Geometric Approach to Symmetries at Finite Temperature

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University of Pennsylvania

Based On

hep-th/2503.16427 w/ Hübner and Murdia

hep-th/2501.17911 w/ Cvetič, Hübner and Murdia

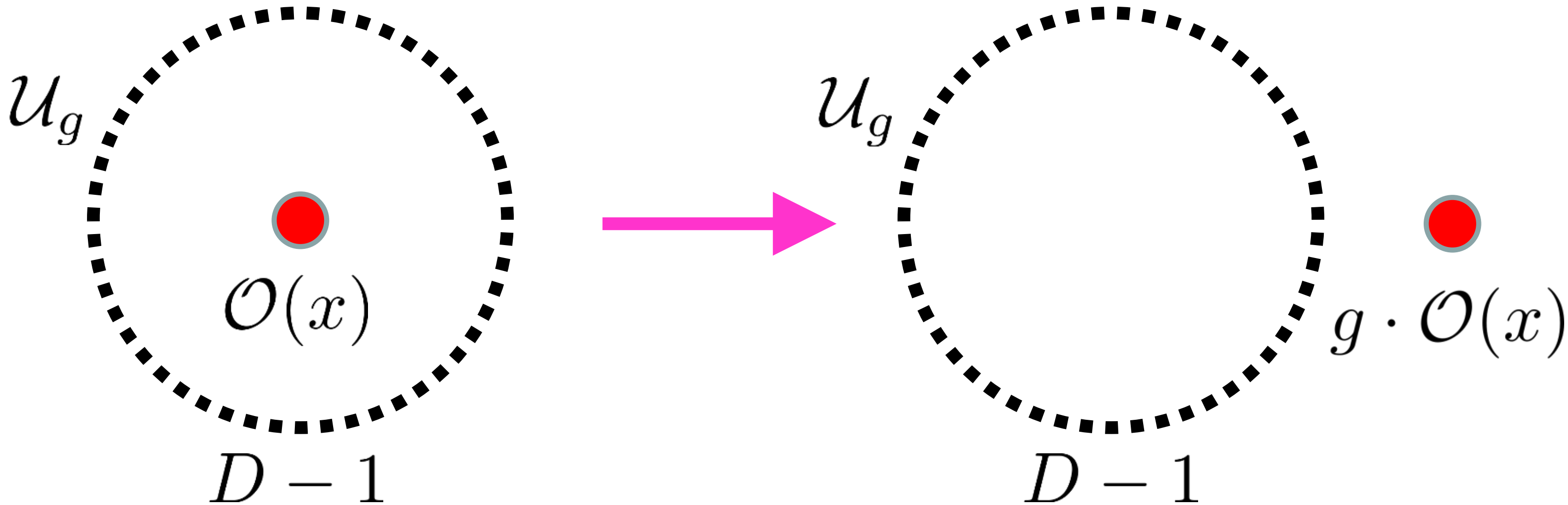
Gen$^{\text{ed}}$ Symms in QFT

Gaiotto Kapustin Seiberg Willett '14

Main Idea: Global Symms are *Topological*

Consider D-dim^l QFT & Charged Local Op:

"Zero-Form Symmetry"



