SymTFTs and Non-invertible Symmetries of 6d SCFTs from M-theory

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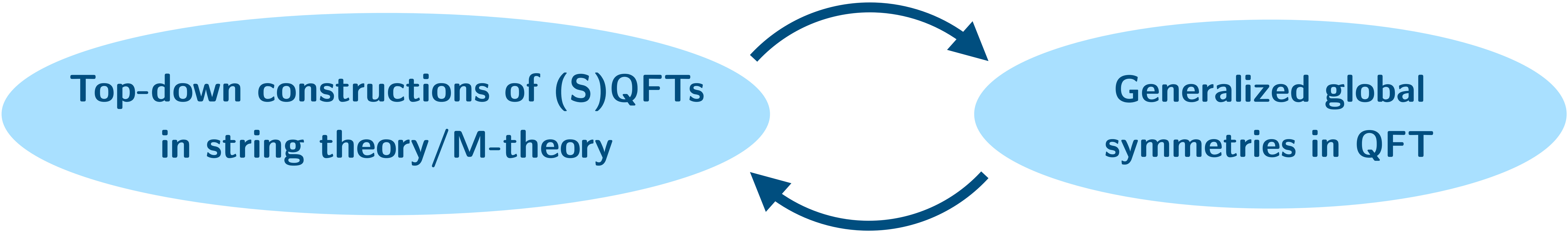
Introduction and motivation

- One of the most striking results of string/M-theory is the prediction of **non-trivial UV fixed points** in 5d and 6d
- They are hard to study with more conventional field-theoretic techniques
- Intrinsically **strongly coupled**, elude textbook Lagrangian formulation based on weak-coupling
- Rich landscape of interacting models in lower dimensions via dimensional reductions
- Laboratory to explore the space of (supersymmetric) QFTs beyond Lagrangian models

Introduction and motivation



- Rich interplay between SQFT constructions in string/M-theory and generalized global symmetries
    - Symmetry structures provide robust data that guides our analysis of SQFTs from string/M-theory
    - String/M-theory constructions offer unique chances to study symmetry structures beyond Lagrangian theories

Introduction and motivation

- Case study for this talk: **6d (2,0) SCFTs of type D**
- We leverage their realization in **M-theory** to uncover their symmetry structures

    1. Derivation of their SymTFT from M-theory
    2. Derivation of topological operators of the SymTFT from M2- and M5-branes
    3. Different global variants exhibit non-trivial symmetry structures
        - Mixed 't Hooft anomaly
        - Higher-group
        - Non-invertible symmetry

Outline

1. Introduction
2. Brief reminder on generalized symmetries and the SymTFT construction
3. SymTFT for 6d SCFTs of type D
4. Application: (non-)invertible symmetries
5. Conclusions and outlook

Brief reminder on generalized symmetries and the SymTFT construction

Generalized global symmetries

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- The textbook notion of global symmetry has been vastly generalized building on a key idea

Global symmetries $\leftrightarrow$ *Topological operators*

[Gaiotto, Kapustin, Seiberg, Willet 14]

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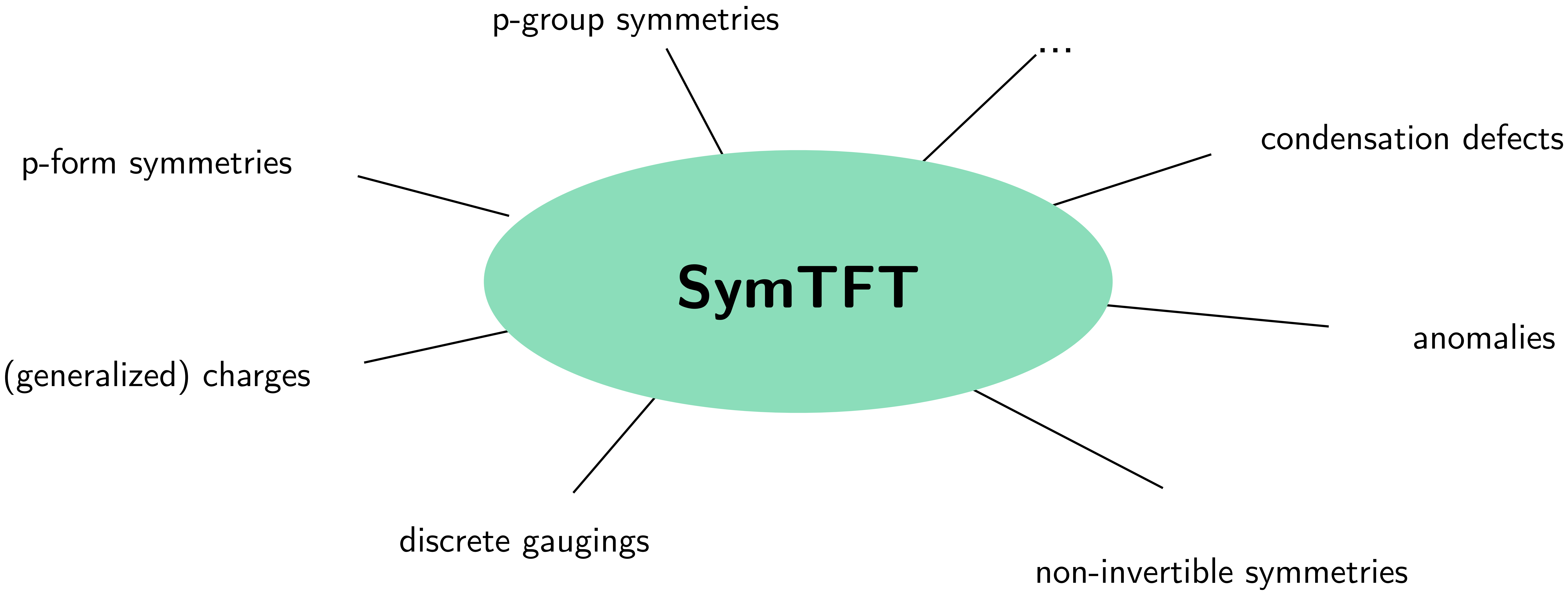
- Usual symmetries (0-form) are associated to codimension-1 topological operators labeled by a group element $D^{(g)}_{d-1}$
- We can allow for:
  - Topological operators of different dimensions $D^{(g)}_{d-p-1}$ $\leftrightarrow$ p-form symmetries
  - Non-trivial "mixtures" of p-form symmetries of different degrees $\leftrightarrow$ higher-group structures
  - Topological operators whose fusion algebra is richer than group-like $\leftrightarrow$ non-invertible symmetries

$$D^{(a)}_{d-p-1} \otimes D^{(b)}_{d-p-1} \cong \sum_c \mathscr{T}_c \, D^{(c)}_{d-p-1}$$

e.g reviews [Cordova, Dumitrescu, Intriligator, Shao 22; McGreevy 22; Gomes 23; Schafer-Nameki 23; Brennan, Hong 23; Bhardwaj, Bottini, Fraser-Taliente, Gladden, Gould, Platschorre, Tillim 23; Shao 23; Carqueville, Del Zotto, Runkel 23]

The SymTFT: an organizing principle

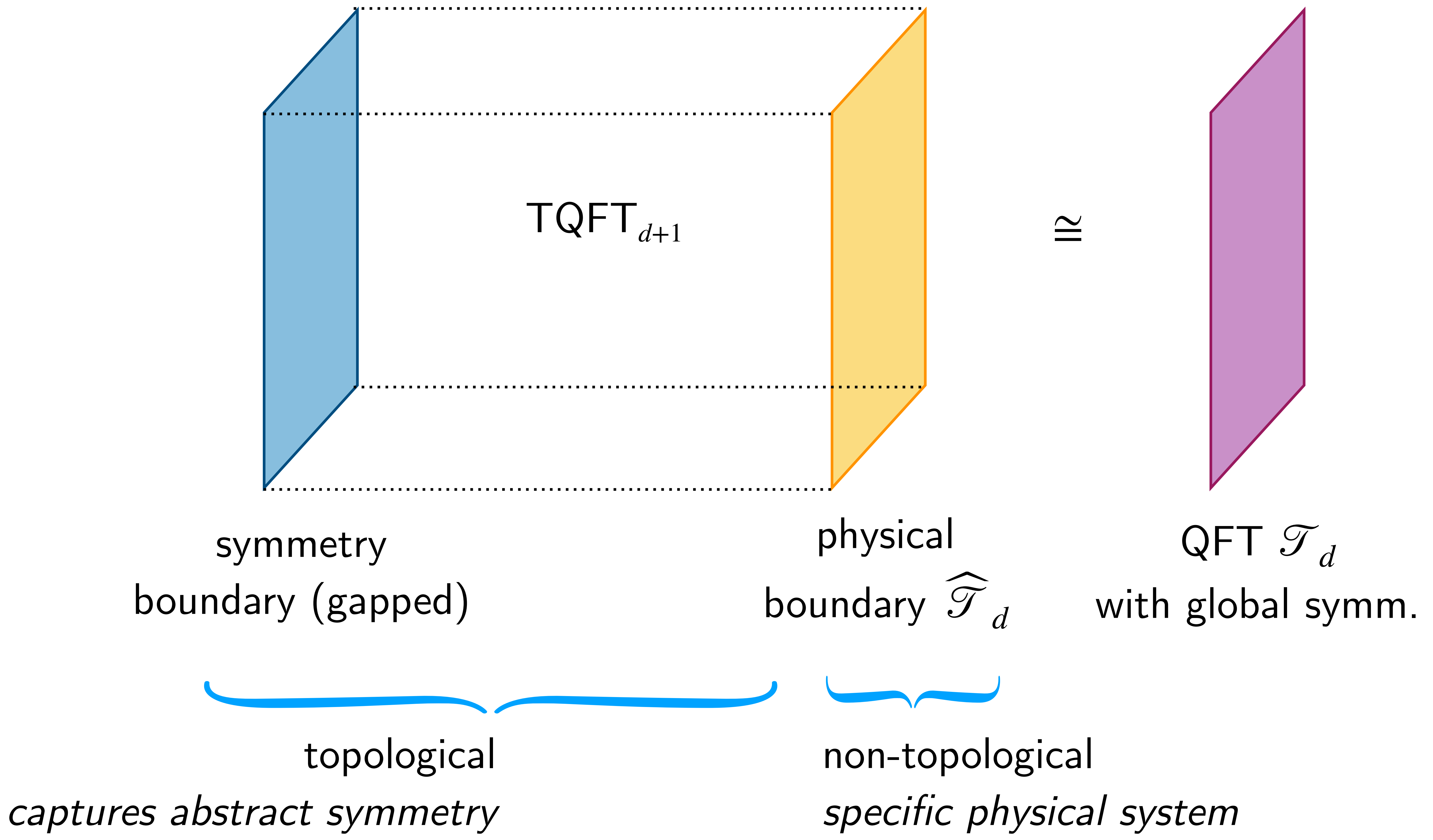
physical QFT $\mathscr{T}$ in d dimensions $\rightarrow$ symmetry topological field theory in $d+1$



[Freed, Teleman 12; Freed 14; Ji, Wen 19; Gaiotto, Kulp 20; Apruzzi, **FB**, García Etxebarria, Hosseini, Schäfer-Nameki 21; Freed, Moore, Teleman 22; ...]

Sandwich construction

[Freed, Teleman 12; Freed 14; Ji, Wen 19; Gaiotto, Kulp 20; Apruzzi, **FB**, García Etxebarria, Hosseini, Schäfer-Nameki 21; Freed, Moore, Teleman 22; ...]



