Exploring Utopia

Based on work with:

Noah Braeger, Arun Debray, Jonathan J. Heckman, Miguel Montero

— to appear —

And earlier works with: A. Debray, J.J. Heckman, M. Montero, E. Torres

— Strings & Geometry —

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Markus Dierigl

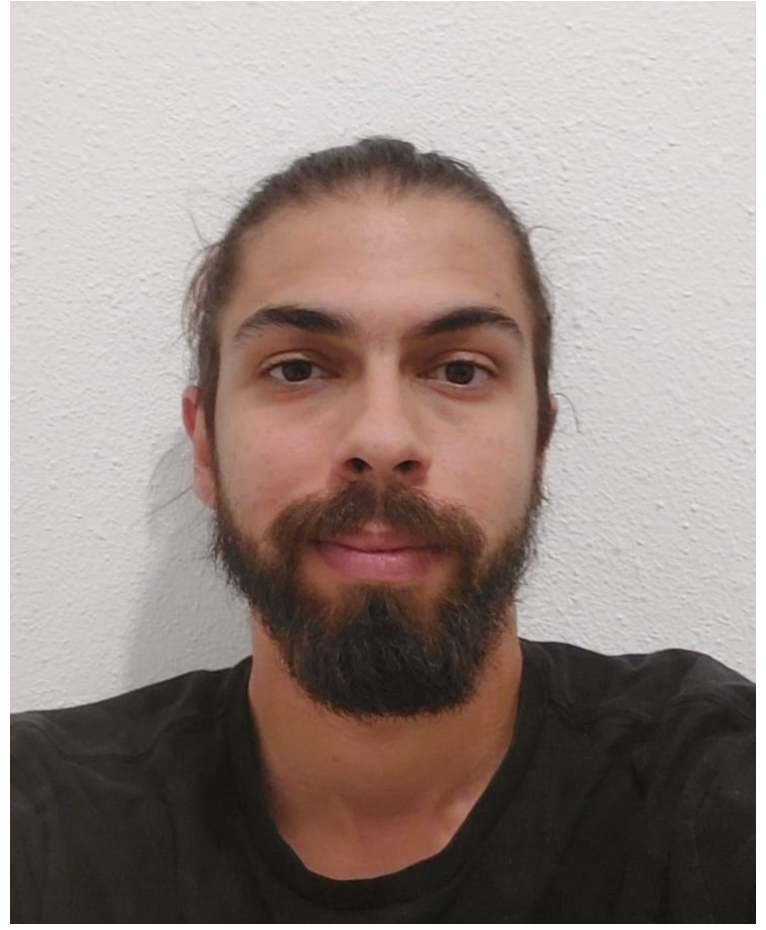
Discrete anomaly cancellation (in 6d)

Quick advertisement: arxiv:2504.02934

(Quadratically) Refined Discrete Anomaly Cancellation

Markus Dierigl[1], Michelangelo Tartaglia[2]

[1] *Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland*
[2] *Instituto de Física Teórica IFT-UAM/CSIC, C/ Nicolás Cabrera 13-15, Campus de Cantoblanco, 28049 Madrid, Spain*



Ask Michelangleo!

What would you do if I give you a theory?

For example the Lagrangian:

$$\mathcal{L} = \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}$$

Probably:

- **Equations of motion:** $\partial_\mu F^{\mu\nu} = 0$
- **Bianchi identities:** $\partial_{[\mu} F_{\nu\rho]} = 0$

(from local definition: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$)

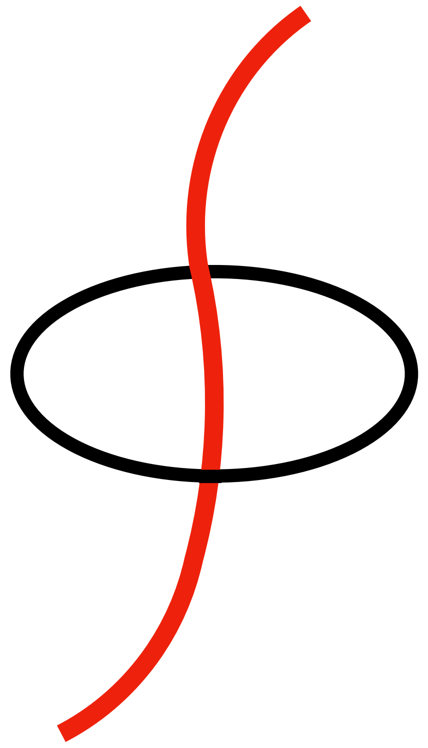
- **Quantize (canonically):**

γ

What would you do if I give you a theory?

Maybe: Analyze the symmetries

0-form U(1) gauge symmetry: $A_\mu \to A_\mu + \partial_\mu \lambda$



1-form U(1) global symmetries:

[Gaiotto, Kapustin, Seiberg, Willett '14]

Maybe: Add **charged fields** ψ_q and couple, e.g., $\overline{\psi}_q i\gamma^\mu D_\mu \psi_q$

Then: Check for anomalies

Everything pretty action oriented / model dependent

What if instead I give you the following data

I want to investigate a theory:

- **that is D-dimensional**
- **allows for fermions**
- **has a U(1) gauge theory**

Way more general: $(F_{\mu\nu}F^{\mu\nu})^n$

$$j_e^\mu\,, j_m^\mu\,, D_\mu\overline{\phi}D^\mu\phi\,, \overline{\psi_q}i\gamma^\mu D_\mu\psi_q\,, \ldots$$

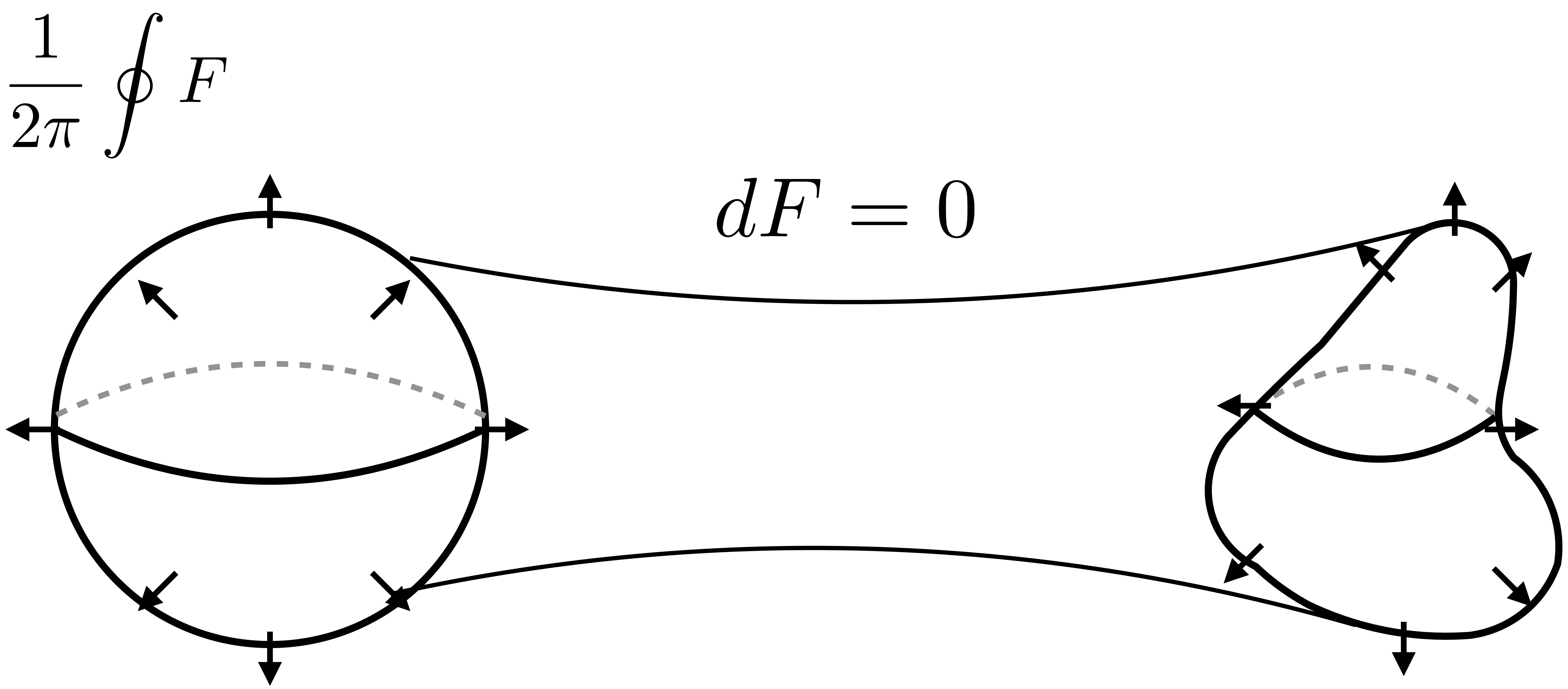
➞ **Additionally:** **allow for dynamics of spacetime**

(**EFT** flavor: **smooth** spacetime)

Topological gauge data

We still have the U(1) gauge field: A

And a Bianchi identity: $dF = d(dA) = 0$



$$\frac{1}{2\pi}\oint F$$

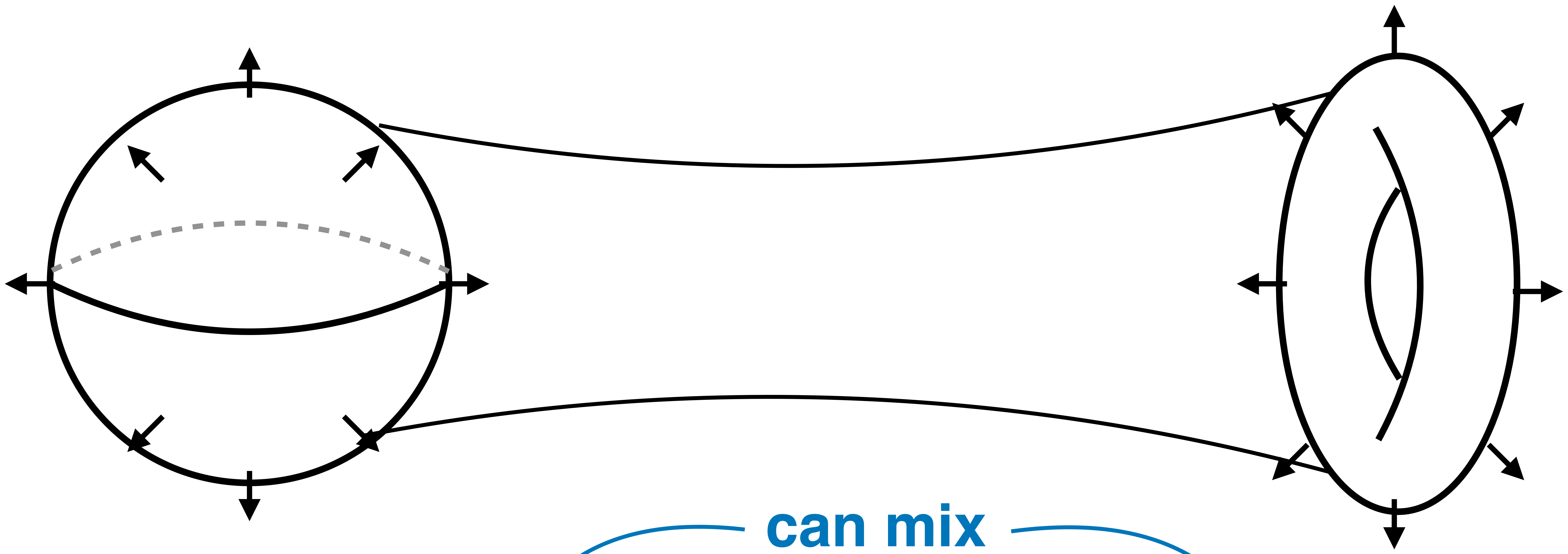
$$dF = 0$$

cannot deform away

→ conserved **topological charges**

Combine with gravitational data

corresponds to topology of spacetime manifold



can one deform smoothly
from left to right?

can mix

gravity part

gauge part

Bordism groups $\Omega_d^{\xi}(BG)$

dimension

classify conserved charges
(= global symmetries)

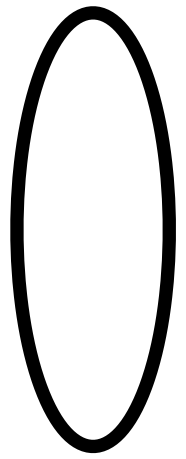
Example

Let's go back to our example:

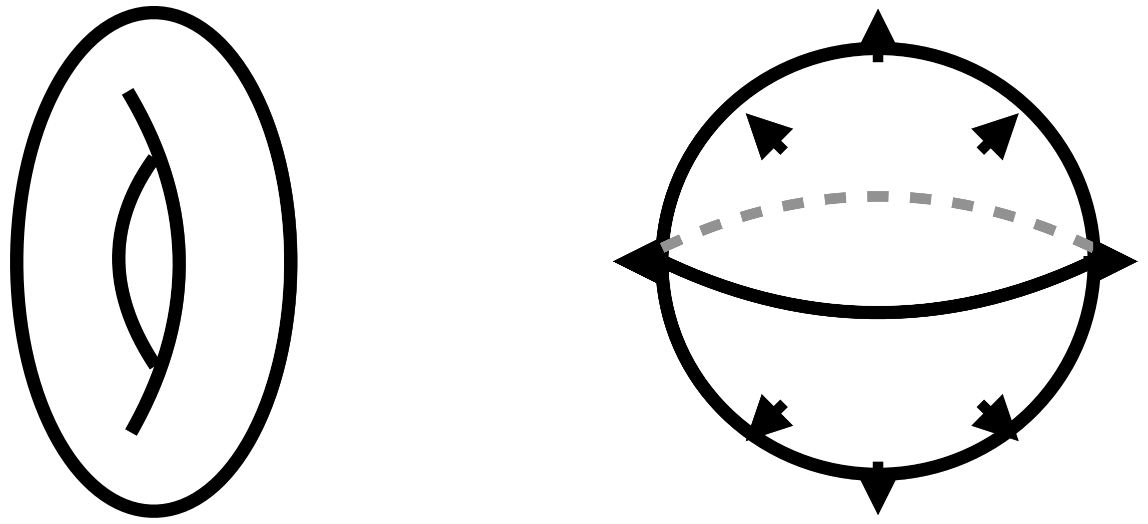
- that is D-dimensional
- allows for fermions
- has a U(1) gauge theory

$$\Omega_d^{\mathrm{Spin}}\big(B\mathrm{U}(1)\big)$$

d	$\Omega_d^{\mathrm{Spin}}\big(\mathrm{U}(1)\big)$
1	$\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}$
3	0
4	$\mathbb{Z} \oplus \mathbb{Z}$
5	0
6	$\mathbb{Z} \oplus \mathbb{Z}$