Gen$^{\text{ed}}$ Symms in QFT

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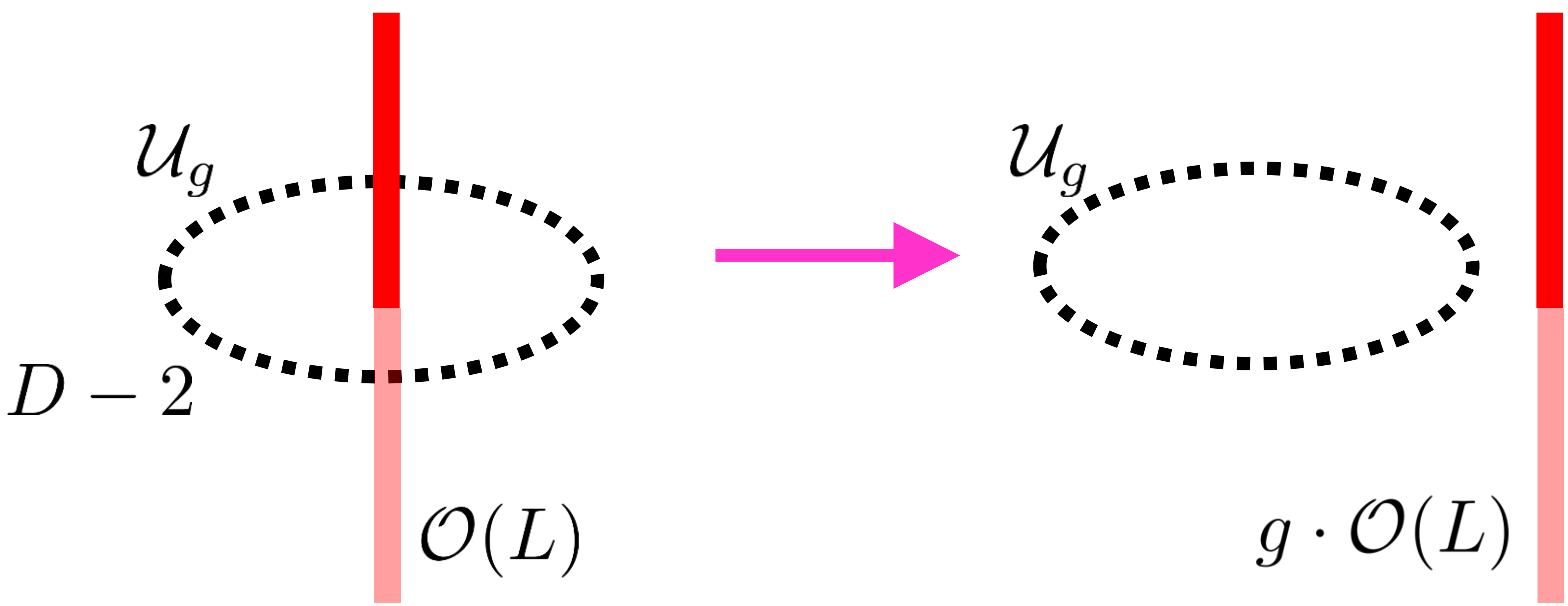

Top Down Questions

- Symmetries of Non-Lagrangian Theories?
  6D SCFTs, 5D SCFTs, Some 4D SCFTs,...

- Coupling to Gravity?

- Computing properties of symmetries
  $\mathrm{Tr}(e^{-\beta H}\mathcal{U}) = ???$

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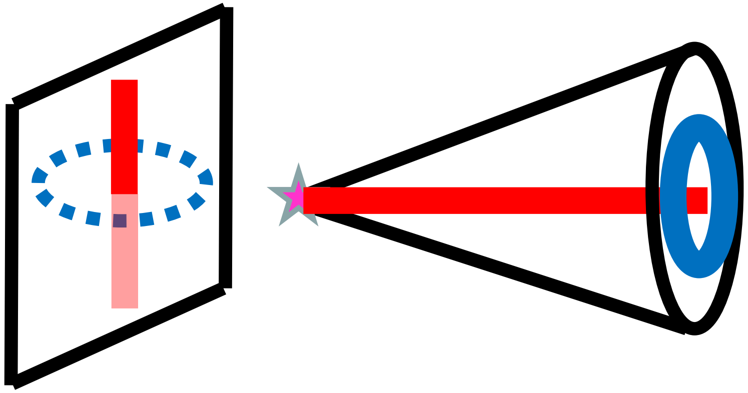
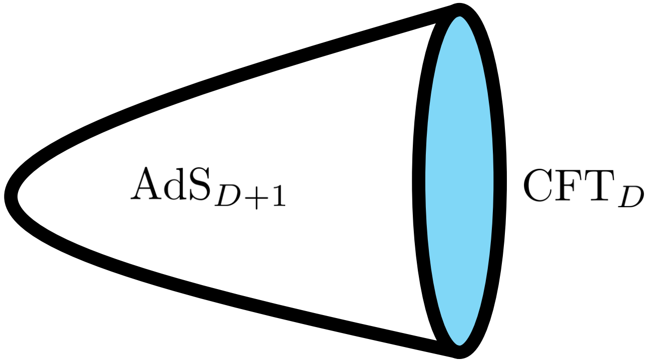
Branes and Symmetries

