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Strings and Geometry 2025

On the classification of 5d conformal matter SCFTs

Based on past and upcoming work in collaboration with:
Michele Del Zotto, Antoine Bourget, Mario De Marco, Michele Graffeo, Julius Grimminger

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5d $\mathcal{N} = 1$ SCFTs

A rich toolbox to investigate the features of 5d SCFTs, and to propose **classification programs**:

- 5-brane webs
- F-theory on elliptic threefolds + dimensional reduction
- Bottom up-constructions of shrinkable CY3 geometries
- M-theory compactification on CY3

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Countless works by (in no particular order):

Intriligator, Morrison, Seiberg, Vafa, Hanany, Jefferson, S. Katz, H-C Kim, Aharony, Kol, Apruzzi, Lawrie, Lin, Schäfer-Nameki, Bhardwaj, Zafrir, Xie, Del Zotto, Closset, Saxena, Carta, Akhond, Sperling, Hayashi, S-S Kim, Tachikawa, Yonekura, Ohmori, Dwivedi, Benini, Benvenuti, Sacchi, Wang, Mu, Zhang, Shimizu, Giacomelli, Yau, Bourget, Collinucci, Valandro, Acharya, Grimminger, De Marco, Graffeo, Lambert, Santilli, Dierigl, Uhlemann, Najjar, Svanes, Tian, Heckman, Meynet, Moscrop, Hübner, Bergman, Oh, De Wolfe, Iqbal, E. Katz, Rodríguez-Gómez, Carreño Bolla, Franco, Arias-Tamargo, Furrer, Magureanu, Bertolini, Mignosa, Honda, Tizzano, Benetti-Genolini, Lee, Taki, Yagi, van Beest, Eckhard, Cabrera, Gaiotto, Witten, Cremonesi, Albertini, García-Etxebarria, Hosseini, Willett, Cvetiç, Torres…

Partial atomic classification of 5d SCFTs

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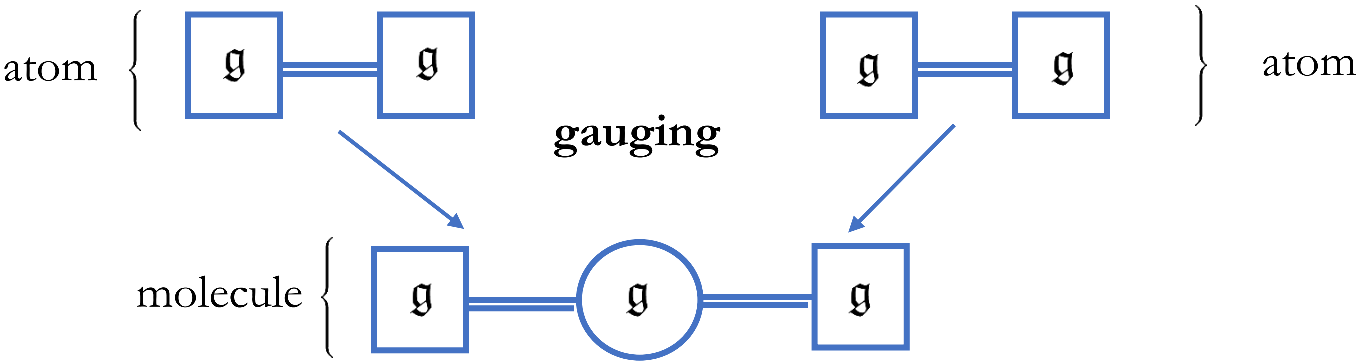
- Characterize the "chemistry" (i.e. the fusion properties) of atoms → form more general 5d SCFTs (**molecules**)



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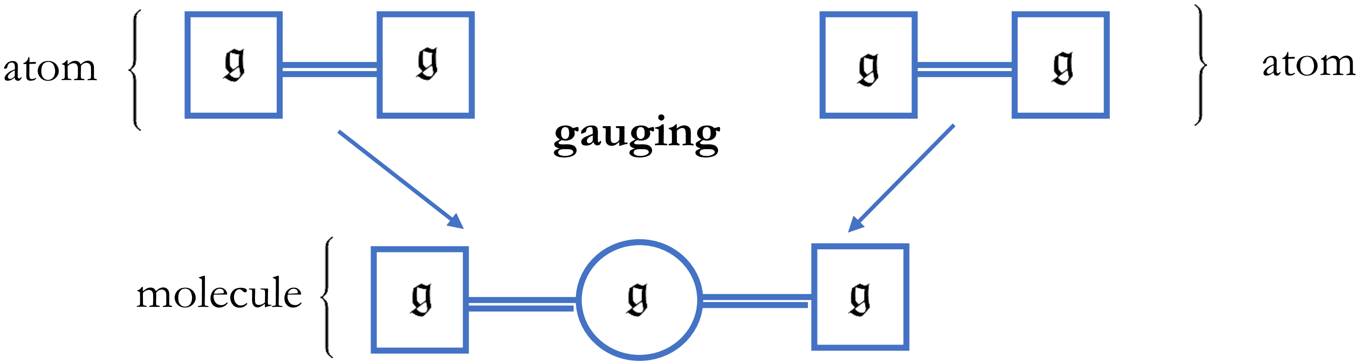

atom $\{\ \mathfrak{g} = \mathfrak{g}$

$\mathfrak{g} = \mathfrak{g}\ \}$ atom

gauging

molecule $\{\ \mathfrak{g} = \mathfrak{g} = \mathfrak{g}$

- Notice: this approach has been extremely successful in the classification of 6d $\mathcal{N}=(1,0)$ SCFTs [Del Zotto, Heckman, Tomasiello, Vafa '14], [Heckman, Morrison, Vafa '14], [Heckman, Morrison, Rudelius, Vafa '15], [Bhardwaj '19]

Summary of results

- Results: (partial) atomic classification of 5d “**conformal matter**” $\mathcal{N}=1$ SCFTs

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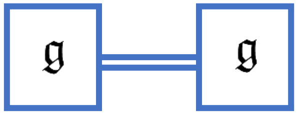
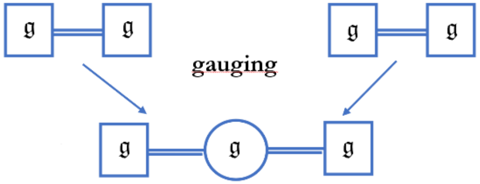
I. **Bifundamental** Conformal Matter theories



$\mathfrak{g} \in ADE$

  - we **identify** and **classify** a finite set of atoms, that can be gauged together to form more complicated molecules [De Marco, Del Zotto, Graffeo, AS, 23], [Bourget, De Marco, Del Zotto, Grimminger, AS, 25]
  - we provide a coherent picture of their Higgs branch, connecting with class-S constructions and magnetic quiver techniques [De Marco, Del Zotto, Grimminger, AS, 25]



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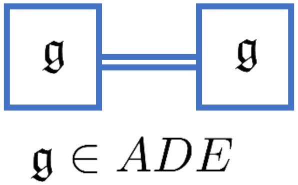

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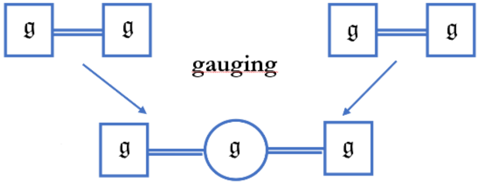
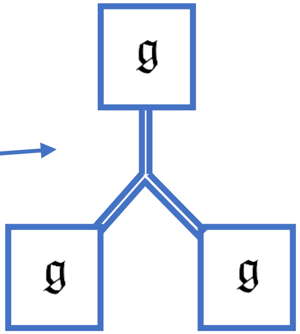


II. **Trinions** and **Tetraons** Conformal matter theories



  - Trinions of type A admit a (well-known) atomic classification
  - A new balancing condition on generalized quivers shows the natural appearance of **trinions** and **tetraons** of type D

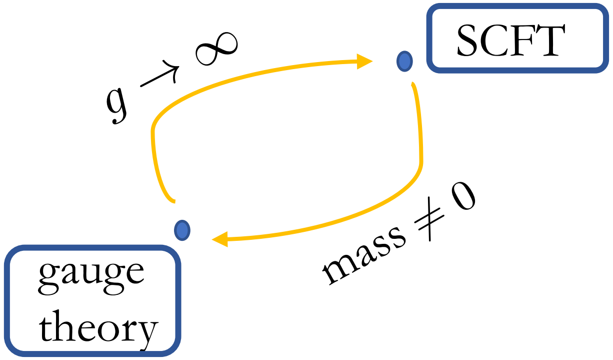
    ⟶ caveat: they are not irreducible, hence not atoms

[De Marco, Del Zotto, Graffeo, AS, 25]

5d $\mathcal{N} = 1$ SCFTs

- Very difficult to treat from a gauge-theoretic perspective:

In 5d: $\left[\frac{1}{g^2}\right] = \text{mass}$



SCFT

$g \to \infty$

mass $\neq 0$

gauge theory

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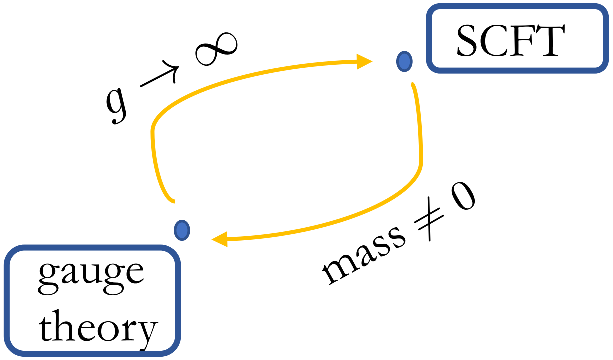
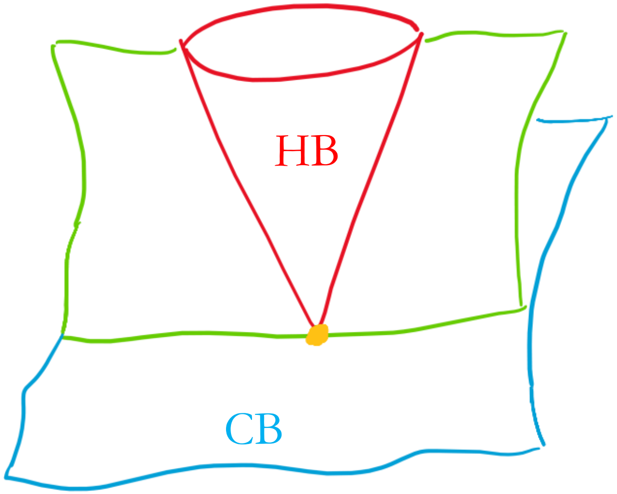


Since the mid-ninenties [Seiberg '96], [Morrison, Seiberg '96][Intriligator, Morrison, Seiberg '97], thriving effort to study of the properties of 5d SCFTs via String-/M-/F-theoretic methods.



Data to characterize the fixed points:

- **Moduli spaces** of vacua
- **RG flows**
- **Symmetries** (gauge, flavor, categorical symmetries…)

Geometric engineering of 5d SCFTs

construct 5d SCFTs employing M-theoretic setups on non-compact CY3

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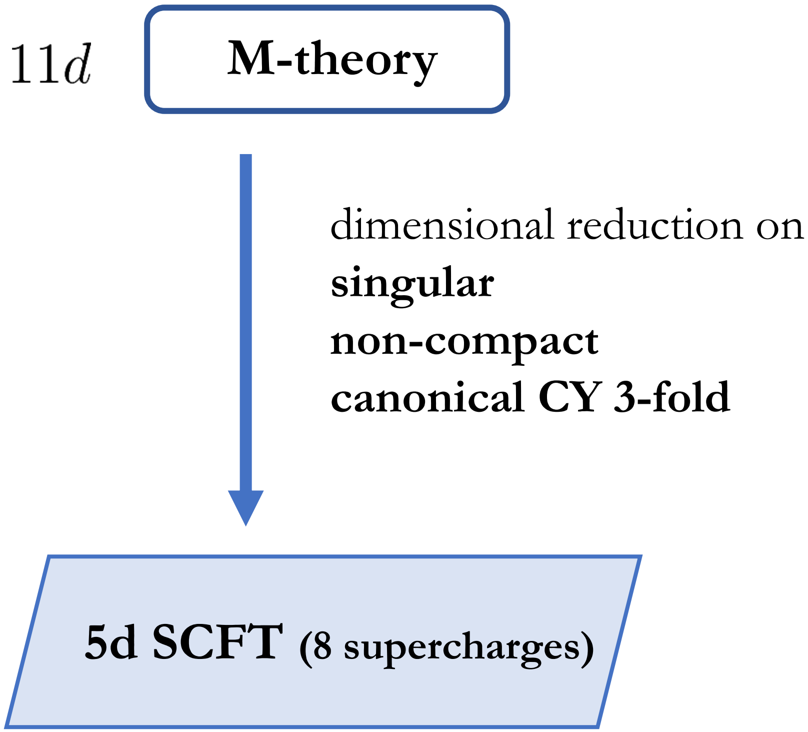