$\psi \to +\psi$



$\frac{1}{2\pi}\int F$

All of this from **minimal input**

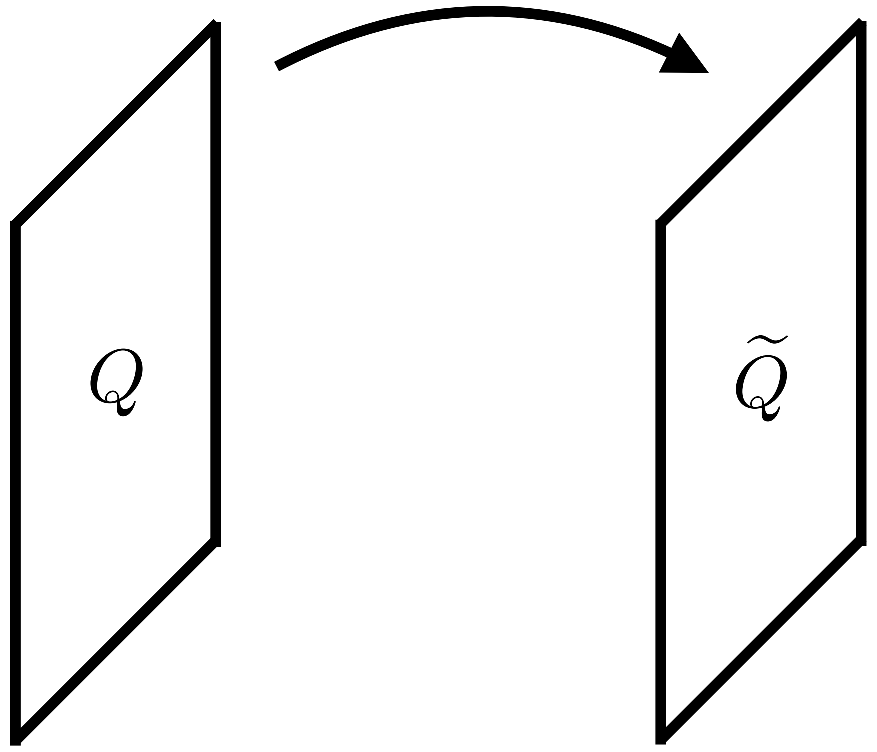
(losing information but sensitive to universal features)

➞ but there is a **problem**

No global symmetries in quantum gravity

See e.g. [Banks, Dixon ’88], [Banks, Seiberg ’11], [Harlow, Ooguri ’18], [Harlow, Shaghoulian ’20], [Bah, Chen, Maldacena ’22], [Heckman, Hübner, Murdia ’24], …

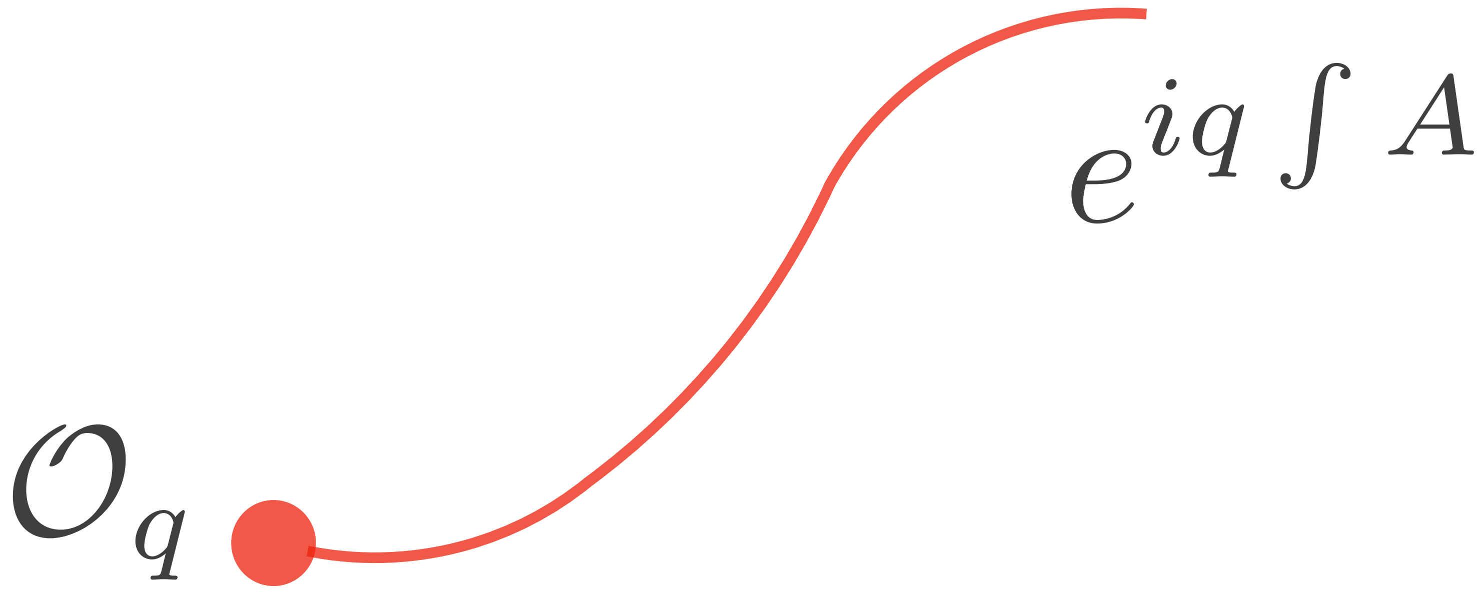
broken



charge not conserved

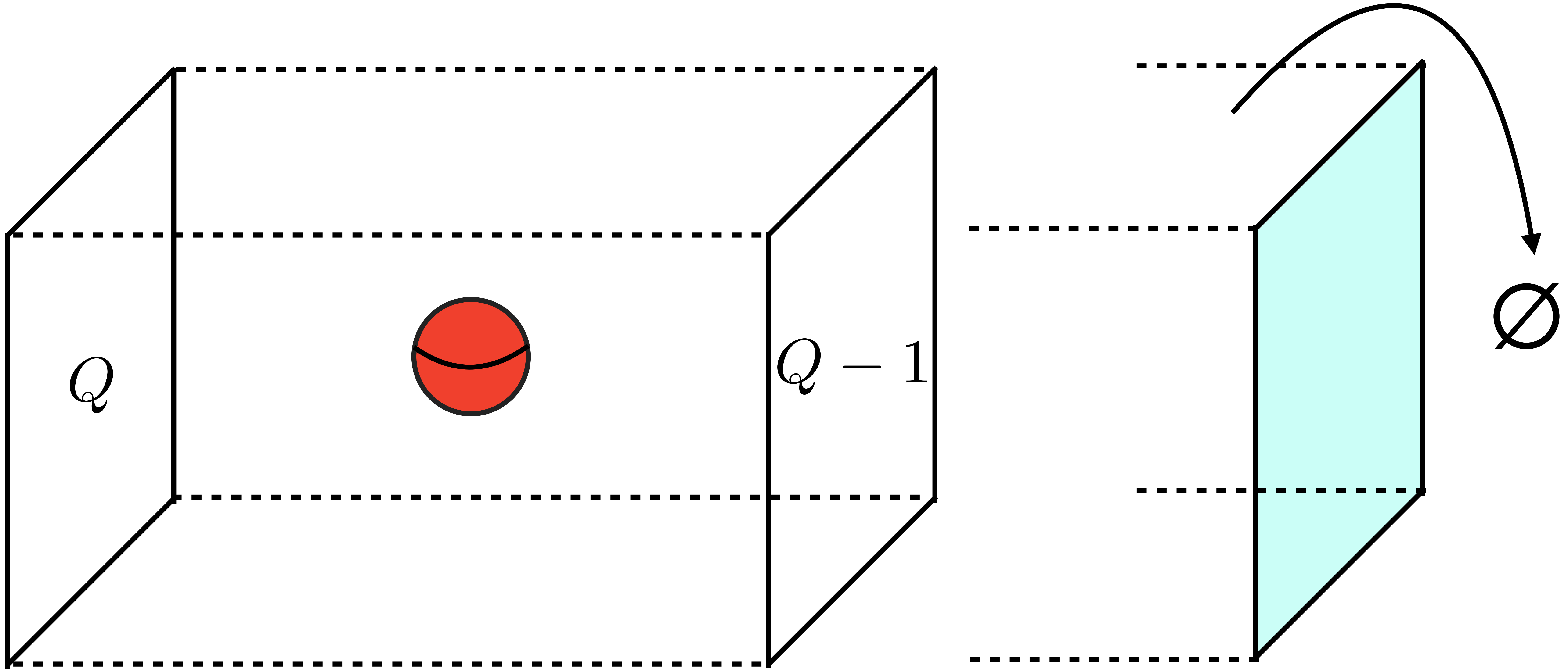
Violation typically **at least** $e^{-M_{Pl}^2/\Lambda^2}$

see also [Daus, Hebecker, Leonhardt, March-Russell ’20]



Symmetry-breaking objects

backgrounds associated with **new symmetry-breaking defects**



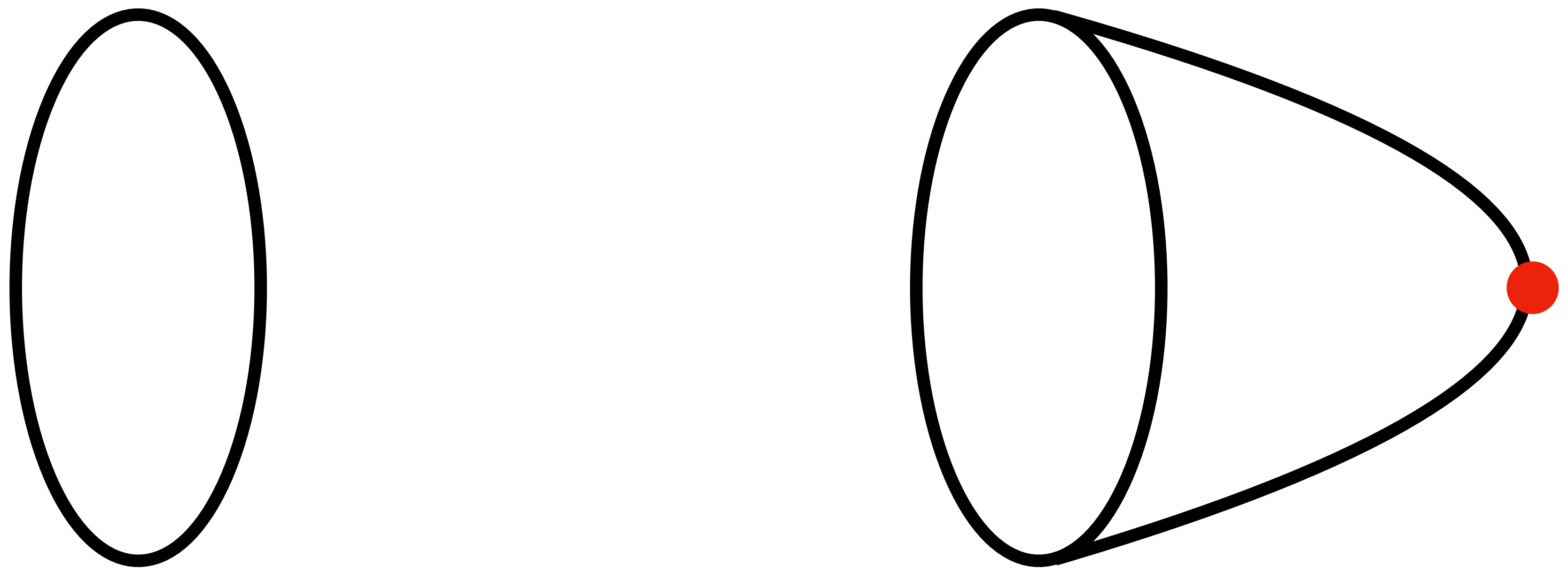
$$\Omega_d^{\mathrm{QG}} = 0$$

Swampland Cobordism Conjecture

[McNamara, Vafa ’19]

Can look **singular** at **low-energies**, but have **finite mass** (tension)

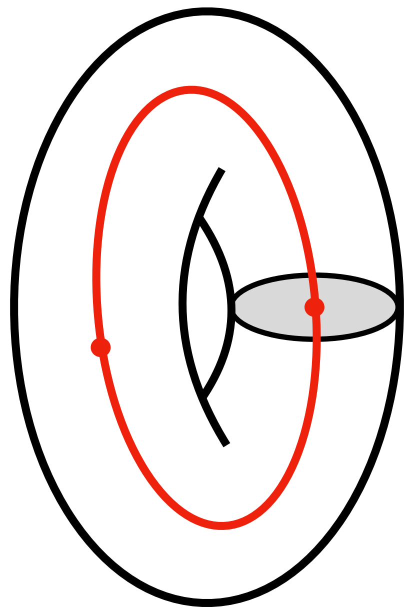
Example



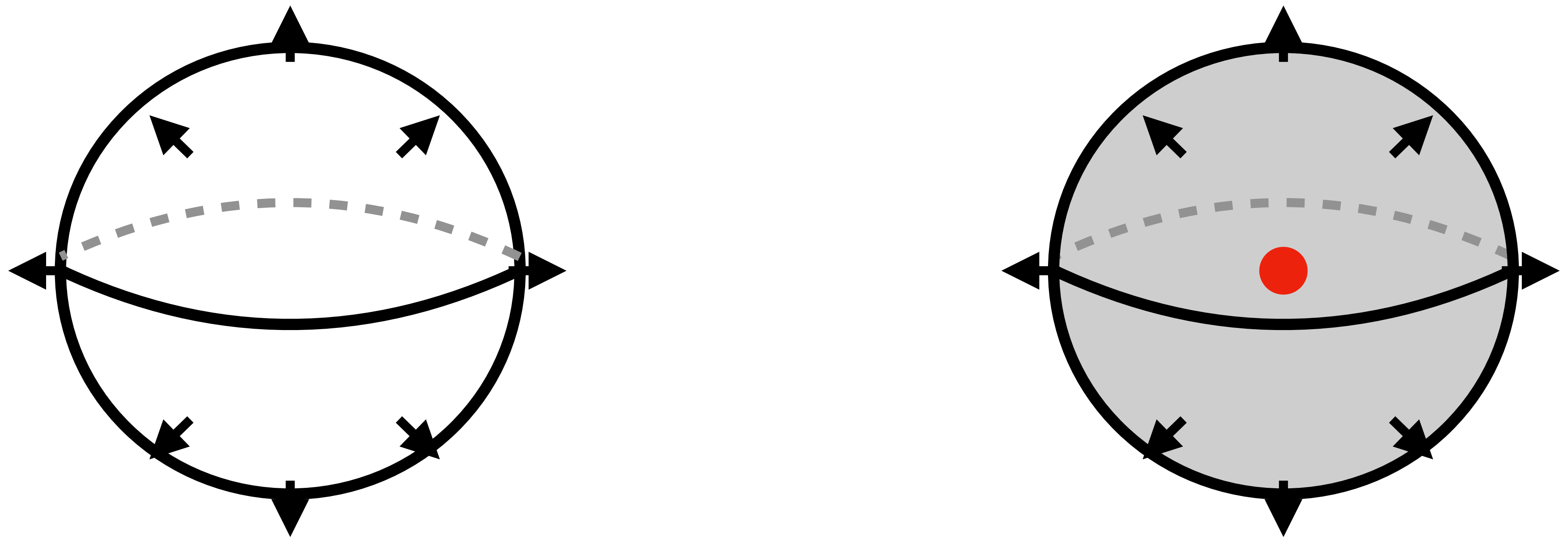
codimension-two Spin defect

[McNamara, Vafa ’19], [Hamada, Hamada, Kimura ’25]