Symmetries from Branes
(Top Down)



Branes from Symmetries
(Bottom Up)

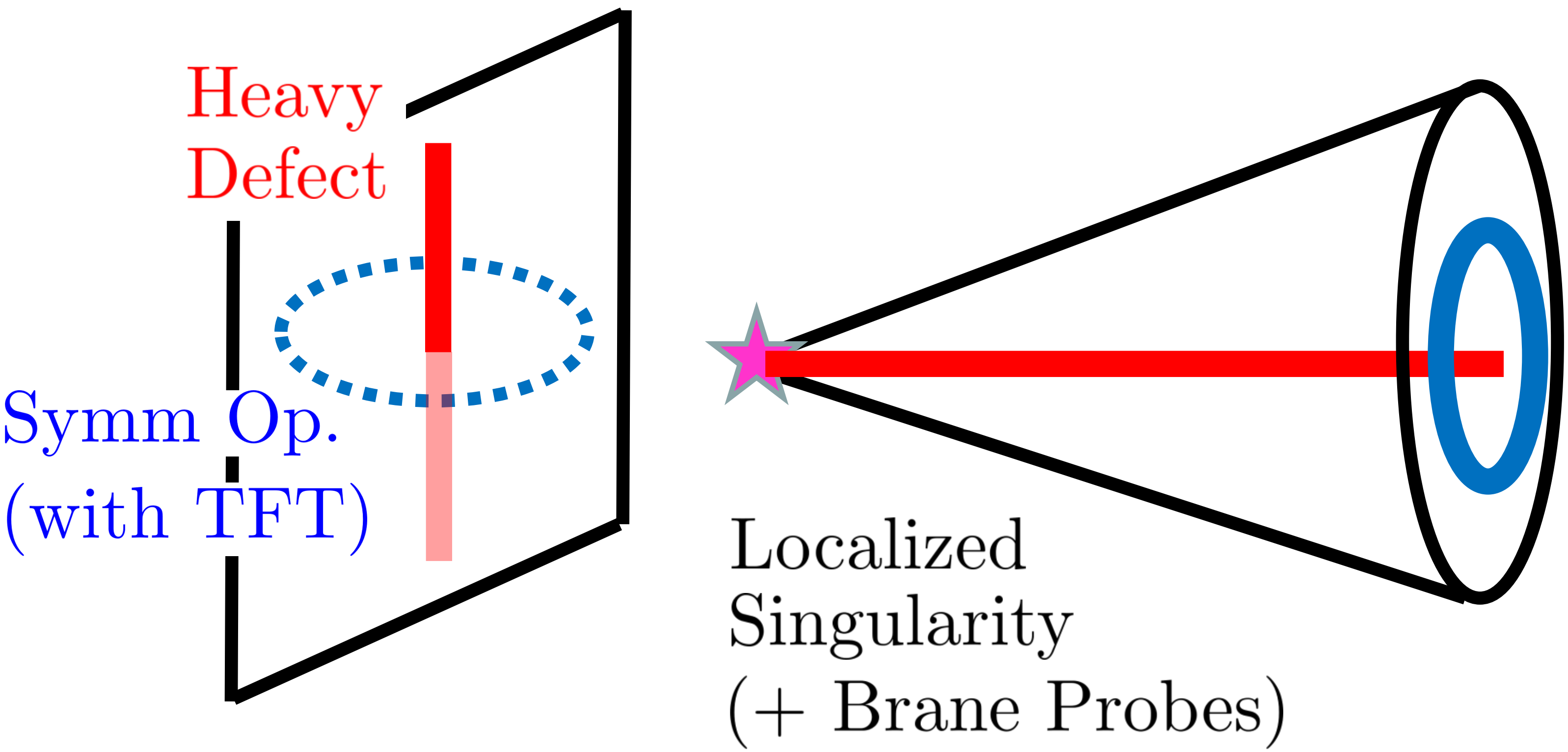


... Many ...

