Worldsheet boundary states and extended operators in field theory

Work in progress w/
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Motivations

Generalized symmetries made clear that non-local operators play a crucial role in field theory.
[Gaiotto, Kapustin, Seiberg, Willet ’14]

In String Theory, the massless modes of open strings describe gauge fields living on the world volume of branes. Gauge fields are local operators in the field theory.

We are interested in the String Theory origin of Generalized Symmetries.

Vast literature on holographic realization of generalized symmetries [Cordova, Dumitrescu, Intriligator ’18, Bergman, Tachikawa, Zafrir ’20, Apruzzi, van Beest, Gould, Schafer-Nameki, ’21, Apruzzi, Bonetti, Garcia Etxebarria, Hosseini, Schafer-Nameki ’22, Apruzzi, Bah, Bonetti, Schafer-Nameki ‘22, Garcia Extebarria ’22, Heckman, Hubner, Torres, Zhang, ’22, Heckman, Hubner, Torres, Yu, Zhang, ’22, Antinucci, Benini, Copetti, Galati, Rizi ’22, Antinucci, Copetti, Galati, Rizi ’22, Bah, Leung, Waddleton ’23, Apruzzi, Bonetti, Gould, Schafer-Nameki ’23, Bergman, Garcia-Valdecasas, Mignosa, Rodriguez-Gomez ’24, Calvo, Mignosa, Rodrioguez-Gomez ’25, …]

In particular, we are interested in their *worldsheet* realization.

Motivations

We are interested in the worldsheet origin of extended operators [Gutperle, Li, Rathore, Roumpedakis ’24, Bharadwaj, Niro, Roumpedakis ’24, Angius, Giaccari, Volpato ’24].

Warning: as examples of generalized symmetries, non-invertible ones (usually) do not survive the genus expansion of string theory [Heckman, McNamara, Montero, Sharon, Vafa, Valenzuela ‘24, Kaidi, Tachikawa, Zhang ‘24].

We focus on the limit where only the sphere diagram survive, i.e. the large N or planar limit.

$$N \to \infty \ , \quad g_{YM}^2 N \sim g_s N \sim \frac{R^4}{l_s^4} \quad \text{fixed}$$

Plan

1. Setup and goal
   - Worldsheet descriptions for $\mathrm{AdS}_3 \times S^3 \times T^4$; specific limit for $\mathrm{AdS}_5 \times S^5$
   - Extended operators in free $\mathcal{N} = 4$ SYM
2. (Would be) Proposal: boundary states for extended operators
   - Brane realization of GW operators in 4d;
   - Strategy: boundary states
3. Outlook

Setup and goal

For capturing field theory operators from the worldsheet (WS), we need a WS description in the first place.

- The $S_N(T^4)$ orbifold CFT_2 at large N is dual to strings on $\mathrm{AdS}_3 \times S^3 \times T^4$ with $k = 1$ NS flux.

  The WS is described by a $\mathfrak{psu}(1,1\,|\,2)_1$ WZW model, which admits a free-field realization [Eberhardt, Gaberdiel, Gopakumar '18].

- The string dual to free $\mathcal{N} = 4$ SYM in 4d is proposed in [Gaberdiel, Gopakumar '21]. The description consists of free fields that realize the $\mathfrak{psu}(2,2\,|\,4)_1$ algebra, inspired by the AdS_3 case.

