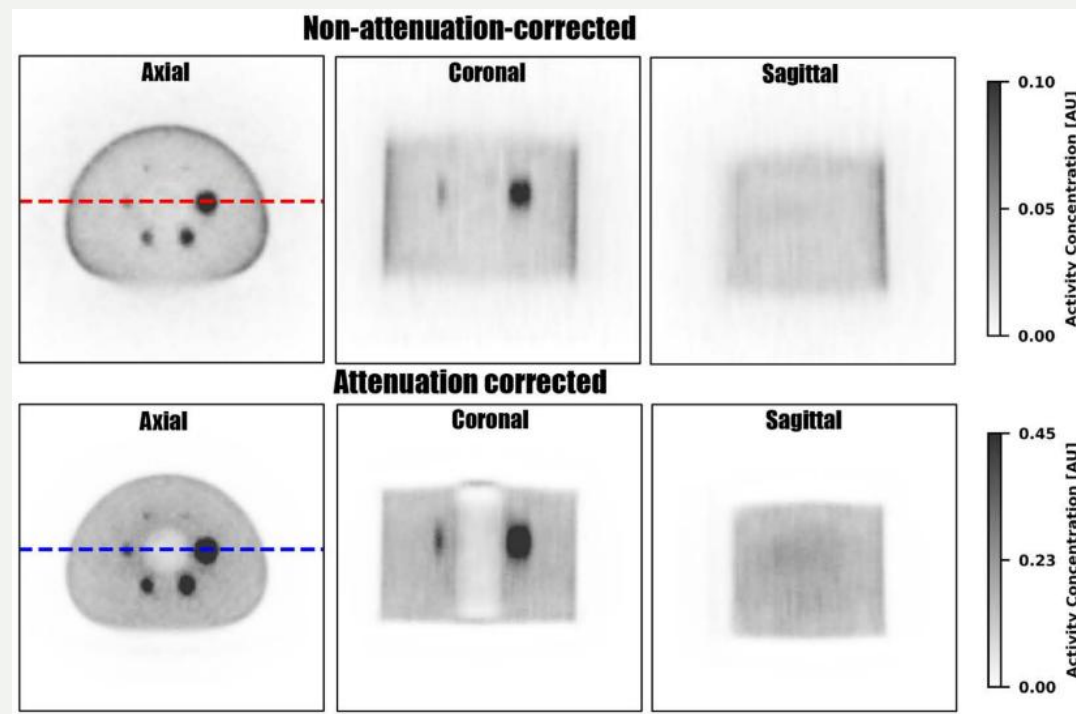
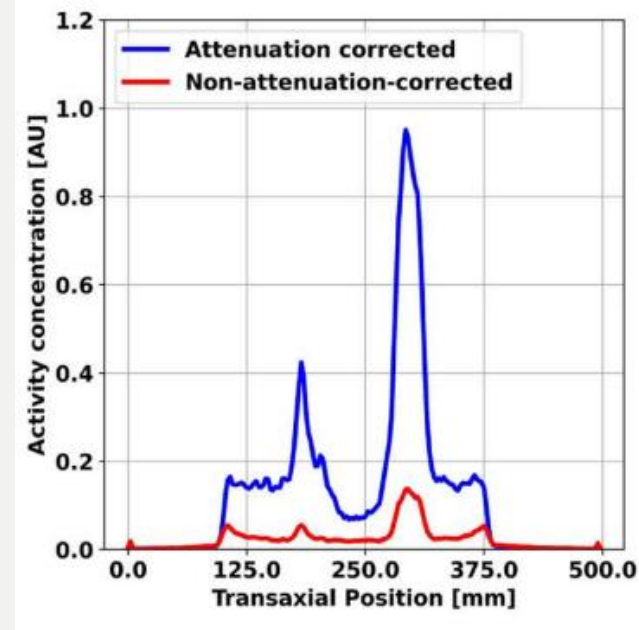
Absorption correction

Absorption of gamma quanta in tissues



Underestimation of radiopharmaceutical concentration in deeper tissues

- J-PET: CT-based attenuation correction for two-gamma imaging
- Standard attenuation maps - not sufficient for three-gamma imaging



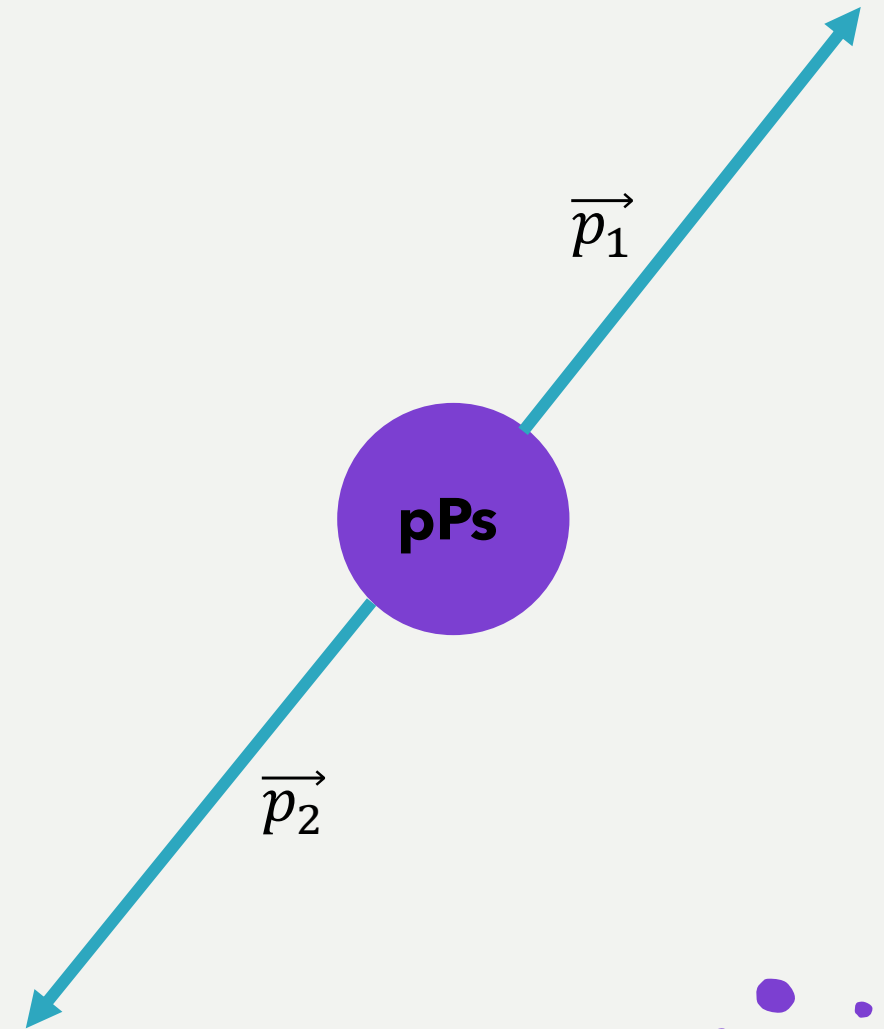
Simulation of pPs decay

Para-positronium – singlet state 1S_0

Isotropic decay into 2 back-to-back photons with energies of 511 keV

Decay simulation:

1. Randomizing the direction of $\vec{p}_1$
2. Assigning the energy of 511 keV
3. Determining the momentum $\vec{p}_2 = -\vec{p}_1$



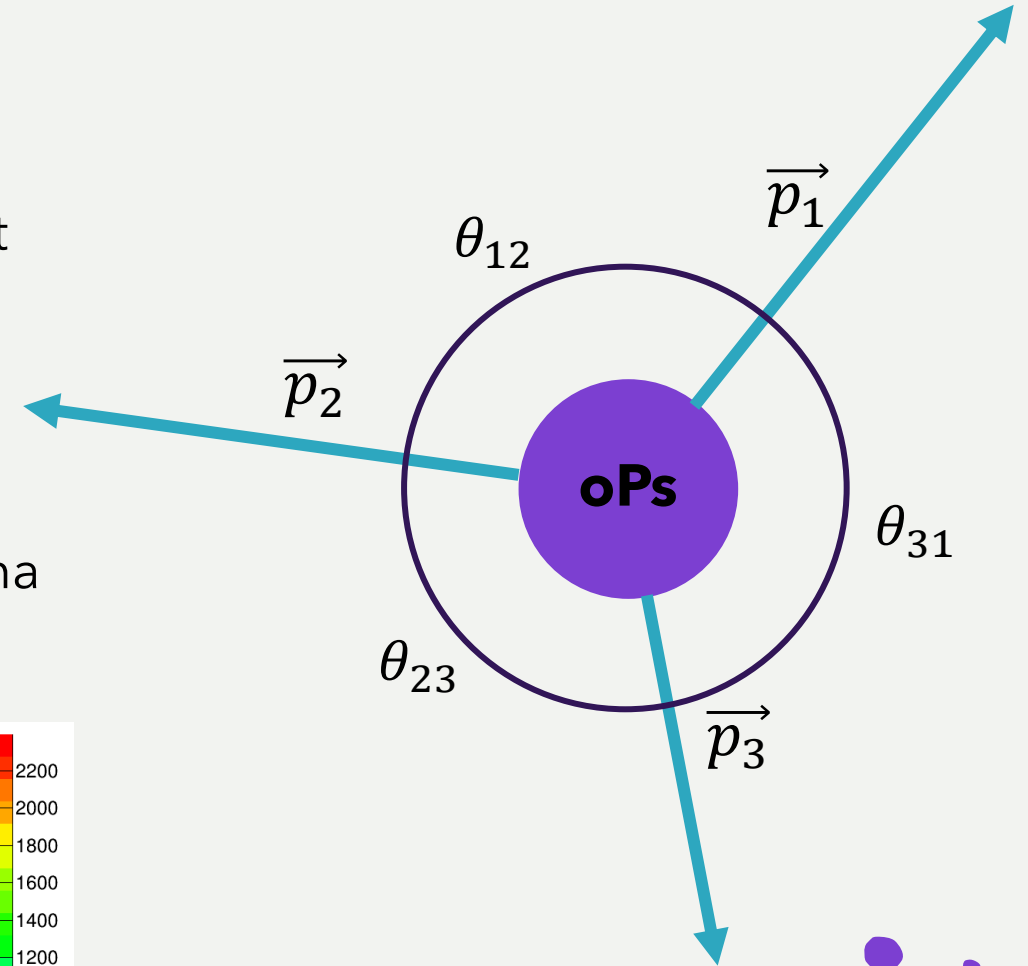
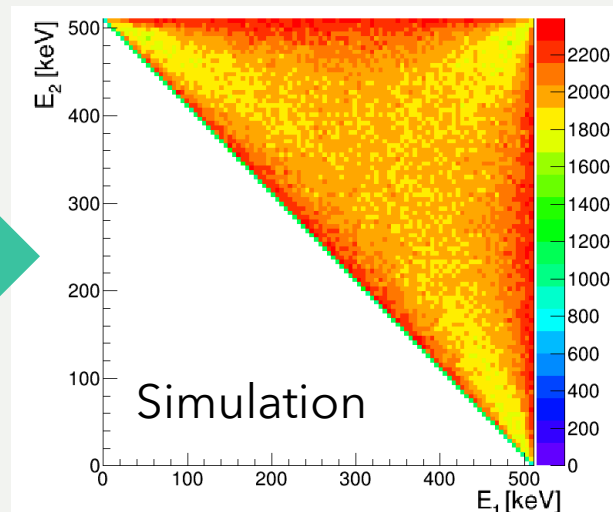
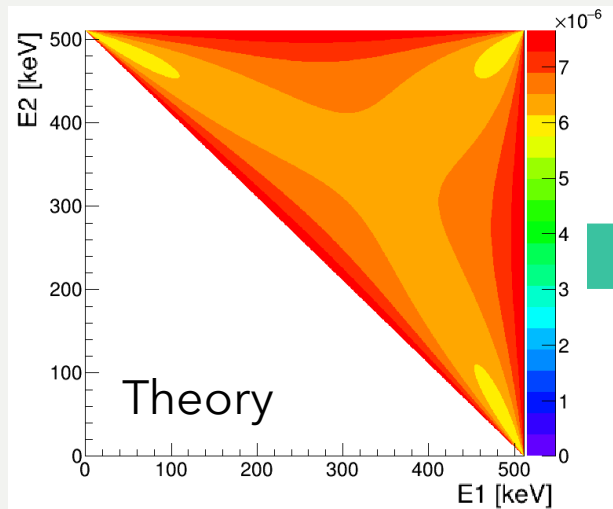
Simulation of oPs decay