- The physical theory of interest is realized as interval compactification of a TQFT in one higher dimension with one gapped boundary and one physical boundary (non necessarily gapped)

Sandwich construction

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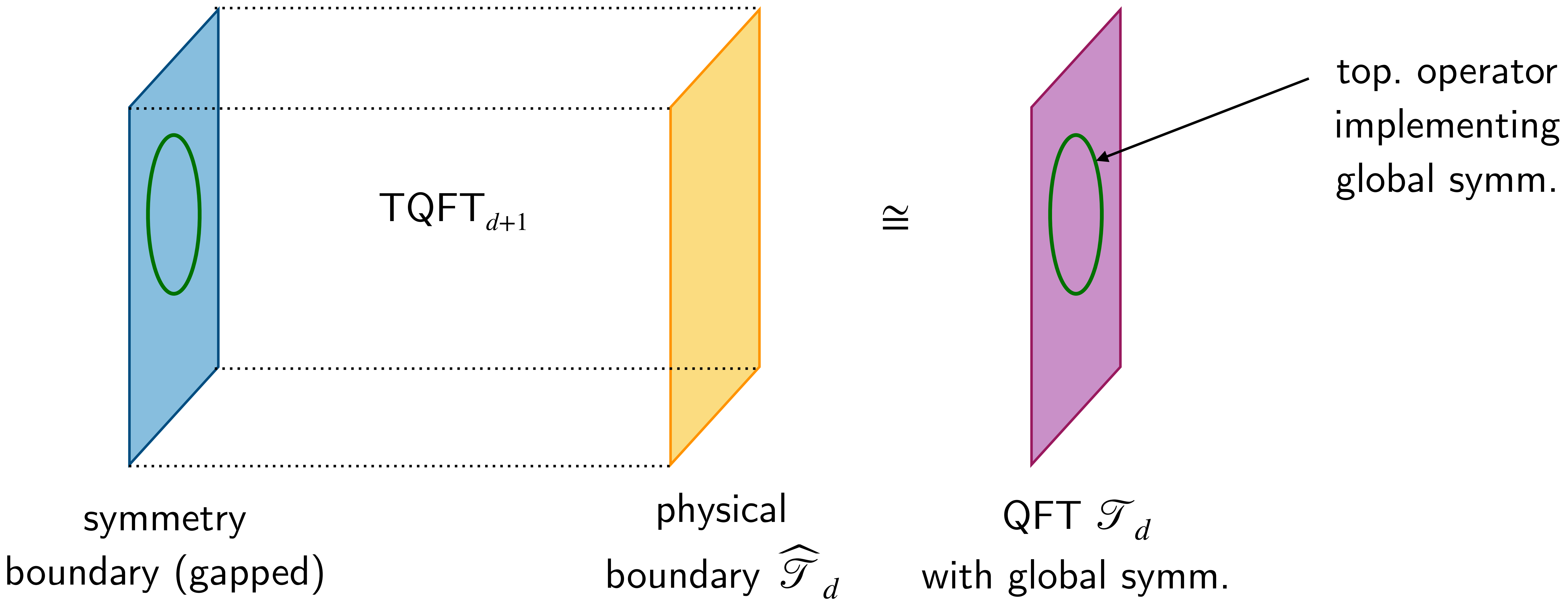
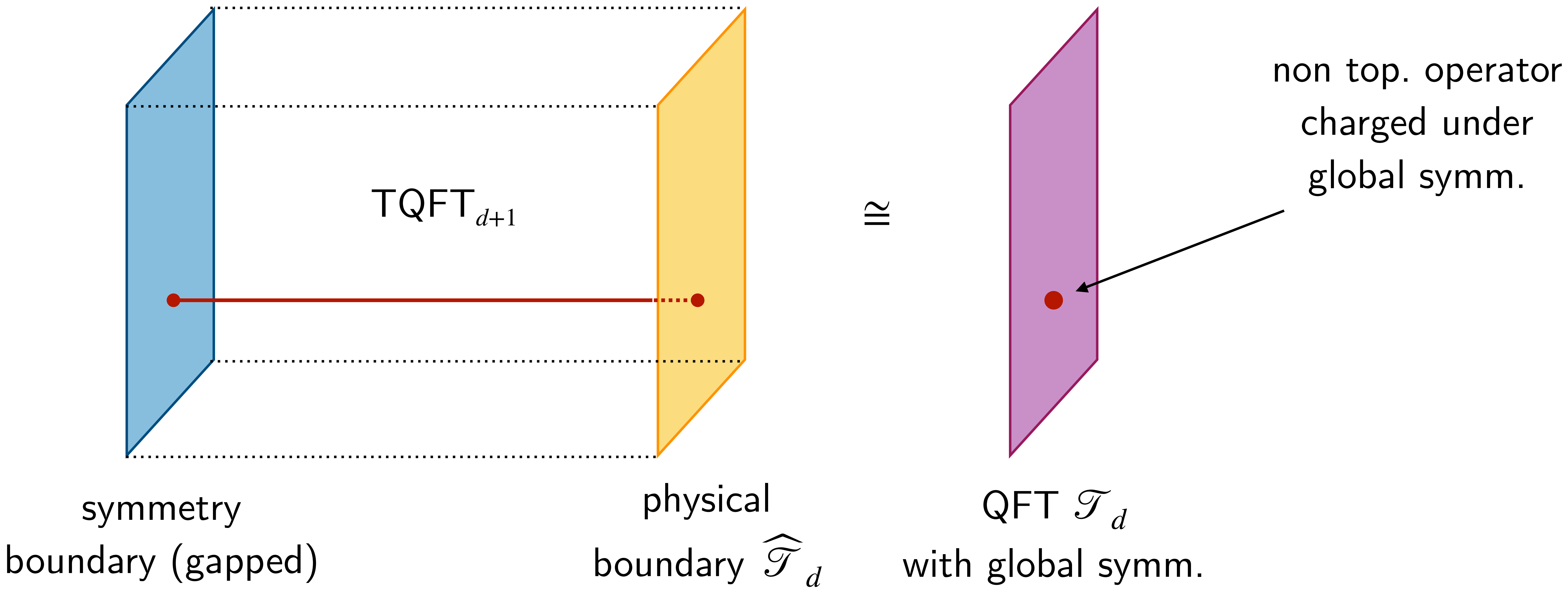


- Topological operators parallel to the boundaries are mapped to the topological operators that implement the global symmetries of the QFT $\mathscr{T}_d$

Sandwich construction

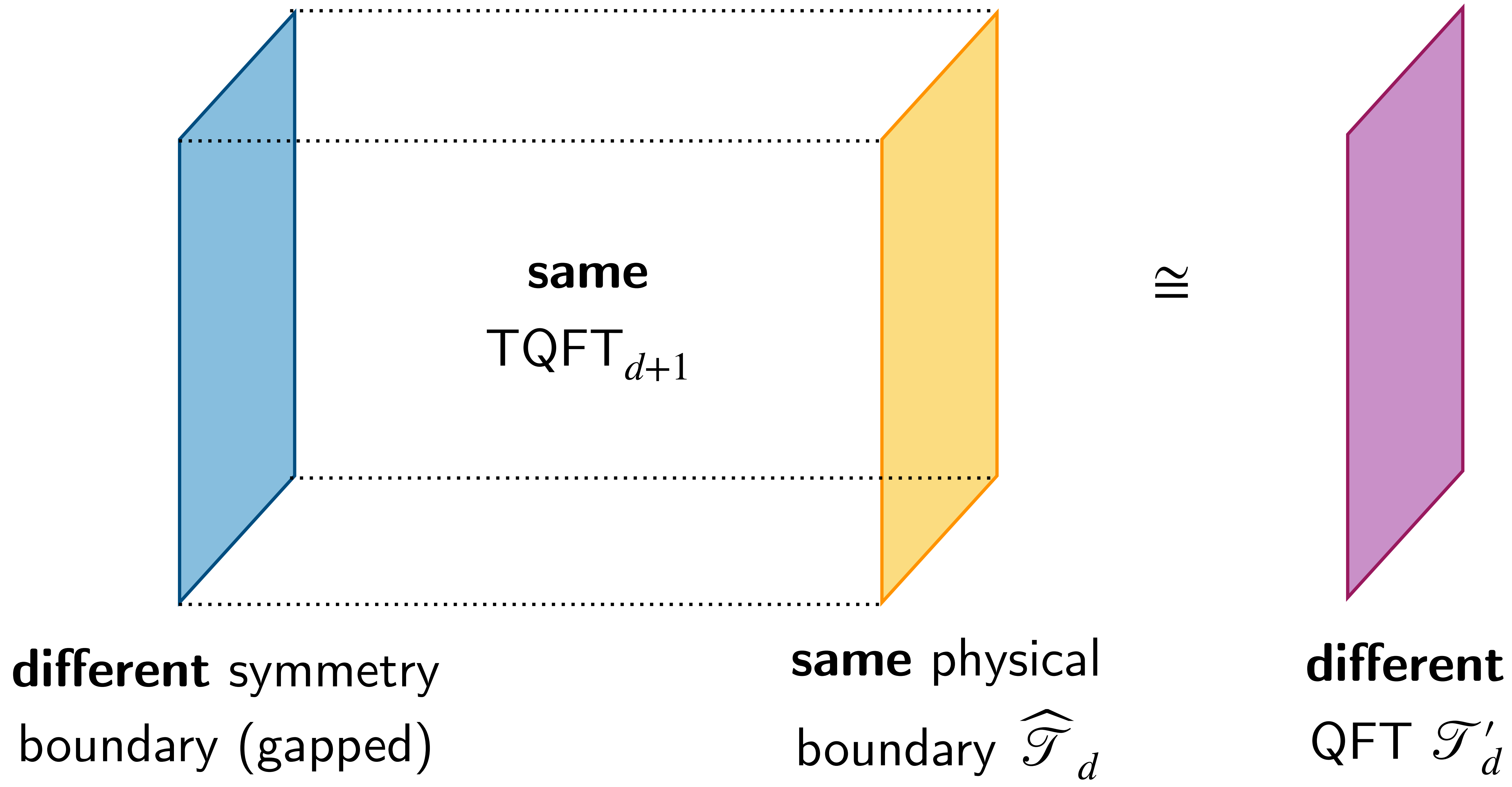
[Freed, Teleman 12; Freed 14; Ji, Wen 19; Gaiotto, Kulp 20; Apruzzi, **FB**, García Etxebarria, Hosseini, Schäfer-Nameki 21; Freed, Moore, Teleman 22; ...]



- Topological operators stretched between the two boundaries yield non-topological operators in $\mathscr{T}_d$ that are charged under the global symmetry

Sandwich construction

[Freed, Teleman 12; Freed 14; Ji, Wen 19; Gaiotto, Kulp 20; Apruzzi, **FB**, García Etxebarria, Hosseini, Schäfer-Nameki 21; Freed, Moore, Teleman 22; ...]



