Emergent Strings in type II$_b$ Limits of Type IIB String Theory

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Higher Structures, Moduli Spaces and Integrability

based on:

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(in collaboration with
B. Friedrich, J. Monnee, and T. Weigand)

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Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Motivation

Geometric compactifications of String Theory

$\rightarrow$ good testing ground for quantum gravity:

properties of resulting gravitational theories $\longleftrightarrow$ *geometry of the compactification manifold*

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properties of resulting gravitational theories $\longleftrightarrow$ *geometry of the compactification manifold*

Geometric description particularly powerful in the absence of quantum corrections

$\rightarrow$ *due to non-renormalization theorems*

and/or

$\rightarrow$ *gravitational weak-coupling limits in which* $\dfrac{\Lambda_{QG}}{M_{\text{pl}}} \ll 1$

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Questions: **1.** What kind of gravitational weak-coupling limits can exist?

2. How are these characterized?

3. What is the low-energy description, degrees of freedom, symmetries, …?

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3. What is the low-energy description, degrees of freedom, symmetries, …?

In geometric compactifications of string theory:

1. Does the geometry constrain the possible weak-coupling limits?

2. What degrees of freedoms arise in weak coupling description?

3. Reverse question: Does the physics constrain the geometry?

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  2. Additional light states can collectively be described by a dual theory.

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- Summarized in the Distance and Emergent String Conjectures

At infinite distances in the moduli space of a consistent theory of gravity, there is a tower of states becoming light as [Ooguri, Vafa '06]

$$\frac{m_{\text{tower}}}{M_{\text{pl}}} \rightarrow \exp(-\alpha\Delta_{dist})$$

and this tower of states is either [Lee, Lerche, Weigand '19]

i) a KK-tower signaling a decompactification limit

or

ii) the tower of excitations of a fundamental string.

ESC: What happened so far …

- Usual test of emergent string conjecture (in string compactifications):

E.g. [Lee, Lerche, Weigand '18-'21; Baume, Marchesano, MW '19; Xu '20; Kläwer, Lee, Weigand, MW '20; Alvarez-Garcia, Kläwer, Weigand '21; Basile '22; Blumenhagen, Gligovic, Paraskevopoulou '23; Alvarez-Garcia, Lee, Weigand '23; Aoufia, Basile, Leone '24; …]

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2. Identify the tower of light states → either KK states of the original geometry, or wrapped D-/NS5-branes

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- For emergent string limits *(tensionless p-branes on (p-1)-cycles):*

→ *use known string dualities to argue for criticality of string*

AND/OR

→ *establish criticality from the worldsheet perspective by reducing worldvolume theory of p-brane on (p-1)-cycle in* $\mathscr{M}$

ESC without candidate branes for emergent strings

- **This talk:** Vector multiplet moduli space of Type IIB compactifications on CY3.

    - *Effective 4d N=2 theory of supergravity*

    - $\mathscr{M}_{\mathrm{V}}^{\mathrm{IIB}} \mathrel{\hat{=}}$ *complex structure deformations of CY3-fold also at quantum level*

    - *No light 4d BPS strings from wrapped branes!*

$\rightarrow$ **Puzzle!!**

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- Setups previously investigated in the context of the Distance Conjecture

[Grimm, Palti, Valenzuela '18, Grimm, Li, Palti '18]

  - *Candidates for towers of light BPS states from wrapped D3-branes*
  - *Open: Origin of emergent string mirror dual to Type IIA on CY3?*

[Lee, Lerche, Weigand '19]

Goals:

1. Establish the existence of a tower of BPS states becoming light in type II limits!

2. Show criticality of tensionless string in type II limit!

3. New prediction on geometry from Emergent String Conjecture and BPS State Counting on A-Model Side!

Low-energy Effective Action Perspective

- Low-energy effective action of Type IIB on Calabi-Yau threefold V

$$S_4 = \int \frac{1}{2} M_{\mathrm{Pl}}^2 R \star 1 + G_{i\bar{j}}\, \mathrm{d}u^i \wedge \star\, \mathrm{d}u^{\bar{j}} + \frac{1}{4} \mathcal{I}_{IJ} F^I \wedge \star F^j + \frac{1}{4} \mathcal{R}_{IJ} F^I \wedge F^J .$$

$u^i,\, i = 1,\ldots, h^{2,1}$
vector multiplet moduli

$F^I = dA^I,\, I = 0,\ldots, h^{2,1}$
gauge fields

- Couplings determined by Hodge inner product and Hodge norm on $H^3(V)$

$$\langle v, w \rangle = \int_V v \wedge \star \bar{w}\,, \quad \|v\|^2 = \langle v, v \rangle\,, \quad v, w \in H^3(V)$$

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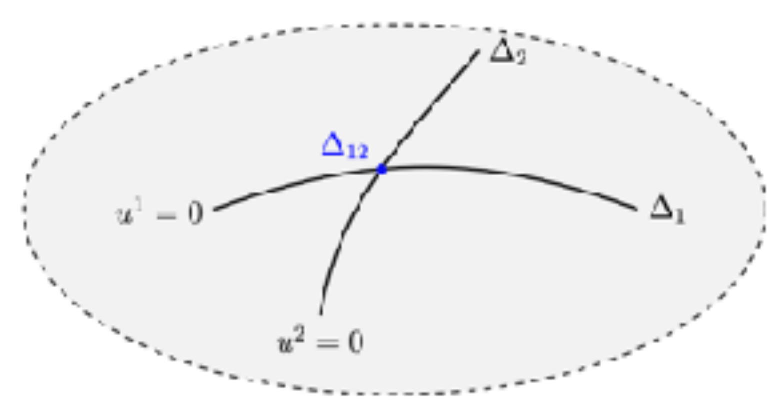
- Extreme limits correspond to singularities of $\mathscr{M}_{\mathrm{V}}^{\mathrm{IIB}}$: $\quad \Delta_{k_1,\ldots,k_r} = \{u^{k_1} = \ldots = u^{k_r} = 0\}$



- Behavior of couplings $\rightarrow$ behavior of $H^3(V)$ as $u^{k_i} \rightarrow 0$.

$\rightarrow$ asymptotic Hodge theory

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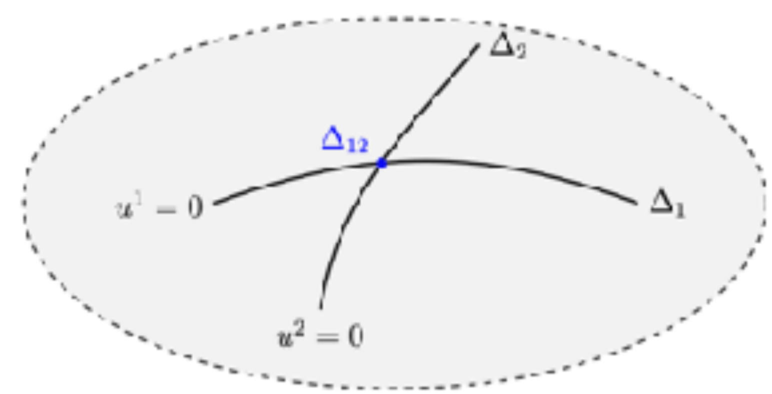


- *In this **talk**, focus on one-parameter limits $u \rightarrow 0$ (more general case, see paper or ask us later)*

[Friedrich, Monnee, Weigand, MW '25]

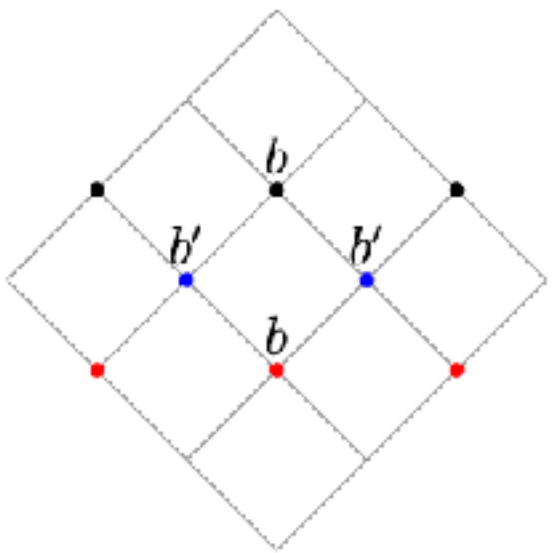
Type II$_b$ Limits in $\mathscr{M}_{\mathrm{V}}^{\mathrm{IIB}}$

- Near $\Delta = \{u = 0\}$: mixed Hodge structure by decomposing the middle cohomology

for review/introduction to the topic, see
[van de Heisteeg '22, Monnee '24]

$$H^3(V, \mathbb{C}) = \bigoplus_{0 \le p,q \le 3} I^{p,q}(\Delta)$$

- Different limits distinguished by the dimensions $i^{p,q} = \dim(I^{p,q})$



- Focus here: $i^{3,1} = 1 \qquad i^{3,q\neq 1} = 0 \;\rightarrow$ "type II limits"

  $\rightarrow$ free parameter the dimension $i^{1,1} \equiv b$.

