Discrete Gauging of 6d SCFTs and Wreathed 3d $\mathcal{N} = 4$ Quivers

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Based on: 2504.03830 w/ A. Mininno and T. Lepper

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Motivation: Classifications of SCFTs

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e.g. non-compact 6d manifold → "String Theory" → 4d QFT

physical properties here depend on geometric properties here

"geometrization"

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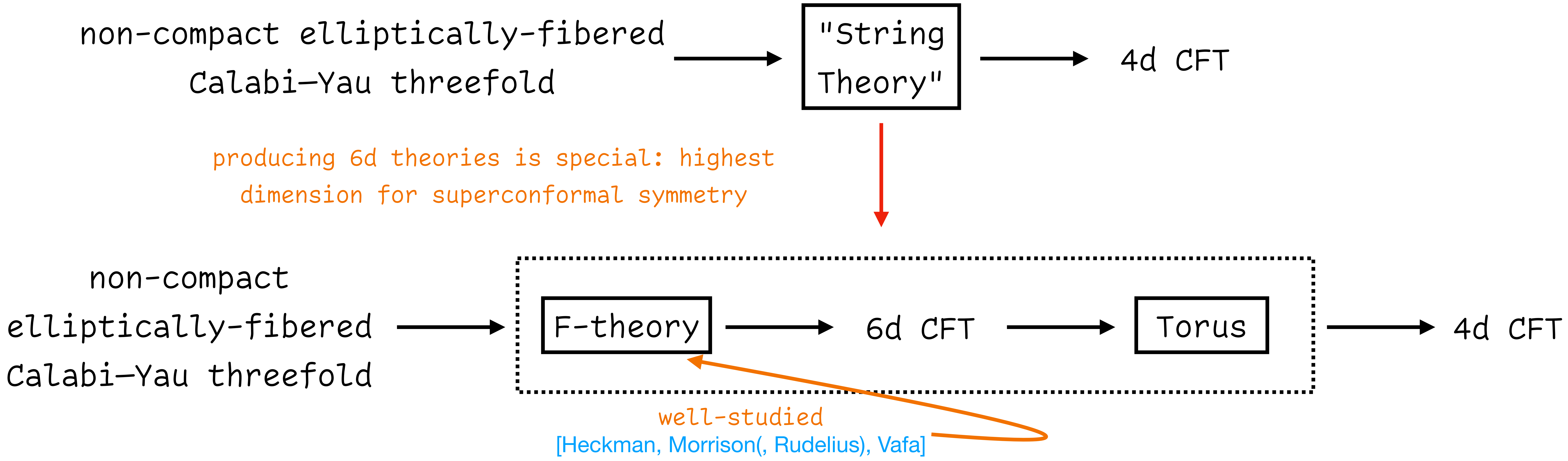
e.g.



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example: 4d $\mathcal{N} = 2$

conjecture: $(2a - c)$ is a sum over Coulomb branch operators [Shapere, Tachikawa]

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violated when discrete-gauging!

[Argyres, Lu, Martone], [Argyres, Martone], [Bourget, Pini, Rodriguez-Gomez], [Bourton, Pini, Pomoni]

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cf, Hanany's "characterization of symplectic singularities"

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technique: find a 3d $\mathcal{N}=4$ Lagrangian gauge theory $\mathscr{T}_M$ such that

$$\mathrm{HB}\left[\mathscr{T}\right] \cong \mathrm{CB}\left[\mathscr{T}_M\right]$$

"magnetic quiver for the Higgs branch of $\mathscr{T}$"

[Ferlito, Hanany, Mekareeya, Zafrir], [Hanany, Witten], [Hanany, Zaffaroni]

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quiver subtraction/decay and fission [Cabrera, Hanany], [Bourget, Sperling, Zhong]^2, [CL, Mansi, Sperling, Zhong]

monopole formula for Coulomb branch Hilbert series [Cremonesi, Hanany, Zaffaroni]

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use tools to study CB of Lagrangian quivers!

Proposal

let $\mathcal{T}$ be a 6d $(1,0)$ SCFT with discrete symmetry Γ
and magnetic quiver for the Higgs branch $\mathcal{T}_M$

known for many families of 6d SCFTs

[Cabrera, Hanany, Sperling]^2, [Fazzi, Giri],
[Hanany, Mekareeya], [Hanany, Zafrir], [CL, Mansi]^2,
[CL, Mansi, Sperling, Zhong], [Mekareeya, Ohmori, Tachikawa, Zafrir]

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evidence in lower dimensions: [Arias-Tamargo, Bourget, Pini, Rodriguez-Gomez], [Arias-Tamargo, Rodriguez-Gomez], [Bourget, Hanany, Miketa], [Giacomelli, Harding, Mekareeya, Mininno], [Grimminger, Harding, Mekareeya], [Hanany, Kumaran, Li, Liu, Sperling], [Hanany, Sperling]

Wreathing?

let G be a (reductive) Lie group, then

$$G \wr S_n = \left(\prod_{i=1}^{n} G \right) \rtimes S_n$$

wreath product

acts on the G factors by permutation

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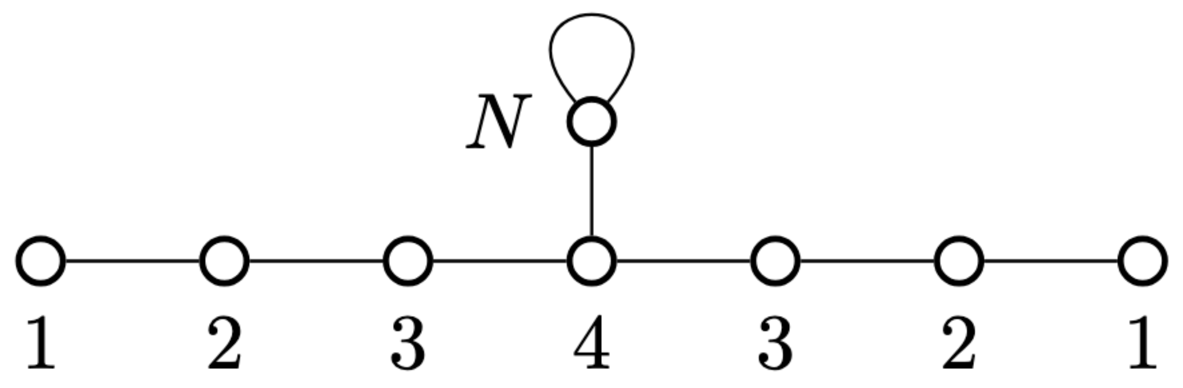
consider a Lagrangian quiver with gauge group $G \supseteq G_w \times G_w$
and $\mathbb{Z}_2$ quiver automorphism that exchanges the G_w



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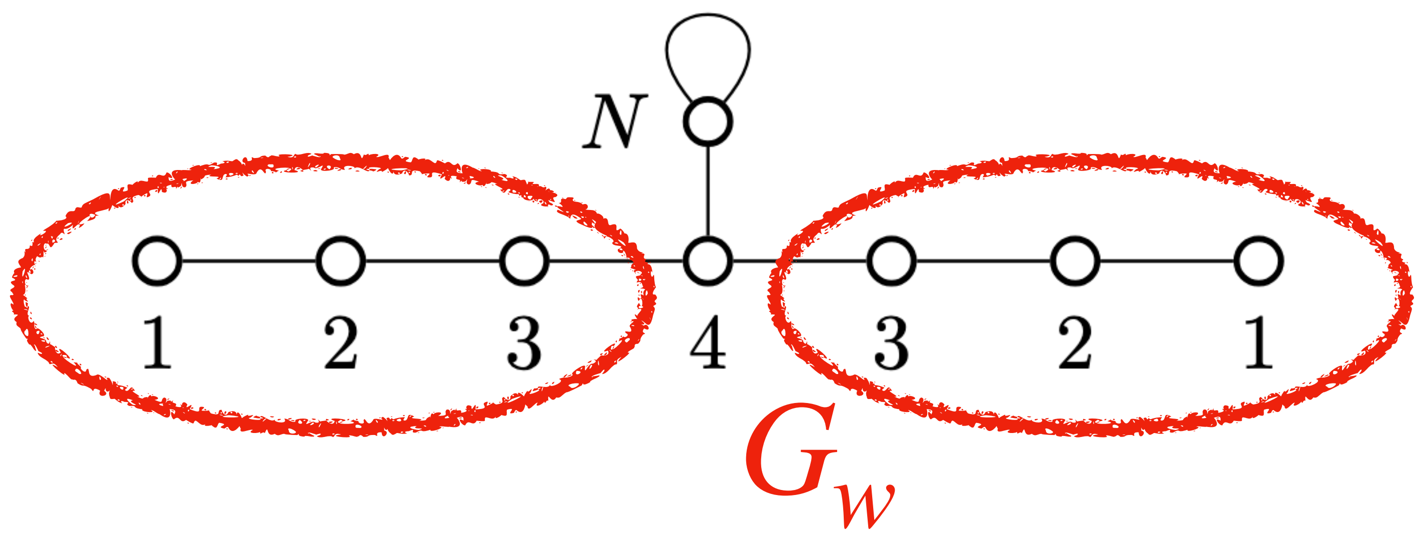