- If we keep the same bulk TQFT and the same physical boundary, but change the gapped boundary, we obtain a new QFT $\mathscr{T}'_d$
- How is $\mathscr{T}'_d$ related to $\mathscr{T}_d$? It is obtained via a topological manipulation
  e.g. **gauging a finite global symmetry** of $\mathscr{T}_d$

SymTFT for 6d SCFTs of type D

6d (2,0) SCFTs

- Main focus of this talk: 6d (2,0) SCFTs
- Labeled by a Lie algebra $\mathfrak{g}$ of ADE type: A_{N-1}, D_N, E_6, E_7, E_8
- Non-trivial dynamics in 6d and starting point of vast program of dimensional reduction
  - e.g. 4d $\mathcal{N} = 2$ (class S), etc... [Witten 97; ...; Gaiotto 09; Gaiotto, Moore, Neitzke 09; Gaiotto, Maldacena 09; ...]
- Shed light on surprising phenomena in lower-dimensional field theories (e.g. 6d picture of S-duality of 4d $\mathcal{N} = 4$ SYM)
- All ADE models admit a realization in Type IIB on a singularity $\mathbb{C}^2/\Gamma_{\text{ADE}}$ [Witten 95; ...]
- The A and D type SCFTs can be accessed in M-theory

6d (2,0) SCFTs as relative QFTs

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- In general, 6d (2,0) SCFTs are best understood as **relative** quantum field theories

[Witten 98; Moore 04; Freed, Teleman 12; Monnier 17; Heckmann, Tizzano 17; Gukov, Hsin, Pei 20]

Absolute QFT

- Partition function on closed manifold is a number
- Spectrum of mutually local extended operators

Relative QFT

- Vector of partition functions on closed manifold ("conformal blocks")
- Spectrum contains mutually non-local extended operators ("defect group")

[Del Zotto, Heckman, Park, Rudelius 15]

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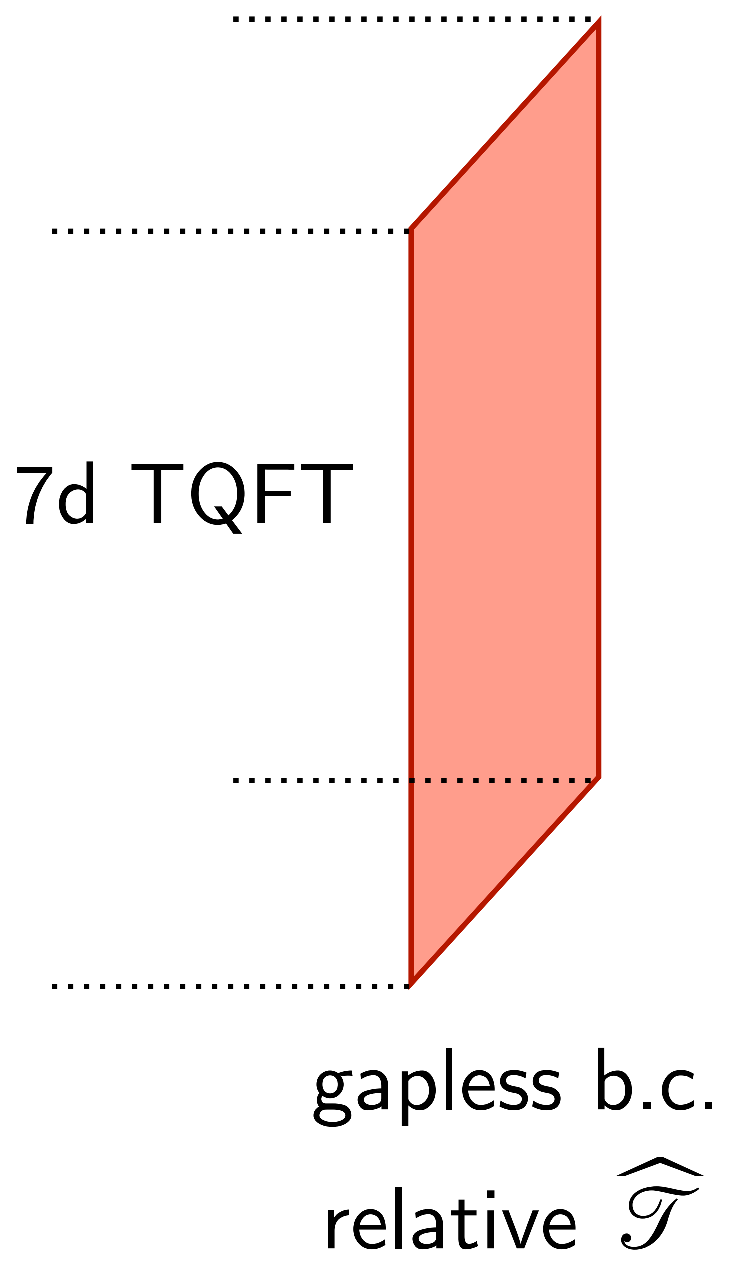
▸ Spectrum of mutually local extended operators	▸ Spectrum contains mutually non-local extended operators ("defect group")
	[Del Zotto, Heckman, Park, Rudelius 15]

- 7d/6d **bulk/boundary description**: 6d interacting degrees of freedom are realized as gapless boundary of a gapped (topological) 7d system

- $\widehat{\mathscr{T}}$: local operators of interacting 6d (2,0) SCFT of type $\mathfrak{g}$



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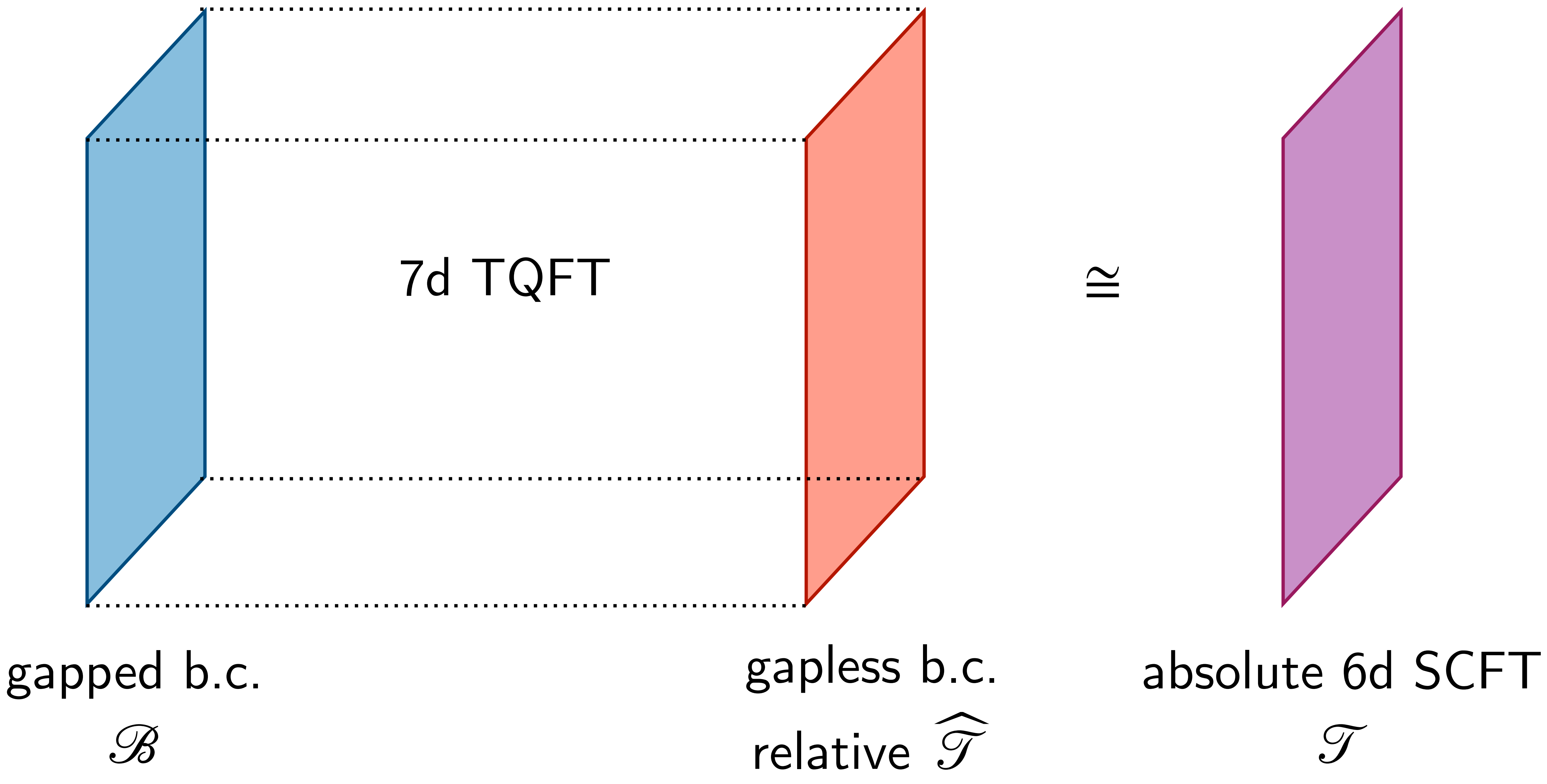
- If the 7d TQFT admits **gapped** boundary conditions, we can construct an **absolute 6d (2,0) SCFT**
- Gapped boundary conditions ↔ absolute global variants
- Absolute global variants can be labeled by the lattice of string charges in 6d



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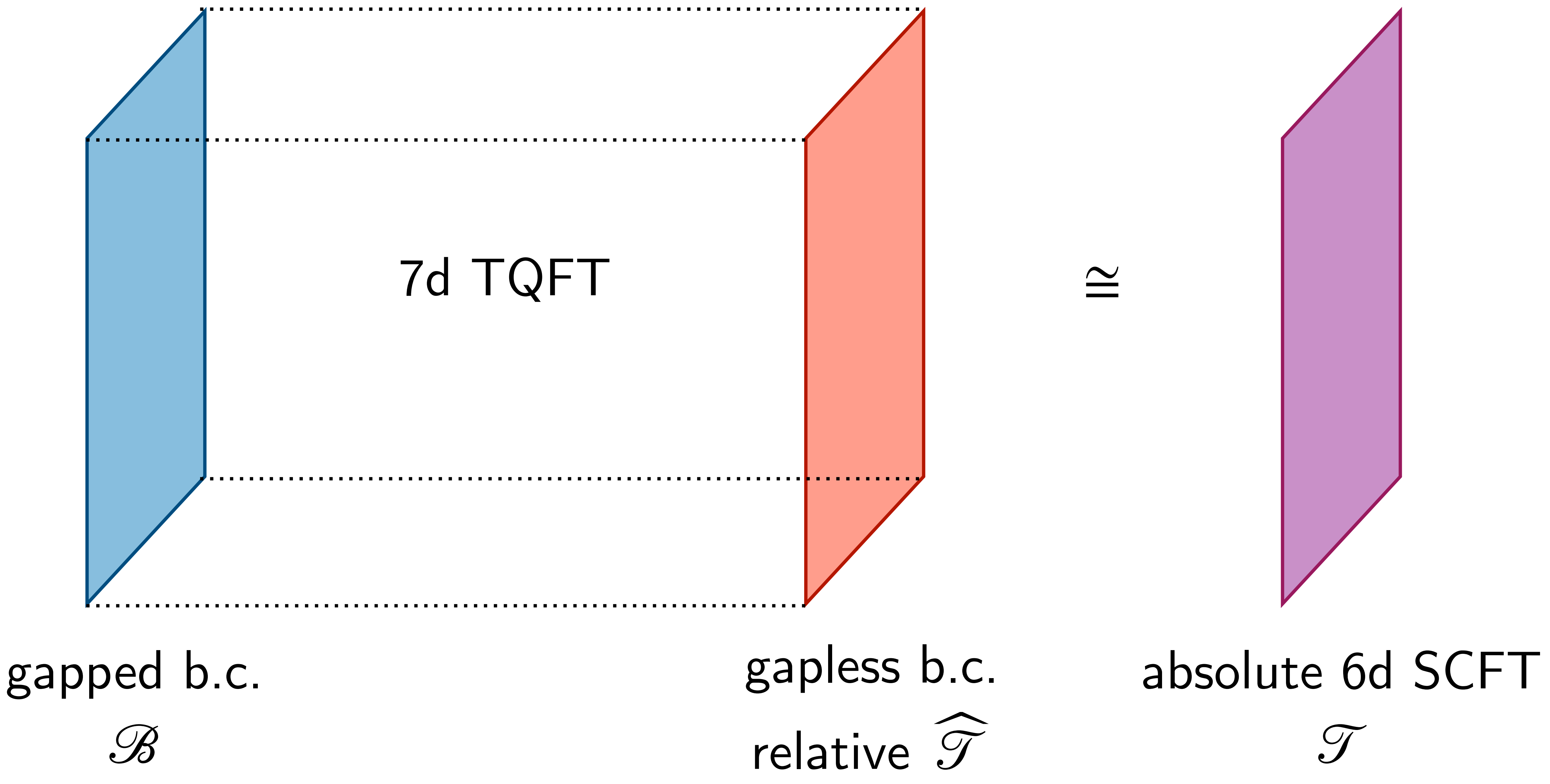