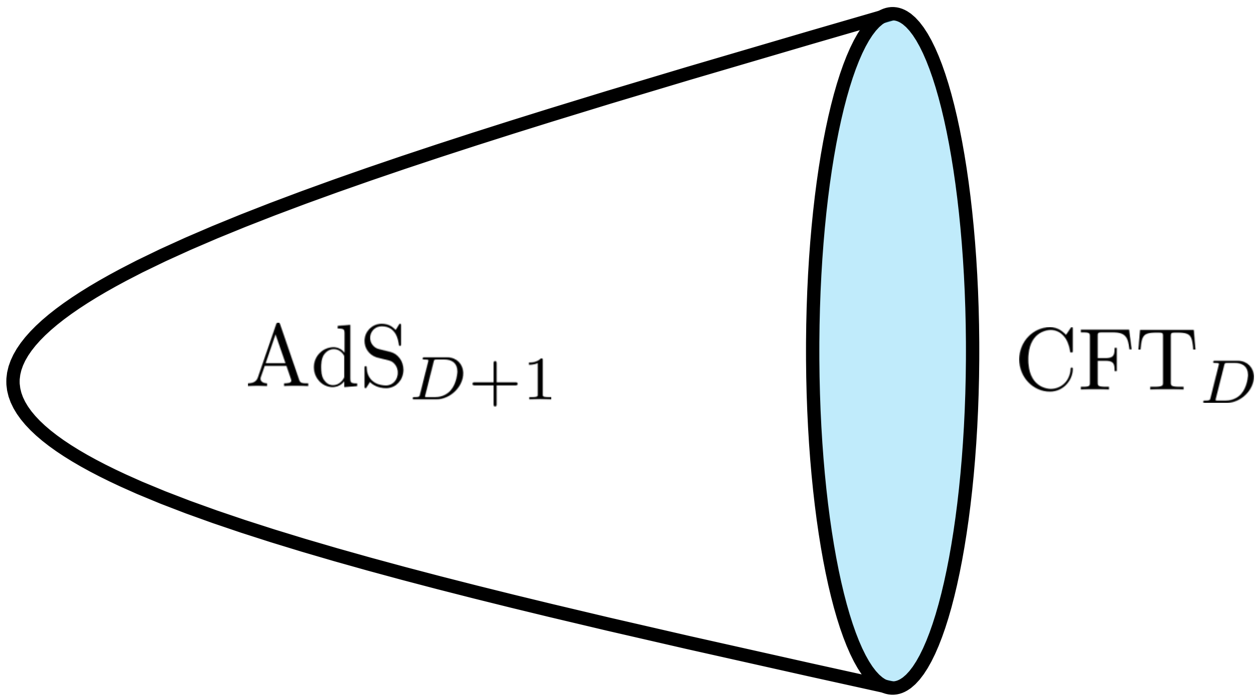
Apruzzi Bah Bonetti Schafer Nameki '22; Garcia Etxebarria '22; JJH Hubner Torres Zhang '22

Symmetries from Branes

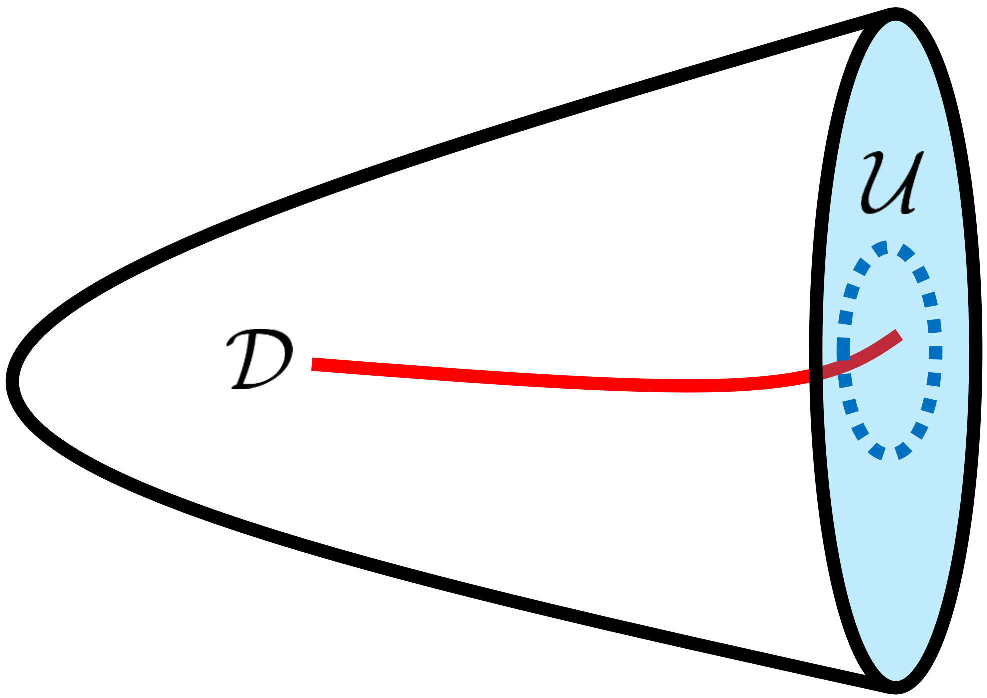
String backgrounds of the form:



Branes from Symmetries

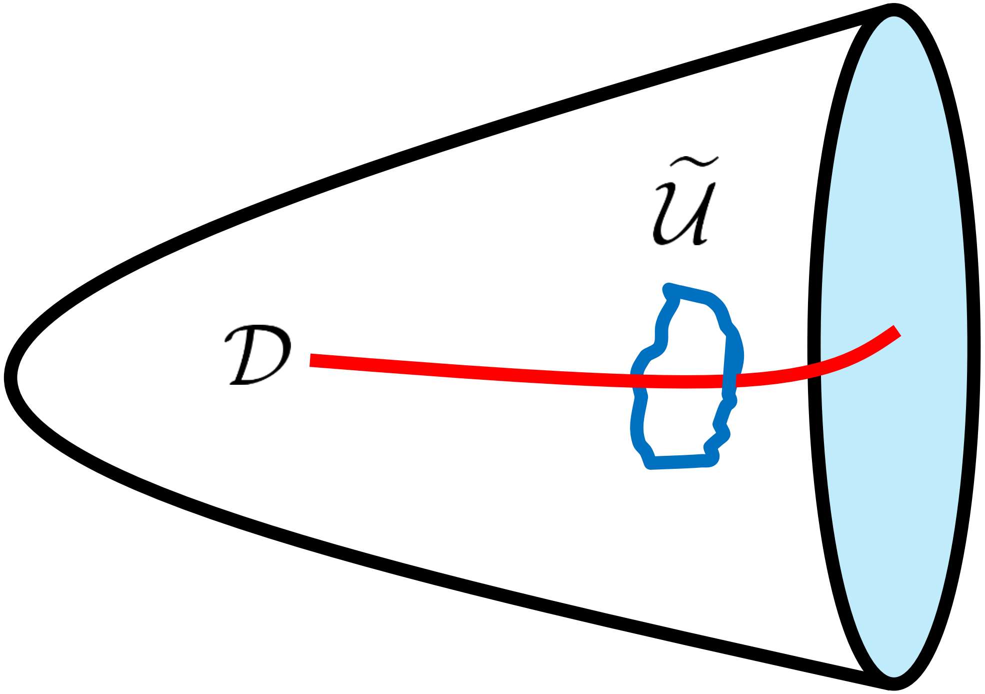


Branes from Symmetries



$\widetilde{\mathcal{U}}$ is a Fluctuating Brane

JJH Hübner Murdia '24



Corollary: No Global Symmetries

(see also Harlow Ooguri '18)

Symmetries $\Rightarrow$ Branes

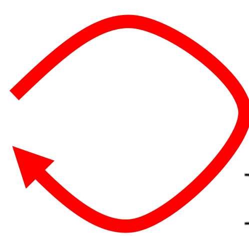
Sometimes “known objects” (e.g., D-branes)

Apruzzi Bah Bonetti Schafer Nameki ’22; Garcia Etxebarria ’22; JJH Hubner Torres Zhang ’22

Sometimes “new objects”
(e.g., R7-Branes / Charge Conjugation)

Dierigl JJH Montero Torres ’22; Debray Dierigl JJH Montero ’23; Dierigl JJH Montero Torres ’23

