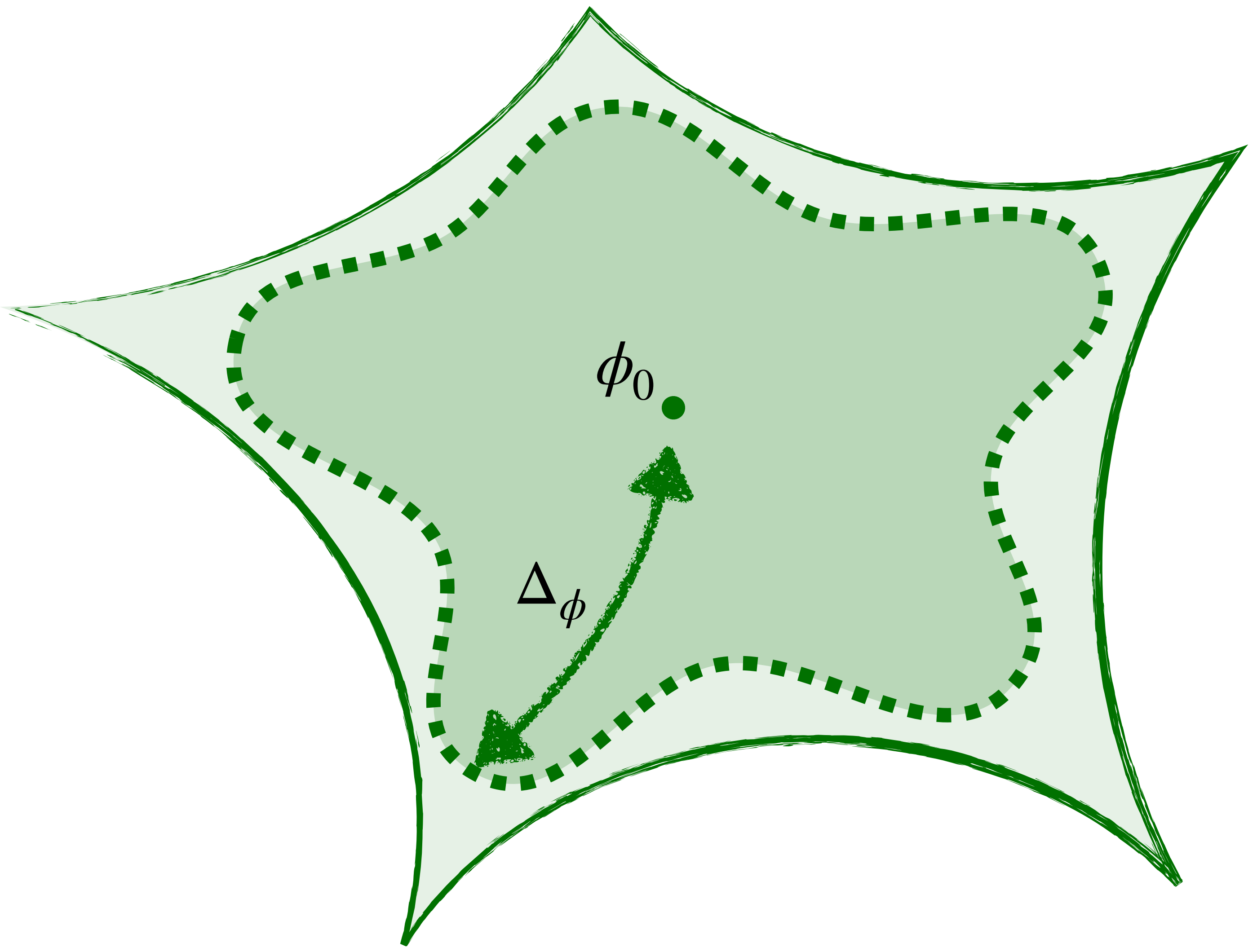
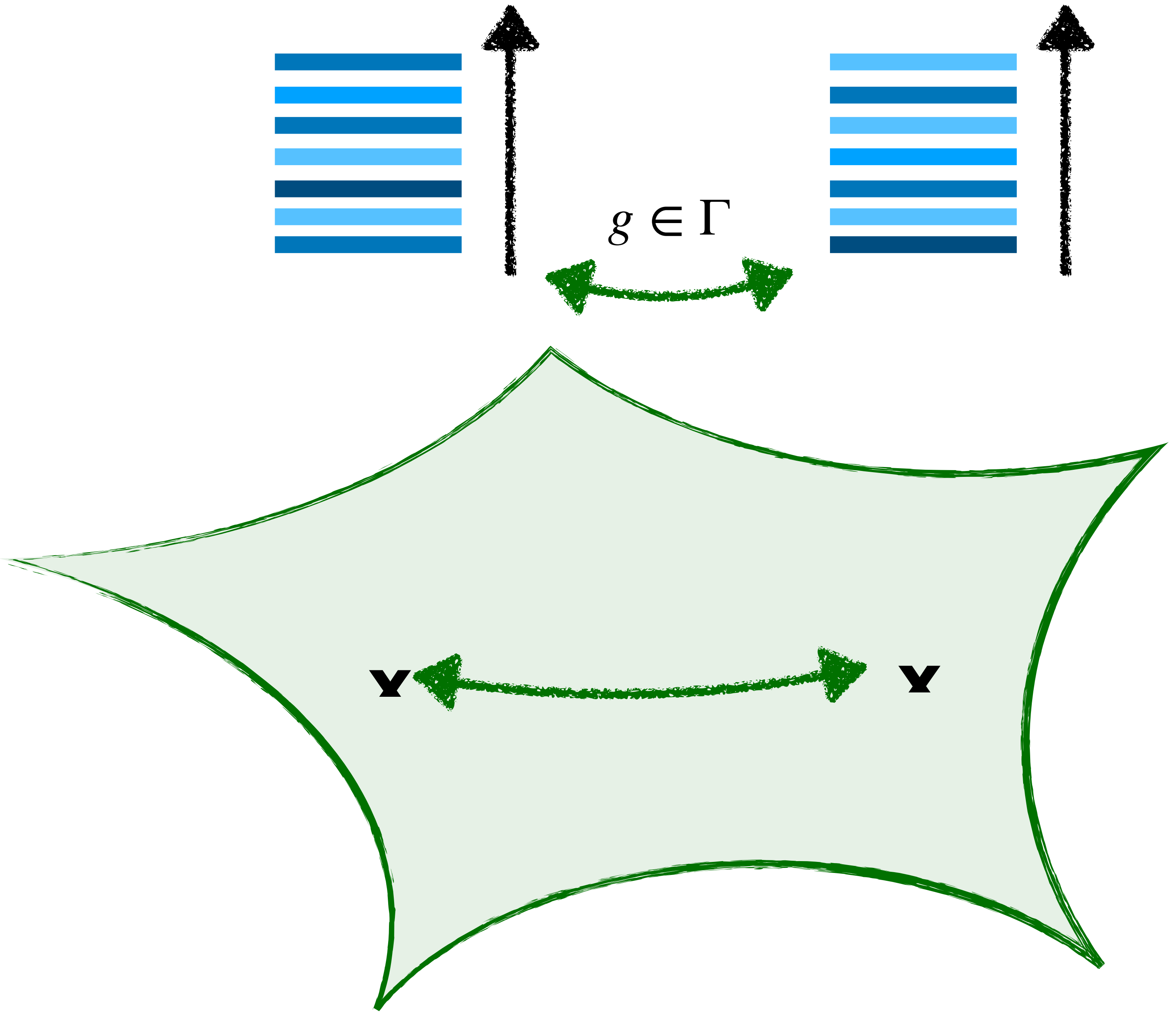
Finiteness & the Emergence of Dualities



Damian van de Heisteeg



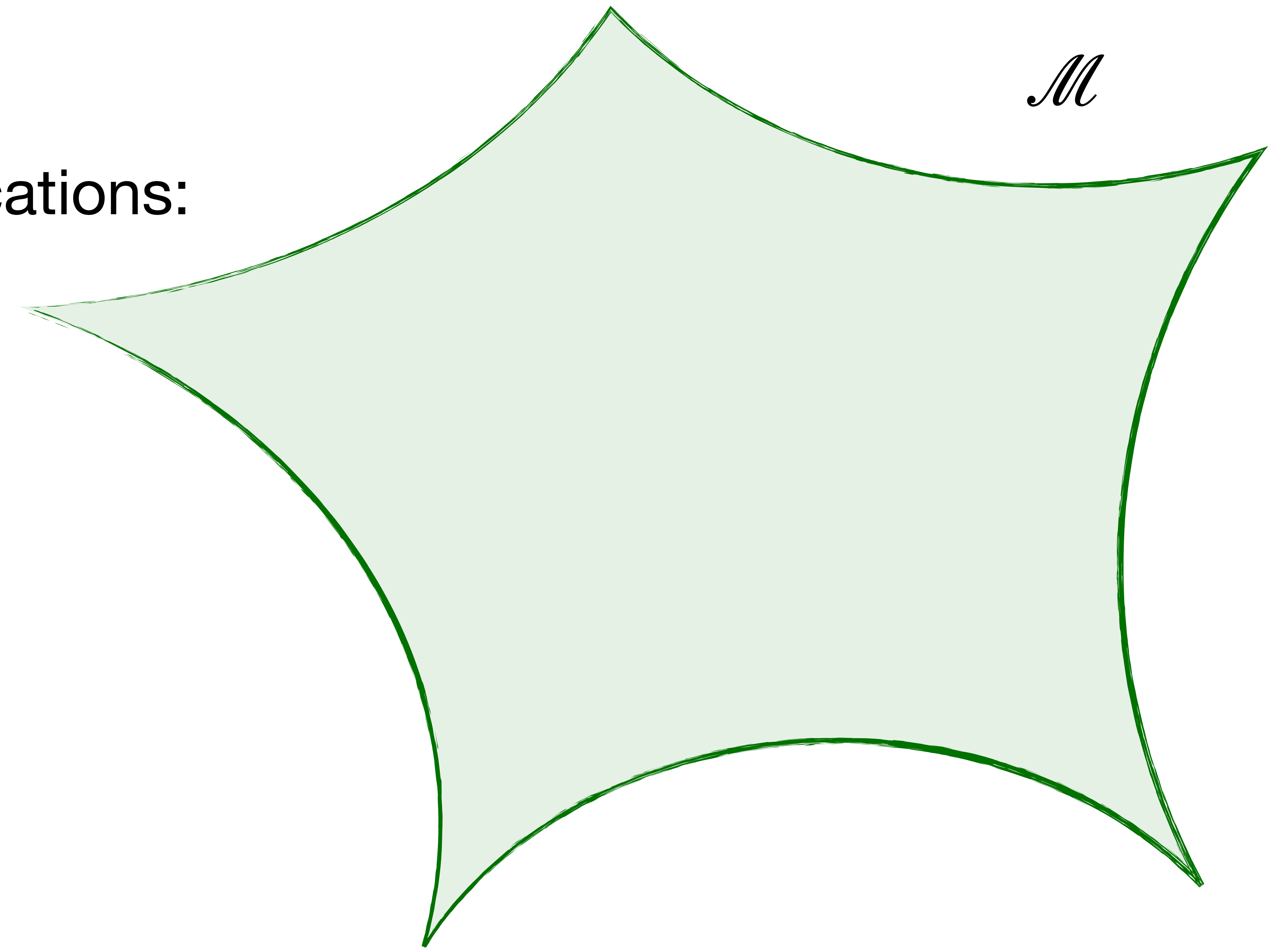
Based on: 2412.03640,
with **Matilda Delgado**, **Sanjay Raman**,
Ethan Torres, **Cumrun Vafa** & **Kai Xu**

Strings & Geometry
ICTP, April 9th

Stringy EFTs

Typical effective action arising from string compactifications:

$$\mathscr{L} = R - g_{IJ}\partial_\mu \phi^I \partial^\mu \phi^J - e^{a\phi}|F|^2 - \dots$$



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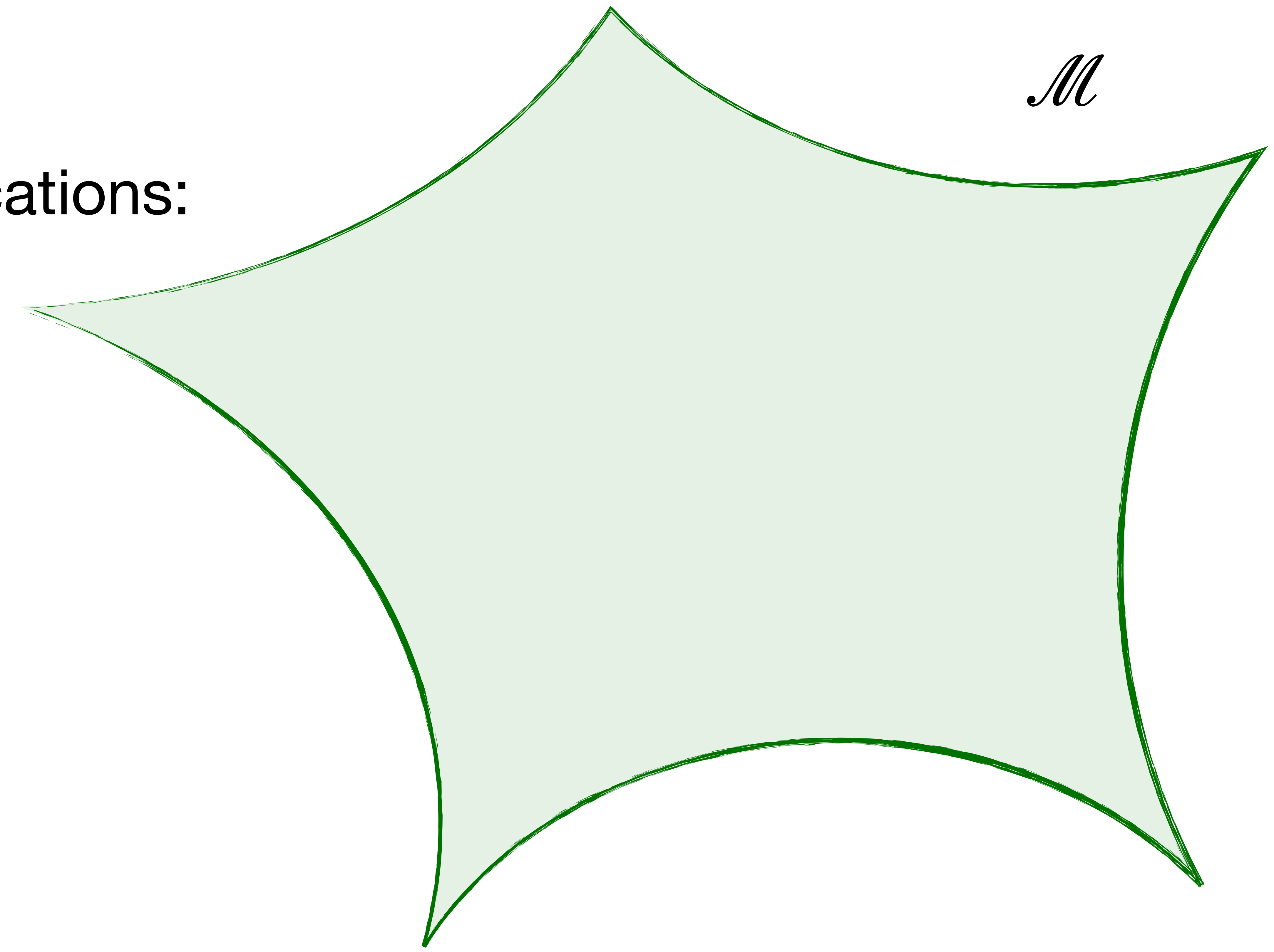

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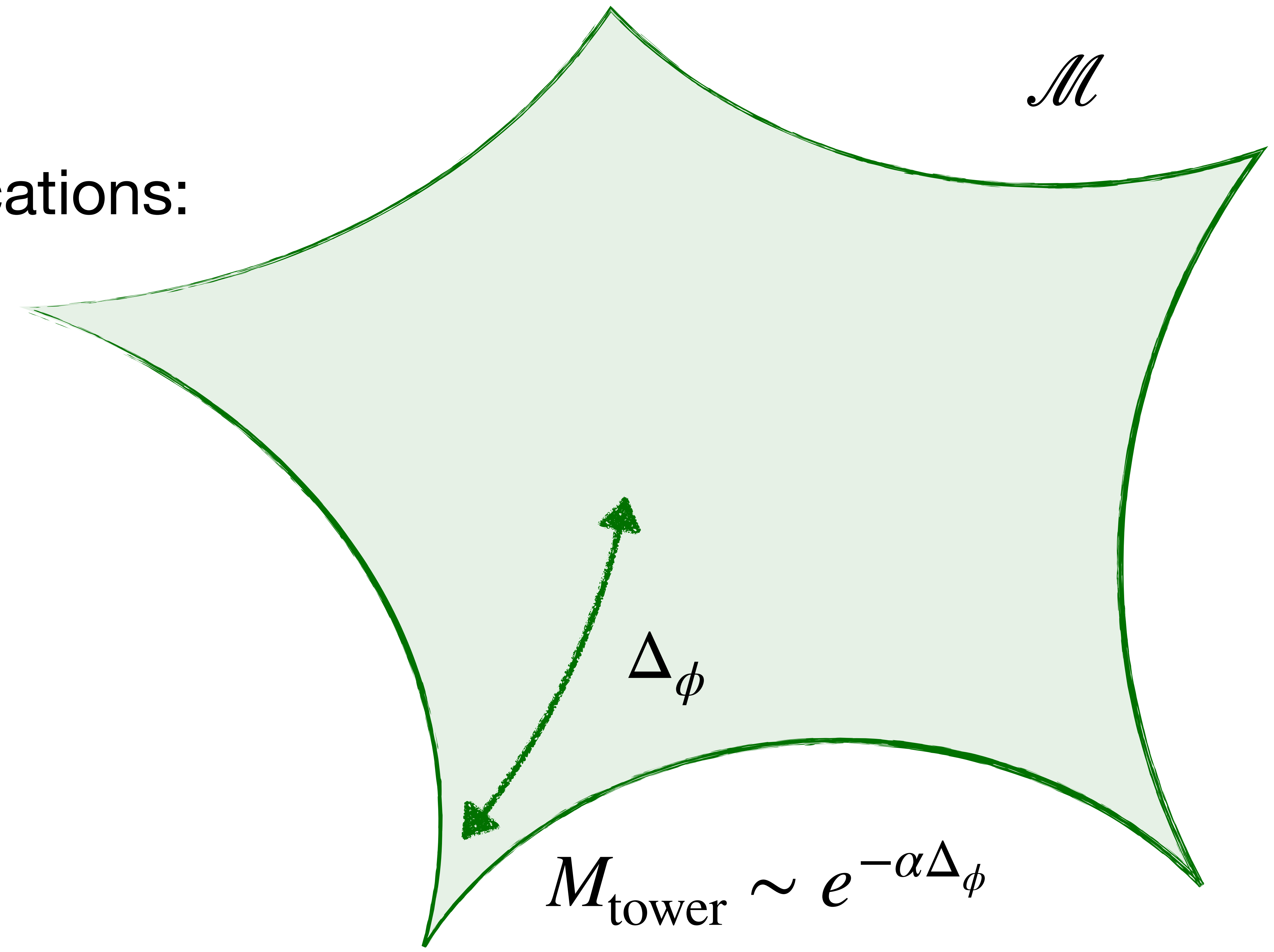
- **Distance Conjecture** [Ooguri, Vafa, '06]

infinite towers of states become exponentially light



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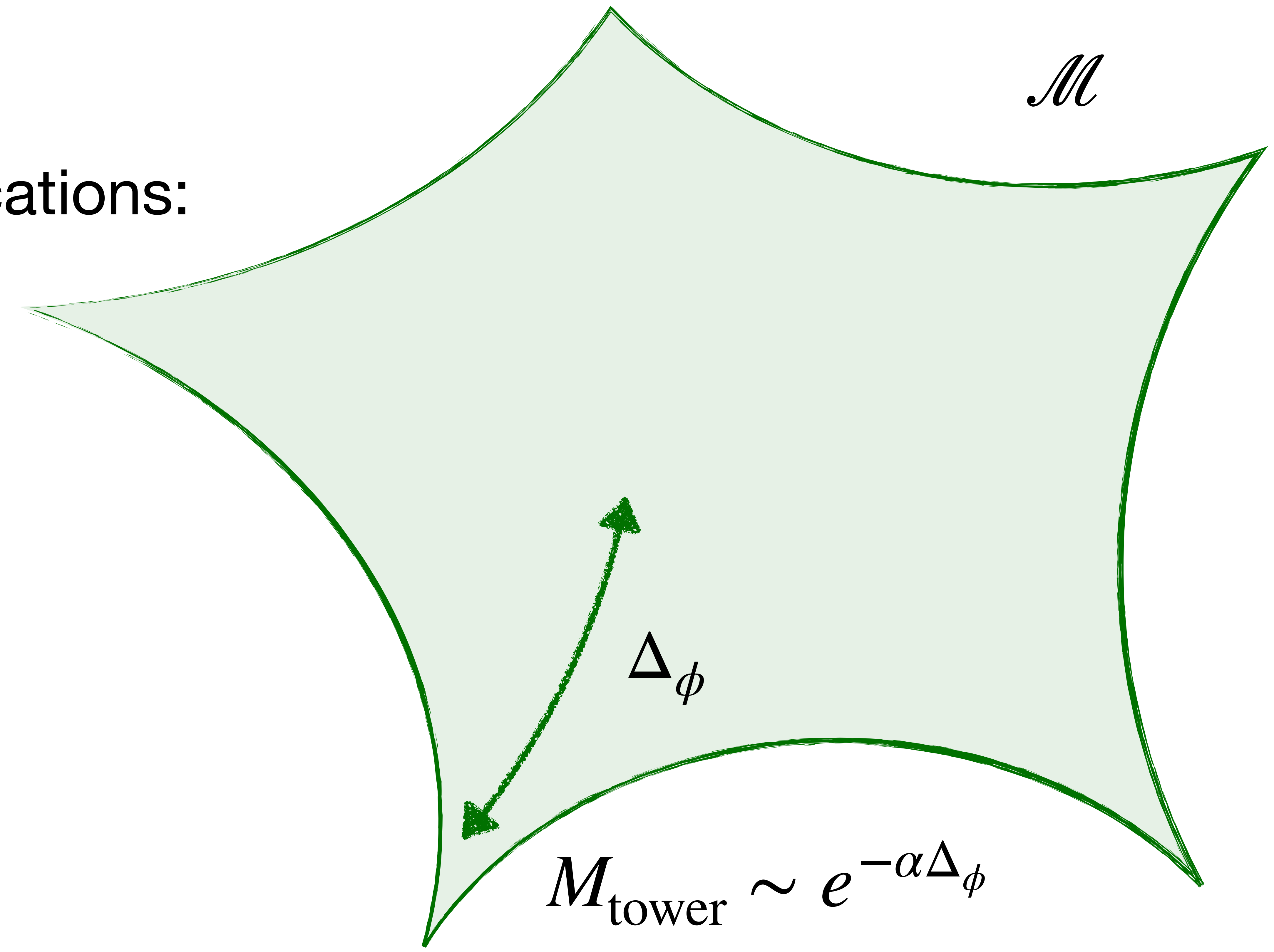

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infinite towers of states become exponentially light

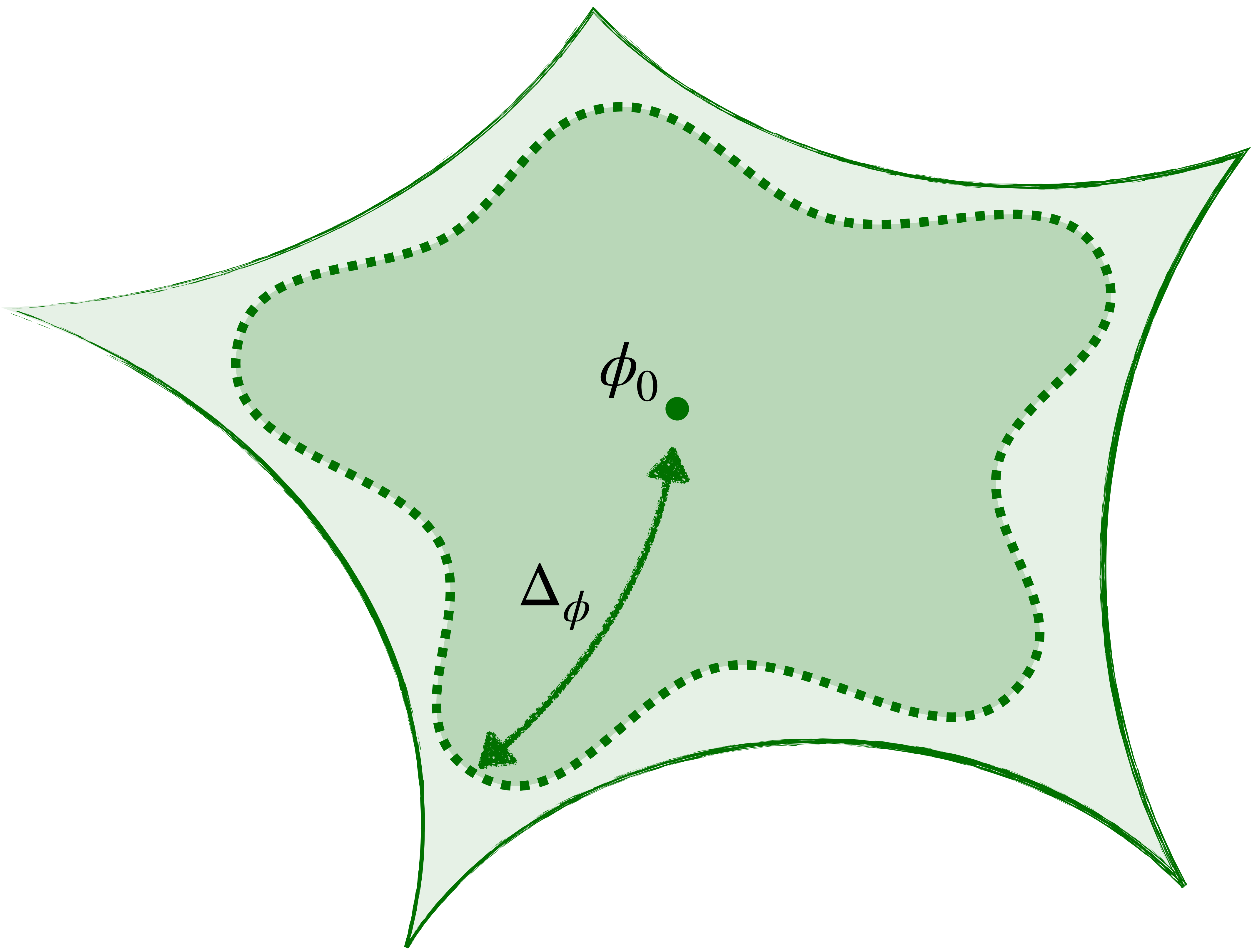
- strong evidence from string compactifications

[Grimm, Palti, Valenzuela '18; Lee, Lerche, Weigand, '19; ….]