Ortho-positronium - triplet state 3S_1

Decay into 3 photons described with the matrix element $M_{oPs \rightarrow 3\gamma}$:

$$M_{oPs \rightarrow 3\gamma} = \left(\frac{m_e - E_1}{E_2 E_3} \right)^2 + \left(\frac{m_e - E_2}{E_1 E_3} \right)^2 + \left(\frac{m_e - E_3}{E_1 E_2} \right)^2 ,$$

where m_e - electron mass, E_1, E_2, E_3 - energies of gamma quanta.



Absorption of gamma quanta in matter

Probability that a photon with energy E_i is **not** absorbed:

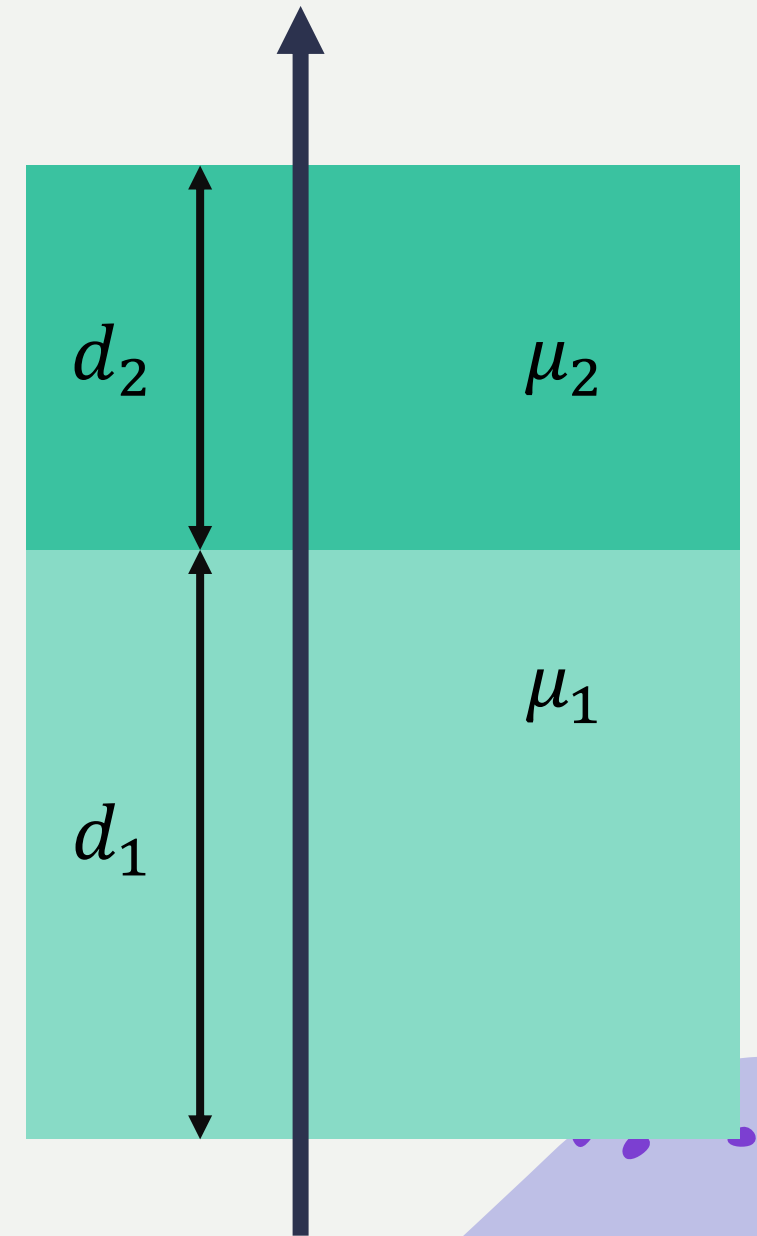
$$P_i = \prod_m e^{-\mu_m(E_i)d_m}$$

distance

linear attenuation coefficient

Total probability that **none** of the photons are absorbed:

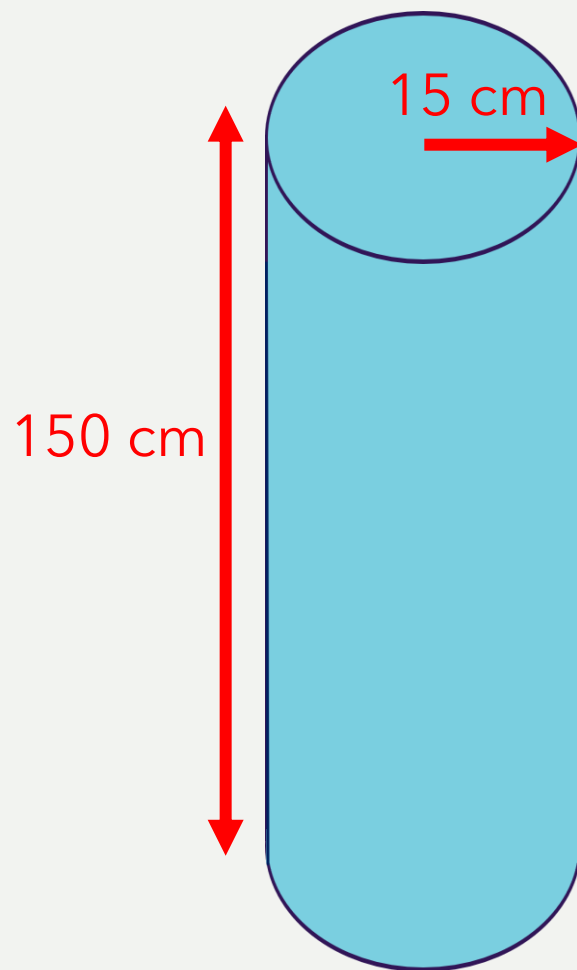
$$P_{tot} = \prod_i P_i$$



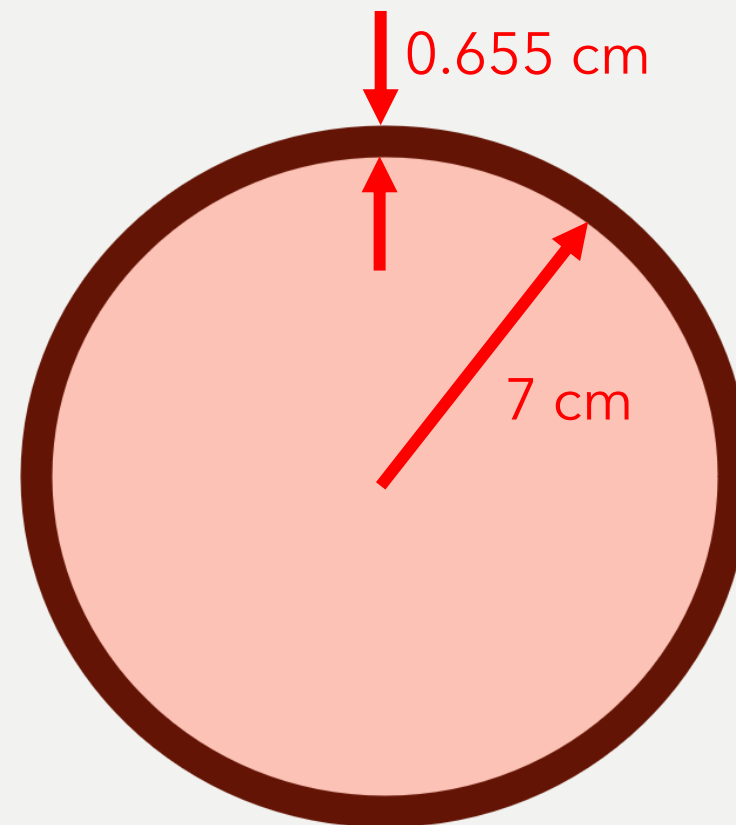
Simplified phantom models



Water sphere



Water cylinder



Brain sphere + layer of bone

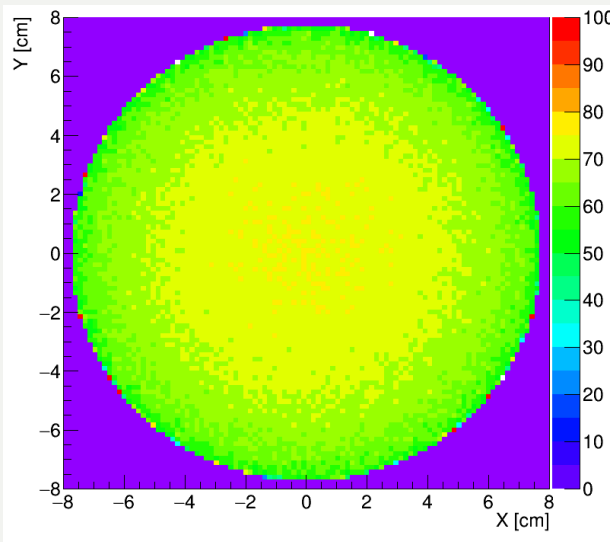
Absorption map – head model

- Percentage of gamma quanta pairs or triplets, from which **at least one** of the photons **reacted** with the model
- Specific to the annihilation point
- 10^7 events