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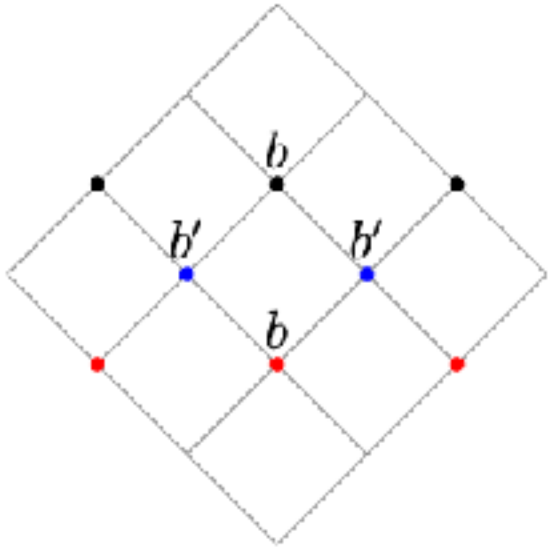

- To determine the behavior of the couplings, define the graded spaces

$$\mathrm{Gr}_\ell(\Delta) = \bigoplus_{p+q=\ell} I^{p,q}(\Delta)$$

- For Type II limits: $\quad H^3(V, \mathbb{C}) = \mathrm{Gr}_2 \oplus \mathrm{Gr}_3 \oplus \mathrm{Gr}_4\,, \qquad \mathrm{Gr}_2 \cong \mathrm{Gr}_4$ most relevant for this talk!

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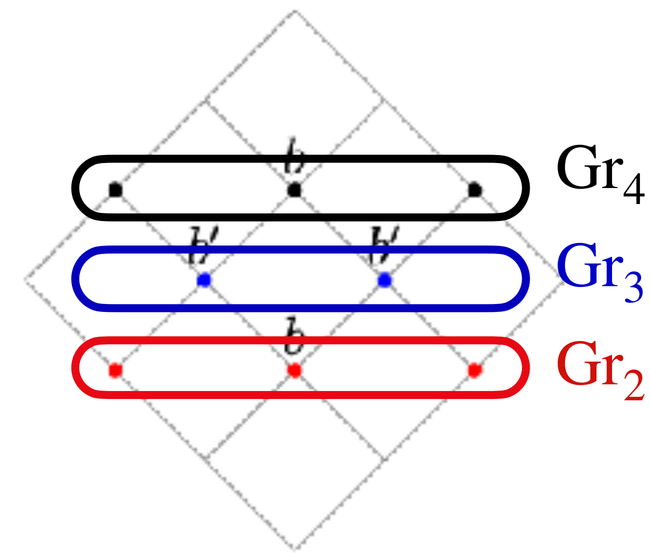

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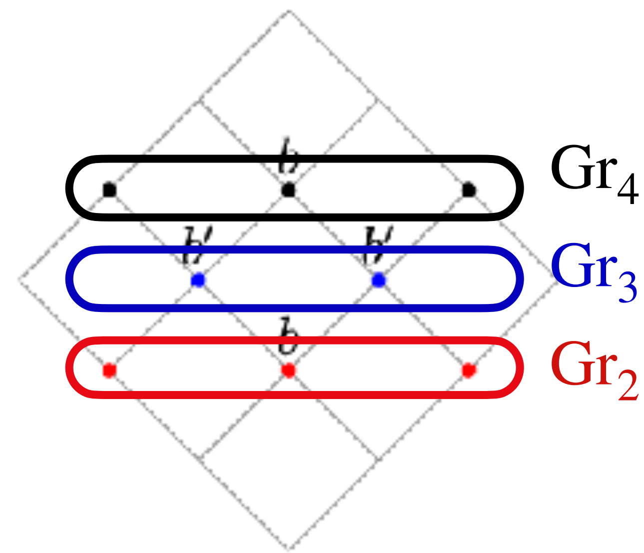

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  - Define coordinate $\qquad t = \dfrac{\log u}{2\pi \mathrm{i}} = a + \mathrm{i}s$

  - Then (for $u \to 0 \;\leftrightarrow\; s \to \infty$) $\quad e^{-K} = \|\Omega\| \sim s \qquad$ and for $q \in \mathrm{Gr}_2(\Delta)$: $\quad \|q\|^2 \sim s^{-1}$

Follows from growth theorem [Schmid '73, Cattani, Kaplan, Schmid '86]

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- D3-branes with charge $q \in H^3(V, \mathbb{Z}) \cap \mathrm{Gr}_2$.
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  [Lanza, Marchesano, Martucci, Valenzuela '20, '21]
- For string realizing type II limit, the tension scales as:

$$\frac{T(s)}{M_{\text{pl}}^2} = \frac{1}{s}$$

Comparison of scales: $\frac{m_{\text{tower}}}{M_{\text{Pl}}} \sim \frac{\sqrt{T}}{M_{\text{Pl}}}$

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This behavior is indicative of emergent string limits!
→ type II$_b$ limits are *candidates* of emergent string limits.

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- **Need:** additional information about the UV completion

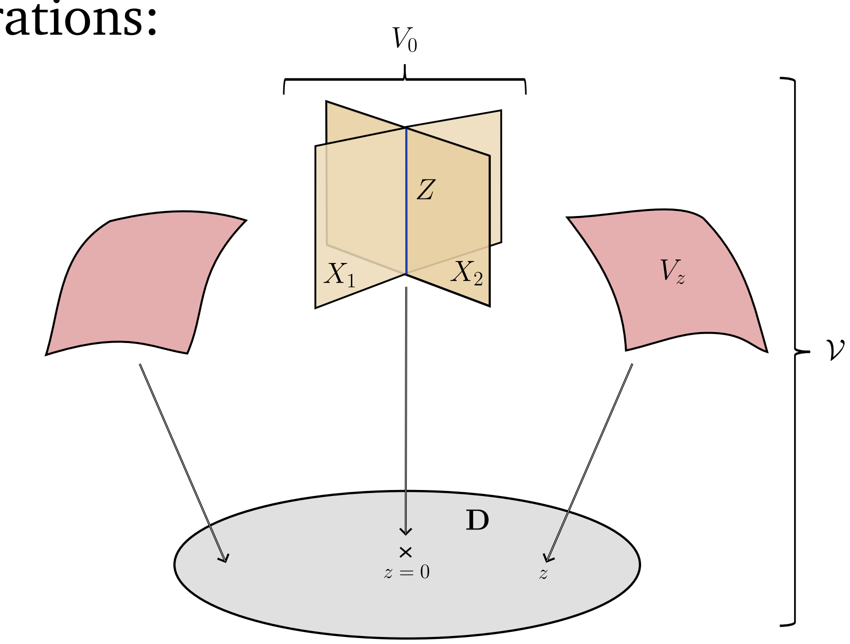
  $\rightarrow$ *geometry* of the degenerate Calabi-Yau threefold arising at Δ .

- **First:** simple class of geometries realizing type II degenerations:

$$V \longrightarrow V_0 = X_1 \cup_Z X_2$$ [Tyurin '04]

quasi-Fano threefolds

K3-surface



Goals for this talk:

1. Establish the existence of a tower of BPS states becoming light in type II limits corresponding to Tyurin degenerations!