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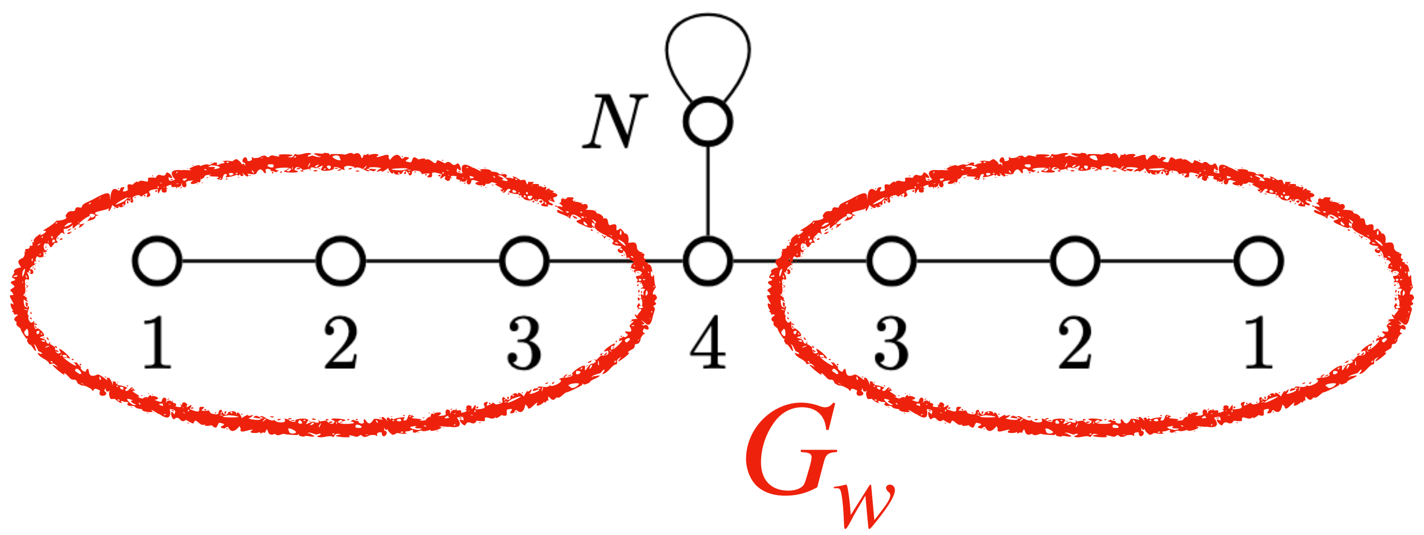
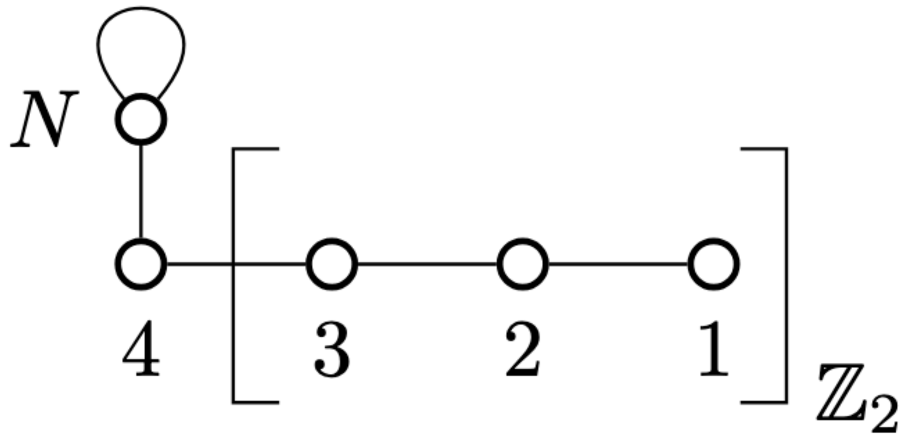
⟶ quiver admits a $\mathbb{Z}_2$ wreathing, which replaces



$$G_w \times G_w \to G_w \wr \mathbb{Z}_2$$



How to Build a 6d SCFT

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In Type IIB/F-theory, a 6d SCFT is associated to

a non-compact elliptically-fibered Calabi–Yau threefold $\pi : Y \to B$

with

no non-minimal singularities in the fiber

and such that

all compact curves C_i in B can be simultaneously contracted

[Heckman, Morrison, Vafa]

[Heckman, Morrison, Rudelius, Vafa]

$\mathrm{vol}(C_i) > 0$ -> tensor branch; $\mathrm{vol}(C_i) = 0$ -> origin of tensor branch = where the SCFT lives

How to Build a 6d SCFT

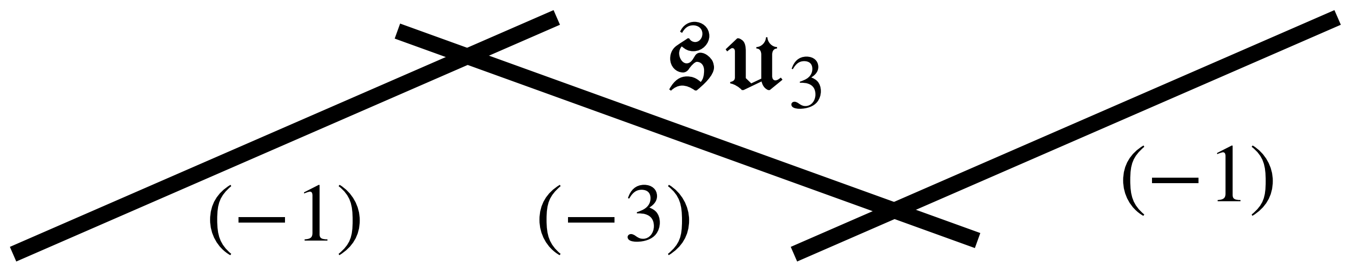
These conditions are very restrictive:

All compact curves are smooth rational: $C_i \cong \mathbb{P}^1$

Negative-definite intersection matrix: $C_i \cdot C_j \prec 0$

Singular fiber over C_i -> simple Lie algebra $\mathfrak{g}_i$ ← gauge algebra with coupling $\propto \frac{1}{\mathrm{vol}(C_i)}$

Example: three curves with $\mathfrak{g}_1 = \mathfrak{g}_3 = \emptyset, \; \mathfrak{g}_2 = \mathfrak{su}_3$



$$C_i \cdot C_j = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & -1 \end{pmatrix}_{ij}$$

$$\begin{array}{ccc} & \mathfrak{su}_3 & \\ 1 & 3 & 1 \end{array}$$

How to Build a 6d SCFT

Tuned non-Higgsable clusters

All configurations satisfying these conditions can be built from “building blocks”

“non-Higgsable clusters”

$$\overset{\mathfrak{su}_3}{3}\,,\quad \overset{\mathfrak{so}_8}{4}\,,\quad \overset{\mathfrak{f}_4}{5}\,,\quad \overset{\mathfrak{e}_6}{6}\,,\quad \overset{\mathfrak{e}_7}{7}\,,\quad \overset{\mathfrak{e}_7}{8}\,,\quad \overset{\mathfrak{e}_8}{12}\,,\quad \overset{\mathfrak{su}_2}{2}\;\overset{\mathfrak{g}_2}{3}\,,\quad 2\;\overset{\mathfrak{su}_2}{2}\;\overset{\mathfrak{g}_2}{3}\,,\quad \overset{\mathfrak{su}_2}{2}\;\overset{\mathfrak{so}_7}{3}\;\overset{\mathfrak{su}_2}{2}$$

$$\underbrace{2\cdots2}_{N-1}\,,\quad \underbrace{2\cdots2}_{N-3}\overset{2}{2}2\,,\quad 22\overset{2}{2}22\,,\quad 222\overset{2}{2}22\,,\quad 2222\overset{2}{2}22$$

[Morrison, Vafa]
[Morrison, Taylor]

- Can tune the gauge algebra $\mathfrak{g} \rightarrow \tilde{\mathfrak{g}} \supset \mathfrak{g}$
- Only specific combinations of $n = -C \cdot C$ and $\mathfrak{g}$ are allowed
- Anomaly cancellation + pair $(n, \mathfrak{g})$ almost always fixes hypermultiplet spectrum

How to Build a 6d SCFT

E-strings + tuning + gauging

- Non-Higgsable clusters can be gauged via E-strings

$$\overset{}{1} \qquad \overset{\mathfrak{g}}{1}$$

E-string   Tuned E-string

E_8 flavor symmetry: gauging a subalgebra joins together non-Higgsable clusters

Example: $\overset{\mathfrak{su}_3}{3} + 1 + \overset{\mathfrak{su}_3}{3} = \overset{\mathfrak{su}_3}{3}\,1\,\overset{\mathfrak{su}_3}{3}$

- $\mathfrak{su}_3 \oplus \mathfrak{su}_3 \subset \mathfrak{e}_8$ ✓
- Intersection matrix is negative definite ✓
- Fibers are minimal ✓

How to Build a 6d SCFT

Summary

[Heckman, Morrison, Vafa]
[Heckman, Morrison, Rudelius, Vafa]

Combinatorics of combining building blocks via gauging

-> algorithmic to write down billions of 6d (1,0) SCFTs

In fact, they are all arranged into
a small number of infinite families
[Heckman, Rudelius, Tomasiello]

Families of 6d SCFTs: Higgs Branch RG-flows

- Complicated tensor branch configurations → can be neatly ordered via Higgs branch flows from progenitor theories

[Heckman, Morrison, Rudelius, Vafa], [Heckman, Rudelius, Tomasiello], …

- The rank N $(\mathfrak{g}, \mathfrak{g})$ conformal matter theories [del Zotto, Heckman, Tomasiello, Vafa]

$$\underbrace{\overset{\mathfrak{g}}{2}\cdots\overset{\mathfrak{g}}{2}}_{N-1}$$

← Worldvolume of N M5-branes probing a $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$ orbifold

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- Theories have an $\mathfrak{g} \oplus \mathfrak{g}$ flavor symmetry

- Each $\mathfrak{g}$ can be Higgsed by a choice of nilpotent orbit of $\mathfrak{g}$

$$A_{N-1}^{\mathfrak{g}}(O_L, O_R)$$

Green--Schwarz Automorphisms

6d (1,0) SCFTs have discrete global symmetry Γ if Γ is a "Green--Schwarz automorphism" of the corresponding Calabi--Yau

[Apruzzi, Heckman, Rudelius]

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$A^{\mathfrak{g}}_{N-1}(O_L, O_R)$ is

$$\begin{array}{ccccccccccc} \mathfrak{su}(k_1) & \mathfrak{su}(k_2) & & \mathfrak{su}(k_K) & \mathfrak{su}(K) & & \mathfrak{su}(K) & \mathfrak{su}(k'_K) & & \mathfrak{su}(k'_2) & \mathfrak{su}(k'_1) \\ 2 & 2 & \cdots & 2 & 2 & \cdots & 2 & 2 & \cdots & 2 & 2 \\ {[m_1]} & {[m_2]} & & {[m_K]} & & & & {[m'_K]} & & {[m'_2]} & {[m'_1]} \end{array}$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{N-2K-1}$$