Example: $\mathfrak{g} = A_{N-1}$

- admits one absolute global variant if $N = n^2$

$$SU(n^2)/\mathbb{Z}_n$$

- no absolute variant if $N \neq n^2$

[Gukov, Hsin, Pei 20]

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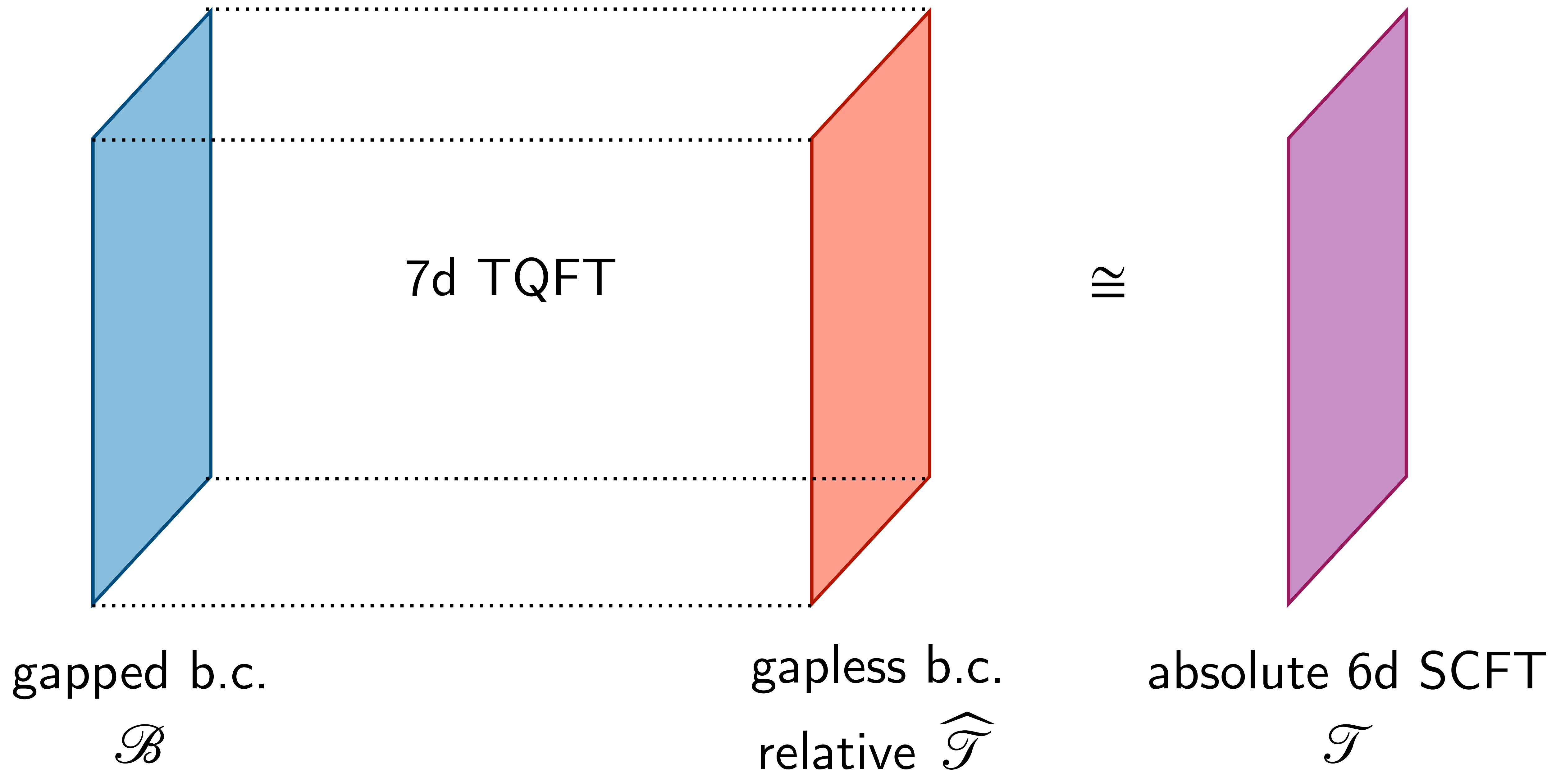

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Example: $\mathfrak{g} = D_N$

- admits one absolute global variant if $N \neq 4k$

$$SO(2N)$$

- admits three absolute global variants for $N = 4k$

$$SO(8k),\ Sc(8k),\ Ss(8k)$$

M-theory realization of 6d (2,0) SCFTs of type D

- M-theory is symmetric under **parity** (reflection of odd number of spatial directions)
  - Recall: the 3-form gauge field C_3 has negative parity: $C_3 \to -C_3$
- We start with a stack of $2N$ M5-branes at the origin of flat space $\mathbb{R}^{1,10} = \mathbb{R}^{1,5} \times \mathbb{R}^5$
- We perform an involution by the $\mathbb{Z}_2$ parity action that flips the five directions of $\mathbb{R}^5$ (**OM5-plane**)
  - The worldvolume theory on the M5-brane stack still preserves 6d (2,0) supersymmetry
  - The overall center-of-mass mode of the stack is removed by the $\mathbb{Z}_2$ involution
  - The resulting 6d (2,0) worldvolume theory is an interacting theory of type D_N
- Near-horizon limit (Maldacena limit)
  - The gravity dual is M-theory on $AdS_7 \times \mathbb{RP}^4$

$\mathbb{RP}^4 = S^4/$*antipodal identification*

[Dasgupta, Mukhi 95; Witten 95; Hori 98; Ahn, Kim, Yang 98; Gimon 98; Hanany, Kol 00]

SymTFT result — 2-form symmetry sector

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- Some aspects of the 7d TQFT associated to 6d (2,0) SCFT of type $\mathfrak{g}$ are common to all ADE types
- The 7d TQFT contains a **3-form CS theory**

$$S = \frac{1}{4\pi} K_{ij} \int_{M_7} c_3^{(i)} \wedge dc_3^{(j)} \qquad K_{ij} \text{ symmetric and integral}$$

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$$S = \frac{1}{4\pi} K_{ij} \int_{M_7} c_3^{(i)} \wedge dc_3^{(j)} \qquad K_{ij} \text{ symmetric and integral}$$

- For $\mathfrak{g} = A_{N-1}$ and D_{N-1}
  - $\mathfrak{g} = A_{N-1}$: $\quad S = \frac{N}{4\pi} \int_{M_7} c_3 \wedge dc_3$
  - $\mathfrak{g} = D_N$: $\quad S = \frac{1}{2\pi} \int_{M_7} \left[2b_3 \wedge dc_3 - \frac{N}{2} c_3 \wedge dc_3 \right]$ ⟵ *we focus on this case*
- Caveat: for odd N these are not ordinary bosonic TQFT, but rather require additional structure (such as spin structure)

[Witten 98; Monnier 17; Heckman, Tizzano 17; Gukov, Hsin, Pei 20]

SymTFT result — including discrete 0-form symmetry

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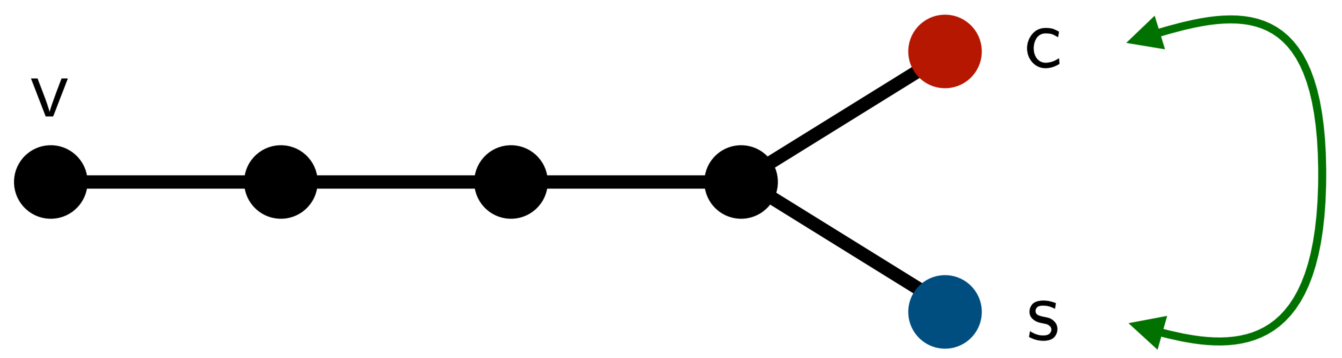
- In the D series we can enrich the 7d TQFT by adding a $\mathbb{Z}_2$ 1-form gauge field and a $\mathbb{Z}_2$ 5-form gauge field
- Associated to the $\mathbb{Z}_2$ **outer automorphism** of the D_N Lie algebra
- Crucially, this $\mathbb{Z}_2$ will be directly accessible from the holographic perspective



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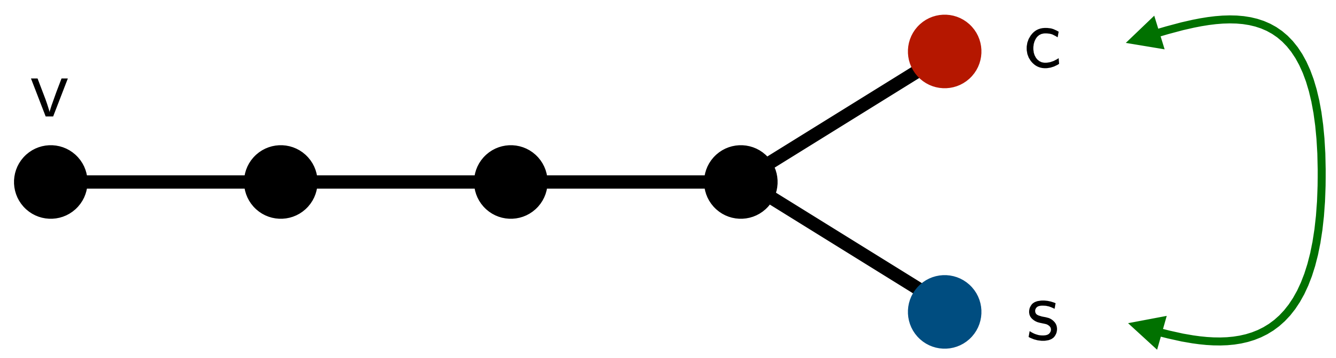

- Associated to the $\mathbb{Z}_2$ **outer automorphism** of the D_N Lie algebra
- Crucially, this $\mathbb{Z}_2$ will be directly accessible from the holographic perspective
- Full action in discrete formulation

$$S = 2\pi \int_{M_7} \left[\frac{1}{2} b_3 \cup \delta c_3 - \frac{N}{8} c_3 \cup \delta c_3 + \frac{1}{2} a_5 \cup \delta a_1 + \frac{1}{4} a_1 \cup c_3 \cup c_3 \right]$$

- Fields are written as $\mathbb{Z}_2$ cochains (translation from the continuum via $c_3/(2\pi) \to c_3/2$)
- Enriching with a_1 allows us to access a **non-trivial cubic coupling** in the 7d TQFT

Caveat: more properly, it is understood as $\frac{1}{2} a_1 \cup Q(c_3)$ where Q is a $\mathbb{Z}_2$-valued quadratic refinement of $x_3 \cup y_3$

[Browder 69; Brown 72; ... Gukov, Hsin, Pei 20; Hsin, Ji, Jian, 21]

Topological operators in 7d

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- A class of topological operators in the 7d TQFT originates from **holonomies** of the $\mathbb{Z}_2$ gauge fields c_3, b_3, a_1, a_5
- The holonomies of c_3 and a_1 are standard

$$Q_3(\Sigma_3) = \exp \pi i \int_{\Sigma_3} c_3 \qquad Q_1(\Sigma_1) = \exp \pi i \int_{\Sigma_1} a_1$$

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- ... but the holonomies of b_3, a_5 need to be **dressed by** suitable localized **topological field theories**

$$\widehat{Q}_3(\Sigma_3) = \left(\exp \pi i \int_{\Sigma_3} b_3\right) T_3[a_1, c_3] \qquad T_3[a_1, c_3] = \int [D\phi_0 D\phi_2] \, \exp 2\pi i \int_{\Sigma_3} \left[\frac{1}{2}\phi_0 \cup \delta\phi_2 + \frac{1}{2}\phi_2 \cup a_1 + \frac{1}{2}\phi_0 \cup c_3\right]$$

$$\widehat{Q}_5(\Sigma_5) = \left(\exp \pi i \int_{\Sigma_5} a_5\right) T_5[c_3] \qquad T_5[c_3] = \int [D\varphi_2] \, \exp 2\pi i \int_{\Sigma_5} \left[\frac{1}{4}\varphi_2 \cup \delta\varphi_2 + \frac{1}{2}\varphi_2 \cup c_3\right]$$

- The topological operators $\widehat{Q}_3$, $\widehat{Q}_5$ are **non-invertible** in the 7d bulk TQFT

cfr. [Kaidi, Ohmori, Zheng 21, 22; Bhardwaj, Bottini, Schafer-Nameki, Tiwari 22; Choi, Lam, Shao 22; Kaidi, Nardoni, Zafrir, Zheng 23; ...]