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Setup and goal

The proposed string dual to free $\mathcal{N}=4$ SYM in 4d [Gaberdiel, Gopakumar '21] consists of

- Two pairs of symplectic bosons $(\lambda^\alpha, \mu^\dagger_\alpha)$, $(\mu^{\dot\alpha}, \lambda^\dagger_{\dot\alpha})$, $\alpha, \dot\alpha = 1{,}2$ with commutators

$$[\lambda^\alpha_m, (\mu^\dagger_\beta)_n] = \delta^\alpha_\beta\, \delta_{m+n,0} \quad , \qquad [\mu^{\dot\alpha}_m, (\lambda^\dagger_{\dot\beta})_n] = \delta^{\dot\alpha}_{\dot\beta}\, \delta_{m+n,0}$$

- Four complex fermions $(\psi^a, \psi^\dagger_a)$, $a = 1{,}2{,}3{,}4$ with anti-commutators

$$\{\psi^a_m, (\psi^\dagger_b)_n\} = \delta^a_b\, \delta_{m+n,0}$$

They generate the $\mathfrak{psu}(2{,}2\,|\,4)_1$ algebra and spectrum of local operators of free $\mathcal{N}=4$ SYM.

Setup and goal

In [Antinucci, Galati, Rizi '22], they argue for the UV limit of 4d $SU(N)$ YM to be related to the gauge theory $U(1)^{N-1} \rtimes S_N$. For our purposes: symmetries and the spectrum of gauge-invariant operators match:

- Local operators: S_N - invariant combinations of F_i , $i = 1,\ldots,N$
- Line operators: S_N - invariant combinations of lines in the abelian theory.

E.g. for Wilson lines $\mathscr{V}(\mathbf{n})[\gamma] = \sum_{\sigma \in S_N} \sigma^{-1} \cdot W(\mathbf{n})[\gamma]$, $\quad W(\mathbf{n})[\gamma] = \prod_{j=1}^{N-1} \exp\left(i\, n_j \oint A_j \right) \quad n_j \in \mathbb{Z}$

- Surface operators: S_N - invariant combinations of surface ops. in the abelian theory. $\quad \alpha_j \in [0,2\pi]$

E.g. for Gukov-Witten ops. $T(\alpha)[\Sigma] = \sum_{\sigma \in S_N} \sigma^{-1} \cdot \mathscr{D}(\alpha)[\Sigma]$, $\quad \mathscr{D}(\alpha)[\Sigma] = \prod_{j=1}^{N-1} \exp\left(i\, \alpha_j \int_\Sigma \frac{\star F_j}{e^2} \right)$

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Setup and goal

In the free-field realization of the WS for $\mathcal{N} = 4$,

we want to realize the dual of extended operators in the field theory.

The strategy consists of defining suitable boundary states on the WS.

Today, we discuss the realization of surface operators.

Boundary states

Suppose we want to define boundary states that preserve a symmetry algebra $\mathfrak{g}$. The set of states is organized in representations of $\mathfrak{g}$.

Being S_n , $\bar{S}_n$ the modes for the symmetry generators, impose the gluing conditions

$$\left(S_n - (-1)^h \rho(\bar{S}_{-n})\right) \|i\rangle\rangle = 0 \,, \quad \forall n$$

The solutions are coherent states called Ishibashi states.

Suitable combinations of these states (Cardy condition, consistent overlap) define a boundary state that preserves the desired symmetry algebra.

Boundary states

for surface operators

4d $\mathcal{N} = 4$ SYM lives on the WV of D3 branes, whose near horizon geometry is $\mathrm{AdS}_5 \times S^5$.

Gukov-Witten operators [Gukov, Witten '08] are realized holographically as D3' branes embedded as

$$\mathrm{AdS}_3 \times S^1 \subset \mathrm{AdS}_5 \times S^5$$

Such that symmetries are broken down as

$$SU(2,2) \times SO(6) \to SO(2,2) \times SO(2) \times (\ SO(4) \times SO(2)\ )$$

Boundary states

for surface operators

Notice that:

- The free fields WS realization of free $\mathcal{N} = 4$ is made of double the field for the WS realization for $\mathrm{AdS}_3 \times S^3 \times T^4$ $k = 1$.

- In [Bachas, Petropoulos '01, Rajaraman, Rozali '01, Lee, Ooguri, Park '01, Gaberdiel, Knighton, Vosmera '21], they construct boundary states for $\mathrm{AdS}_3 \times S^3$ that define specific branes and preserve the full algebra $sl(2,\mathbb{R}) \oplus su(2)$. We are following a similar route.