$11d$ **M-theory**

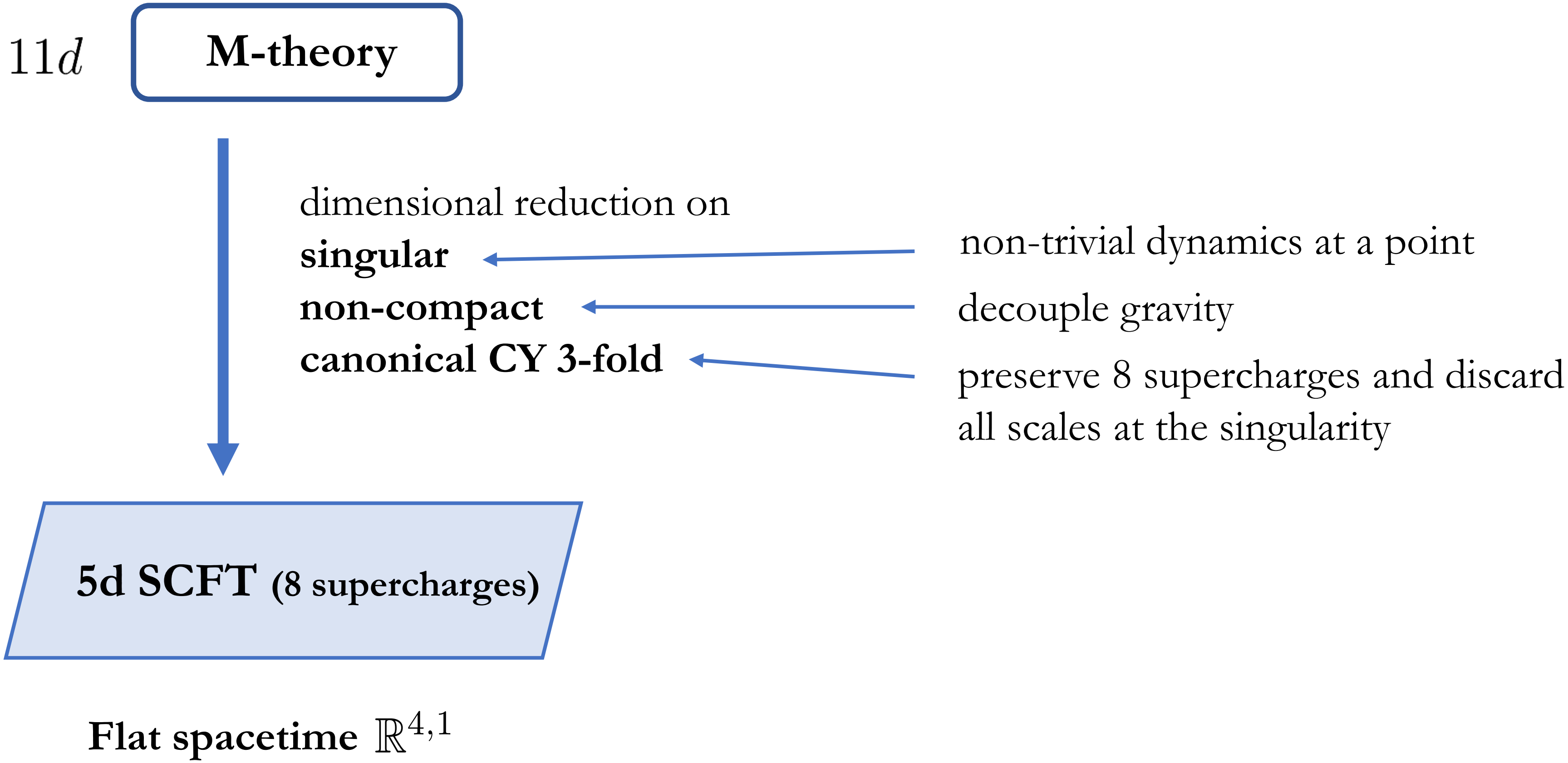
dimensional reduction on
singular
non-compact
canonical CY 3-fold

5d SCFT (8 supercharges)

Flat spacetime $\mathbb{R}^{4,1}$

Geometric engineering of 5d SCFTs

construct 5d SCFTs employing M-theoretic setups on non-compact CY3



$11d$ **M-theory**

dimensional reduction on

singular ← non-trivial dynamics at a point

non-compact ← decouple gravity

canonical CY 3-fold ← preserve 8 supercharges and discard all scales at the singularity

5d SCFT **(8 supercharges)**

Flat spacetime $\mathbb{R}^{4,1}$

M-theory/5d SCFTs dictionary

DICTIONARY

5d SCFT	**CY threefold X**
Coulomb branch parameters	Compact divisors ⟷ $H_4(\tilde{X},\mathbb{R})$
Mass parameters	Non-compact divisors ⟷ $H_2(\tilde{X},\mathbb{R}) - H_4(\tilde{X},\mathbb{R})$
Higgs branch parameters	Normalizable complex structure deformations ⟷ $H_3(\hat{X},\mathbb{R})$

($\tilde{X}$ is the resolved CY3, $\hat{X}$ is the deformed CY3)

Bifundamental 5d CM SCFTs

- **Motivating question:** find analogue of 6d conformal matter (bifundamental matter with non-trivial gauge dynamics) in 5d setting

UV flavor symmetry $\supseteq \mathfrak{g} \times \mathfrak{g}, \quad \mathfrak{g} \in ADE$

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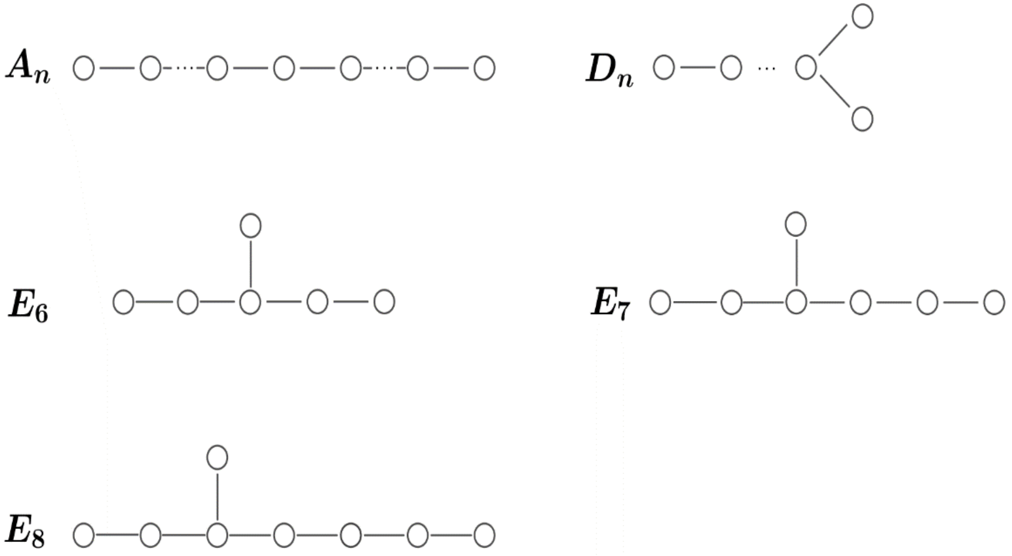
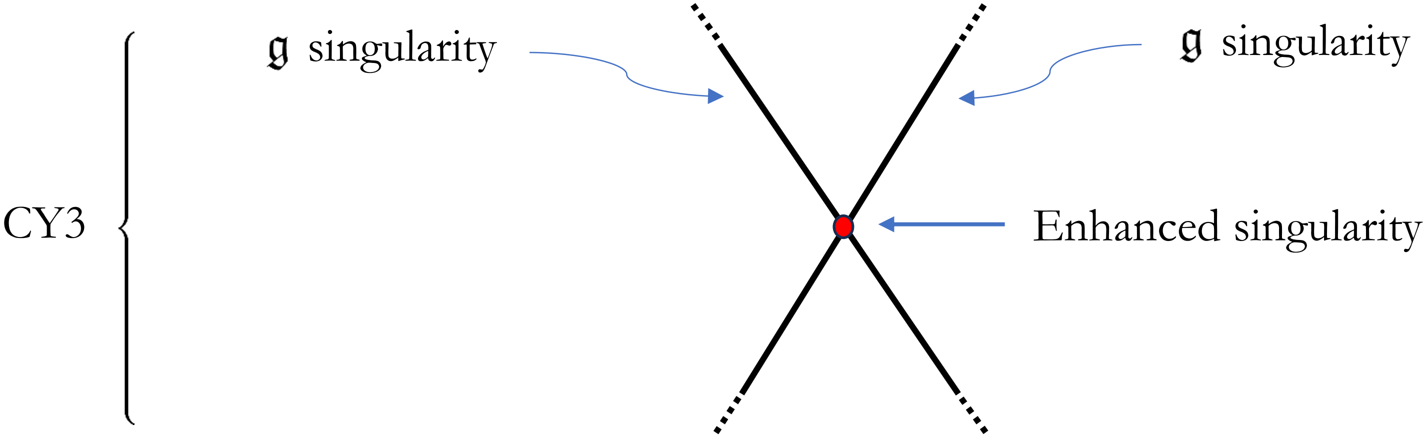
- **Geometric answer:** **5d conformal matter** can be constructed from M-theory geometric engineering

$$\begin{cases} \boldsymbol{A_n}: & x^2+y^2+z^{n+1}=0 \\ \boldsymbol{D_n}: & x^2+zy^2+z^{n-1}=0 \\ \boldsymbol{E_6}: & x^2+y^3+z^4=0 \\ \boldsymbol{E_7}: & x^2+y^3+yz^3=0 \\ \boldsymbol{E_8}: & x^2+y^3+z^5=0 \end{cases}$$



Atoms of bifundamental 5d CM SCFTs



$\mathfrak{g}$ singularity

$\mathfrak{g}$ singularity

CY3

Enhanced singularity

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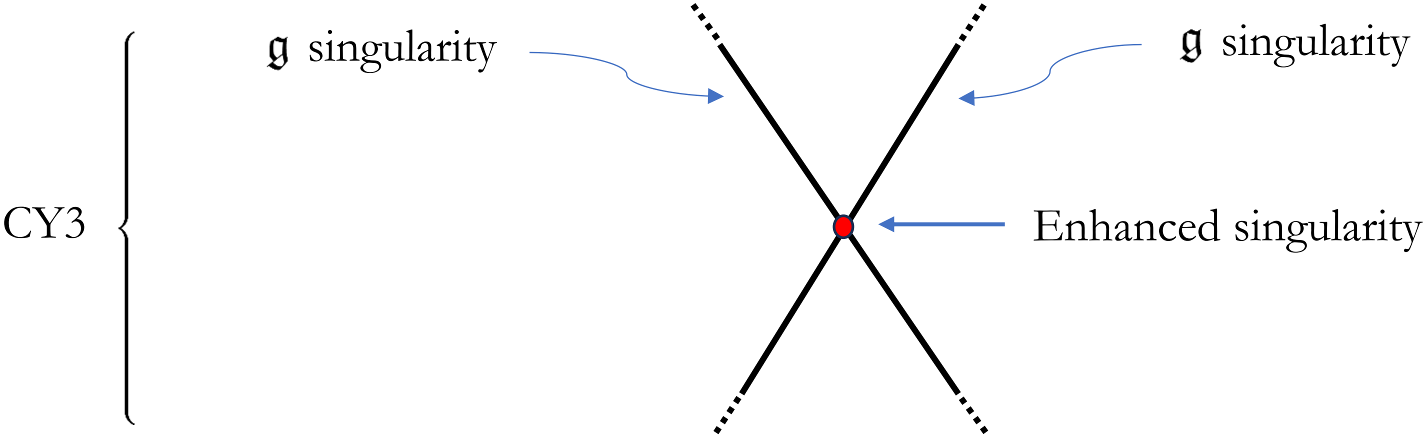

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- **Example:**

5d conformal matter $\boldsymbol{(E_6, E_6)}$ can be built from M-theory geometric engineering on:

$$\textbf{CY3:} \begin{cases} x^2 + y^3 + z^4 = 0 \\ x = uv \end{cases}$$

base change

E_6 singularity

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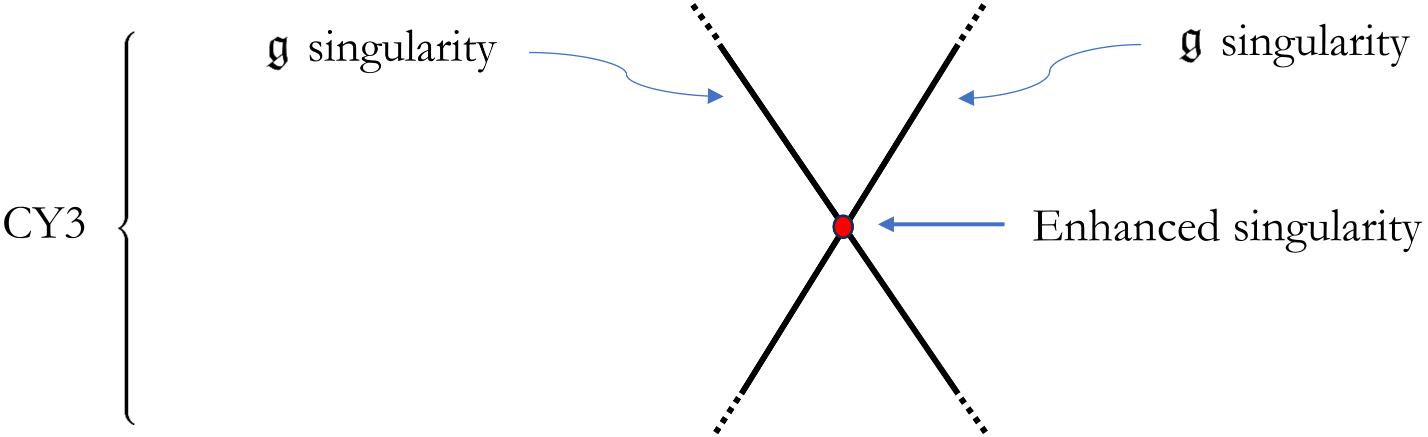

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Singular lines:
$$u = y = z = 0 \quad E_6 \text{ type}$$
$$v = y = z = 0 \quad E_6 \text{ type}$$