No issue in IIB: [Debray, MD, Heckman, Montero ’23]



also takes care of this
(compactified on circle)



magnetic monopole

Once upon a time in IIBordia

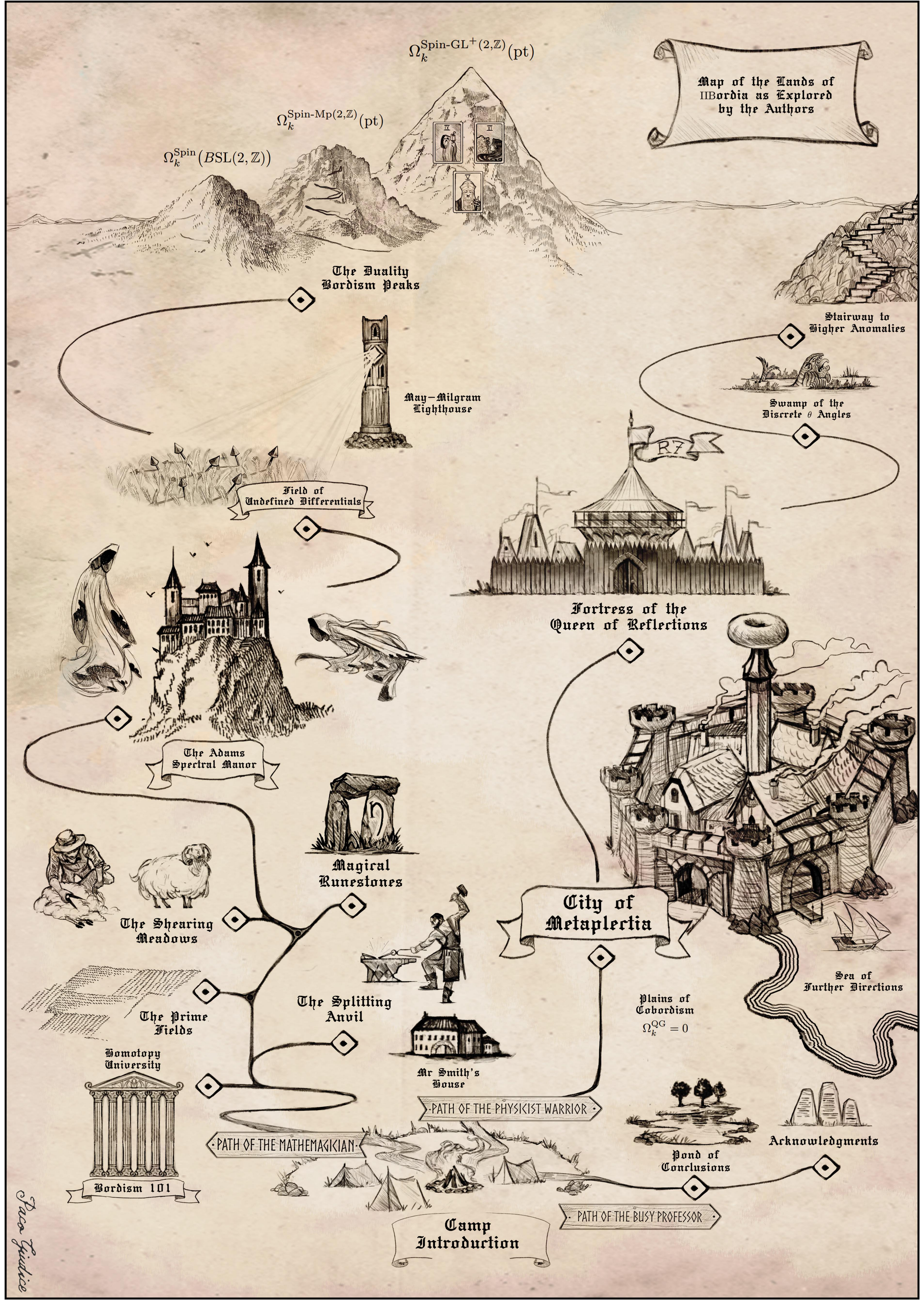
[Debray, MD, Heckman, Montero ’21 & ’23]

Type IIB supergravity:

- properties of **spacetime**
- S-duality

Expectation:

$$\Omega_d^{\mathrm{IIB}} = 0$$



(Big thanks to **Paco Giudice** for the map)

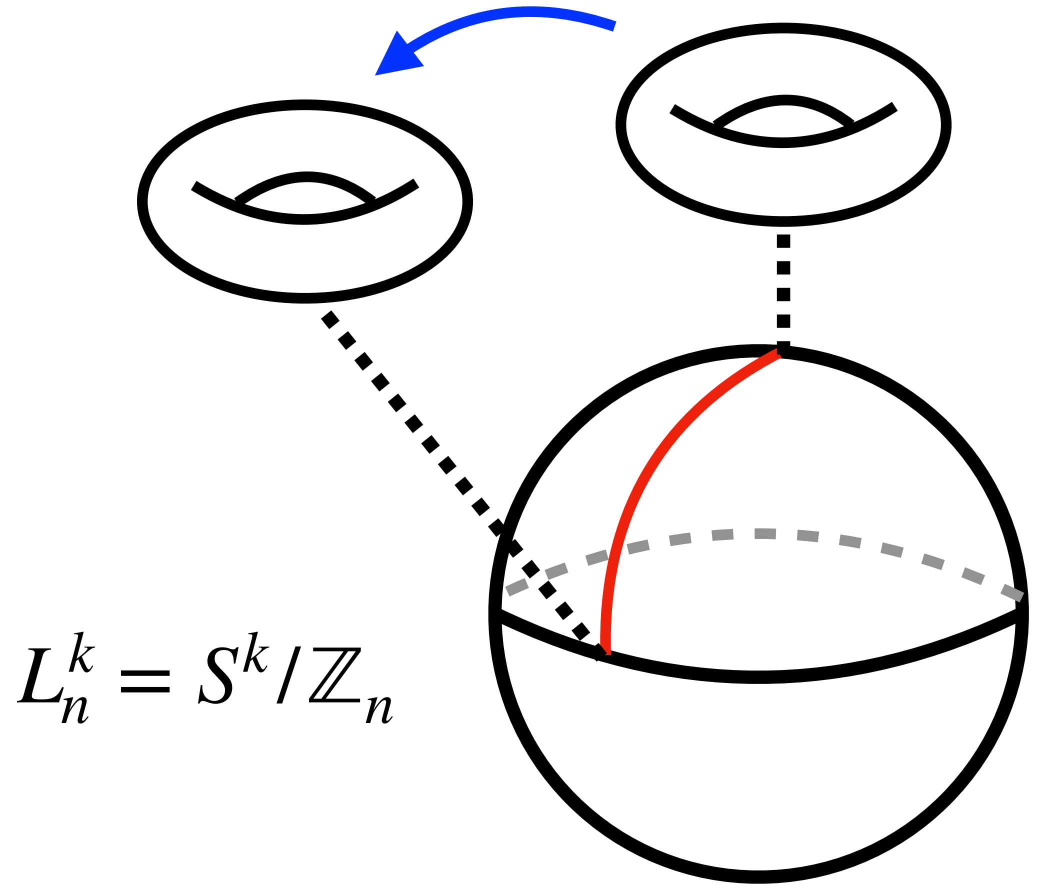
d	$\Omega_d^{\mathrm{Spin}}\big(B\mathrm{SL}(2,\mathbb{Z})\big)$	$\Omega_d^{\mathrm{Spin}\text{-}\mathrm{Mp}(2,\mathbb{Z})}$	$\Omega_d^{\mathrm{Spin}\text{-}\mathrm{GL}^+(2,\mathbb{Z})}$
	§4	§5	§6
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2$
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
5	$\mathbb{Z}_{36}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

many potential global symmetries

[Debray, MD, Heckman, Montero ’21 & ’23]

Symmetry-breaking defects

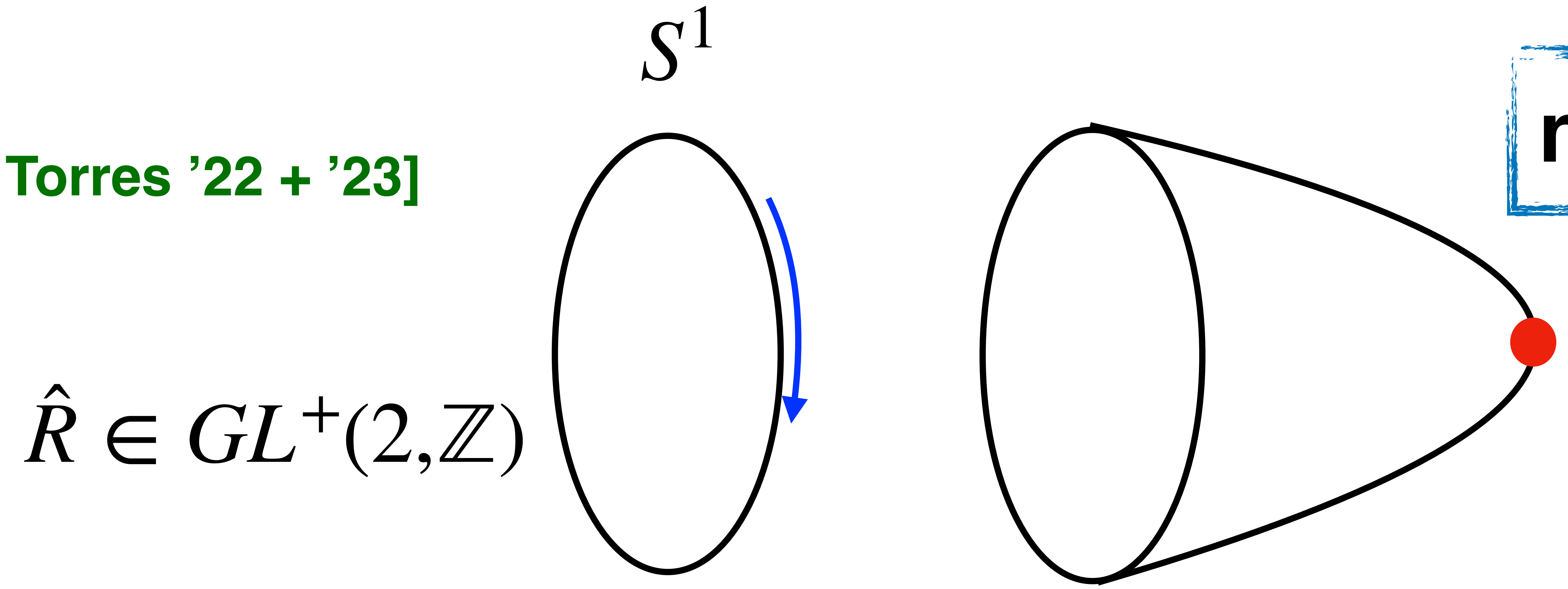
String theory takes care of it → interesting corners



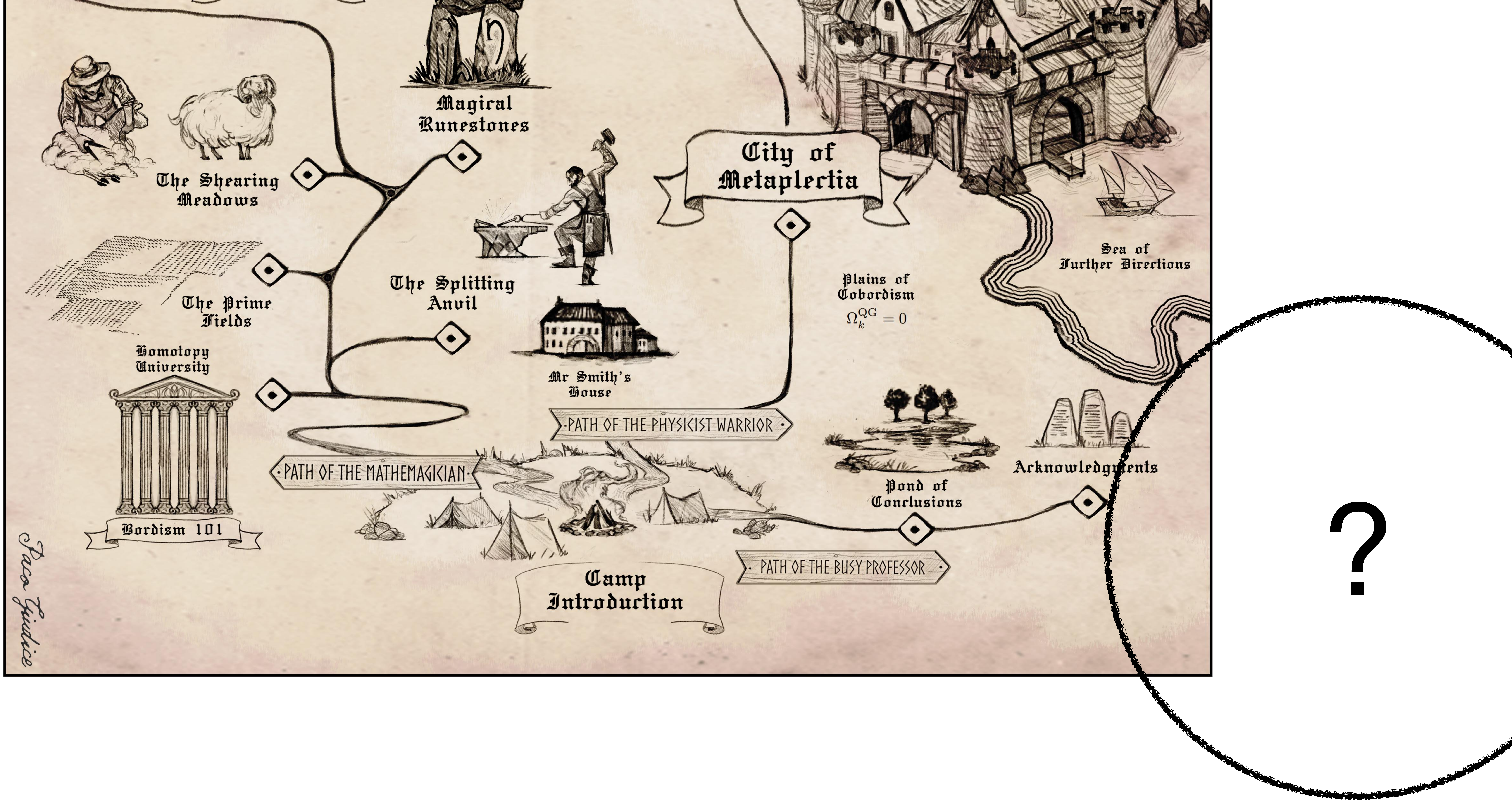
- **Non-Higgsable clusters**
  [Morrison, Taylor ’12]
- $\mathcal{N} = 3$ **S-folds**
  [Garcia-Etxebarria, Regalado ’15]
- **[p,q]-7-branes**