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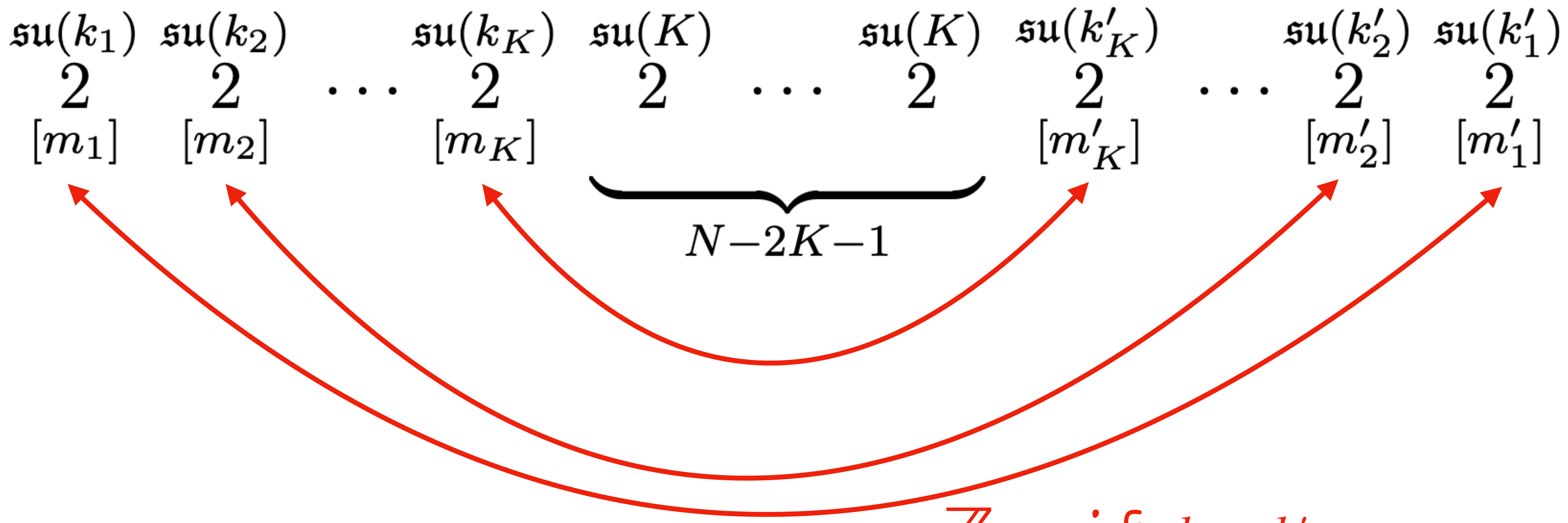

$\mathbb{Z}_2$ if $k_i = k'_i$

Green--Schwarz Automorphisms

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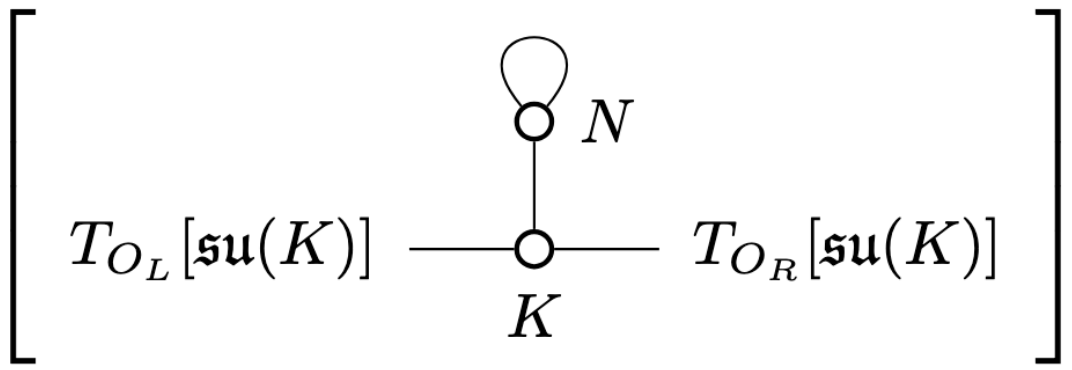
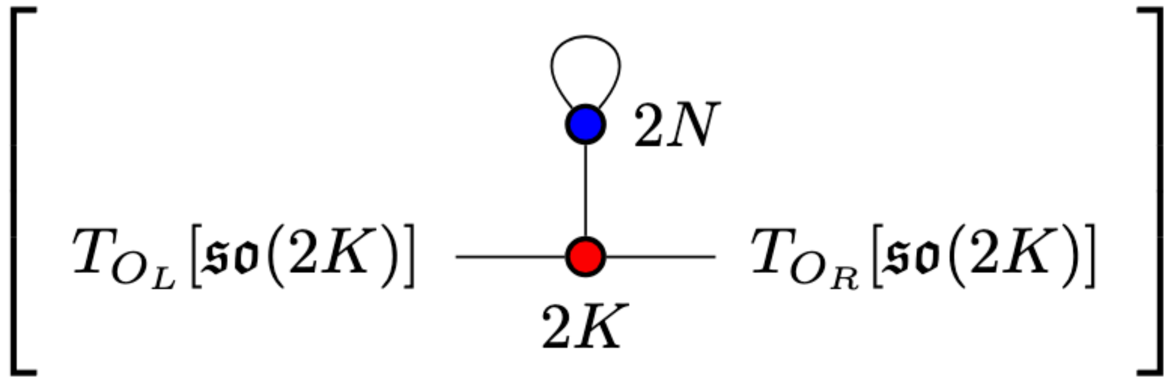
the magnetic quivers for the Higgs branch of conformal matter are

$$\mathrm{HB}\left[A_{N-1}^{\mathfrak{su}(K)}(O_L,O_R)\right] \cong \mathrm{CB}$$



$$\mathrm{HB}\left[A_{N-1}^{\mathfrak{so}(2K)}(O_L,O_R)\right] \cong \mathrm{CB}$$



[Hanany, Sperling], [Hanany, Zafrir]

first hint: $\mathbb{Z}_2$ GS automorphism $\Rightarrow$ $O_L = O_R$

$\Rightarrow$ magnetic quiver admits $\mathbb{Z}_2$ wreathing

Proposal

let $\widetilde{\mathscr{T}}$ be obtained from conformal matter, $\mathscr{T}$, via gauging $\mathbb{Z}_2$

proposal: $\widetilde{\mathscr{T}}_M = \mathscr{T}_M \wr \Gamma$

check this proposal by:

1) conjecture flavor symmetry of $\widetilde{\mathscr{T}}$ by acting with $\mathbb{Z}_2$ on Higgs branch chiral ring generators
2) compute Coulomb branch Hilbert series of $\mathscr{T}_M \wr \mathbb{Z}_2$

→ they are consistent!

Flavor Algebras of a 6d SCFT

- Two origins:

“E-string flavor”

“Classical flavor”

Flavor Algebras of a 6d SCFT

Classical flavor

k hypermultiplets in representation $\mathbf{R}$ rotated by flavor algebra

$$\mathfrak{f} = \begin{cases} \mathfrak{su}_k & \text{if } \mathbf{R} \text{ is complex} \\ \mathfrak{so}_{2k} & \text{if } \mathbf{R} \text{ is quaternionic} \\ \mathfrak{usp}_{2k} & \text{if } \mathbf{R} \text{ is real} \end{cases}$$

Example:

Anomaly cancellation -> $\overset{\mathfrak{su}_2}{1}$ has 10 fundamental hypermultiplets

$\longrightarrow$ $\mathfrak{f} = \mathfrak{so}_{20}$ $\frac{1}{2}(\mathbf{2}, \mathbf{20})$

Flavor Algebras of a 6d SCFT

E-string flavor

Tensor branch:

$$\cdots \overset{\mathfrak{g}_L}{n_L} \; 1 \; \overset{\mathfrak{g}_R}{n_R} \cdots$$

Subalgebra $\mathfrak{g}_L \oplus \mathfrak{g}_R$ of E-string flavor symmetry is gauged:

$$\rho : \mathfrak{g}_L \oplus \mathfrak{g}_R \to \mathfrak{e}_8$$

Residual flavor symmetry: $\mathfrak{f} \supseteq \text{Commutant}(\rho, \mathfrak{e}_8, \mathfrak{g}_L \oplus \mathfrak{g}_R)$

Example: $1 \; \overset{\mathfrak{su}_3}{3} \; 1$ has $\mathfrak{f} = \mathfrak{e}_6 \oplus \mathfrak{e}_6$

Flavor Algebras of a 6d SCFT

E-string flavor: a subtlety

[Distler, Kang, CL]

- Consider the tensor branch:

$$\overset{\mathfrak{so}_7}{3}\ 1\ \overset{\mathfrak{so}_7}{3}$$

- There are two inequivalent embeddings $\mathfrak{so}_7 \oplus \mathfrak{so}_7 \subset \mathfrak{e}_8$

- They have different commutants!

$$\text{Commutant}(\rho_1, \mathfrak{e}_8, \mathfrak{so}_7 \oplus \mathfrak{so}_7) = \mathfrak{u}_1 \qquad \text{Commutant}(\rho_2, \mathfrak{e}_8, \mathfrak{so}_7 \oplus \mathfrak{so}_7) = \varnothing$$

- $\overset{\mathfrak{so}_7}{3}\ 1\ \overset{\mathfrak{so}_7}{3}$ is TWO 6d (1,0) SCFTs with different flavor symmetry

On T^2 both are in class $\mathcal{S}$ of type $\mathfrak{so}_8$ with pairs of *very even* punctures

Flavor Algebras of a 6d SCFT

Example

$$1\,2\,\overset{\mathfrak{su}_2}{2}\,\overset{\mathfrak{g}_2}{3}\,1\,\overset{\mathfrak{g}_2}{3}$$

Flavor Algebras of a 6d SCFT

Example

$$[\mathfrak{e}_8]\ 1\ 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ \underset{[\mathfrak{su}_2]}{1}\ \overset{\mathfrak{g}_2}{3}\ [\mathfrak{usp}_2]$$

E-string flavor

Anomaly cancellation:
1 fundamental hyper of $\mathfrak{g}_2$
-> classical flavor

Flavor Symmetry after Discrete Gauging

we need to conjecture the flavor symmetry in $\widetilde{\mathscr{T}}$

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(A, A) $\quad \overset{\mathfrak{su}(k_1)}{\underset{[m_1]}{2}} \; \overset{\mathfrak{su}(k_2)}{\underset{[m_2]}{2}} \; \cdots \; \overset{\mathfrak{su}(k_q)}{\underset{[m_q]}{2}} \; \overset{\mathfrak{su}(k_{q+1})}{\underset{[m_{q+1}]}{2}} \; \overset{\mathfrak{su}(k_q)}{\underset{[m_q]}{2}} \; \cdots \; \overset{\mathfrak{su}(k_2)}{\underset{[m_2]}{2}} \; \overset{\mathfrak{su}(k_1)}{\underset{[m_1]}{2}}$

classical flavor symmetry: $\mathfrak{su}(m_{q+1}) \oplus \bigoplus_{i=1}^{q} \mathfrak{su}(m_i)^2 \oplus \mathfrak{u}(1)^\ell$

don't discuss $\mathfrak{u}(1)$s today due to ABJ anomalies

[Apruzzi, Fazzi, Heckman, Rudelius, Zhang]
[Lee, Regalado, Weigand]

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classical flavor symmetry: $\mathfrak{su}(m_{q+1}) \oplus \bigoplus_{i=1}^{q} \mathfrak{su}(m_i)^2 \oplus \mathfrak{u}(1)^\ell$

$$\mathfrak{su}(m_{q+1}) \supset \mathfrak{su}\left(\frac{m_{q+1}}{2}\right) \oplus \mathfrak{su}\left(\frac{m_{q+1}}{2}\right)$$

$\mathbb{Z}_2$ identifies these moment maps

$\mathbb{Z}_2$ identifies left and right moment maps

$$\bigoplus_{i=1}^{q} \mathfrak{su}(m_i)$$

what happens to the additional moment maps in the adjoint of $\mathfrak{su}(m_{q+1})$?