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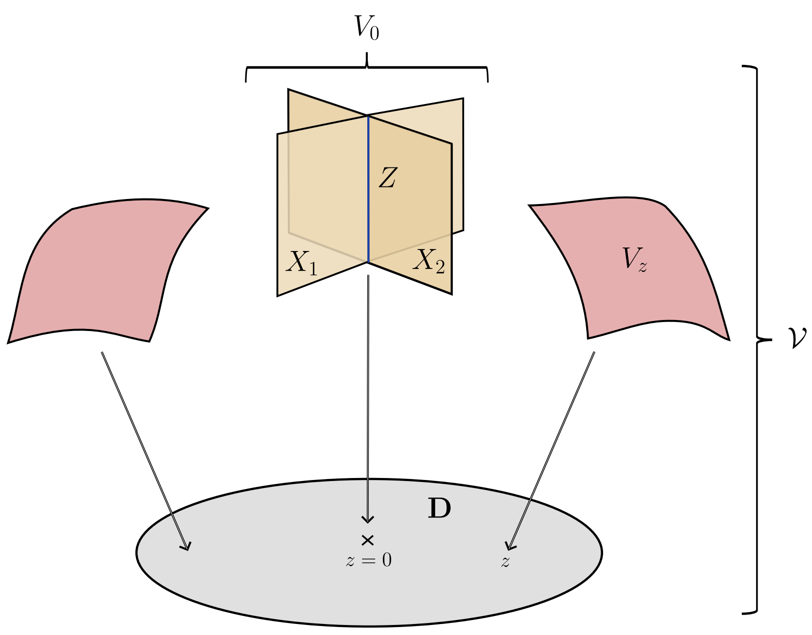

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- **Strategy**: identify a charge $q_0 \in H^3(V, \mathbb{Z}) \cap \mathrm{Gr}_2$ for which dual 3-cycle $\Gamma_0 \in H_3(V)$ gives a **bound state** upon *multi-wrapping D3-branes* on it
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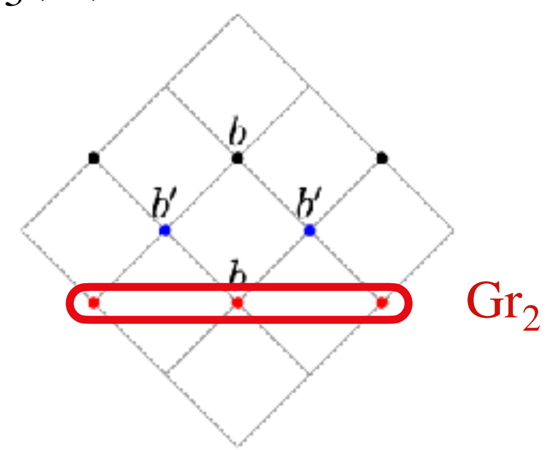

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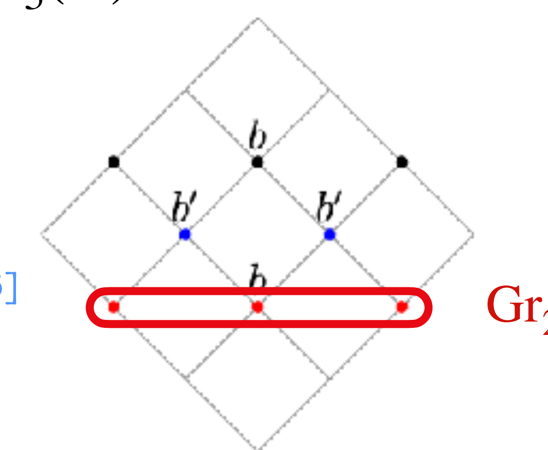

- **Insight:** Geometry of Tyurin degeneration identifies cf. [Doran, Harder, Thompson '16]

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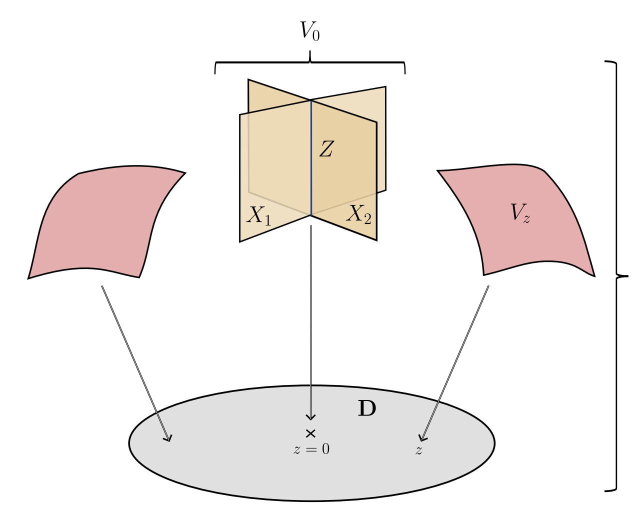

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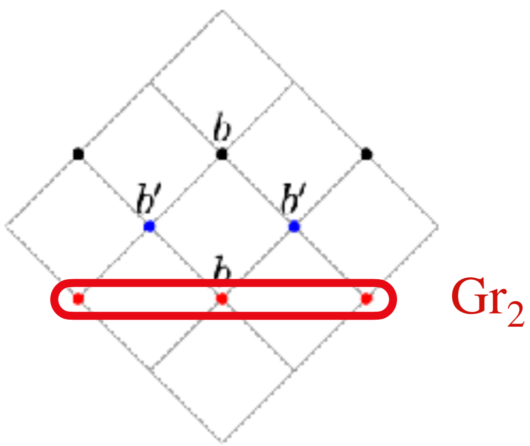

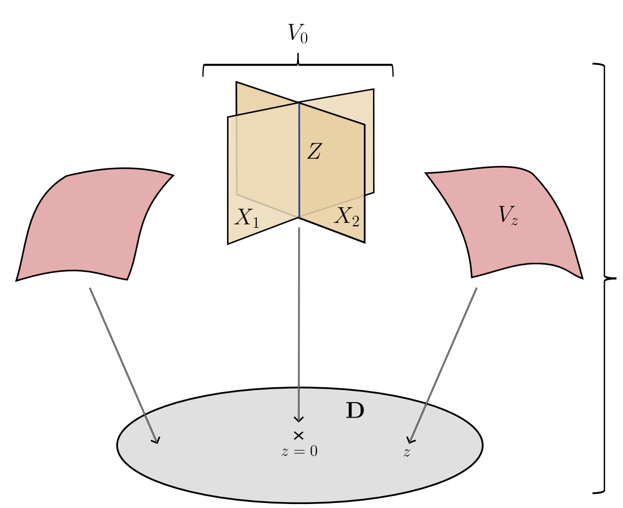

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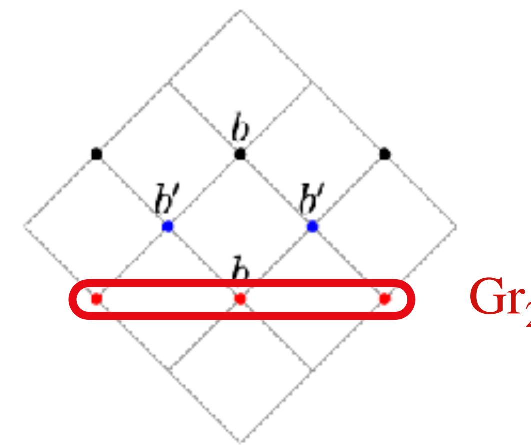
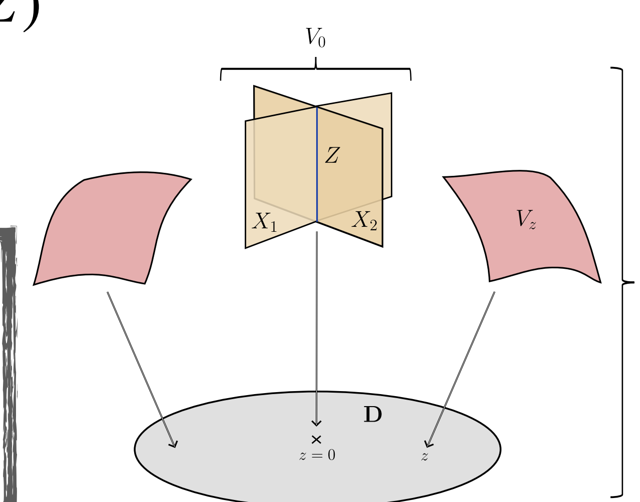


- **Locally at degeneration** *(via Mayer-Vieotris):*
three-cycle Γ_0 dual to $q_0 \in H^3(V,\mathbb{Z}) \cap \mathrm{Gr}_2$ $\leftrightarrow$ S^1 fibration over $C_0 \in H_2(Z)$

S^1 shrinking at degeneration.



BPS **bound state** with charge nq_0 exists $\leftrightarrow$ Γ_0 is *special Lagrangian* and super-QM of n D3-branes on Γ_0 have enough *scalars to break* $U(n) \rightarrow U(1)$.