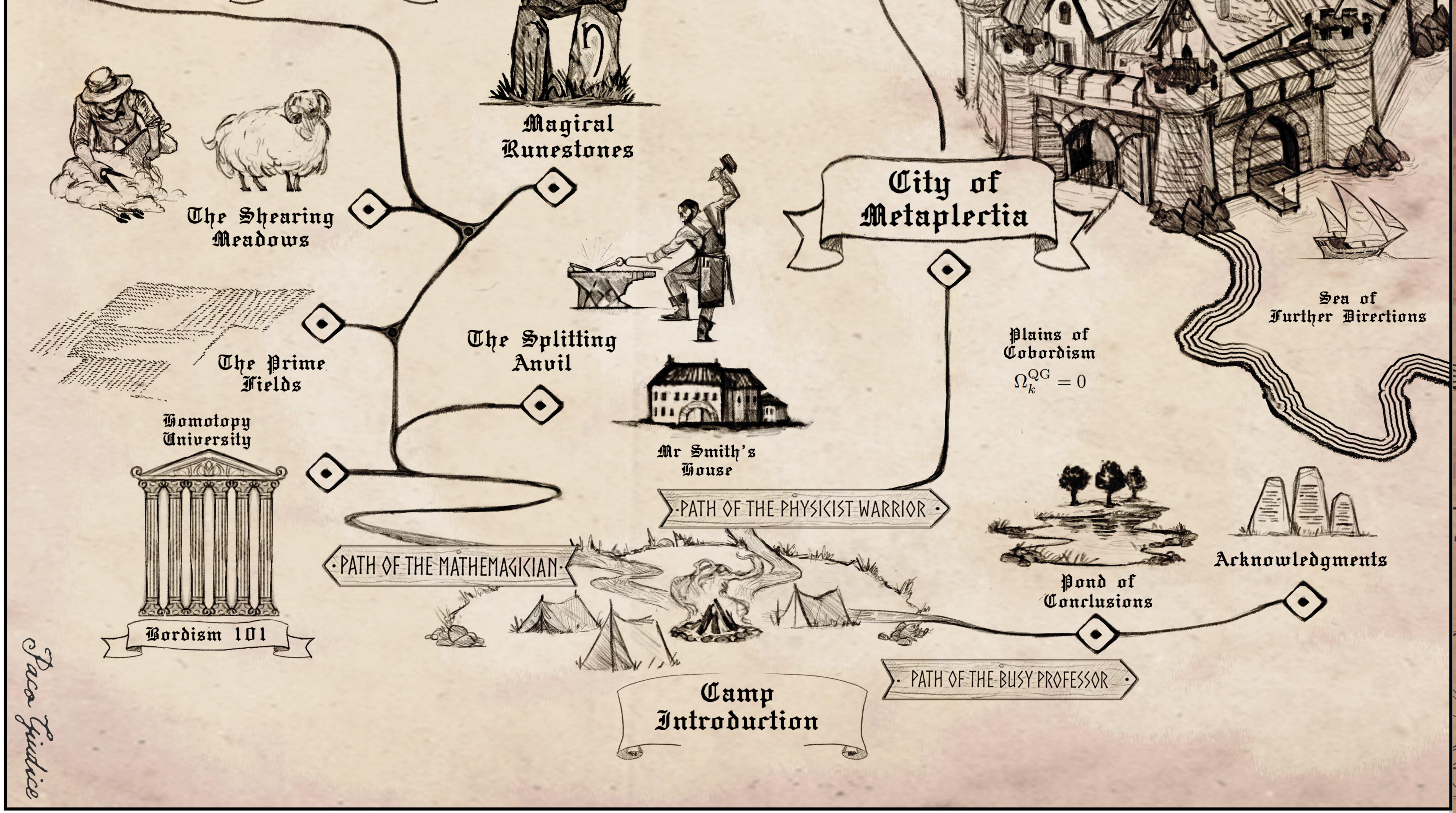
One remains

[MD, Heckman, Montero, Torres ’22 + ’23]



new 7-brane


Magical Runestones
The Shearing Meadows
The Prime Fields
The Splitting Anvil
Homotopy University
Bordism 101
City of Metaplectia
Sea of Further Directions
Plains of Cobordism
$\Omega_k^{\mathrm{QG}} = 0$
Mr Smith's House
PATH OF THE PHYSICIST WARRIOR
PATH OF THE MATHEMAGICIAN
Pond of Conclusions
Acknowledgments
PATH OF THE BUSY PROFESSOR
Camp Introduction
Paco Giudice
?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Let’s explore Utopia

Supergravity with:

- **32 supercharges**
- Spacetime
- **U-duality**

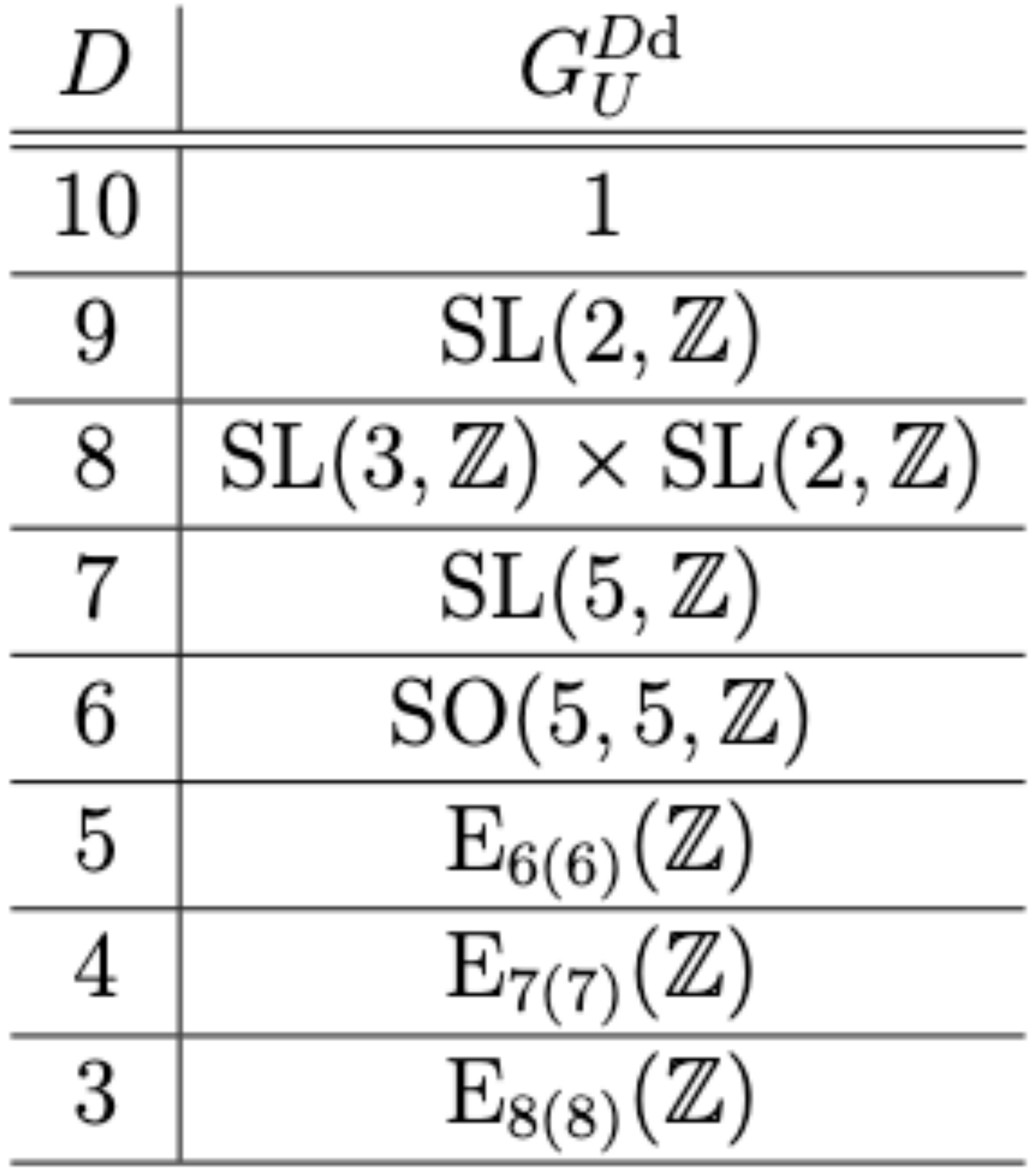
(Another huge thanks to Paco Giudice)

U-duality

See e.g. [Obers, Pioline ’98]

U-duality in (11-k) dimensions:

$$\mathrm{SL}(k;\mathbb{Z}) \bowtie \mathrm{SO}(k-1,k-1;\mathbb{Z})$$



D	$G_U^{D\mathrm{d}}$
10	1
9	$\mathrm{SL}(2,\mathbb{Z})$
8	$\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$
7	$\mathrm{SL}(5,\mathbb{Z})$
6	$\mathrm{SO}(5,5,\mathbb{Z})$
5	$\mathrm{E}_{6(6)}(\mathbb{Z})$
4	$\mathrm{E}_{7(7)}(\mathbb{Z})$
3	$\mathrm{E}_{8(8)}(\mathbb{Z})$

comes from:

M-theory on T^k

$$\rotatebox{90}{$\simeq$}$$

type IIB on T^{k-1}

Surprise I: Sparse codimension-two

In general one has:

$$\Omega_1^{\mathrm{Spin}}(BG_U) = \mathbb{Z}_2 \oplus \mathrm{Ab}(G_U)$$

Abelianization

$$\mathrm{Ab}(G_U) = \frac{G_U}{[G_U, G_U]}$$

$$\mathrm{Ab}\big(\mathrm{SL}(3,\mathbb{Z})\big) = 0$$

$$\mathrm{Ab}\big(G_U^{D\mathrm{d}}\big) = 0\,, \quad D < 8$$

no extra defects needed in codim-two

Still **interesting exotic branes**

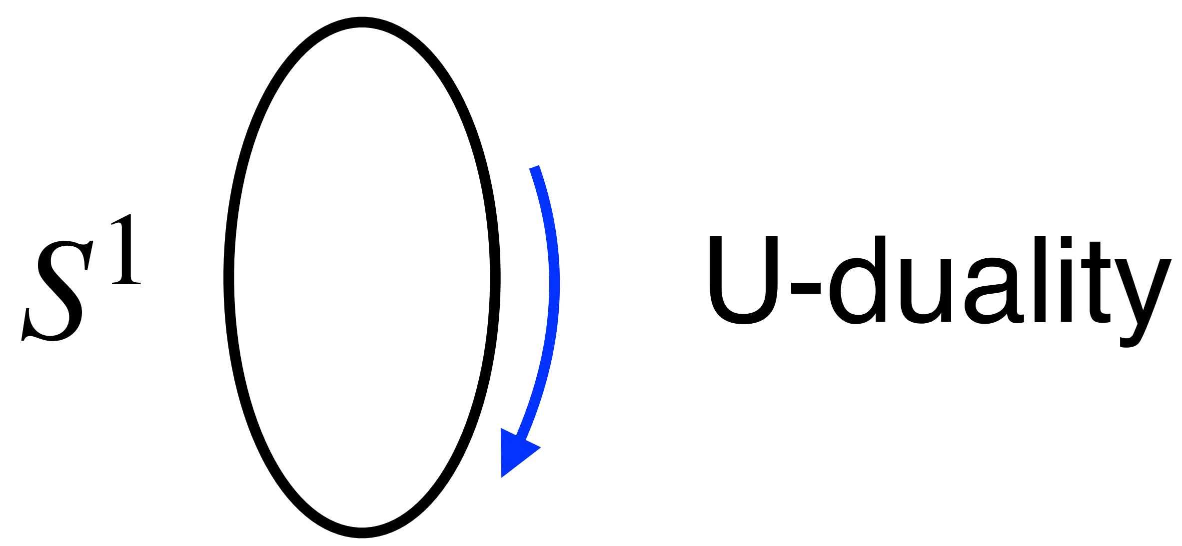
[deBoer, Shigemori '12]

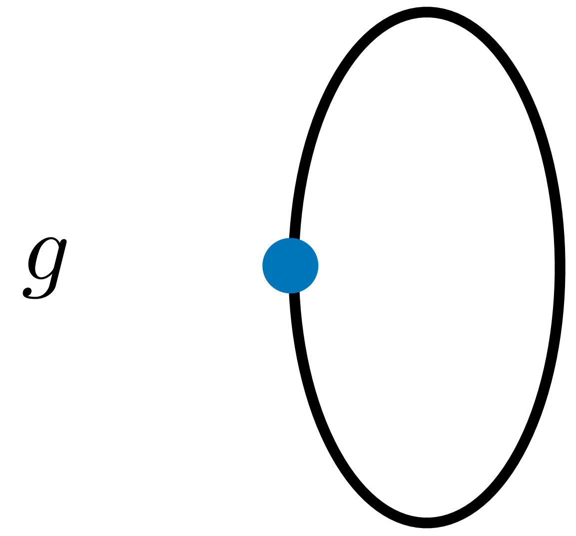
Why are they not required?

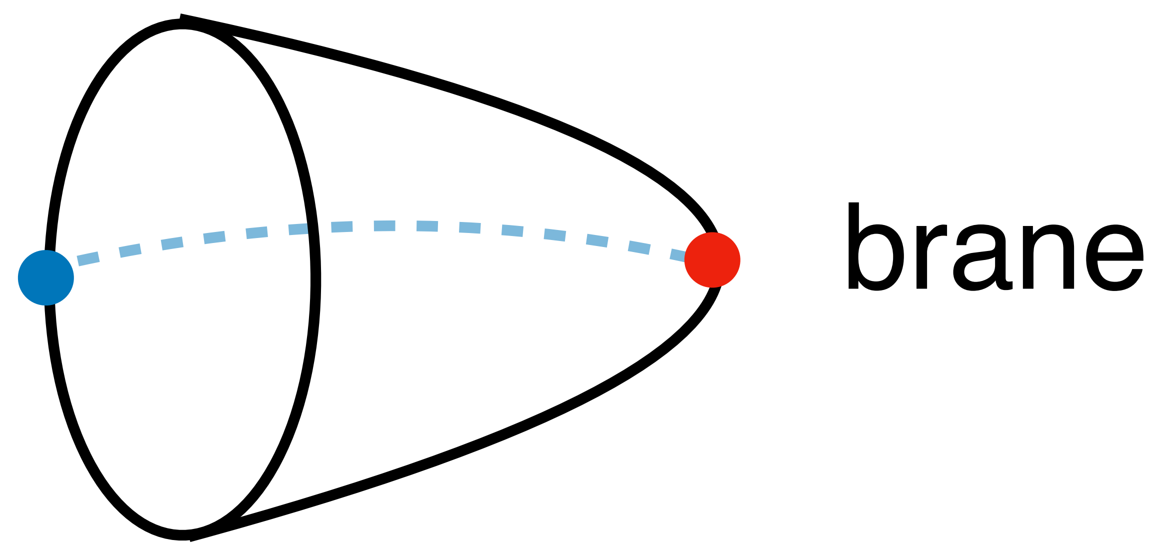
Gravitational solitons

[McNamara '21], [Debray, MD, Heckman, Montero '23]

Gravitational solitons do the job (smooth geometries)