Flavor Symmetry after Discrete Gauging

we need to conjecture the flavor symmetry in $\mathscr{T}$

(A, A)

$$\begin{array}{ccccccccc} \mathfrak{su}(k_1) & \mathfrak{su}(k_2) & & \mathfrak{su}(k_q) & \mathfrak{su}(k_{q+1}) & \mathfrak{su}(k_q) & & \mathfrak{su}(k_2) & \mathfrak{su}(k_1) \\ 2 & 2 & \cdots & 2 & 2 & 2 & \cdots & 2 & 2 \\ [m_1] & [m_2] & & [m_q] & [m_{q+1}] & [m_q] & & [m_2] & [m_1] \end{array}$$

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(A, A)

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$\mathbb{Z}_2$ identifies these moment maps

$$\mathfrak{su}(m_{q+1}) \;\to\; \mathfrak{su}\left(\frac{m_{q+1}}{2}\right) \oplus \mathfrak{su}\left(\frac{m_{q+1}}{2}\right)$$

$$\mathbf{adj} \;\to\; (\mathbf{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{adj}) \oplus (\mathbf{1}, \mathbf{1}) \oplus \left(\frac{\boldsymbol{m_{q+1}}}{\mathbf{2}}, \overline{\frac{\boldsymbol{m_{q+1}}}{\mathbf{2}}}\right) \oplus \left(\overline{\frac{\boldsymbol{m_{q+1}}}{\mathbf{2}}}, \frac{\boldsymbol{m_{q+1}}}{\mathbf{2}}\right)$$

what happens to the additional moment maps in the adjoint of $\mathfrak{su}(m_{q+1})$?

$\Rightarrow \mathfrak{su}\left(\frac{m_{q+1}}{2}\right)$ with moment maps in the

$$\mathbf{adj} \oplus \mathbf{Sym}^2 \oplus \overline{\mathbf{Sym}^2} \oplus \mathbf{1}$$

$\Rightarrow \mathfrak{usp}(m_{q+1})$ enhanced flavor!

Flavor Symmetry after Discrete Gauging

we need to conjecture the flavor symmetry in $\widetilde{\mathscr{T}}$

(A, A)

$$\begin{array}{ccccccccc} \mathfrak{su}(k_1) & \mathfrak{su}(k_2) & & \mathfrak{su}(k_q) & \mathfrak{su}(k_{q+1}) & \mathfrak{su}(k_q) & & \mathfrak{su}(k_2) & \mathfrak{su}(k_1) \\ 2 & 2 & \cdots & 2 & 2 & 2 & \cdots & 2 & 2 \\ {[m_1]} & {[m_2]} & & {[m_q]} & {[m_{q+1}]} & {[m_q]} & & {[m_2]} & {[m_1]} \end{array}$$

classical flavor symmetry: $\mathfrak{su}(m_{q+1}) \oplus \bigoplus_{i=1}^{q} \mathfrak{su}(m_i)^2 \oplus \mathfrak{u}(1)^\ell$

classical flavor symmetry after discrete gauging: $\mathfrak{usp}(m_{q+1}) \oplus \bigoplus_{i=1}^{q} \mathfrak{su}(m_i)$

(ignoring $\mathfrak{u}(1)$s)

Flavor Symmetry after Discrete Gauging

we can do the same thing for (D, D) conformal matter

$$\overset{\mathfrak{usp}(2k_1)}{\underset{[m_1]}{1}}\ \overset{\mathfrak{so}(k_2)}{\underset{[m_2]}{4}}\ \overset{\mathfrak{usp}(2k_3)}{\underset{[m_3]}{1}}\ \cdots\ \overset{\mathfrak{so}(k_{q-1})}{\underset{[m_{q-1}]}{4}}\ \overset{\mathfrak{usp}(2k_q)}{\underset{[m_q]}{1}}\ \overset{\mathfrak{so}(k'_{q-1})}{\underset{[m_{q-1}]}{4}}\ \cdots\ \overset{\mathfrak{usp}(2k_3)}{\underset{[m_3]}{1}}\ \overset{\mathfrak{so}(k_2)}{\underset{[m_2]}{4}}\ \overset{\mathfrak{usp}(2k_1)}{\underset{[m_1]}{1}}$$

Flavor Symmetry after Discrete Gauging

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$$\overset{\mathfrak{usp}(2k_1)}{\underset{[m_1]}{1}}\ \overset{\mathfrak{so}(k_2)}{\underset{[m_2]}{4}}\ \overset{\mathfrak{usp}(2k_3)}{\underset{[m_3]}{1}}\ \cdots\ \overset{\mathfrak{so}(k_{q-1})}{\underset{[m_{q-1}]}{4}}\ \overset{\mathfrak{usp}(2k_q)}{\underset{[m_q]}{1}}\ \overset{\mathfrak{so}(k'_{q-1})}{\underset{[m_{q-1}]}{4}}\ \cdots\ \overset{\mathfrak{usp}(2k_3)}{\underset{[m_3]}{1}}\ \overset{\mathfrak{so}(k_2)}{\underset{[m_2]}{4}}\ \overset{\mathfrak{usp}(2k_1)}{\underset{[m_1]}{1}}$$

$\mathbb{Z}_2$ identifies these moment maps

$$\bigoplus_{i=1}^{q-1} \mathfrak{f}_i(m_i)^{\oplus 2} \quad \to \quad \bigoplus_{i=1}^{q-1} \mathfrak{f}_i(m_i)$$

$$\mathfrak{f}_i(m_i) = \begin{cases} \mathfrak{so}(m_i) & \text{if } i \text{ odd,} \\ \mathfrak{usp}(m_i) & \text{if } i \text{ even} \end{cases}$$

Flavor Symmetry after Discrete Gauging

we can do the same thing for (D, D) conformal matter

$$\underset{[m_1]}{\overset{\mathfrak{usp}(2k_1)}{1}}\ \underset{[m_2]}{\overset{\mathfrak{so}(k_2)}{4}}\ \underset{[m_3]}{\overset{\mathfrak{usp}(2k_3)}{1}}\ \cdots\ \underset{[m_{q-1}]}{\overset{\mathfrak{so}(k_{q-1})}{4}}\ \underset{[m_q]}{\overset{\mathfrak{usp}(2k_q)}{1}}\ \underset{[m_{q-1}]}{\overset{\mathfrak{so}(k'_{q-1})}{4}}\ \cdots\ \underset{[m_3]}{\overset{\mathfrak{usp}(2k_3)}{1}}\ \underset{[m_2]}{\overset{\mathfrak{so}(k_2)}{4}}\ \underset{[m_1]}{\overset{\mathfrak{usp}(2k_1)}{1}}$$

the only subtle case is

what happens on the central (-1)-curve?

$$\mathfrak{so}\left(\frac{m_q}{2}\right) \oplus \mathfrak{so}\left(\frac{m_q}{2}\right) \quad \rightarrow \quad \mathfrak{so}(m_q)$$

$\mathbb{Z}_2$ identifies these moment maps

Flavor Symmetry after Discrete Gauging

we can do the same thing for (D, D) conformal matter

$$\underset{[m_1]}{\overset{\mathfrak{usp}(2k_1)}{1}}\ \underset{[m_2]}{\overset{\mathfrak{so}(k_2)}{4}}\ \underset{[m_3]}{\overset{\mathfrak{usp}(2k_3)}{1}}\ \cdots\ \underset{[m_{q-1}]}{\overset{\mathfrak{so}(k_{q-1})}{4}}\ \underset{[m_q]}{\overset{\mathfrak{usp}(2k_q)}{1}}\ \underset{[m_{q-1}]}{\overset{\mathfrak{so}(k'_{q-1})}{4}}\ \cdots\ \underset{[m_3]}{\overset{\mathfrak{usp}(2k_3)}{1}}\ \underset{[m_2]}{\overset{\mathfrak{so}(k_2)}{4}}\ \underset{[m_1]}{\overset{\mathfrak{usp}(2k_1)}{1}}$$

the only subtle case is

what happens on the central (-1)-curve?

$$\mathfrak{so}\left(\frac{m_q}{2}\right) \oplus \mathfrak{so}\left(\frac{m_q}{2}\right) \rightarrow \mathfrak{so}(m_q)$$

$\mathbb{Z}_2$ identifies these moment maps

$$\mathfrak{so}(m_{q+1}) \rightarrow \mathfrak{so}\left(\frac{m_q}{2}\right) \oplus \mathfrak{so}\left(\frac{m_q}{2}\right)$$

$$\mathbf{adj} \rightarrow (\mathbf{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{adj}) \oplus \left(\frac{\boldsymbol{m_q}}{\mathbf{2}}, \frac{\boldsymbol{m_q}}{\mathbf{2}}\right)$$