Intersection: $u = v = y = z = 0$

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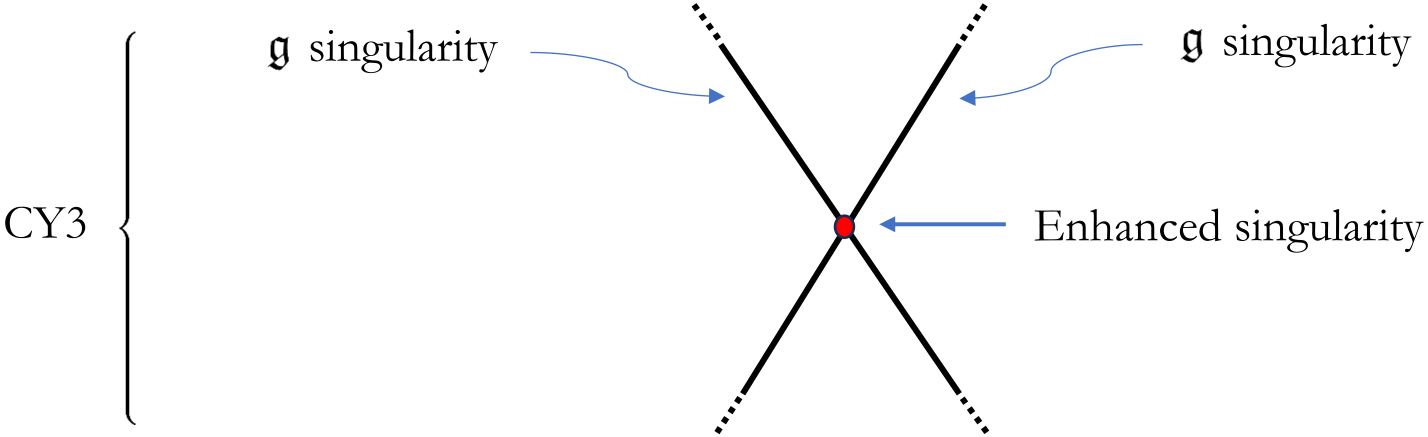

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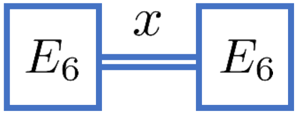

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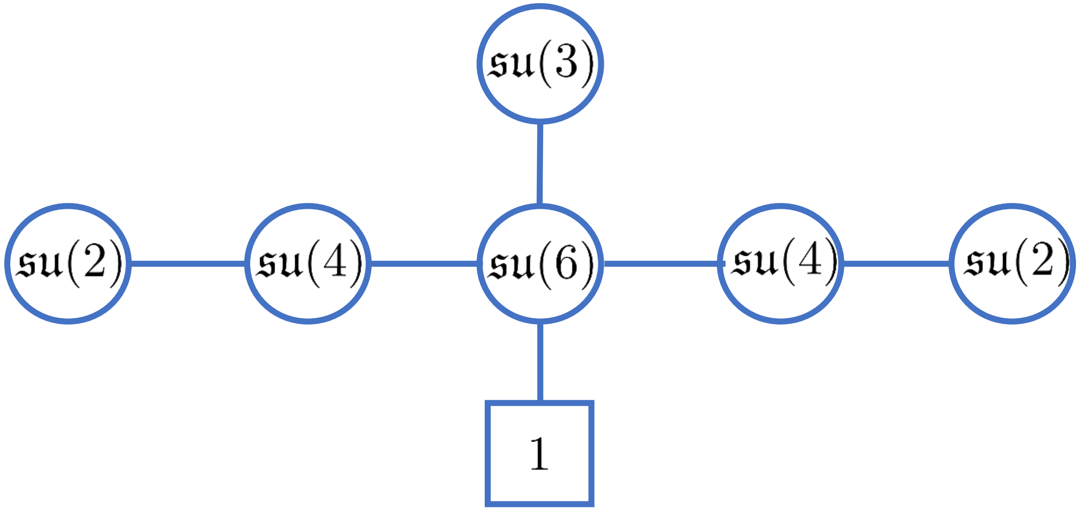
- Resolution: **low-energy quiver gauge theory phase**



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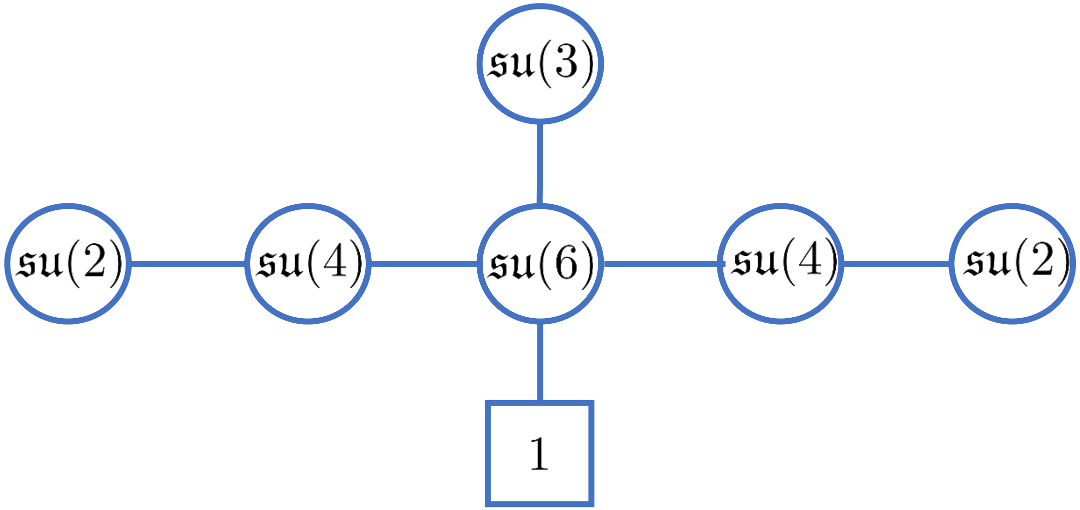

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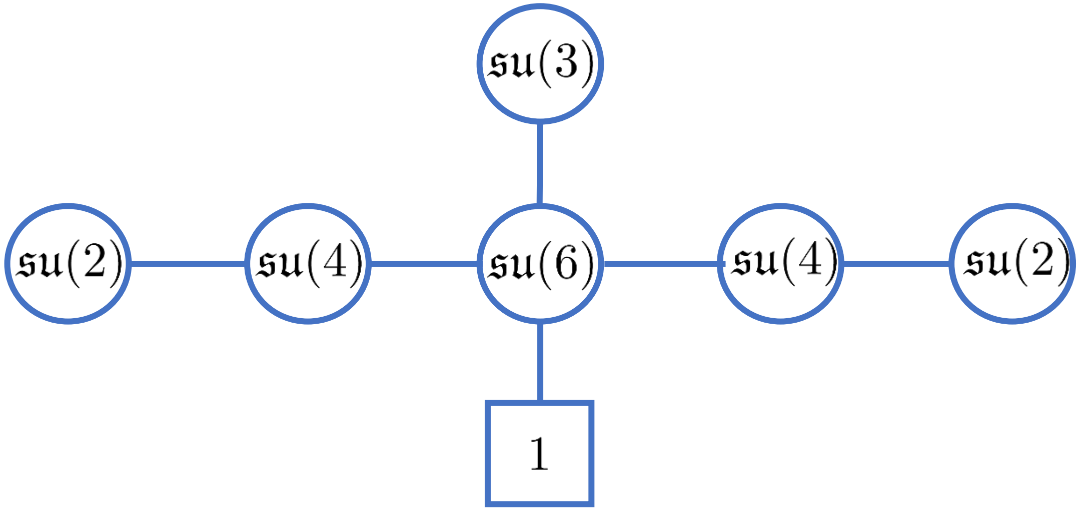

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The quiver above has E_6 shape and it is balanced → the fixed point has at least $E_6 \times E_6$ flavor symmetry

→ Independent **field-theoretic check** that the M-theory geometry gives rise to the expected flavor symmetry

(partial) Atomic Classification of 5d CM SCFTs

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5d conformal matter $(\boldsymbol{E_6}, \boldsymbol{E_6})$ can be built from M-theory geometric engineering on:

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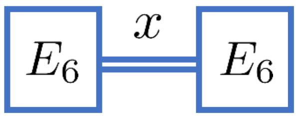

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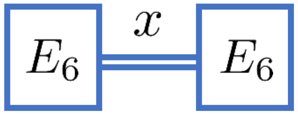

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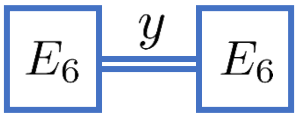

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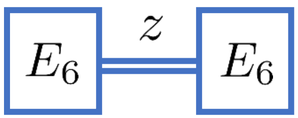

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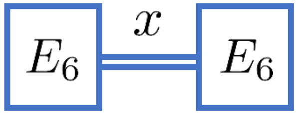

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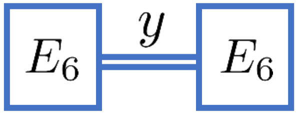

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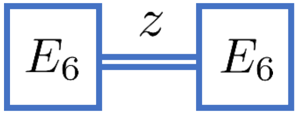

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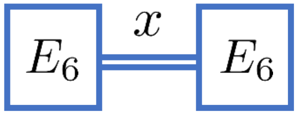

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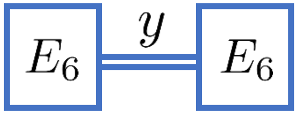

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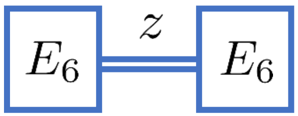

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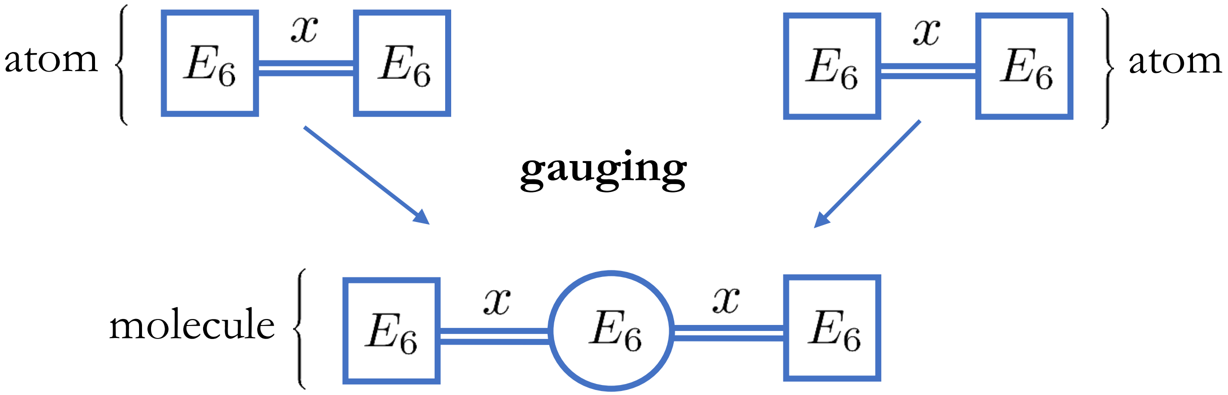
- All these theories exhibit at least a $E_6 \times E_6$ factor in the UV flavor symmetry
- We can employ the same technology to identify bifundamental atoms for all ADE algebras

Molecules of 5d CM SCFTs

- **5d conformal matter atoms** can be used as fundamental blocks to construct more complicated SCFTs of conformal matter type



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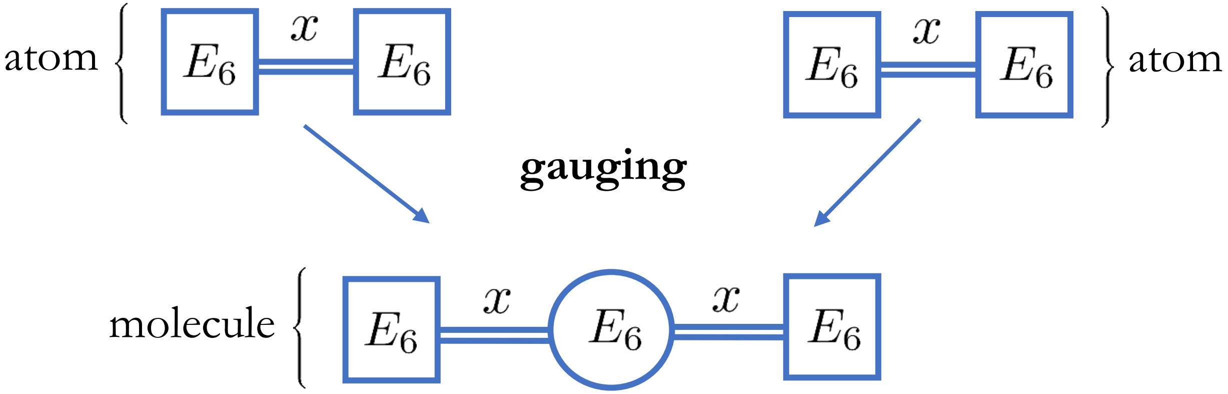

- **Geometric counterpart:**

CY3:
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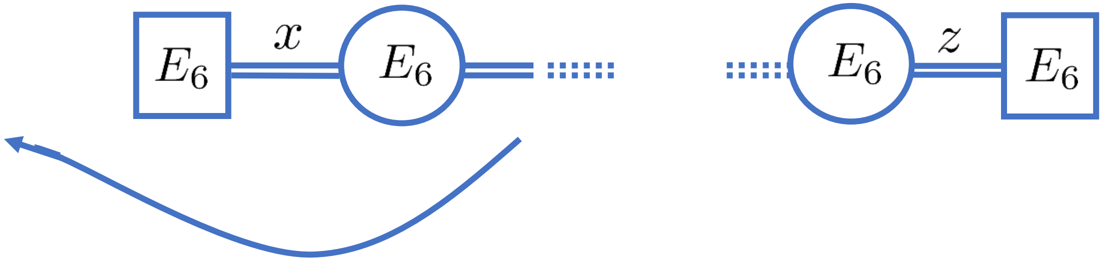
- **Most general molecule:**

CY3:
$$\begin{cases} x^2 + y^3 + z^4 = 0 \\ x^{n_1} y^{n_2} z^{n_3} = uv \end{cases}$$



contracting all $\mathbb{P}^1$'s

Novel 5d UV dualities

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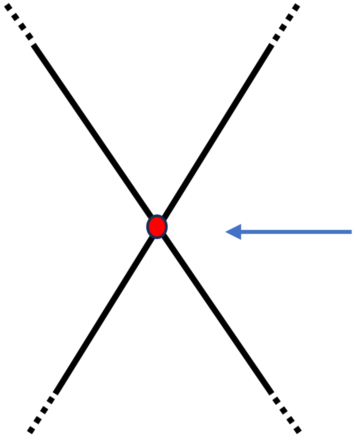