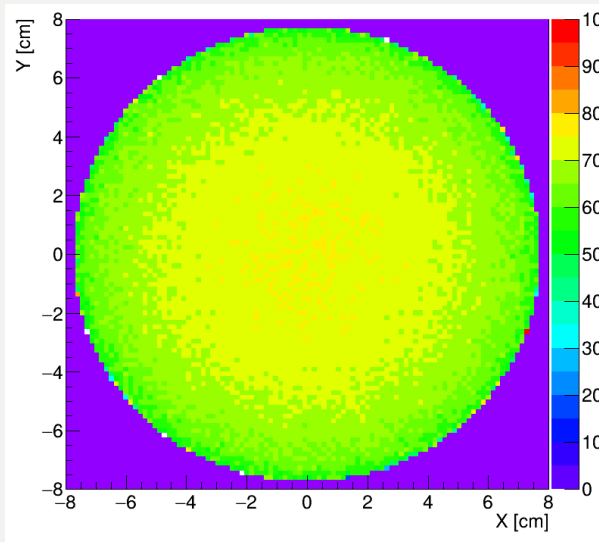
Surviving photon multiplets within the whole model

	Toy MC	GATE
pPs	$30.156 \pm 0.017\%$	$30.052 \pm 0.017\%$
oPs	$11.879 \pm 0.011\%$	$11.919 \pm 0.011\%$
$3\gamma/2\gamma$	0.39392 ± 0.00043	0.39392 ± 0.00043

p-Ps $\rightarrow 2\gamma$

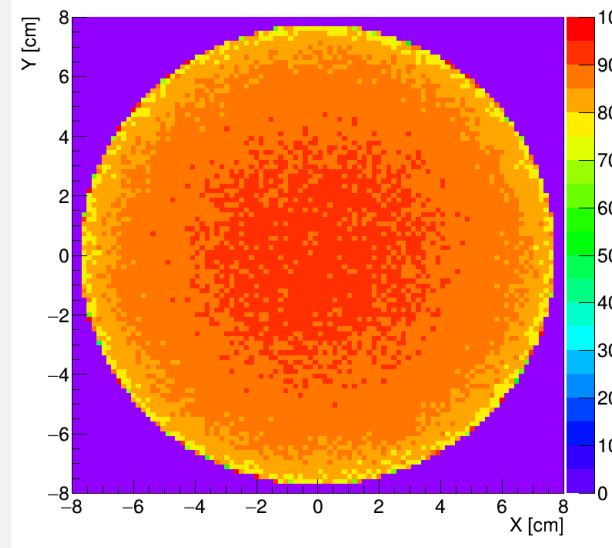


Toy MC

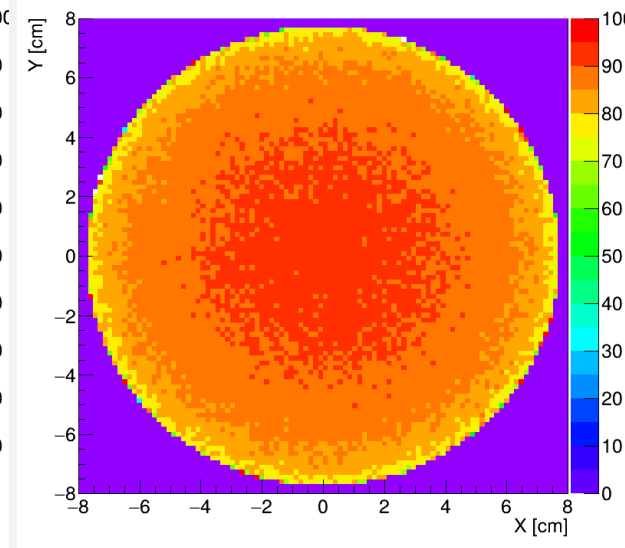


GATE

o-Ps $\rightarrow 3\gamma$



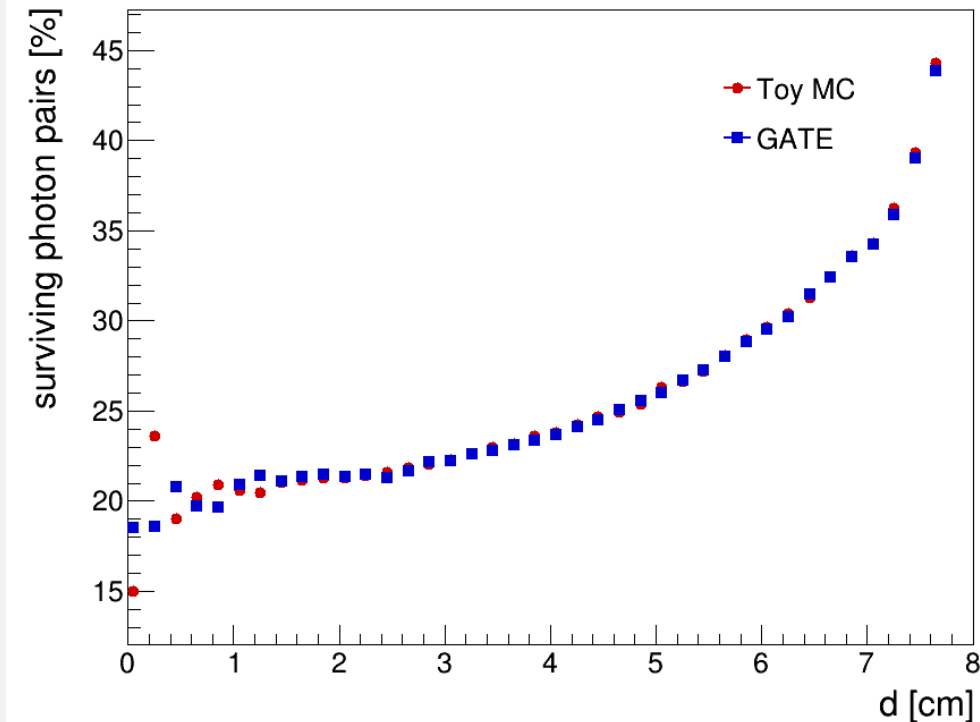
Toy MC



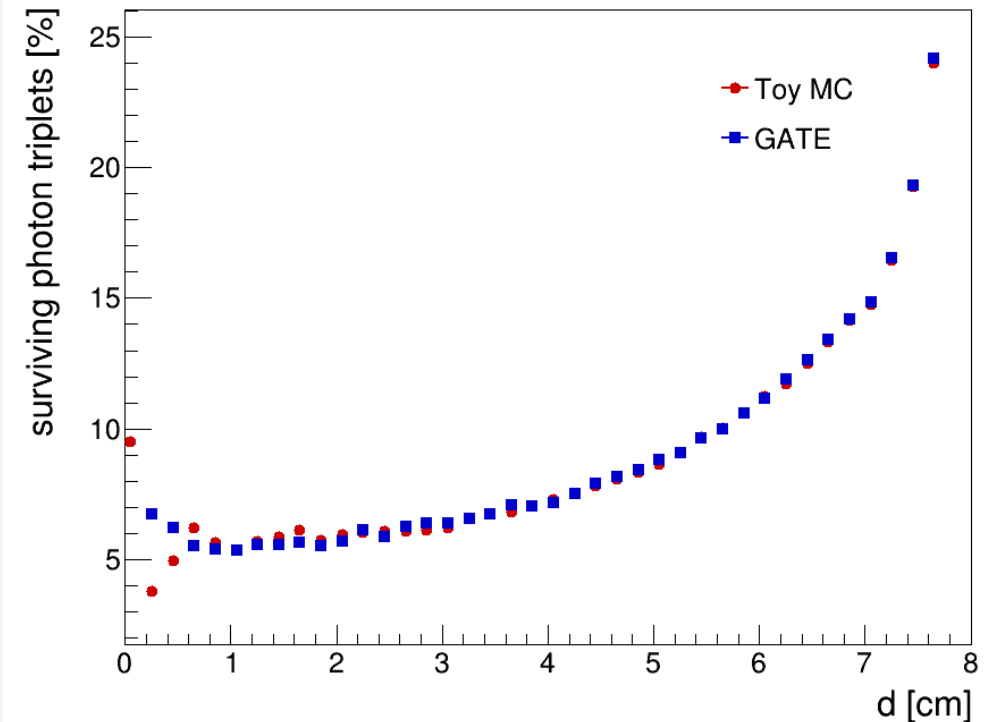
GATE

Dependence of the percentage of **surviving** photon multiplets on the distance from the center of the head (d)