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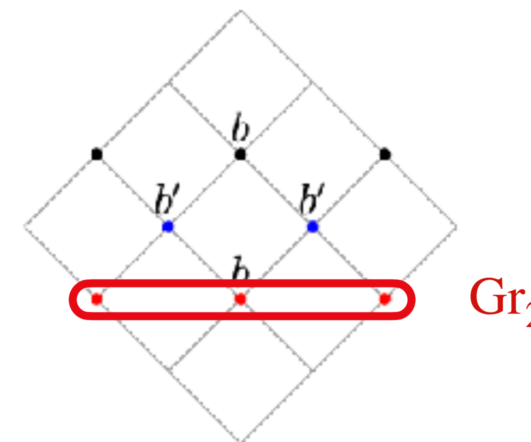

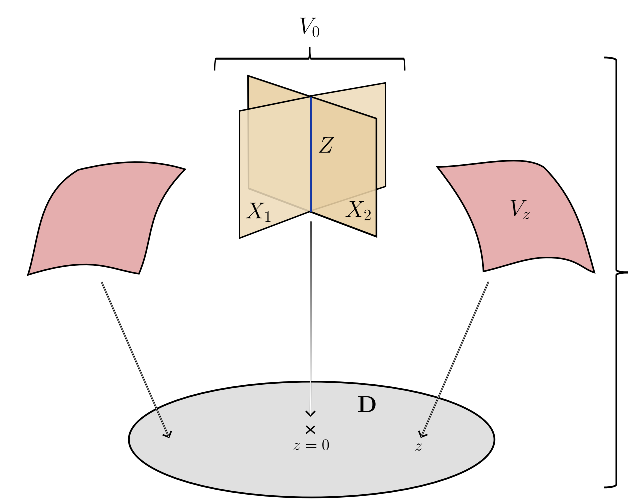

Tower of BPS States

[Friedrich, Monnee, Weigand, MW '25]

1. Establish the existence of a tower of BPS states becoming light in type II limits corresponding to Tyurin degenerations!

$$\mathrm{Gr}_2 \cong \frac{H^2(Z)}{\mathrm{im}(i^* H^2(X_1)) + \mathrm{im}(j^* H^2(X_2))}$$

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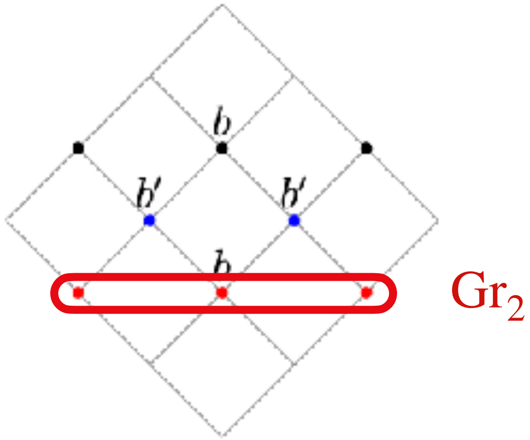

- Elements of $\mathrm{Gr}_2 \cong \Lambda_{\mathrm{trans}}(Z)$ with signature $(2,b)$

$\rightarrow \exists$ (at least) two curve classes $C_0^{(k)} \in H_2(Z)$ with $C_0^{(k)} \cdot_Z C_0^{(k)} > 0$

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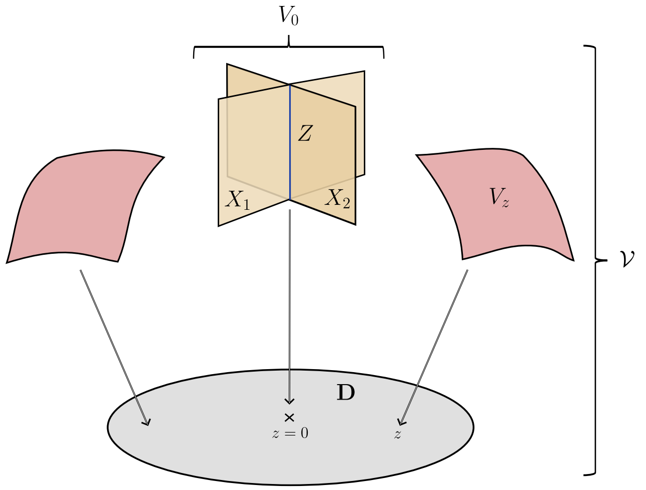

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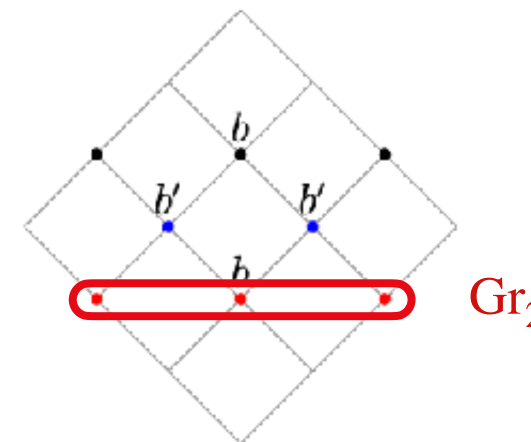

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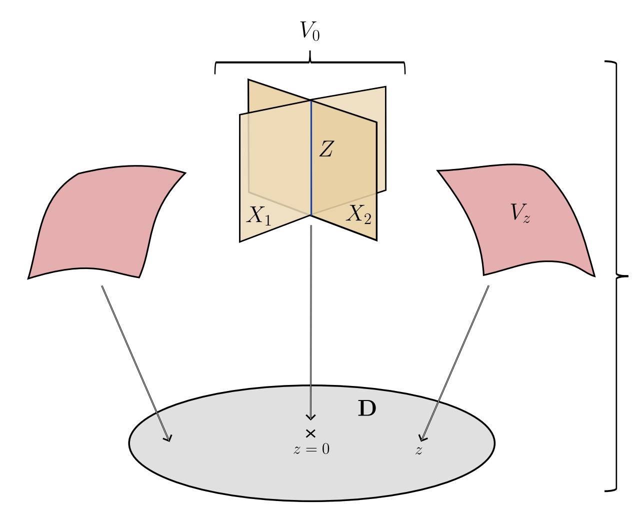

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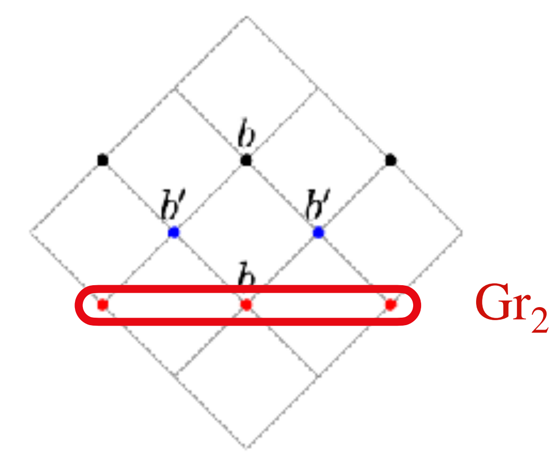

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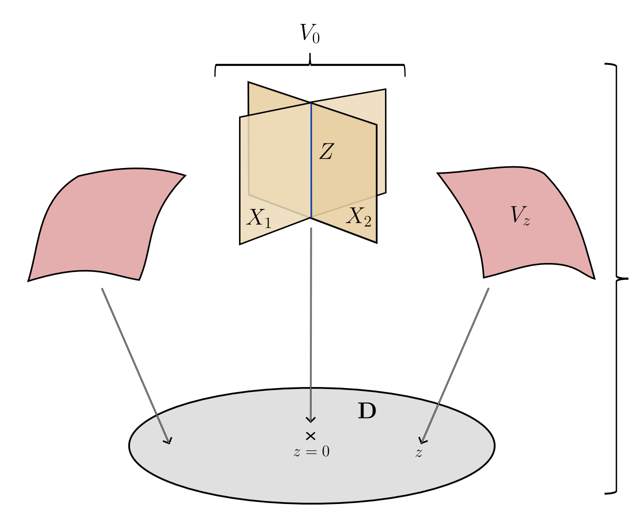