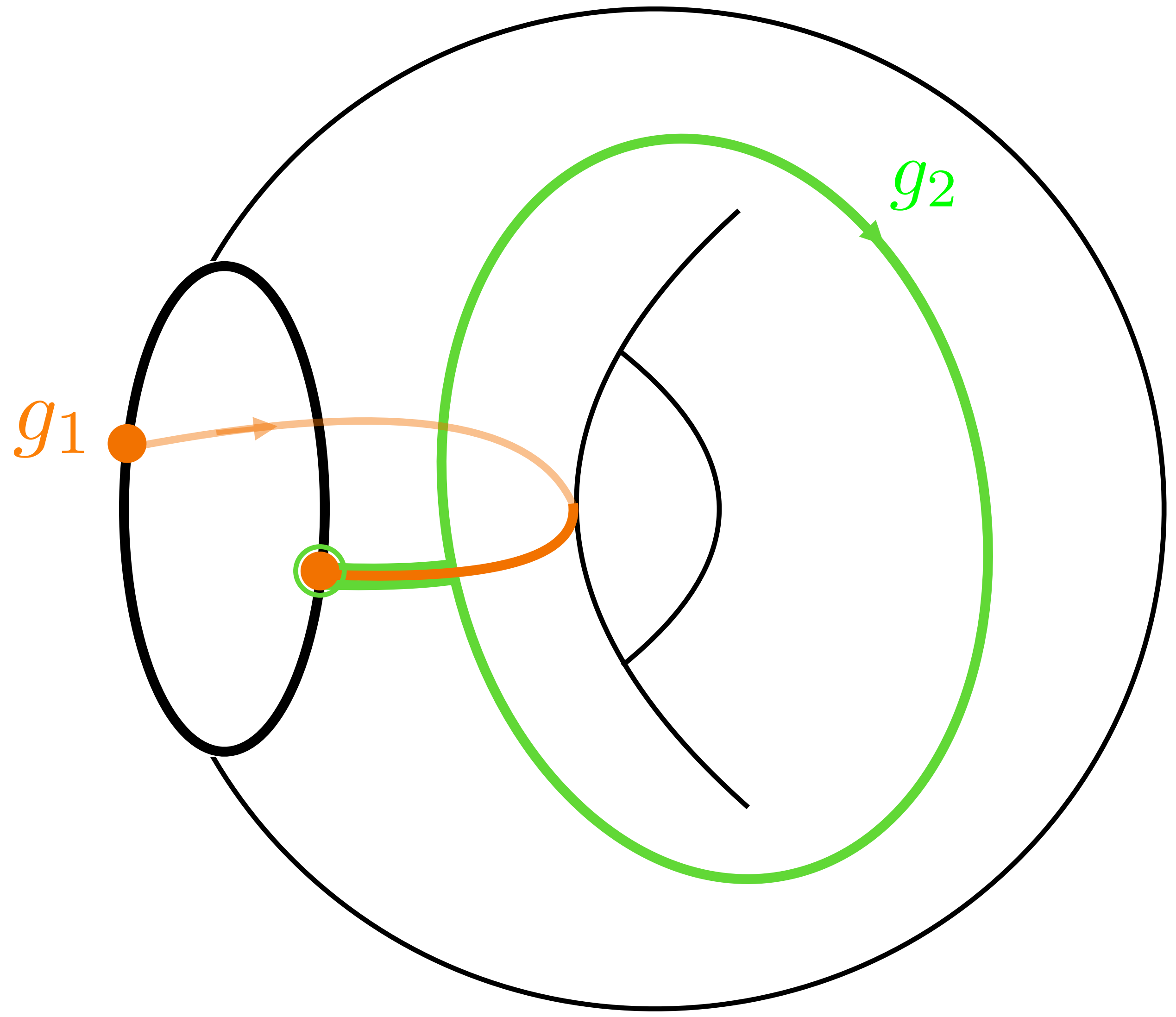




If

$$g = [g_1, g_2] = g_1 g_2 g_1^{-1} g_2^{-1}$$

in commutator subgroup

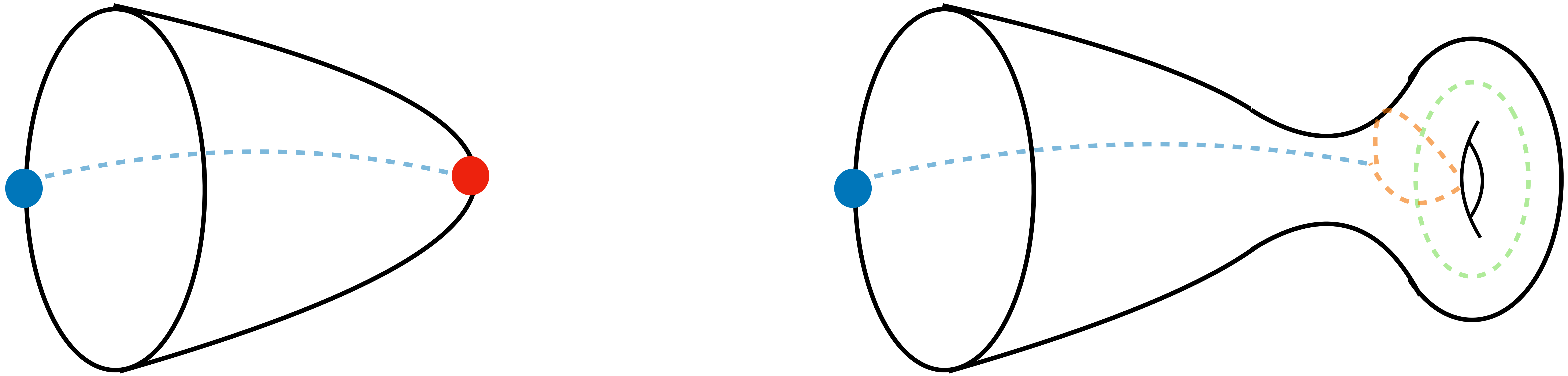


no singular object, but **non-trivial topology**

Gravitational solitons

See also [Ruiz '24]

Gravitational solitons = wormholes



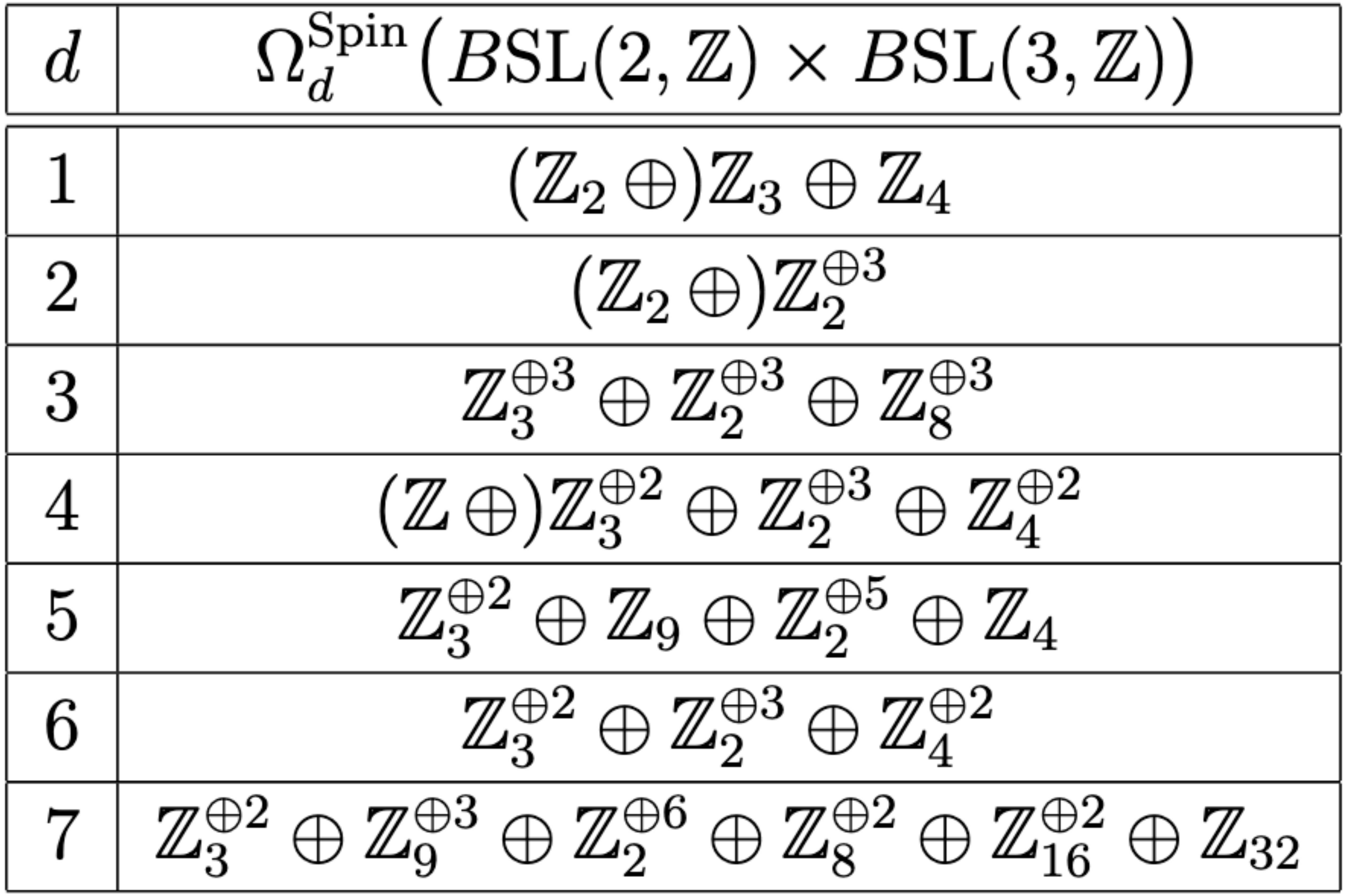
Non-perturbative breaking in quantum gravity $e^{-M_{pl}^2}$

8d Supergravity

$\mathcal{N} = 2$ **supergravity** in eight dimensions with **U-duality**

$$\boxed{\mathrm{SL}(2,\mathbb{Z}) \times \mathrm{SL}(3,\mathbb{Z})}$$

$$\boxed{\rightarrow \ \Omega_d^{\mathrm{Spin}}\big(B(\mathrm{SL}(2,\mathbb{Z}) \times \mathrm{SL}(3,\mathbb{Z}))\big)}$$



d	$\Omega_d^{\text{Spin}}(BSL(2,\mathbb{Z}) \times BSL(3,\mathbb{Z}))$
1	$(\mathbb{Z}_2 \oplus)\mathbb{Z}_3 \oplus \mathbb{Z}_4$
2	$(\mathbb{Z}_2 \oplus)\mathbb{Z}_2^{\oplus 3}$
3	$\mathbb{Z}_3^{\oplus 3} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_8^{\oplus 3}$
4	$(\mathbb{Z} \oplus)\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_4^{\oplus 2}$
5	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_2^{\oplus 5} \oplus \mathbb{Z}_4$
6	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_4^{\oplus 2}$
7	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_9^{\oplus 3} \oplus \mathbb{Z}_2^{\oplus 6} \oplus \mathbb{Z}_8^{\oplus 2} \oplus \mathbb{Z}_{16}^{\oplus 2} \oplus \mathbb{Z}_{32}$

a LOT of
topological charges

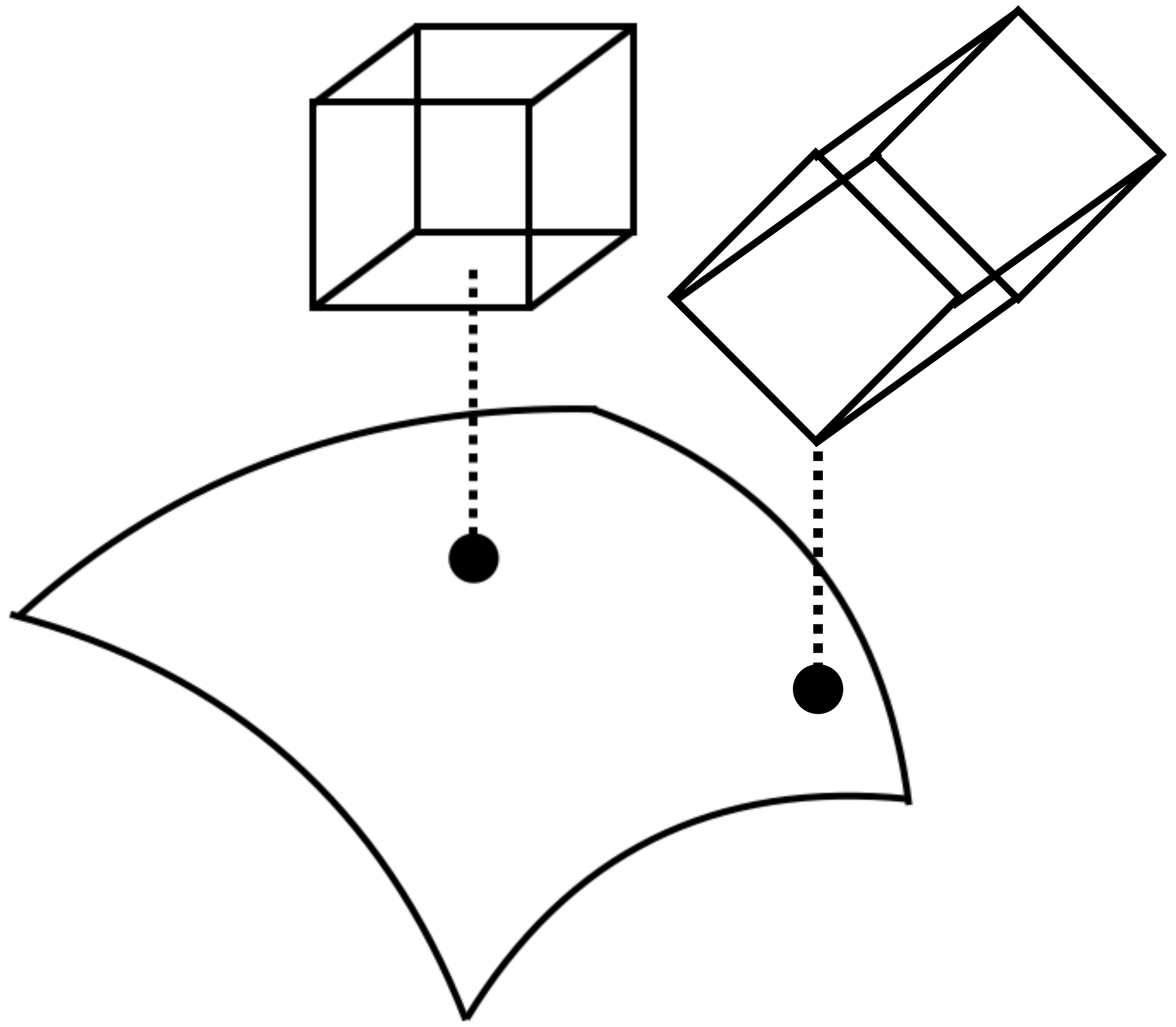
What are the objects?