Compactifiability

Region within distance Δ:

$$\mathscr{M}_\Delta(\phi_0) = \{\phi \in \mathscr{M} \mid d(\phi, \phi_0) \leq \Delta\}$$



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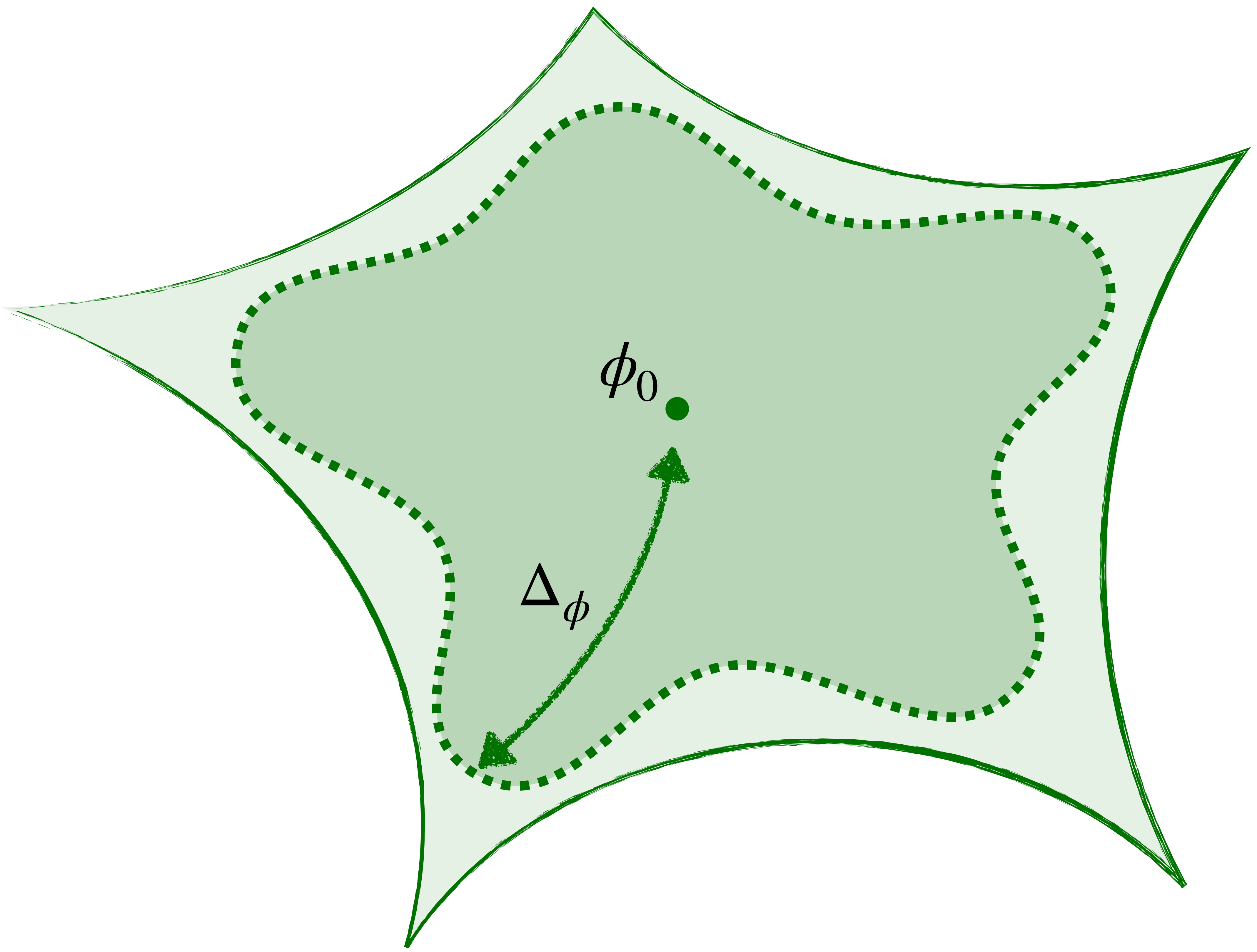

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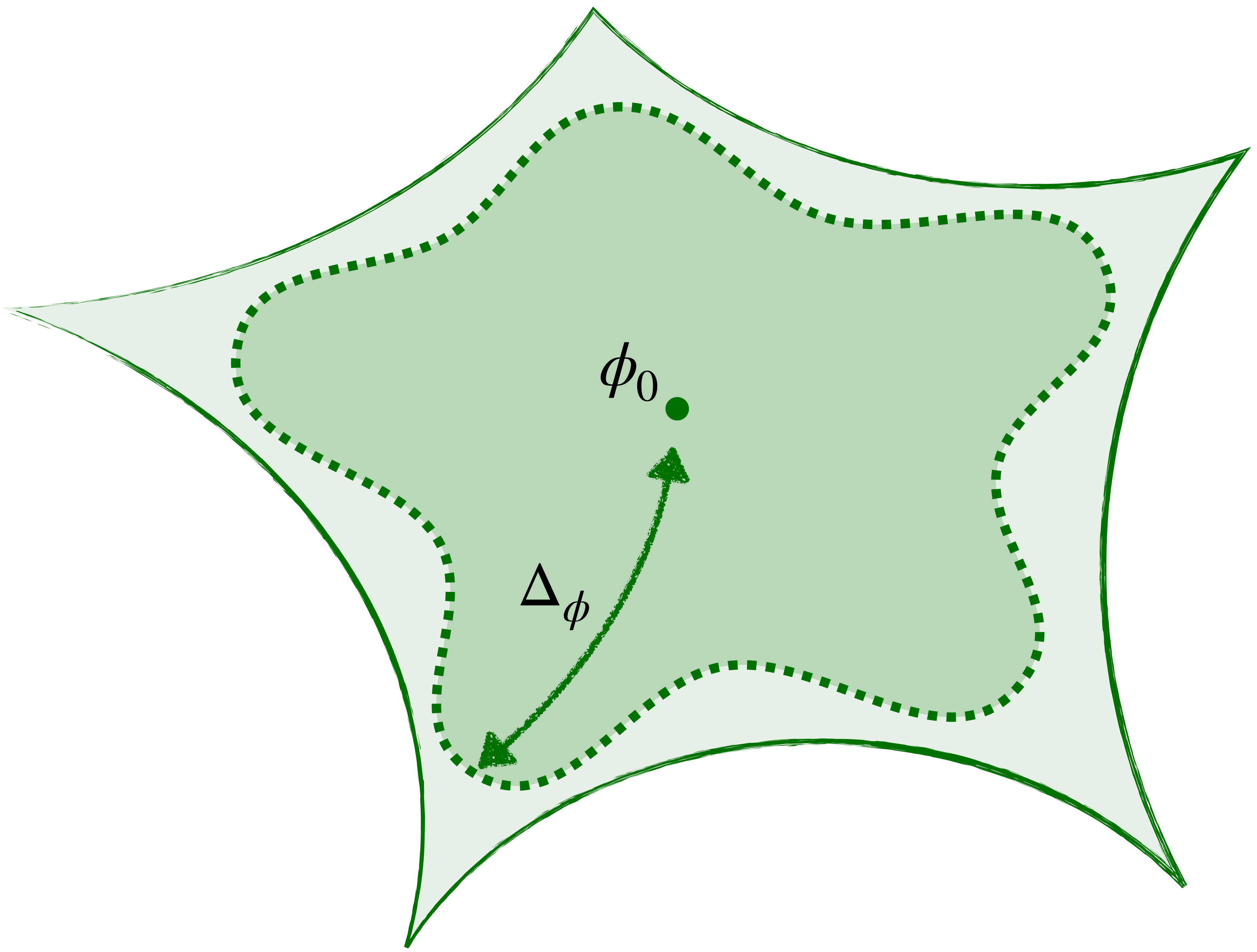

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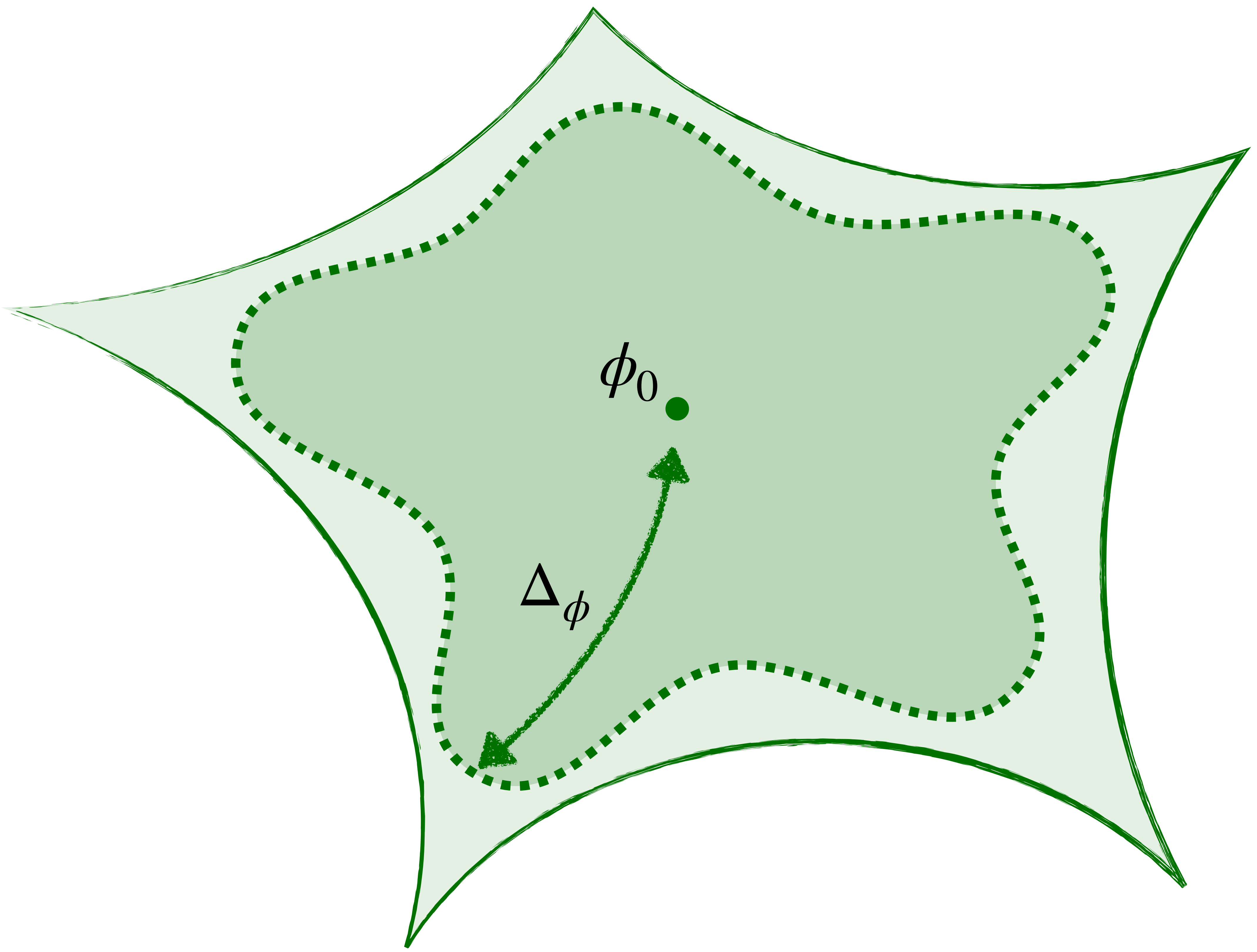

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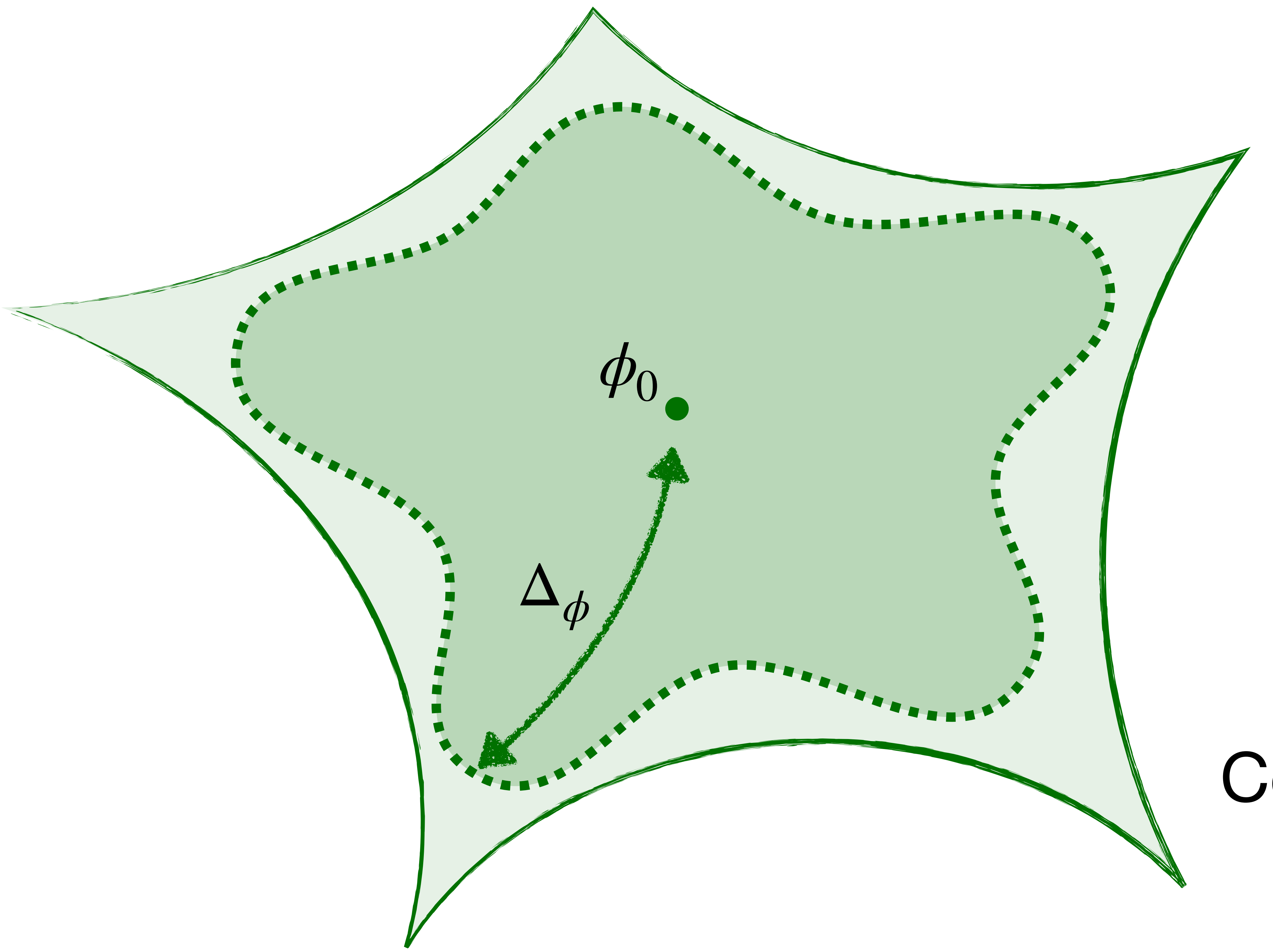

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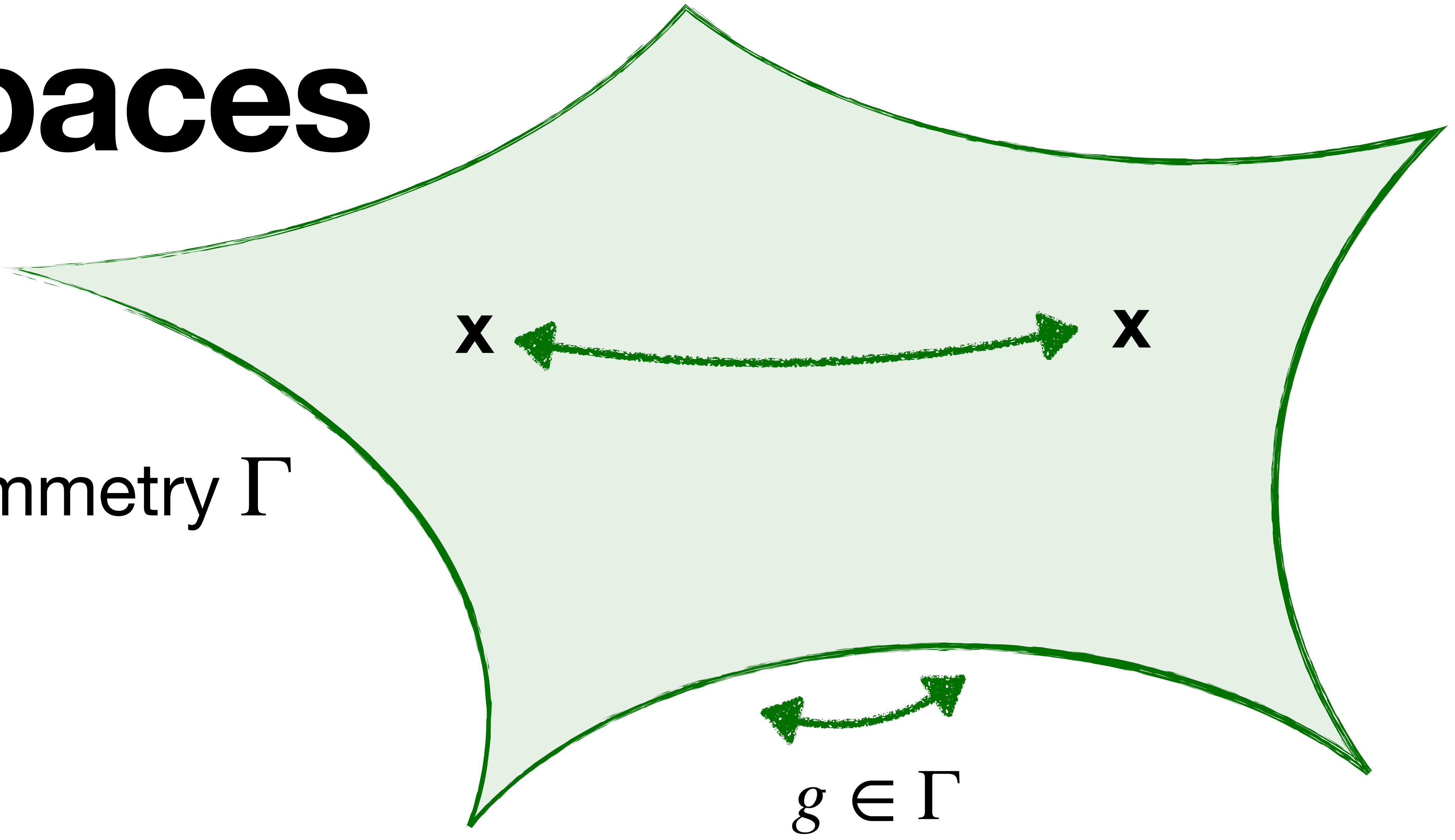
Complementary version: tame Euclidean embedding

[Grimm, Prieto, van Vliet, '25]

Dualities and moduli spaces

Self-dualities:

discrete, spontaneously broken, 0-form gauge symmetry Γ



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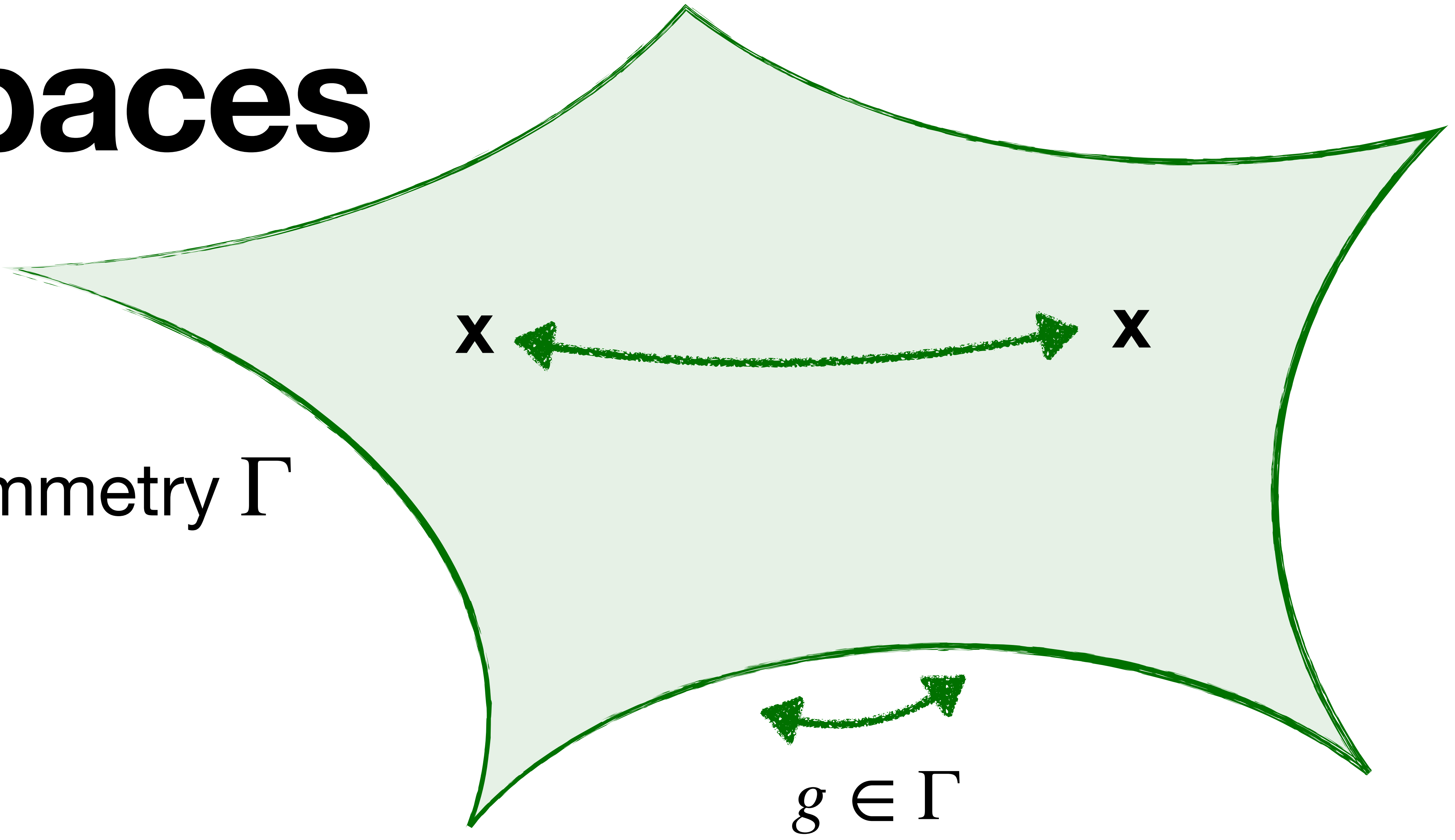

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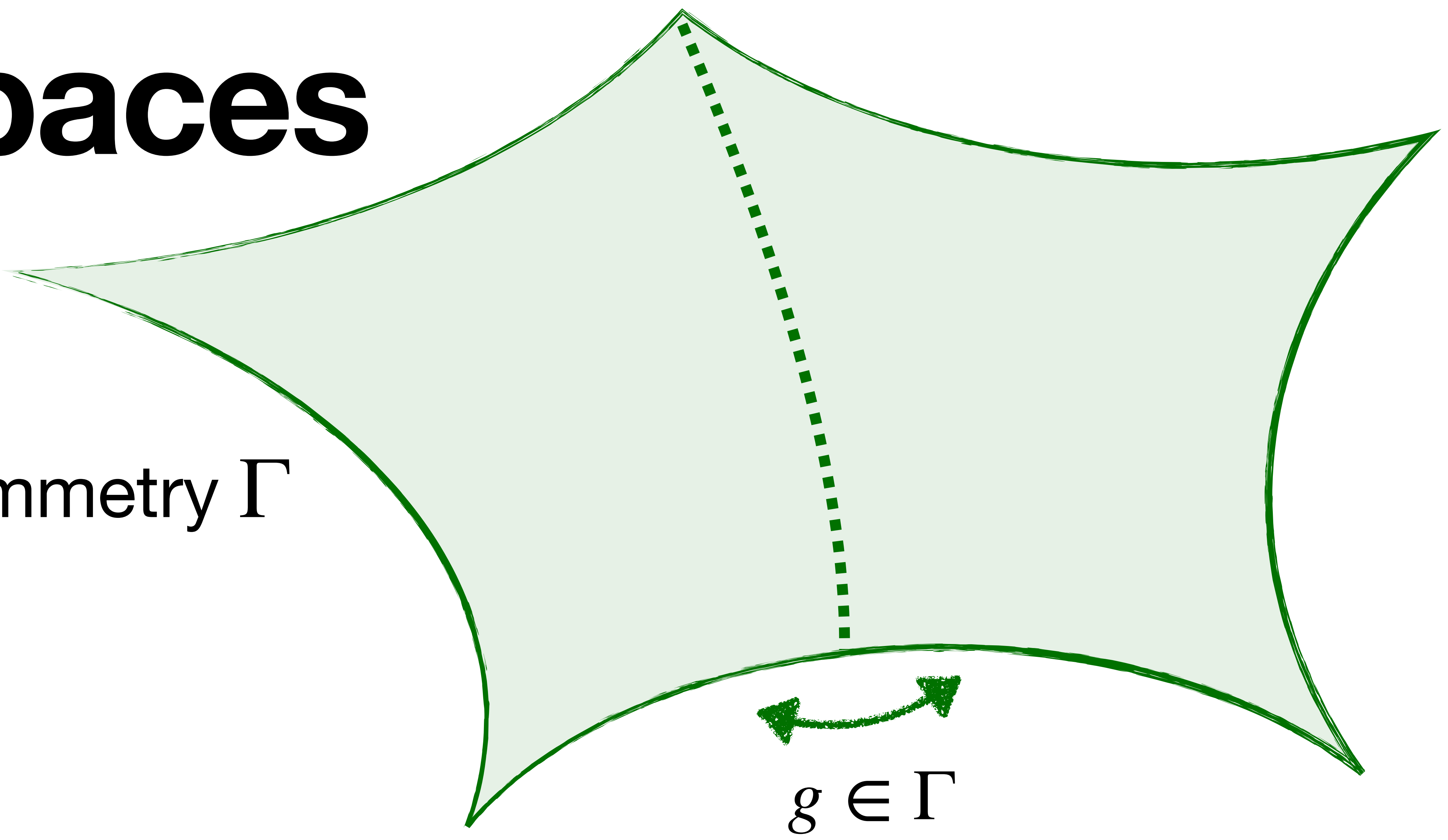

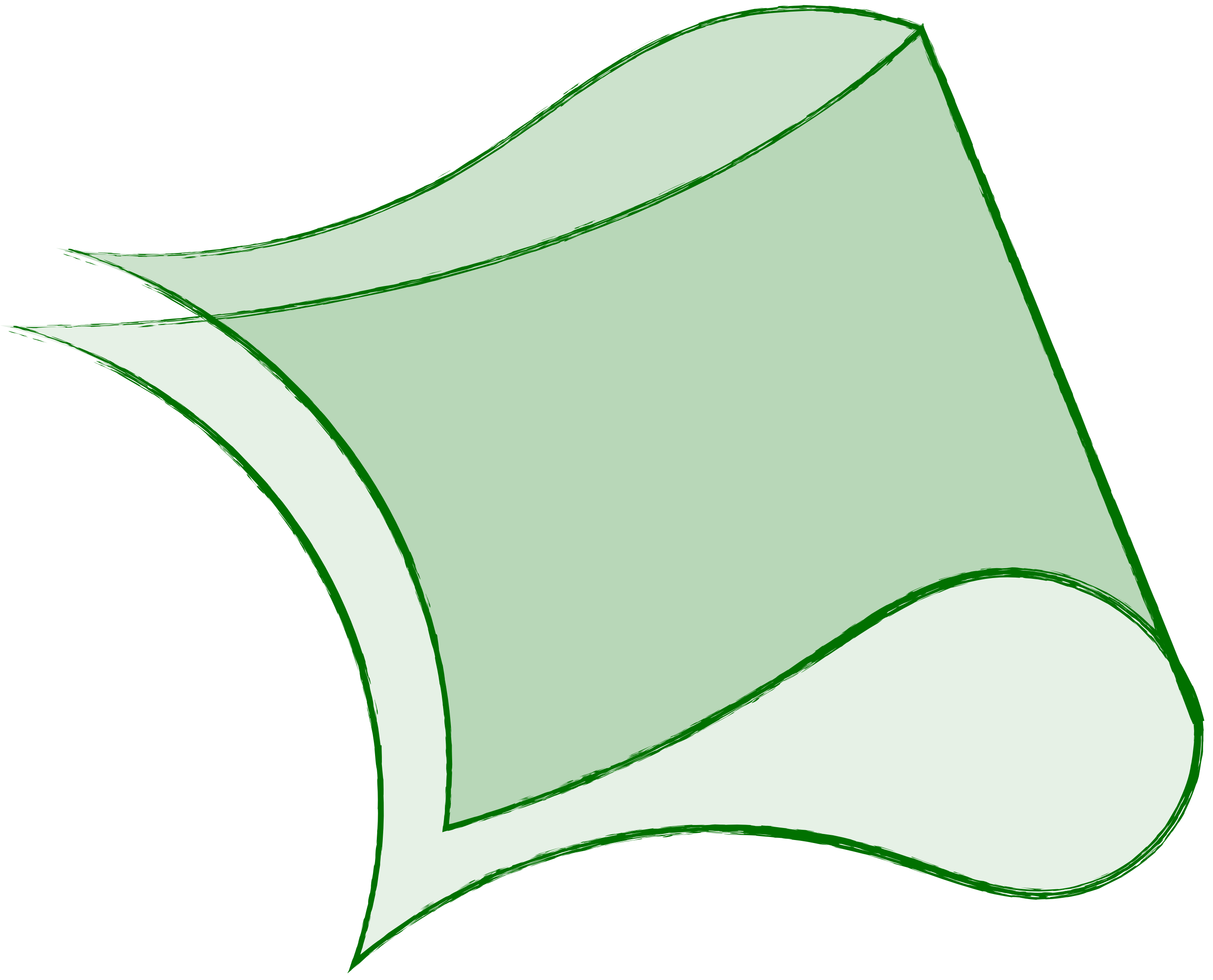

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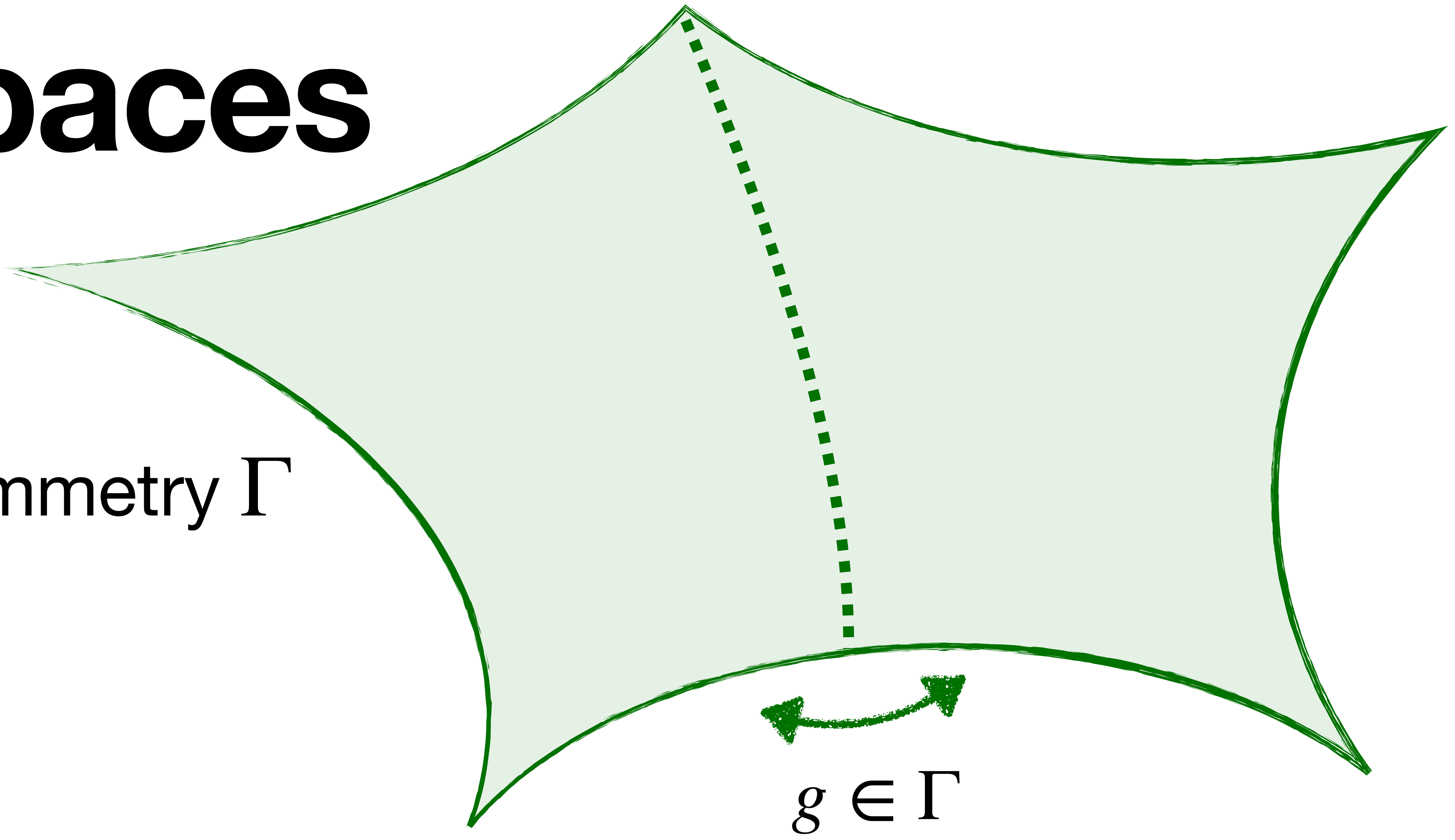

