Planar abelian mirror duals of 3d $\mathcal{N} = 2$ CS quiver theories

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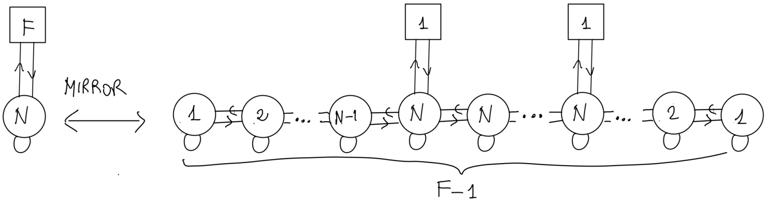
based on [2411.05620] and [2504.XXXX]
with S. Benvenuti, R. Comi, G. Pedde-Ungureanu, S. Rota, A. Shri.

The main takeaway of this talk is that a very very large class of 3d $\mathcal{N} = 2$ CS theories admits a purely abelian and planar dual.

Such abelian-planar duals can be generated via $\mathcal{N} = 4 \to \mathcal{N} = 2$ breaking real mass deformations starting from known $\mathcal{N} = 4$ mirror pairs.

Since this process can be quite subtle and technically involved, we develop an algorithmic approach to systematically generate the abelian-planar duals of 3d 3d $\mathcal{N} = 2$ CS chiral quiver theories.

Let's consider the mirror pair for the $\mathcal{N}=4$ $U(N)$ SQCD:
[Intriligator-Seiberg"96]



- SQCD: $SU(F)_{\text{flav}} \times U(1)_{\text{top}}$.
- MirrorSQCD: $U(1)_{\text{top}}^{F-1} \times U(1)_{\text{flav}} \xrightarrow{\text{IR}} SU(F)_{\text{top}} \times U(1)_{\text{flav}}$.
- Monopoles $\leftrightarrow$ Mesons.

We work in the $\mathcal{N}=2^*$ set-up with:

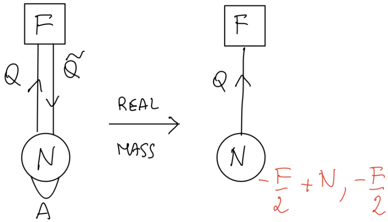
$$U(1)_R = U(1)_{C+H} \qquad \text{and} \quad U(1)_\tau = U(1)_{H-C},$$

where $U(1)_C \in SU(2)_C$ and $U(1)_H \in SU(2)_H$ of the non-Abelian $\mathcal{N}=4$ R-symmetry $SU(2)_H \times SU(2)_H$ [Tong'00].

We turn on a $\mathcal{N}=4 \to \mathcal{N}=2$ breaking real mass deformation for $U(1)_\tau$ assigning charges $1, -\frac{1}{2}, -\frac{1}{2}$ to $A, Q, \tilde{Q}$.

Naively this integrates out all the matter fields leaving a TQFT.

We move on the Coulomb branch so to reach an interacting vacua, for example we can land on the chiral SQCD:



- All the fund. chirals remain massles.
- Each antifund. is integrated out yielding a $-1/2$ CS-level for $U(N)$.
- The adjoint is integrated out yielding a N CS level only for $SU(N) \subset U(N)$.

In the mirror dual we can move in many many ways on the Coulomb branch (many gauge groups) to save some matter fields...

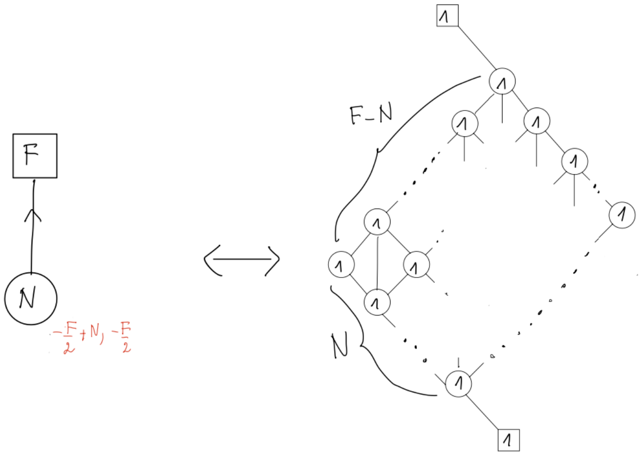
...How to can we be sure we are on the dual of the deformed theory?