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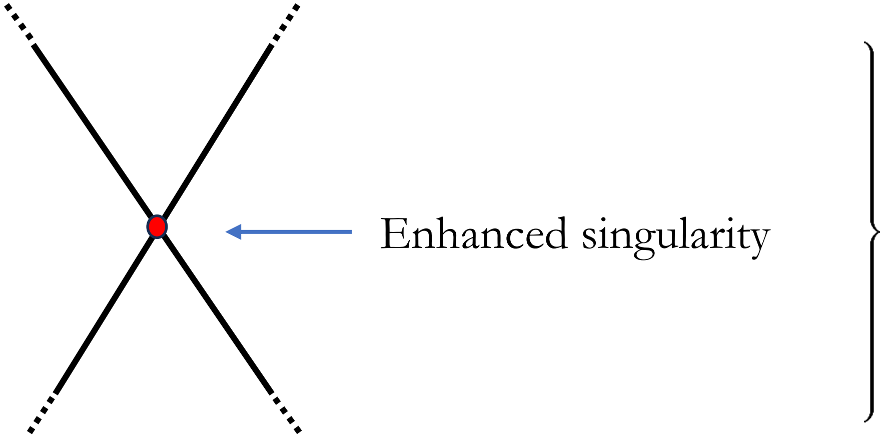
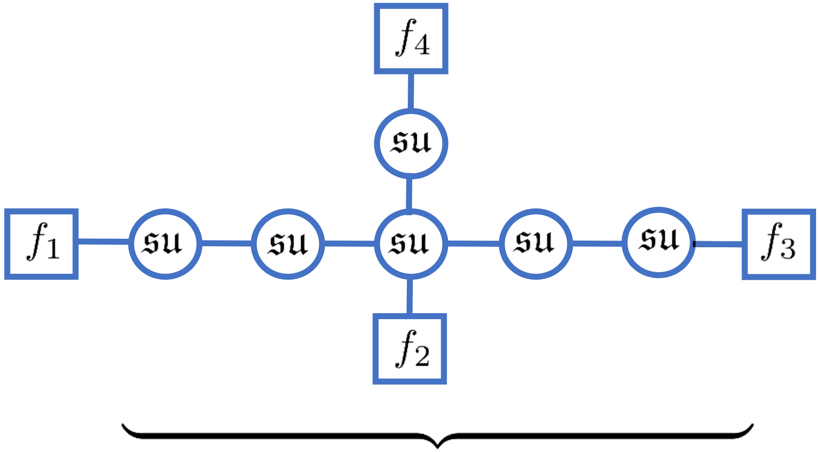


Enhanced singularity

SCFT phase

Resolution



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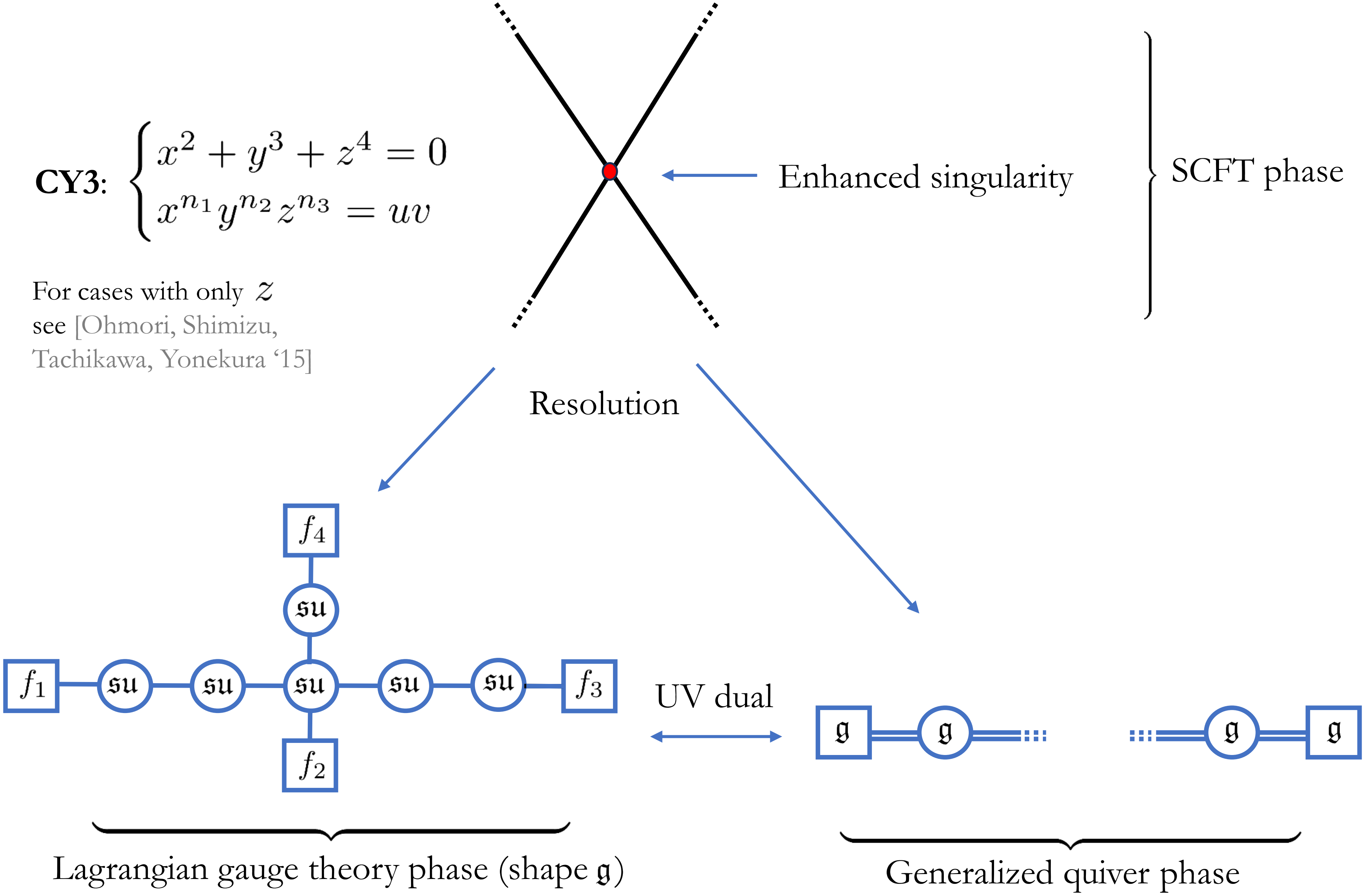

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Generalized quiver phase

On the classification of 5d bifundamental CM

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- **Surprising aspect:** there exist 5d CM SCFTs which are neither atoms nor molecules (i.e. they are not gaugings of atoms, but they can be Higgsed to molecules)
  → we call these theories 5d CM **hybrids**

We obtain a full classification of atoms and hybrids for all ADE algebras

(partial) classification of 5d bifundamental CM

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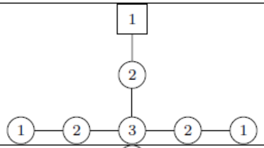
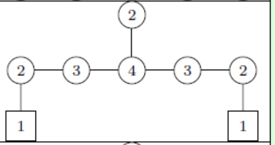
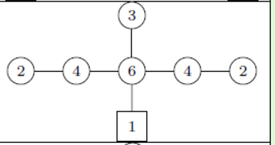
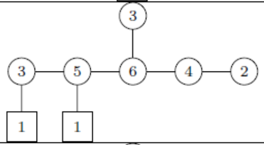
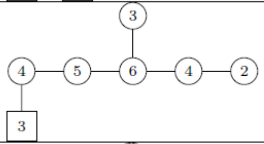
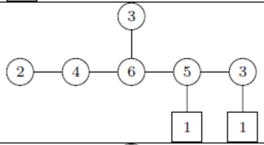
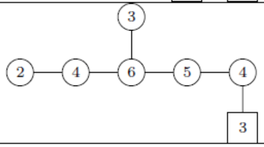
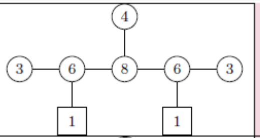
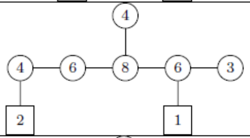
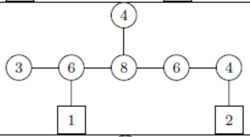
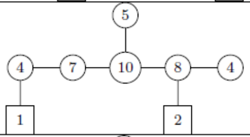
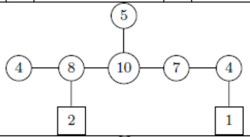
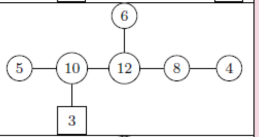
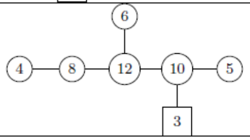
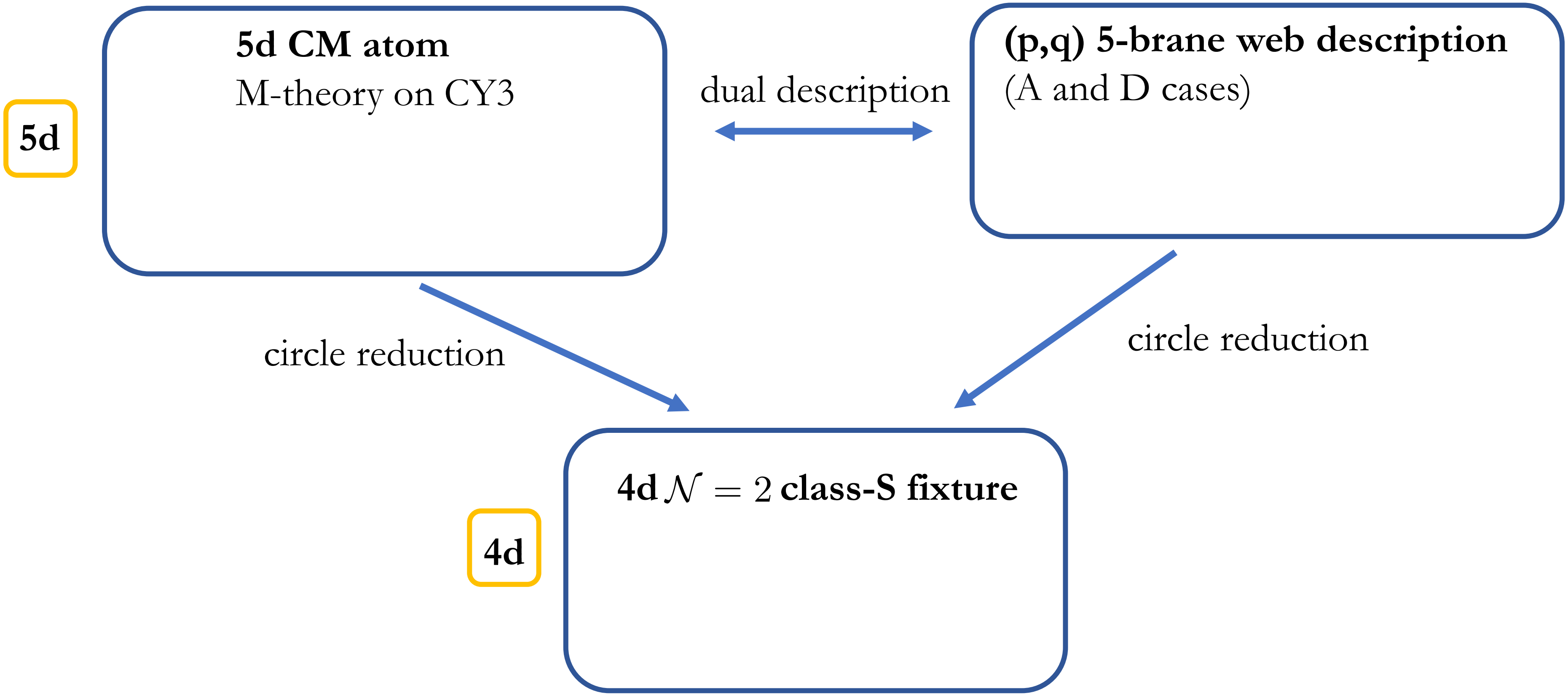
Example:

(E_6, E_6) 5d CM

Coweight	Dynkin Quiver	Type	CY3
$\begin{bmatrix} & & 1 & & \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$		Atom	$z = uv$
$\begin{bmatrix} & & 0 & & \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$		Atom	$y = uv$
$\begin{bmatrix} & & 0 & & \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$		Atom	$x = uv$
$\begin{bmatrix} & & 0 & & \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$		Atom	$x + iz^2 + cyz = uv$
$\begin{bmatrix} & & 0 & & \\ 3 & 0 & 0 & 0 & 0 \end{bmatrix}$		Atom	$x + iz^2 = uv$
$\begin{bmatrix} & & 0 & & \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$		Atom	$x - iz^2 + cyz = uv$
$\begin{bmatrix} & & 0 & & \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$		Atom	$x - iz^2 = uv$

Coweight	Dynkin Quiver	Type	CY3
$\begin{bmatrix} & & 0 & & \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$		Hybrid	$z^3 + cy^2 = uv$
$\begin{bmatrix} & & 0 & & \\ 2 & 0 & 0 & 1 & 0 \end{bmatrix}$		Hybrid	$z\left(x + iz^2\right) + cy^2 = uv$
$\begin{bmatrix} & & 0 & & \\ 0 & 1 & 0 & 0 & 2 \end{bmatrix}$		Hybrid	$z\left(x - iz^2\right) + cy^2 = uv$
$\begin{bmatrix} & & 0 & & \\ 1 & 0 & 0 & 2 & 0 \end{bmatrix}$		Hybrid	$y\left(x - iz^2\right) + cx^2 = uv$
$\begin{bmatrix} & & 0 & & \\ 0 & 2 & 0 & 0 & 1 \end{bmatrix}$		Hybrid	$y\left(x + iz^2\right) + cx^2 = uv$
$\begin{bmatrix} & & 0 & & \\ 0 & 3 & 0 & 0 & 0 \end{bmatrix}$		Hybrid	$\left(x + iz^2\right)^2 + cz^5 = uv$
$\begin{bmatrix} & & 0 & & \\ 0 & 0 & 0 & 3 & 0 \end{bmatrix}$		Hybrid	$\left(x - iz^2\right)^2 + cz^5 = uv$