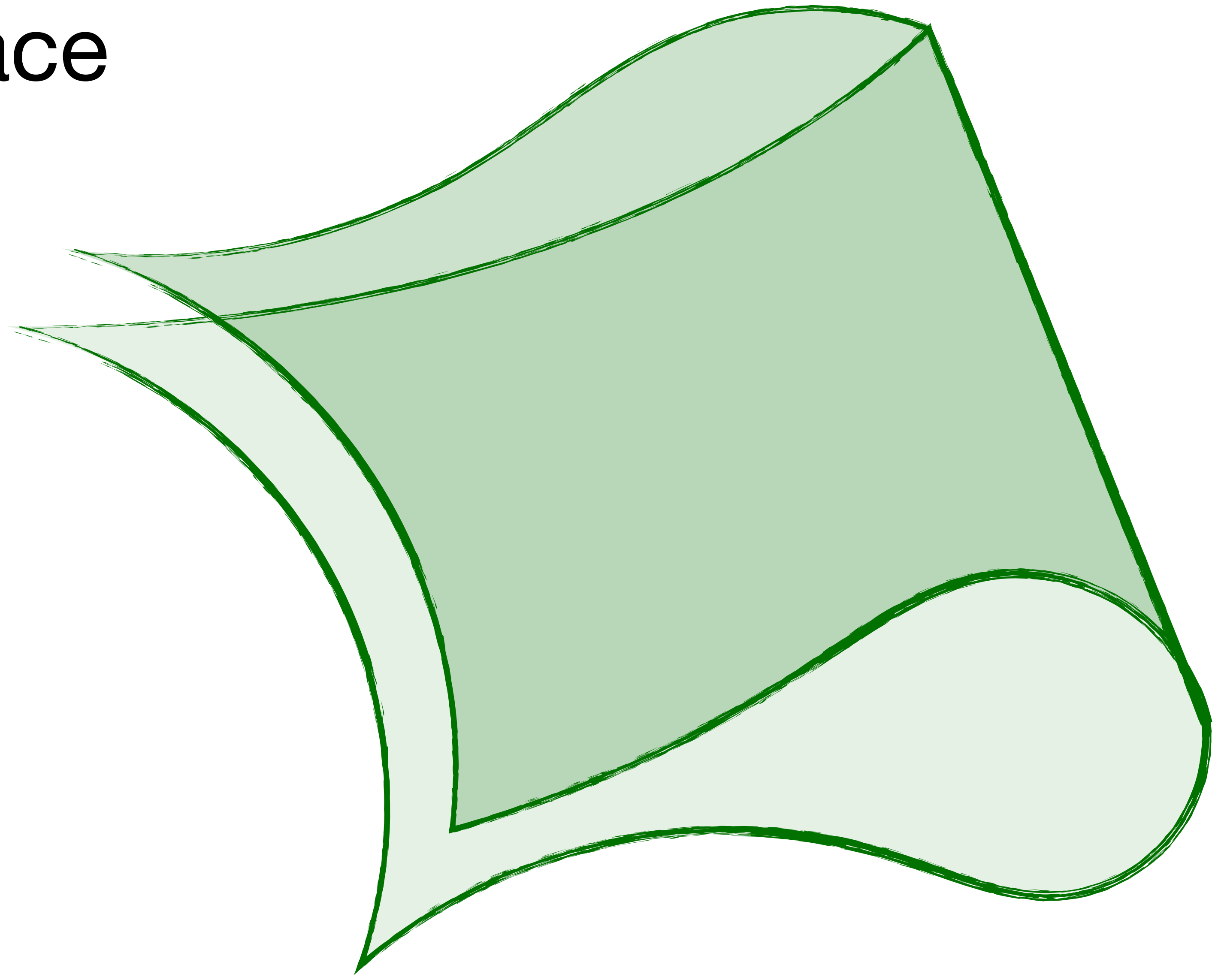

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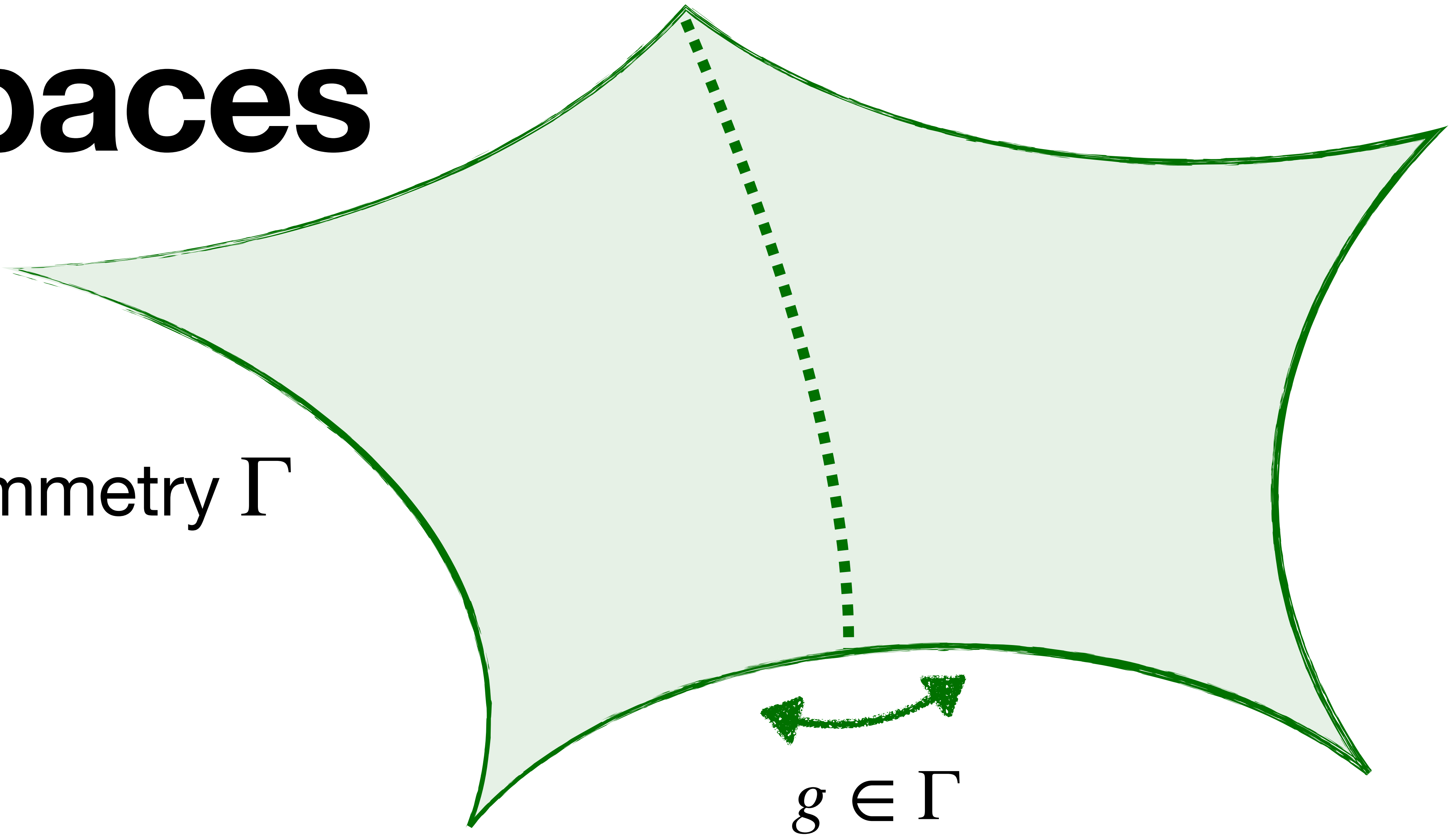

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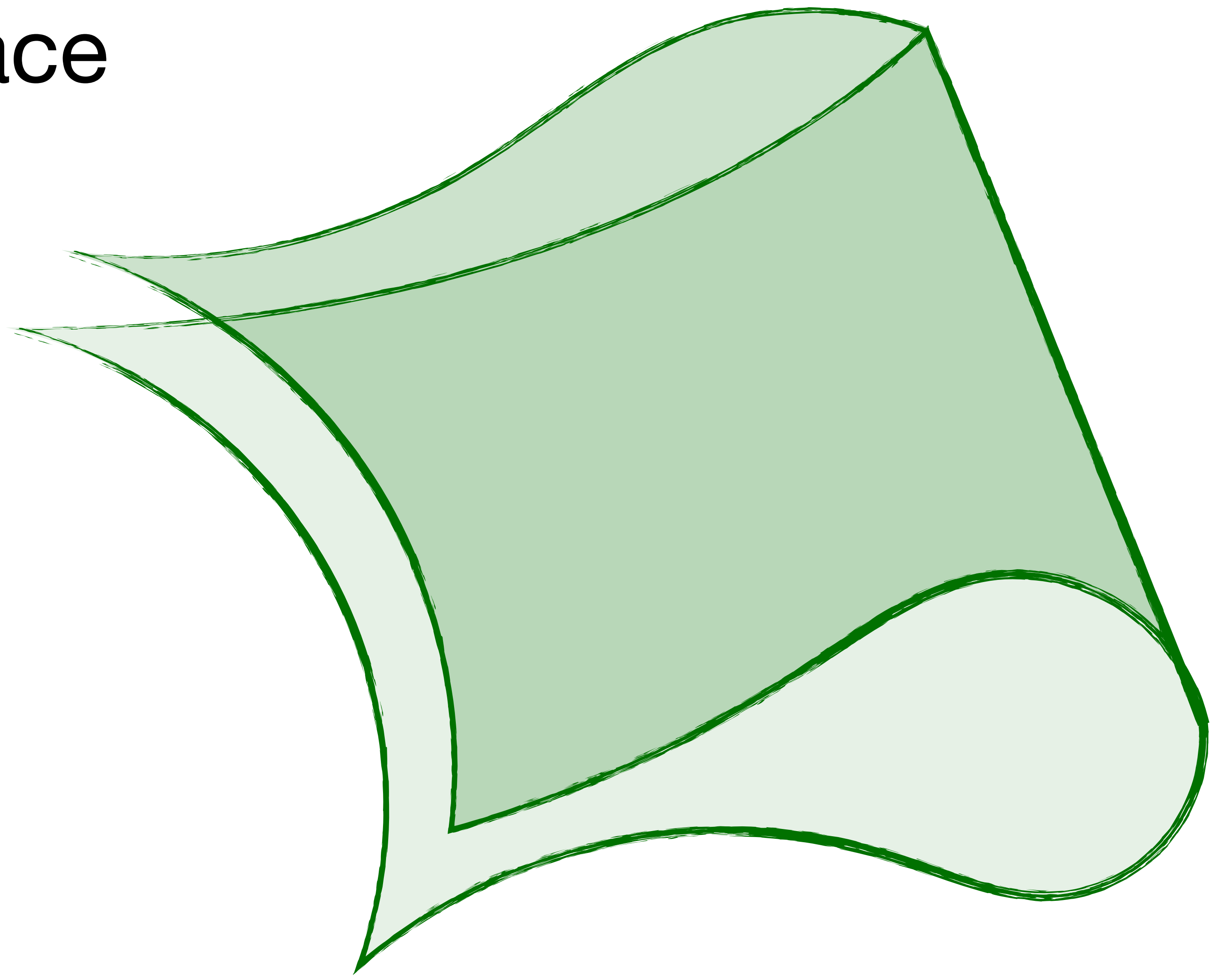

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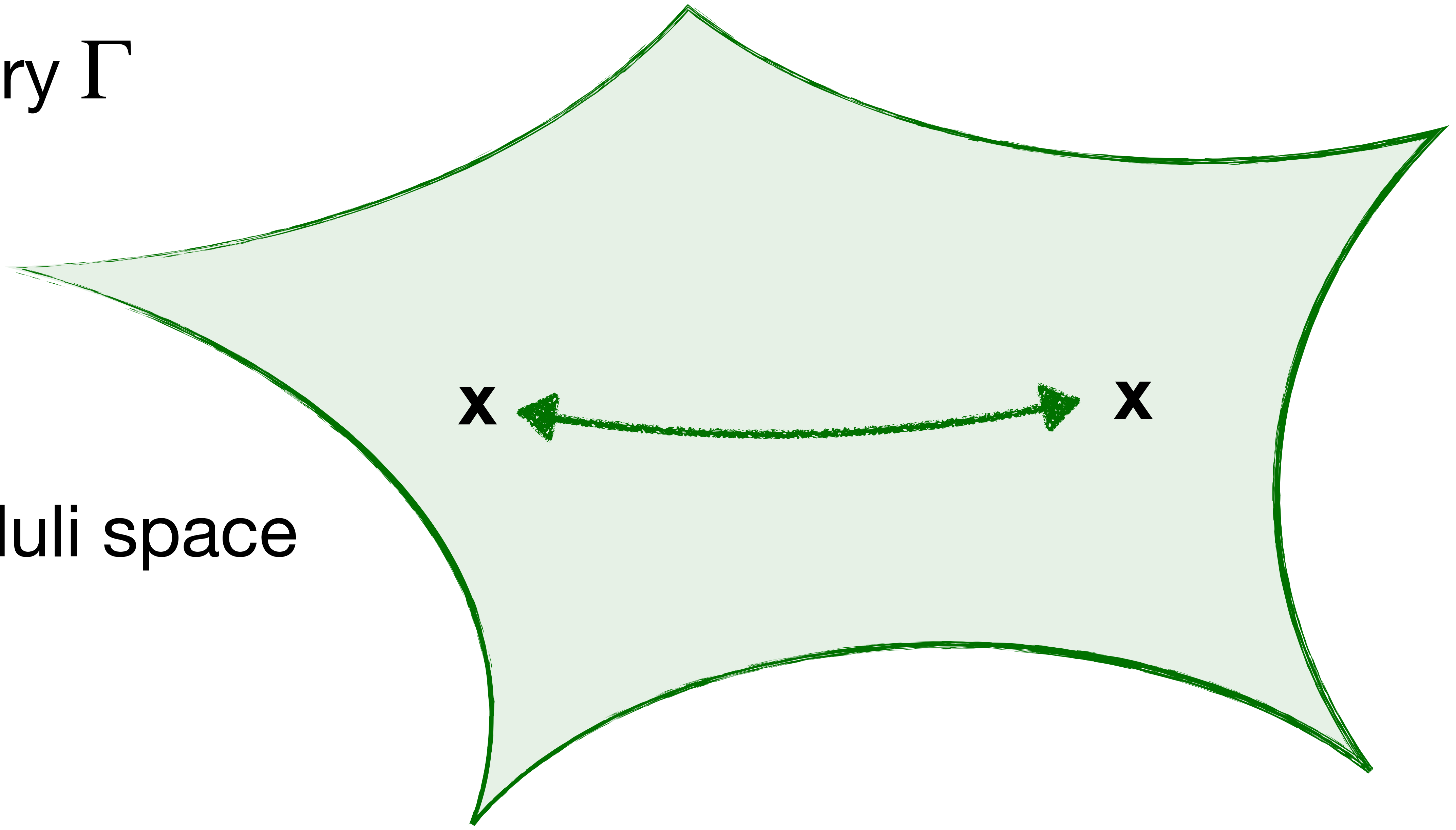

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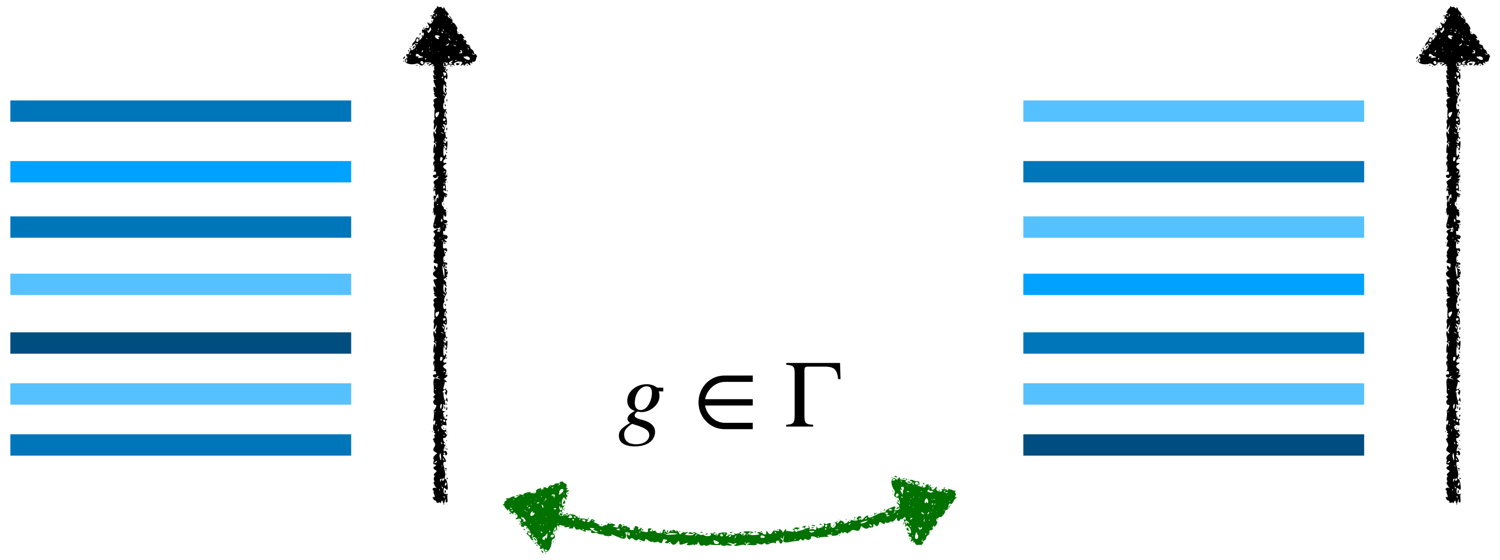

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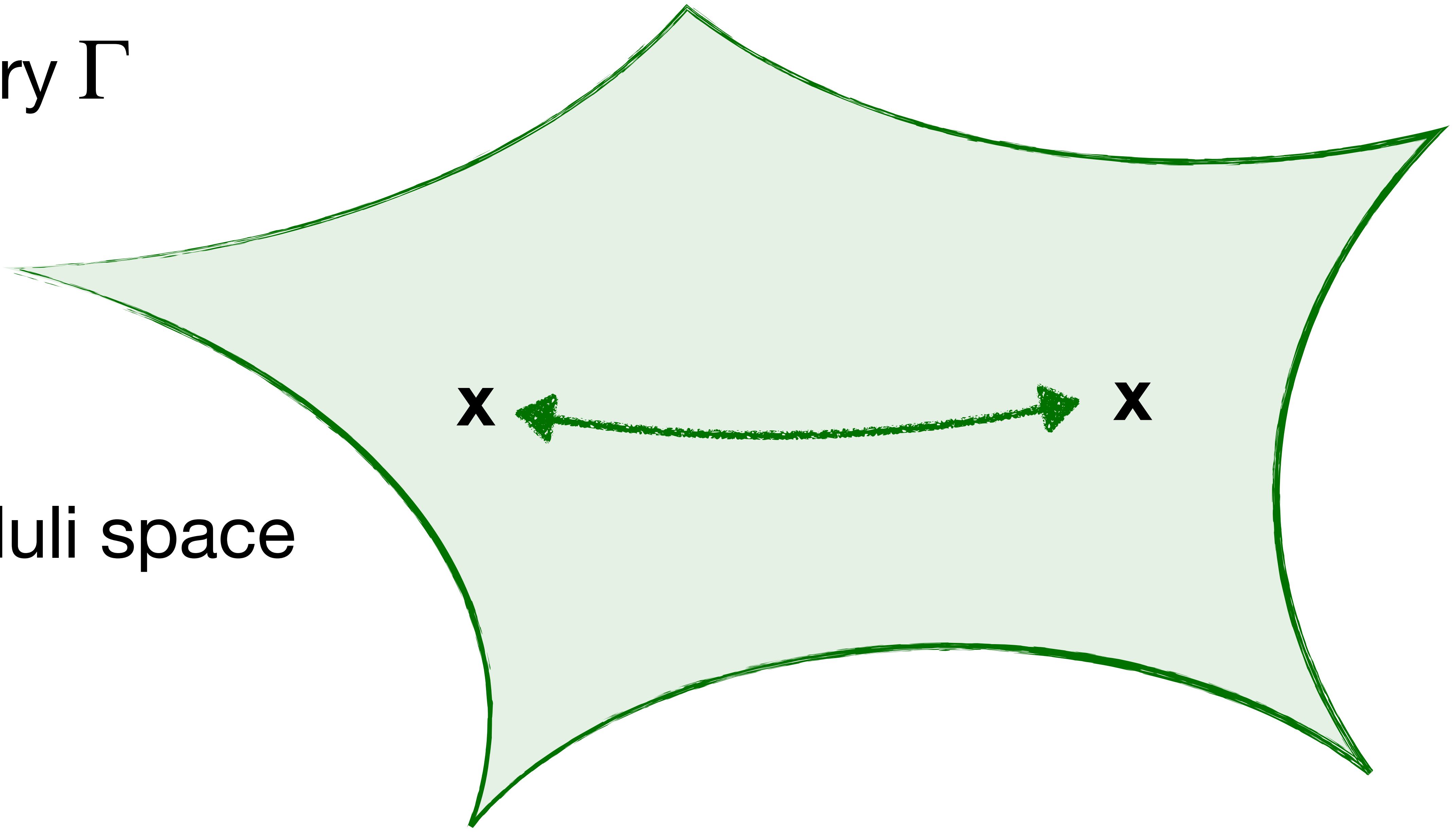

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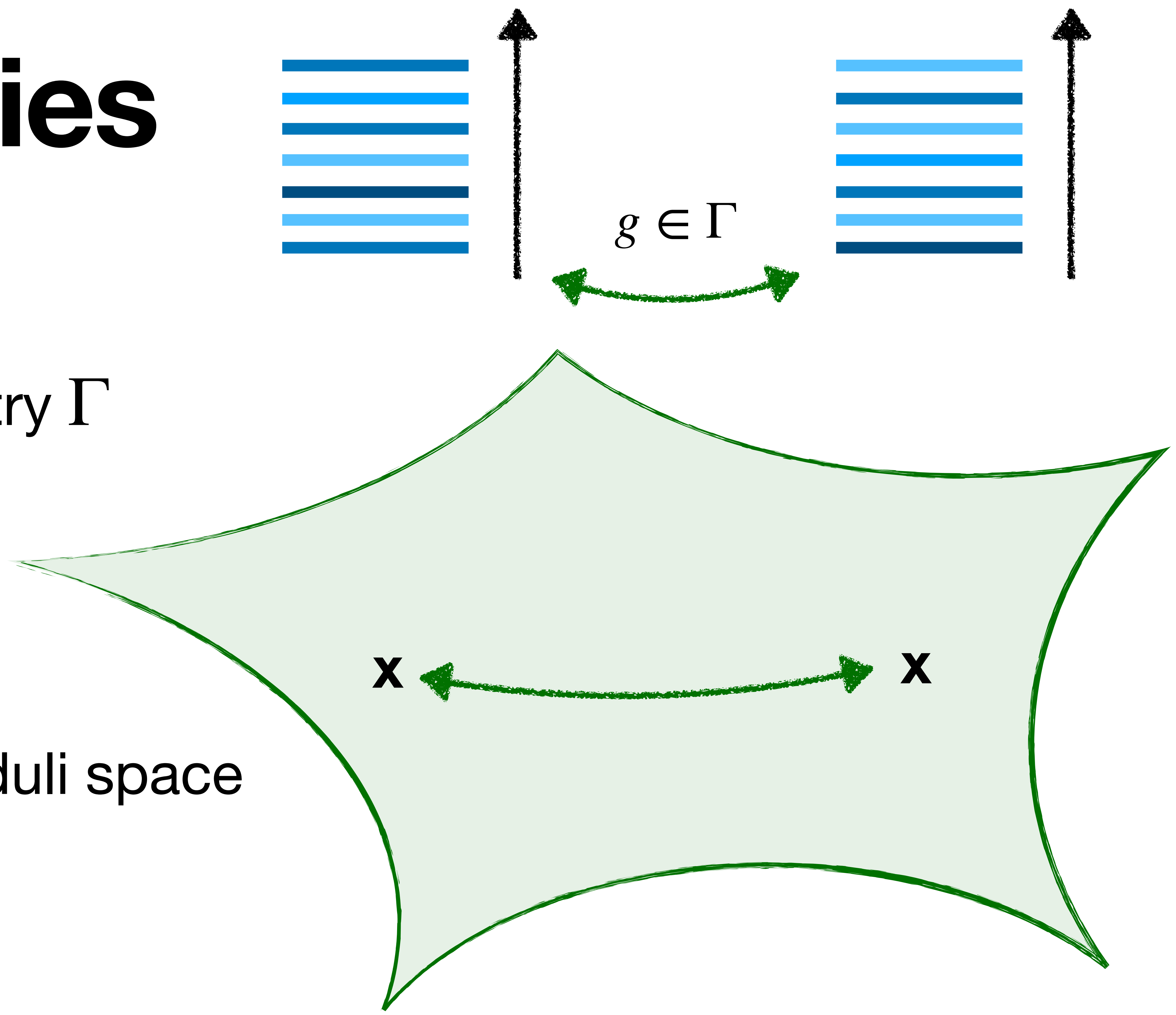

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- Duality vortices: codim-2 defects that implement the duality as you wind around

(7-branes in 10d Type IIB, axionic strings in 4d supergravity)

The Plan

Today: explore the role of *moduli space volumes* and *dualities* in Quantum Gravity/String Theory.

1. **Warm-up examples:** How does the volume grow?
   How do dualities act?

2. **4d $\mathcal{N} = 2$ CYs compactifications:** What is the representation of duality groups? What do these duality groups *explicitly* look like?

3. **Bottom-up argument for Compactifiability:** How do ground states see the moduli space? Is their finiteness related to the volume?