Derivation of 7d TQFT from M-theory: strategy

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Two complementary approaches:

A. 11d low-energy effective action on $\mathbb{RP}^4$

- We recover the discrete gauge fields c_3, b_3, a_1, a_5 from expanding C_3 and C_6 onto torsional cycles in $\mathbb{RP}^4$
- 11d couplings can be used to infer 7d couplings

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B. Topological operators from "branes at infinity"

- We realize directly the topological operators of the 7d TQFT
- They corresponds to "branes at infinity" i.e. approaching the conformal boundary of AdS_7
- Non-topological modes on branes freeze-out in this limit. We keep track of topological couplings

The brane approach (B) and its interplay with (A) have proven very useful in vast classes of examples

[Apruzzi, Bah, **FB**, Schafer-Nameki 22; Garcia-Etxebarria 22; Heckman, Hübner, Torres, Zhang 22; Heckman, Hubner, Torres, Yu, Zhang 22; ... Bah, Leung, Waddleton 23; Apruzzi, **FB**, Gould, Schafer-Nameki 23; Heckman, Hübner, Murdia 24; Braeger, Chakrabhavi, Heckman, Hübner 24; Etheredge, Garcia-Etxebarria, Heidenreich, Rauch 23; Waddleton 24; Bergman, Garcia-Valdecasas, Mignosa, Rodriguez-Gomez 24; Tian, Wang 24; Najjar, Santilli, Wang 24; Calvo, Mignosa, Rodriguez-Gomez 25; Heckman, Hübner, Murdia 25; ...]

An overview of the M-theory derivation

1. Identification of the **torsional cycles** yielding the 7d discrete gauge fields c_3, b_3, a_1, a_5
2. Identification of which **M2- and M5-branes** give rise to the relevant topological operators in 7d
3. M-theory origin of the **quadratic couplings** in the 7d action
   - non-trivial canonical commutators from branes [Freed, Moore, Segal 06; ... Albertini, Del Zotto, Garcia Etxebarria, Hosseini 20]
4. M-theory origin of the **cubic coupling**
   - dimensional reduction of interaction term in 11d using techniques from [Apruzzi, **FB**, Garcia Etxebarria, Hosseini, Schafer-Nameki 21]

Some features of $\mathbb{RP}^4$

- Some remarks on the $AdS_7 \times \mathbb{RP}^4$ background:
  - $\mathbb{RP}^4$ is a **non-orientable** space. It is not Spin, but it is **Pin**$^+$ $(w_1 \neq 0,\ w_2 = 0)$
  - $\mathbb{RP}^4$ is supported by $N - 1/2$ units of G_4-flux: $\displaystyle\int_{\mathbb{RP}^4} \frac{G_4}{2\pi} = N - \frac{1}{2}$
  - **Half-integral quantization** of G_4-flux is necessary for consistency because $\mathbb{RP}^4$ has non-trivial w_4

$$\int_{\mathbb{RP}^4} \frac{2G_4}{2\pi} = \int_{\mathbb{RP}^4} w_4 = 1 \mod 2$$

[Witten 96; ... Freed, Hopkins 19]

Torsional cycles in $\mathbb{RP}^4$ and origin of c_3, b_3, a_1, a_5

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- Since $\mathbb{RP}^4$ is non-orientable, we distinguish between twisted and untwisted cycles
- C_3 is odd under parity: we expand it onto **twisted** cycles
- The dual potential C_6 is even under parity: we expand it onto **untwisted** cycles

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dim	twisted cycles	untwisted cycles
0	$\widetilde{\text{pt}}$	pt
1		$\mathbb{RP}^1$
2	$\mathbb{RP}^2$	
3		$\mathbb{RP}^3$
4	$\mathbb{RP}^4$	

$$c_3 = \int_{\widetilde{\text{pt}}} C_3$$

$$b_3 = \int_{\mathbb{RP}^3} C_6$$

$$a_1 = \int_{\mathbb{RP}^2} C_3$$

$$a_5 = \int_{\mathbb{RP}^1} C_6$$

Brane origins of 7d topological operators

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TQFT operators

$$Q_3(\Sigma_3) = \exp \pi i \int_{\Sigma_3} c_3 \quad , \quad Q_1(\Sigma_1) = \exp \pi i \int_{\Sigma_1} a_1 \quad , \quad \widehat{Q}_3(\Sigma_3) = \left(\exp \pi i \int_{\Sigma_3} b_3 \right) T_3[a_1, c_3] \quad , \quad \widehat{Q}_5(\Sigma_5) = \left(\exp \pi i \int_{\Sigma_5} a_5 \right) T_5[c_3]$$

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Brane origin in M-theory

$Q_3(\Sigma_3) = \text{M2 on } \Sigma_3 \times \widetilde{\text{pt}}$

$Q_1(\Sigma_1) = \text{M2 on } \Sigma_1 \times \mathbb{RP}^2$

$\widehat{Q}_3(\Sigma_3) = \text{M5 on } \Sigma_3 \times \mathbb{RP}^3$

$\widehat{Q}_5(\Sigma_5) = \text{M5 on } \Sigma_5 \times \mathbb{RP}^1$

Remarks:

- Invertible operators originate from M2-branes, non-invertible operators from M5-branes
- The 3d, 5d TQFTs T_3, T_5 that dress the non-invertible operators $\widehat{Q}_3$, $\widehat{Q}_5$ can be derived from the worldvolume theory of the M5-brane coupled to the bulk C_3

M-theory origin of the cubic coupling

- We can derive the 7d cubic coupling $\frac{1}{4}a_1c_3c_3$ from the 11d cubic coupling $C_3G_4G_4$ in the M-theory effective action
- The strategy is very similar to a standard **Kaluza-Klein reduction**:
  - C_3 is expanded onto non-trivial cohomology classes of the internal space $\mathbb{RP}^4$
  - We plug back the expansion in the 11d term $C_3G_4G_4$ and we integrate over $\mathbb{RP}^4$
  - Technical aspects: we use the formalism of **differential cohomology** to account for torsional classes

Application: (non-)invertible symmetries

Canonical gapped boundary condition

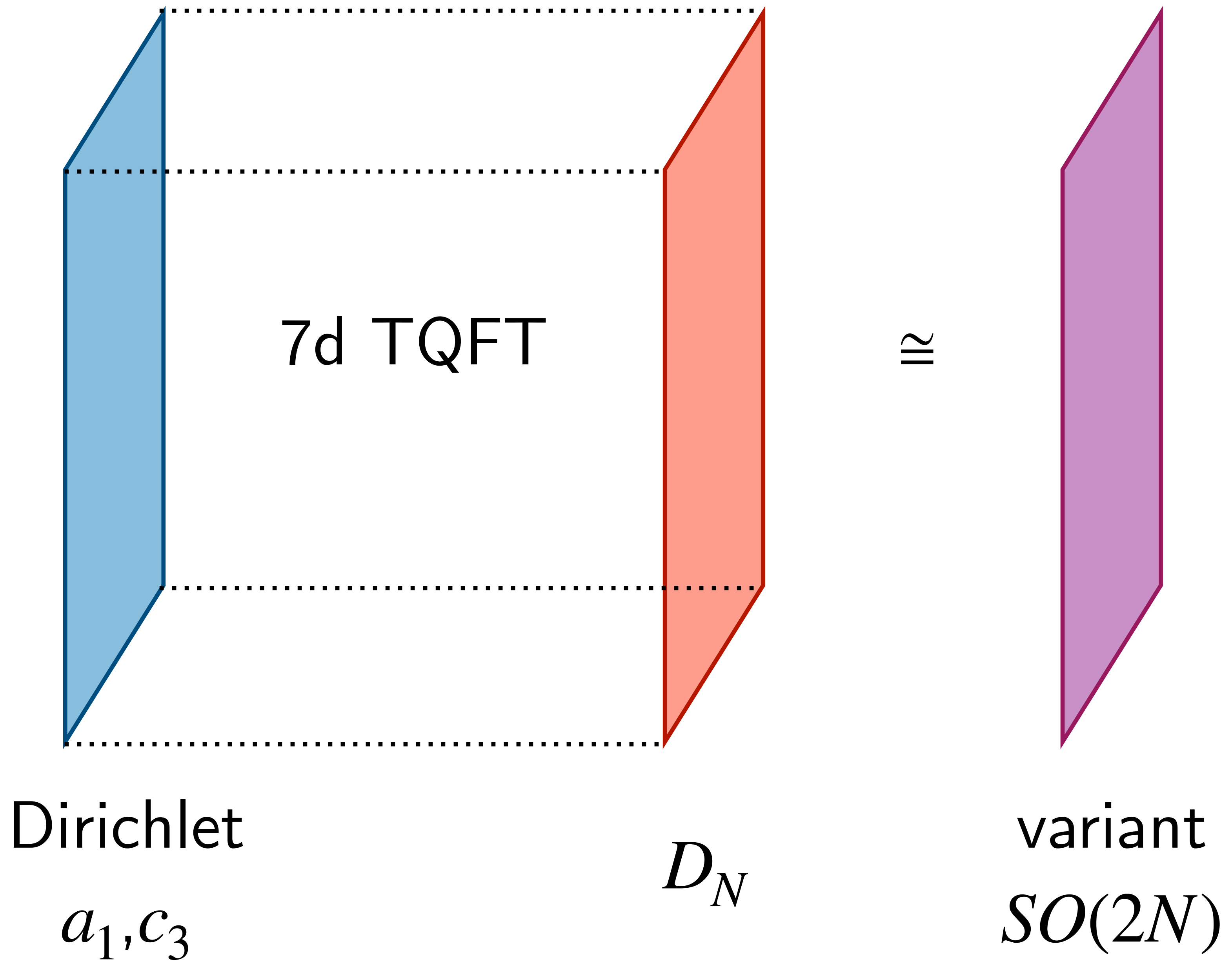
- For any value of N there is a canonical choice of gapped boundary conditions for the 7d TQFT

Dirichlet b.c. for (a_1, c_3)

- This corresponds to the global variant $SO(2N)$
- This variant has **invertible** global symmetries

symm	backgr
$\mathbb{Z}_2$ 0-form	a_1
$\mathbb{Z}_2$ 2-form	c_3

$$S = 2\pi \int_{M_7} \left[\frac{1}{2} b_3 \cup \delta c_3 - \frac{N}{8} c_3 \cup \delta c_3 + \frac{1}{2} a_5 \cup \delta a_1 + \frac{1}{4} a_1 \cup c_3 \cup c_3 \right]$$