$\Rightarrow \mathfrak{so}\left(\frac{m_{q+1}}{2}\right)$ with moment maps in the

$$\mathbf{adj} \oplus \mathbf{Sym}^2 \oplus \mathbf{1}$$

Flavor Symmetry after Discrete Gauging

we can do the same thing for (D, D) conformal matter

$$\begin{array}{ccccccccccccc} \mathfrak{usp}(2k_1) & \mathfrak{so}(k_2) & \mathfrak{usp}(2k_3) & & \mathfrak{so}(k_{q-1}) & \mathfrak{usp}(2k_q) & \mathfrak{so}(k'_{q-1}) & & \mathfrak{usp}(2k_3) & \mathfrak{so}(k_2) & \mathfrak{usp}(2k_1) \\ 1 & 4 & 1 & \cdots & 4 & 1 & 4 & \cdots & 1 & 4 & 1 \\ [m_1] & [m_2] & [m_3] & & [m_{q-1}] & [m_q] & [m_{q-1}] & & [m_3] & [m_2] & [m_1] \end{array}$$

the only subtle case is

what happens on the central (-1)-curve?

$$\mathfrak{so}(m_{q+1}) \to \mathfrak{so}\left(\frac{m_q}{2}\right) \oplus \mathfrak{so}\left(\frac{m_q}{2}\right)$$

$$\mathbf{adj} \to (\mathbf{adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{adj}) \oplus \left(\frac{\boldsymbol{m_q}}{\mathbf{2}}, \frac{\boldsymbol{m_q}}{\mathbf{2}}\right)$$

$$\mathfrak{so}\left(\frac{m_q}{2}\right) \oplus \mathfrak{so}\left(\frac{m_q}{2}\right) \to \mathfrak{so}(m_q)$$

$\mathbb{Z}_2$ identifies these moment maps

$\Rightarrow \mathfrak{so}\left(\frac{m_{q+1}}{2}\right)$ with moment maps in the $\mathbf{adj} \oplus \mathbf{Sym}^2 \oplus \mathbf{1}$

$\Rightarrow \mathfrak{u}(m_{q+1})$ enhanced flavor!

Flavor Symmetry after Discrete Gauging

in this way: we propose the flavor symmetry for almost all discretely-gauged Higgsed (A, A) and (D, D) conformal matter

(there are precisely 3 examples which require a more subtle analysis)

Higgsed conformal matter where the magnetic quiver is "sufficiently small"

#	Conformal Matter	Tensor Branch	$\mathfrak{f}$	$\mathfrak{f}_{\mathbb{Z}_2}$
1	$A_3^{\mathfrak{so}(6)}([3,1^3],[3,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(4)$	$\mathfrak{usp}(4)$
2	$A_1^{\mathfrak{so}(6)}([1^6],[1^6])$	$\overset{\mathfrak{su}(4)}{2}$	$\mathfrak{su}(8)$	$\mathfrak{usp}(8)$
3	$A_5^{\mathfrak{so}(6)}([3^2],[3^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
4	$A_2^{\mathfrak{so}(8)}([3^2,1^2],[3^2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{su}(3)^{\oplus 2}$	?
5	$A_4^{\mathfrak{so}(8)}([5,1^3],[5,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ 1\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
6	$A_4^{\mathfrak{so}(8)}([5,3],[5,3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)$	?
7	$A_3^{\mathfrak{so}(8)}([4,2,1^2],[4,2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{so}(8)}{4}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{so}(2)^{\oplus 4}$	$\mathfrak{so}(2)^{\oplus 2}$
8	$A_2^{\mathfrak{so}(10)}([3^3,1],[3^3,1])$	$\overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{g}_2}{3}$	$\mathfrak{so}(6)$	$\mathfrak{u}(3)$
9	$A_4^{\mathfrak{so}(10)}([5,3,1^2],[5,3,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{u}(1)^{\oplus 3}$	$\mathfrak{u}(1)^{\oplus 2}$
10	$A_4^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(4)$	$\mathfrak{u}(2)$
11	$A_3^{\mathfrak{so}(10)}([4,3,2,1],[4,3,2,1])$	$\overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(10)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{g}_2}{3}$	$\mathfrak{su}(2)^{\oplus 2}$	$\mathfrak{su}(2)$
12	$A_5^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(10)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(2)^{\oplus 2}$	$\mathfrak{so}(2)$
13	$A_6^{\mathfrak{so}(10)}([7,1^3],[7,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
14	$A_6^{\mathfrak{so}(10)}([7,3],[7,3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{u}(1)$	$\mathfrak{u}(1)$
15	$A_1^{\mathfrak{su}(3)}([1^3],[1^3])$	$\overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(6)$	$\mathfrak{usp}(6)$
16	$A_3^{\mathfrak{su}(3)}([2,1],[2,1])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
17	$A_3^{\mathfrak{su}(4)}([2,1^2],[2,1^2])$	$\overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(2)^{\oplus 3}\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$

Monopole Formula for Wreathed Quivers

Given a 3d quiver $\mathcal{X}$, the Coulomb branch Hilbert series is

[Cremonesi, Hanany, Zaffaroni]

$$\mathrm{HS}[\mathrm{CB\ of\ }\mathcal{X}](t) = \frac{1}{|W|} \sum_{\mathbf{m}} \sum_{\gamma \in W(\mathbf{m})} \frac{t^{2\Delta(\mathbf{m})}}{\det(\mathbb{1} - t^2 \gamma)}$$

Weyl group of gauge group

conformal dimension of monopole operator with magnetic flux $\mathbf{m}$

$$\Delta(\mathbf{m}) = -\sum_{\alpha \in \Delta_+} |\alpha(\mathbf{m})| + \frac{1}{2} \sum_{i=1}^{N_f} \sum_{\rho_i \in \mathcal{R}_i} |\rho_i(\mathbf{m})|$$

vectors hypers

prefactor $$P_G(t, \mathbf{m}) = \frac{1}{|W|} \sum_{\gamma \in W(\mathbf{m})} \frac{1}{\det(\mathbb{1} - t^2 \gamma)}$$

counts gauge-invariant operators of gauge group unbroken by $\mathbf{m}$

Monopole Formula for Wreathed Quivers

Given a wreathed 3d quiver $\widetilde{\mathcal{X}}$, the Coulomb branch Hilbert series is

[Bourget, Hanany, Miketa]

$$\mathrm{HS}\left[\mathrm{CB\ of\ }\widetilde{\mathcal{X}}\right](t) = \frac{1}{|W_\Gamma|}\sum_{\mathbf{n}}\sum_{\gamma\in W_\Gamma(\mathbf{n})}\frac{t^{2\Delta(\mathbf{n})}}{\det(\mathbb{1}-t^2\gamma)}$$

wreathed Weyl group $W \wr \Gamma$

conformal dimension of monopole operator with magnetic flux $\mathbf{n}$

wreathed prefactor $P_{G\wr\Gamma}(t,\mathbf{n}) = \frac{1}{|W_\Gamma(\mathbf{n})|}\sum_{\gamma\in W_\Gamma(\mathbf{n})}\frac{1}{\det\left(\mathbb{1}-t^2\gamma\right)}$

prefactors for $\mathbb{Z}_2$ wreathing involving $U(K)$, $SO(K)$, and $USp(2K)$ nodes

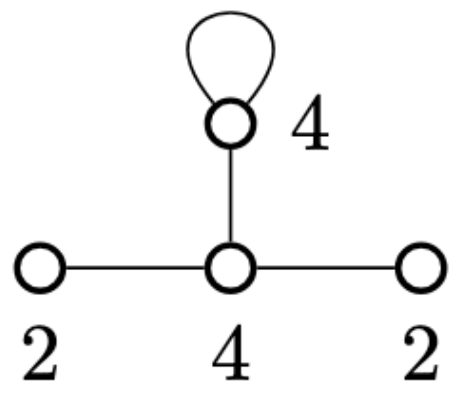
[CL, Lepper, Mininno]

Monopole Formula for Wreathed Quivers

let's consider a unitary example [CL, Lepper, Mininno]

Monopole Formula for Wreathed Quivers

let's consider a unitary example [CL, Lepper, Mininno]



has

$$\mathrm{HS}(t,x,y,z) =$$

$$\mathrm{PE}\left[t^2\left(1+\chi^{\mathfrak{su}(2)}_{[2]}(x)+\chi^{\mathfrak{su}(2)}_{[2]}(y)+\left(z^2+\frac{1}{z^2}\right)\chi^{\mathfrak{su}(2)}_{[1]}(x)\chi^{\mathfrak{su}(2)}_{[1]}(y)\right)+\mathcal{O}(t^4)\right]$$

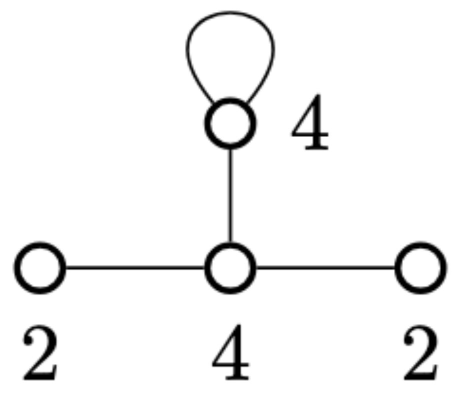
this is the adjoint representation of $\mathfrak{su}(4)$ under

$$\mathfrak{su}(4) \to \mathfrak{su}(2)_x \oplus \mathfrak{su}(2)_y \oplus \mathfrak{u}(1)_z\,,$$

$$\mathbf{15} \to (\mathbf{2},\mathbf{2})_2 \oplus (\mathbf{3},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{2},\mathbf{2})_{-2}$$