1. Warm-up examples

Example: 10d Type IIA string theory

Moduli space – $\mathscr{M} = \mathbb{R}$ real line parametrized by $g_s = e^{\phi}$

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$\implies$ Volume within distance Δ: Vol$(\mathscr{M}_\Delta) = 2\Delta$

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$\Longrightarrow$ Volume within distance Δ: Vol$(\mathscr{M}_\Delta) = 2\Delta$

Aside: the EFT with cut-off $\Lambda \leq \Lambda_{\text{species}}(\phi)$ has a moduli space of **finite diameter**

[DvdH, Vafa, Wiesner, Wu, '23]

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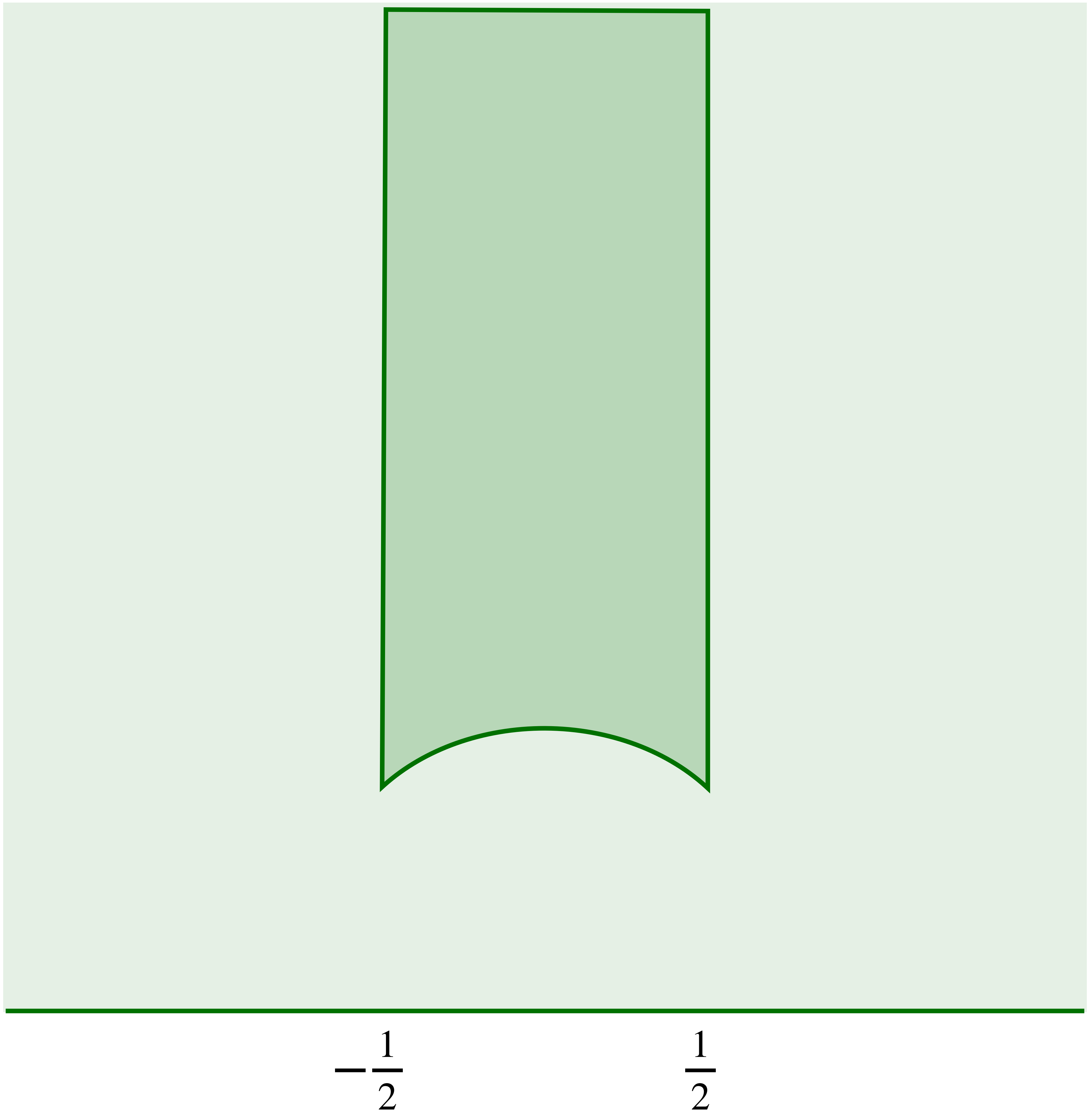

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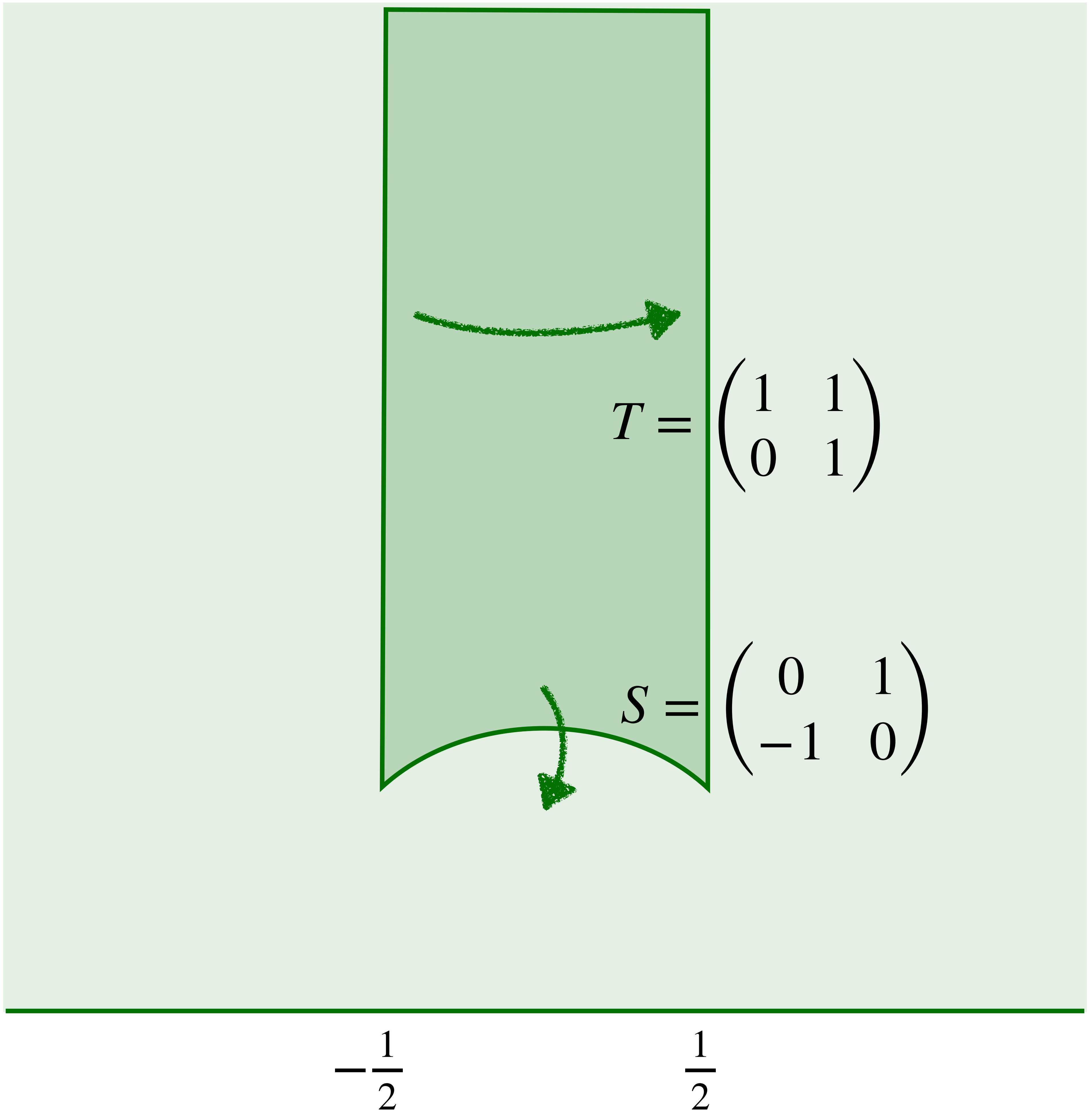

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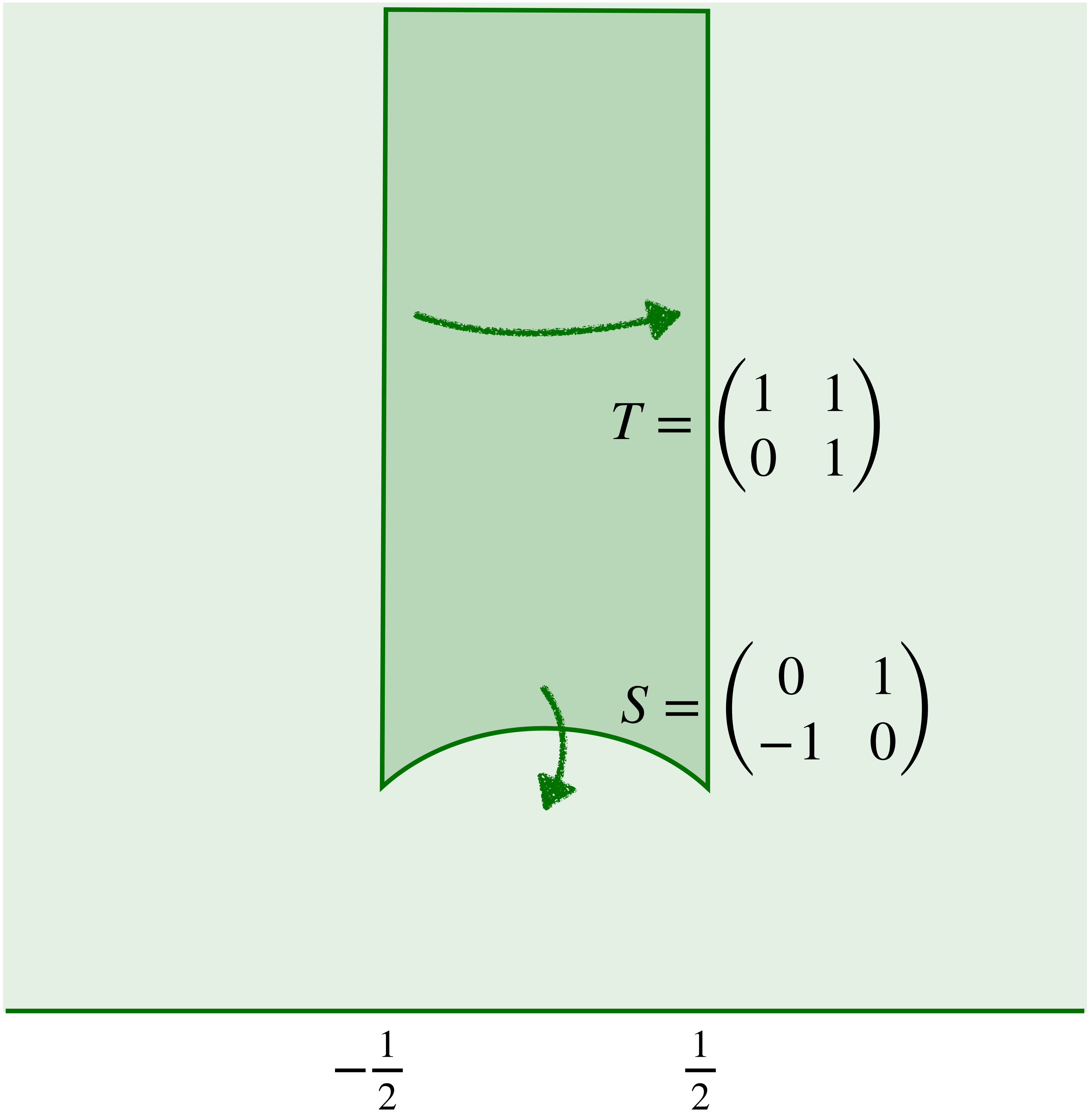

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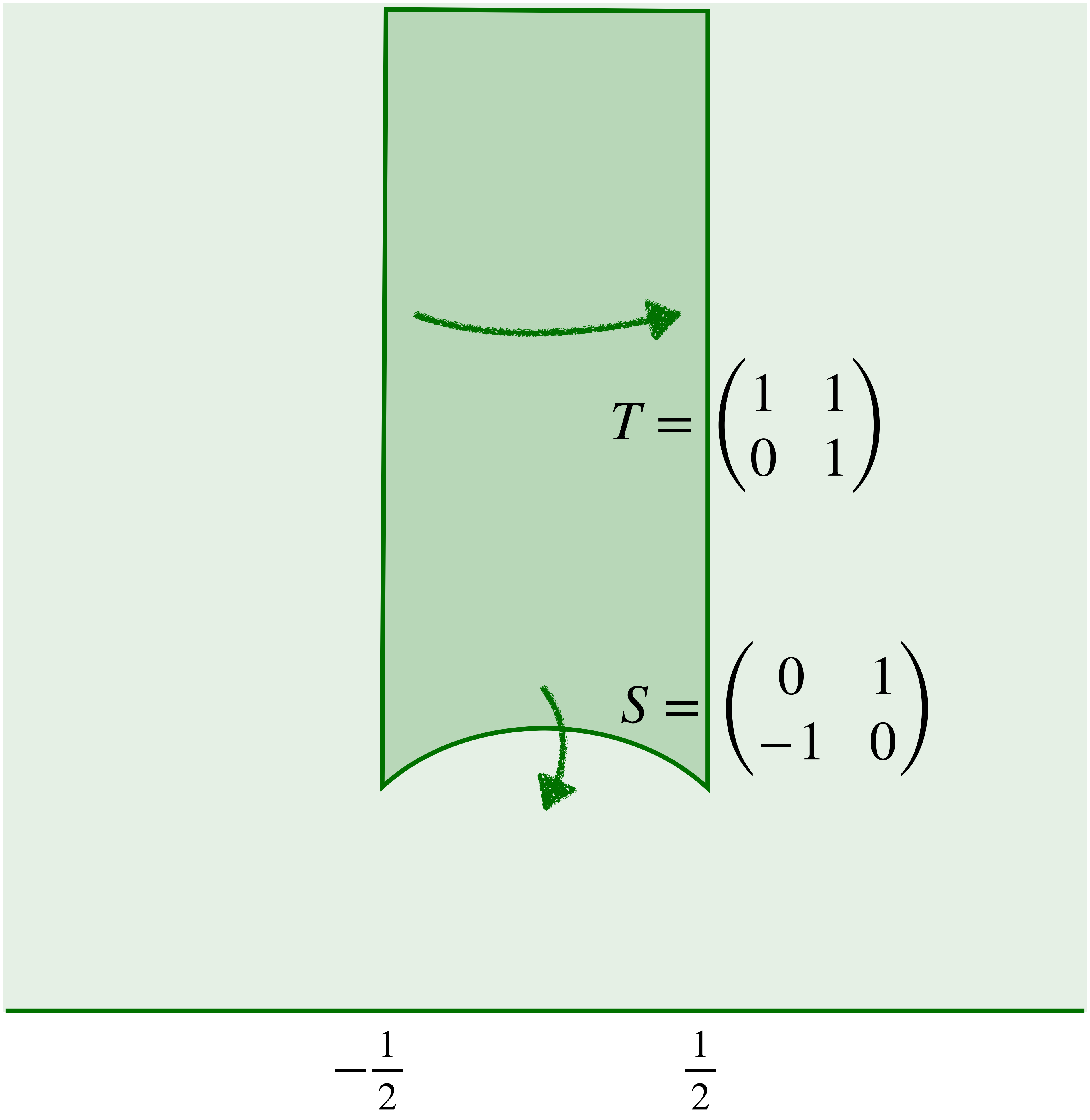
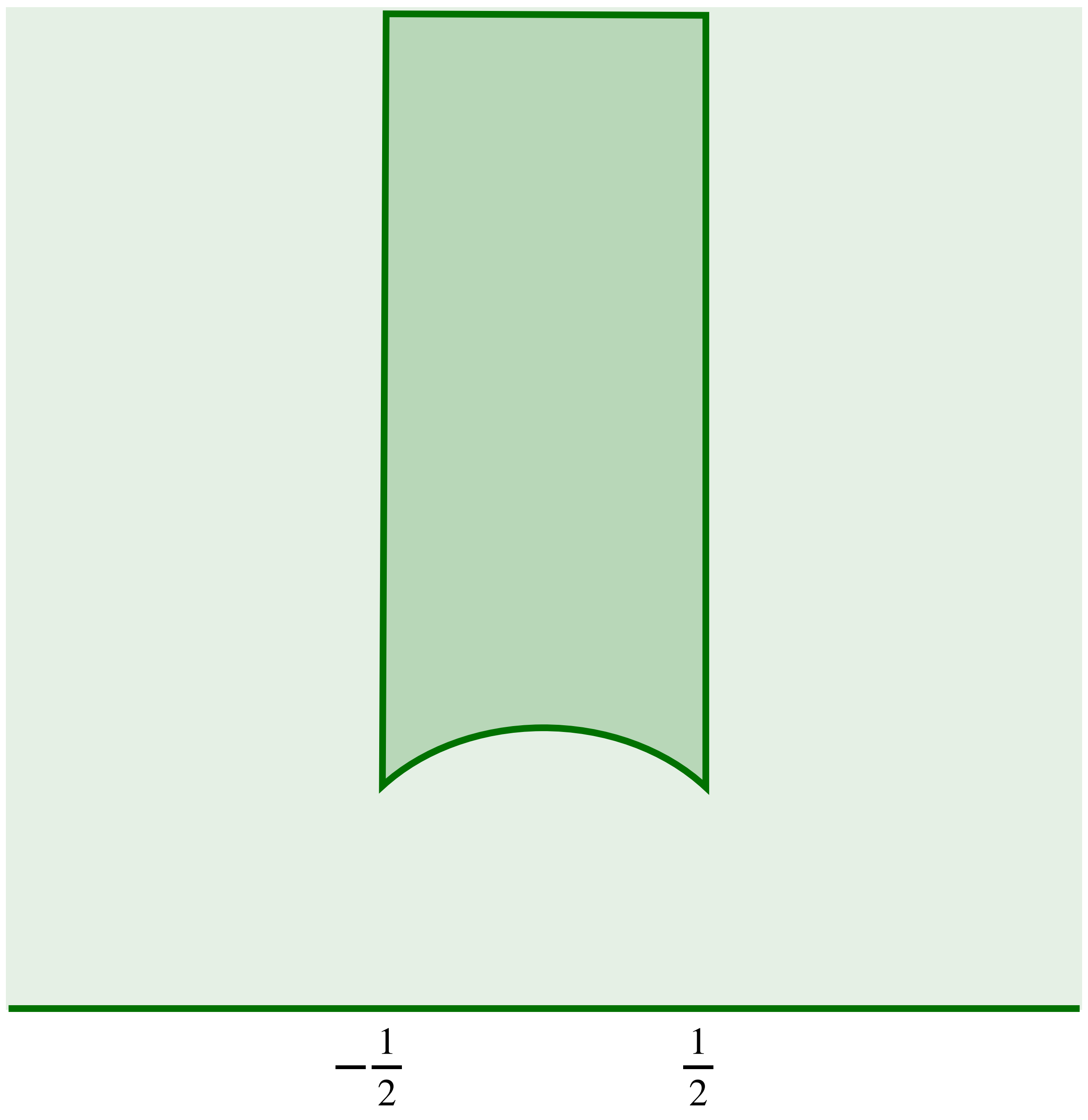
Duality vortices — (p,q) 7-branes:

$$T_{p,q} = g_{p,q}^{-1} T g_{p,q} = \begin{pmatrix} 1 + pq & p^2 \\ -q^2 & 1 - pq \end{pmatrix}$$



Type IIB comparison

$\Gamma = \mathrm{SL}(2,\mathbb{Z})$



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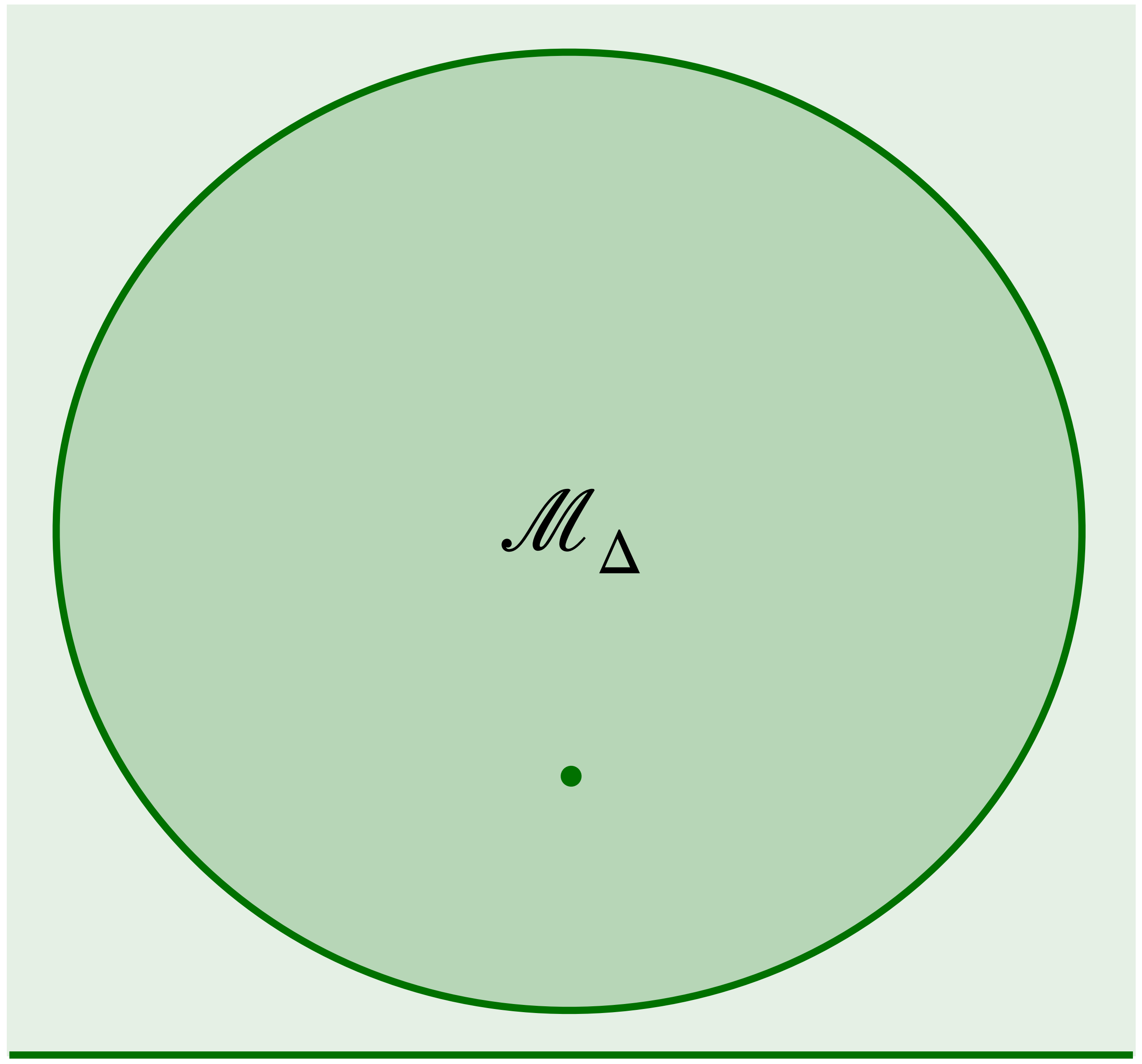

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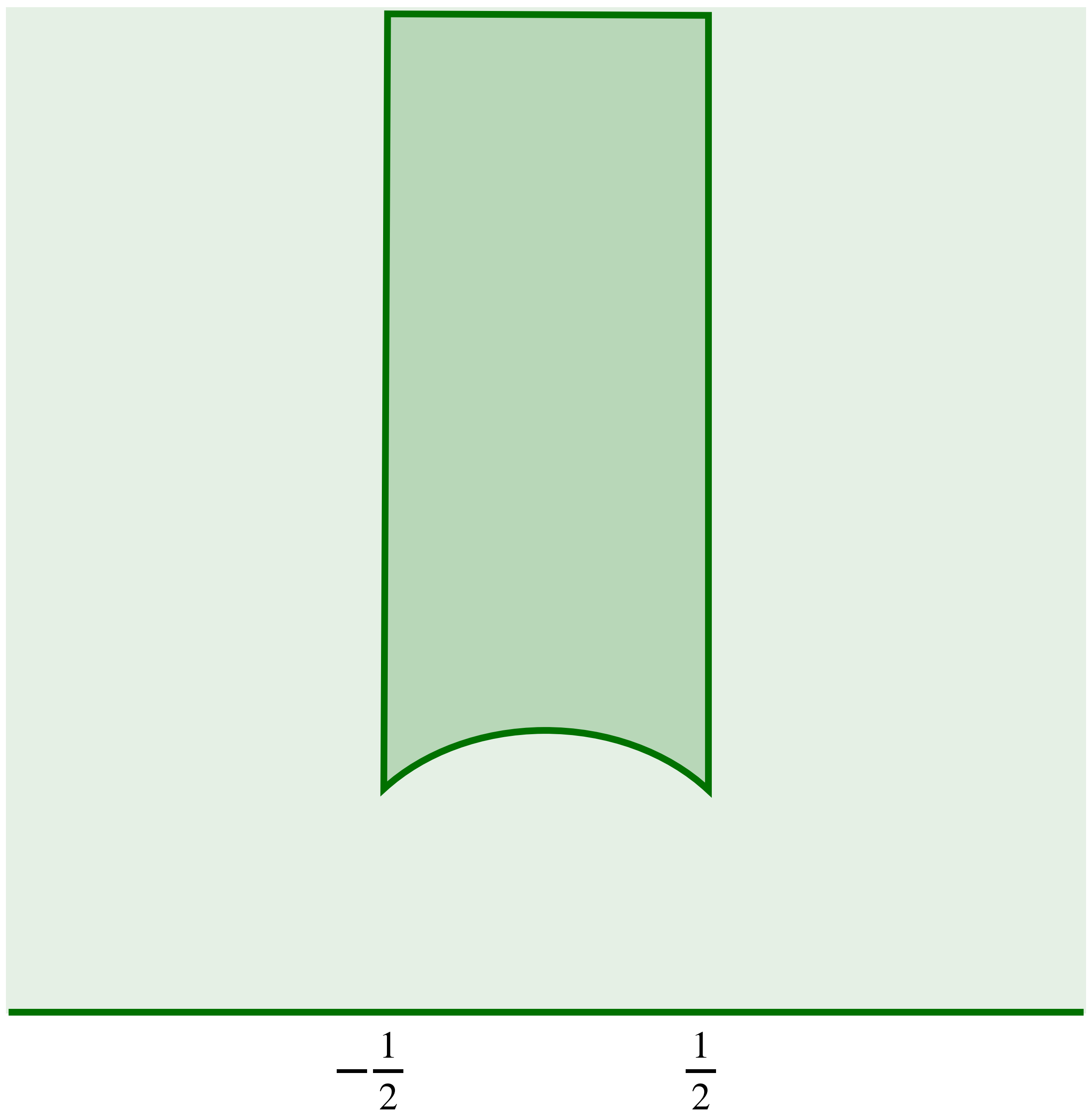

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*any finite-index $\Gamma \subset \mathrm{SL}(2,\mathbb{Z})$ works

(expect only genus-zero modular curves [Dierigl, Heckman '20])

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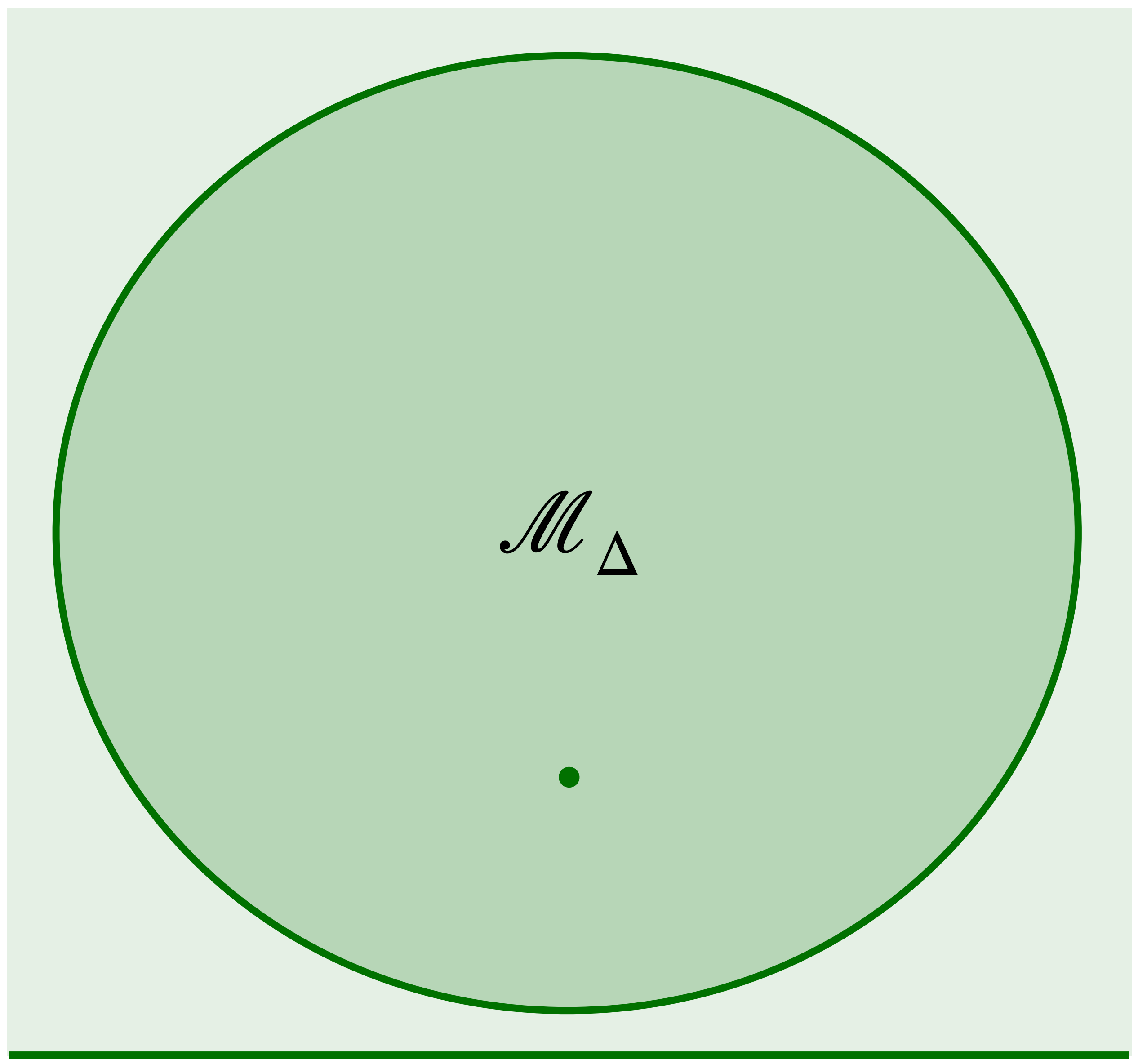

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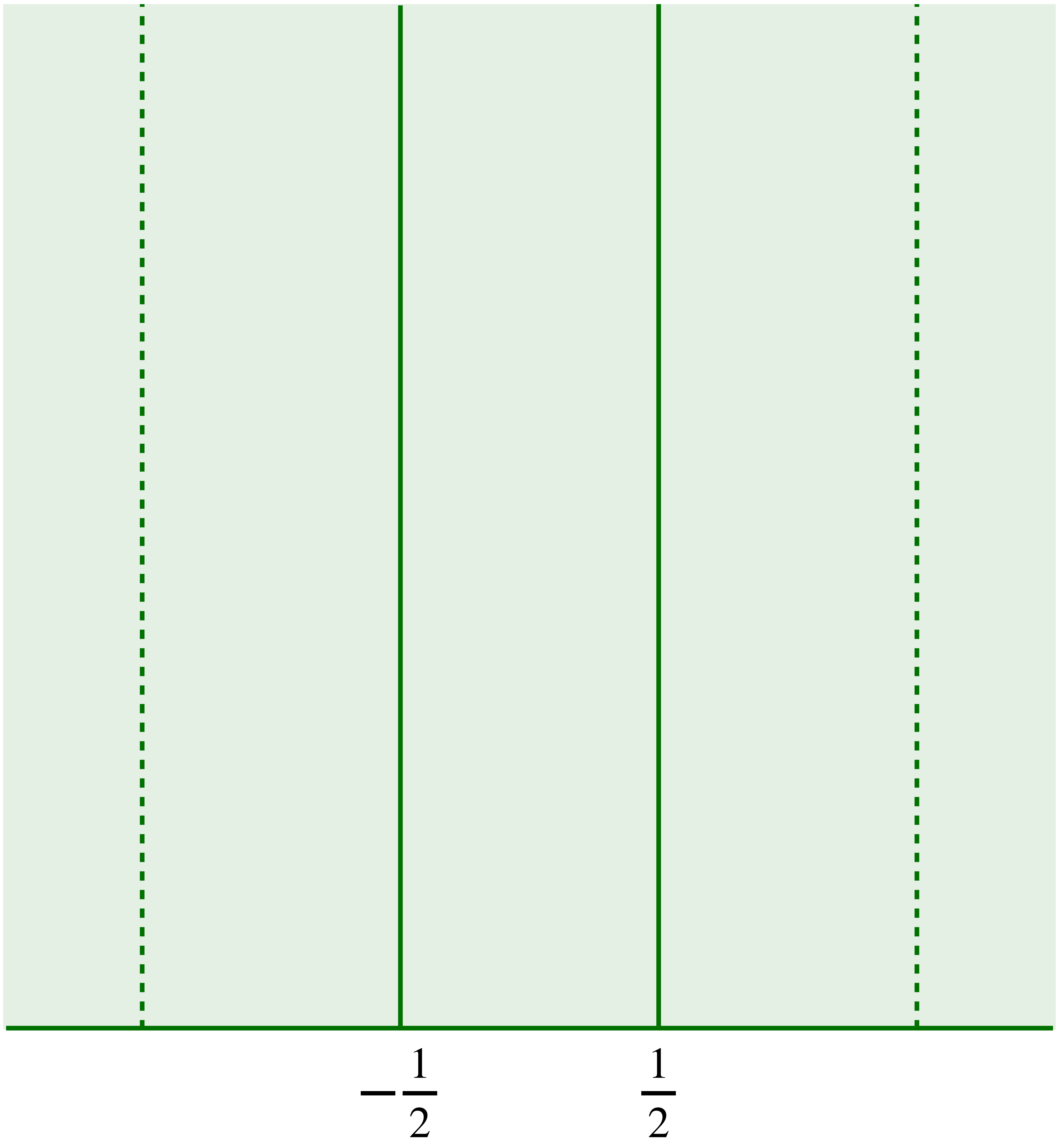
$$\Gamma_{\text{uni}} = \begin{pmatrix} 1 & \mathbb{Z} \\ 0 & 1 \end{pmatrix}$$



