# Defects and phases of abelian GLSM

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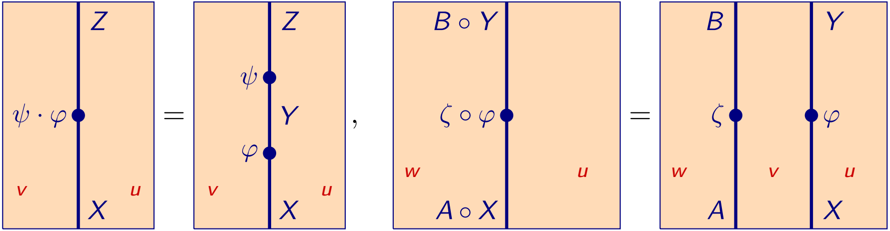
based on 2412.05172 with D. Roggenkamp and C. Schneider

# Properties of (topological) defects in 2 dimensions

- Defect separates theories $u$ and $v$
- Local operators can be constrained to live on the defect $(\psi, \varphi)$
- Defects can be merged.

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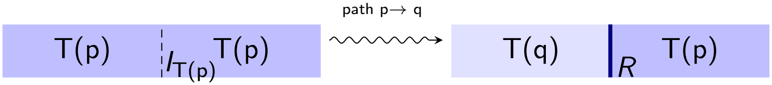
- Special case 1: Theory $u$ may be empty: Defect becomes a boundary
- Special case 2: $u, v, w$ are the same theories, defect can be 'trivial': identity defect.

# Defects, moduli spaces and perturbations

- Defect in 2 dimensional theory: 1 dimensional line, connecting 2 different theories.
- Often in physics: Families of theories
- Defects can be used to connect theories at different points in a moduli space $\mathcal{M}$
- $p, q \in \mathcal{M}$, $\mathcal{M}$ moduli space
- $T(p), T(q)$: Theories at points $p$ and point $q$.

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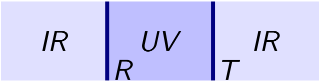
- Defect $R$ connects theories at different points in moduli space.
- Depends on path $\gamma$ connecting points $p$ and $q$.
- Beyond moduli spaces: relevant perturbations
- Flow defects connecting a UV to an IR theory.

IB+Roggenkamp 2007, Gaiotto 2012

# Features of deformed identities

- In physical theories, they are not topological.
- Fusion with other defects is highly singular.
- Favorable situations: SUSY and topological subsectors
- Fusion in one direction yields identity:




$R \otimes T = id_{IR}$

- ...and a projector in the other direction


$T \otimes R = P_{UV}$

# Gauged linear sigma models

- UV theory: $G = U(1)^k$ gauge theory, charged matter multiplets $Y_i$, superpotential, $N = (2,2)$ supersymmetry
- Potential for scalars

$$U = \sum_{i=1}^{n} \left| \sum_{a=1}^{k} Q_i^a \sigma_a y_i \right|^2 + \frac{e^2}{2} \sum_{a=1}^{k} \left( \sum_{i=1}^{n} Q_i^a |y_i|^2 - r^a \right)^2 + \sum_{i=1}^{n} \left| \frac{\partial W}{\partial y_i}(y_1, \ldots, y_n) \right|^2 . \tag{3}$$

- Classical vacuum manifold: $U = 0/_{gauge-transformations}$
- Moduli: Complexified Fayet-Iliopoulos Parameter [1]
- Model may exhibit several phases, characterized by a (partial) breaking of the gauge symmetry.
- Geometric phases/orbifold phases

[1] may run

# Example: Orbifold singularity

- Orbifold $\mathbb{C}^2/\mathbb{Z}_N$ arises as a phase that also exhibits a geometric (resolved) phase
- Matter content in a GLSM description

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $\dots$ | | $X_{N-1}$ | $X_N$ | $X_{N+1}$ |
|---|---|---|---|---|---|---|---|---|---|
| $Q_{1X_i}$ | 1 | -2 | 1 | 0 | $\dots$ | 0 | 0 | 0 | 0 |
| $Q_{2X_i}$ | 0 | 1 | -2 | 1 | $\dots$ | 0 | 0 | 0 | 0 |
| $\vdots$ | $\vdots$ | | | | | | | | $\vdots$ |
| $Q_{N-2X_i}$ | 0 | 0 | 0 | 0 | $\dots$ | 1 | -2 | 1 | 0 |
| $Q_{N-1X_i}$ | 0 | 0 | 0 | 0 | $\dots$ | 0 | 1 | -2 | 1 |

- no superpotential
- no running

# Example: Singularity in LG framework

- Superpotential in GLSM: $W = X_0^d X_1^{d-1} X_2^{d-2} \dots X_{d-2}^2$

| | $X_0$ | $X_1$ | $X_2$ | $X_3$ | $\dots$ | $\dots$ | $X_{d-3}$ | $X_{d-2}$ |
|---|---|---|---|---|---|---|---|---|
| $U(1)_0$ | $(d-1)$ | $-d$ | $0$ | $\dots$ | $\dots$ | $\dots$ | $\dots$ | $0$ |
| $U(1)_1$ | $1$ | $-2$ | $1$ | $0$ | $\dots$ | $\dots$ | $\dots$ | $0$ |
| $U(1)_2$ | $0$ | $1$ | $-2$ | $1$ | $0$ | $\dots$ | $\dots$ | $0$ |
| $U(1)_3$ | $0$ | $0$ | $1$ | $-2$ | $1$ | $0$ | $\dots$ | $0$ |
| $\vdots$ | $\vdots$ | | $\ddots$ | $\ddots$ | $\ddots$ | $\ddots$ | $\ddots$ | $\vdots$ |
| $U(1)_{d-4}$ | $0$ | $\dots$ | $\dots$ | $0$ | $1$ | $-2$ | $1$ | $0$ |
| $U(1)_{d-3}$ | $0$ | $\dots$ | $\dots$ | $\dots$ | $0$ | $1$ | $-2$ | $1$ |