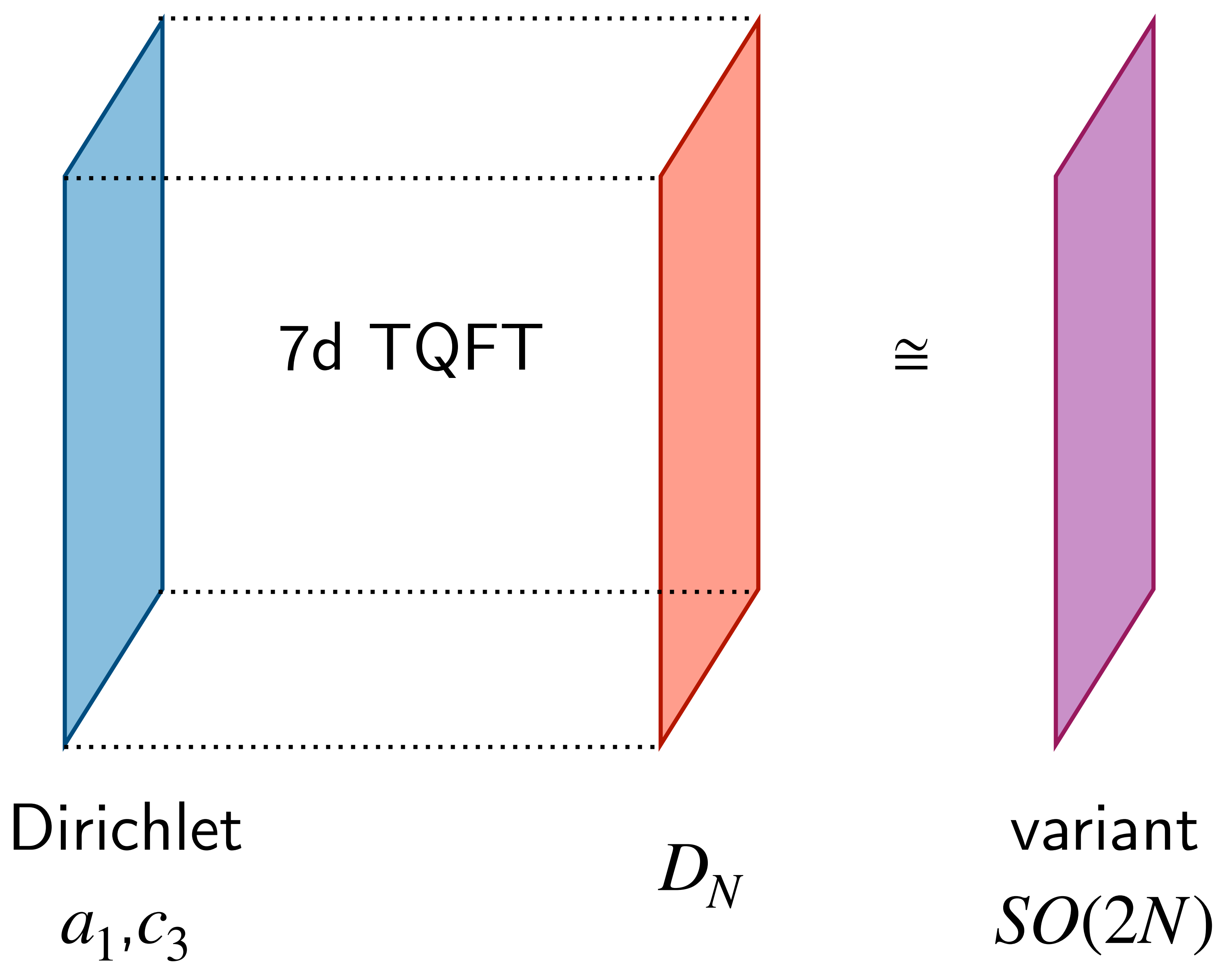
[Gukov, Hsin, Pei 20]

Canonical gapped boundary condition

Dirichlet b.c. for (a_1, c_3)

$$S = 2\pi \int_{M_7} \left[\frac{1}{2} b_3 \cup \delta c_3 - \frac{N}{8} c_3 \cup \delta c_3 + \frac{1}{2} a_5 \cup \delta a_1 + \frac{1}{4} a_1 \cup c_3 \cup c_3 \right]$$

- The highlighted terms are interpreted as **'t Hooft anomalies** of the global symmetries of the absolute variant $SO(2N)$



$$\text{anomaly of absolute } SO(2N) = \exp 2\pi i \int_{M_7} \left[-\frac{N}{4} c_3 \cup \beta c_3 + \frac{1}{4} a_1 \cup c_3 \cup c_3 \right]$$

here a_1 and c_3 are understood as the values at the Dirichlet boundary (non dynamical background fields of $SO(2N)$ variant)

$\beta = \delta/2$ is Bockstein homomorpshims of $0 \to \mathbb{Z}_2 \to \mathbb{Z}_4 \to \mathbb{Z}_2 \to 0$

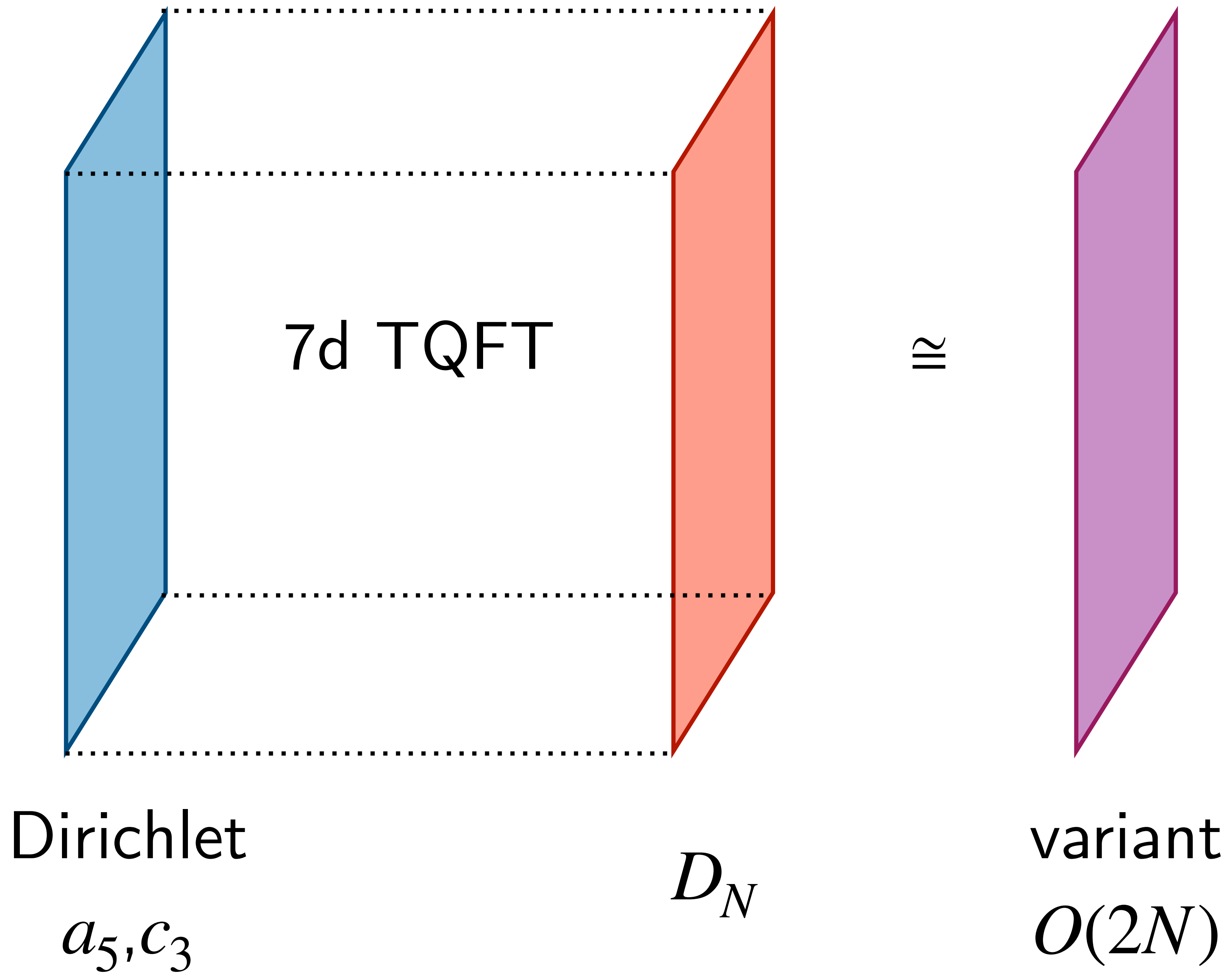
Gauging a_1: higher-group in $O(2N)$ variant

$$S = 2\pi \int_{M_7} \left[\frac{1}{2} b_3 \cup \delta c_3 - \frac{N}{8} c_3 \cup \delta c_3 + \frac{1}{2} a_5 \cup \delta a_1 + \frac{1}{4} a_1 \cup c_3 \cup c_3 \right]$$

- For any N we can gauge the $\mathbb{Z}_2$ 0-form symmetry of the $SO(2N)$ absolute variant
- We get a variant that we can denote $O(2N)$
- It corresponds to the gapped boundary condition

  Dirichlet b.c. for (a_5, c_3)

- The $O(2N)$ variant has 4-form and 2-form global symmetries
- These mix into a **higher-group** : $\delta a_5 = \frac{1}{2} c_3 \cup c_3$



[Tachikawa 17; ..., Cordova, Dumitrescu, Intriligator 18; Benini, Cordova, Hsin 18; ...]

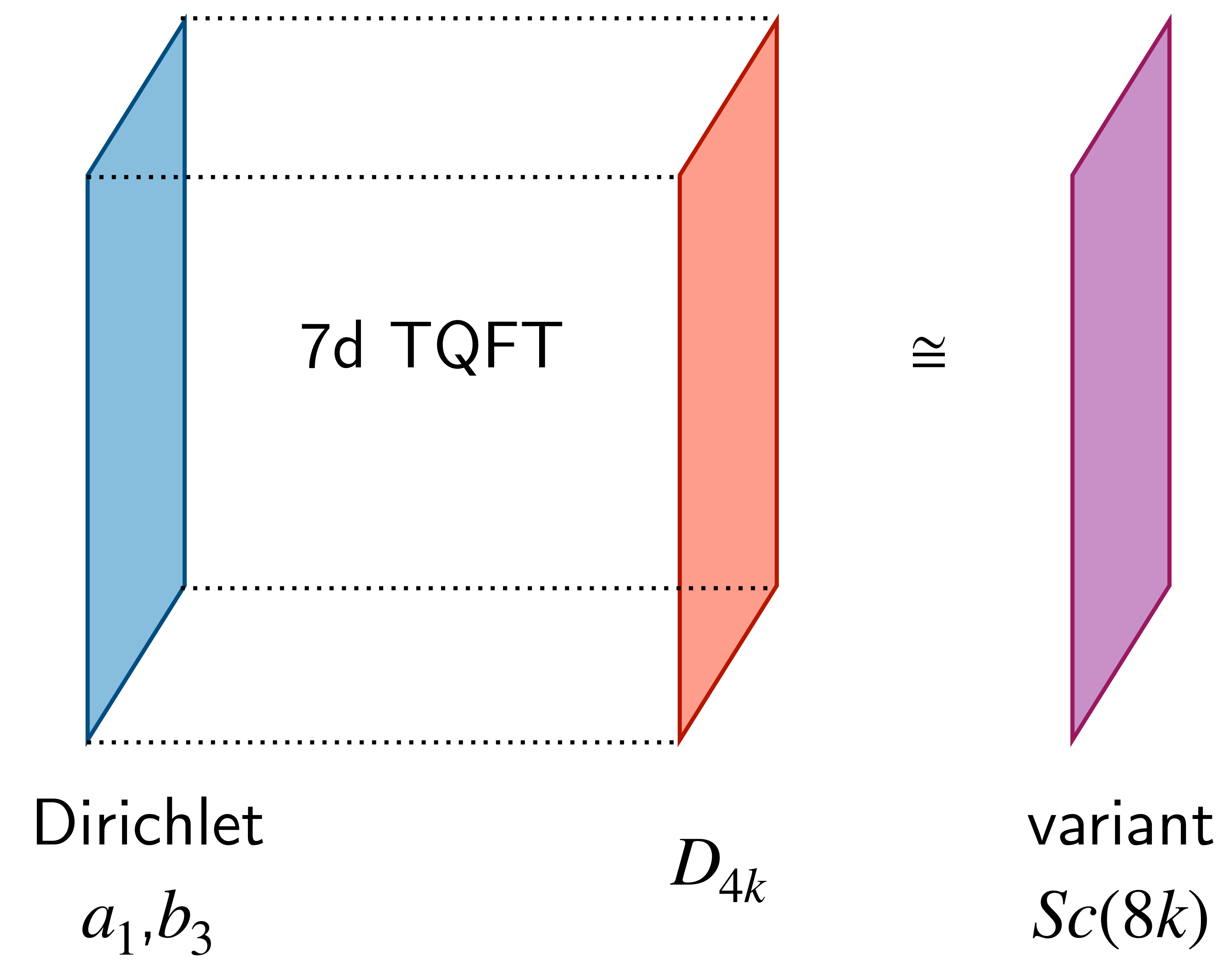
The case $N = 4k$: $Sc(8k)$ and $Ss(8k)$ variants