An important help comes from the study of the S_b^3 partition function [Kapustin-Willett-Yaakov '10, Hama-Hosomichi-Lee '11]:

$$
\begin{array}{ccc}
\lim\limits_{\tau\to\infty} Z^{\mathcal{N}=4}_{SQCD}(\vec{X}, Y, \tau) & = & \lim\limits_{\tau\to\infty} \hat{Z}^{\mathcal{N}=4}_{SQCD}(\vec{X}, Y, \tau) \\
\| & & \| \\
e^{i\Phi(\tau)} Z^{\mathcal{N}=2}_{SQCD}(\vec{X}, Y) & = & e^{i\Phi(\tau)} \hat{Z}^{\mathcal{N}=2}_{SQCD}(\vec{X}, Y)
\end{array}
$$

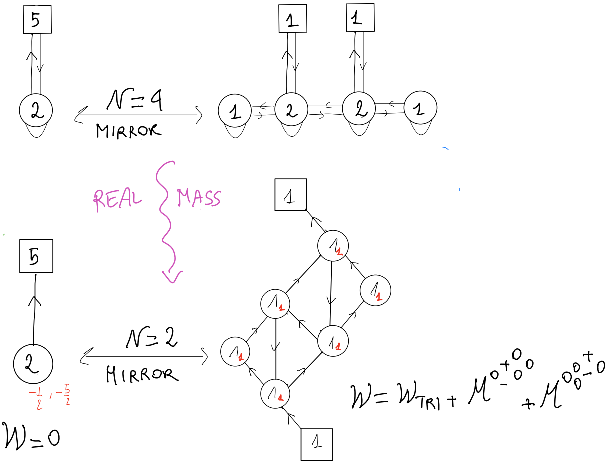
Following [Benini-Closset-Cremonesi '11, Aharony-Razamat-Seiberg-Willett '13] we identify the dual vacuum as the one with matching highly oscillating phase!

Surprise: the mirror dual of the chiral SQCD is a completely abelian and planar quiver theory!



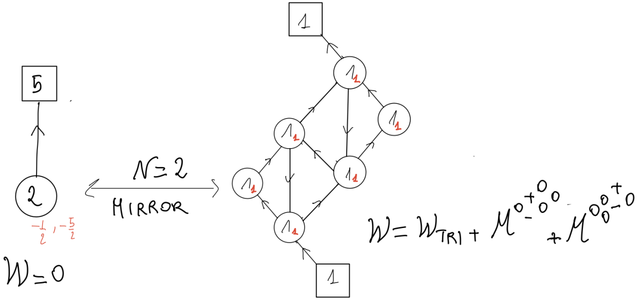
- This is the dual with the matching phase $\Phi(\tau)$.
- Every $U(k)$ gauge group is Higgsed to $U(1)^k$.
- Match of the the superconformal index for low values of N, F (easy thanks to the knowledge of exact parameter map),

Let's consider the $U(2)$ SQCD with $F = 5$ flavors:



- All the $U(N)$ nodes are Higgsed to $U(1)^N$, a monopole superpotential is generated.
- $\mathcal{W}_{\text{TRI}}$ are cubic couplings for chirals in a triangular loop.
- All nodes have CS-level 1 and there are mixed CS couplings.

The chiral SQCD has a manifest $SU(5)_{\text{flav}} \times U(1)_{\text{top}}$ global symmetry:

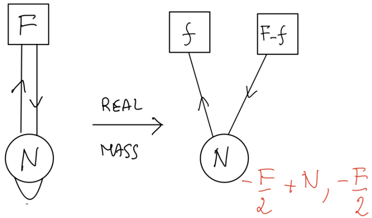


On the mirror side this is realized as:

- Flavor symmetry reduced by $\mathcal{W}_{\text{TRI}}$ and gauge transf. to $U(1)_{\text{flav}}$.
- Monopole superpotential breaks $U(1)^{k}_{\text{top}} \to U(1)_{\text{top}}$ in each column. Then there is enhancement: $U(1)^{F-1}_{\text{top}} \to SU(F)_{\text{top}}$.