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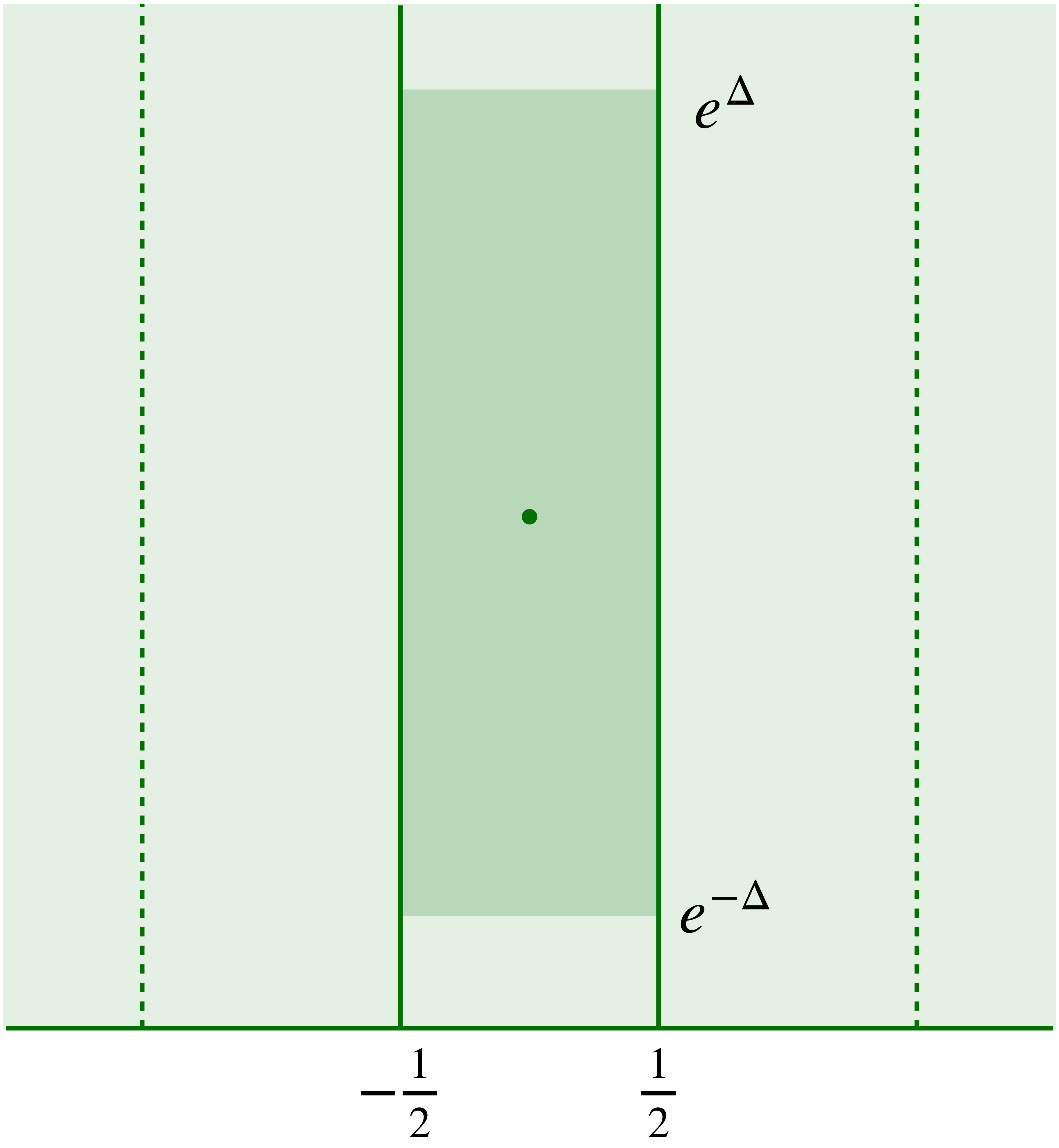

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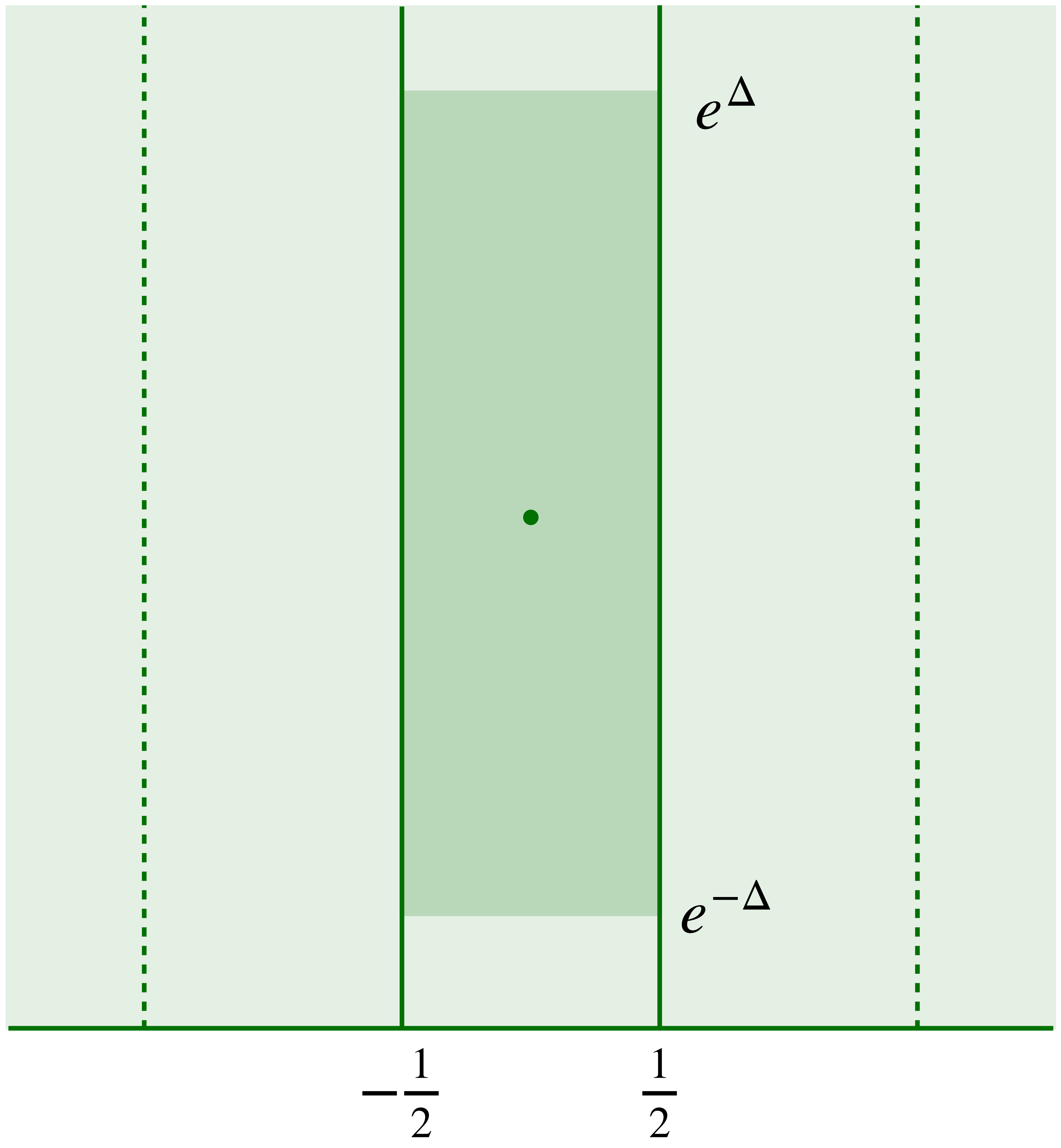

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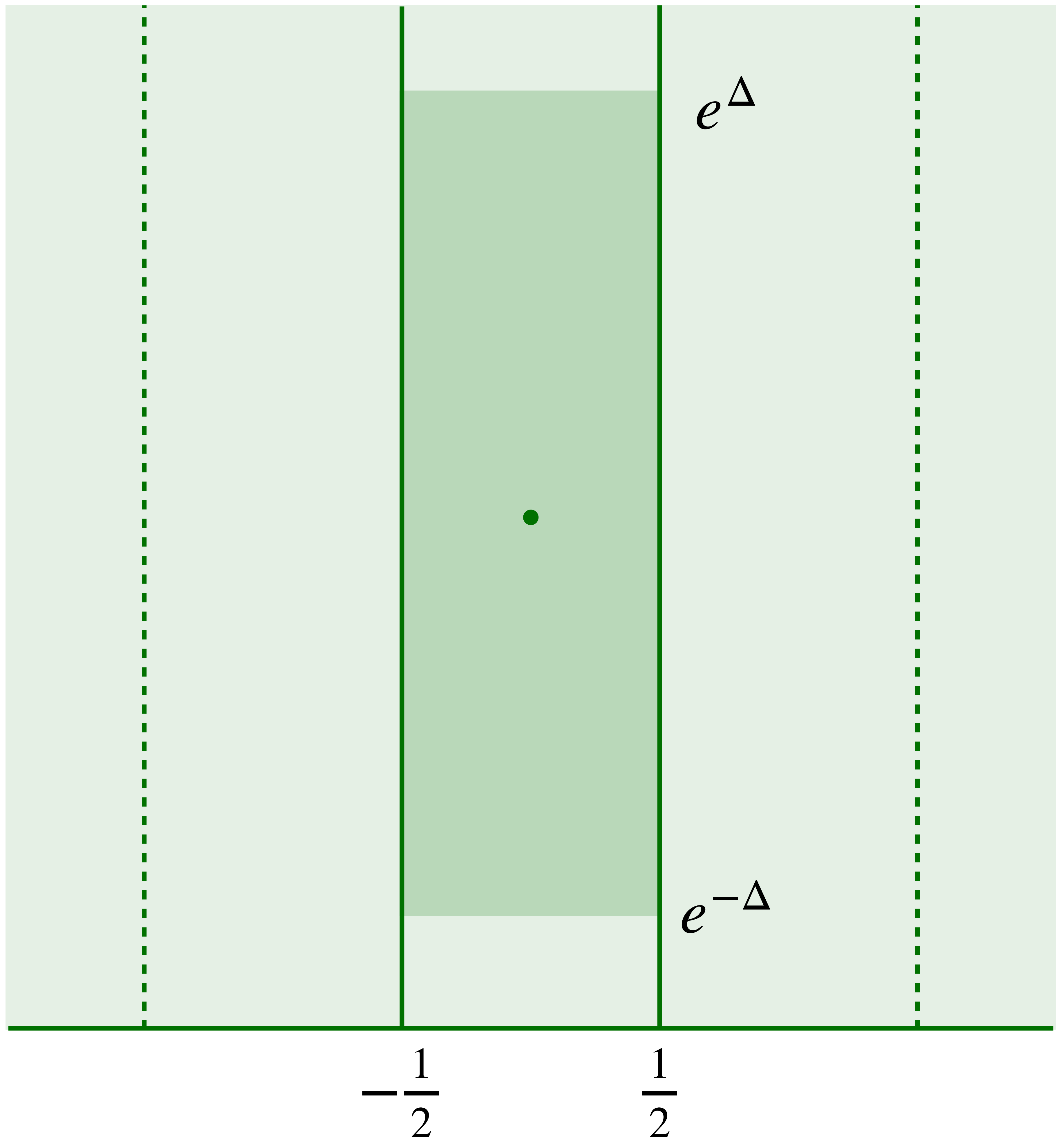

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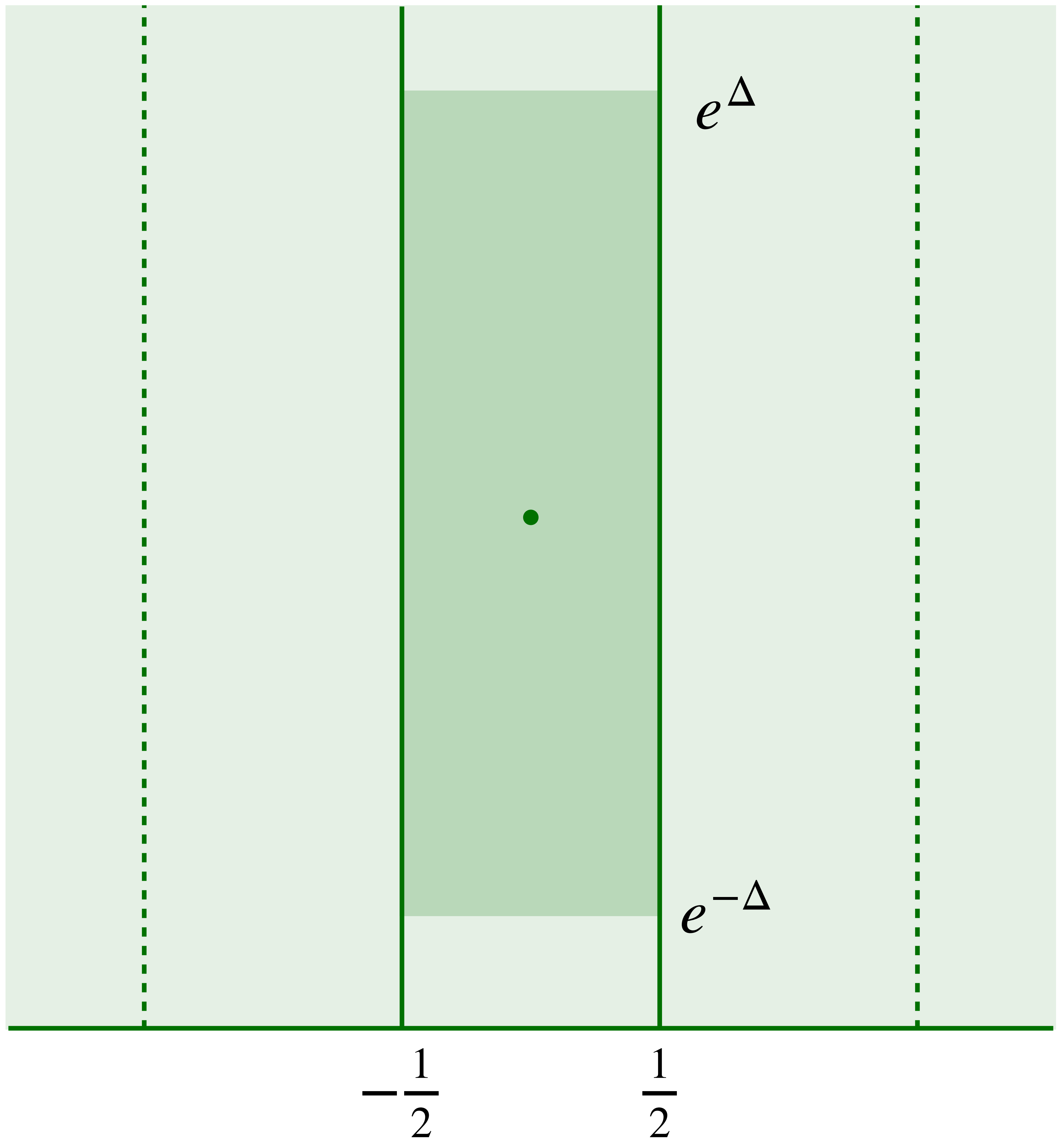
$\Longrightarrow$ Γ_{uni} is a *bad type* of duality group



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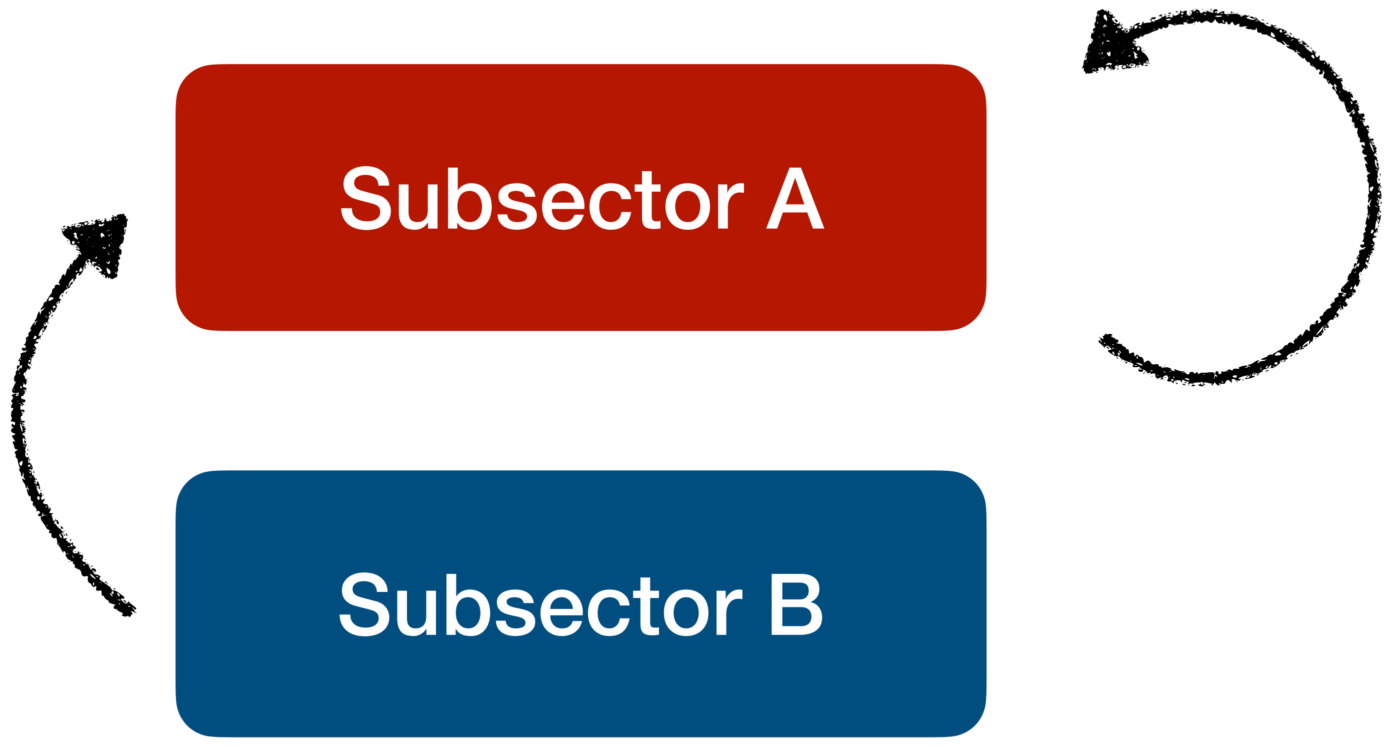

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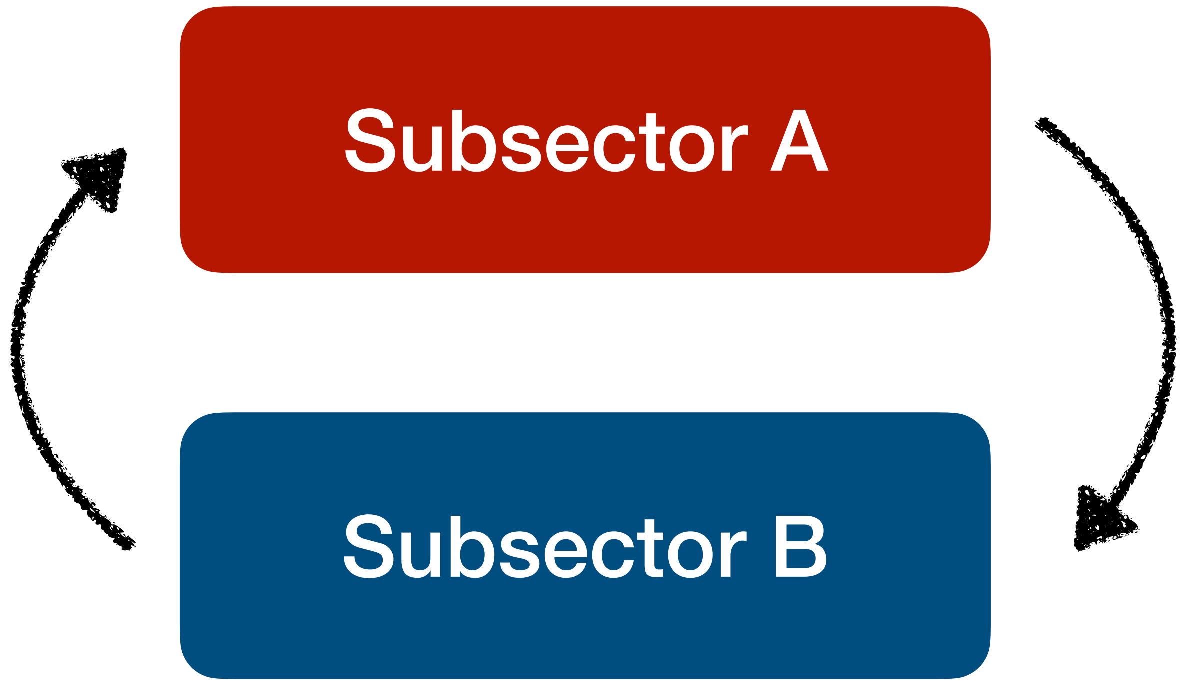

Semisimple

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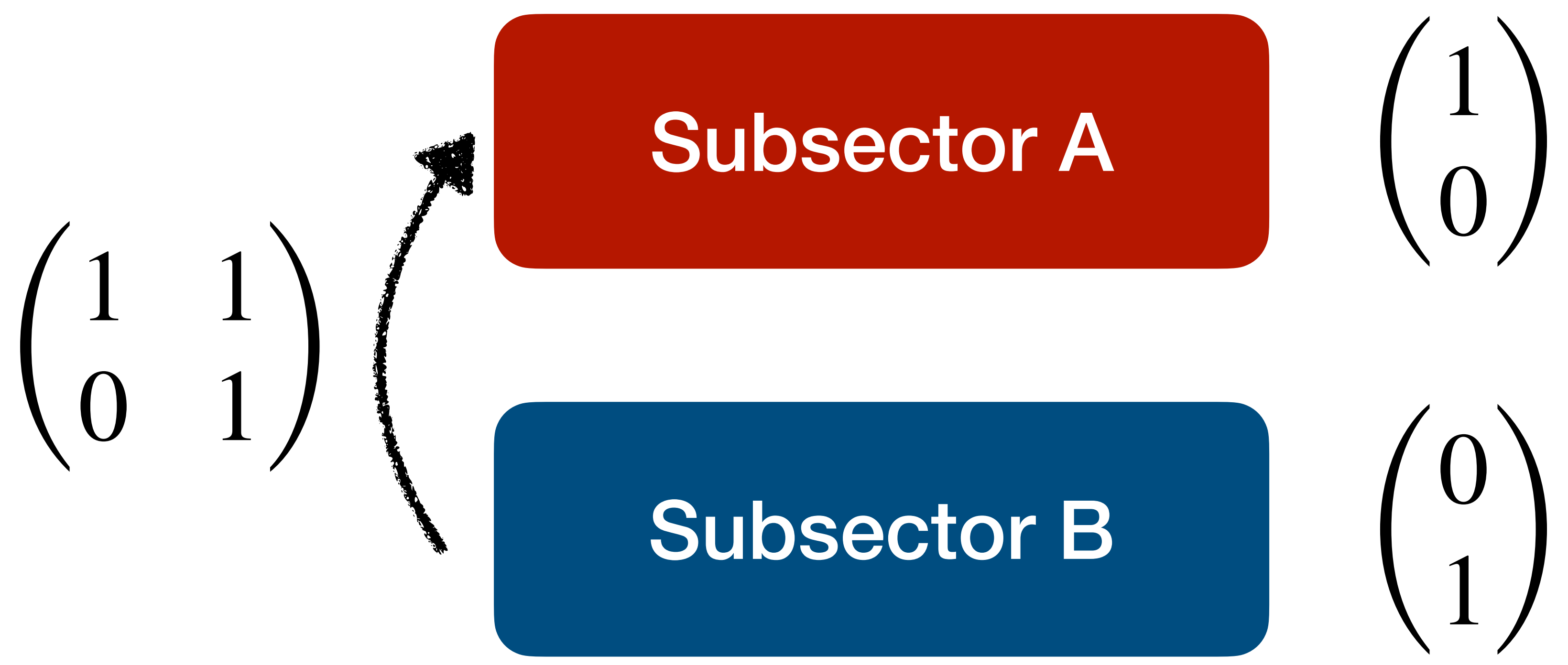

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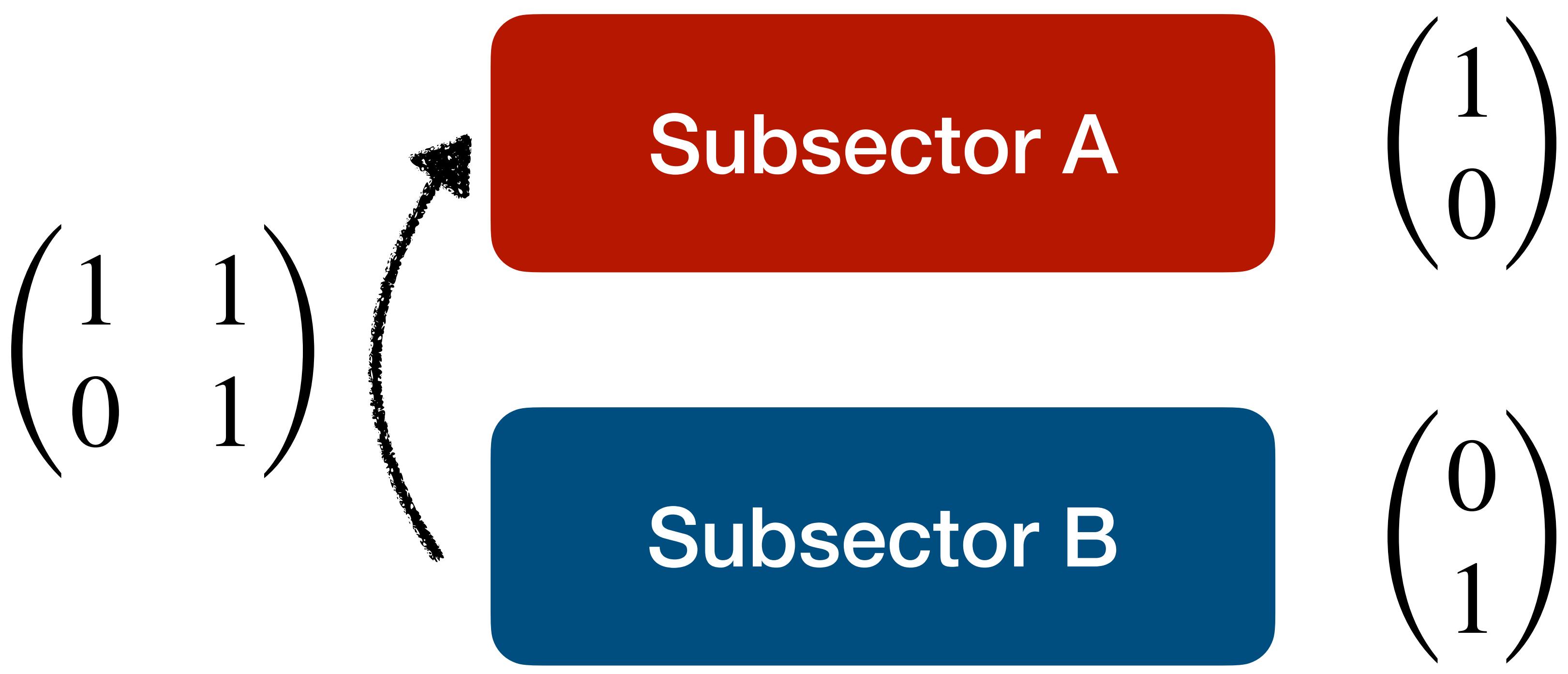
spectrum of states (codim>2) → V

irreducible subrepresentations → V_i

in Type IIB: (p,q)-strings, D3-branes, 5-branes, …



Non-semisimple

Can we link **semisimple dualities** and **compactifiable moduli spaces** in more involved examples?

2. 4d $\mathcal{N} = 2$ CY compactifications

Type IIB on Calabi-Yau threefolds

4d $\mathcal{N} = 2$ supergravity sector $\implies$ vector multiplet sector

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  mirror dual: $\mathbf{q} = (q_{D0}, q_{D2}, q_{D4}, q_{D6})$

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- **Duality vortices —** axionic strings, [Lanza, Marchesano, Martucci, Valenzuela, '21; …]

  e.g. from wrapping NS5-branes on divisors (in Type IIA)

  Type IIB: [Friedrich, Monnee, Weigand, Wiesner '25] Talk by Max!