- In this case the anomaly $\frac{N}{4} c_3 \cup \beta c_3$ trivializes
- We can access a **larger set of gapped boundary conditions** which involve the field b_3

Dirichlet b.c. for (a_1, b_3) or $(a_1, b_3 + c_3)$

- This corresponds to gauging the $\mathbb{Z}_2$ 2-form symmetry of the $SO(8k)$ variant (with or without discrete torsion)
- This yields variants known as $Sc(8k)$ and $Ss(8k)$

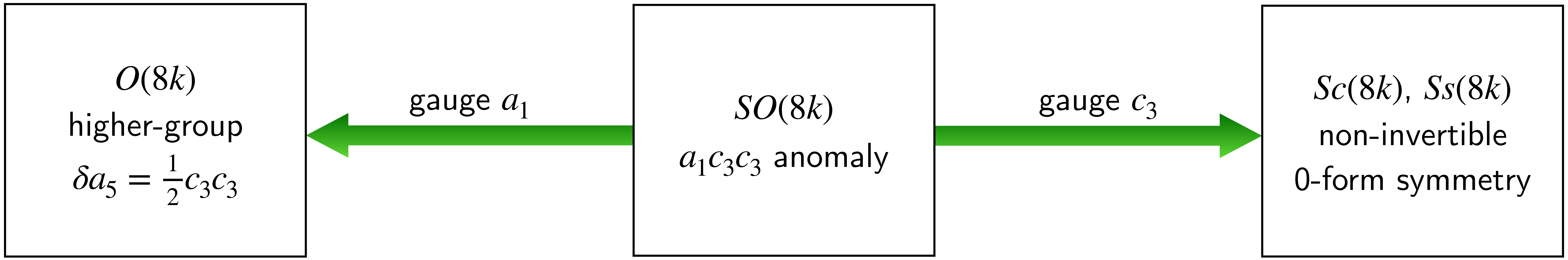
$$S = 2\pi \int_{M_7} \left[\frac{1}{2} b_3 \cup \delta c_3 - \frac{N}{8} c_3 \cup \delta c_3 + \frac{1}{2} a_5 \cup \delta a_1 + \frac{1}{4} a_1 \cup c_3 \cup c_3 \right]$$



[Gukov, Hsin, Pei 20]

Non-invertible symmetry from mixed 't Hooft anomaly

- The $SO(8k)$ variant has a mixed $a_1 c_3 c_3$ anomaly
- Upon gauging c_3, the 0-form symmetry associated to a_1 becomes **non-invertible** [Tachikawa 17; Kaidi, Ohmori, Zheng 21]



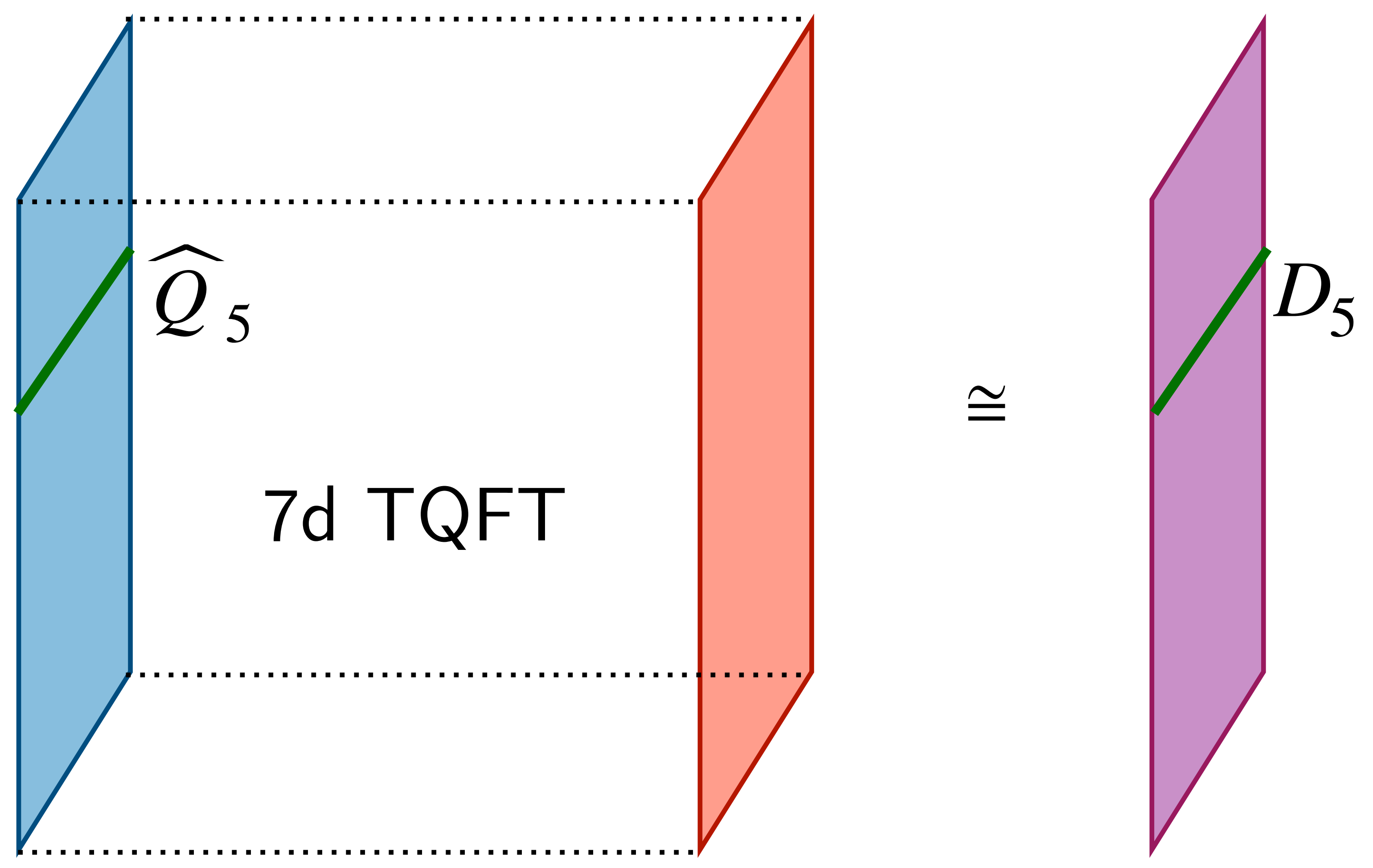
- Cfr. non-invertible symmetries in 6d SCFTs in [Lawrie, Yu, Zhang 23; Apruzzi, Schafer-Nameki, Warman 24]

Alternative viewpoint on the non-invertible symmetry

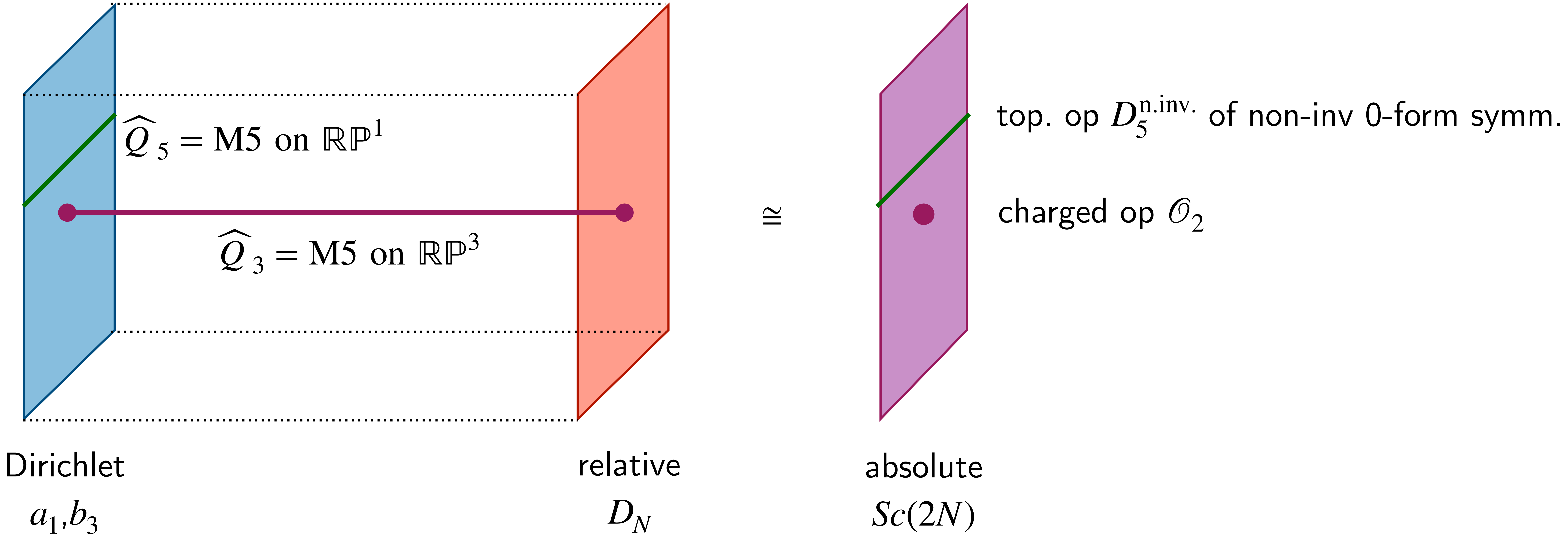
- We have noticed that the 7d bulk TQFT contains non-invertible topological operators

$$\widehat{Q}_5(\Sigma_5) = \left(\exp \pi i \int_{\Sigma_5} a_5\right) T_5[c_3] \qquad \widehat{Q}_5(\Sigma_5) = \text{M5 on } \Sigma_5 \times \mathbb{RP}^1$$

- Upon closing the sandwich, these operators yield invertible or non-invertible symmetries depending on the choice of topological boundary conditions
  - In the $SO(8k)$ and $O(8k)$ variants: Dirichlet boundary conditions for c_3 → invertible
  - In the $Sc(8k)$ and $Ss(8k)$ variants: c_3 fluctuates at the boundary → non-invertible