Monopole Formula for Wreathed Quivers

let's consider a unitary example [CL, Lepper, Mininno]



has

$$\mathrm{HS}(t,x,y,z) = \mathrm{PE}\left[t^2\left(1+\chi^{\mathfrak{su}(2)}_{[2]}(x)+\chi^{\mathfrak{su}(2)}_{[2]}(y)+\left(z^2+\frac{1}{z^2}\right)\chi^{\mathfrak{su}(2)}_{[1]}(x)\chi^{\mathfrak{su}(2)}_{[1]}(y)\right)+\mathcal{O}(t^4)\right]$$

this is the adjoint representation of $\mathfrak{su}(4)$ under

$$\mathfrak{su}(4) \to \mathfrak{su}(2)_x \oplus \mathfrak{su}(2)_y \oplus \mathfrak{u}(1)_z\,,$$
$$\mathbf{15} \to (\mathbf{2},\mathbf{2})_2 \oplus (\mathbf{3},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{2},\mathbf{2})_{-2}$$

wreathing the $\mathfrak{u}(2)$s gives

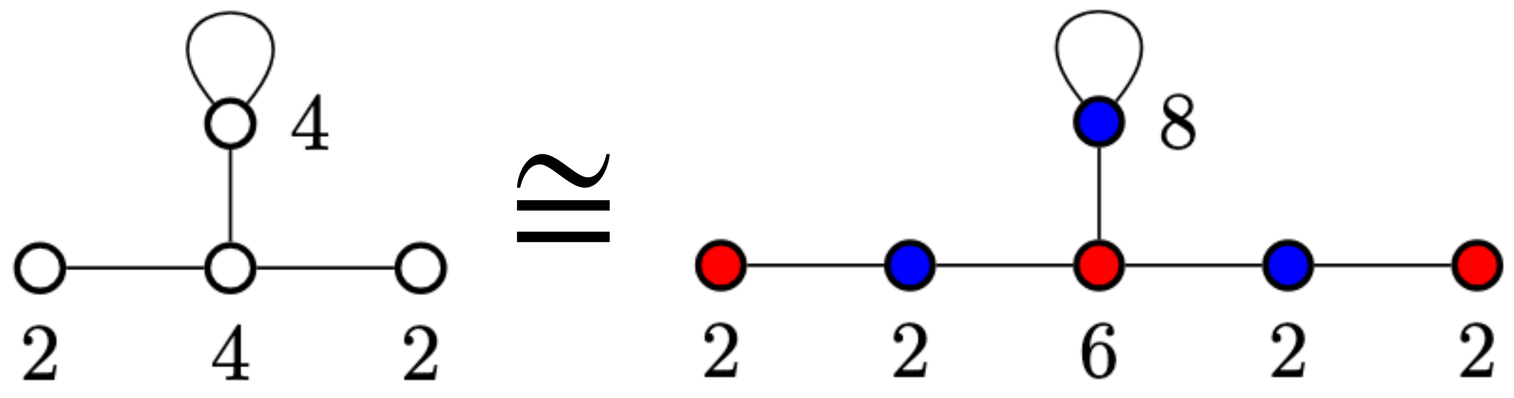
$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,x,z) = \mathrm{PE}\left[t^2\left(1+\chi^{\mathfrak{su}(2)}_{[2]}(x)+\left(z^2+\frac{1}{z^2}\right)\chi^{\mathfrak{su}(2)}_{[2]}(x)\right)+\mathcal{O}(t^4)\right]$$

this is the adjoint representation of $\mathfrak{usp}(4)$ under

$$\mathfrak{usp}(4) \to \mathfrak{su}(2) \oplus \mathfrak{u}(1)\,,$$
$$\mathbf{10} \to \mathbf{3}_2 \oplus \mathbf{3}_{-2} \oplus \mathbf{3}_0 \oplus \mathbf{1}_0$$

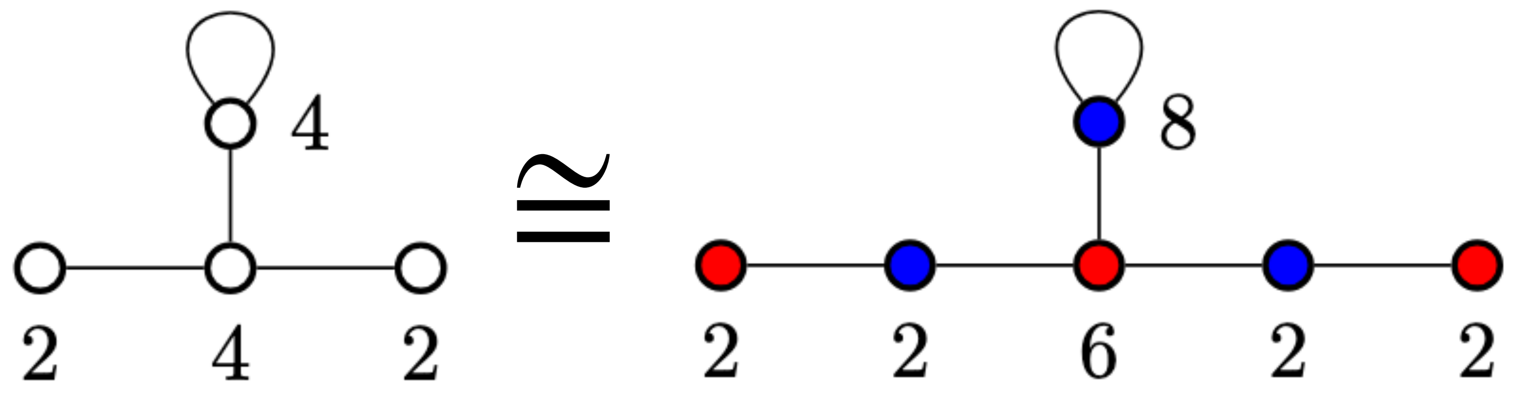
Monopole Formula for Wreathed Quivers

using $\mathfrak{su}(4) \cong \mathfrak{so}(6)$ we also have an orthosymplectic description [CL, Lepper, Mininno]



Monopole Formula for Wreathed Quivers

using $\mathfrak{su}(4) \cong \mathfrak{so}(6)$ we also have an orthosymplectic description [CL, Lepper, Mininno]



unwreathed Hilbert series: $\mathrm{PE}\left[t^2(7+8\omega) + t^4(13+16\omega) + \mathcal{O}(t^6)\right]$

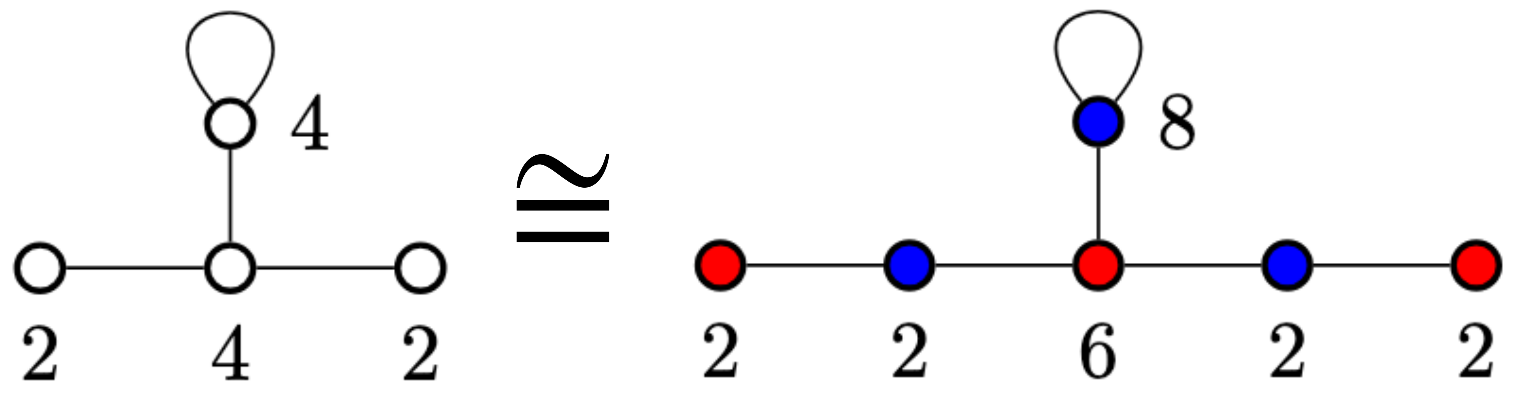
dimension of $\mathfrak{su}(4)$

wreathed Hilbert series: $\mathrm{PE}\left[t^2(4+6\omega) + t^4(19+18\omega) + \mathcal{O}(t^6)\right]$

dimension of $\mathfrak{usp}(4)$

Monopole Formula for Wreathed Quivers

using $\mathfrak{su}(4) \cong \mathfrak{so}(6)$ we also have an orthosymplectic description [CL, Lepper, Mininno]



unwreathed Hilbert series: $\mathrm{PE}\left[t^2(7+8\omega)+t^4(13+16\omega)+\mathcal{O}(t^6)\right]$

dimension of $\mathfrak{su}(4)$

wreathed Hilbert series: $\mathrm{PE}\left[t^2(4+6\omega)+t^4(19+18\omega)+\mathcal{O}(t^6)\right]$

dimension of $\mathfrak{usp}(4)$

but this is precisely the magnetic quiver for a conformal matter theory: $A_3^{\mathfrak{so}(6)}([3,1^3],[3,1^3])$!

#	Conformal Matter	Tensor Branch	$\mathfrak{f}$	$\mathfrak{f}_{\mathbb{Z}_2}$
1	$A_3^{\mathfrak{so}(6)}([3,1^3],[3,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(4)$	$\mathfrak{usp}(4)$
2	$A_1^{\mathfrak{so}(6)}([1^6],[1^6])$	$\overset{\mathfrak{su}(4)}{2}$	$\mathfrak{su}(8)$	$\mathfrak{usp}(8)$
3	$A_5^{\mathfrak{so}(6)}([3^2],[3^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
4	$A_2^{\mathfrak{so}(8)}([3^2,1^2],[3^2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{su}(3)^{\oplus 2}$	?
5	$A_4^{\mathfrak{so}(8)}([5,1^3],[5,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ 1\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
6	$A_4^{\mathfrak{so}(8)}([5,3],[5,3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)$	?
7	$A_3^{\mathfrak{so}(8)}([4,2,1^2],[4,2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{so}(8)}{4}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{so}(2)^{\oplus 4}$	$\mathfrak{so}(2)^{\oplus 2}$
8	$A_2^{\mathfrak{so}(10)}([3^3,1],[3^3,1])$	$\overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{g}_2}{3}$	$\mathfrak{so}(6)$	$\mathfrak{u}(3)$
9	$A_4^{\mathfrak{so}(10)}([5,3,1^2],[5,3,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{u}(1)^{\oplus 3}$	$\mathfrak{u}(1)^{\oplus 2}$
10	$A_4^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(4)$	$\mathfrak{u}(2)$
11	$A_3^{\mathfrak{so}(10)}([4,3,2,1],[4,3,2,1])$	$\overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(10)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{g}_2}{3}$	$\mathfrak{su}(2)^{\oplus 2}$	$\mathfrak{su}(2)$
12	$A_5^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(10)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(2)^{\oplus 2}$	$\mathfrak{so}(2)$
13	$A_6^{\mathfrak{so}(10)}([7,1^3],[7,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
14	$A_6^{\mathfrak{so}(10)}([7,3],[7,3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{u}(1)$	$\mathfrak{u}(1)$
15	$A_1^{\mathfrak{su}(3)}([1^3],[1^3])$	$\overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(6)$	$\mathfrak{usp}(6)$
16	$A_3^{\mathfrak{su}(3)}([2,1],[2,1])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
17	$A_3^{\mathfrak{su}(4)}([2,1^2],[2,1^2])$	$\overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(2)^{\oplus 3}\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$