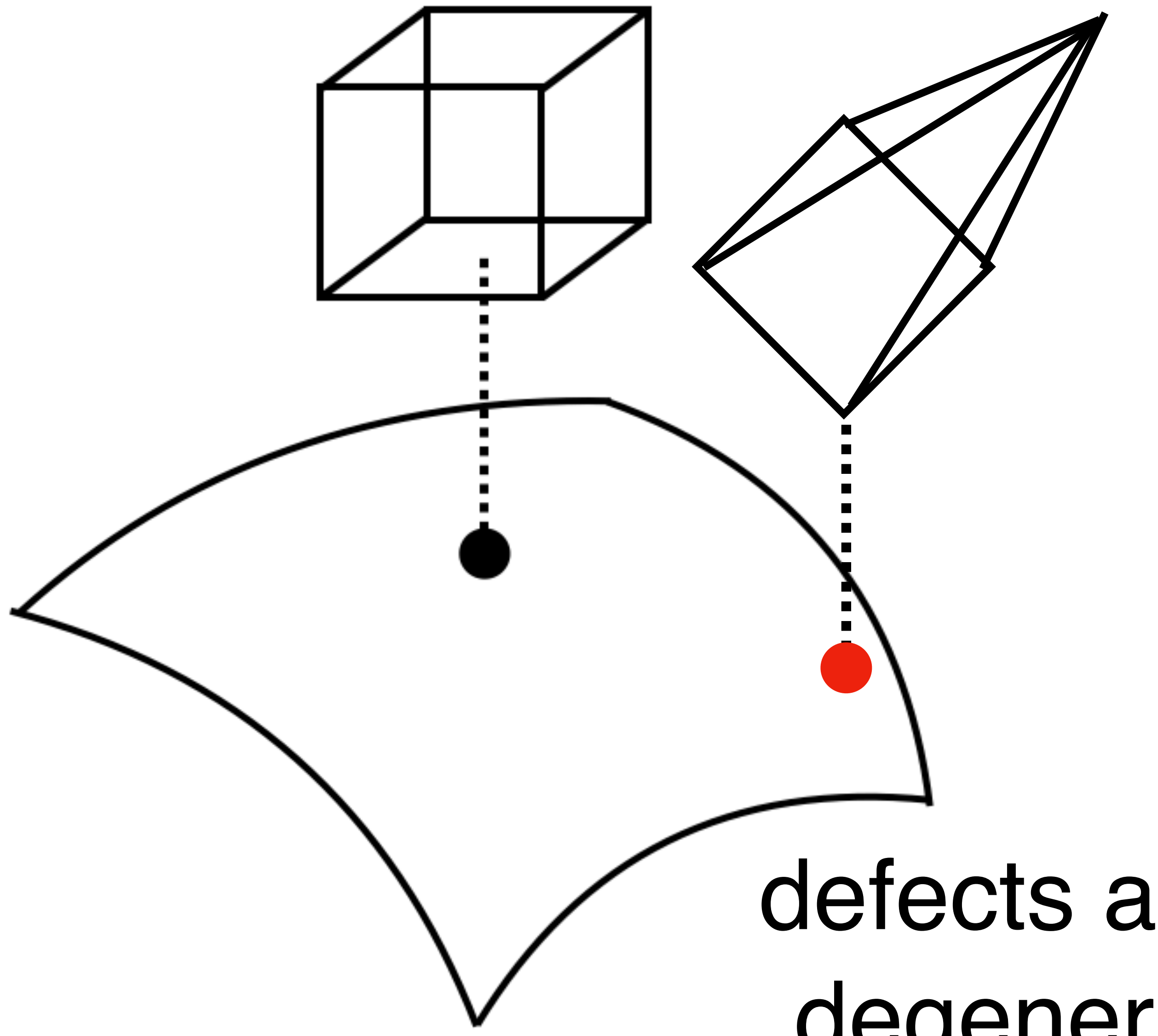
What do we expect?

We do know some ‘geometric’ UV completions:

- **Type IIB and F-theory configurations**
- **M-theory**
- **Compactifications of defects**
- **Non-geometric defects**

  mixing diffeos and T-duality

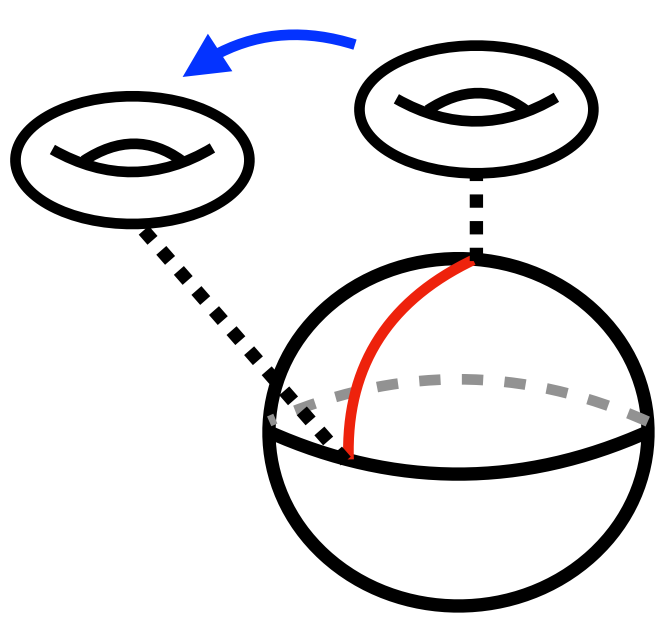




defects at fiber degeneration

What we find (examples) (confirmation of what I told you last year)

Type IIB:



$\mathbb{Z}_3 \in \Omega_3^{\mathrm{Spin}}(BG_U)$

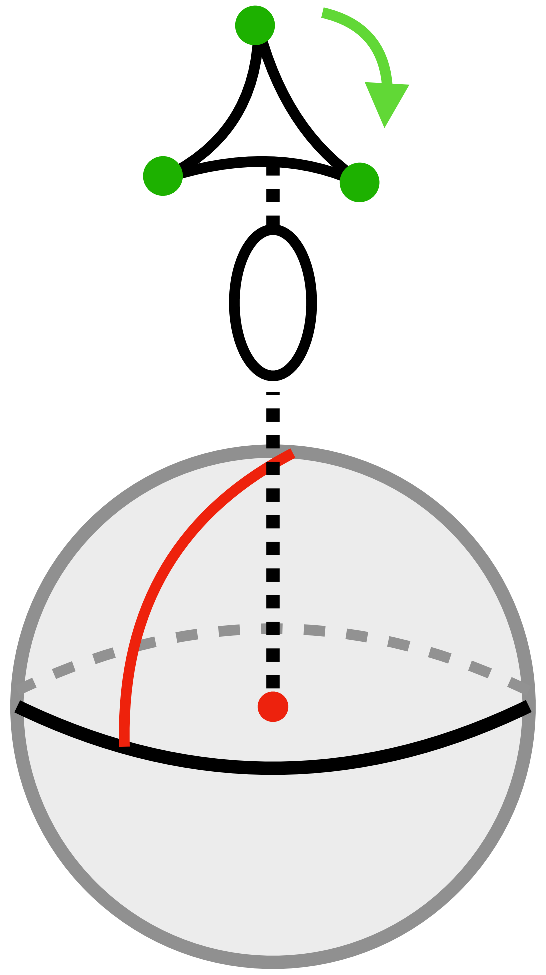
non-Higgsable cluster compactified on torus

(a descendant of IIBorida)

➡ **Argyres-Douglas theory** $D_3(\mathrm{SU}(2))$

[del Zotto, Vafa, Xie ’15], [Carta, Giacomelli, Mekareeya, Mininno ’23]

M-theory:



$\mathbb{Z}_3 \in \Omega_3^{\mathrm{Spin}}(BG_U)$

three sector of E_0 theory (M-theory on $\mathbb{C}^3/\mathbb{Z}_3$)

[Seiberg ’96], [Morrison, Seiberg ’97]

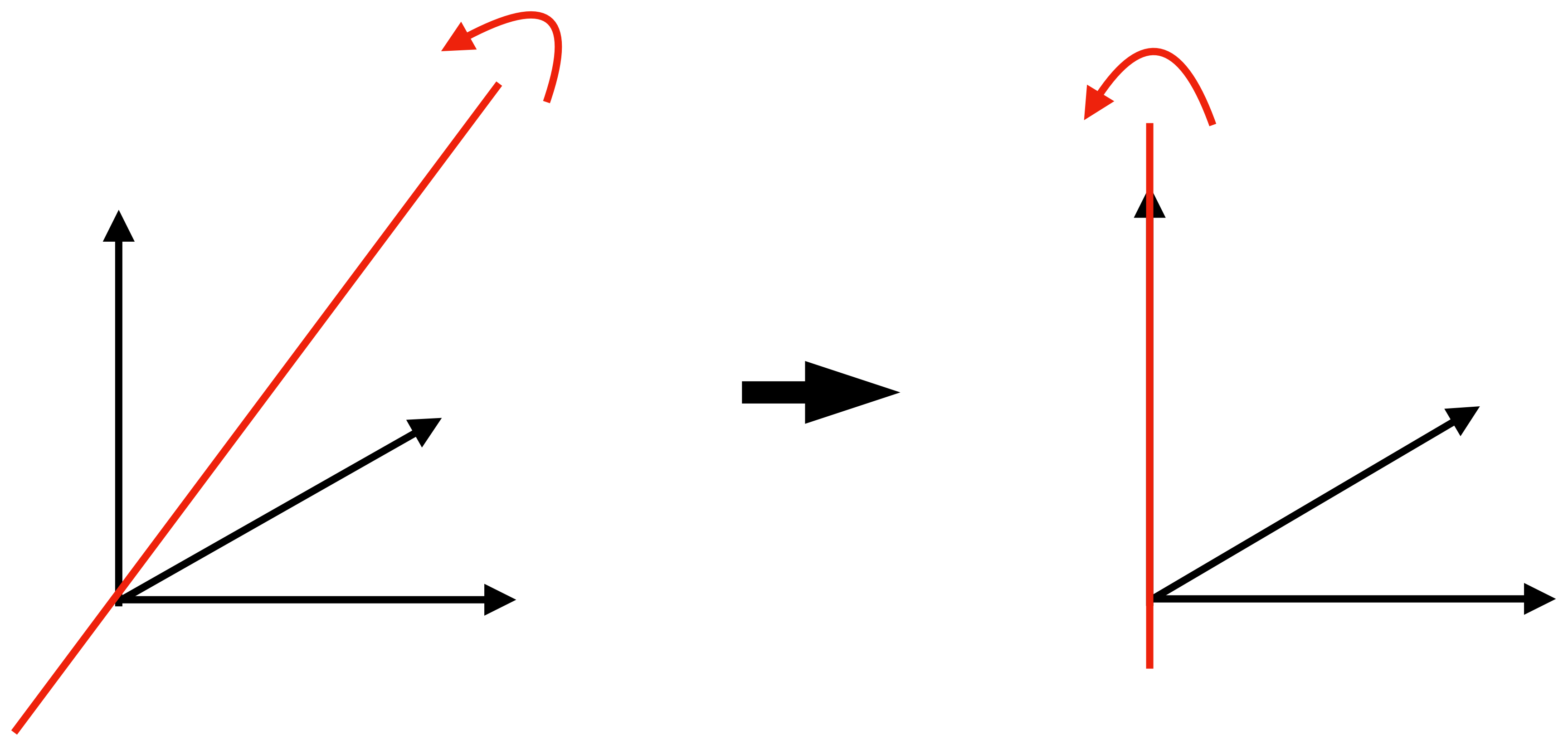
$$\begin{tikzcd} (\mathbb{C}^3/\mathbb{Z}_3) \arrow[rr, bend right] & \oplus & (\mathbb{C}^3/\mathbb{Z}_3) \arrow[rr, bend right] & \oplus & (\mathbb{C}^3/\mathbb{Z}_3) \arrow[llll, bend right] \end{tikzcd}$$

See also U-folds, e.g., [Kumar, Vafa ’96], [Liu, Minasian ’97], …

Surprise II: Z vs Q

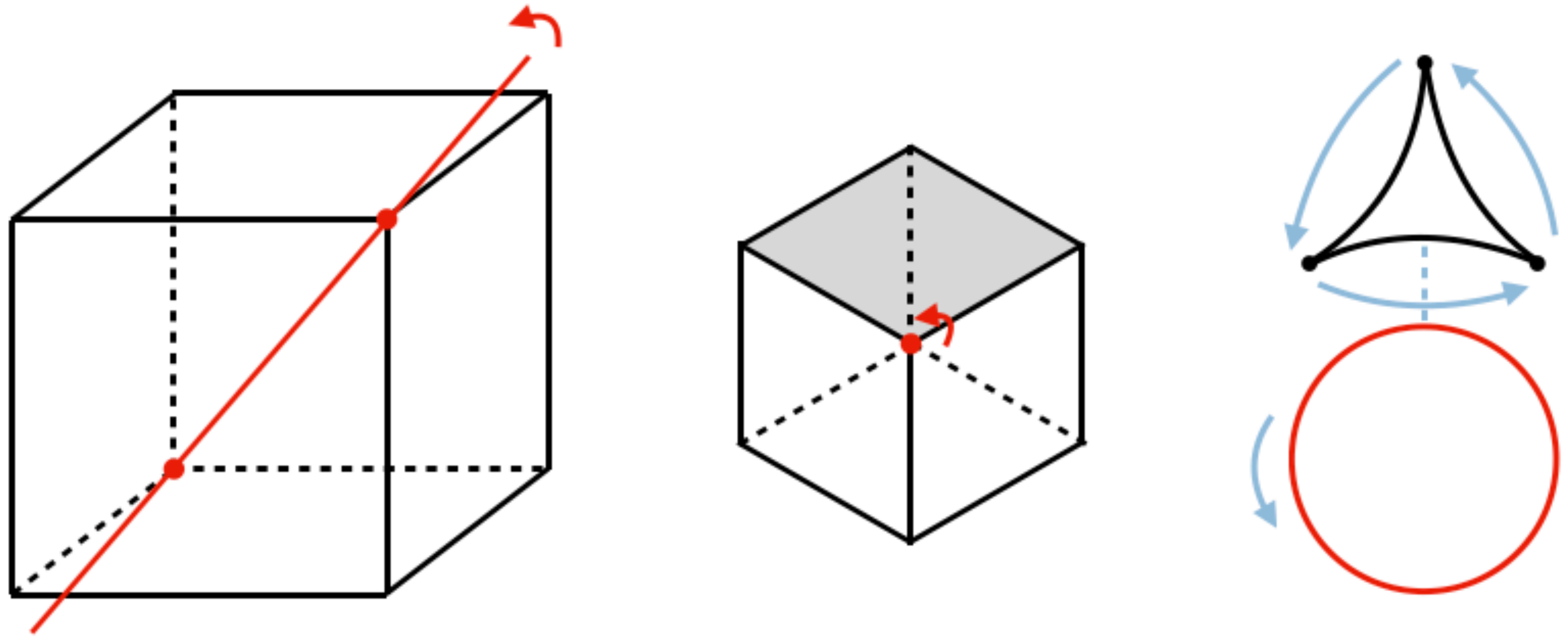
$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \in \mathrm{SO}(3)$$

Why not ‘rotate’ to an easier frame?