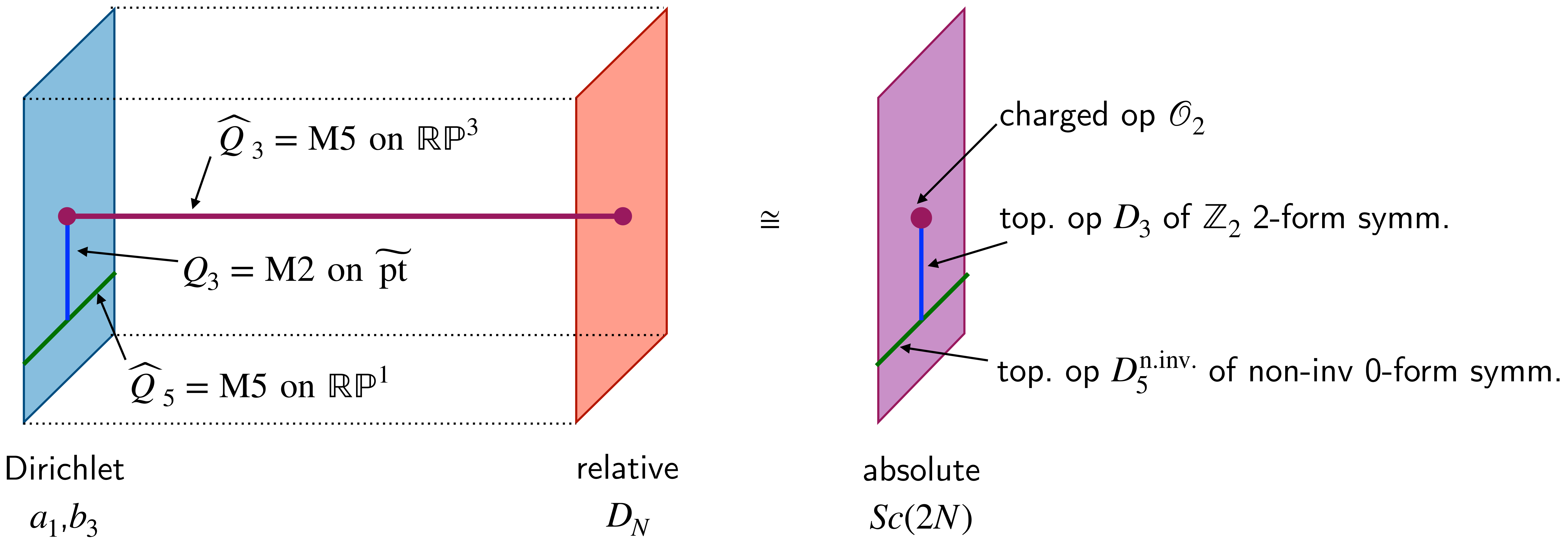
Non-invertible action and Hanany-Witten effect



- We focus on the $Sc(8k)$ variant and its non-invertible 0-form symmetry
- The 5d top. op. implementing the non-invertible 0-form symmetry originates from an M5-brane
- The theory also has non-topological 2d operators that are charged under the $\mathbb{Z}_2$ 2-form symmetry. They also come from M5-branes

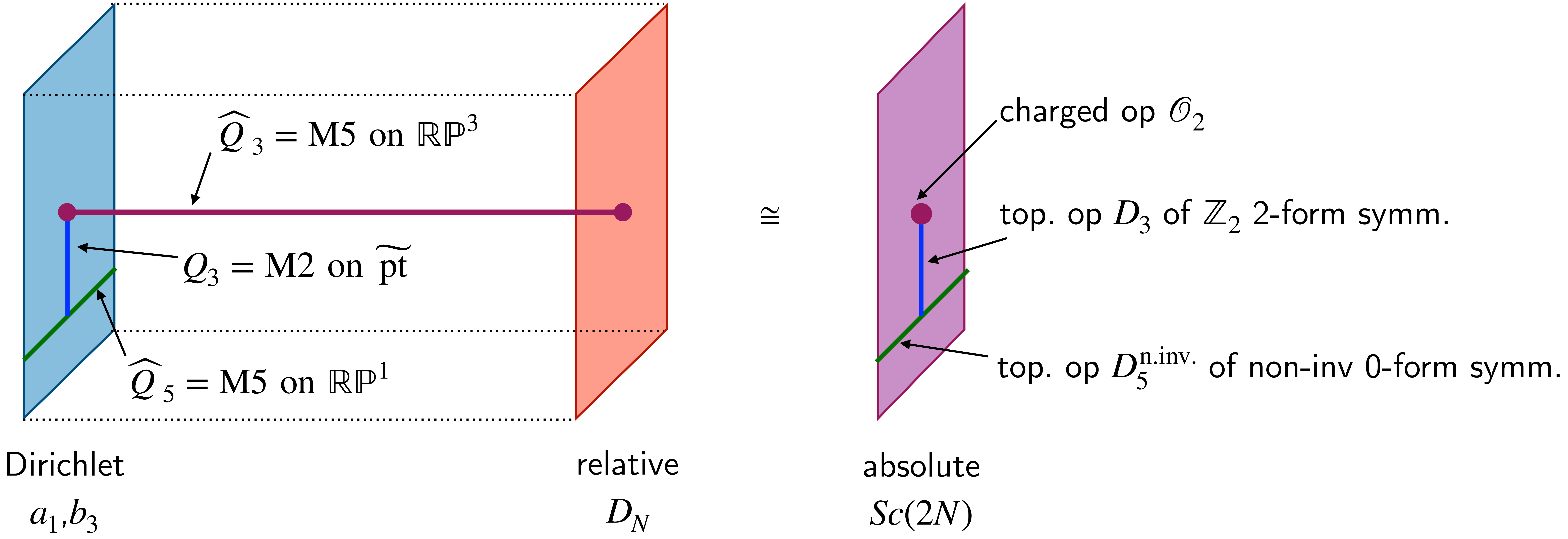
Non-invertible action and Hanany-Witten effect

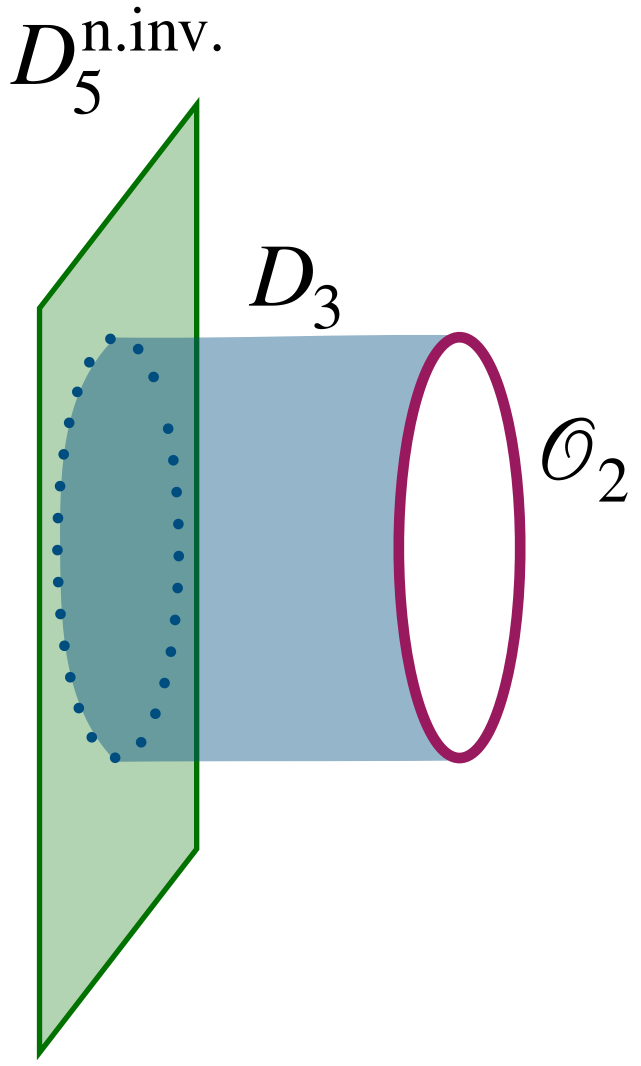
Non-invertible action and Hanany-Witten effect



- Hanany-Witten effect: if we pass the M5 on $\mathbb{RP}^3$ across the M5 on $\mathbb{RP}^1$, and M2 is created
  [Hanany, Witten 96]

Non-invertible action and Hanany-Witten effect





- Hanany-Witten effect: if we pass the M5 on $\mathbb{RP}^3$ across the M5 on $\mathbb{RP}^1$, and M2 is created
  [Hanany, Witten 96]

- In field theory: if we drag $D_5^{\text{n.inv.}}$ past the charged operator $\mathcal{O}_2$, a topological operator D_3 is generated
  [Choi, Lam, Shao 22; ...]

- HW effect and symmetries from branes

cfr. [Apruzzi, Bah, **FB**, Schafer-Nameki 22; Garcia-Etxebarria 22; Heckman, Hübner, Torres, Yu, Zhang 22; Apruzzi, **FB**, Gould, Schafer-Nameki 23]

A consistency check: reduction on a torus

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- The 6d (2,0) SCFT of type $SO(8k)$ reduced on T^2 reproduces 4d $\mathcal{N} = 4$ $SO(8k)$ super Yang-Mills

- We can reduce our 7d TQFT on T^2 and get a **5d TQFT**

- The latter matches the known SymTFT for 4d $\mathcal{N} = 4$ $SO(8k)$ super Yang-Mills

[Bergman, Hirano 22;
Etheredge, Garcia Etxebarria, Heidenreich, Rauch 23]

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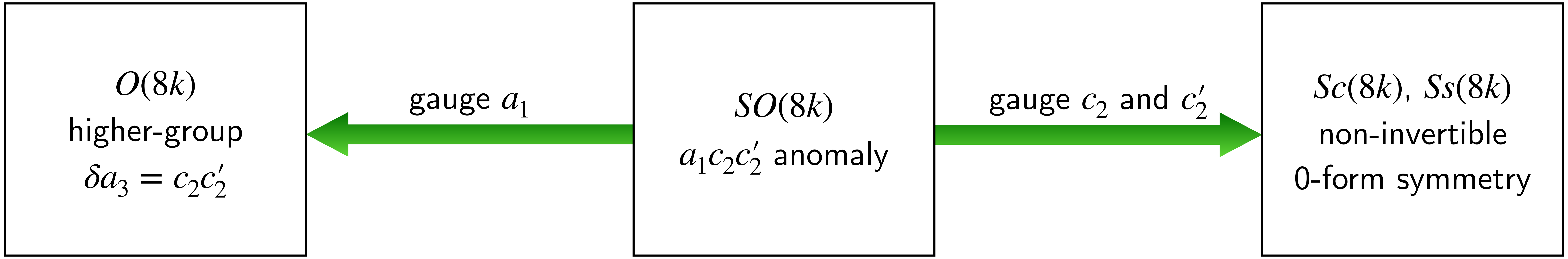
- We can reduce our 7d TQFT on T^2 and get a **5d TQFT**
- The latter matches the known SymTFT for 4d $\mathcal{N} = 4$ $SO(8k)$ super Yang-Mills
- This includes a mixed anomaly of the form $a_1 c_2 c_2'$ that originates from $a_1 c_3 c_3$ in 7d

[Bergman, Hirano 22; Etheredge, Garcia Etxebarria, Heidenreich, Rauch 23]



[Bhardwaj, Bottini, Schafer-Nameki, Tiwari 22]

Conclusions and outlook

Conclusions

- We revisited the holographic description of 6d (2,0) SCFTs of type D
- We derived the associated 7d SymTFT from M-theory and identified the brane origin of topological operators
- The SymTFT includes a cubic coupling that affects the symmetry structures of the absolute global variants

mixed anomaly *higher-group* *non-invertible symmetry*

Outlook

- Explore implications of this non-invertible symmetry of type D (e.g. in compactifications to 4d on a Riemann surface)
- Revisit global variants of type A to identify possible cubic terms in their SymTFTs

Thank you!