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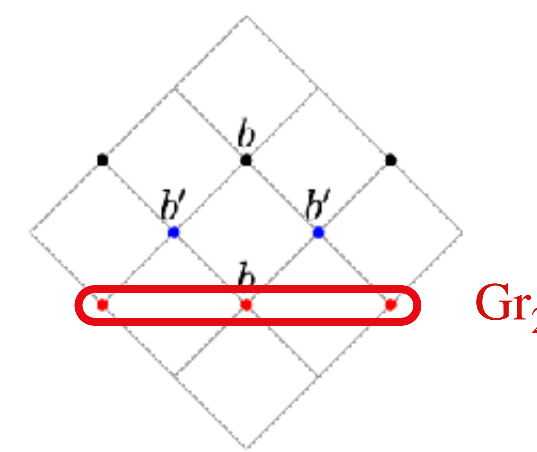
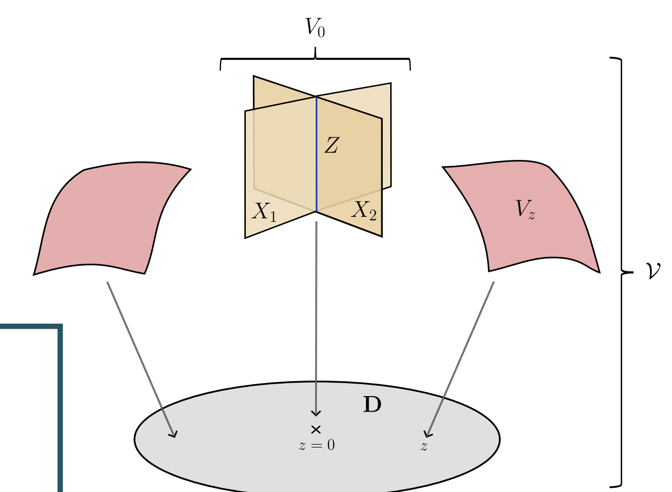


$$C_0^{(k)} \cdot_Z C_0^{(k)} > 0 \to g(C_0^{(k)}) \geq 1 \to b_1(C_0^{(k)}) \geq 2$$

- Local fibration structure of $\Gamma_0^{(k)} \to b_1(\Gamma_0^{(k)}) \geq 3$:
  $\to n$ D3-branes on $\Gamma_0^{(k)}$ have sufficient scalar d.o.f. to break $U(n) \to U(1)$.



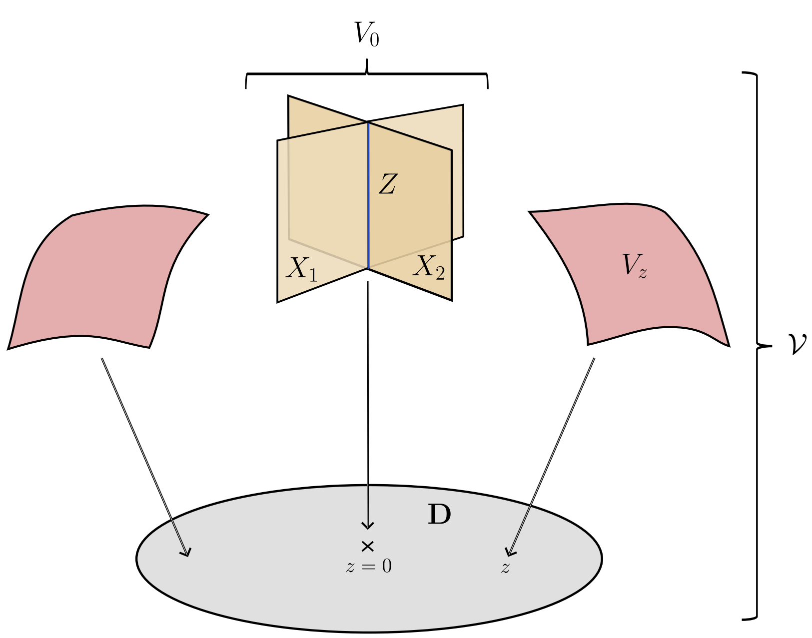
Tower of BPS bound states with mass scaling as $\dfrac{m_{\mathrm{tower}}}{M_{\mathrm{Pl}}} \sim \dfrac{1}{\sqrt{s}}$

Goals for this talk:

[Friedrich, Monnee, Weigand, MW '25]

1. Establish the existence of a tower of BPS states becoming light in type II limits corresponding to Tyurin degenerations!

2. Show criticality of tensionless EFT string!



Criticality of the Light String

[Friedrich, Monnee, Weigand, MW '25]

2. Show that the EFT string realizing these limits and becoming tensionless is a critical string and determine the perturbative dual frame.

Strategy:

I. Determine the spectrum of massless degrees of freedoms along the string.

II. Show central charges on the string are $c_L = 24\,, \quad c_R = 12$

$\rightarrow$ *heterotic string in light-cone gauge.*

III. Study the worldsheet interactions to distinguish free vs. interacting fields.

IV. Use this to identify the perturbative dual frame:

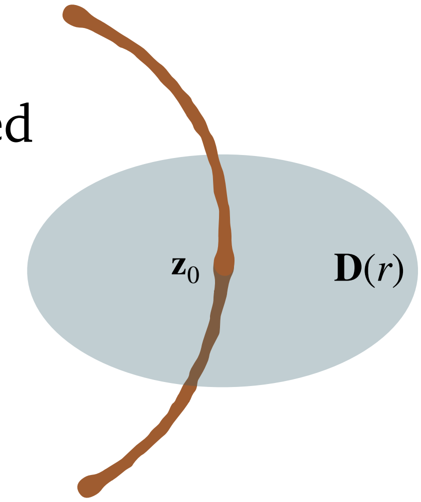
$\rightarrow$ *heterotic string on* $(T^2 \rightarrow \mathbb{P}^1) \times T^2$ *with perturbative gauge group of rank* $2 + b$ *for a limit of type* II_b.

I. Worldsheet Degrees of Freedom

[Friedrich, Monnee, Weigand, MW '25]

- Take the BPS string solutions of [Lanza, Marchesano, Martucci, Valenzuela '20, '21]

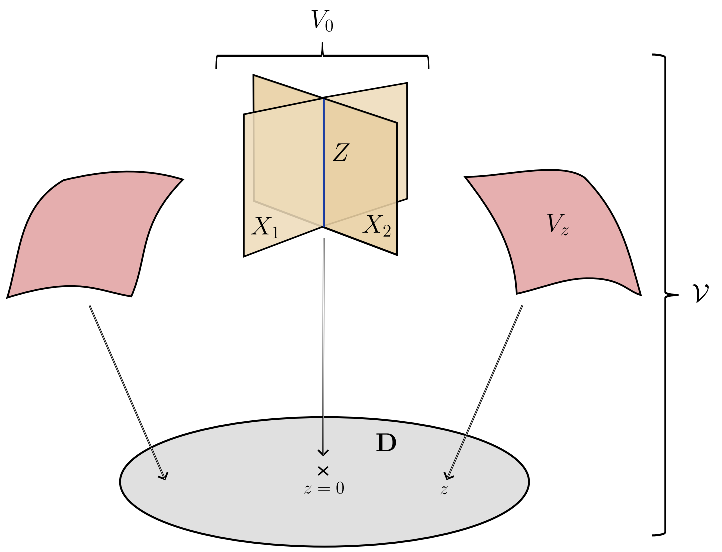
  → zoom in to the string core: string can be approximated as infinitely extended



$$ds^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + e^{2D}\mathrm{d}z\mathrm{d}\bar{z}\,, \qquad \text{with} \quad e^{2D} = f_0 e^{-K}\,,$$

$$t(z) = \mathrm{i}\bar{s} + \frac{1}{2\pi \mathrm{i}}\log\left(\frac{z}{r}\right)\,,$$

- BPS string solution of 10d Type IIB supergravity → string worldsheet preserves $\mathcal{N} = (0{,}4)$ supersymmetry in 2d.