Compactifiability and semisimple dualities

General proof for **semisimple dualities** from **compactifiability** in Hodge theory:

[Schmid, '70]

(same asymptotic Hodge theory machinery as [Grimm, Palti, Valenzuela, '18; …])

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$\mathscr{M}_{\text{vector}}$

$\Longrightarrow$

Semisimple electromagnetic dualities

$\Gamma_{\text{EM}} \subseteq Sp(2n_V + 2, \mathbb{Z})$

Finite-volume proven for CY3 moduli spaces

[Todorov, '04; Lu, Sun '05]

Sketch of the proof [W. Schmid, ’70]

How does this proof of semisimplicity roughly work?

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for any Γ-invariant subspace $V \subset W$, there is a complementary Γ-invariant subspace V' s.t. $V \oplus V' = W$

(subrepresentation) $W = \mathbb{R}^{2n_V+2}$ (subrepresentation)

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(For Type IIB: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ vs $\frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}$)

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- Hodge product $\int_{Y_3} v \wedge \star w$: positive definite, but moduli-dependent $\Longrightarrow$ need compactifiability!

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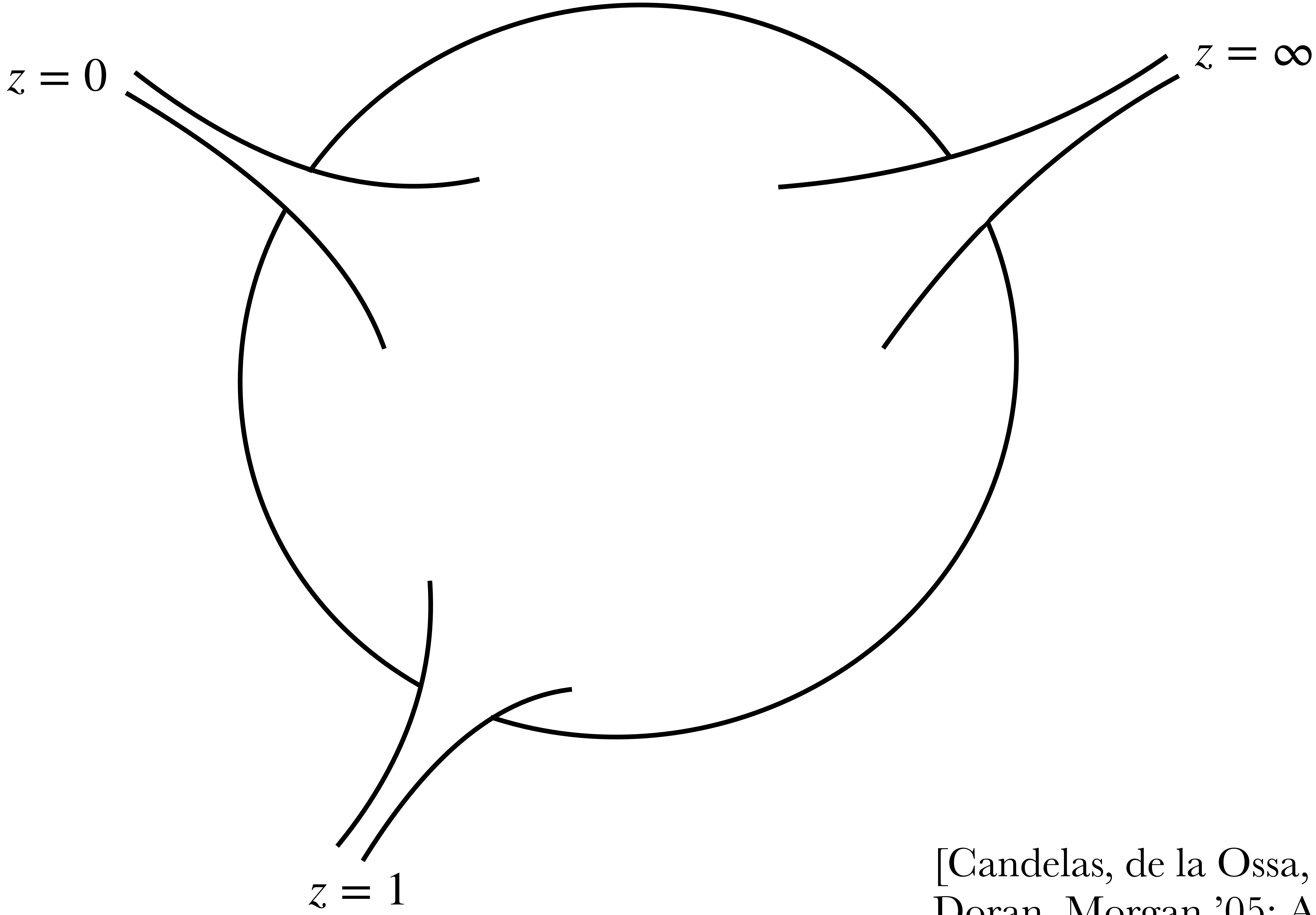
Calabi-Yau threefolds with $\mathscr{M}_{\rm cs} = \mathbb{P}^1 - \{0,1,\infty\}$



[Candelas, de la Ossa, Green, Parkes '93;…; Doran, Morgan '05; Almkvist, van Enckevort, van Straten, Zudilin, '05]

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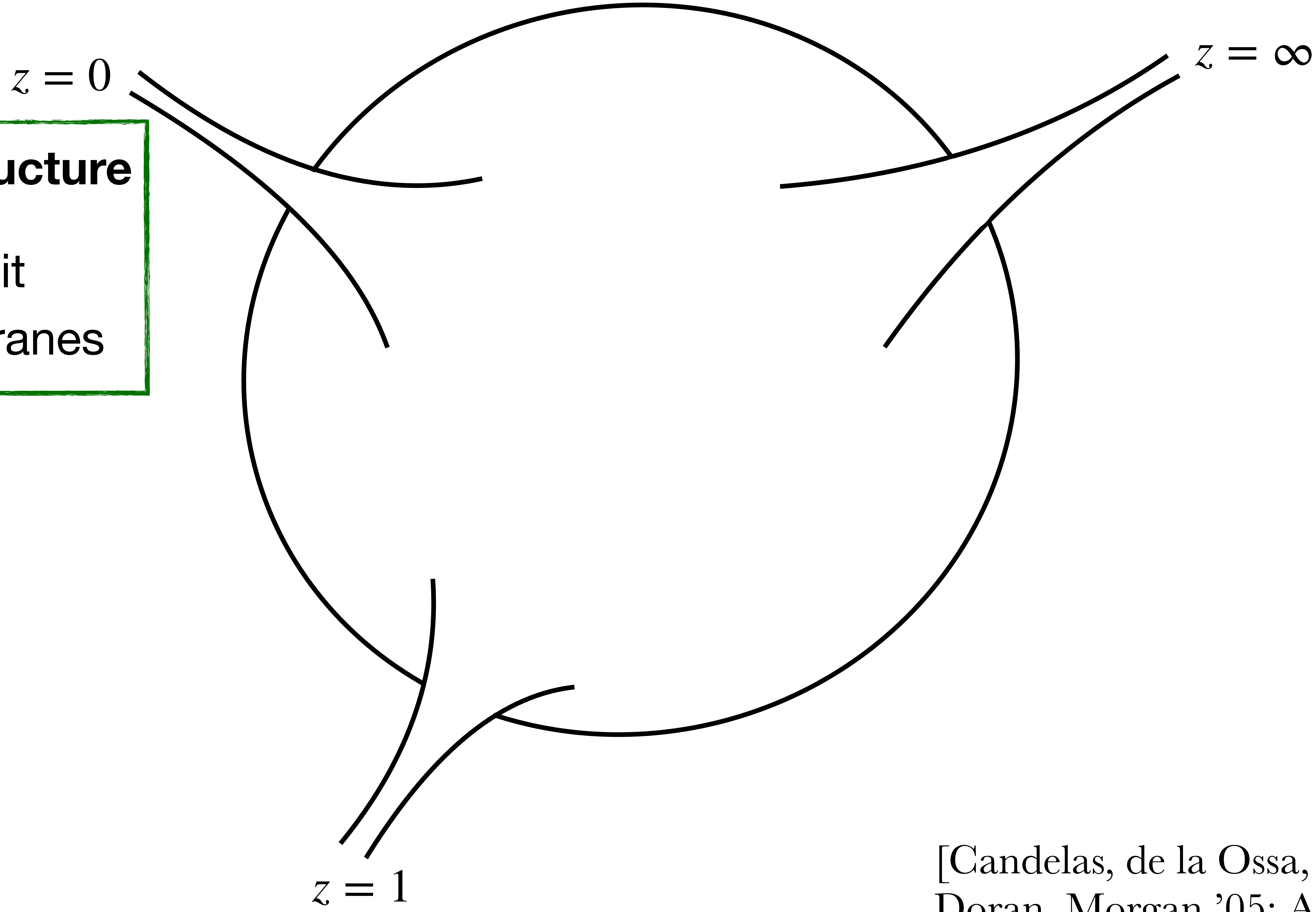
Large complex structure

Infinite distance limit

Tower: mirror D0-branes



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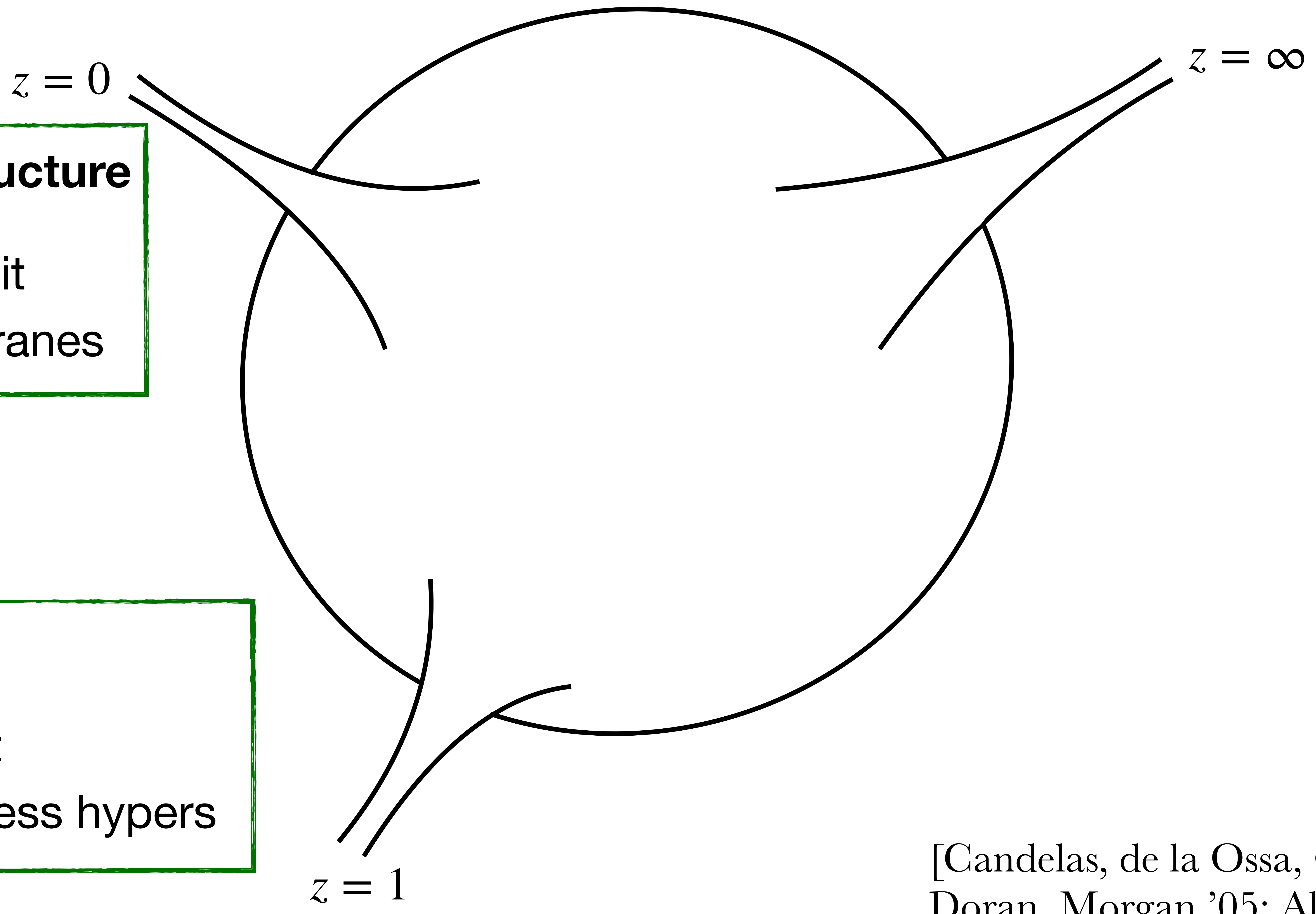

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Conifold point

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Light states: massless hypers

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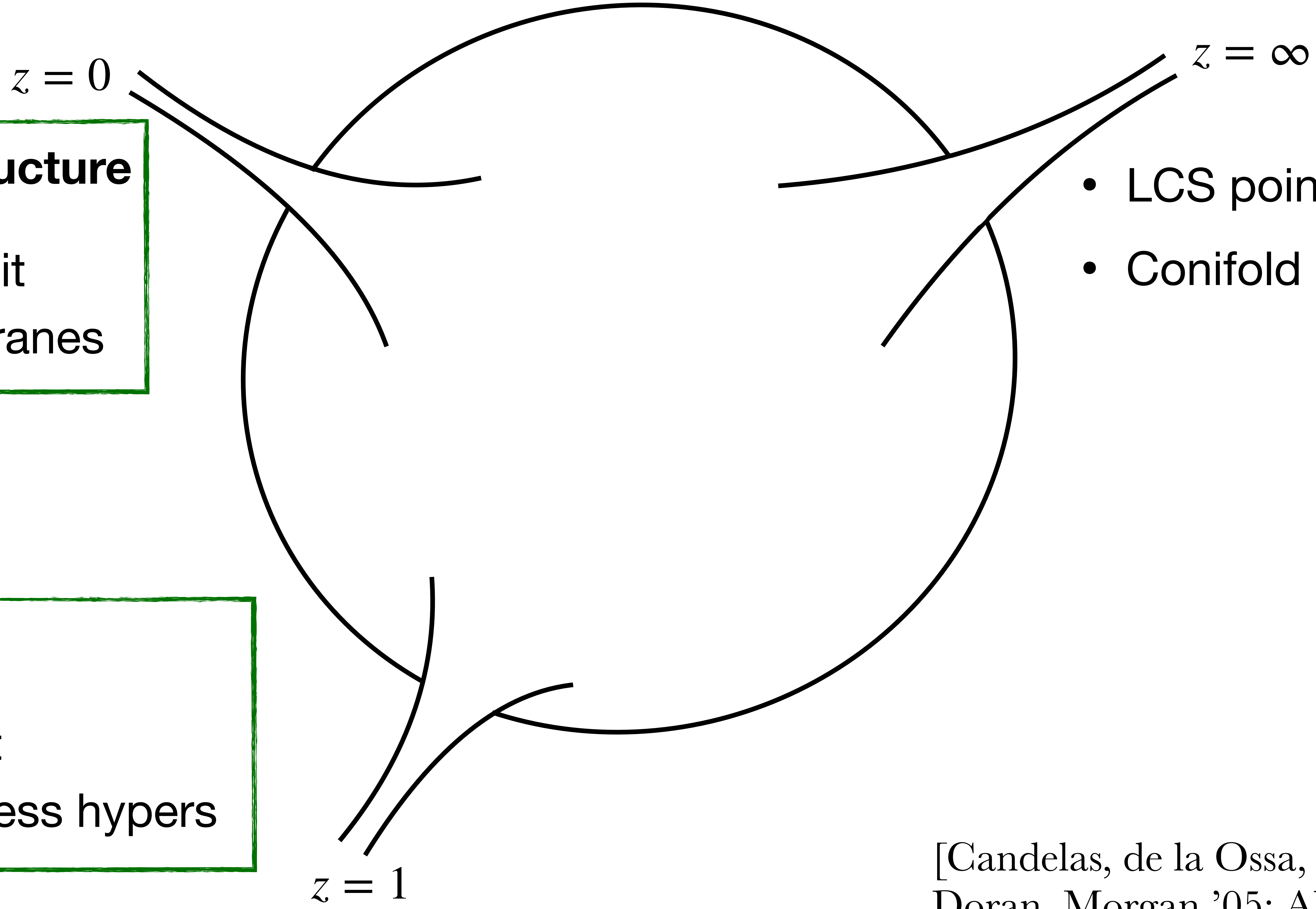

- LCS point (1x)
- Conifold point (3x)

Conifold point

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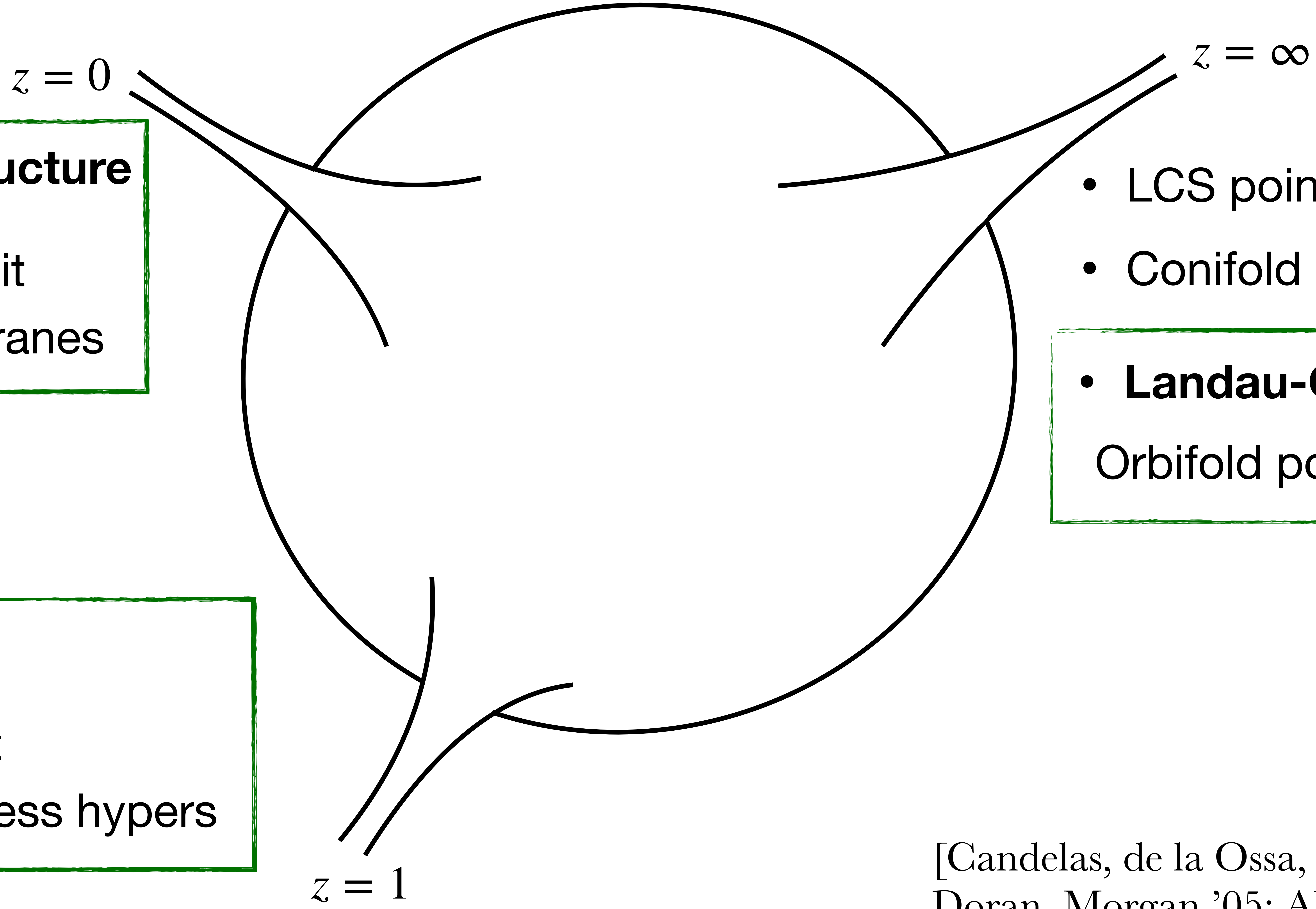

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Orbifold point, no light states

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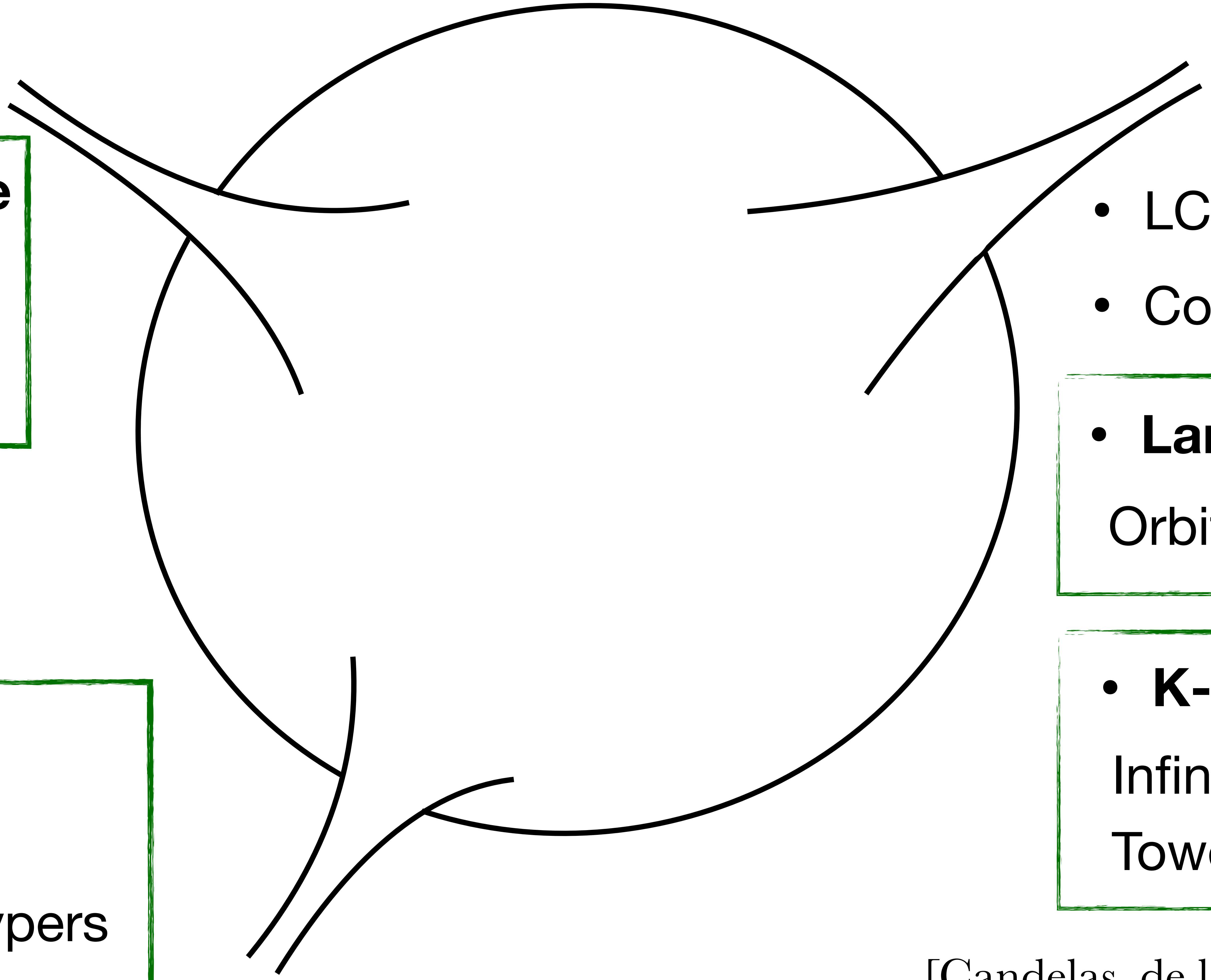

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- Conifold point (3x)

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Orbifold point, no light states

- **K-point** (3x)

Infinite distance limit

Tower: tensionless string

[Candelas, de la Ossa, Green, Parkes ’93;…; Doran, Morgan ’05; Almkvist, van Enckevort, van Straten, Zudilin, ’05]

Monodromies and volume

Large complex structure

$$M_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -\frac{\kappa}{2} - \sigma & -\kappa & 1 & 0 \\ \frac{c_2 + 2\kappa}{12} & \frac{\kappa}{2} - \sigma & -1 & 1 \end{pmatrix}$$

Conifold point

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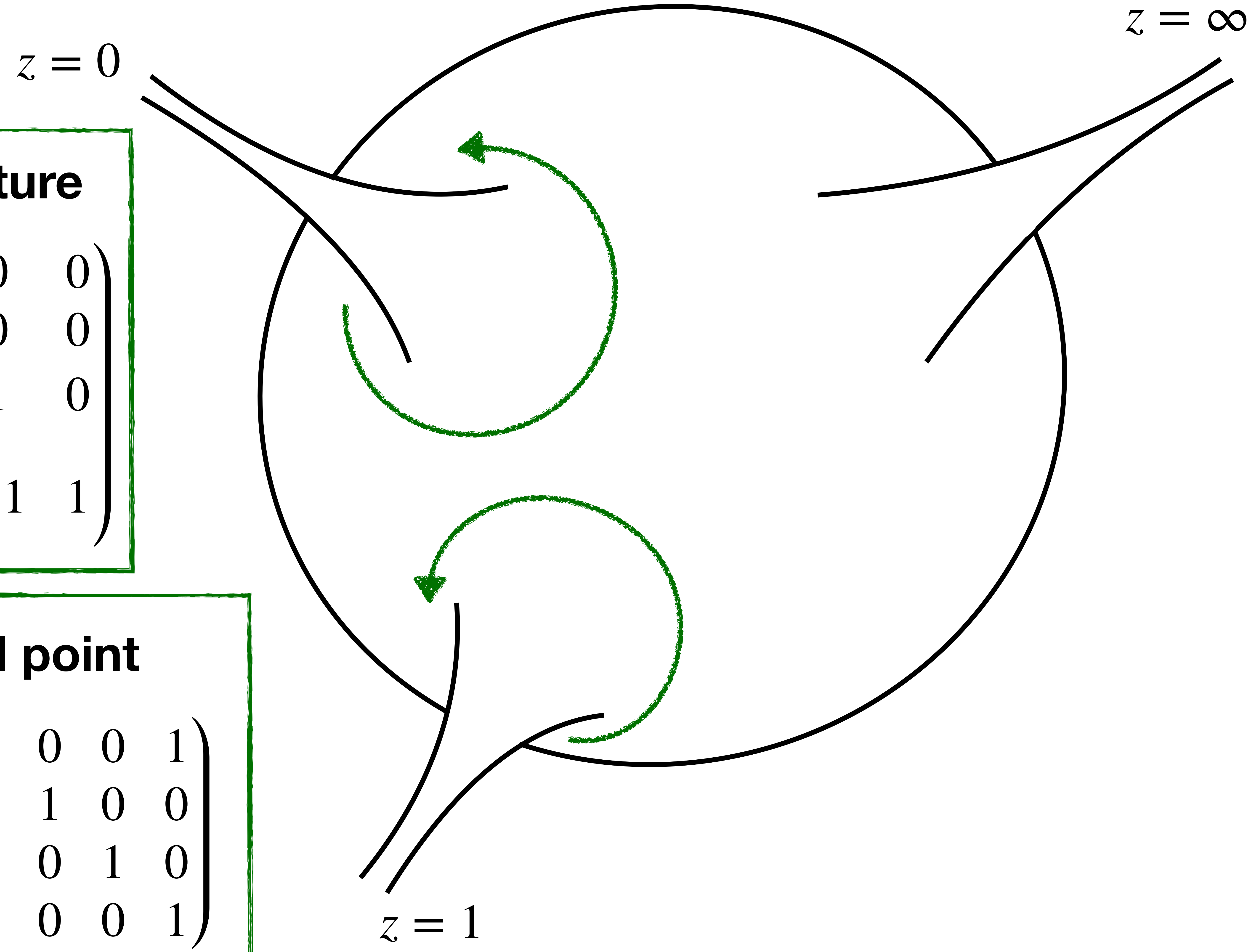

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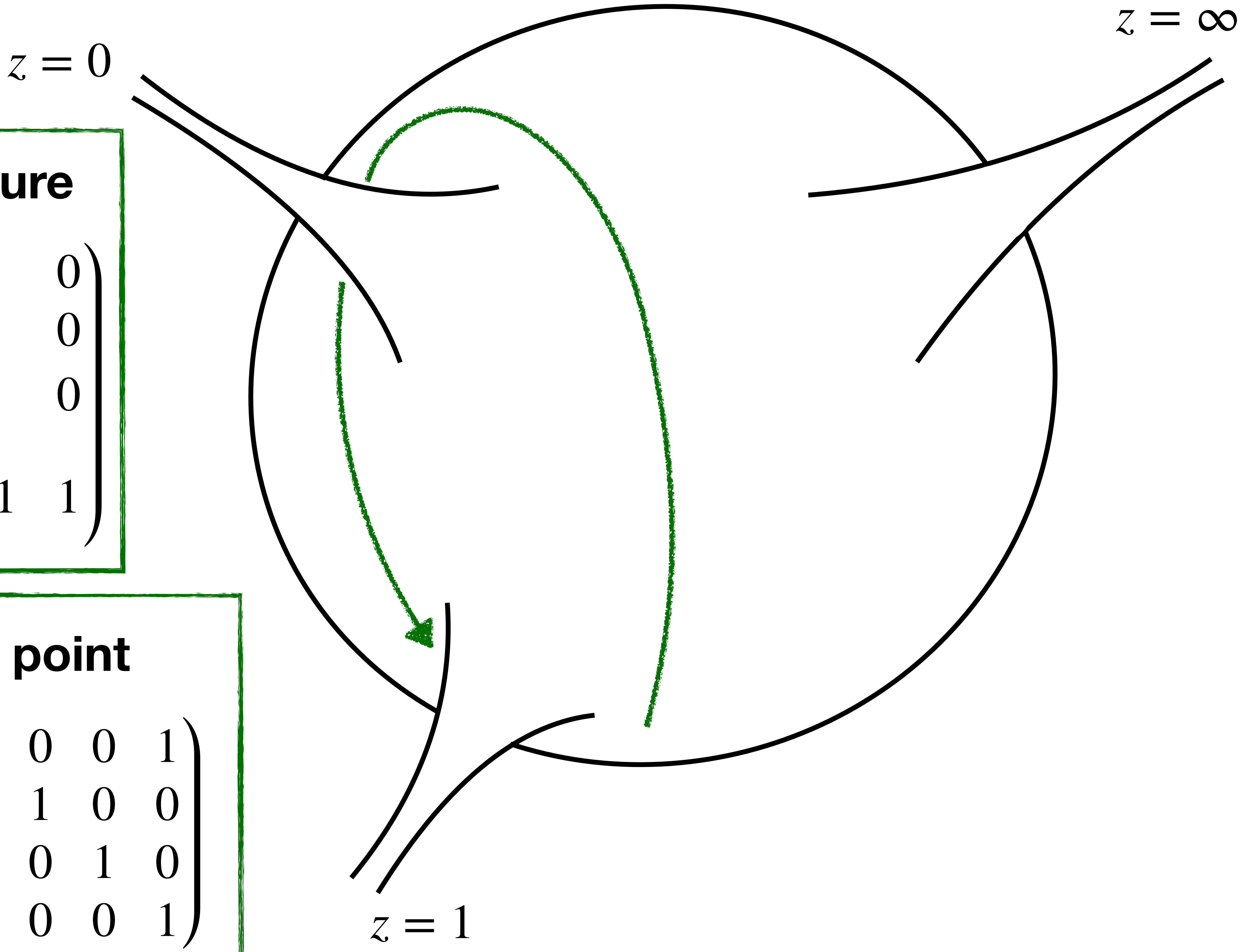