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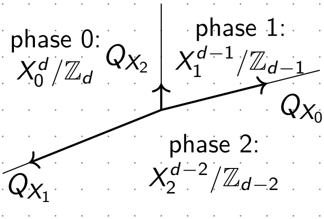
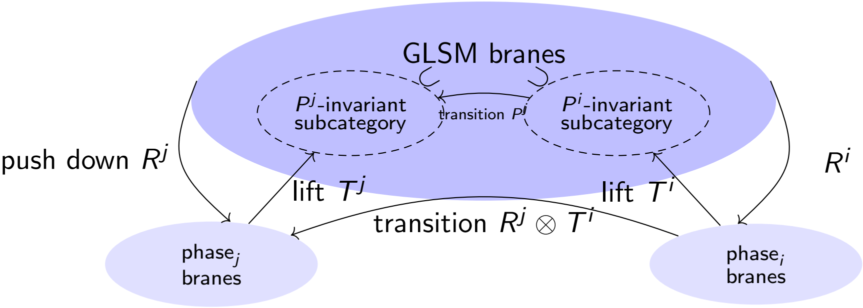
- Different Landau-Ginzburg Orbifold phases, $W = X^{d-i}/\mathbb{Z}_{d-i}$.
- LG model captures physics of the singularity

# 2-parameter model with 2 LG phases

- $U(1)^2$, 3 chiral fields, $W = X_0^d X_1^{d-1} X_2^{d-2}$.
- 3 Landau-Ginzburg phases
- Phase diagram



# Task




- Defects that lift branes from phases to GLSM and vice versa?
- Action on branes: merge defect with boundary
- Functors relating brane categories of phases and GLSM

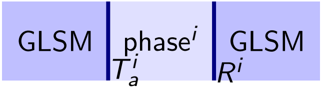
# Setting and strategy

- Consider gauged linear sigma models with different phases.
- Go to a topological sector (B-type SUSY)
- Decouple gauge degrees of freedom. Remnant: Equivariance
- Explicit description of brane categories known!
  - Branes in a geometric phase: Derived category of coherent sheaves.
  - Branes in LG phase: (equivariant) category of matrix factorizations of the superpotential, finite rank
  - GLSM $\rightarrow U(1)^k$ equivariant LG model.
- Defects
  - Description of defects in phases is known
  - In particular, we know the explicit form of the 'invisible' defect
  - Want defects between phases, and between GLSMs and phases

# Properties of $T^i$, $R^i$

- For a fixed phase $i$, $R^i$ and $T^i_a$ can be used to embed the phase into the GLSM
- $R^i \otimes T^i_a = id^i$

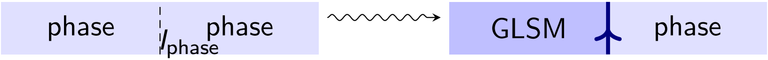

- $T^i_a \otimes R^i = P^i_a$



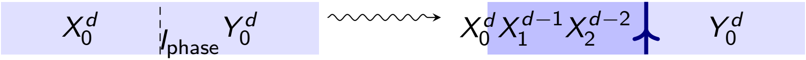

- $P^i_a$ is a projector and realizes the brane category of the phase inside the GLSM.
- Merge defects for different phases $i$, $j$

# Construction

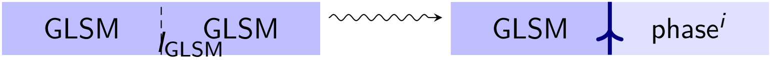
- Main players: Identity defects of phase and GLSM
- "Lift" on one side to GLSM




- Example: $U(1)^2, W = X_0^d X_1^{d-1} X_2^{d-2}$, LG orbifold phases



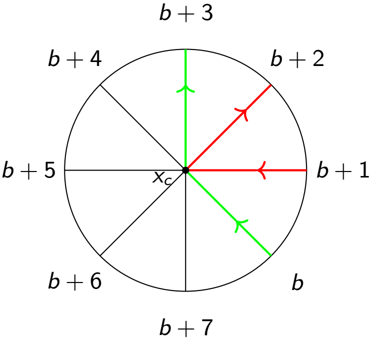

- A priori (too) many lifts
- Pick those that one can obtain from the GLSM



# Example: Abelian GLSM with $W = X_0^d X_1^{d-1} \ldots$

Mirror perspective on phases

- LG orbifold $X^d/\mathbb{Z}_d$ is mirror to LG model with $W = X^d$.
- A-branes: described by straight lines emanating from a critical point, reality condition on $W$. Hori, Iqbal, Vafa
- A-brane corresponds to thimble bounded by two rays
- RG flow: relevant perturbation by lower order polynomial
- FIgure: $W = X^8$



# RG flows and defects

- Under a perturbation, the critical point splits up and some (elementary) branes decouple.
- 'Wedges' collapse in picture
- The defect describing the flow contains precisely the information on which branes decouple
- In our approach, it is obtained as:


- Merging yields a defect between different LG-orbifold models that correctly reproduces the behavior of branes under the flow.

# Conclusions

- Construction of functors between brane categories in different phases of a GLSM.
- Match algebraic data specifying the functor with paths.
- Functors are given in terms of defects, e.g. $T$ between phase and GLSM.
- Uses rigidity of SUSY and defect constructions.
- Explicit functor!
- In agreement with results obtained by other methods: analyticity of hemisphere partition function, boundary potentials Herbst-Hori-Page, Hori-Romo, Knapp-Romo-Scheidegger...
- (In particular: Reproducing the 'grade restriction rule')