pPs

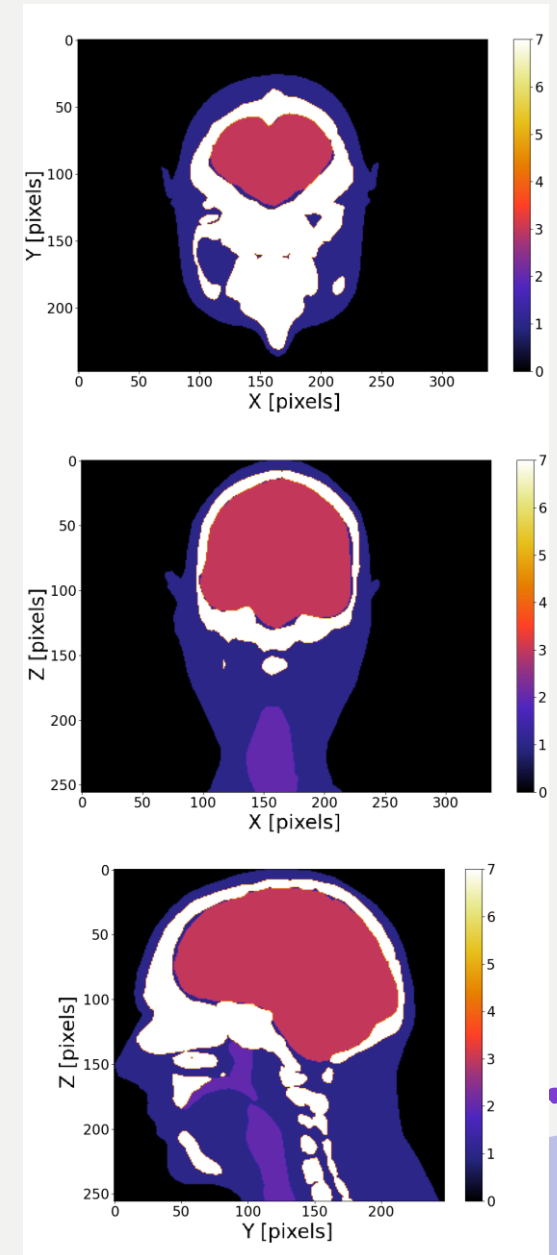
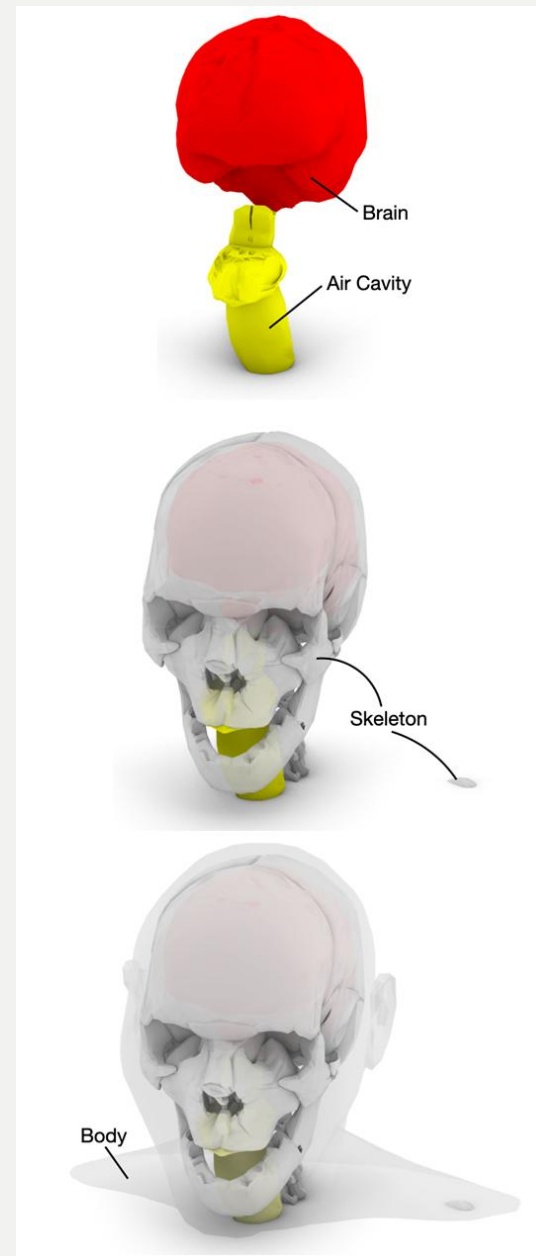


oPs



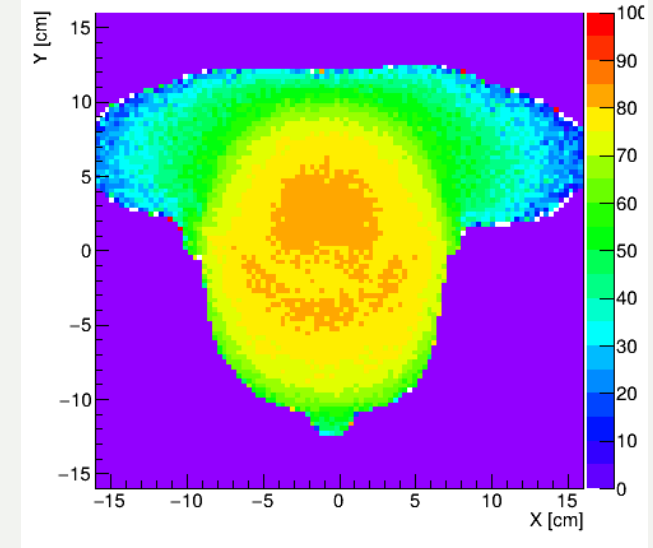
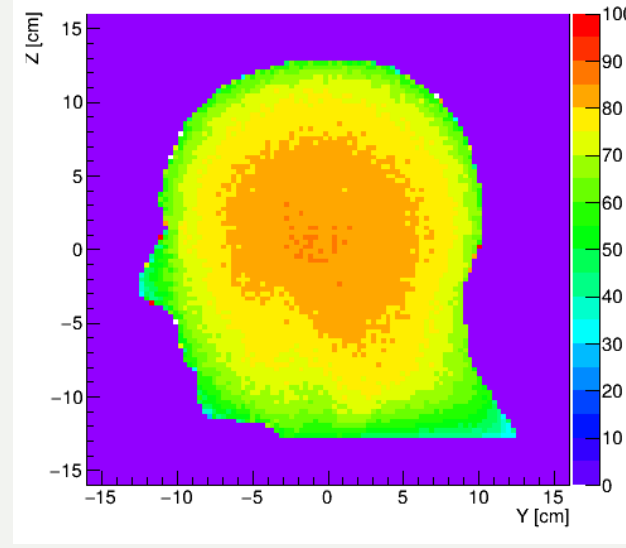
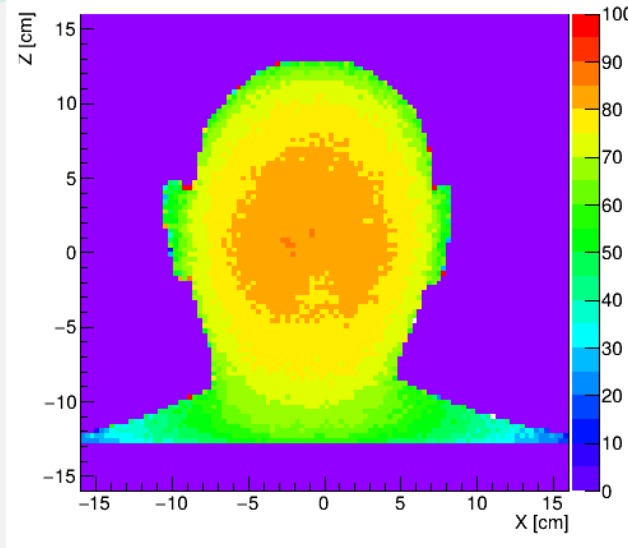
XCAT human phantom

- Mesh-based XCAT phantom (**mesh50_XCAT**) developed and maintained by Auer Benjamin
- Highly detailed male anatomy for subject in 50th percentile
- Source - voxelized version of the phantom
- Head Only model (computational limitations)

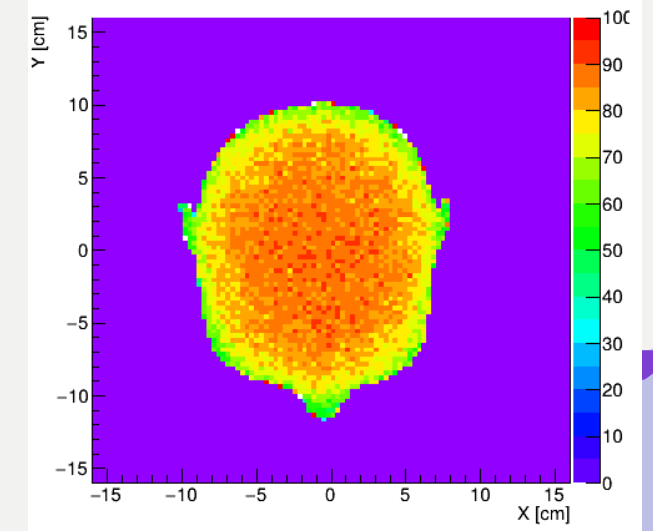
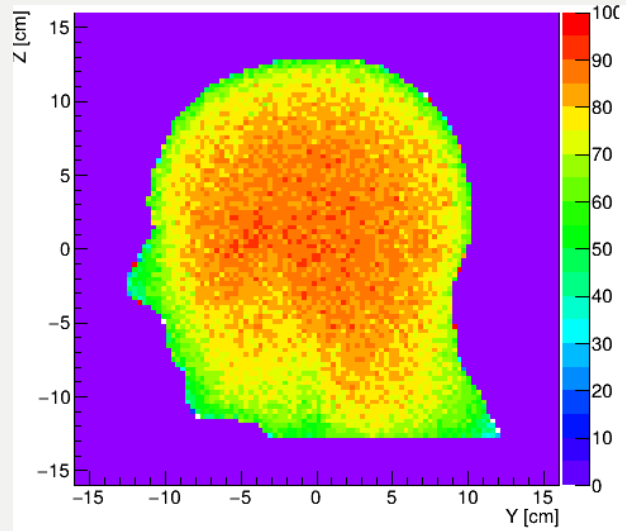
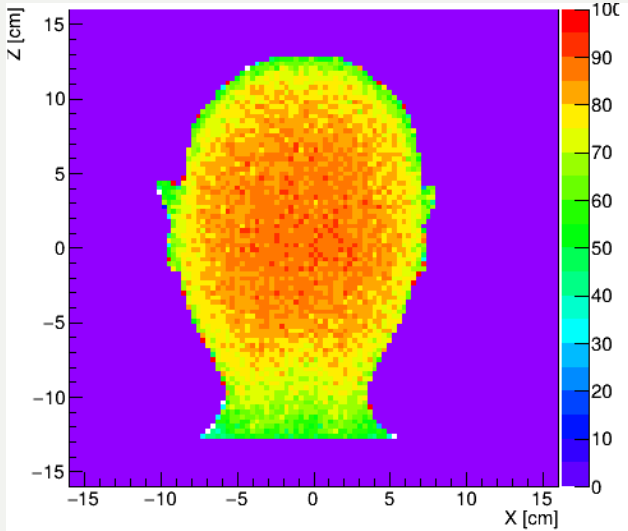


XCAT Phantom absorption maps – p-Ps (10^7 events, $24.922 \pm 0.016\%$ pairs not absorbed)