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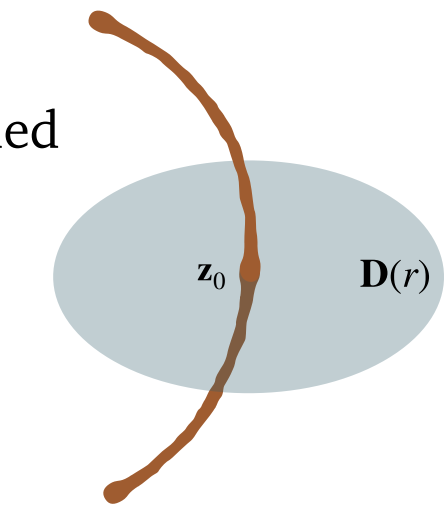

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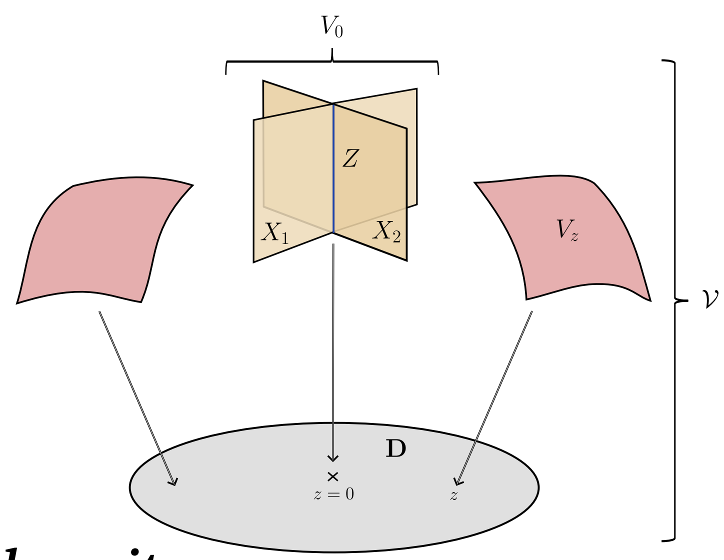
- Worldsheet degrees of freedom have two origins:

  1. Geometric moduli of string:

  - *Position modulus $\mathbf{z}_0 \in \mathbf{D}(r)$ → two left- & two right-moving scalars on the string worldsheet.*

  - *Modulus Φ of $Z \in V_0$ → one left- & one right-moving scalar on string WS.*



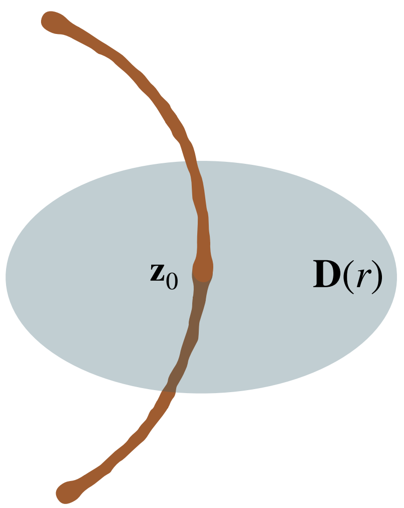
  2. Components of Type IIB p-forms localized to string and propagating along it.

I. Worldsheet Degrees of Freedom

[Friedrich, Monnee, Weigand, MW '25]

2. *Components of Type IIB p-forms localized to string and propagating along it.*

- Consider p-form of Type IIB string theory C_p .



- It gives rise to a massless field propagating along the string if there exists a harmonic $(p-2)$-form localized on the string.

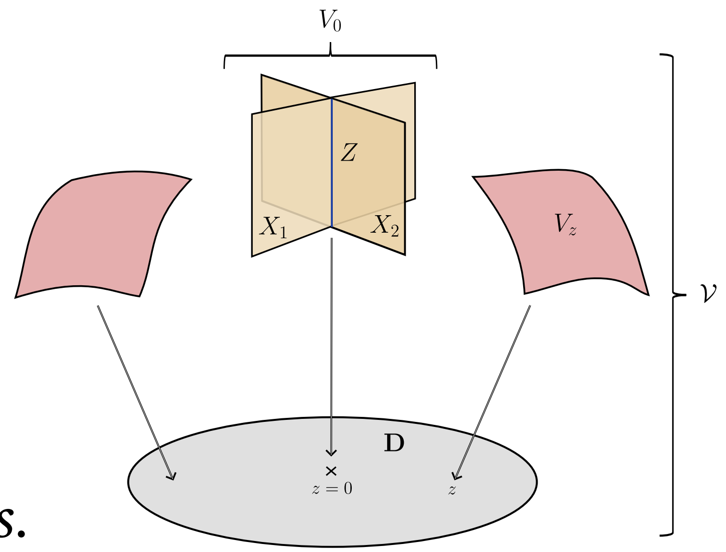
$$C_p \supset B_2^a \wedge \omega_a^{p-2} \qquad\qquad \omega_a \in H^{p-2}(Z)$$

Transverse components give scalar fields propagating along string

- In our setup, we get localized modes from C_4 and (B_2, C_2)

  - *E.g.: from C_4 we get:* $\quad C_4 = B_2^\alpha \wedge \omega_\alpha \qquad \omega_\alpha \in H^2(Z)$

  - $\mathrm{sgn}(H^2(Z)) = (3{,}19) \rightarrow$ *3 right- and 19 left-moving WS scalars.*



II. Criticality of the String

[Friedrich, Monnee, Weigand, MW '25]

- Summary of bosonic degrees of freedom:

		n_L^{bos}	n_R^{bos}
- Geometric Moduli	$\mathbf{z}_0$	2	2
	$\boldsymbol{\Phi}$	1	1
- (B_2, C_2) :	$\mathbf{b}^0, \widetilde{\mathbf{b}}^0$	2	2
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+ fermionic superpartners
for (0,4) WS SUSY

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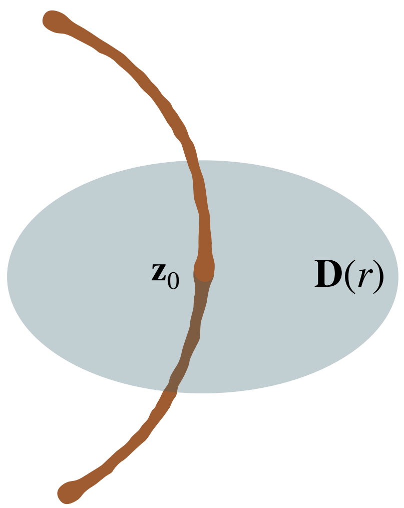
EFT string emerging at type II$_b$ limit is a heterotic string! → ***which one?***

III. Interactions on the Worldsheet

[Friedrich, Monnee, Weigand, MW '25]

- Specifically, we aim to distinguish free fields on the WS from interacting fields.



- **Strategy:** Consider the kinetic terms for the scalar fields on the string

$$S_{\mathrm{WS,kin}} = \frac{1}{2}\int \mathrm{d}^2\sigma\, g_{\mu\nu}(\phi)\, \partial_\sigma \phi^\mu \partial^\sigma \phi^\nu$$

Field space metric on the string

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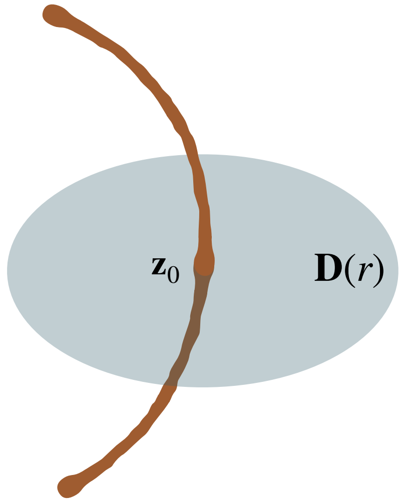

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metric is field-independent
$\leftrightarrow$ ϕ^{μ_0} free scalar

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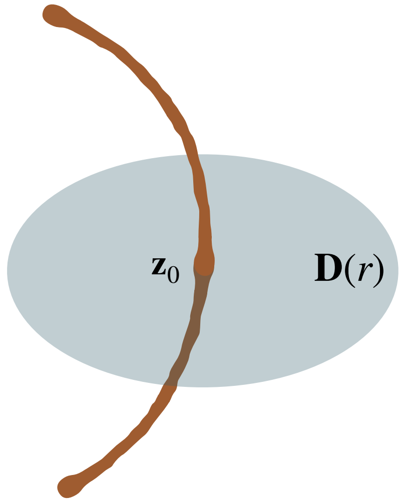

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- For the modes arising, e.g., from C_4:

$$C_4 = B_2^\alpha \wedge \omega_\alpha$$

$$S_{10\mathrm{d}} = \int \mathrm{d}C_4 \wedge \star_{10d}\, \mathrm{d}C_4 \supset \int_{\mathbf{D}} \mathrm{d}z\, \mathrm{d}\bar{z}\, \mathcal{S}(z, \bar{z}, b^\alpha)$$

- Extract $S_{\mathrm{WS,kin}}$ as:

$$\delta^{(2)}(z - \mathbf{z}_0)\, S_{\mathrm{WS,kin}}(\mathbf{z}_0, \bar{\mathbf{z}}_0, b^\alpha) = \delta^{(2)}(z - \mathbf{z}_0)\, \mathcal{S}(z - \mathbf{z}_0, \bar{z} - \bar{\mathbf{z}}_0, b^\alpha)$$