✔ $\mathscr{T}_M = \mathscr{T}_M \wr \Gamma$ ✔

✓ $\mathscr{T}_M = \mathscr{T}_M \wr \Gamma$ ✓

#	Conformal Matter	Tensor Branch	$\mathfrak{f}$	$\mathfrak{f}_{\mathbb{Z}_2}$	
1	$A_3^{\mathfrak{so}(6)}([3,1^3],[3,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(4)$	$\mathfrak{usp}(4)$	
2	$A_1^{\mathfrak{so}(6)}([1^6],[1^6])$	$\overset{\mathfrak{su}(4)}{2}$	$\mathfrak{su}(8)$	$\mathfrak{usp}(8)$	
3	$A_5^{\mathfrak{so}(6)}([3^2],[3^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$	
4	$A_2^{\mathfrak{so}(8)}([3^2,1^2],[3^2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{su}(3)^{\oplus 2}$	?	
5	$A_4^{\mathfrak{so}(8)}([5,1^3],[5,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ 1\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$	$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega)=\mathrm{PE}\left[4t^2+t^4(6\omega+11)+\mathcal{O}(t^5)\right]$
6	$A_4^{\mathfrak{so}(8)}([5,3],[5,3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)$	?	
7	$A_3^{\mathfrak{so}(8)}([4,2,1^2],[4,2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{so}(8)}{4}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{so}(2)^{\oplus 4}$	$\mathfrak{so}(2)^{\oplus 2}$	$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega)=\mathrm{PE}\left[2t^2+t^4(15+6\omega)+\mathcal{O}(t^5)\right]$
8	$A_2^{\mathfrak{so}(10)}([3^3,1],[3^3,1])$	$\overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{g}_2}{3}$	$\mathfrak{so}(6)$	$\mathfrak{u}(3)$	$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega)=\mathrm{PE}\left[9t^2+6\omega t^3+40t^4+\mathcal{O}(t^5)\right]$
9	$A_4^{\mathfrak{so}(10)}([5,3,1^2],[5,3,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{u}(1)^{\oplus 3}$	$\mathfrak{u}(1)^{\oplus 2}$	$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega)=\mathrm{PE}\left[2t^2+9t^4+\mathcal{O}(t^5)\right]$
10	$A_4^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(4)$	$\mathfrak{u}(2)$	$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega)=\mathrm{PE}\left[4t^2+2\omega t^3+10t^4+\mathcal{O}(t^5)\right]$
11	$A_3^{\mathfrak{so}(10)}([4,3,2,1],[4,3,2,1])$	$\overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(10)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{g}_2}{3}$	$\mathfrak{su}(2)^{\oplus 2}$	$\mathfrak{su}(2)$	$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega)=\mathrm{PE}\left[3t^2+22t^4+\mathcal{O}(t^5)\right]$
12	$A_5^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(10)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(2)^{\oplus 2}$	$\mathfrak{so}(2)$	$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega)=\mathrm{PE}\left[t^2+8t^4+\mathcal{O}(t^5)\right]$
13	$A_6^{\mathfrak{so}(10)}([7,1^3],[7,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$	$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega)=\mathrm{PE}\left[4t^2+9t^4+\mathcal{O}(t^5)\right]$
14	$A_6^{\mathfrak{so}(10)}([7,3],[7,3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{u}(1)$	$\mathfrak{u}(1)$	$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega)=\mathrm{PE}\left[t^2+4t^4+\mathcal{O}(t^5)\right]$
15	$A_1^{\mathfrak{su}(3)}([1^3],[1^3])$	$\overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(6)$	$\mathfrak{usp}(6)$	
16	$A_3^{\mathfrak{su}(3)}([2,1],[2,1])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$	
17	$A_3^{\mathfrak{su}(4)}([2,1^2],[2,1^2])$	$\overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(2)^{\oplus 3}\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	

#	Conformal Matter	Tensor Branch	$\mathfrak{f}$	$\mathfrak{f}_{\mathbb{Z}_2}$
1	$A_3^{\mathfrak{so}(6)}([3,1^3],[3,1^3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{su}(4)}{2}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(4)$	$\mathfrak{usp}(4)$
2	$A_1^{\mathfrak{so}(6)}([1^6],[1^6])$	$\overset{\mathfrak{su}(4)}{2}$	$\mathfrak{su}(8)$	$\mathfrak{usp}(8)$
3	$A_5^{\mathfrak{so}(6)}([3^2],[3^2])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{su}(3)}{2}\;\overset{\mathfrak{su}(4)}{2}\;\overset{\mathfrak{su}(3)}{2}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
4	$A_2^{\mathfrak{so}(8)}([3^2,1^2],[3^2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\;1\;\overset{\mathfrak{su}(3)}{3}$	$\mathfrak{su}(3)^{\oplus 2}$	?
5	$A_4^{\mathfrak{so}(8)}([5,1^3],[5,1^3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{so}(7)}{3}\;1\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
6	$A_4^{\mathfrak{so}(8)}([5,3],[5,3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{g}_2}{3}\;1\;\overset{\mathfrak{g}_2}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)$	?
7	$A_3^{\mathfrak{so}(8)}([4,2,1^2],[4,2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\;1\;\overset{\mathfrak{so}(8)}{4}\;1\;\overset{\mathfrak{su}(3)}{3}$	$\mathfrak{so}(2)^{\oplus 4}$	$\mathfrak{so}(2)^{\oplus 2}$
8	$A_2^{\mathfrak{so}(10)}([3^3,1],[3^3,1])$	$\overset{\mathfrak{g}_2}{3}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{g}_2}{3}$	$\mathfrak{so}(6)$	$\mathfrak{u}(3)$
9	$A_4^{\mathfrak{so}(10)}([5,3,1^2],[5,3,1^2])$	$\overset{\mathfrak{su}(3)}{3}\;1\;\overset{\mathfrak{so}(9)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(9)}{4}\;1\;\overset{\mathfrak{su}(3)}{3}$	$\mathfrak{u}(1)^{\oplus 3}$	$\mathfrak{u}(1)^{\oplus 2}$
10	$A_4^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(4)$	$\mathfrak{u}(2)$
11	$A_3^{\mathfrak{so}(10)}([4,3,2,1],[4,3,2,1])$	$\overset{\mathfrak{g}_2}{3}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(10)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{g}_2}{3}$	$\mathfrak{su}(2)^{\oplus 2}$	$\mathfrak{su}(2)$
12	$A_5^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(10)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(2)^{\oplus 2}$	$\mathfrak{so}(2)$
13	$A_6^{\mathfrak{so}(10)}([7,1^3],[7,1^3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{so}(7)}{3}\;1\;\overset{\mathfrak{so}(9)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(9)}{4}\;1\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
14	$A_6^{\mathfrak{so}(10)}([7,3],[7,3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{g}_2}{3}\;1\;\overset{\mathfrak{so}(9)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(9)}{4}\;1\;\overset{\mathfrak{g}_2}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{u}(1)$	$\mathfrak{u}(1)$
15	$A_1^{\mathfrak{su}(3)}([1^3],[1^3])$	$\overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(6)$	$\mathfrak{usp}(6)$
16	$A_3^{\mathfrak{su}(3)}([2,1],[2,1])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{su}(3)}{2}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
17	$A_3^{\mathfrak{su}(4)}([2,1^2],[2,1^2])$	$\overset{\mathfrak{su}(3)}{2}\;\overset{\mathfrak{su}(4)}{2}\;\overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(2)^{\oplus 3}\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$

$$\mathcal{T}_M = \mathcal{T}_M \wr \Gamma$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[4t^2 + t^4(6\omega+11) + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[2t^2 + t^4(15+6\omega) + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[9t^2 + 6\omega t^3 + 40t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[2t^2 + 9t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[4t^2 + 2\omega t^3 + 10t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[3t^2 + 22t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[t^2 + 8t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[4t^2 + 9t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[t^2 + 4t^4 + \mathcal{O}(t^5)\right]$$