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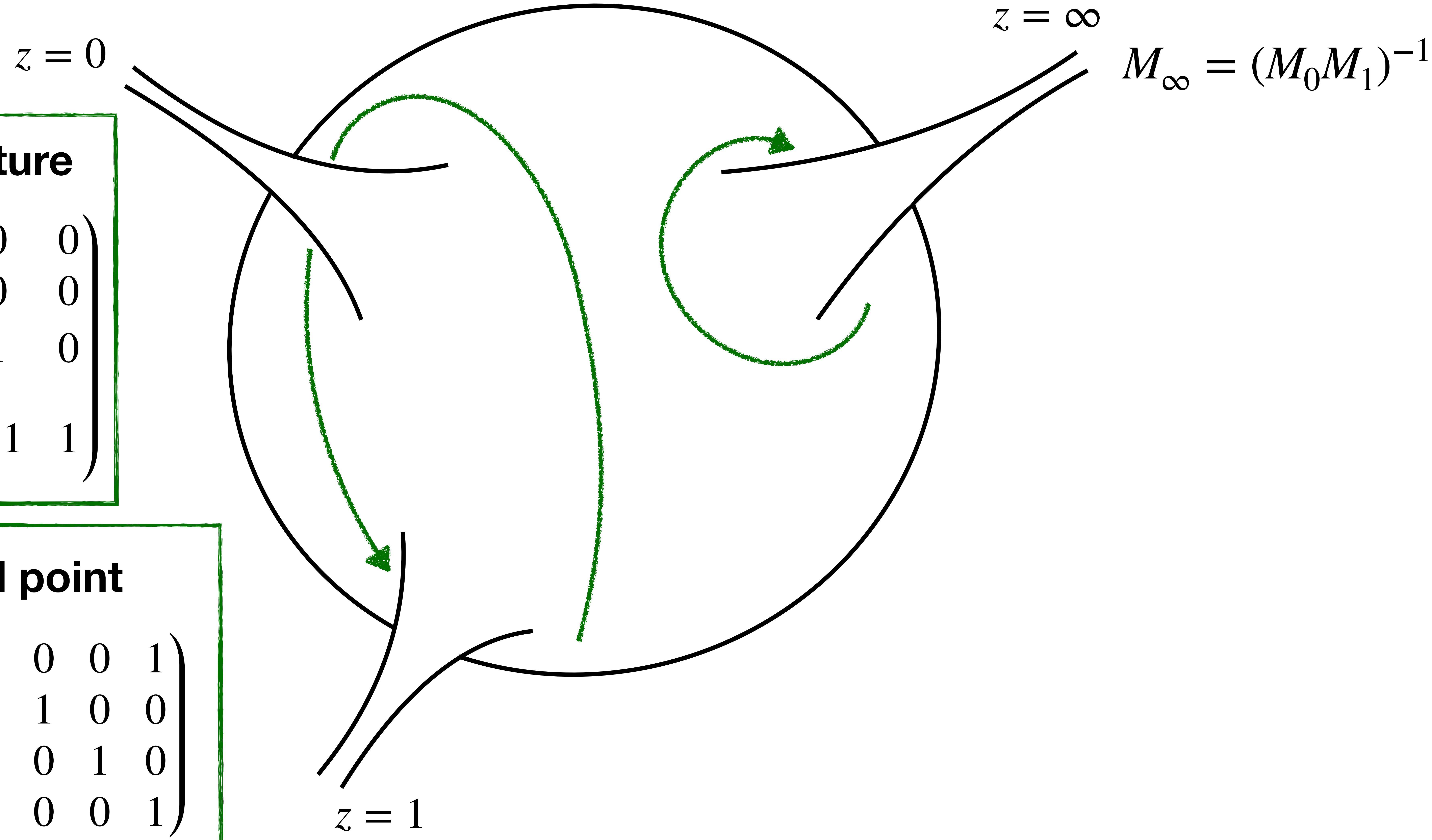

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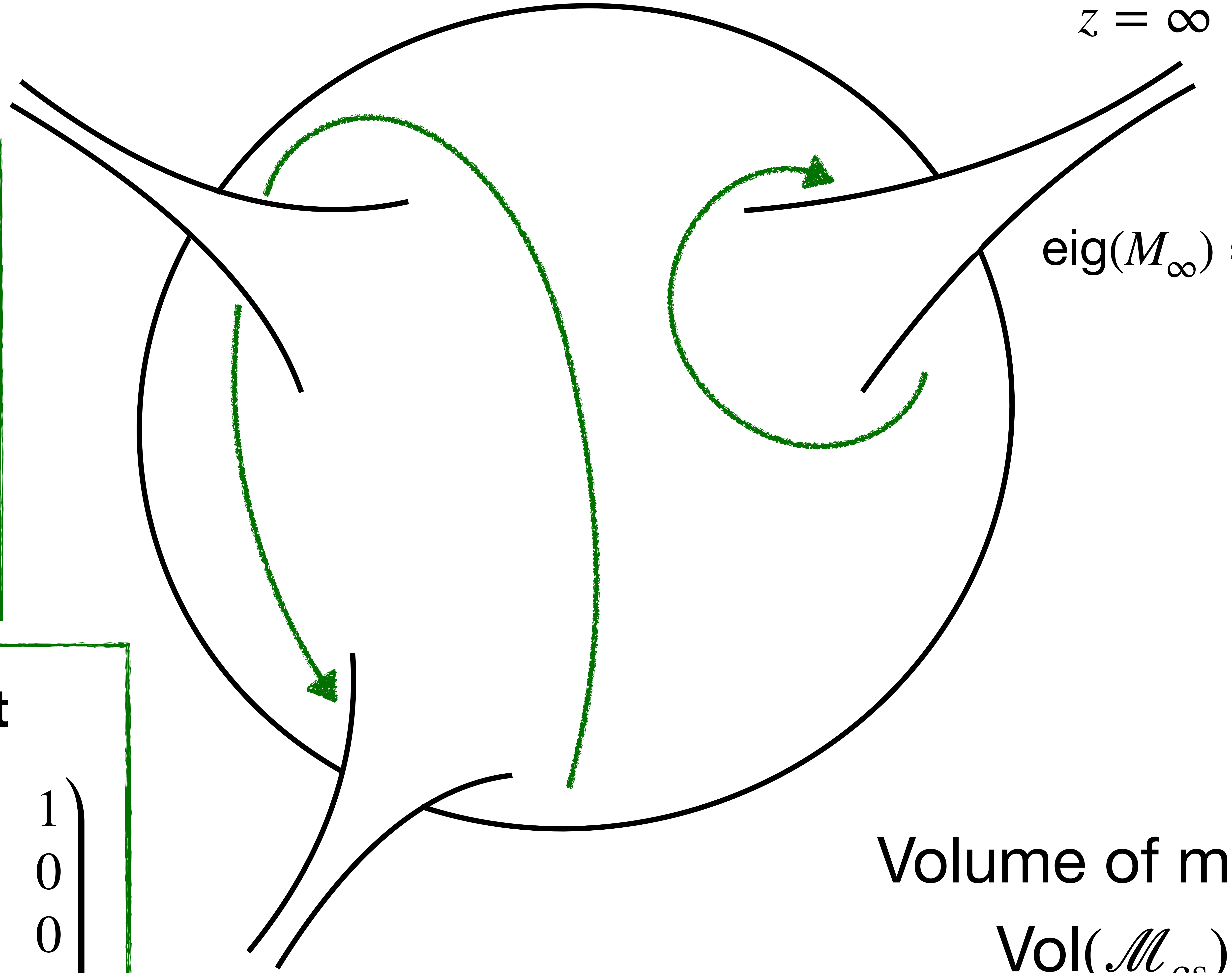

$z = 1$

$z = \infty$

$M_\infty = (M_0 M_1)^{-1}$

$\text{eig}(M_\infty) = (e^{2\pi i a_1}, e^{2\pi i a_2}, e^{2\pi i a_3}, e^{2\pi i a_4})$

Volume of moduli space:

$\text{Vol}(\mathscr{M}_{\text{cs}}) = 2\pi a_1$

Monodromy groups as amalgamated products

[Brav, Thomas; '12]

For 7 CY3s with $\mathscr{M}_{\rm cs} = \mathbb{P}^1 - \{0,1,\infty\}$:

(Reminiscent of Type IIB: $SL(2,\mathbb{Z}) = \mathbb{Z}_4 *_{\mathbb{Z}_2} \mathbb{Z}_6$)

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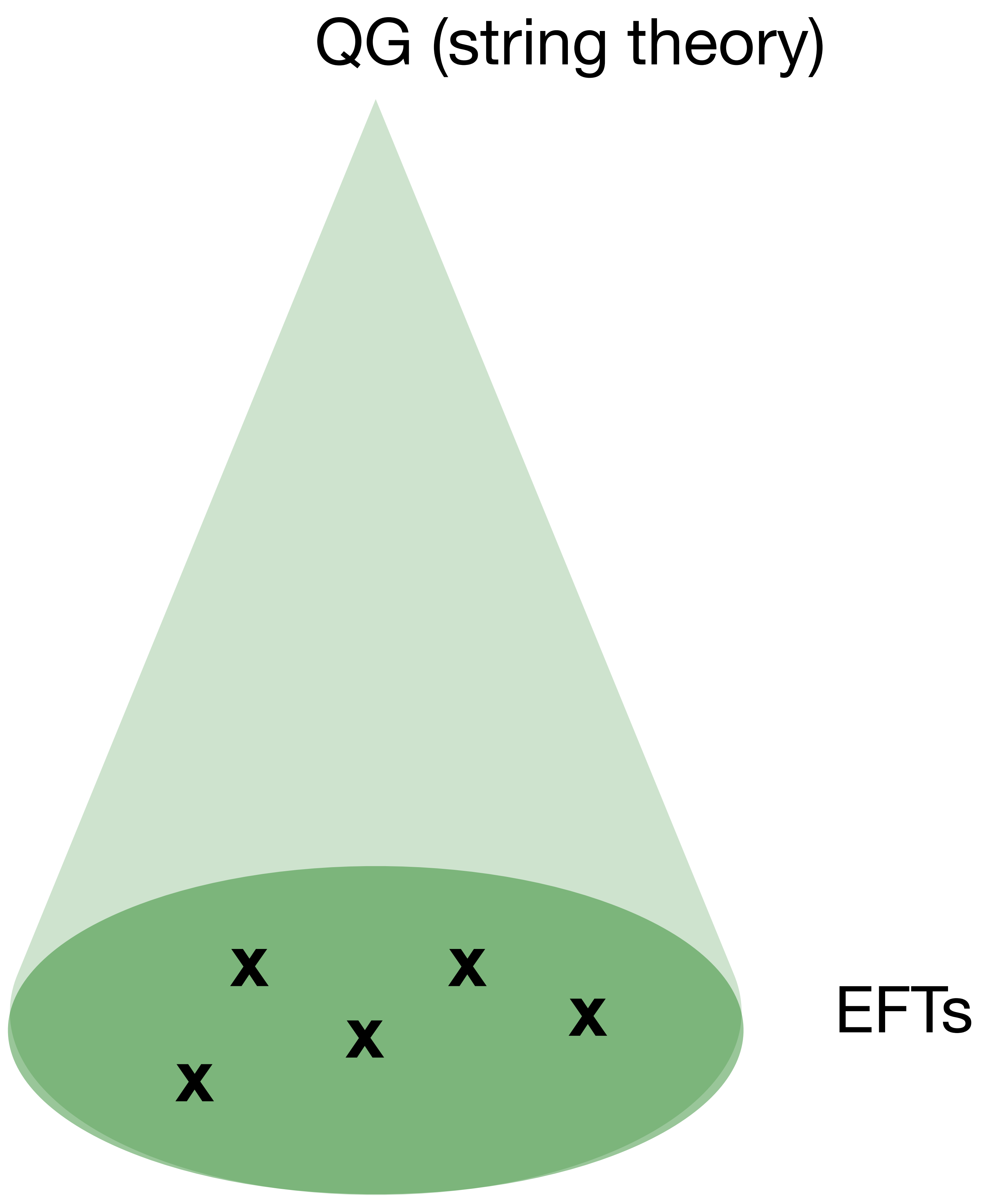
3. Bottom-up Argument for Compactifiability

Finiteness

Landscape of string theory vacua is expected to be **finite**

[Vafa ’05; Douglas ’05; Acharya, Douglas, ’06]



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- Finiteness of Calabi-Yau manifolds?

  ✓ elliptic Calabi-Yau manifolds [Gross, '93; Birkar, Cerbo, Svaldi, '24]

- Finiteness of 6d supergravity landscape

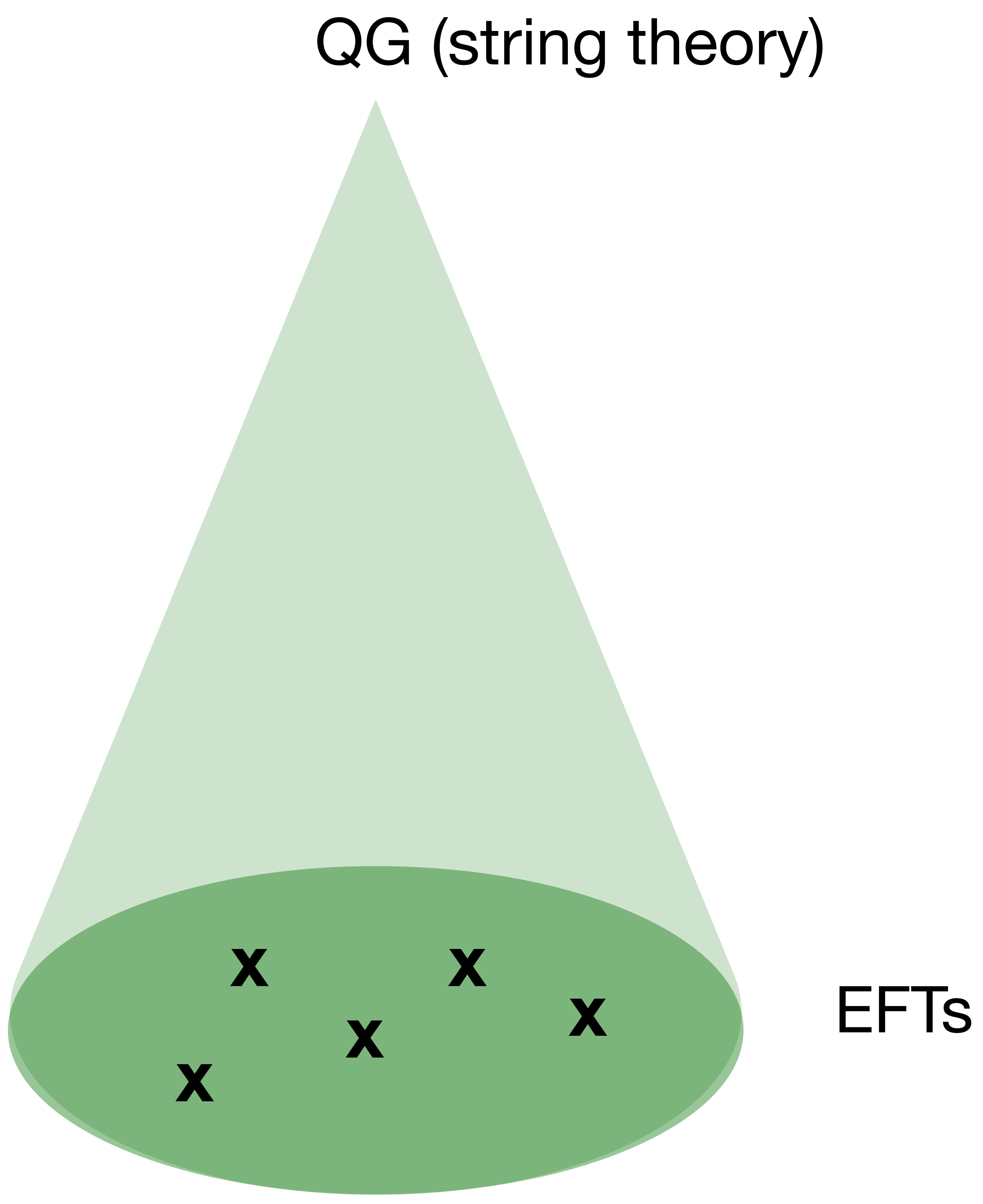
  [Kumar, Taylor '09; …, Kim, Vafa, Xu, '24]

- Finiteness of self-dual flux vacua

  [Bakker, Grimm, Schnell, Tsimerman '21]

- Finiteness properties of QG theories by reduction to **1d quantum-mechanical systems**

  [Hamada, Montero, Vafa, Valenzuela '21; Delgado, DvdH, Raman, Torres, Vafa, Xu '24]



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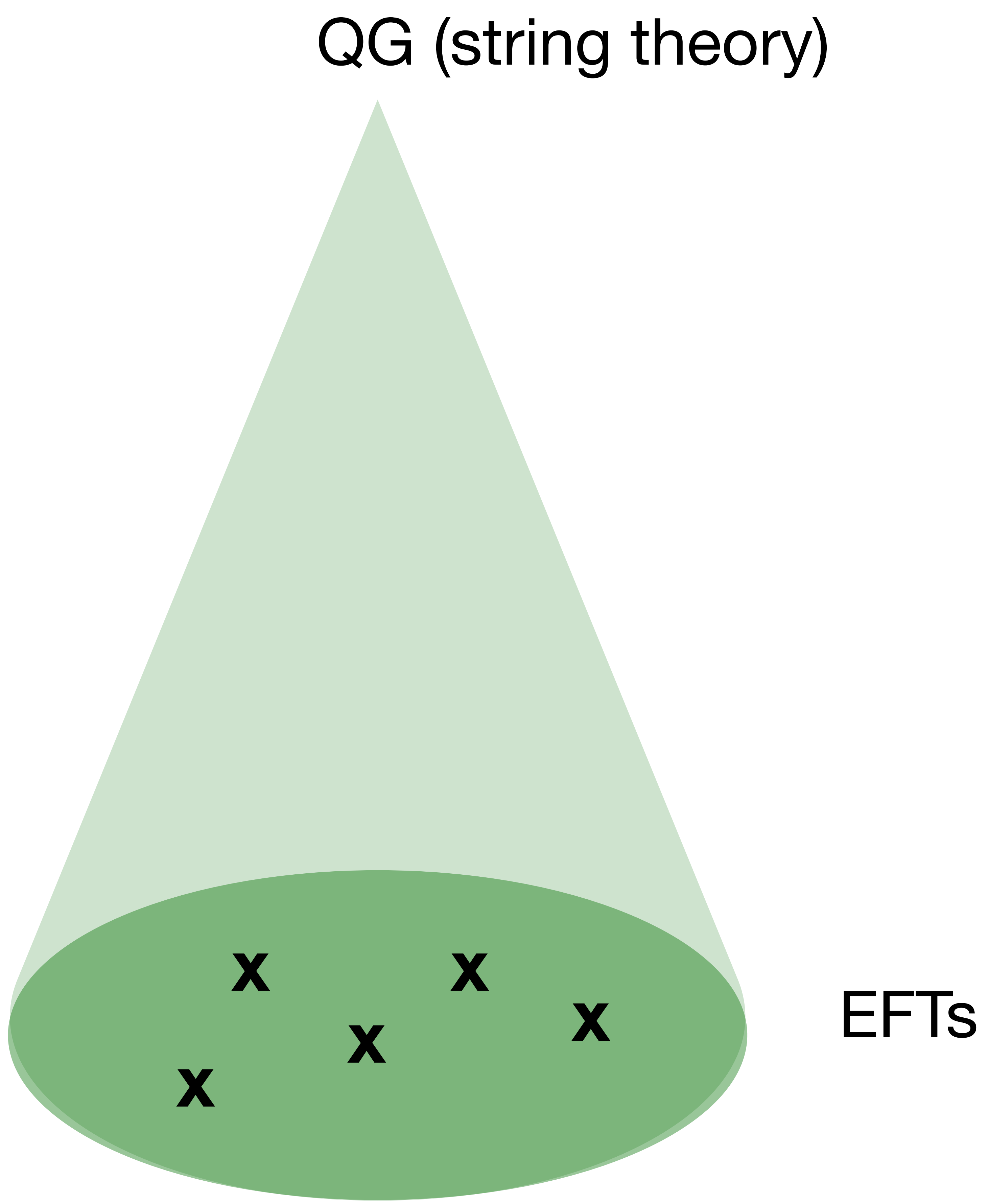

Finiteness of vacua

Main criterion

Compactify QG theory to 1d $\implies$ number of ground states should be finite

(similar idea: [Hamada, Montero, Vafa, Valenzuela ’21])

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Compactify QG theory to 1d $\Longrightarrow$ number of ground states should be finite

(similar idea: [Hamada, Montero, Vafa, Valenzuela ’21])

Why?

- Entropy of system diverges even at zero temperature:

$$S(T=0) \sim \log(\#\ \text{ground states}) \to \infty$$

- Partition function diverges at finite temperature:

$$Z = \text{Tr}e^{-\beta H} = \sum_n e^{-\beta E_n} \to \infty$$

Ground states

d-dim susy QG theory

Compactify on T^{d-1}

$\Longrightarrow$

1d SUSY Quantum Mechanics

Ground states

d-dim susy QG theory

Compactify on T^{d-1}

$\Longrightarrow$

1d SUSY Quantum Mechanics

Idea: consider zero-modes coming fluctuations on the moduli space [Witten; '82]

Harmonic, normalizable p-forms

$$f(\phi) d\phi^1 \wedge \ldots \wedge d\phi^p$$

$\Longleftrightarrow$

Ground states

$$f(\phi)\psi_0^1 \ldots \psi_0^p \, |0\rangle$$

Ground states

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Harmonic, normalizable p-forms $f(\phi)d\phi^1 \wedge \ldots \wedge d\phi^p$ $\Longleftrightarrow$ Ground states $f(\phi)\psi_0^1 \ldots \psi_0^p \,|\, 0\rangle$

Can we relate the growth of Vol($\mathscr{M}$) to this ground state spectrum?

Ground states and volume growth

Fact: For metrics of the form $ds^2 = dr^2 + r^{2+\epsilon} d\text{Vol}(\partial \mathscr{M})^2$ (dimension $2k$), there are infinitely many harmonic, normalizable k-forms when $\epsilon > 0$

[Atiyah, Patodi, Singer ’75; Dodziuk ’79; Mazzeo ’88; Lott ’97]

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Example: Type IIB with **no** duality group

Harmonic, normalizable one-forms on $\mathbb{H}$: $\omega_n = e^{-\frac{2\pi\tau_2}{n}}(\cos(2\pi\tau_1/n)d\tau_1 + \sin(2\pi\tau_1/n)d\tau_2)$

Mixing with T^{d-1} moduli

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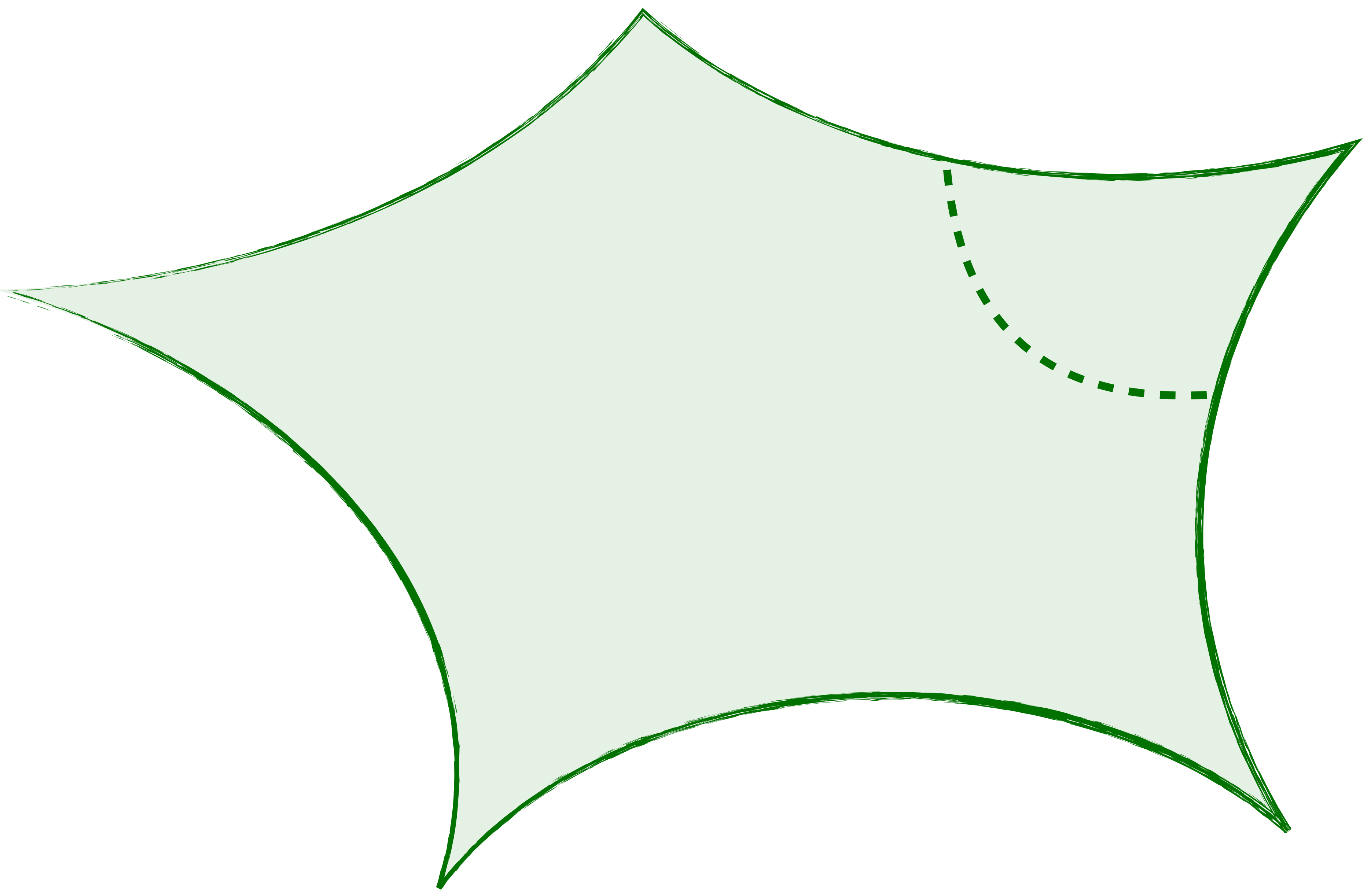

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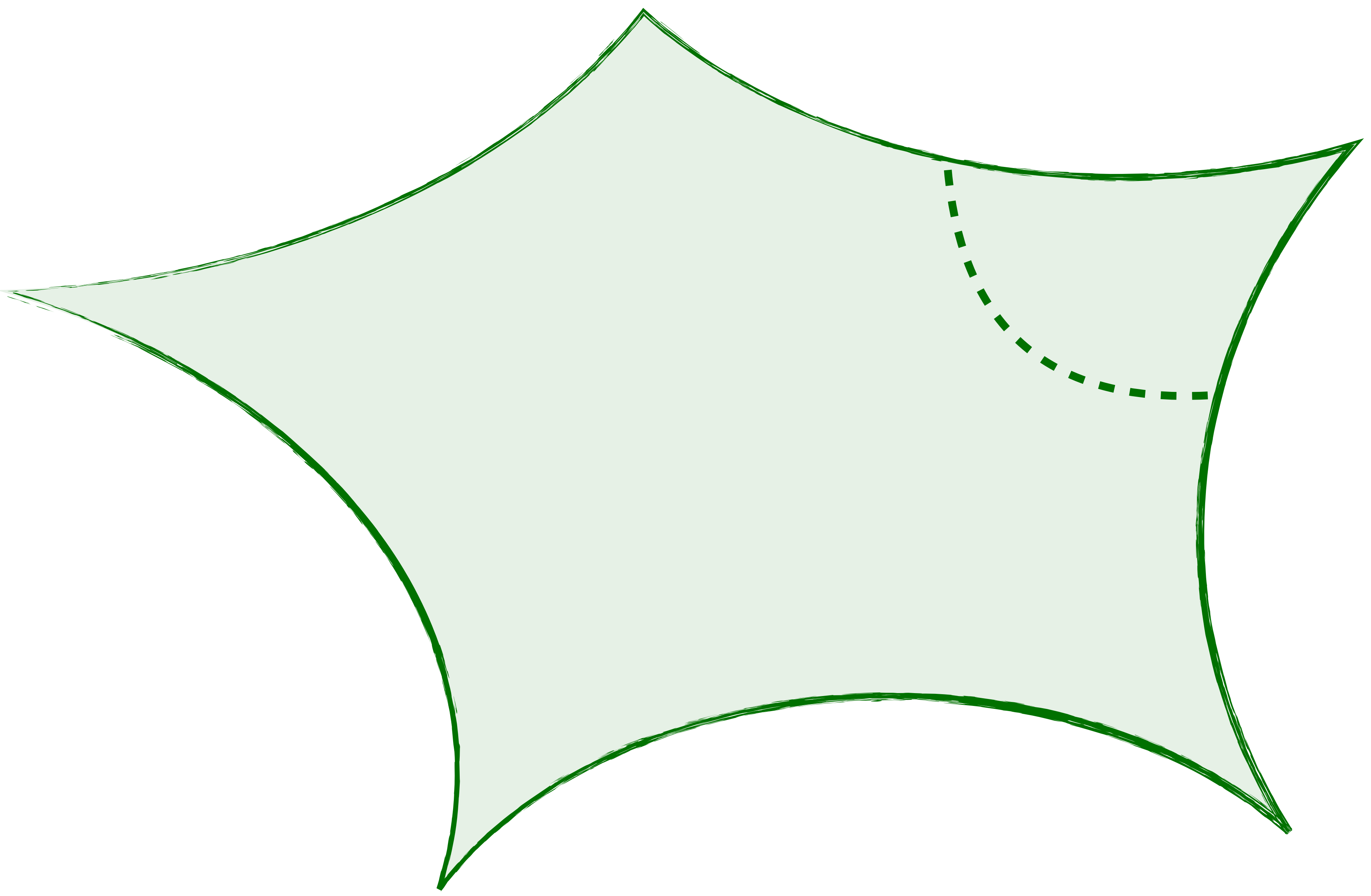