What about the Higgs branch? (atoms)



5d

5d CM atom

M-theory on CY3

dual description

(p,q) 5-brane web description

(A and D cases)

circle reduction

circle reduction

4d

4d $\mathcal{N}=2$ class-S fixture

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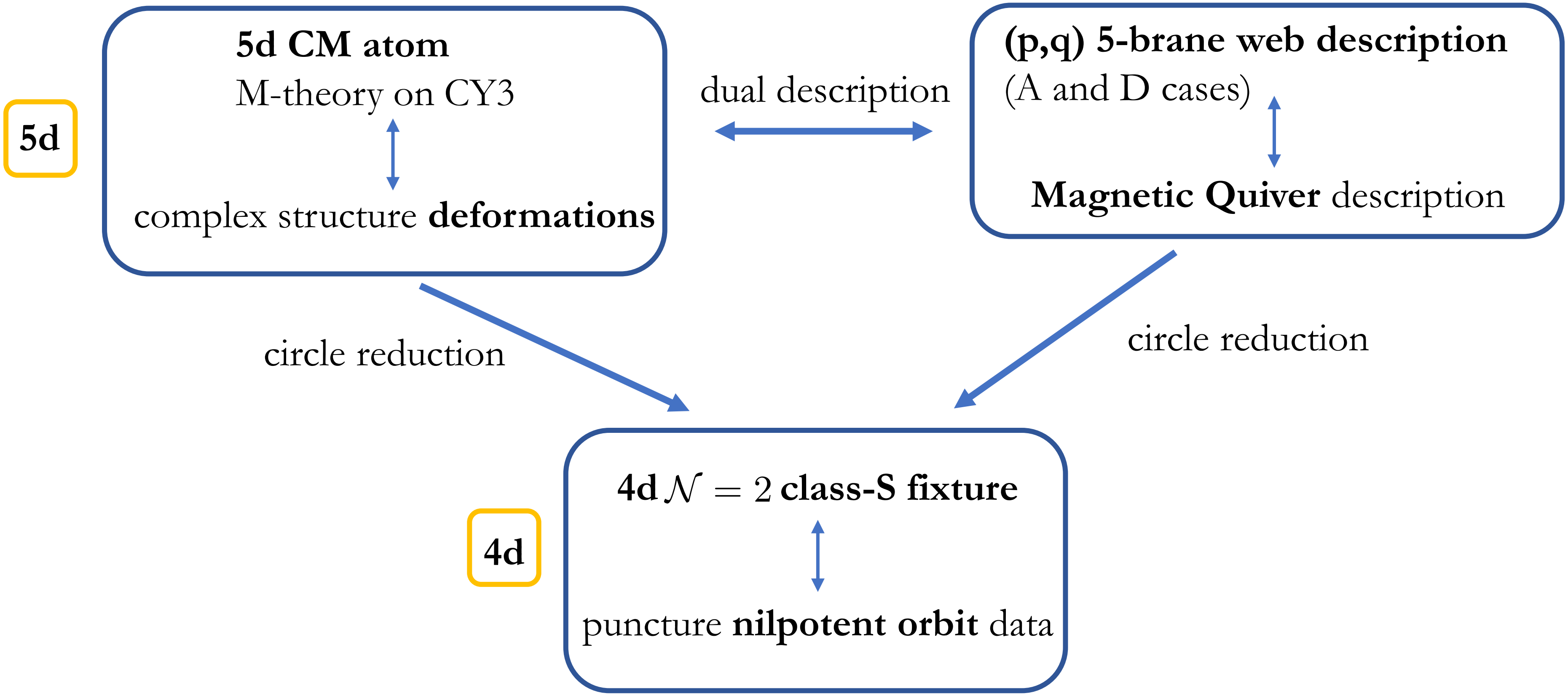

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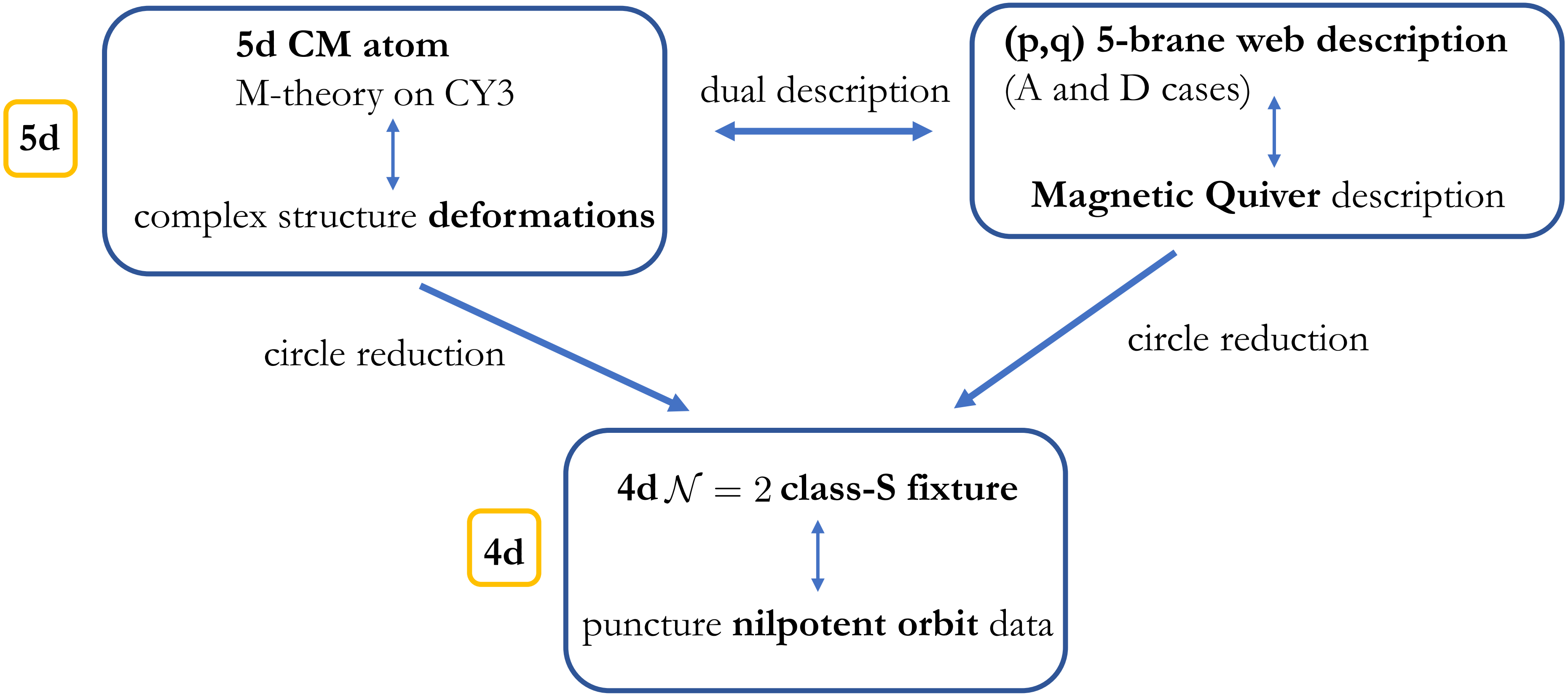

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For **5d CM atom SCFTs** all these techniques perfectly agree, yielding the dimension of the Higgs branch at the UV fixed point

What about the Higgs branch? (molecules)

5d

5d CM molecule
M-theory on CY3
↕
complex structure **deformations**

dual description ↔

(p,q) 5-brane web description
(A and D cases)
↕
Magnetic Quiver description

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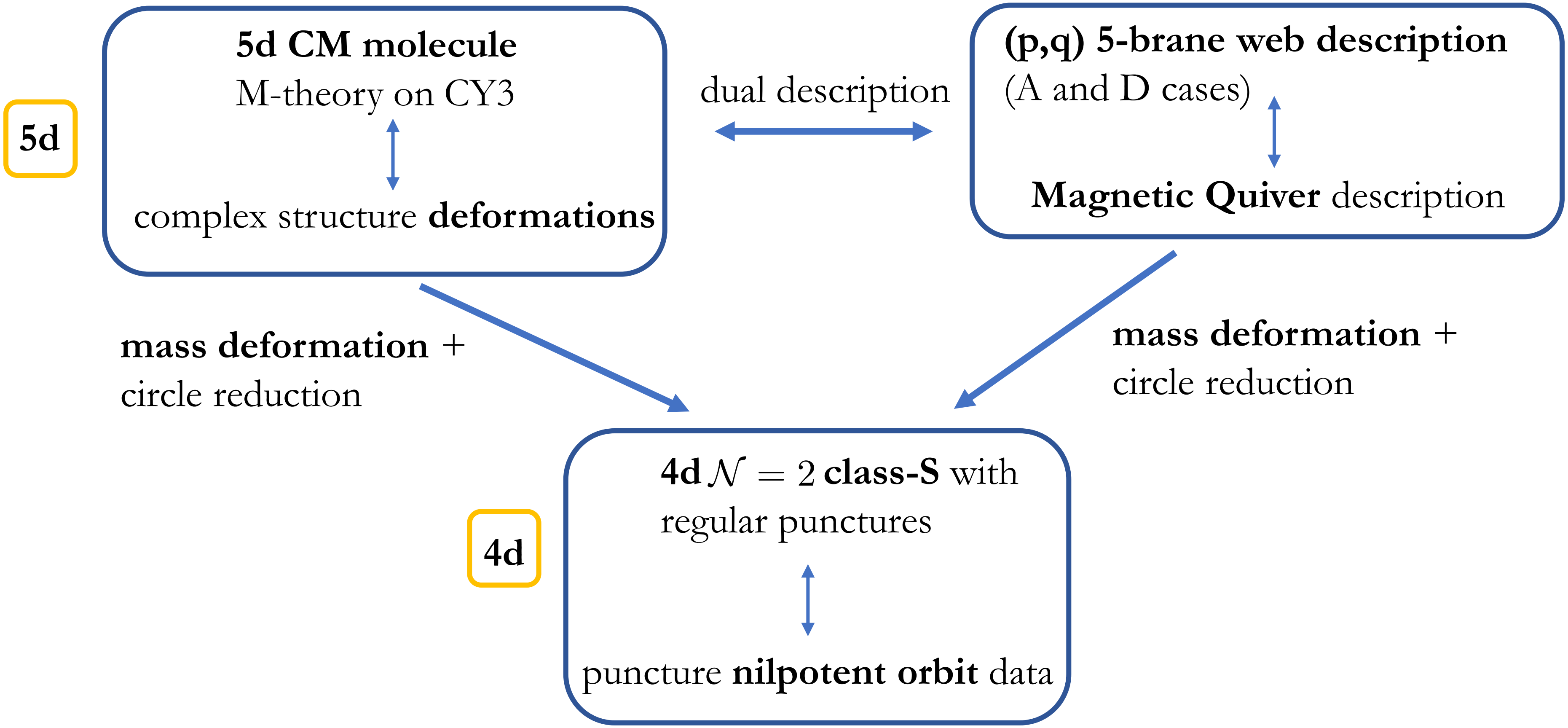

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mass deformation +
circle reduction

mass deformation +
circle reduction

4d

4d $\mathcal{N}=2$ **class-S** with
regular punctures
↕
puncture **nilpotent orbit** data

- For **5d CM molecules SCFTs** the magnetic quiver and CY3 perspective perfectly agree
- The HB from the class-S phase undergoes enhancement at the UV fixed point

(See [Ohmori, Shimizu, Tachikawa, Yonekura ‘15])

6d origin of bifundamental 5d CM SCFTs

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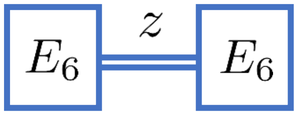

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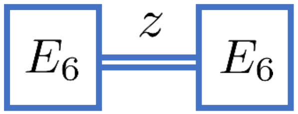

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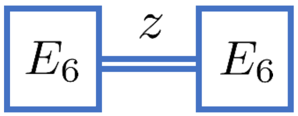

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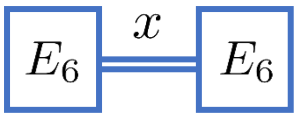

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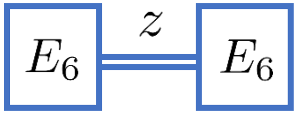

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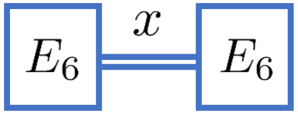

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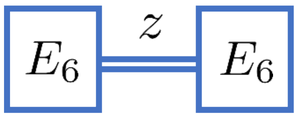

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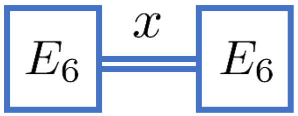

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$$\begin{cases} x^2+y^3+z^4=0 \\ z^2=uv \end{cases} \xrightarrow{\text{decompactify and deform}} \begin{cases} x^2+y^3+z^4=0 \\ z^2+cx=uv \end{cases} \xrightarrow{\text{discard irrelevant terms}} \begin{cases} x^2+y^3+z^4=0 \\ x=uv \end{cases}$$

Extending the (partial) atomic classification

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- **So far:** partial classification of 5d **bifundamental** CM SCFTs

Sketch of the state of the art:

6d $\mathcal{N} = (1,0)$ SCFTs: (almost) complete classification in terms of **bifundamental** theories

[Del Zotto, Heckman, Tomasiello, Vafa ’14], [Heckman, Morrison, Vafa ‘14], [Heckman, Morrison, Rudelius, Vafa ‘15], [Bhardwaj ‘19]

4d $\mathcal{N} = 2$ SCFTs: partial classification in terms of **trinions** (fixtures) + exotics

[Chacaltana, Distler ’10, ‘11], [Chacaltana, Distler, Tachikawa ’12, ‘12], [Chacaltana, Distler, Trimm ’13, ‘15], [Chacaltana, Distler, Zhu ’17, ‘18], [Argyres, Douglas ‘95] [Argyres, Lotito, Lü, Martone ‘15]

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5d $\mathcal{N} = 1$ SCFTs: partial atomic classification of **bifundamental** theories. What about **trinions/tetraons/**…? Do they exhibit a similar "atomic" structure?