Boundary states

for surface operators

The strategy is to embed $\mathrm{AdS}_3 \times S^1$ as target space inside the free-field WS description:

We need to define boundary states that preserve the algebra $sl(2,\mathbb{R}) \oplus u(1)$:

- Start with WS of [Gaberdiel, Gopakumar '21], which realizes the $\mathfrak{psu}(2,2|4)_1$ algebra;
- Realize the above algebra, i.e. construct the generators;
- Impose gluing conditions over those to define the Ishibashi states;
- Define consistent boundary states;

Boundary states

for surface operators

A general $\mathfrak{psu}(2,2|4)$ has the form [Beisert '11]

$$X = \left(\begin{array}{cc|c} L^{\alpha}{}_{\beta} & P^{\dot{\alpha}}{}_{\beta} & -iQ^{a}{}_{\beta} \\ K^{\alpha}{}_{\dot{\beta}} & \dot{L}^{\dot{\alpha}}_{\dot{\beta}} & -i\dot{S}^{a}_{\dot{\beta}} \\ \hline S^{\alpha}{}_{b} & \dot{Q}^{\dot{\alpha}}_{b} & R^{a}{}_{b} \end{array}\right)$$

each entry expressed in WS free fields, e.g. $P^{\dot{\alpha}}{}_{\beta} = \mu^{\dot{\alpha}}\mu^{\dagger}_{\beta}$

where $L, \dot{L}, R$ generate the bosonic subalgebra $su(2) \oplus su(2) \oplus so(6)$, P, K generate spacetime translations and $Q, \dot{Q}, S, \dot{S}$ generate susy.

Boundary states

for surface operators

To embed $\mathrm{AdS}_3 \times S^1$ we choose

$$
X = \left(\begin{array}{cc|c}
\begin{pmatrix} -L^1{}_1 & 0 \\ 0 & 0 \end{pmatrix} & \color{red}{\begin{pmatrix} iP^{\mathrm{i}}{}_1 & 0 \\ 0 & 0 \end{pmatrix}} & i\begin{pmatrix} Q^a{}_1 \\ 0 \end{pmatrix} \\
\color{green}{\begin{pmatrix} iK^1{}_{\mathrm{i}} & 0 \\ 0 & 0 \end{pmatrix}} & \begin{pmatrix} \dot{L}^{\mathrm{i}}_{\mathrm{i}} & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} \dot{S}^a_{\mathrm{i}} \\ 0 \end{pmatrix} \\
\hline
-\begin{pmatrix} S^1{}_b & 0 \end{pmatrix} & i\begin{pmatrix} \dot{Q}^{\mathrm{i}}_b & 0 \end{pmatrix} & \frac{1}{4}\begin{pmatrix} -R^1{}_1 & 0 & 0 & 0 \\ 0 & -R^2{}_2 & 0 & 0 \\ 0 & 0 & R^3{}_3 & 0 \\ 0 & 0 & 0 & R^4{}_4 \end{pmatrix}
\end{array}\right)
$$

to reproduce the algebra $sl(2,\mathbb{R}) \oplus u(1)$, with generators J^3, $\color{red}{J^+}$, $\color{green}{J^-}$, K. The choice is compatible with the spectral flow.