Projection



Central slice

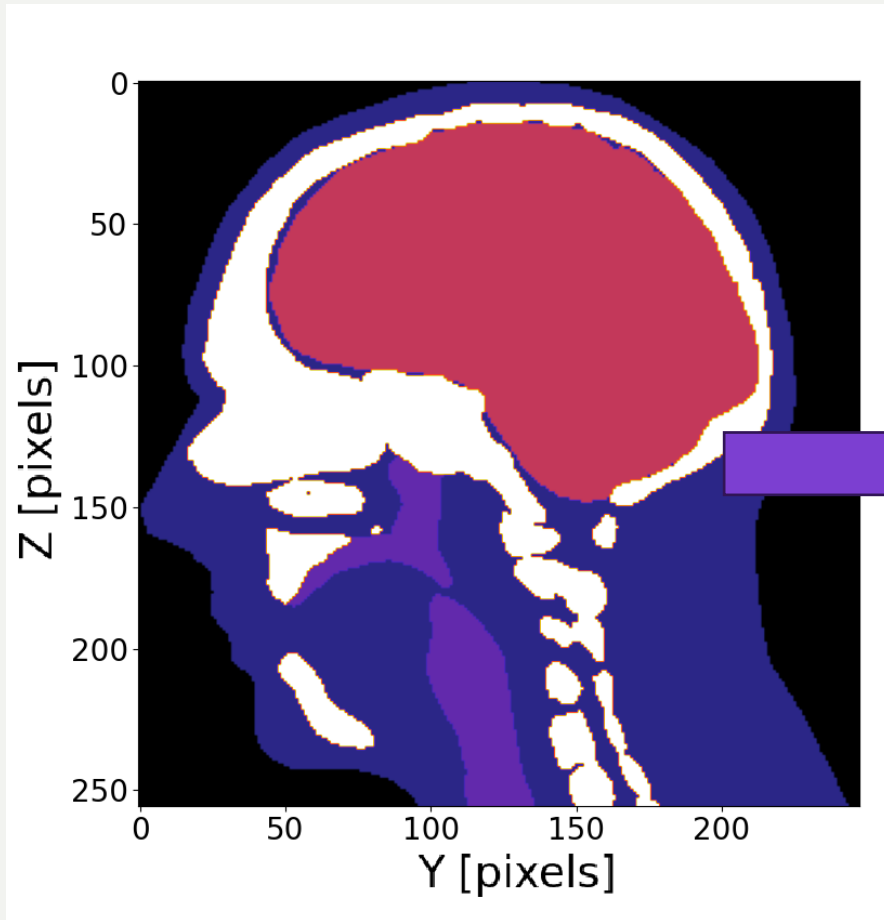


Coronal

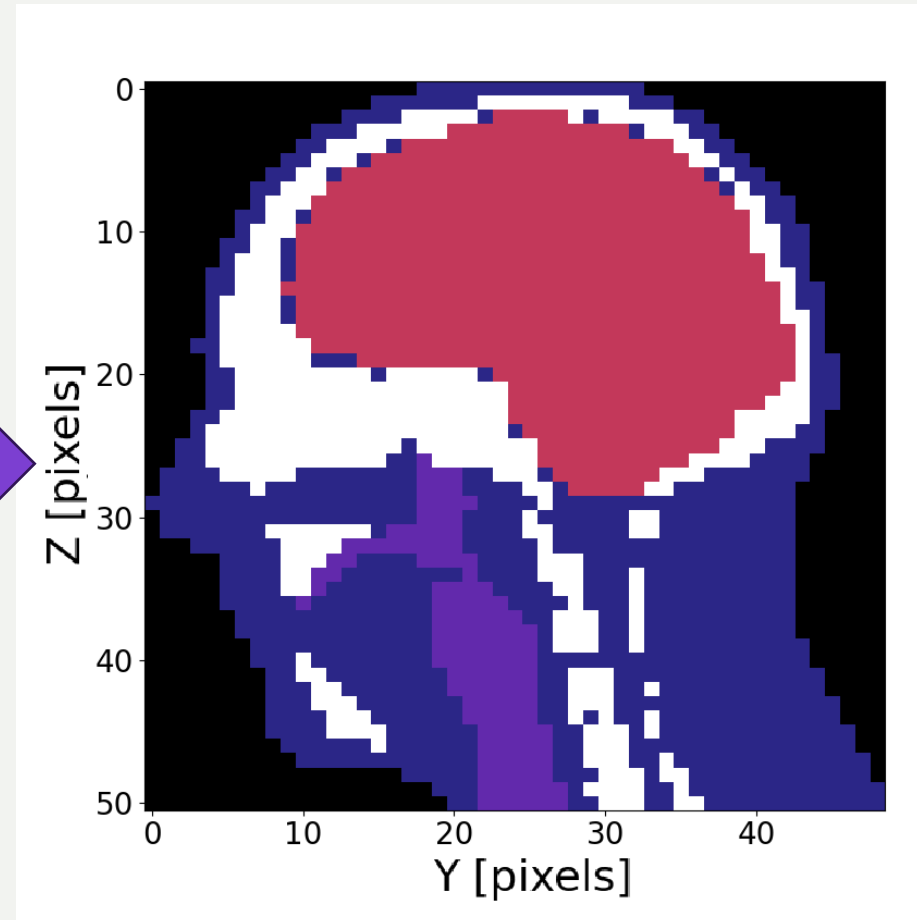
Sagittal

Transverse

oPs source



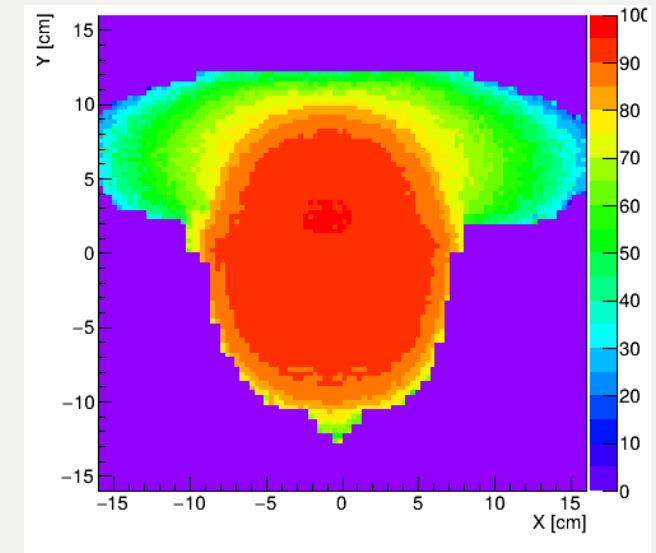
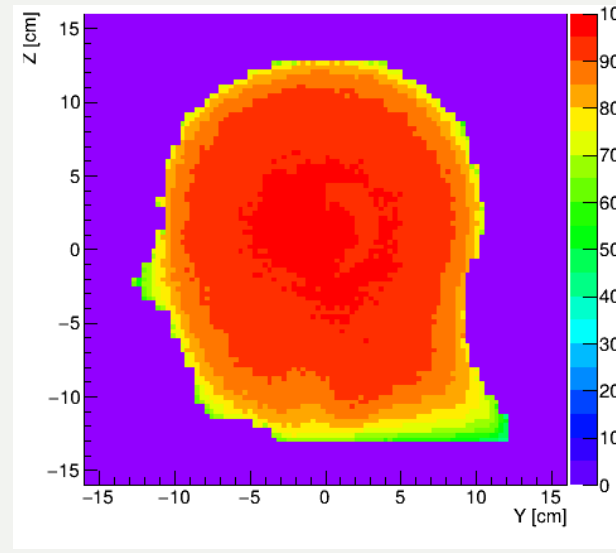
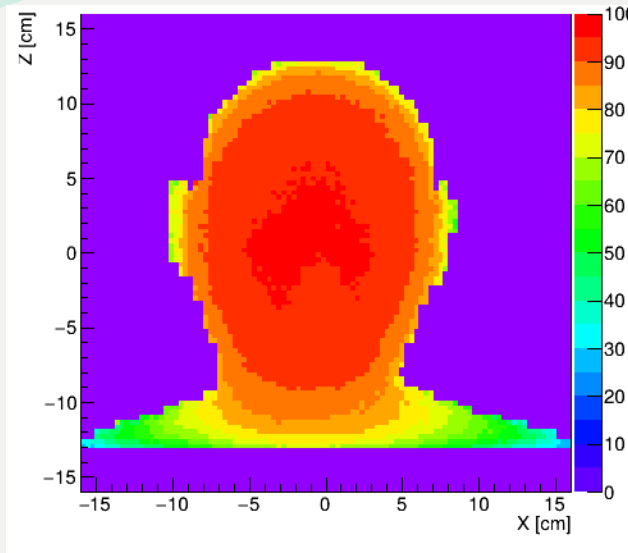
Voxel 1x1x1 mm



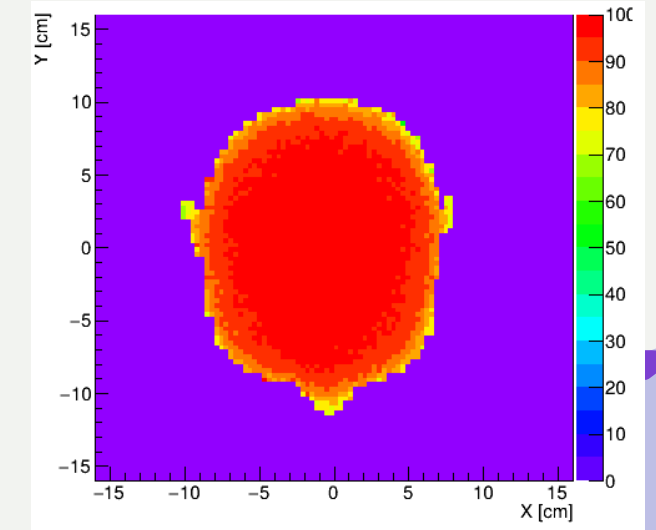
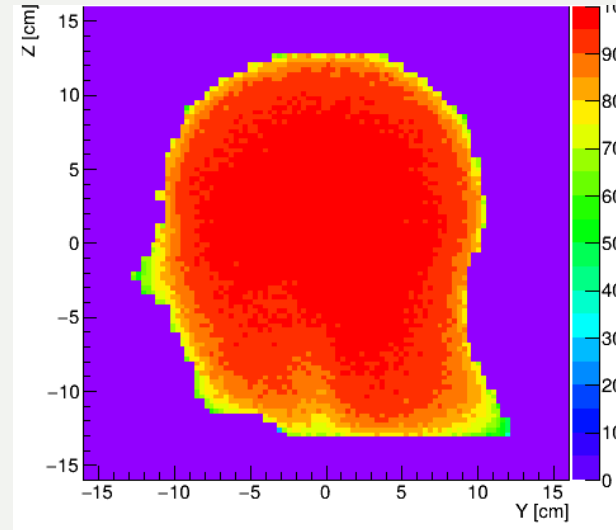
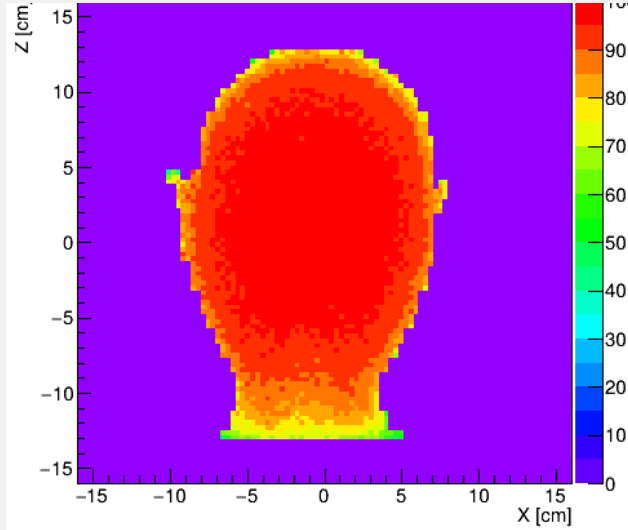
Voxel 5x5x5 mm