Short answer: for case A → atomic classification (well-known)
for case D → trinions and tetraons exist, but they are not irreducible (**new UV flavor enhancements**)

4d $\mathcal{N} = 2$ SCFTs: partial classification in terms of **trinions** (fixtures) + exotics
[Chacaltana, Distler '10, '11], [Chacaltana, Distler, Tachikawa '12, '12], [Chacaltana, Distler, Trimm '13, '15], [Chacaltana, Distler, Zhu '17, '18], [Argyres, Douglas '95] [Argyres, Lotito, Lü, Martone '15]

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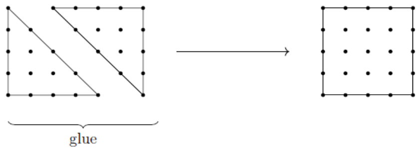
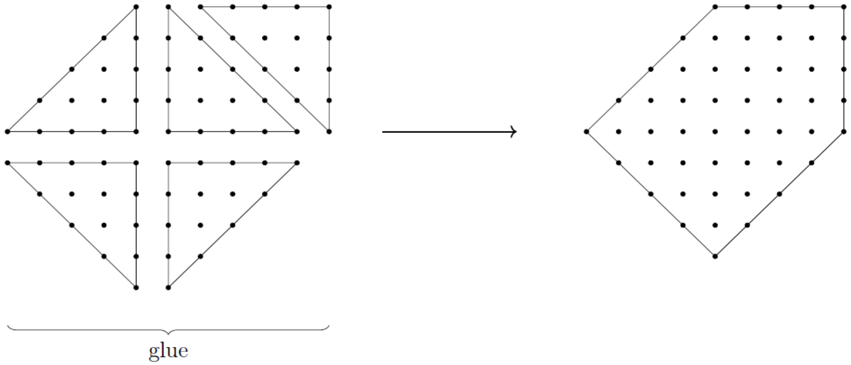


UV flavor $\supseteq A^{\oplus 4}$



UV flavor $\supseteq A^{\oplus 5}$

- See also [Katz, Vafa ‘96] for rank-0 constructions

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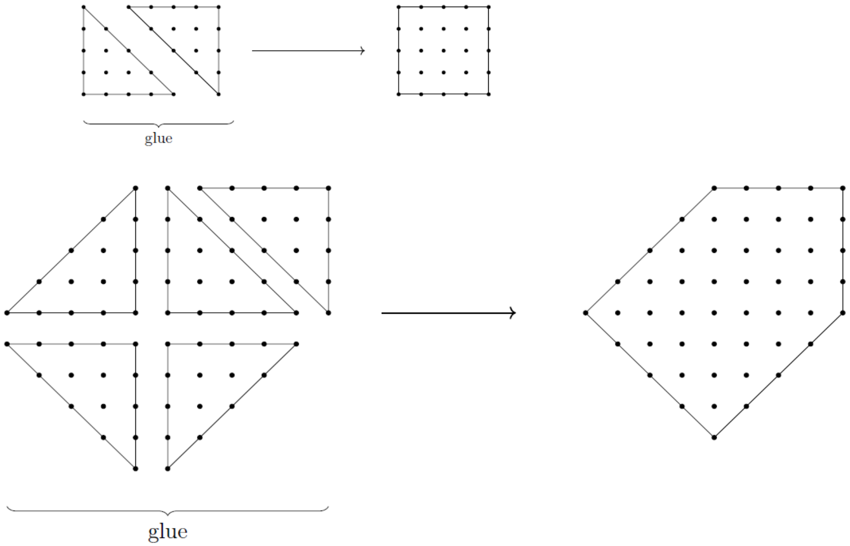
- It is well-known that 5d SCFTs with plenty of A UV flavor factors exist
- The building blocks are $\mathbb{C}^3/(\mathbb{Z}_p \times \mathbb{Z}_q)$ [Benini, Benvenuti, Tachikawa '09]: they can be gauged together to form interacting 5d SCFTs with an arbitrary number of A lines



$\text{UV flavor} \supseteq A^{\oplus 4}$

$\text{UV flavor} \supseteq A^{\oplus 5}$

- See also [Katz, Vafa '96] for rank-0 constructions
- Punchline: the philosophy of the atomic classification holds for the A trinions

5d CM trinions and tetraons

- Claim: there exist **trinion** and **tetraon** 5d SCFTs with UV flavor group $D^{\oplus 3}$ and $D^{\oplus 4}$

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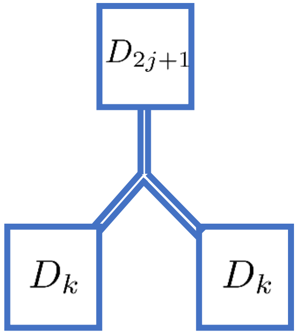

Trinion $(\boldsymbol{D_{2j+1}, D_k, D_k})$:

$$\begin{cases} x^2 + zy^2 + z^{k-1} = 0 \\ z = u(u^{2j-1} + v^2) \end{cases}$$

Singularities:

$x = y = u = 0$	D_k type
$x = y = u^{2j-1} + v^2 = 0$	D_k type
$x = u = v = 0$	D_{2j+1} type

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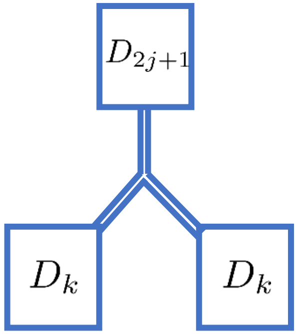

Tetraon (D_{2j}, D_k, D_k, D_k) :

$$\begin{cases} x^2 + zy^2 + z^{k-1} = 0 \\ z = u(u^{j-1} + v)(u^{j-1} - v) \end{cases}$$

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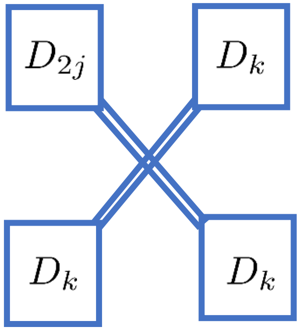

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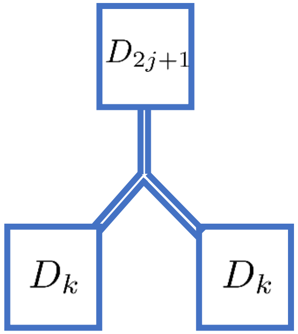

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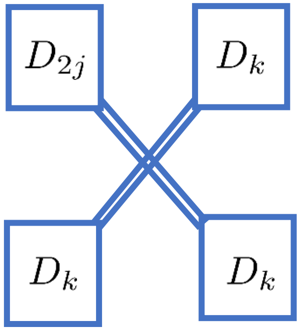

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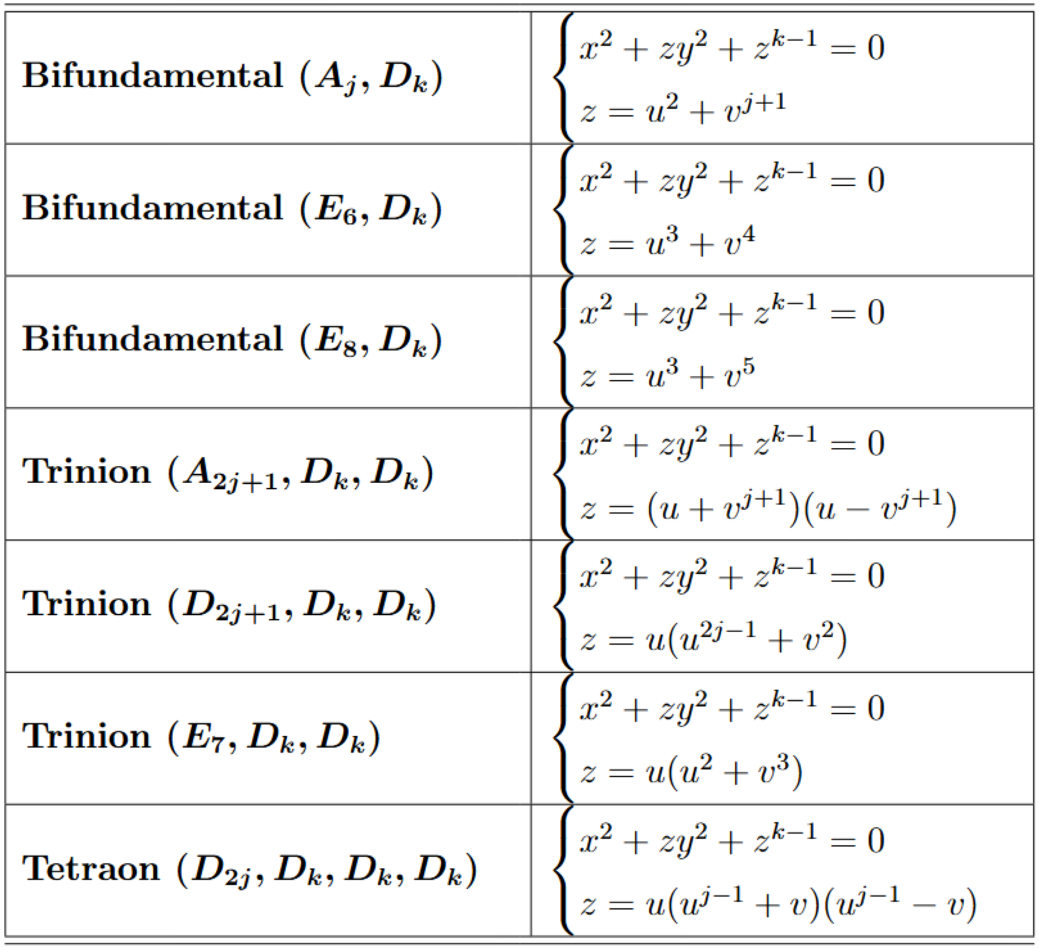
No straightforward way to construct trinions and tetraons of type E_6, E_7, E_8

5d CM trinions and tetraons

- List of canonical CY3 that can be constructed with this method:

- They are all of type $(\mathfrak{g}, D_k, \ldots)$ for some $\mathfrak{g}$



Bifundamental $(\boldsymbol{A_j, D_k})$	$\begin{cases} x^2 + zy^2 + z^{k-1} = 0 \\ z = u^2 + v^{j+1} \end{cases}$
Bifundamental $(\boldsymbol{E_6, D_k})$	$\begin{cases} x^2 + zy^2 + z^{k-1} = 0 \\ z = u^3 + v^4 \end{cases}$
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Trinion $(\boldsymbol{A_{2j+1}, D_k, D_k})$	$\begin{cases} x^2 + zy^2 + z^{k-1} = 0 \\ z = (u + v^{j+1})(u - v^{j+1}) \end{cases}$
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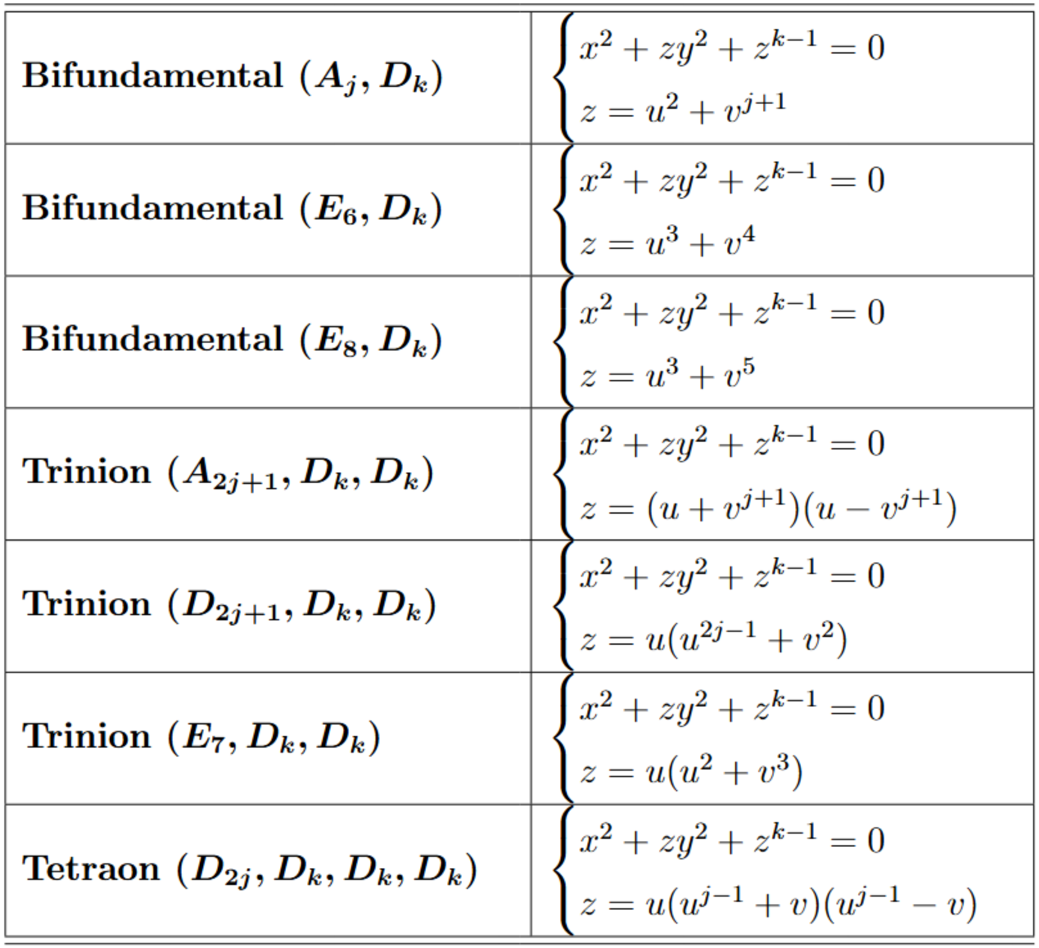