✔ $\widetilde{\mathcal{T}}_M = \mathcal{T}_M \wr \Gamma$ ✔

#	Conformal Matter	Tensor Branch	$\mathfrak{f}$	$\mathfrak{f}_{\mathbb{Z}_2}$
1	$A_3^{\mathfrak{so}(6)}([3,1^3],[3,1^3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{su}(4)}{2}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(4)$	$\mathfrak{usp}(4)$
2	$A_1^{\mathfrak{so}(6)}([1^6],[1^6])$	$\overset{\mathfrak{su}(4)}{2}$	$\mathfrak{su}(8)$	$\mathfrak{usp}(8)$
3	$A_5^{\mathfrak{so}(6)}([3^2],[3^2])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{su}(3)}{2}\;\overset{\mathfrak{su}(4)}{2}\;\overset{\mathfrak{su}(3)}{2}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
4	$A_2^{\mathfrak{so}(8)}([3^2,1^2],[3^2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\;1\;\overset{\mathfrak{su}(3)}{3}$	$\mathfrak{su}(3)^{\oplus 2}$	?
5	$A_4^{\mathfrak{so}(8)}([5,1^3],[5,1^3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{so}(7)}{3}\;1\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
6	$A_4^{\mathfrak{so}(8)}([5,3],[5,3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{g}_2}{3}\;1\;\overset{\mathfrak{g}_2}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)$	?
7	$A_3^{\mathfrak{so}(8)}([4,2,1^2],[4,2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\;1\;\overset{\mathfrak{so}(8)}{4}\;1\;\overset{\mathfrak{su}(3)}{3}$	$\mathfrak{so}(2)^{\oplus 4}$	$\mathfrak{so}(2)^{\oplus 2}$
8	$A_2^{\mathfrak{so}(10)}([3^3,1],[3^3,1])$	$\overset{\mathfrak{g}_2}{3}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{g}_2}{3}$	$\mathfrak{so}(6)$	$\mathfrak{u}(3)$
9	$A_4^{\mathfrak{so}(10)}([5,3,1^2],[5,3,1^2])$	$\overset{\mathfrak{su}(3)}{3}\;1\;\overset{\mathfrak{so}(9)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(9)}{4}\;1\;\overset{\mathfrak{su}(3)}{3}$	$\mathfrak{u}(1)^{\oplus 3}$	$\mathfrak{u}(1)^{\oplus 2}$
10	$A_4^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(4)$	$\mathfrak{u}(2)$
11	$A_3^{\mathfrak{so}(10)}([4,3,2,1],[4,3,2,1])$	$\overset{\mathfrak{g}_2}{3}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(10)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{g}_2}{3}$	$\mathfrak{su}(2)^{\oplus 2}$	$\mathfrak{su}(2)$
12	$A_5^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(10)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(2)^{\oplus 2}$	$\mathfrak{so}(2)$
13	$A_6^{\mathfrak{so}(10)}([7,1^3],[7,1^3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{so}(7)}{3}\;1\;\overset{\mathfrak{so}(9)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(9)}{4}\;1\;\overset{\mathfrak{so}(7)}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
14	$A_6^{\mathfrak{so}(10)}([7,3],[7,3])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{g}_2}{3}\;1\;\overset{\mathfrak{so}(9)}{4}\;\overset{\mathfrak{su}(2)}{1}\;\overset{\mathfrak{so}(9)}{4}\;1\;\overset{\mathfrak{g}_2}{3}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{u}(1)$	$\mathfrak{u}(1)$
15	$A_1^{\mathfrak{su}(3)}([1^3],[1^3])$	$\overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(6)$	$\mathfrak{usp}(6)$
16	$A_3^{\mathfrak{su}(3)}([2,1],[2,1])$	$\overset{\mathfrak{su}(2)}{2}\;\overset{\mathfrak{su}(3)}{2}\;\overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
17	$A_3^{\mathfrak{su}(4)}([2,1^2],[2,1^2])$	$\overset{\mathfrak{su}(3)}{2}\;\overset{\mathfrak{su}(4)}{2}\;\overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(2)^{\oplus 3}\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[t^2(6\omega+5) + t^4\left(26\omega+29\right) + \mathcal{O}\left(t^5\right)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[4t^2 + t^4(6\omega+11) + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[t^2(2\omega+1) + t^4(6\omega+8) + \mathcal{O}\left(t^5\right)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[2t^2 + t^4(15+6\omega) + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[9t^2 + 6\omega t^3 + 40t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[2t^2 + 9t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[4t^2 + 2\omega t^3 + 10t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[3t^2 + 22t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[t^2 + 8t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[4t^2 + 9t^4 + \mathcal{O}(t^5)\right]$$

$$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[t^2 + 4t^4 + \mathcal{O}(t^5)\right]$$

#	Conformal Matter	Tensor Branch	$\mathfrak{f}$	$\mathfrak{f}_{\mathbb{Z}_2}$
1	$A_3^{\mathfrak{so}(6)}([3,1^3],[3,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(4)$	$\mathfrak{usp}(4)$
2	$A_1^{\mathfrak{so}(6)}([1^6],[1^6])$	$\overset{\mathfrak{su}(4)}{2}$	$\mathfrak{su}(8)$	$\mathfrak{usp}(8)$
3	$A_5^{\mathfrak{so}(6)}([3^2],[3^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
4	$A_2^{\mathfrak{so}(8)}([3^2,1^2],[3^2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{su}(3)^{\oplus 2}$	?
5	$A_4^{\mathfrak{so}(8)}([5,1^3],[5,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ 1\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
6	$A_4^{\mathfrak{so}(8)}([5,3],[5,3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)$	?
7	$A_3^{\mathfrak{so}(8)}([4,2,1^2],[4,2,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{so}(8)}{4}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{so}(2)^{\oplus 4}$	$\mathfrak{so}(2)^{\oplus 2}$
8	$A_2^{\mathfrak{so}(10)}([3^3,1],[3^3,1])$	$\overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{g}_2}{3}$	$\mathfrak{so}(6)$	$\mathfrak{u}(3)$
9	$A_4^{\mathfrak{so}(10)}([5,3,1^2],[5,3,1^2])$	$\overset{\mathfrak{su}(3)}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{su}(3)}{3}$	$\mathfrak{u}(1)^{\oplus 3}$	$\mathfrak{u}(1)^{\oplus 2}$
10	$A_4^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(4)$	$\mathfrak{u}(2)$
11	$A_3^{\mathfrak{so}(10)}([4,3,2,1],[4,3,2,1])$	$\overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(10)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{g}_2}{3}$	$\mathfrak{su}(2)^{\oplus 2}$	$\mathfrak{su}(2)$
12	$A_5^{\mathfrak{so}(10)}([5^2],[5^2])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(10)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{so}(2)^{\oplus 2}$	$\mathfrak{so}(2)$
13	$A_6^{\mathfrak{so}(10)}([7,1^3],[7,1^3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{so}(7)}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{so}(7)}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
14	$A_6^{\mathfrak{so}(10)}([7,3],[7,3])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}(9)}{4}\ \overset{\mathfrak{su}(2)}{1}\ \overset{\mathfrak{so}(9)}{4}\ 1\ \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{u}(1)$	$\mathfrak{u}(1)$
15	$A_1^{\mathfrak{su}(3)}([1^3],[1^3])$	$\overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(6)$	$\mathfrak{usp}(6)$
16	$A_3^{\mathfrak{su}(3)}([2,1],[2,1])$	$\overset{\mathfrak{su}(2)}{2}\ \overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(2)}{2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)\oplus\mathfrak{u}(1)$
17	$A_3^{\mathfrak{su}(4)}([2,1^2],[2,1^2])$	$\overset{\mathfrak{su}(3)}{2}\ \overset{\mathfrak{su}(4)}{2}\ \overset{\mathfrak{su}(3)}{2}$	$\mathfrak{su}(2)^{\oplus 3}\oplus\mathfrak{u}(1)^{\oplus 2}$	$\mathfrak{su}(2)^{\oplus 2}\oplus\mathfrak{u}(1)$

✔ $\mathcal{T}_M = \mathcal{T}_M \wr \Gamma$ ✔

prediction!

$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[t^2(6\omega+5) + t^4(26\omega+29) + \mathcal{O}(t^5)\right]$

$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[4t^2 + t^4(6\omega+11) + \mathcal{O}(t^5)\right]$

$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[t^2(2\omega+1) + t^4(6\omega+8) + \mathcal{O}(t^5)\right]$

$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[2t^2 + t^4(15+6\omega) + \mathcal{O}(t^5)\right]$

$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[9t^2 + 6\omega t^3 + 40t^4 + \mathcal{O}(t^5)\right]$

$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[2t^2 + 9t^4 + \mathcal{O}(t^5)\right]$

$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[4t^2 + 2\omega t^3 + 10t^4 + \mathcal{O}(t^5)\right]$

$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[3t^2 + 22t^4 + \mathcal{O}(t^5)\right]$

$\mathrm{HS}_{\wr\mathbb{Z}_2}(t,\omega) = \mathrm{PE}\left[t^2 + 8t^4 + \mathcal{O}(t^5)\right]$

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Conclusion

we studied how the moment maps in the Higgs branch chiral ring of Higgsed (A, A) and (D, D) conformal matter transform under $\mathbb{Z}_2$ GS automorphism

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they agree! $\widetilde{\mathcal{T}}_M = \mathcal{T}_M \wr \Gamma$

Quiver Subtraction Algorithms and Hasse Diagrams?

we now know a magnetic quiver for the Higgs branch of discretely-gauged conformal matter

can we extract the foliation structure of the symplectic singularity?

generalize quiver subtraction/decay and fission to wreathed quivers

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from complex structure deformations of Calabi--Yau geometry in F-theory, we expect certain Higgs branch RG flows to exist

geometry hints at such algorithms

(similarly to [CL, Mansi][2])
[CL, Mansi, Sperling, Zhong]

Non-invertible Symmetries

if a 6d SCFT has a $\mathbb{Z}_2$ zero-form symmetry and a two-form symmetry

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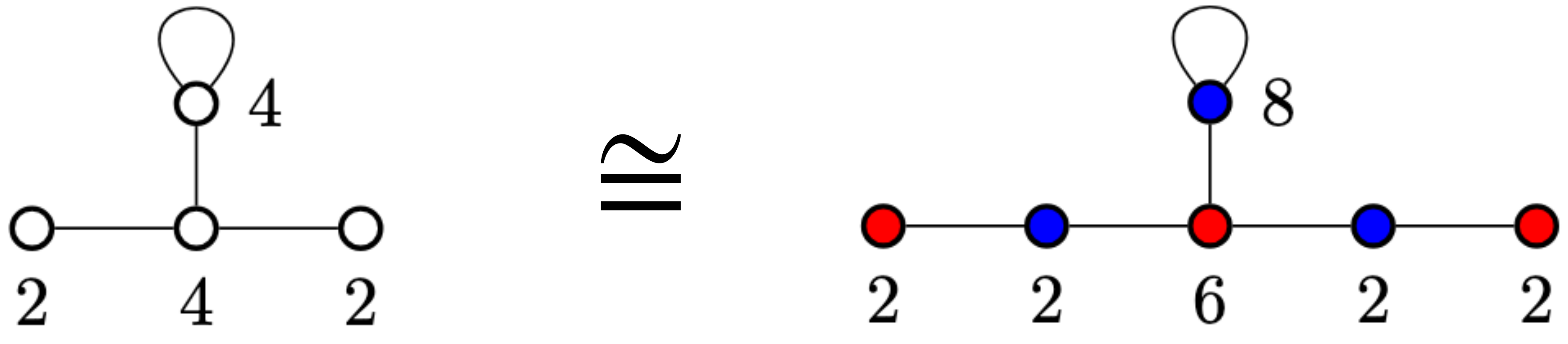

is non-invertible 2-form symmetry reflected

in the wreathed magnetic quiver?

(similar to [Mekareeya, Sacchi] ?)

Thank you!

Flavor algebras of a 6d SCFT

How do we know these flavor symmetries persist at the origin of the tensor branch?

- Not obvious from explicit Weierstrass models [Bertolini, Merkx, Morrison]
- Torus compactification and duality with class $\mathcal{S}$ ✓ [Baume, Kang, CL]

Orbi-instantons, Stiefel--Whitney Twists, and All That