almost done

Boundary states

for surface operators

To embed $\mathrm{AdS}_3 \times S^1$, we impose the gluing conditions on the generators J^3, J^+, J^-, K :

$$\left(J^3_n - \bar{J}^3_{-n}\right) \|i\rangle\rangle = 0$$

$$\left(J^\pm_n + \bar{J}^\mp_{-n}\right) \|i\rangle\rangle = 0$$

$$\left(K_n + \bar{K}_{-n}\right) \|i\rangle\rangle = 0$$

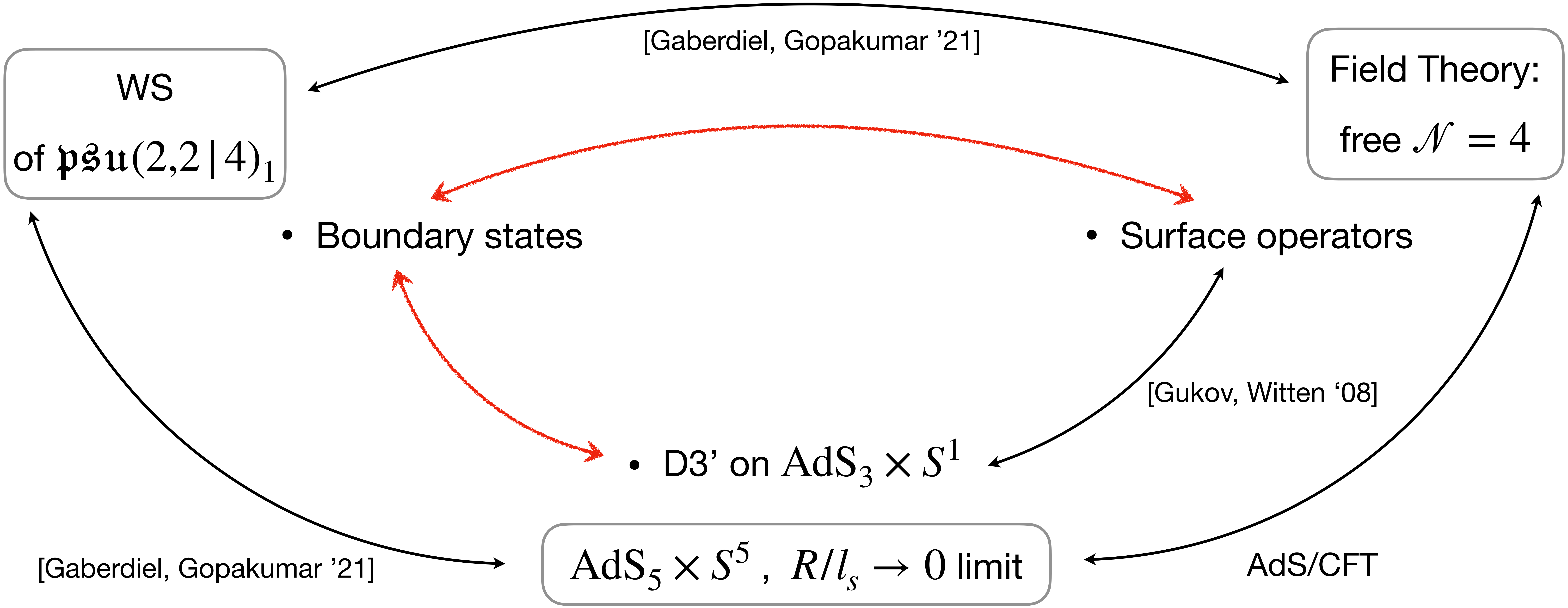
There are questions to be answered yet:

- We are working on identifying the parameter $\alpha \in [0,2\pi]$ of GW operators of gauge theory $U(1)^{N-1} \rtimes S^N$. How does it arise in the WS description?

- Realizing the role of S_N in the construction of the boundary states/operators.

Boundary states

for surface operators



Outlook

What's after?

- Construct line operators, condensation defects;
- Check fusion rules between the full set of operators, i.e. overlap between the boundary states;

Outlook:

- Comparison with $\mathrm{AdS}_3 \times S^3$: fate of non-invertible symmetry?
- Testing of the proposal for the WS realization of free 4d $\mathcal{N} = 4$;
- Insights on WS of weakly coupled 4d $\mathcal{N} = 4$.

Thank you