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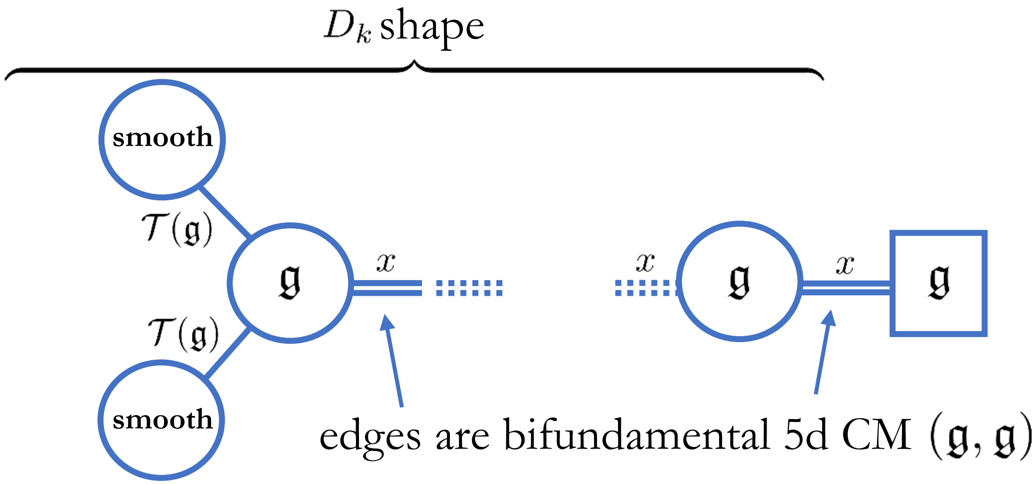
- The UV flavor symmetry can be checked through a **generalized version** of the flavor enhancement theorem by [Yonekura '15]

Novel flavor enhancement conjecture

- Consider the canonical CY3 in the aforementioned list, all of type $(\mathfrak{g}, D_k, \ldots)$

- Resolved phase:



edges are bifundamental 5d CM $(\mathfrak{g}, \mathfrak{g})$

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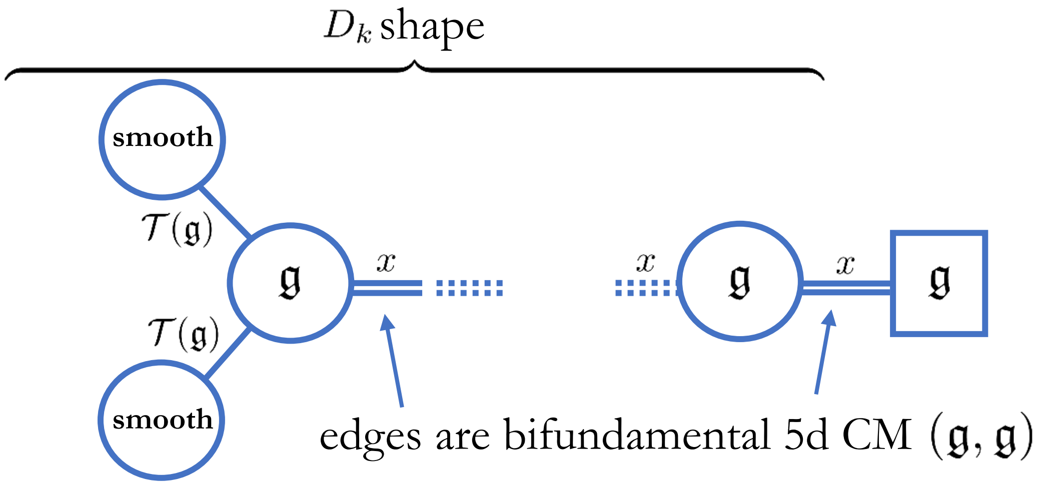

D_k shape

edges are bifundamental 5d CM $(\mathfrak{g}, \mathfrak{g})$

- each gauge node supplies a topological $U(1)_T$
- each double edge supplies as many $U(1)$ flavor factors as the ones in the 5d CM $(\mathfrak{g}, \mathfrak{g})$
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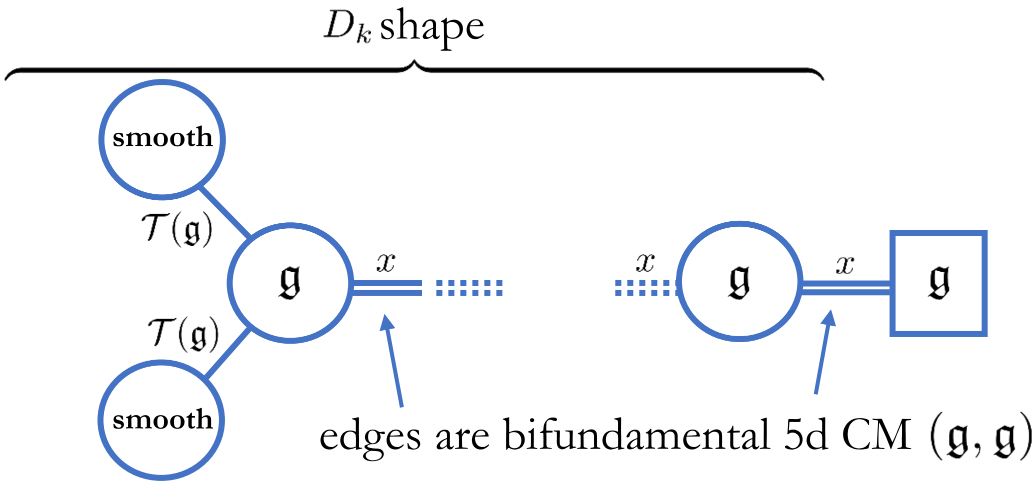

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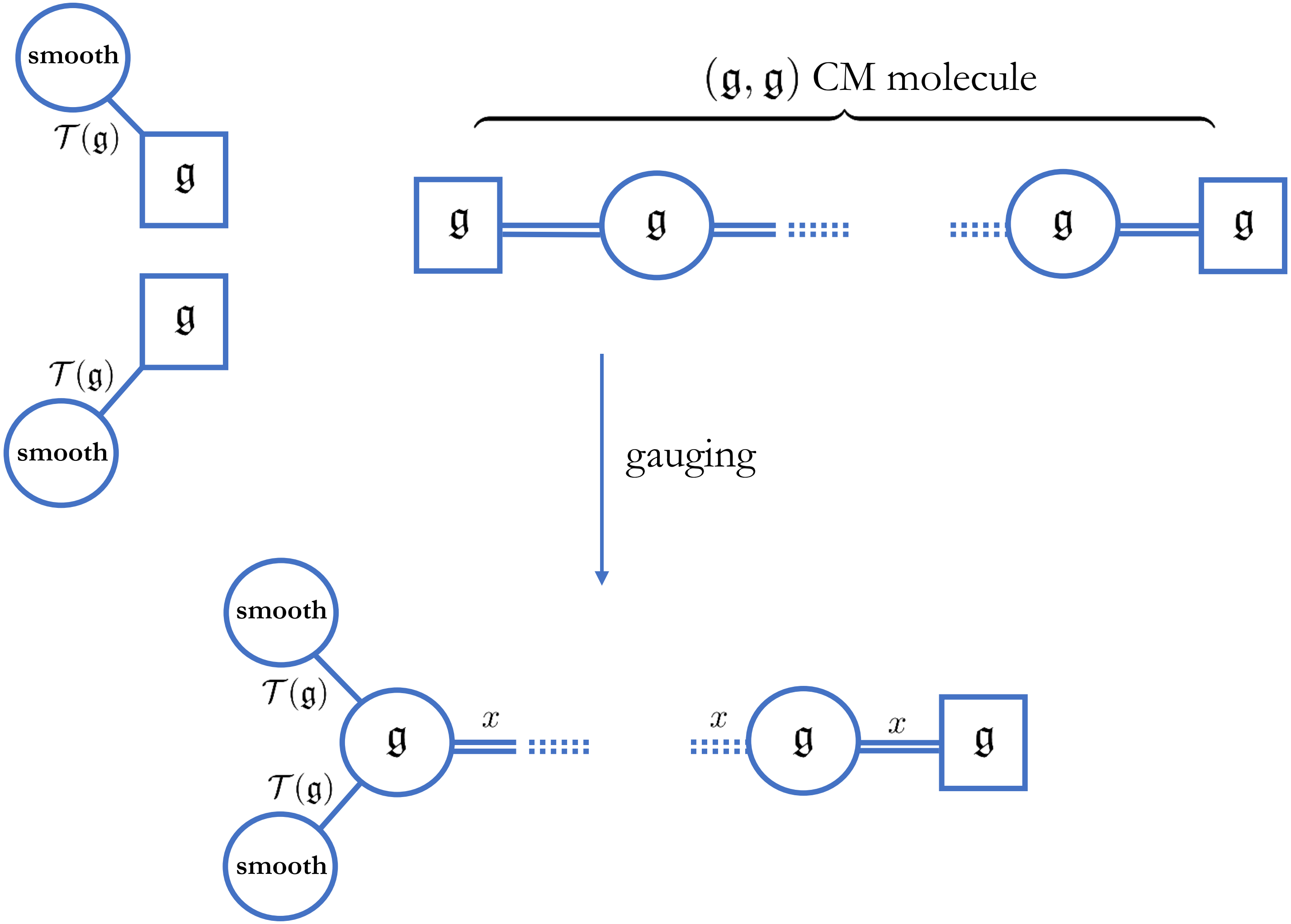
- **Upshot**: the total UV flavor rank from the generalized quiver theory **precisely matches** the flavor that is detected by the singular non-compact lines in the CY3 geometry

→ Example: with $\mathfrak{g} = D_{2j}$ we match the rank of (D_{2j}, D_k, D_k, D_k), in accordance with the expectation from the corresponding canonical CY3

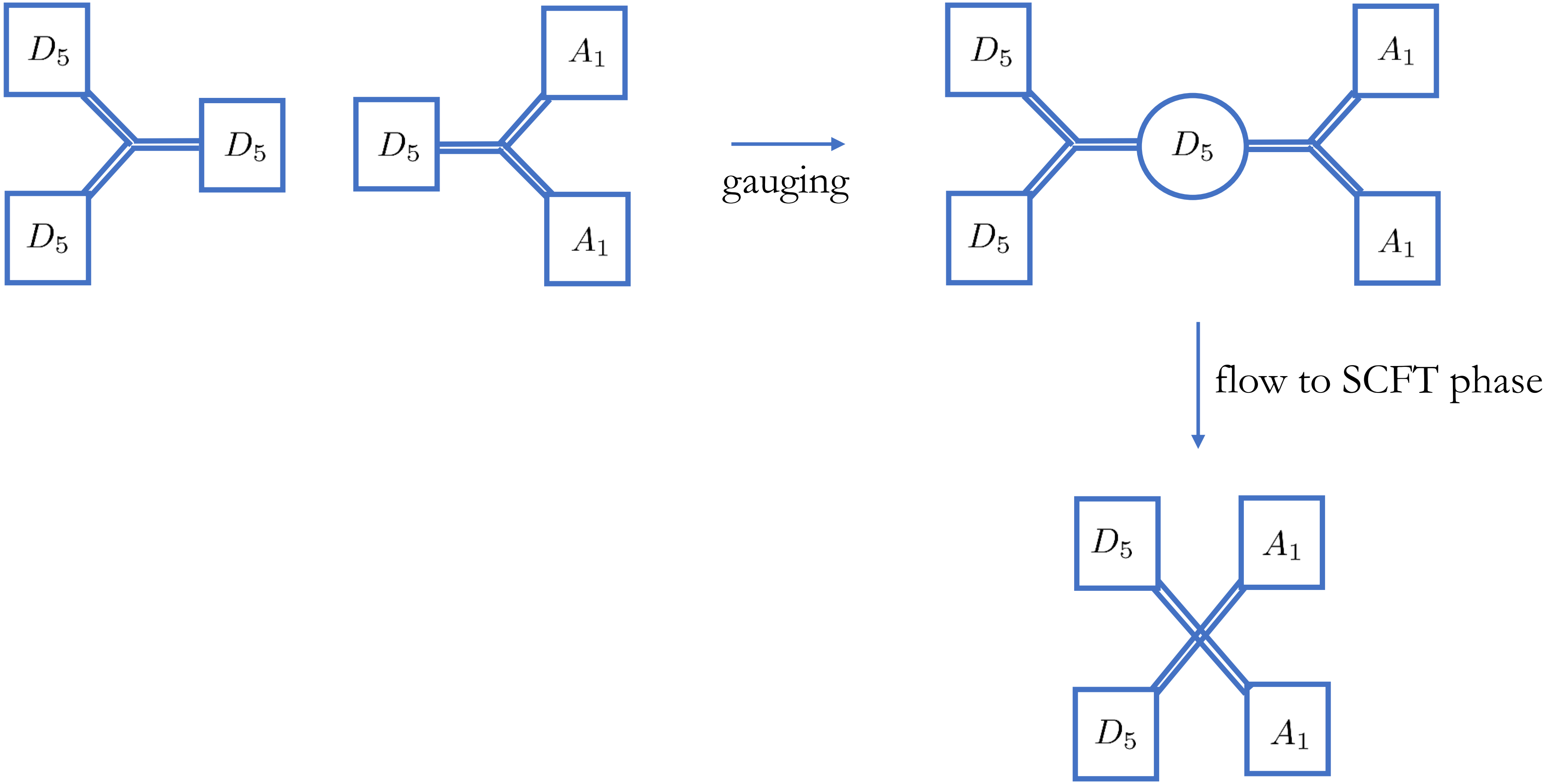
Reducibility of trinions and tetraons

- These new theories are not irreducible → **they are not atoms**



Gauging of 5d CM trinions and tetraons

- Trinions and tetraons admit a very limited set of gaugings that flow to a 5d fixed point