The chiral SQCD has a single gauge invariant (dressed) monopole which maps to the only meson in the dual (up to F-term relations).

Starting from $\mathcal{N} = 4$ SQCD we can take a more general real mass deformation preserving f fund. and $F - f$ antifund. chirals:

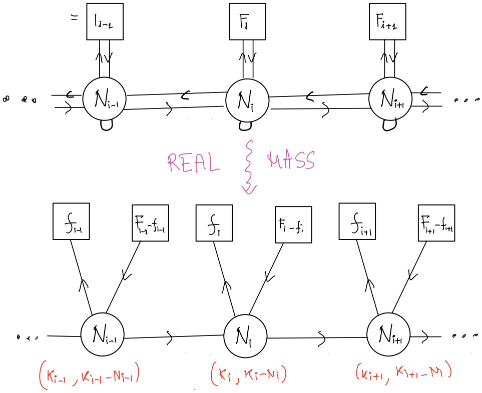


The CS level $(k, k + \ell N)$ is completely fixed by the parent $\mathcal{N} = 4$ theory: each massive (anti)fundamental multiple contributes as $(-\frac{1}{2}, -\frac{1}{2})$, the adjoint as $(N, 0)$ so $k = -\frac{F}{2} + N$ and $\ell = -1$.

To access more general range of CS levels:

- Apply Witten's $SL(2, \mathbb{Z})$ action by gauging/ungaging $U(1) \in U(N)$ with an aribitrary CS level ℓ.
- Perform further real masses and integrate out more chirals to vary k.

So starting from an $\mathcal{N}=4$ mirorr pair of quivers, we perform on the electric side a real mass deformation to a chiral quiver preserving the maximum number of chirals for each node:

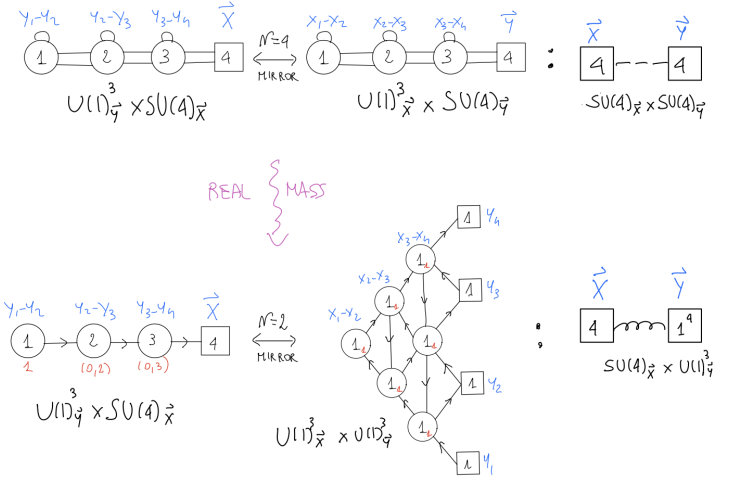


$$k_i = -\frac{(N_{i+1}+N_{i-1}+F_i)}{2} + N_i$$

we can then perform further real masses to access more general CS levels.