XCAT Phantom absorption maps – o-Ps ($4.435 \cdot 10^7$ events, $10.256 \pm 0.010\%$ triplets not absorbed)

Projection



Central slice



Coronal

Sagittal

Transverse

Summary and perspectives

Study of gamma absorption from p-Ps $\rightarrow 2\gamma$ and o-Ps $\rightarrow 3\gamma$ decays in different phantoms – a foundation for developing attenuation maps required for tomographic image correction

Used custom-developed toy MC simulation and the GATE simulation toolkit to create emission-point-specific absorption maps

According to expectations – absorption of photon triplets significantly higher than that of photon pairs due to their lower energies

What's next?

Simulations using whole-body XCAT human phantom

Introducing detector acceptance into the simulation

Simulation using a phantom created based on the patient's CT scan

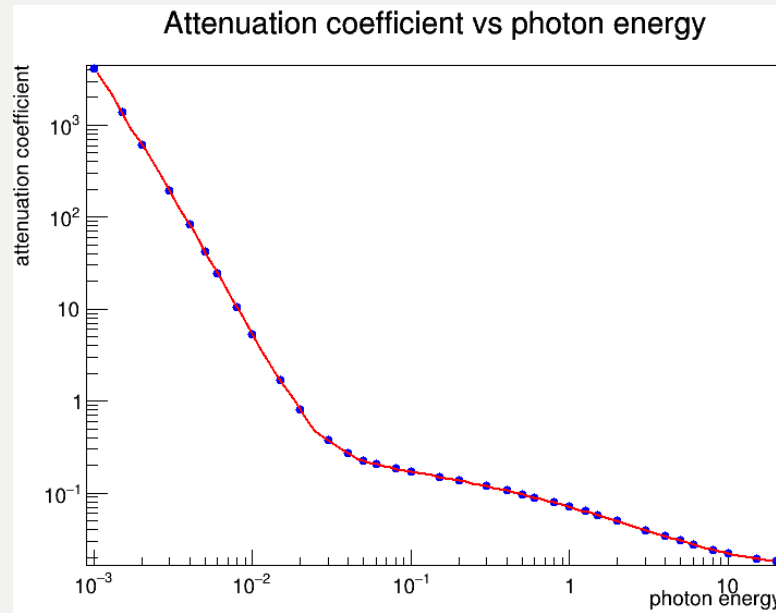


Thank you for your attention!

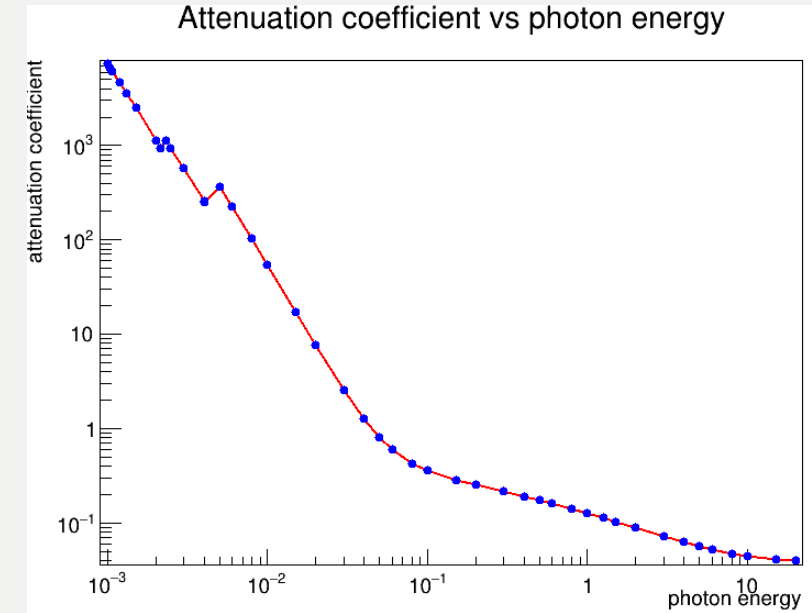
Acknowledgements

I would like to acknowledge Polish high-performance computing infrastructure PLGrid (HPC Center: ACK Cyfronet AGH) for providing computer facilities and support within computational grant no. PLG/2024/017688, and the National Science Center 2021/42/A/ST2/00423.

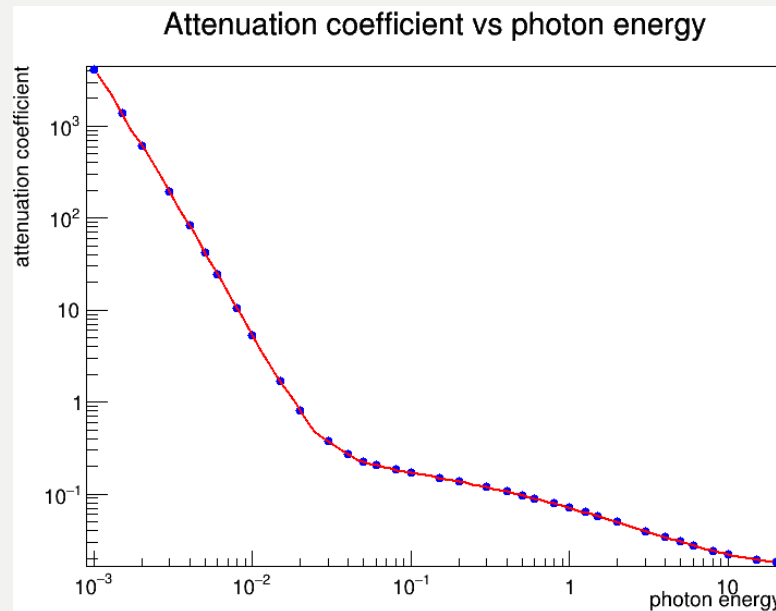
Linear attenuation coefficients for toy MC simulation



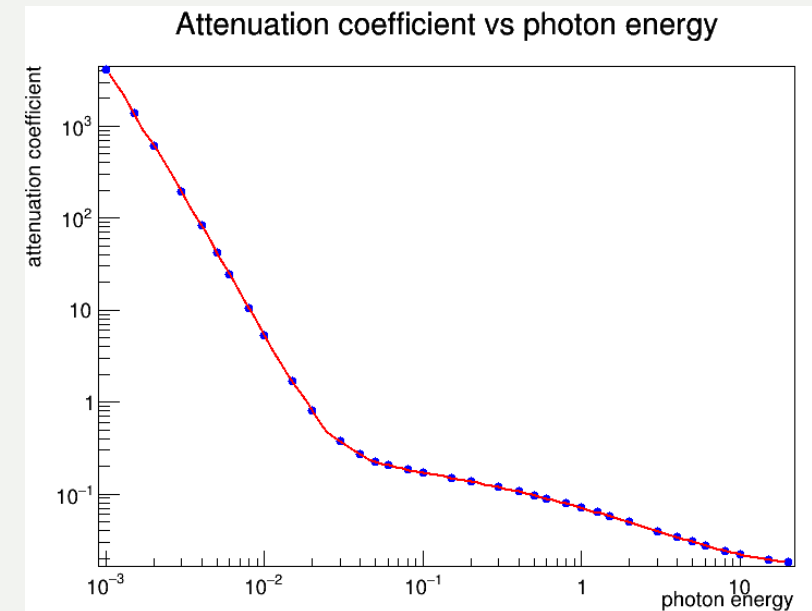
Water



Bone



Brain



Liver

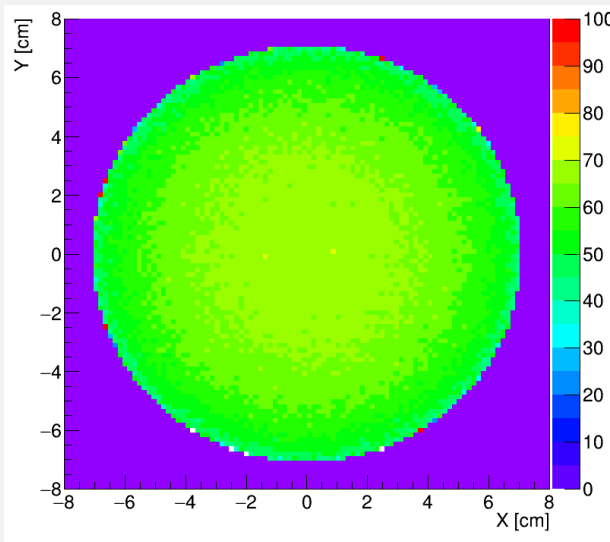
Absorption map – water sphere

- 10^7 events

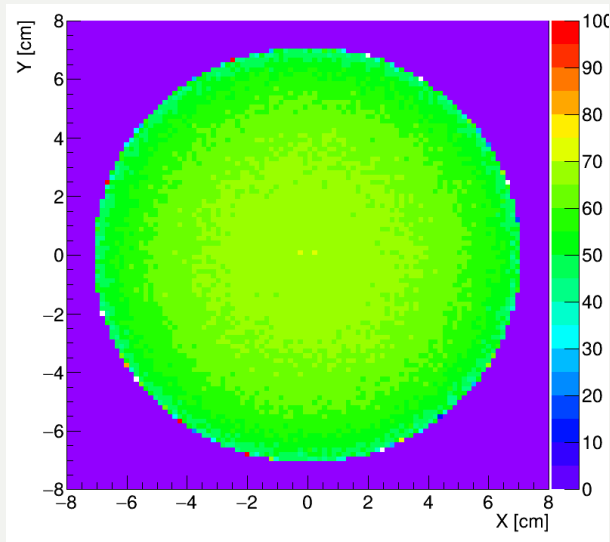
Surviving photon multiplets within the whole model