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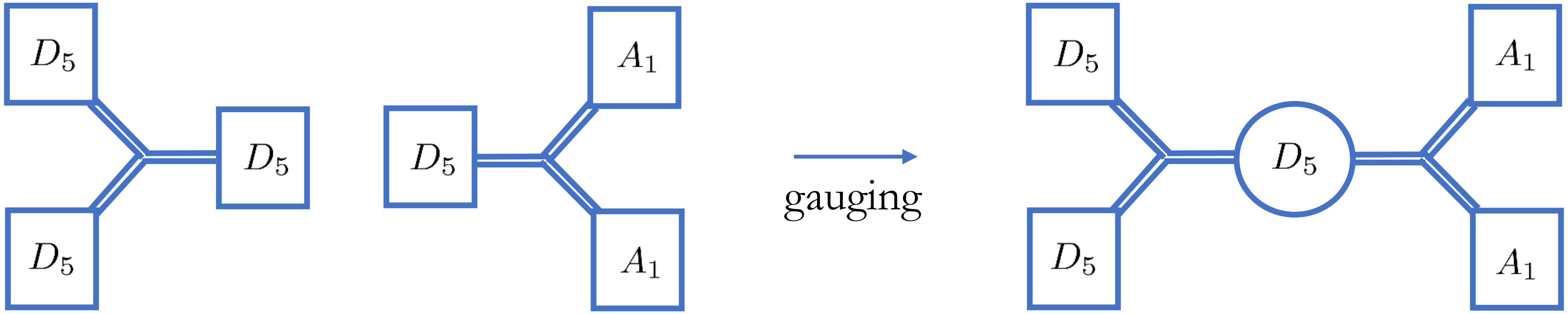

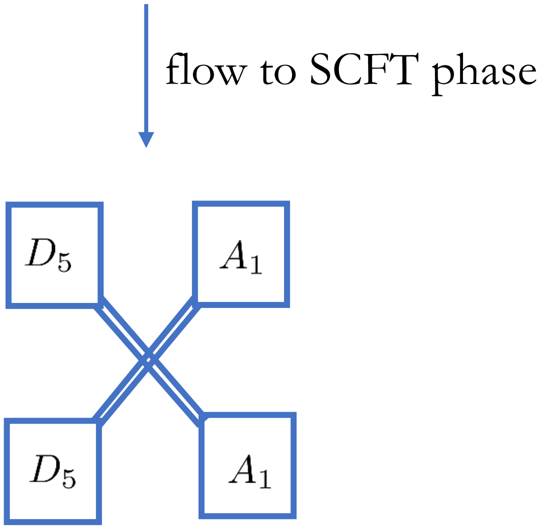

non-toric non-complete intersection canonical CY3: 6 variables and 4 equations

$$\begin{cases} A_1B_2 - A_2B_1 = 0 \\ A_1B_3 - A_3B_1 = 0 \\ A_2B_3 - A_3B_2 = 0 \\ A_1B_1 + B_2(B_3^2 + A_2B_2^2) + B_2A_2^3(A_3^2 + A_2^3)^4 = 0 \end{cases} \longleftrightarrow$$

State of the art on the classification of 5d CM SCFTs

- We have seen that 5d **bifundamental** CM of type $(\mathfrak{g}, \mathfrak{g})$ can be neatly organized into an Atomic Classification scheme (i.e. identify the collection of elementary atoms and their gaugings, in correspondence with singular CY3 geometries) ⟶ more atoms than in 6d

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  I. 5d CM trinions of type A exist and can be gauged together (i.e. they are **atoms**)
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$\downarrow$

- 5d trinions/tetraons of type D admit a very constrained chemistry (they are not irreducible, and we find a **novel UV flavor enhancement** conjecture)

Discussion and outlook

- The formalism of M-theory geometric engineering on **explicit** canonical CY3 seems to be approaching its limit: non-complete intersections with non-isolated singularities most likely dominate the landscape of CY3 $\longrightarrow$ extremely hard to classify

- Do further trinion/tetraon 5d CM of type D, E_6, E_7, E_8 lie in this largely unexplored setting?

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Hint: it has been conjectured [Jefferson, Katz, Kim, Vafa '18], [Bhardwaj '19] that all 5d $\mathcal{N}=1$ SCFTs descend from 6d setups (SCFTs or LSTs)

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Open questions:

- 6d origin of 5d CM trinions and tetraons
- 5d CM SCFTs for non-simply laced flavor groups
- 4d descendants of 5d CM molecules/hybrids/trinions/tetraons

STRINGS AND GEOMETRY 2026

MAY 19-22, 2026

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Thank you for your attention!

Circle reduction of 5d CM atoms

- The 4d $\mathcal{N}=2$ SCFT descendants can be identified for all 5d CM atoms:

Singularity	$\mathcal{O}_{\boldsymbol{3rd}}$	${\mathcal{O}_{\boldsymbol{3rd}}}^{L}$	**rank** $\mathbf{CB}_{\mathcal{T}_{\mathbf{4d}}}$	**flavor sym**
$X^{(1)}_{A_{2j+1}}$	$[2^{j+1}]$	$[(j+1)^2]$	j^2	$A_{2j+1}\times A_{2j+1}\times\mathfrak{u}(1)$
$X^{(1)}_{A_{2j}}$	$[2^{j},1]$	$[j+1,j]$	$j(j-1)$	$A_{2j}\times A_{2j}\times\mathfrak{u}(1)$
$X^{(1)}_{D_{2j+2}}$	$[3^2,2^{2j-2},1^2]$	$[(2j+1)^2,1^2]$	$j(2j+3)$	$D_{2j+2}\times D_{2j+2}\times\mathfrak{u}(1)^2$
$X^{(1)}_{D_{2j+3}}$	$[3^2,2^{2j-2},1^4]$	$[2j+3,2j+1,1^2]$	$j(2j+5)+1$	$D_{2j+3}\times D_{2j+3}\times\mathfrak{u}(1)$
$X^{(2)}_{D_{2j+2}}$	$[2^{2j},1^4]$	$[2j+3,2j+1]$	$2j^2+j-2$	$D_{2j+2}\times D_{2j+2}$
$X^{(2)}_{D_{2j+3}}$	$[2^{2j+2},1^2]$	$[(2j+3)^2]$	$j(2j+3)$	$D_{2j+3}\times D_{2j+3}\times\mathfrak{u}(1)$
$X^{(3)}_{D_j}$	$[3,1^{2j-3}]$	$[2j-3,1^3]$	$j-2$	$D_j\times D_j\times\mathfrak{su}(2)$
$X^{(1)}_{E_6}$	A_2	$E_6(a_3)$	15	$E_6\times E_6$
$X^{(2)}_{E_6}$	$2A_1$	D_5	10	$E_6\times E_6\times\mathfrak{u}(1)$
$X^{(3)}_{E_6}$	A_1	$E_6(a_1)$	5	$E_6\times E_6$
$X^{(1)}_{E_7}$	A_2+A_1	$E_6(a_1)$	31	$E_7\times E_7\times\mathfrak{u}(1)$
$X^{(2)}_{E_7}$	$(3A_1)''$	E_6	20	$E_7\times E_7\times\mathfrak{su}(2)$
$X^{(3)}_{E_7}$	A_1	$E_7(a_1)$	10	$E_7\times E_7$
$X^{(1)}_{E_8}$	A_2+A_1	$E_8(a_4)$	60	$E_8\times E_8$
$X^{(2)}_{E_8}$	$2A_1$	$E_8(a_2)$	38	$E_8\times E_8$
$X^{(3)}_{E_8}$	A_1	$E_8(a_1)$	21	$E_8\times E_8$

Relations to other known constructions

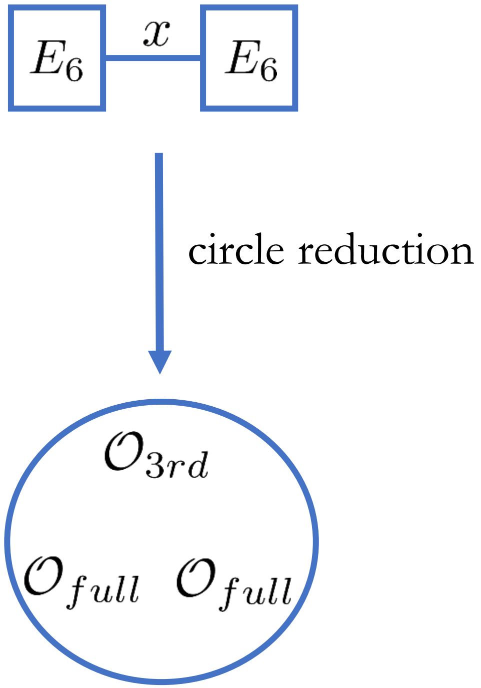
We wish to investigate the 4d $\mathcal{N}=2$ theories obtained reducing 5d CM **atoms** on a circle

- Non-trivially, the resulting 4d $\mathcal{N}=2$ theory is a **SCFT**

$\downarrow$

- It is a **class-S** fixture
  6d $\mathcal{N}=(2,0)$ theory of type $\mathfrak{g}\in ADE$
  on a sphere with **three regular punctures**
- Checks:

$\left\{ \begin{array}{l} \text{matching } \textbf{CB dimension} \\ \text{matching } \textbf{HB dimension} \\ \text{matching } \textbf{flavor symmetry} \end{array} \right.$



- punctures $\mathcal{O}_{full}\times\mathcal{O}_{full}\times\mathcal{O}_{3rd}$
- flavor sym $\mathfrak{g}\oplus\mathfrak{g}\oplus\mathfrak{g}_{rest}$

Circle reduction of 5d CM molecules

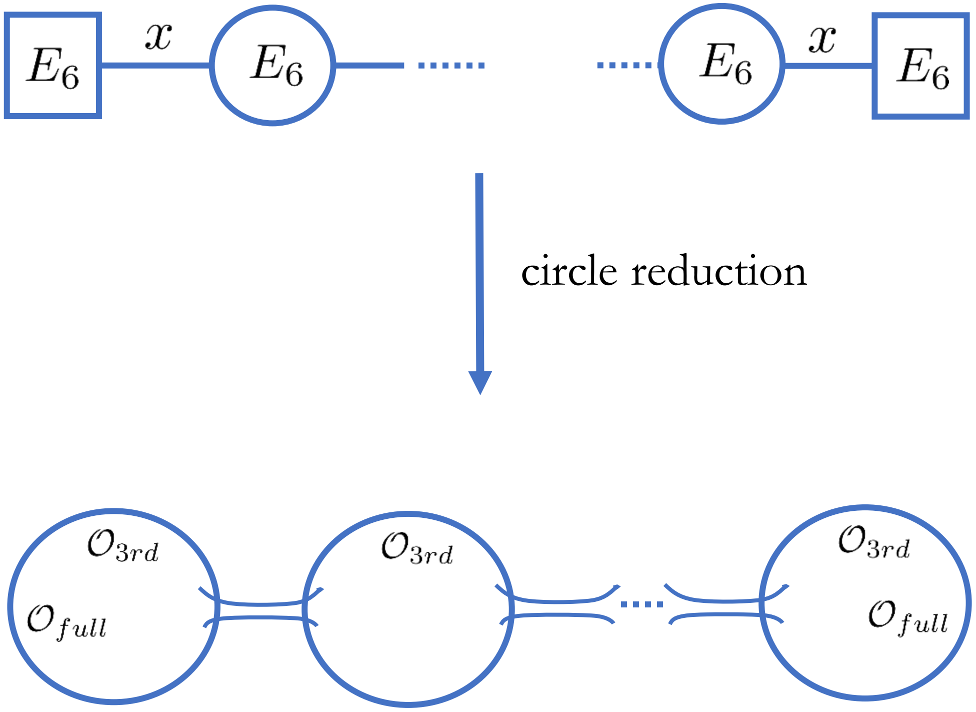
What about the 4d $\mathcal{N}=2$ theories obtained reducing 5d CM **molecules** on a circle

- Molecules admit a 4d $\mathcal{N}=2$ **class-S** description which is **not a SCFT**

↓

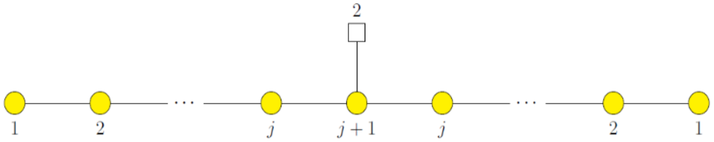
- It corresponds to a low-energy quiver gauge theory phase of the 5d SCFT engineered by:

CY3: $$\begin{cases} x^2+y^3+z^4=0 \\ x^{n_1}y^{n_2}z^{n_3}=uv \end{cases}$$

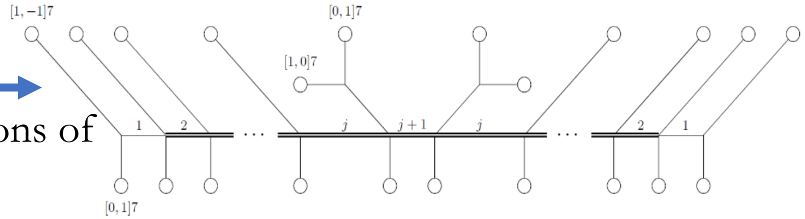


5d CM SCFTs across dimensions (example)

5d

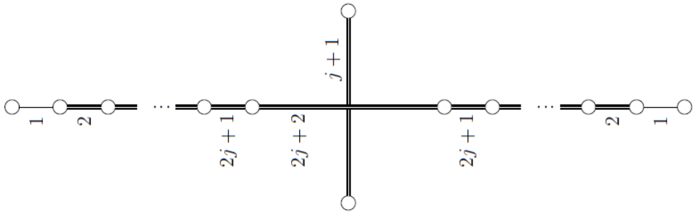


dual descriptions of IR phase



CY3: $\begin{cases} x^2 + y^2 + z^{2j+2} = 0 \\ x = uv \end{cases}$

dual descriptions at UV point



circle reduction

extract MQ

4d

6d $\mathcal{N} = (2,0)$ A_{2j+1} on: $[3, 1^3]$ $[6]$ $[6]$

3d

5d HB data at UV point:

- Flavor sym: $\mathfrak{su}(2j+2) \oplus \mathfrak{su}(2j+2) \oplus \mathfrak{u}(1)$
- HB dimension: $4j^2 + 9j + 4$