Result: $2 + b$ **free modes and** $20 - b$ **interacting modes from** C_4

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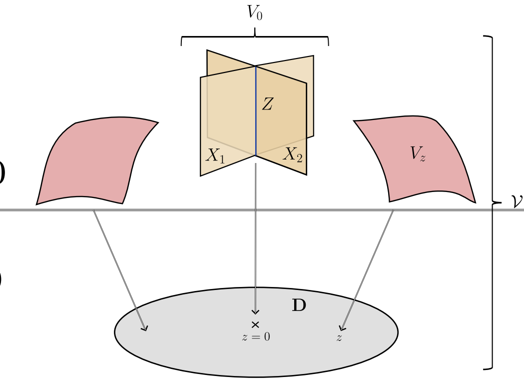

- **Details**: for the modes arising from C_4 have to distinguish:

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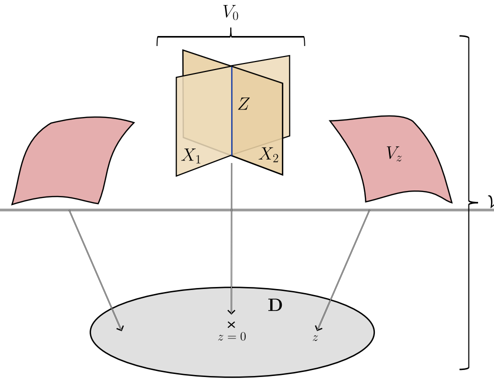

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- Reduce 10d action for C_4

$$S_{10\mathrm{d}} \to \int_{\mathbf{D}} \mathrm{d}z\, \mathrm{d}\bar{z} \int_{\mathbb{R}^{1,1}} \mathrm{d}t\, \mathrm{d}x (\partial b^i)(\partial b^j) e^{2D(z,\bar{z})} \int_{V_z} \bar{\omega}_i \wedge \star\, \bar{\omega}_j$$

- $\bar{\omega}_i$ only have support at $z = \mathbf{z}_0$:

$$\to \qquad \int_{V_z} \bar{\omega}_i \wedge \star\, \bar{\omega}_j = \Omega_{ij}\, \delta^{(2)}(z - \mathbf{z}_0)$$

- We then have:

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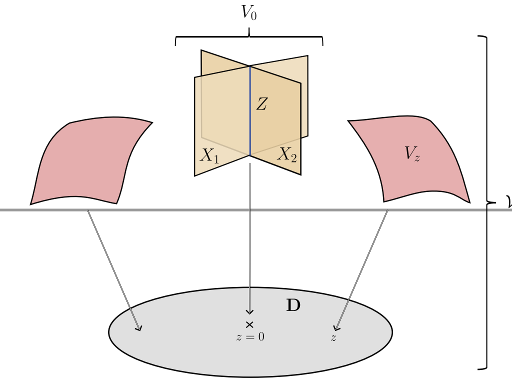

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- For the WS kinetic term this means:

$$S_{\mathrm{WS,kin}}(\mathbf{z}_0, \bar{\mathbf{z}}_0, b^i) = \frac{1}{2} \int \mathrm{d}^2\sigma\, g_{ij}(\mathbf{z}_0, \bar{\mathbf{z}}_0)\, \partial_\sigma b^i \partial^\sigma b^j$$

$$\text{with} \quad g_{ij}(\mathbf{z}_0, \bar{\mathbf{z}}_0) = 2 e^{2D(\mathbf{z}_0, \bar{\mathbf{z}}_0)} \Omega_{ij}$$

Fields b^i non-trivially interact with $\mathbf{z}_0$!

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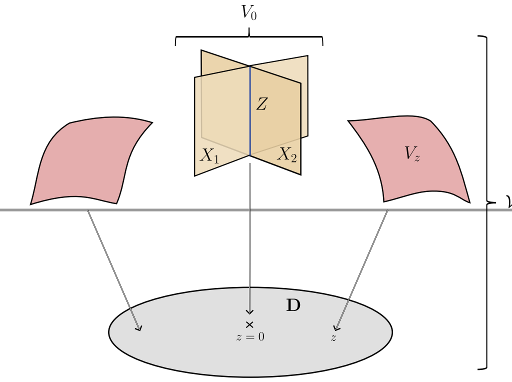

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- Image of ω_a under boundary map non-trivial:

$$0 \neq d^*\omega_a = \gamma_a \in H^3(V_0)$$

- In fact: $\gamma_a \in \mathrm{Gr}_2$ such that $\quad \|\gamma_a\|^2 \sim \dfrac{1}{\log\left|\frac{z - \mathbf{z}_0}{r}\right|} \sim e^K$

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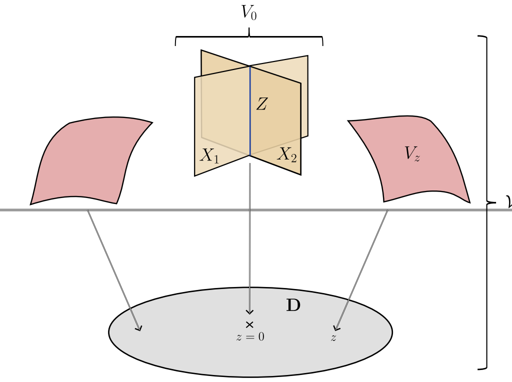

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- In fact: $\gamma_a \in \mathrm{Gr}_2$ such that $\quad \|\gamma_a\|^2 \sim \dfrac{1}{\log|\frac{z-\mathbf{z}_0}{r}|} \sim e^K$

- Reduce 10d action for C_4 → read off kinetic WS term

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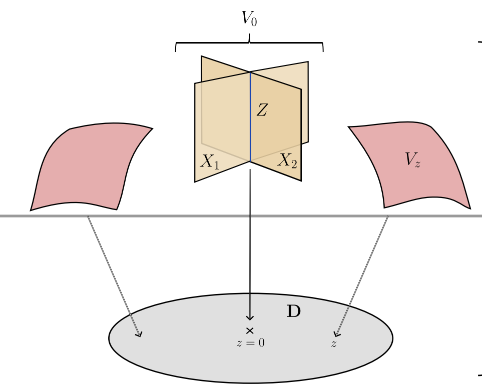

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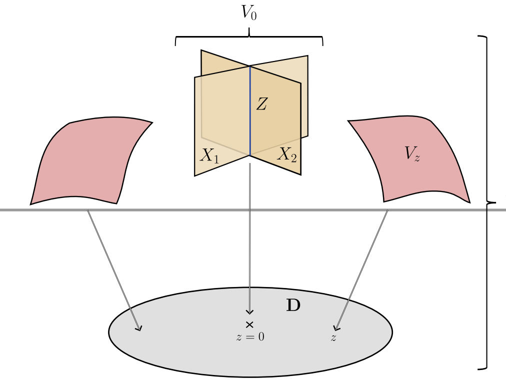

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[Friedrich, Monnee, Weigand, MW '25]

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$$\text{sgn}(\Lambda_{\text{pol}}) = (1, 19 - b)$$

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$19 - b$ interacting left-moving scalars

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Correct for $18 \geq b \geq 2$ — otherwise "target space" slightly different!

1. **Establish the existence of a tower of BPS states becoming light in type II limits corresponding to Tyurin degenerations!** ✓

2. **Show criticality of tensionless EFT string!** ✓

Beyond Tyurin Degenerations

[Friedrich, Monnee, Weigand, MW ’25]

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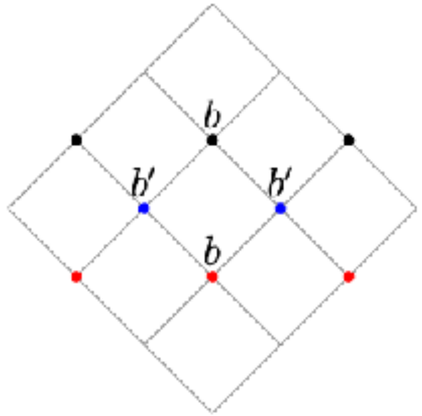
$$V \longrightarrow V_0 = X_1 \cup_Z X_2$$

quasi-Fano threefolds K3-surface



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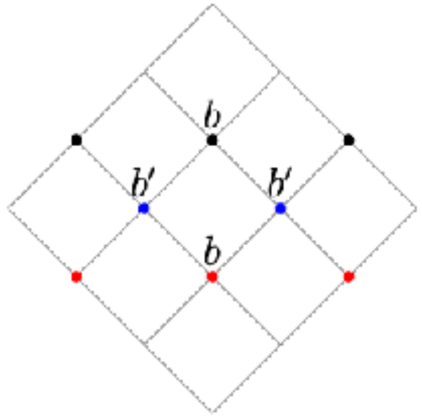