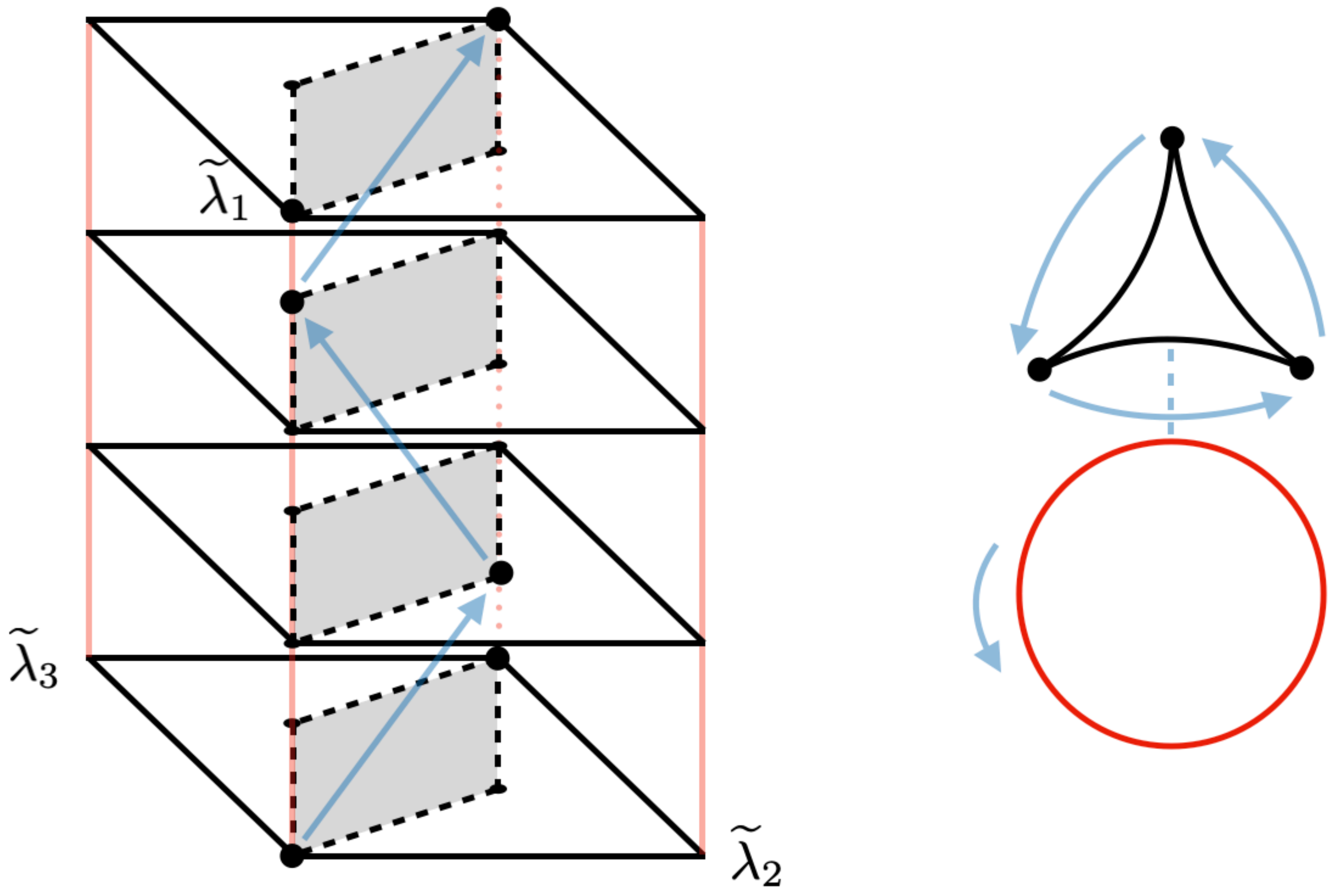


One can do that over $\mathbb{Q}$

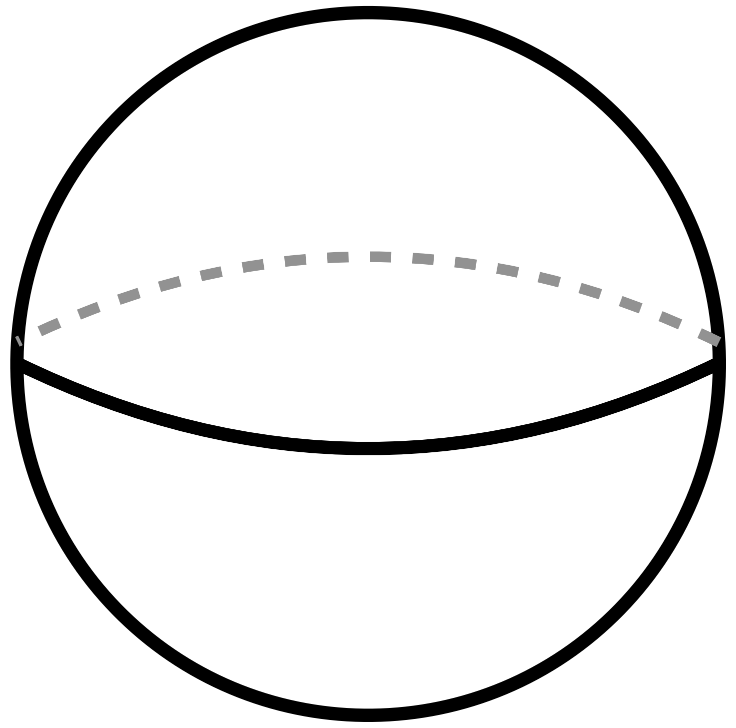
but not over $\mathbb{Z}$



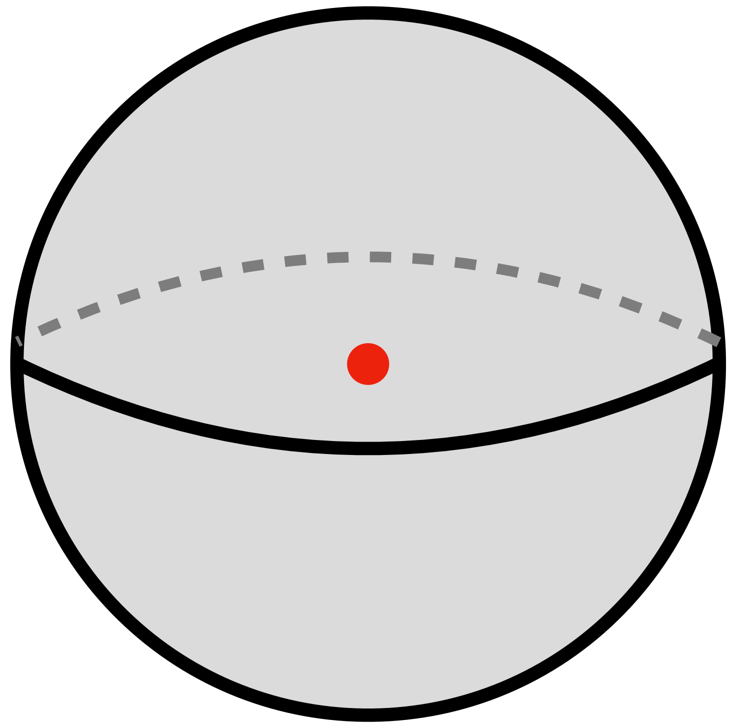
but almost:

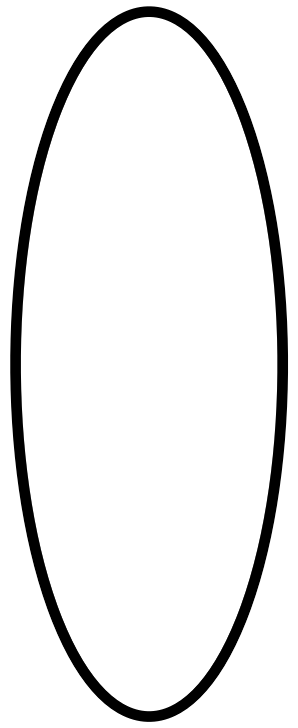


Strings & Non-Geometry

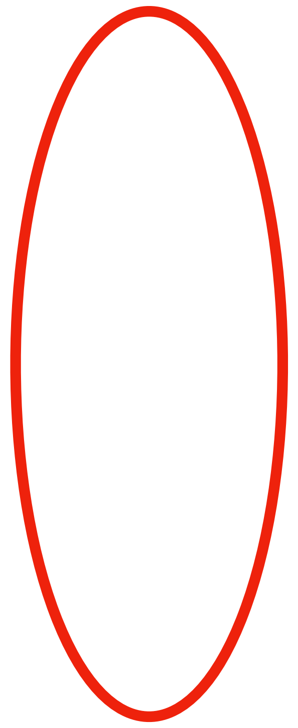


defect might have **geometric** interpretation





But its compactification involves a
Non-geometric action (non-geometric twist)

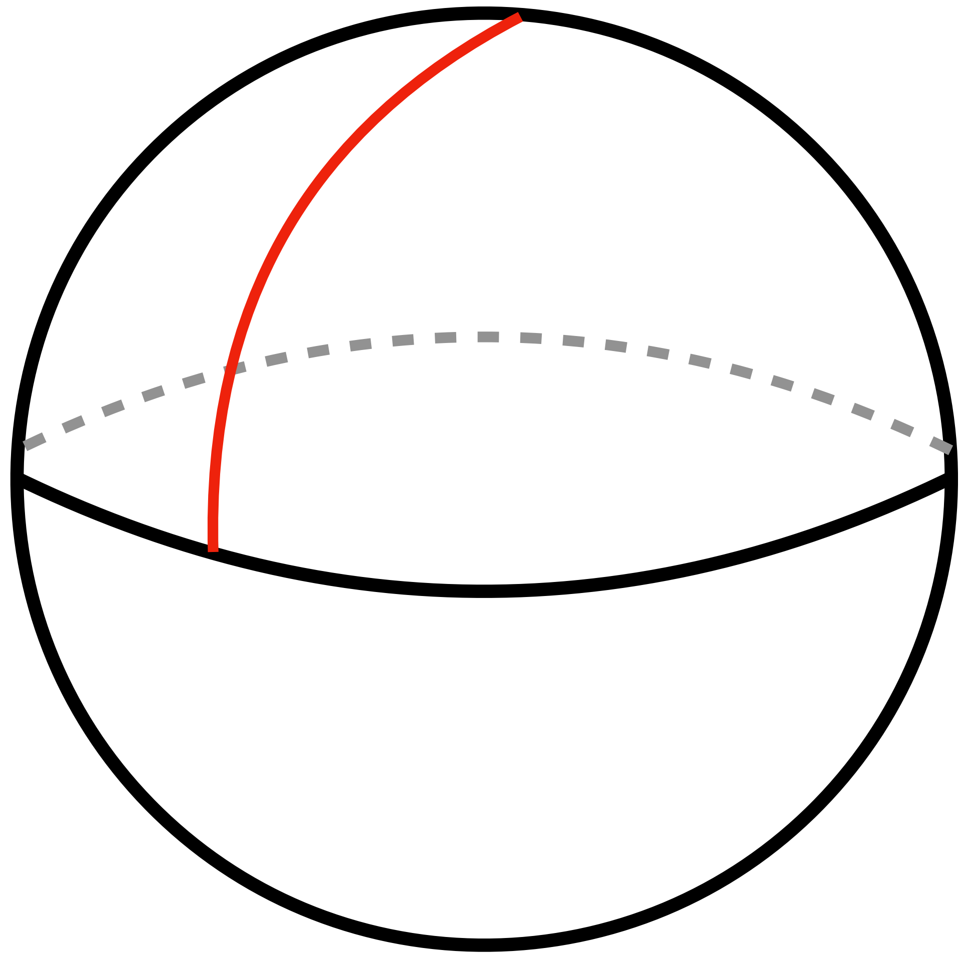


Strings & Non-Geometry

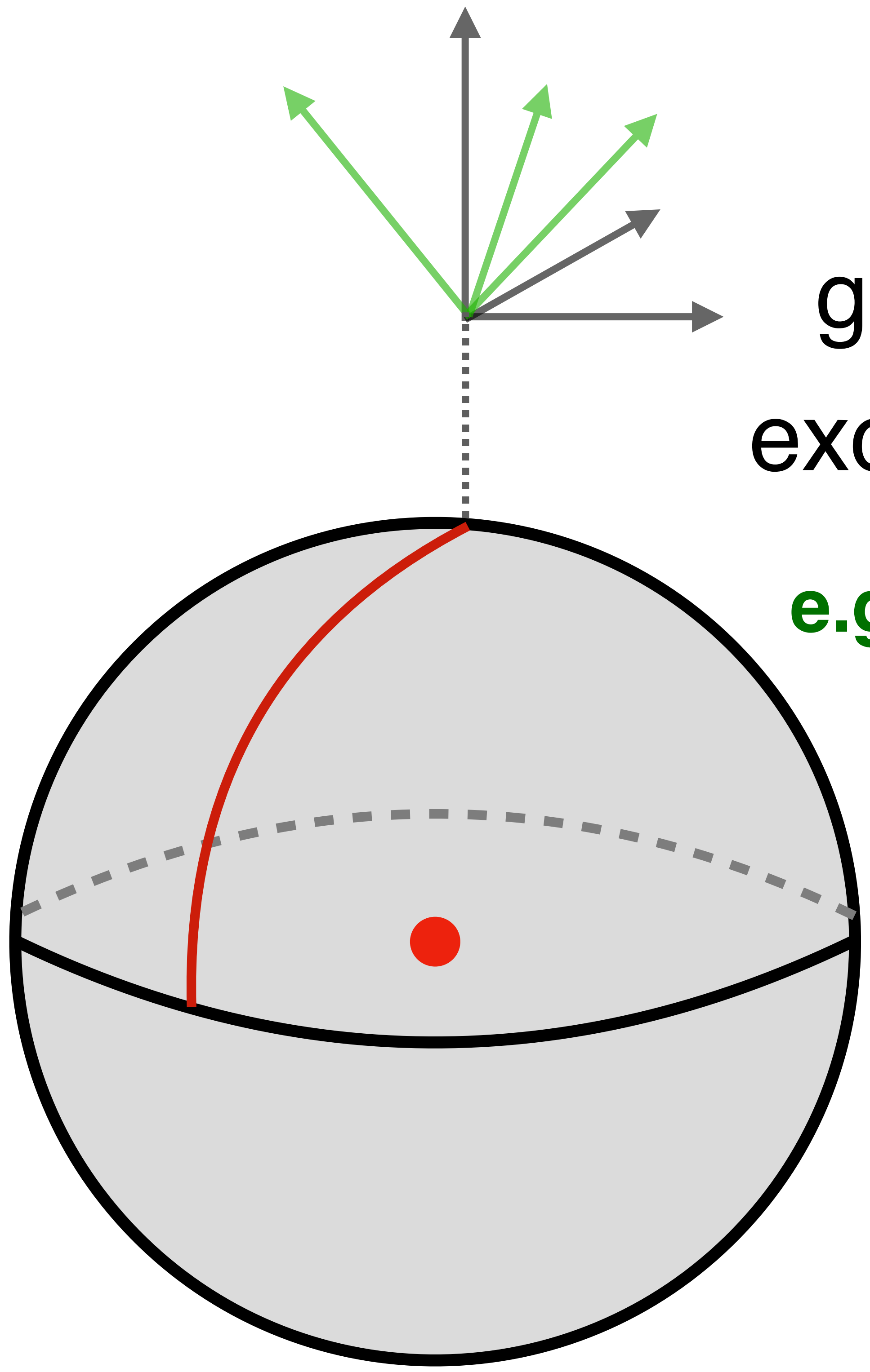
At d = 5 we find non-geometric defects:

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

cannot be conjugated to
a geometric subgroup



$$L_3^5 = \partial(\mathbb{C}^3/\mathbb{Z}_3)$$



geometrization within
exceptional field theory?

e.g. [Hohm, Samtleben '13]