	Toy MC	GATE
pPs	$37.836 \pm 0.019\%$	$37.878 \pm 0.019\%$
oPs	$18.119 \pm 0.013\%$	$18.152 \pm 0.013\%$

p-Ps $\rightarrow 2\gamma$

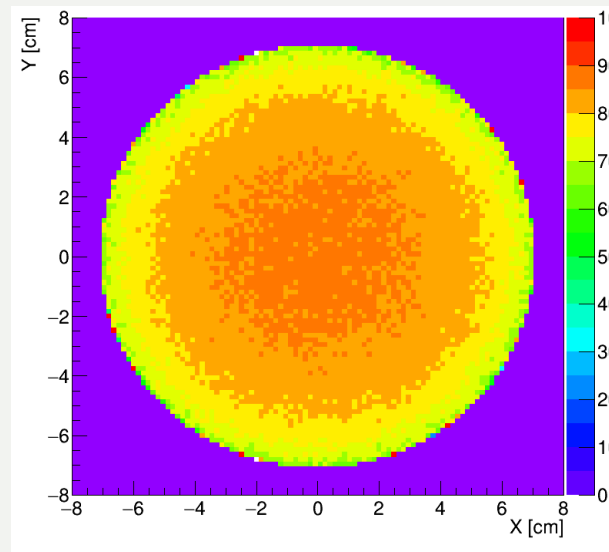


Toy MC

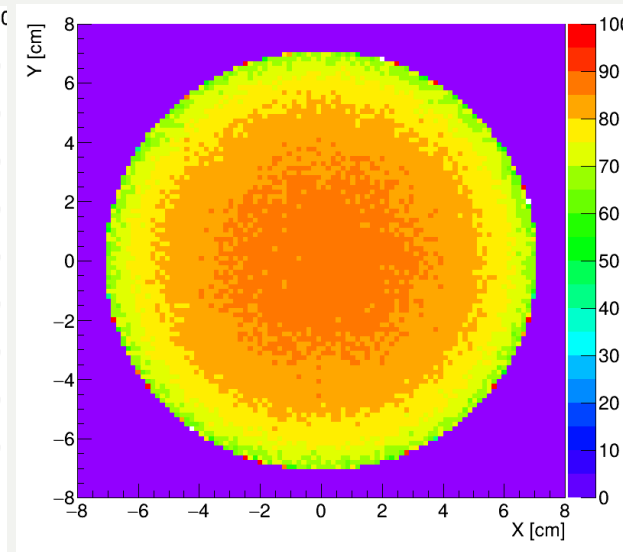


GATE

o-Ps $\rightarrow 3\gamma$



Toy MC



GATE

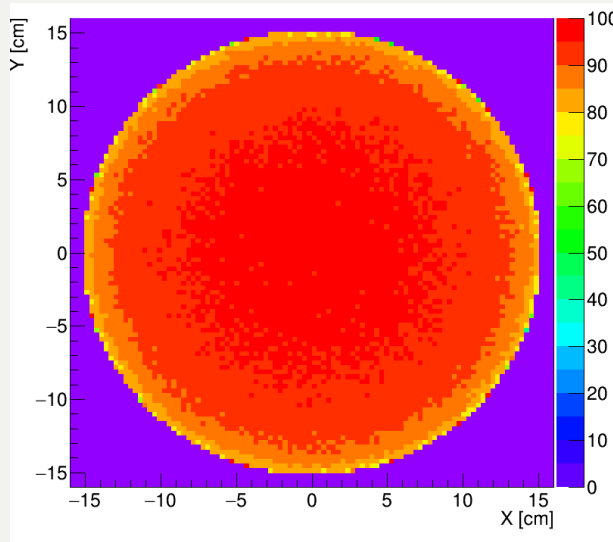
Absorption map – water cylinder – XY plane

- 10^7 events

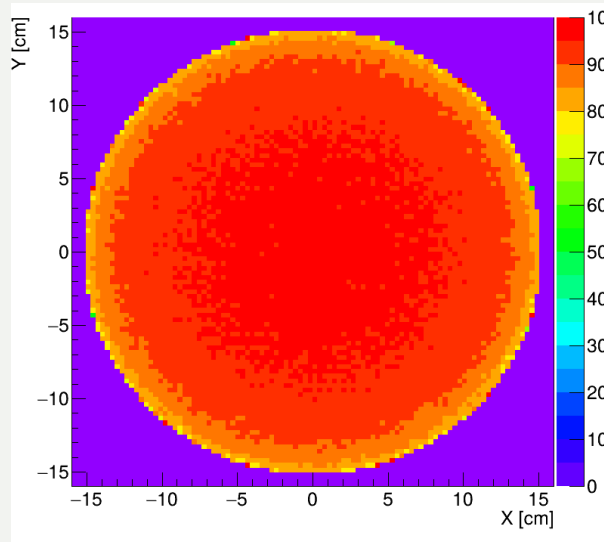
Surviving photon multiplets within the whole model

	Toy MC	GATE
pPs	$7.6063 \pm 0.0087\%$	$7.6176 \pm 0.0087\%$
oPs	$1.6403 \pm 0.0041\%$	$1.6462 \pm 0.0041\%$

p-Ps $\rightarrow 2\gamma$

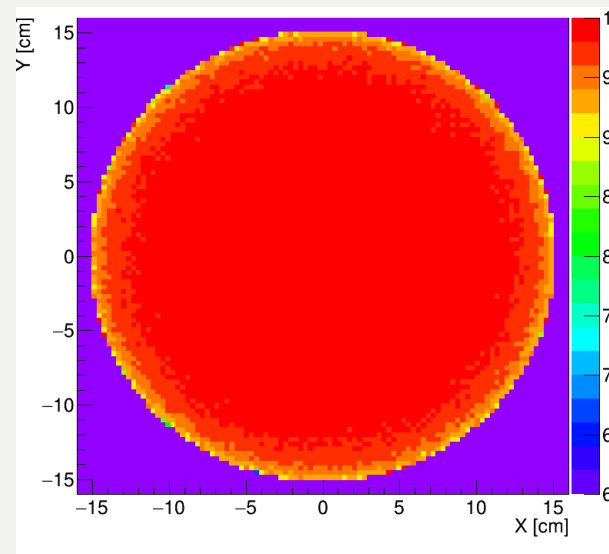


Toy MC

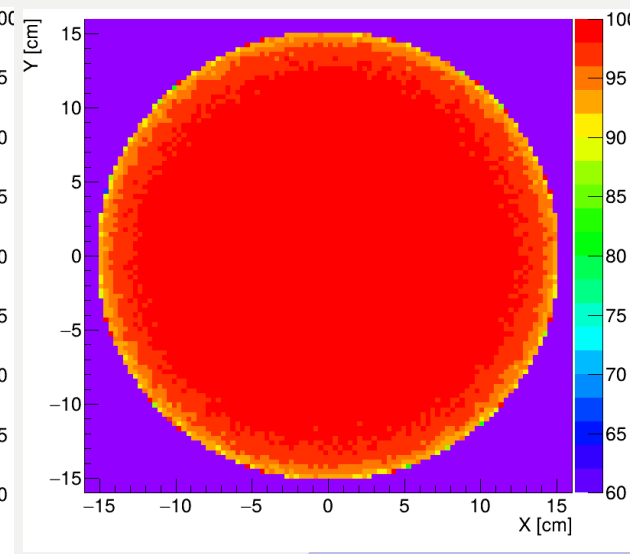


GATE

o-Ps $\rightarrow 3\gamma$



Toy MC



GATE

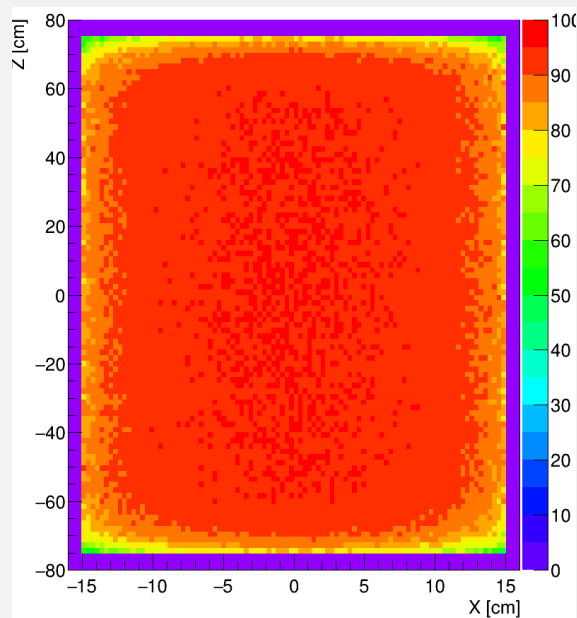
Absorption map – water cylinder – XZ plane

- 10^7 events

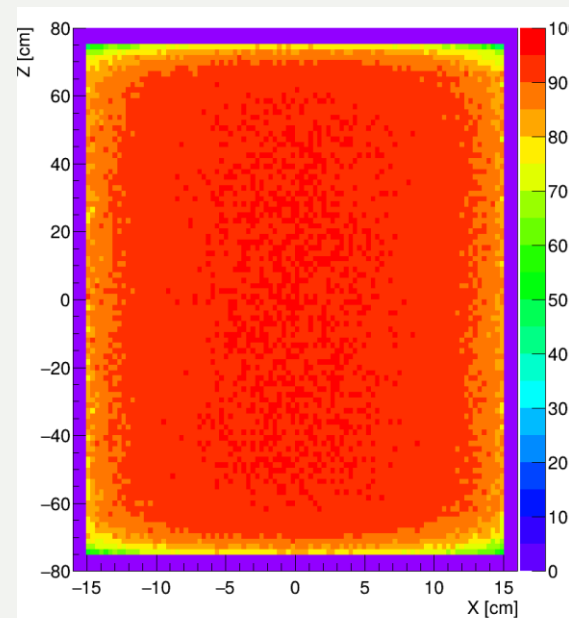
Surviving photon multiplets within the whole model