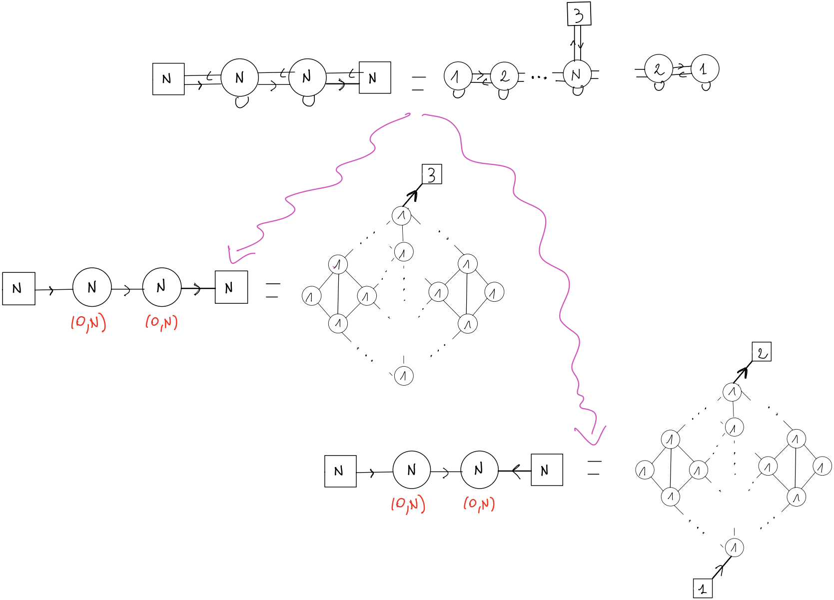
Identifying the corresponding vacuum on the mirror dual side remains quite subtle!

The real mass deformation of the $\mathcal{N}=4$ self-mirror $T[SU(4)]$ theory [Gaiotto-Witten '08] yields a chiral-planar dual pair:



- The chiral global symmetry: $SU(4)_{\text{flav}} \times U(1)^3_{\text{top}}$ is mapped to the enhanced planar $SU(4)_{\text{top}} \times U(1)^3_{\text{flav}}$ symmetry.
- Monopoles $\mathfrak{M}^{+,0,0}, \mathfrak{M}^{0,+,0}, \mathfrak{M}^{0,0,+}$ map to mesons constructed taking the shortest path connecting two adjacent flavor nodes.

More chiral-planar pairs:



The procedure of starting from $\mathcal{N} = 4$ mirror pairs and performing real masses is effective, but undeniably tedious and complicated,

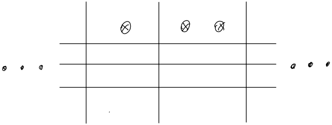
it's like finding a needle in a haystack!

What we seek instead is a more systematic and streamlined strategy to find the planar dual of a chiral quiver.

$\mathcal{N} = 4$ (and recently some $\mathcal{N} = 2$) mirror dualities can be derived by means of the mirror dualization algorithm.

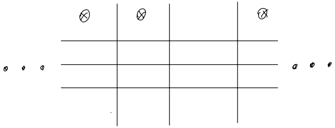
3d $\mathcal{N}=4$ quivers can be engineered on brane setups and mirror dualities follows from type IIB S-duality [Hanany-Witten'97].

We can perform S-duality locally on each 5-brane: [Gaiotto-Witten'08]



$$\dots\ \mathrm{NS}\ \mathrm{D5}\ \mathrm{NS}\ \mathrm{D5}\ \mathrm{D5}\ \mathrm{NS}\ \dots$$

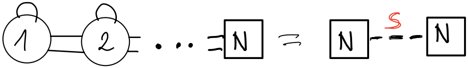
$$\dots\ S\, \underbrace{S^{-1}\,\mathrm{NS}\,S}_{\mathrm{D5}}\, \underbrace{S^{-1}\,\mathrm{D5}\,S}_{\mathrm{NS}}\, \underbrace{S^{-1}\,\mathrm{NS}\,S}_{\mathrm{D5}}\, \underbrace{S^{-1}\,\mathrm{D5}\,S}_{\mathrm{NS}}\, \underbrace{S^{-1}\,\mathrm{D5}\,S}_{\mathrm{NS}}\, \underbrace{S^{-1}\,\mathrm{NS}\,S}_{\mathrm{D5}}\, S^{-1}\ \dots$$



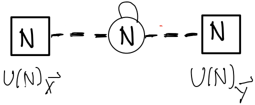
- S IS THE S-DUALITY WALL
- FOR NON-CONST. D3s $\longrightarrow$ HW MOVES

The mirror dualization algorithm implements at the field theory level the local action of S-duality [Hwang-SP-Sacchi '21]:

- $\mathcal{S}$ IS REALIZED AS THE GAIOTTO–WITTEN $T[SU(N)]$:

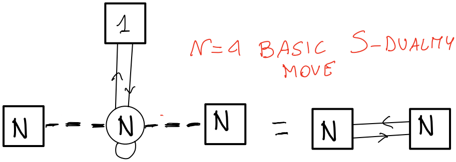


- $\mathcal{S}\mathcal{S}^{-1} = \mathbb{1}$ IS REALIZED AS:

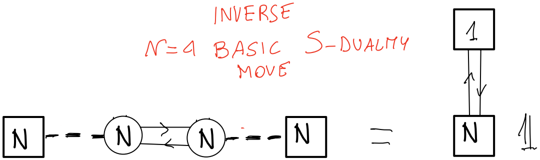


$$= \mathbb{1} \sim \sum_{\sigma \in S_N} \prod_{i=1}^{N} \delta(X_i - Y_{\sigma(i)})$$

- $\boxed{S^{-1}\, D5\, S = NS}$ IS REALIZED AS:

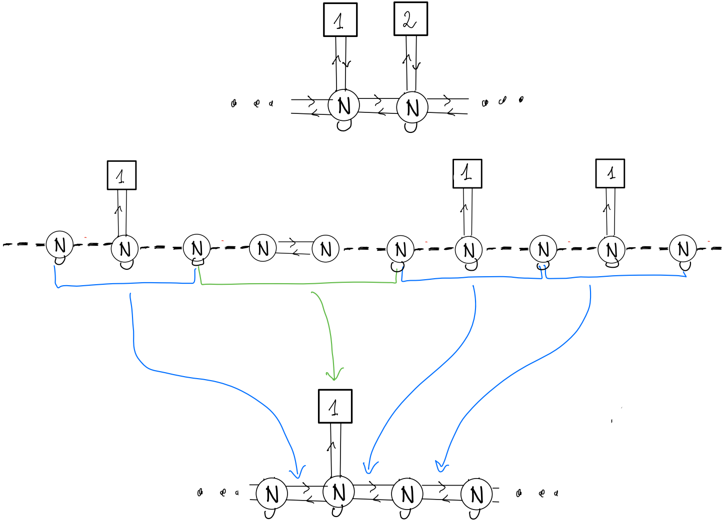


- $\boxed{S^{-1}\, NS\, S = D5}$ IS REALIZED AS:



These are IR dualities which can be derived by iterative applications of the elementary Seiberg-like dualities!

The dualization algorithm at work:



→ FOR NON-CONST. RANKS, IMPEMENT SEQUENTIAL HIGGSING USING HW DUALITY MOVES

The dualization algorithm:

- Provides a purely field-theoretical proof of mirror dualities assuming only basic Seiberg-like dualities.
- Can be implemented at the level of indices/partition functions, yielding integral identities.
- Provides the exact map of all fugacities and all the background CS-couplings.

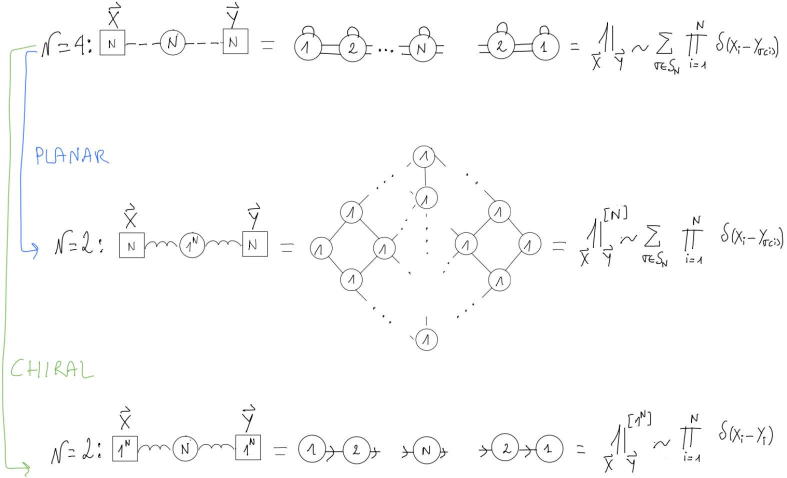
Generalizations:

- Algorithm for $SL(2,\mathbb{Z})$ dualities [Comi-Hwang-Marino-SP-Sacchi '22]
- Algorithm for bad theories $\rightarrow$ multiple good frames, partition functions are distributions! [Giacomelli-Hwang-Marino-SP-Sacchi '23,'24], for USp theories [Comi-Giacomelli-Garavaglia-SP, in progress], for star-shaped quivers [Comi-Giacomelli-Garavaglia-SP-Singh, in progress].
- Algorithm for $\mathcal{N}=2$ dualities involving generalized quivers [Benvenuti-Comi-SP '23].

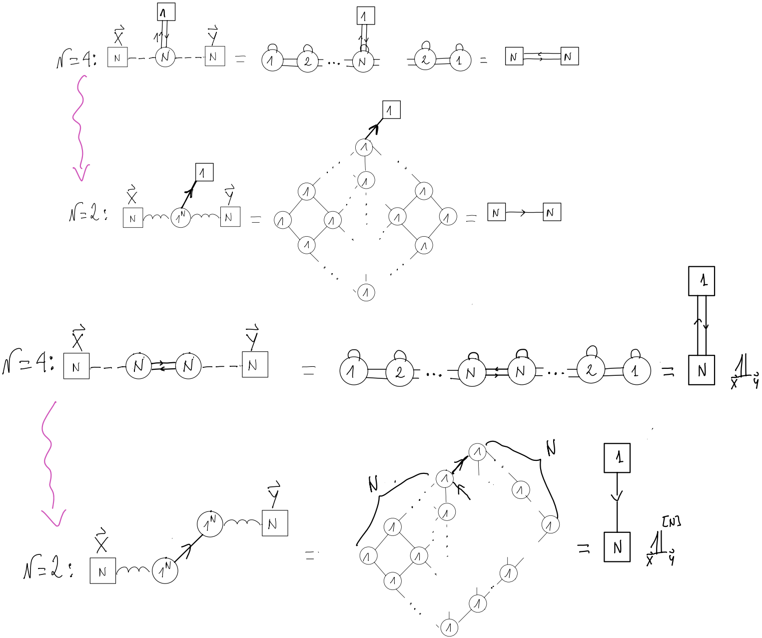
We now establish a dualization algorithm to generate the mirror abelian planar dual of chiral quivers.

It is natural to identify the real mass deformation of the $\mathcal{N}=4$ S-wall, with the chiral-planar S-wall:

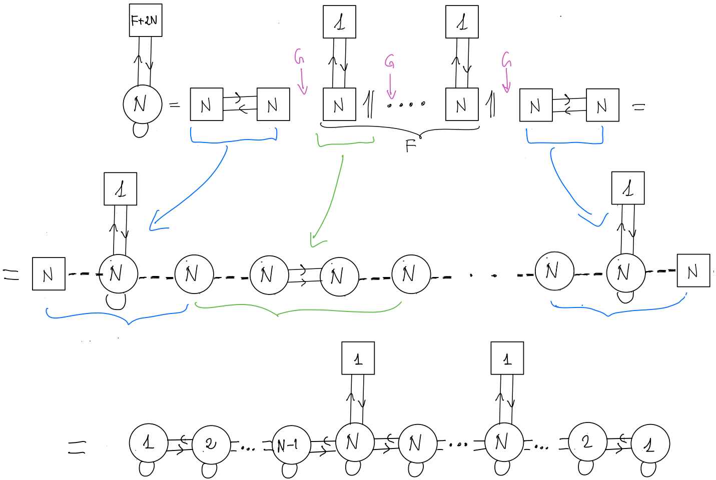
Indeed it satisfies the the fusion-to-identity property:



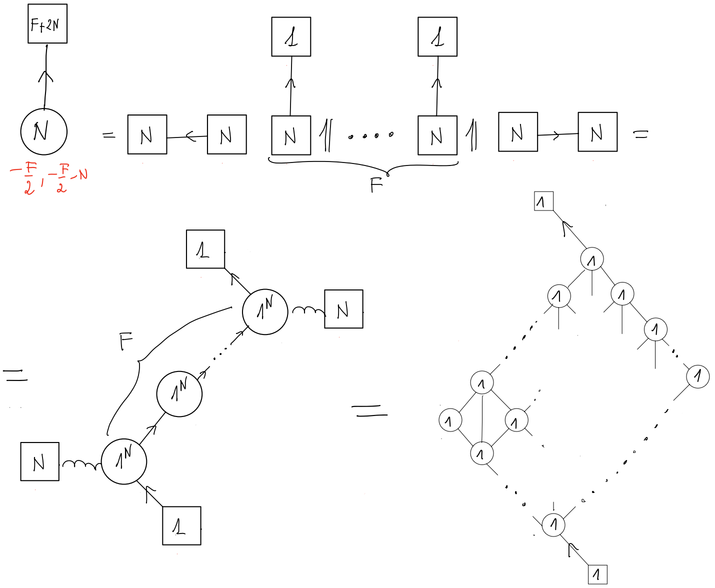
The real mass deformation of the $\mathcal{N} = 4$ basic moves yields the chiral-planar basic moves:



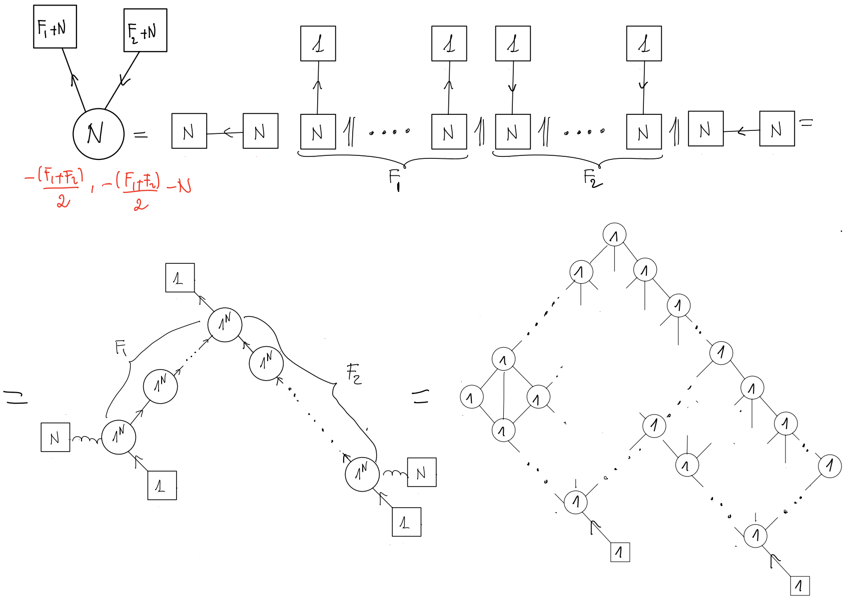
$\mathcal{N} = 4$ SQCD mirror via dualization algorithm:



Chiral-planar dualization of SCQD with $n_f = 2N + F$:



Chiral-planar dualization of SQCD with $n_f = N + F_1$, $n_a = N + F_2$:



Conclusions

We have seen that a very large class of 3d $\mathcal{N} = 2$ CS theories admits a purely abelian planar mirror dual.

These duals can be obtained either by performing real mass deformations on 3d $\mathcal{N} = 4$ mirror pairs, or through the chiral-planar dualization algorithm.

- Can we find an abelian-planar dual for every 3d $\mathcal{N} = 2$ theory?
- We can compactify to 2d, can we find an abelian planar dual for every non-abelian GLSM?
- Is there a stringy realization of chiral-planar dualities, as hinted by the structure of the algorithm?
- Are there non-susy chiral-planar dualities? Yes, stay tuned....

  [S.Benvenuti, R.Comi, G.Pedde-Ungureanu, S.P, S.Rota, A.Shri, to appear]