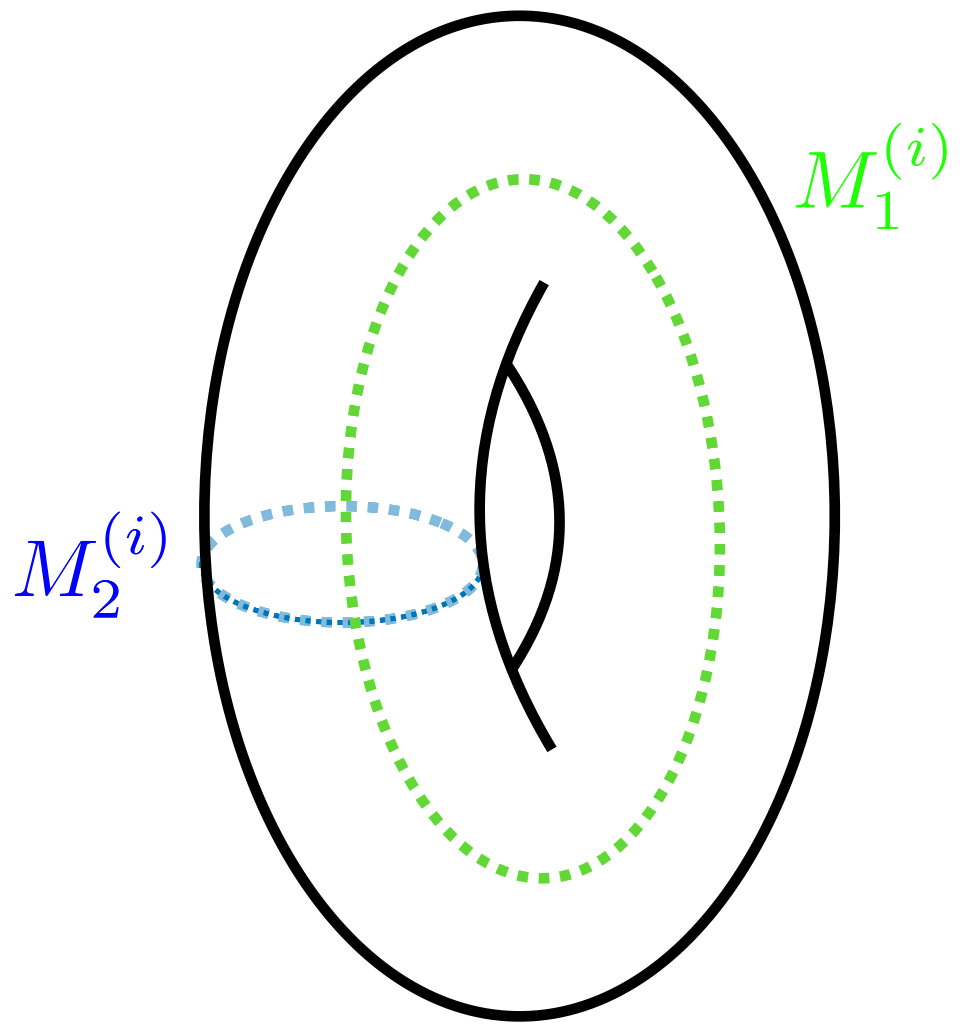
Codimension-six **string defect** with **action on geometry and other moduli**

Non-perturbative generalization of T-fold geometry

[Hull '04, '06]

Surprises III: Resurrection of Codim-2

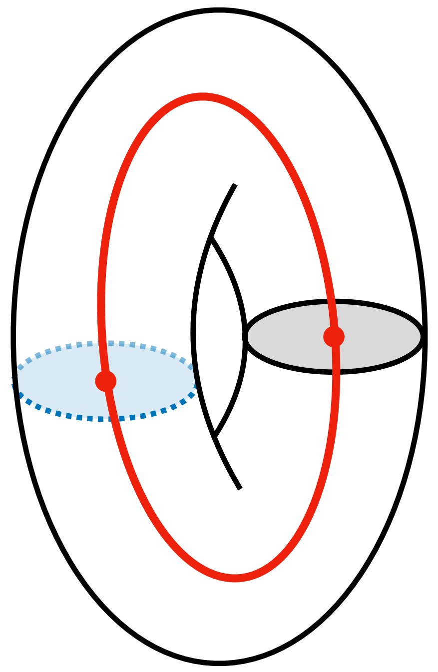
In d = 2 we find:



$$M_1^{(i)}\,, M_2^{(i)} \in \mathrm{SL}(3,\mathbb{Z})\,, \quad [M_1^{(i)}, M_2^{(i)}] = 0$$

Our usual approach would tell us: **New codimension-two brane**

compactified on circle

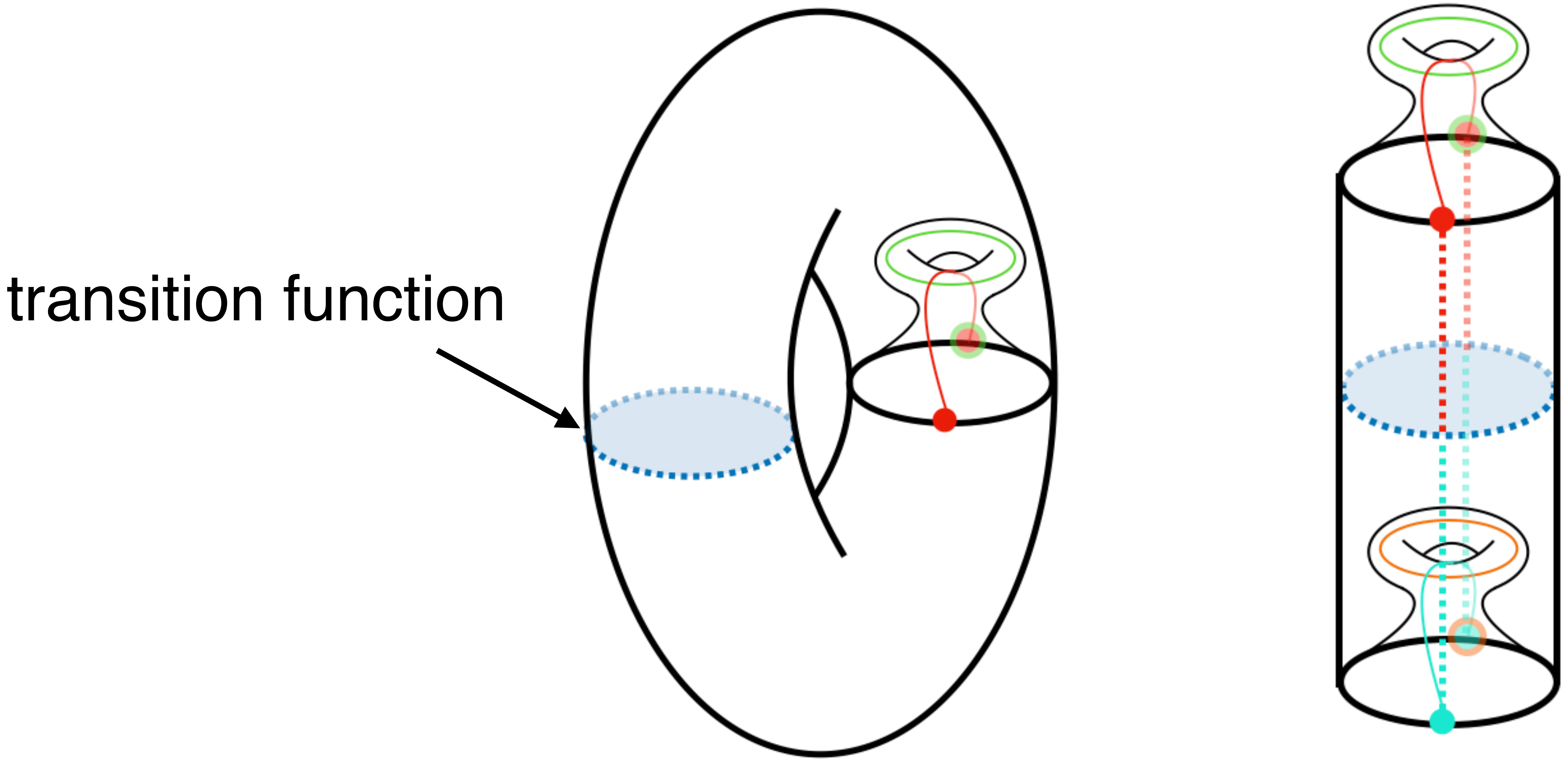


no problem with transition function
since monodromies commute

Why no soliton solution?

Surprises III: Gravitational solitons fail

Try to use the soliton



action incompatible with

$g_1 \, , g_2$

for any choice

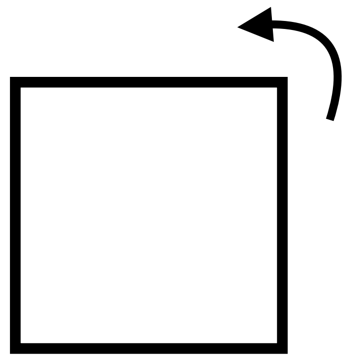
→ **does not lead to well-defined background**

Subtlety of fermions

We use the bosonic version of the duality group

$$\begin{tikzcd} & \mathrm{SL}(2,\mathbb{Z}) \times \mathrm{SL}(3,\mathbb{Z}) & \\ T^2 \arrow[ur] & & T^3 \arrow[ul] \end{tikzcd}$$

geometric action in certain duality frames $T^2 \, , \tau = i : \quad S$



there are fermions: $S^4 \psi = -\psi$

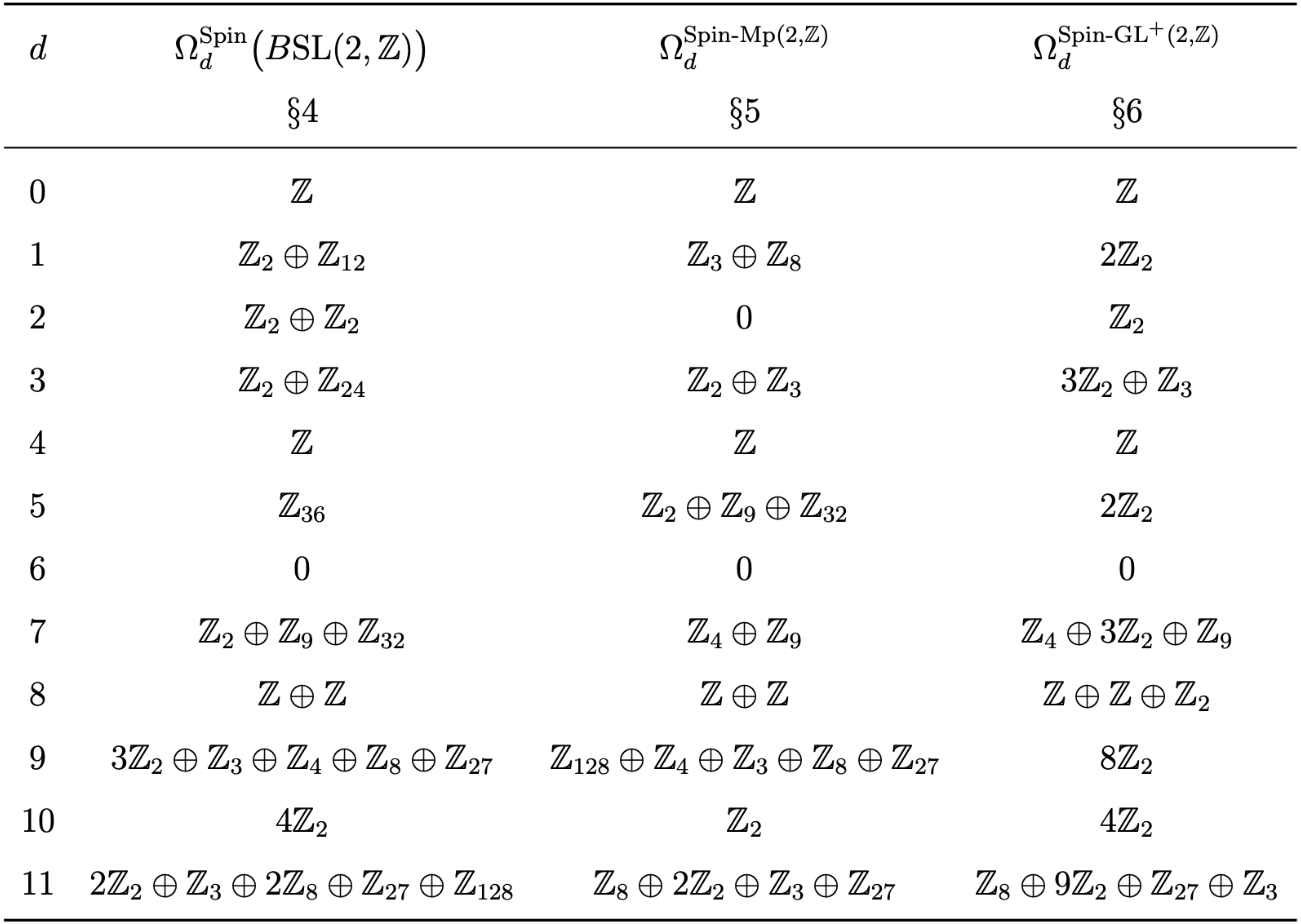
should **affect the full (fermionic) U-duality**

tricky to combine

$\mathrm{Spin}\text{-}\mathrm{GL}^+$ of type IIB

Pin^+ of M-theory

It matters



d	$\Omega_d^{\text{Spin}}\big(B\mathrm{SL}(2,\mathbb{Z})\big)$	$\Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}$	$\Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}$
	§4	§5	§6
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2$
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
5	$\mathbb{Z}_{36}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

Summary & Outlook

(As for type IIB in 10d) this program is able to predict:

- **Many symmetry-breaking defects imposed by U-duality**
- **Interesting worldvolume theories** (SCFTs, twists)
- **Very stringy corners** (non-geometry)

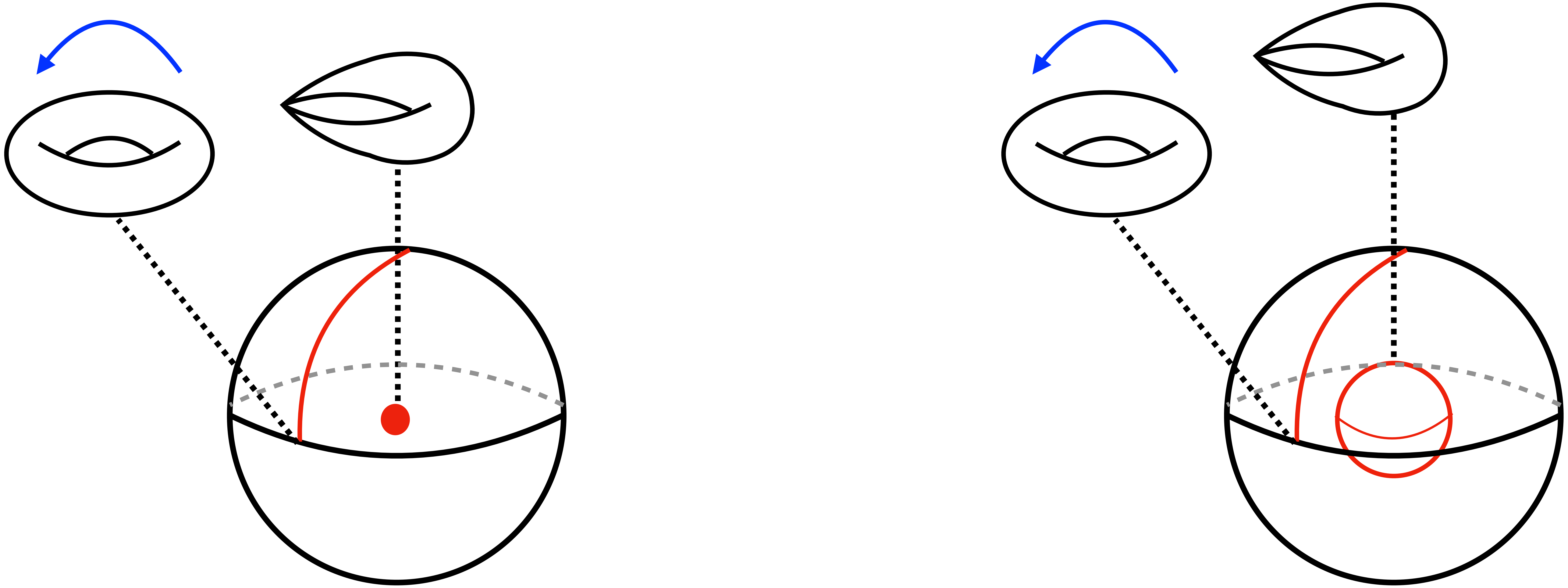
➛ **String theory very complete**

Many things to do:

- **Fermionic lift** (BPS nature)
- **Properties of the defects** (applications, supergravity)
- **Physics in maths**

Non-deformabilty

What does **singular in supergravity mean?**



Maybe can regularize (parts of) spacetime, but still there is a **defect**