	Toy MC	GATE
pPs	$7.6063 \pm 0.0087\%$	$7.6176 \pm 0.0087\%$
oPs	$1.6403 \pm 0.0041\%$	$1.6462 \pm 0.0041\%$

p-Ps $\rightarrow 2\gamma$

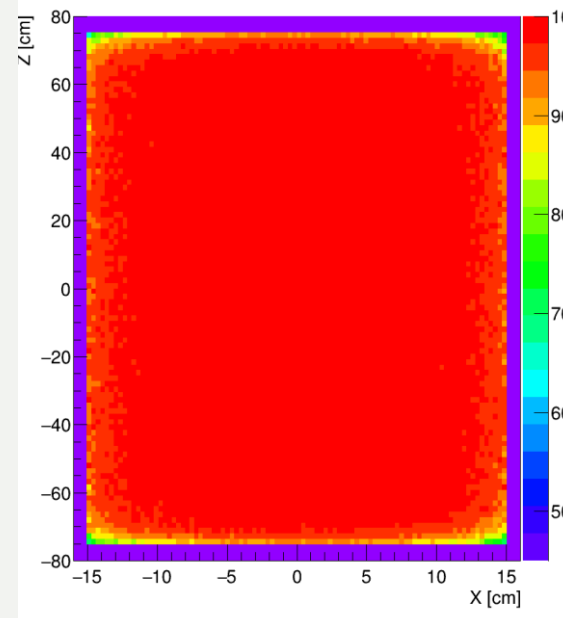


Toy MC

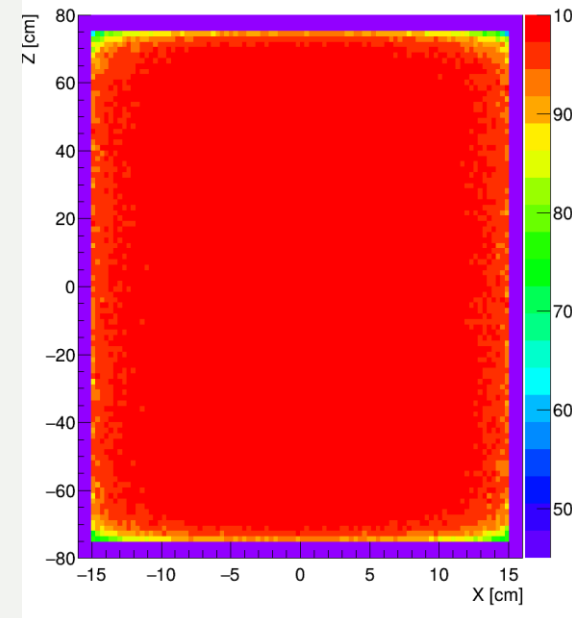


GATE

o-Ps $\rightarrow 3\gamma$

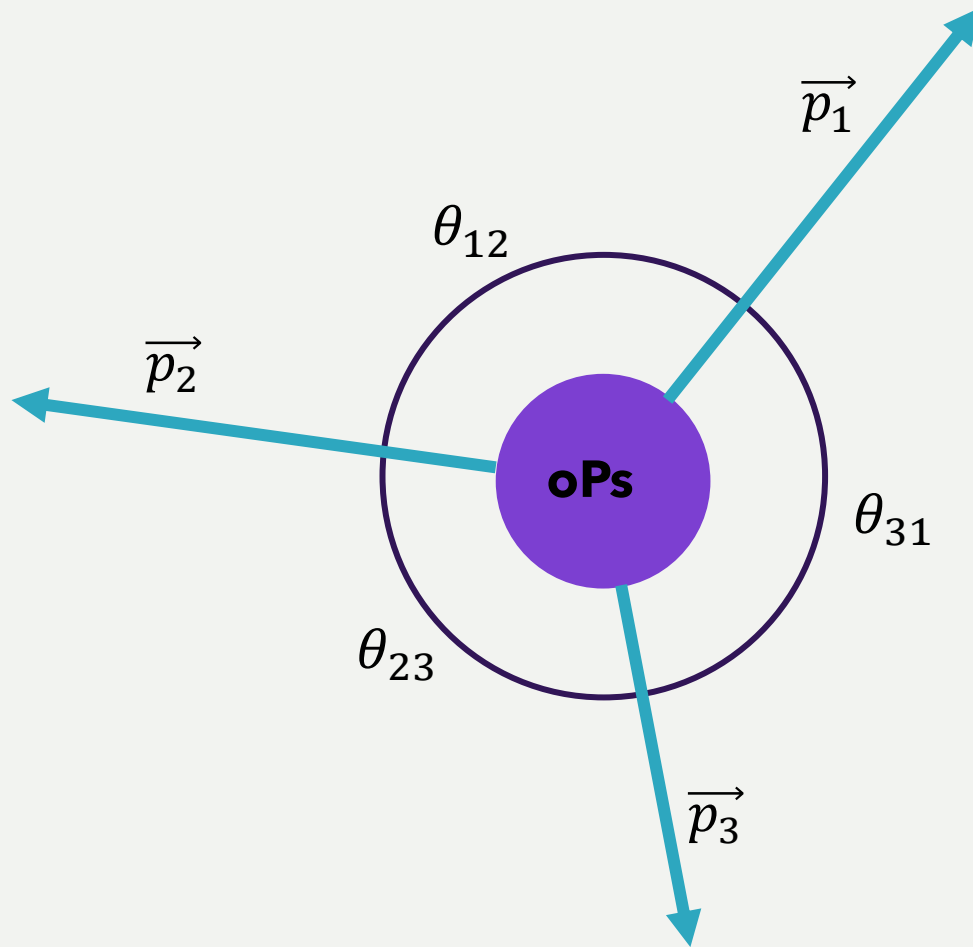


Toy MC



GATE

Simulation of oPs decay



1. Randomizing the energy of photons 1, 2 from the range (0, 511 keV) and assigning:

$$E_3 = M_{Ps} - E_1 - E_2,$$

where $M_{Ps} = 1022$ keV - positronium mass.

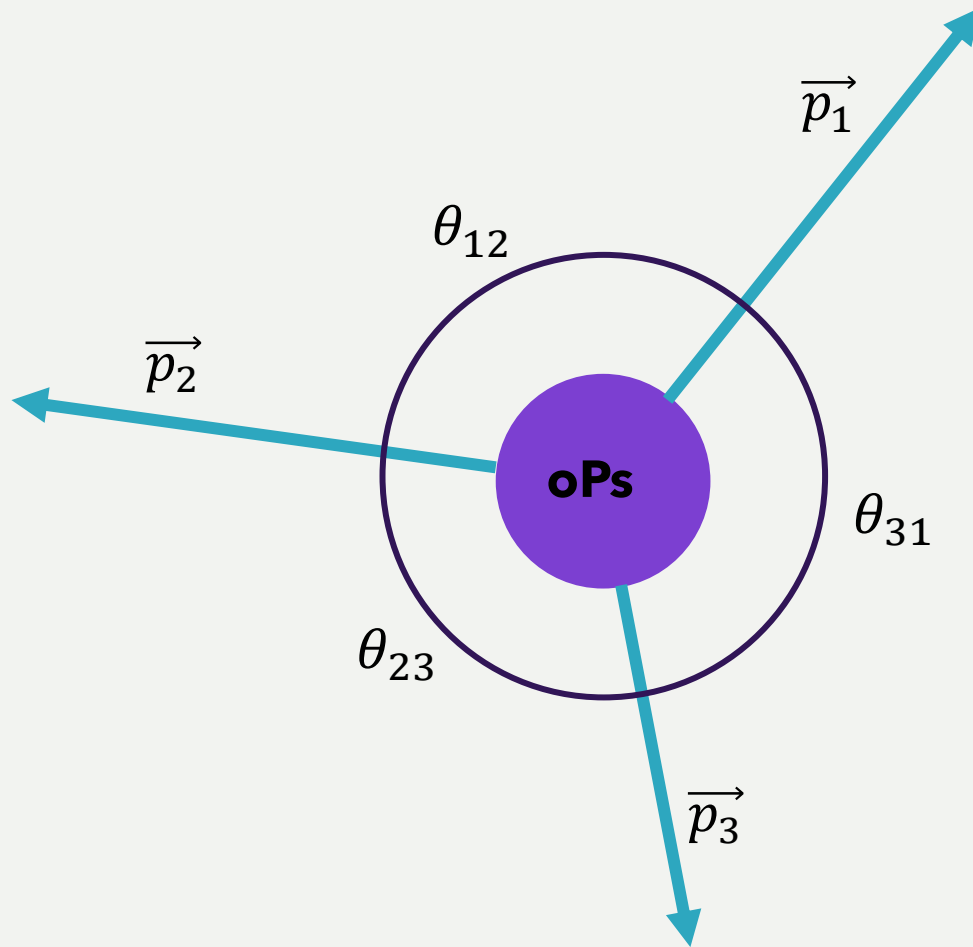
2. Determination of the angles between momentum vectors in plane:

$$\theta_{12} = \arccos\left(\frac{p_3^2 - p_1^2 - p_2^2}{2p_1p_2}\right),$$

$$\theta_{23} = \arccos\left(\frac{p_1^2 - p_2^2 - p_3^2}{2p_2p_3}\right),$$

$$\theta_{31} = \arccos\left(\frac{p_2^2 - p_1^2 - p_3^2}{2p_1p_3}\right).$$

Simulation of oPs decay



3. Determination of the decay plane:

- Rotating $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ at random angle φ relative to the X axis
- Rotating the decay plane using random rotation matrix (M)

4. Monte Carlo hit and miss simulation with the weight specified by the matrix element:

$$M_{oPs \rightarrow 3\gamma} = \left(\frac{m_e - E_1}{E_2 E_3} \right)^2 + \left(\frac{m_e - E_2}{E_1 E_3} \right)^2 + \left(\frac{m_e - E_3}{E_1 E_2} \right)^2$$

Determination of the decay plane:

Generating random rotation matrix using a method based on uniform sampling of unit quaternions:

- Generating three random variables: u_1, u_2, u_3 from a uniform distribution in range $(0, 1)$.
- Constructing a unit quaternion $q = (x, y, z, w)$ using:

$$x = \sqrt{1 - u_1} \cdot \sin(2\pi u_2),$$

$$y = \sqrt{1 - u_1} \cdot \cos(2\pi u_2),$$

$$z = \sqrt{u_1} \cdot \sin(2\pi u_3),$$

$$w = \sqrt{u_1} \cdot \cos(2\pi u_3).$$

- Defining the rotation matrix M :

$$M = \begin{bmatrix} x^2 - y^2 - z^2 + w^2 & 2(xy - zw) & 2(xz + yw) \\ 2(xy + zw) & -x^2 + y^2 - z^2 + w^2 & 2(yz - xw) \\ 2(xz - yw) & 2(yz + xw) & -x^2 - y^2 + z^2 + w^2 \end{bmatrix}$$