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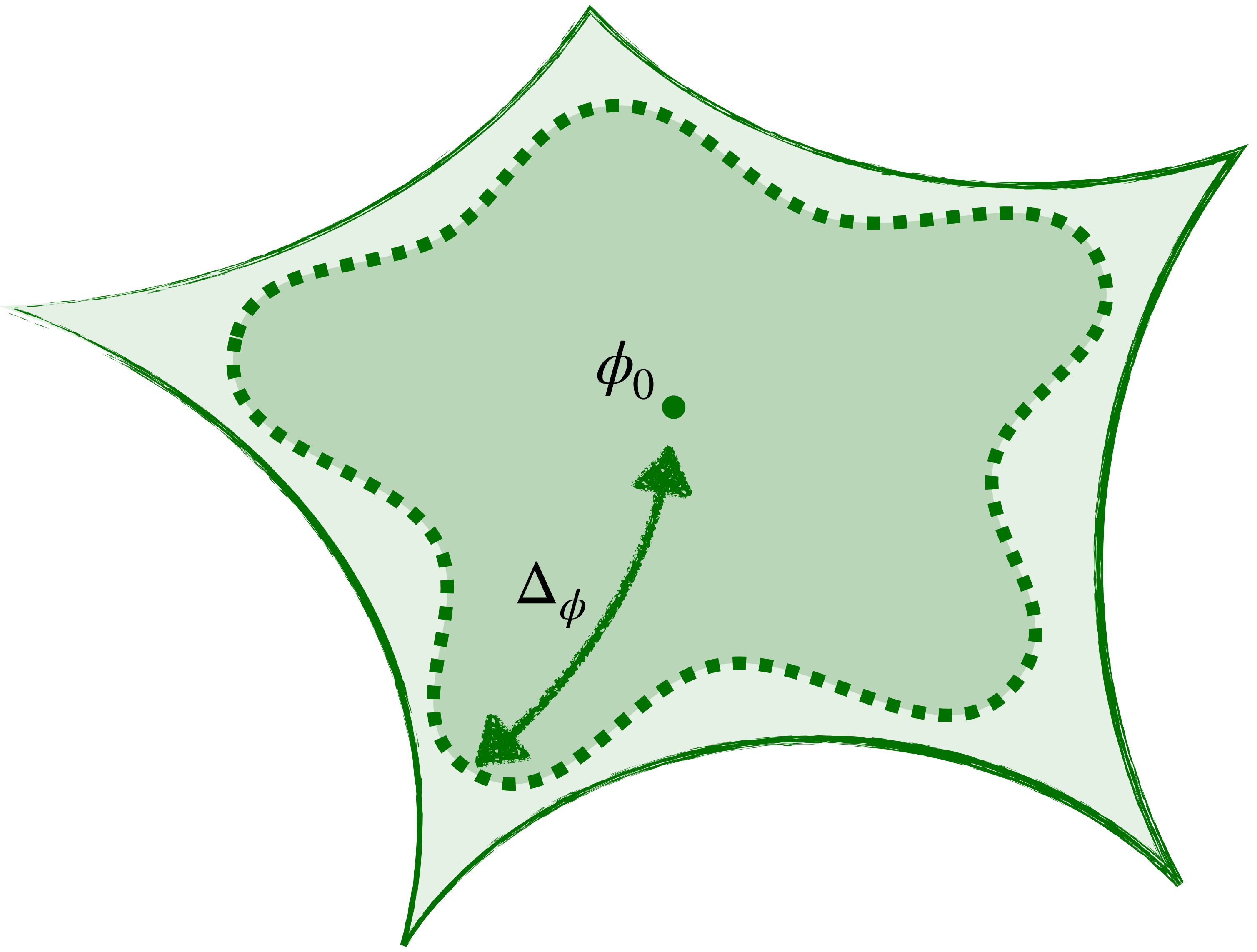
If $\mathscr{M}_{\mathrm{QM}}$ is compactifiable, also the **large-radius region** should be compactifiable

Conclusions

Compactifiability of moduli spaces gives a powerful **bottom-up** principle to constrain EFTs



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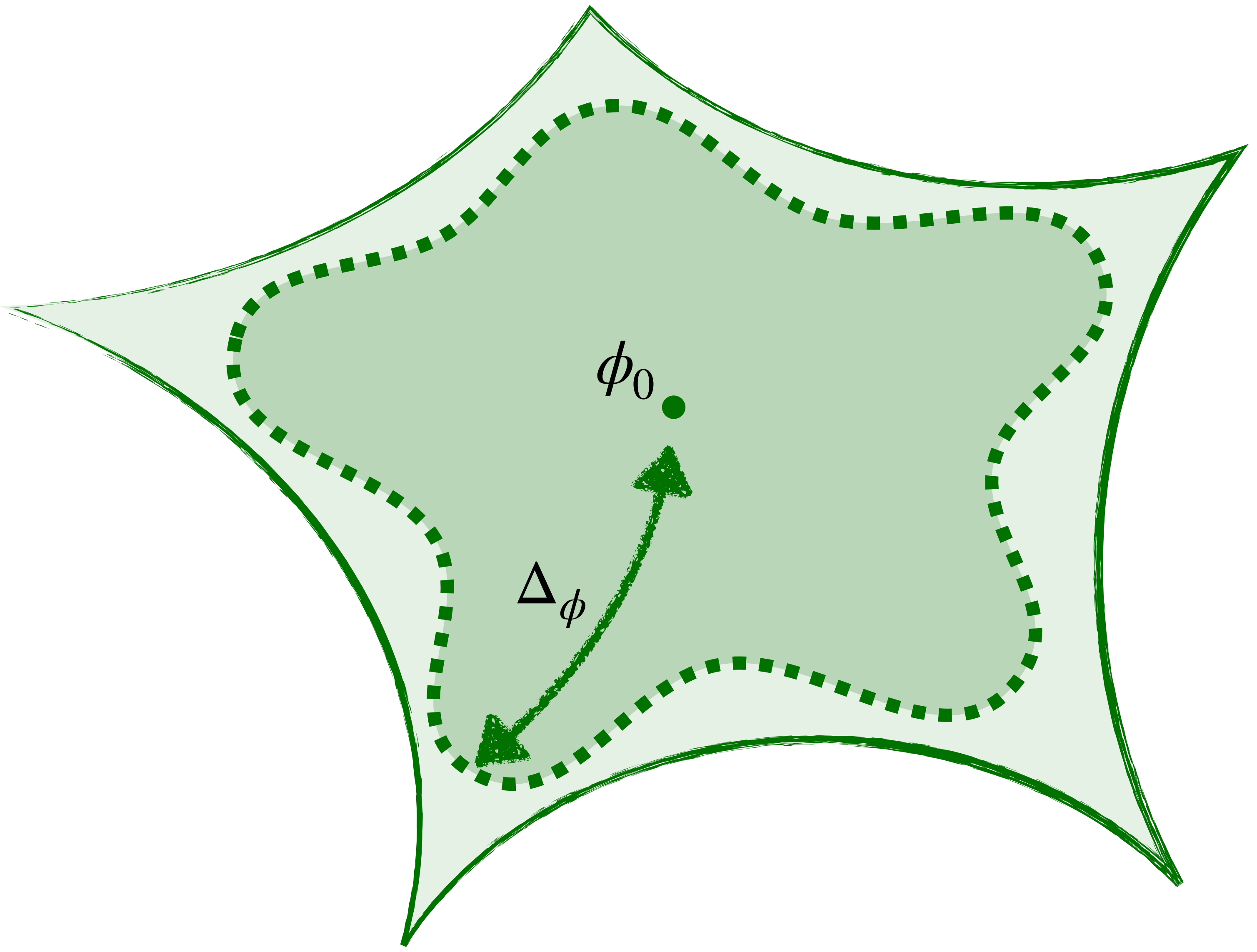

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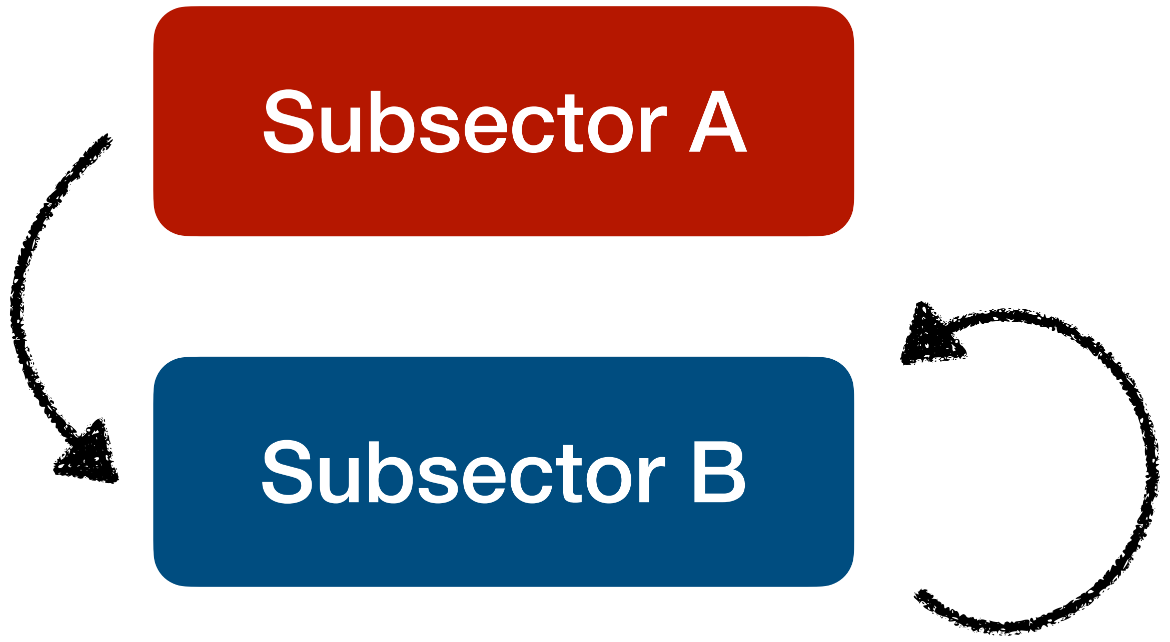
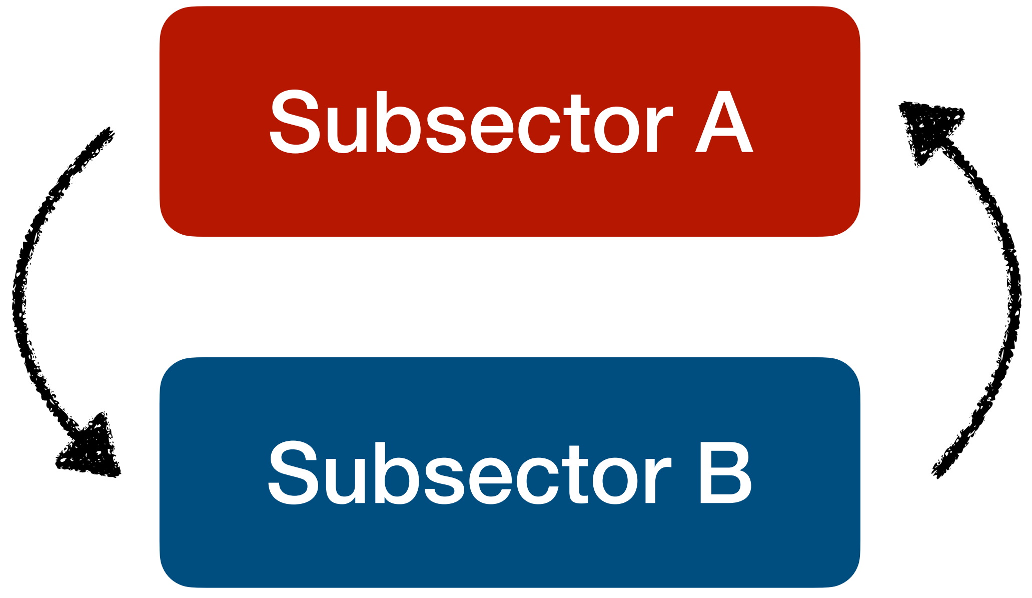
Compactifiability of moduli spaces gives a powerful **bottom-up** principle to constrain EFTs

$\Longrightarrow$ **presence** and **semisimplicity** of dualities





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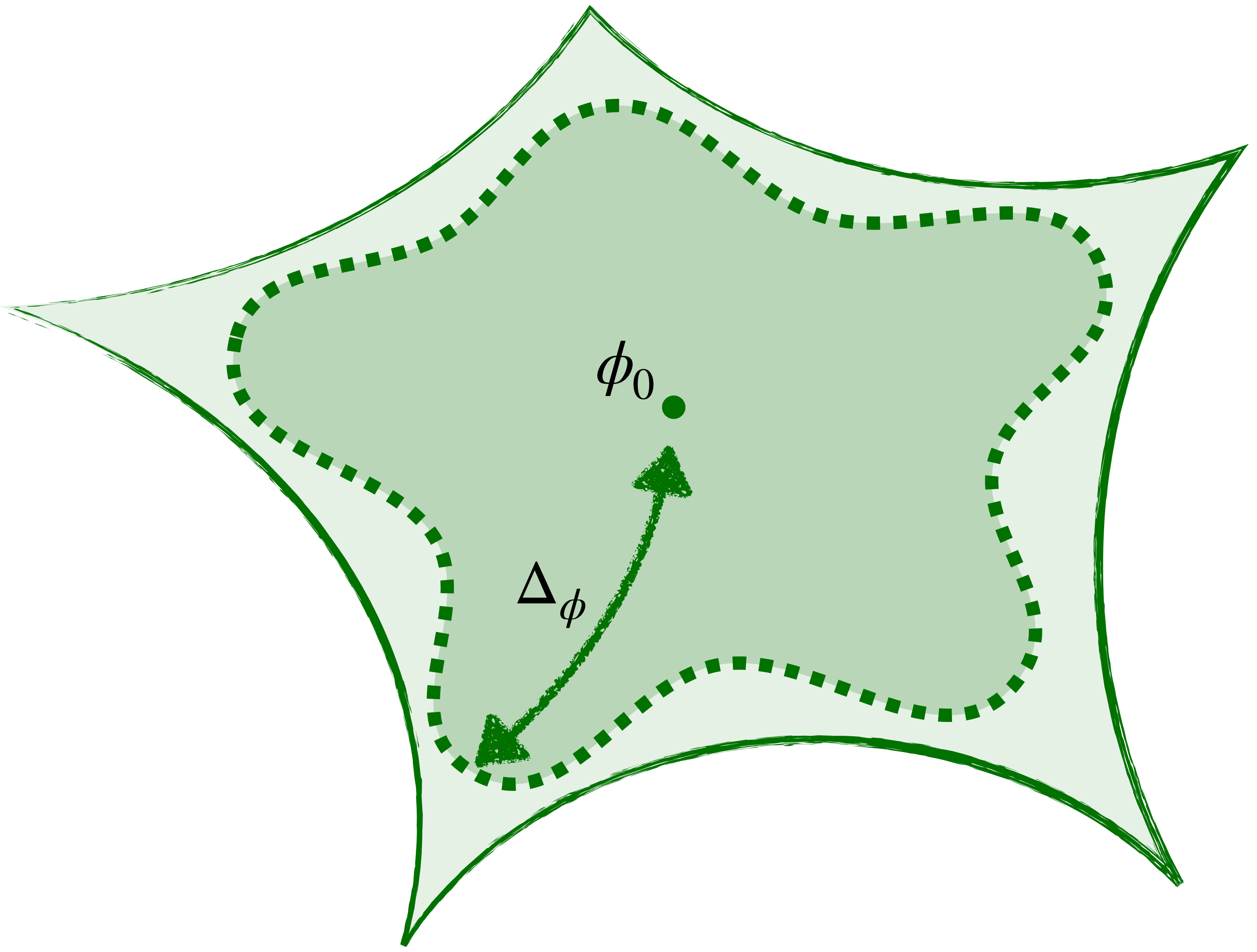

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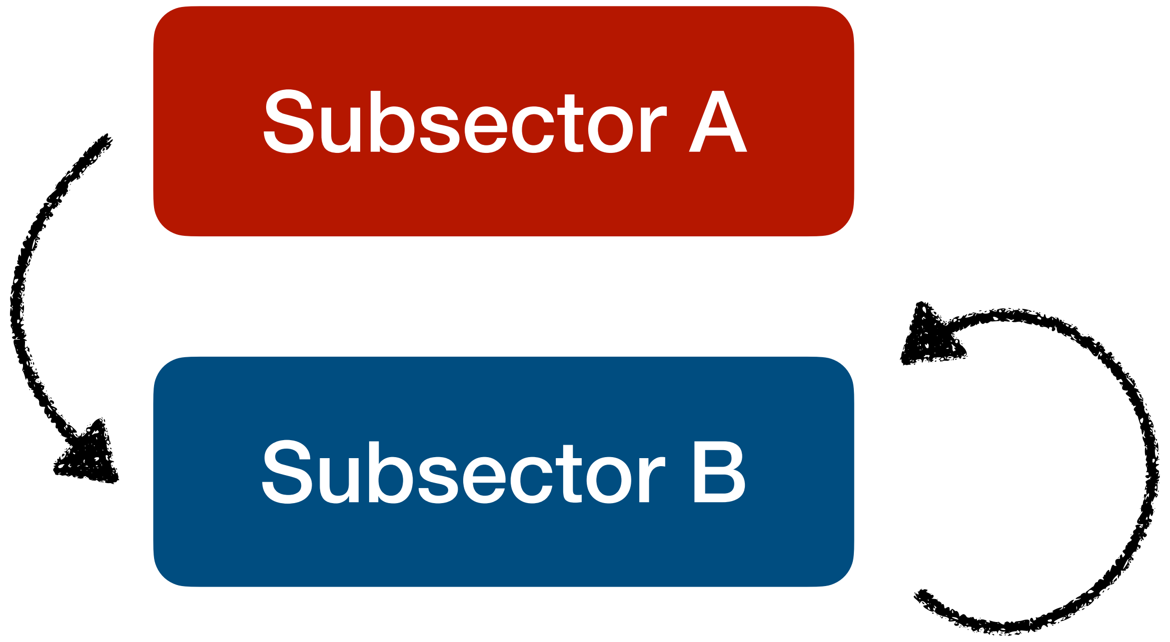

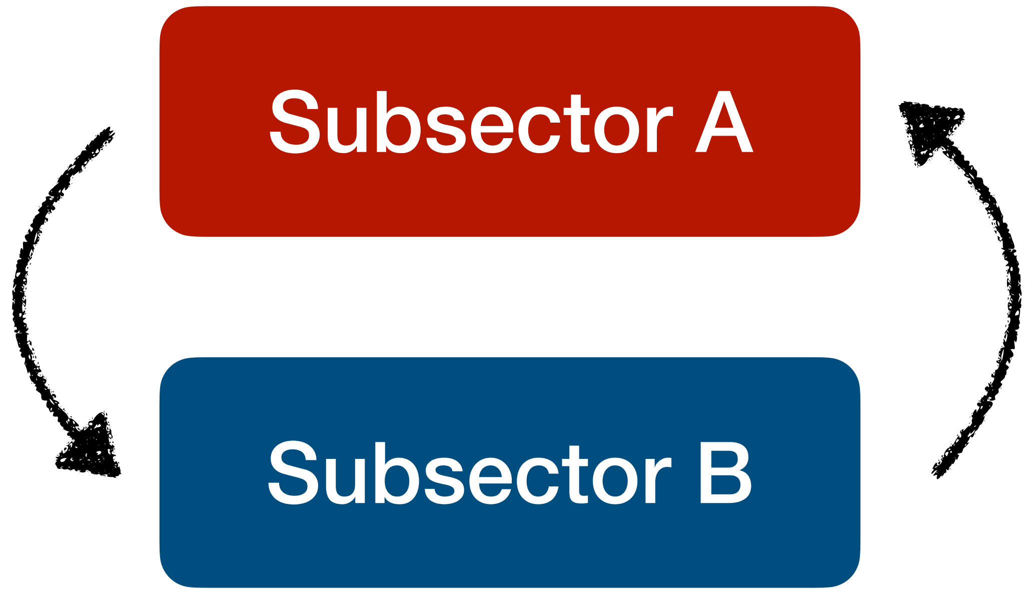

- Is there an independent argument why Quantum Gravity should have semisimple dualities?
- Relation to curvature of moduli space? See also: [Marchesano, Melotti, Paoloni '23; Raman, Vafa '24; Marchesano, Melotti, Wiesner '24; Castellano, Marchesano, Melotti, Paoloni '24]
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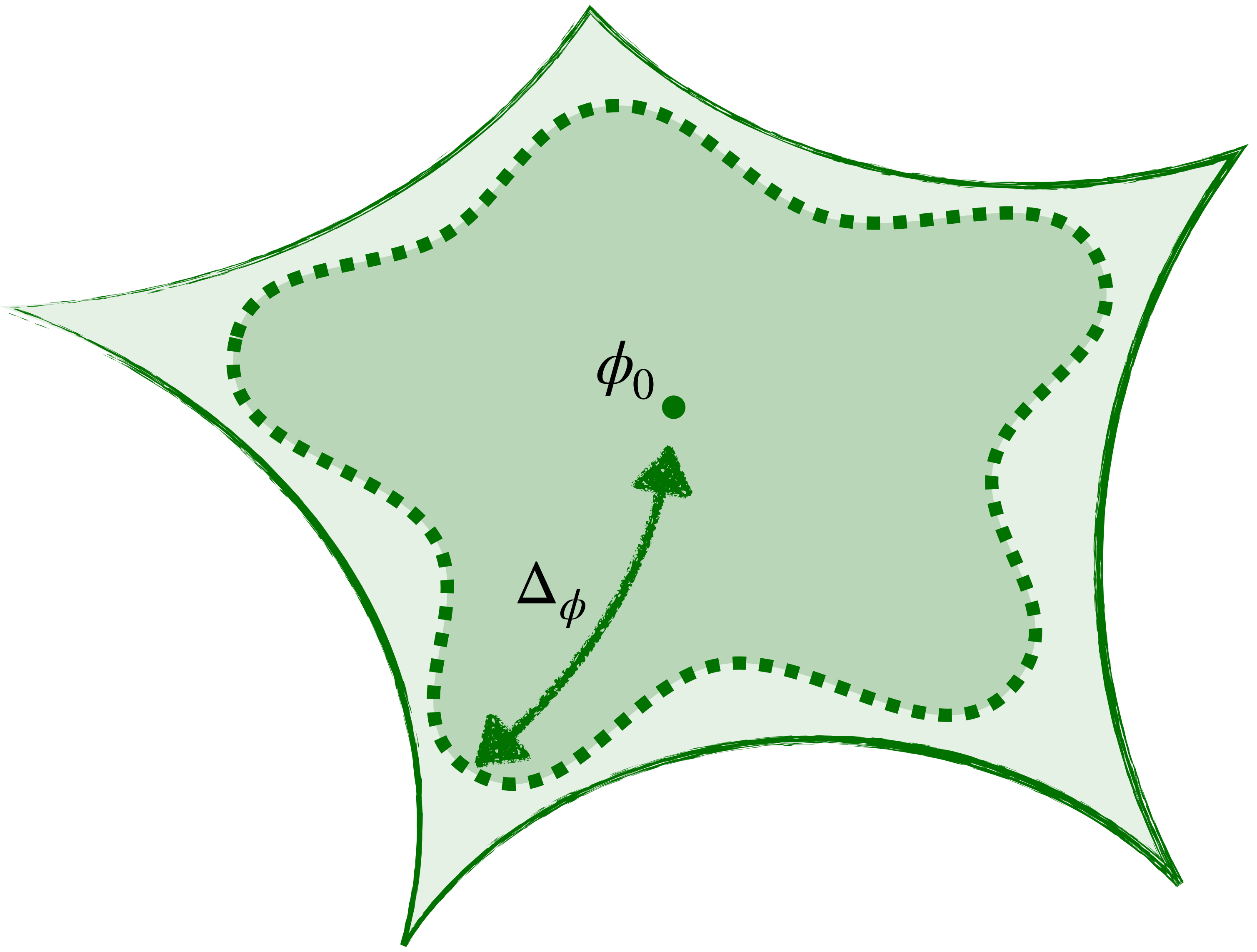

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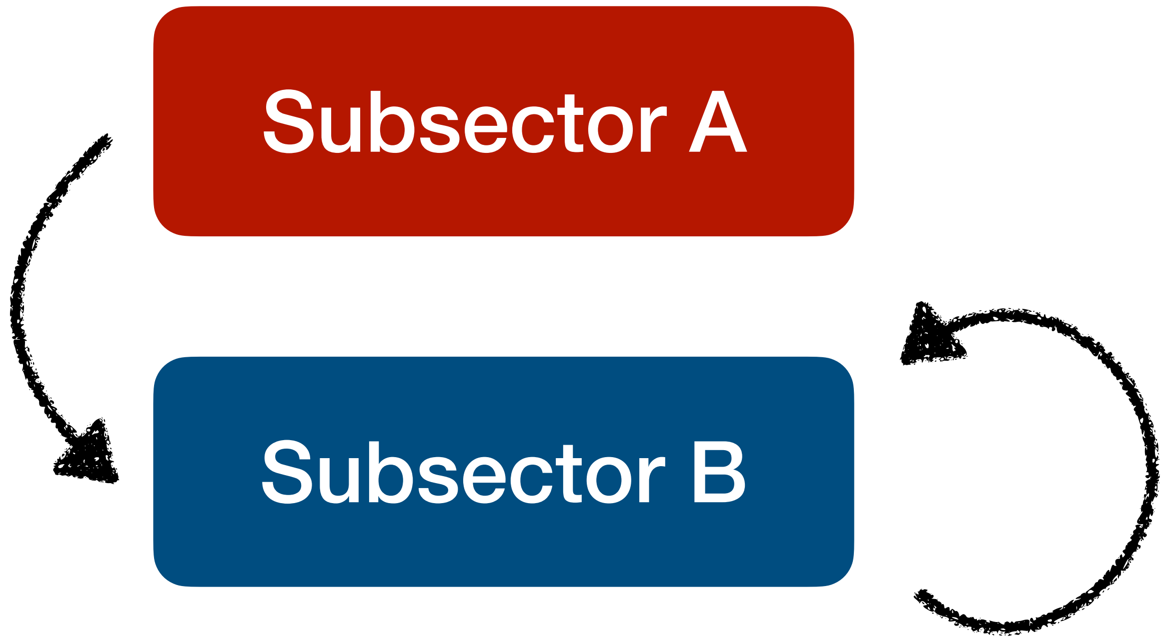

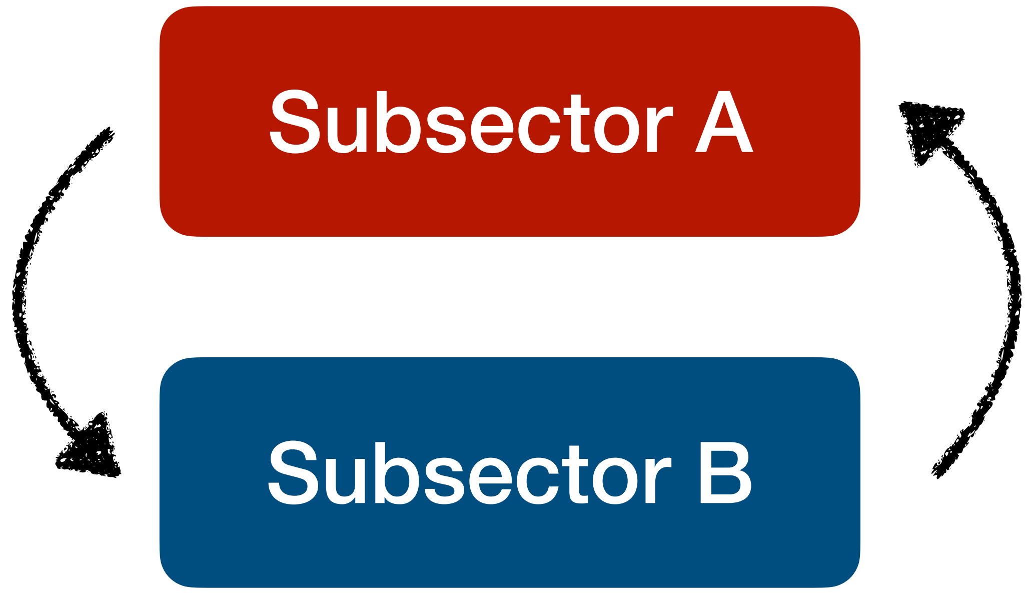

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Thank you!