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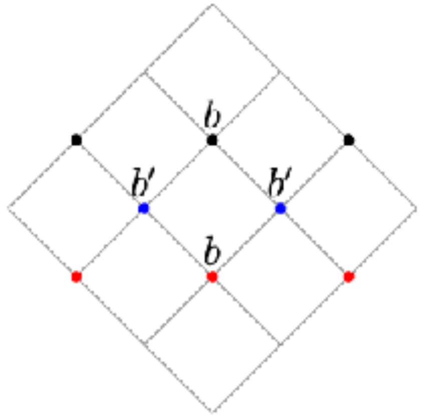

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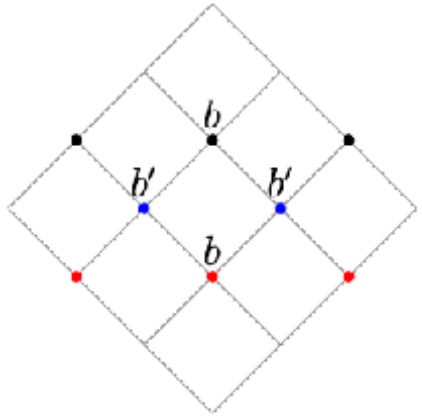

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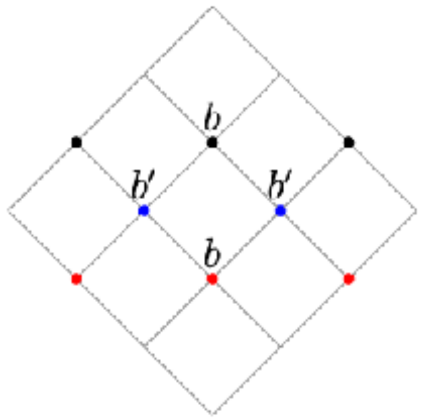

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- **Idea:** *Turn logic around and use Emergent String Conjecture to predict constraints on the possible geometries arising at type II_b singularities.*

Beyond Tyurin Degenerations

[Friedrich, Monnee, Weigand, MW '25]

Conjecture: For a type II_b limit in the complex structure moduli space of a Calabi-Yau threefold, the degenerate threefold, V_0, can be brought into the semi-stable form

$$V_0 = \bigcup_{i=1}^{n} X_i$$

for which the non-vanishing surfaces $X_i \cap X_j$ are either all Abelian surfaces with the same complex structure or all K3 surfaces with the same polarization lattice of rank $(1,19-b)$. In particular, the parameter b characterizing the family of II_b degenerations is bounded as

$$0 \leq b \leq 19$$

- Prediction of Emergent String Conjecture on *geometry*.
- Can be viewed as analogous to the constraints on geometries of Type II Kulikov models of K3 surfaces. [Kulikov '77,'81; Persson '81]
- Can the statement be confirmed from a purely geometric perspective … ?

Conclusions

[Friedrich, Monnee, Weigand, MW '25]

- Successful test of the Emergent String Conjecture in the vector multiplet sector of Type IIB compactifications on Calabi-Yau threefolds.

- higher-dimensional branes cannot provide the critical string in emergent string limits!

  $\leftrightarrow$ "usual" string dualities cannot be used to prove the ESC … .

- Low-energy effective action: type II$_b$ limits are candidates for emergent string limits
  $\rightarrow$ tensionless EFT string as in [Lanza, Marchesano, Martucci, Valenzuela '20, '21]

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In other words: showed heterotic/Type IIB duality from the worldsheet without using mirror symmetry!

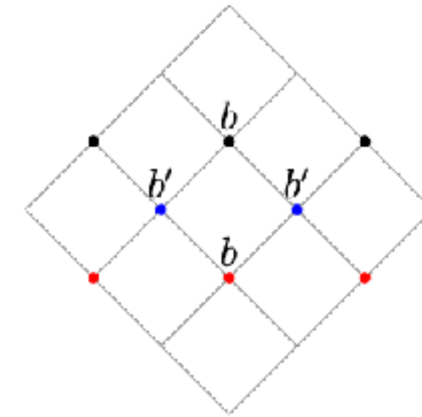
- Furthermore: used ESC to constrain possible other geometries arising at type II_b singularities!

Thank you!

Back-up Slides

Beyond Tyurin Degenerations

[Friedrich, Monnee, Weigand, MW '25]



- Assume we have a type II degeneration $V \to V_0 = \bigcup_{i=1}^{n} X_i \,, \qquad M_{i,j} = X_i \cap X_j$ where (at least) one M_{i_1,j_1} is not a K3 or Abelian surface.

- As before: have a string that is becoming tensionless at the same rate as a tower of BPS states with charge $q \in H^3(V, \mathbb{Z}) \cap \mathrm{Gr}_2$.

- The Type IIB p-forms reduced over localized forms on $M_{i,j}$ give degrees of freedom on the string.

$$C_p \supset B_2^a \wedge \omega_a^{p-2} \qquad\qquad \omega_a \in H^{p-2}(M_{i,j})$$

- One surface M_{i_0,j_0} is a K3/an Abelian surface $\to$ gives right-moving degrees of freedom with $c_R = 12$.

- Since $h^4(M_{i_1,j_1}) \neq 0$ get additional right-moving degrees of freedom $\to$ string has $c_R > 12$! ⚡ criticality of string

- Only way out: all $M_{i,j}$ are equivalent $\to$ counting d.o.f. of each individual one is redundant!

Relationship BPS Tower $\leftrightarrow$ Critical String

[Friedrich, Monnee, Weigand, MW '25]

- We found BPS states obtained from D3-branes on three-cycles Γ_0 associated with a cycle $C_0 \in H_2(Z)$ in the *transcendental lattice* Λ_{trans} of Z.

$$\Lambda_{\text{trans}} \subset U^{\oplus 3} \oplus \Gamma_{16}$$

- Goal: Want to count the BPS invariants associated with Γ_0

$$\Omega_{\text{BPS}}(\Gamma_0) = (-1)^{\dim(\mathscr{M}_{\Gamma_0})} \chi(\mathscr{M}_{\Gamma_0})$$

$\mathscr{M}_{\Gamma_0} \hat{=}$ moduli space of A-brane on Γ_0

Euler characteristic

- Heterotic interpretation of light tower of BPS states: winding and momentum states of T^2 in

$$\mathscr{M} = T^2 \times \text{K3} \times \mathbb{C}^*$$

- Projection of C to $U^{\oplus 2} \subset \Lambda_{\text{trans}}$ counts winding and momentum w.r.t. one-cycles in T^2.

Conjecture: *In type II$_b$ limits associated with a Tyurin degeneration and for a special Lagrangian Γ_0 dual to an element in $H^3(V, \mathbb{Z}) \cap \text{Gr}_2$, there exists a meromorphic mock-modular form*

$$\theta(q) = \sum_{n \in \mathscr{I}} c(n) q^n \quad \textit{such that} \quad \Omega_{\text{BPS}}(\Gamma_0) = c\left(\frac{1}{2} C_0 \cdot_Z C_0\right) .$$

Comparison to Emergent Strings in Type IIA

[Friedrich, Monnee, Weigand, MW '25]

- In Type IIA CY3 compactifications → Emergent strings from NS5-branes on K3-fiber $\hat{Z}$

$$\hat{V} = \hat{Z} \to \mathbb{P}^1_b$$

[Lee, Lerche, Weigand '19]

- Worldsheet theory = Reduction of NS5-brane worldvolume on $\hat{Z}$.

Critical heterotic string since $\hat{Z}$ is K3.

- Polarization lattice $\Lambda_{\text{pol}}(\hat{Z}) = H^2(\hat{Z}) \cap H^{1,1}(CY3)$ gives current algebra → bulk gauge theory

- Target space for heterotic string: $(T^2 \to \mathbb{P}^1_b) \times T^2 \times \mathbb{C}$

	strings in type II_b limits	NS5-brane strings in IIA	Same if
Criticality	$V_0 = X_1 \cup_Z X_2$ K3-surface	$\hat{Z}$ is K3	
Bulk gauge theory	$\Lambda_{\text{trans}}(Z)$	$\Lambda_{\text{pol}}(\hat{Z})$	$\Lambda_{\text{trans}}(Z) = U \oplus \Lambda_{\text{pol}}(\hat{Z})$ (Z and $\hat{Z}$ mirror)
Gauge bundle	embedding $Z \hookrightarrow V$	embedding $\hat{Z} \hookrightarrow \hat{V}$	V and $\hat{V}$ are mirror (at least in absence of het. NS5-branes)

+2