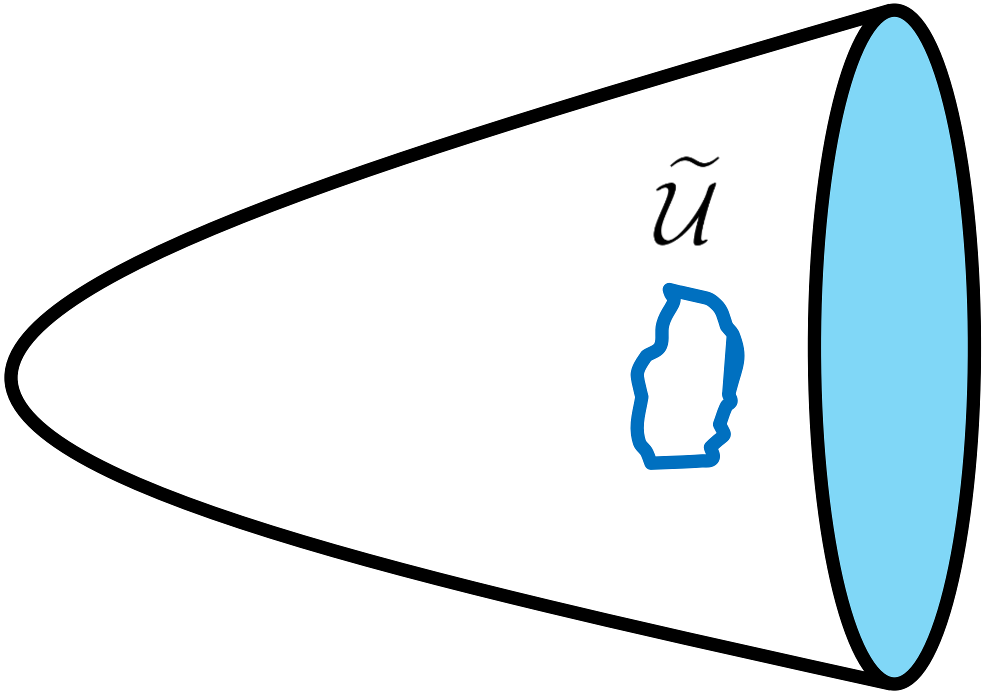
$AdS_7 \times S^4$



Internal Reflection

Symmetries $\Rightarrow$ Branes

¿What is the tension of $\widetilde{\mathcal{U}}$?



$$\mathrm{Tr}(e^{-\beta H}\mathcal{U})$$

QFT_D on $S^1 \times S^{D-1}$

$$\mathcal{U} = \exp\left(i\nu \int_{S^{D-1}} J_{D-1} + \ldots \right)$$

$$\mathrm{Tr}(e^{-\beta H}\mathcal{U}) = \mathrm{Tr}(e^{-\beta H} e^{i\nu Q})$$

"Complexified Chemical Potential"

Main Idea

(Euclidean Gravity Throughout)

$$\frac{\sum_j e^{-I_j[\widetilde{\mathcal{U}}]}}{\sum_j e^{-I_j}} = \langle \mathcal{U} \rangle_\beta = \frac{\mathrm{Tr}(e^{-\beta H}\mathcal{U})}{\mathrm{Tr}(e^{-\beta H})}$$

Main Idea

(Euclidean Gravity Throughout)

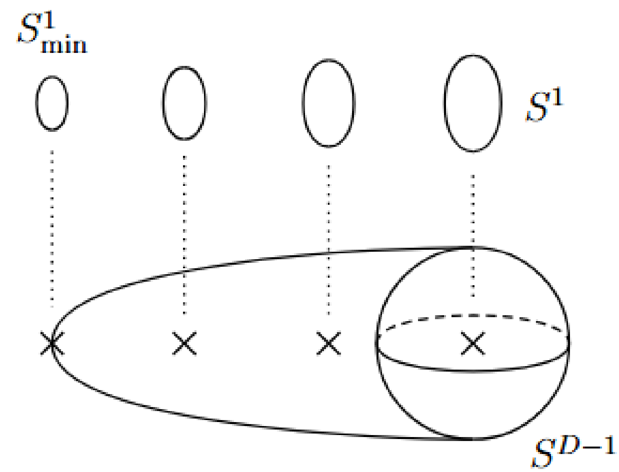
Asymptotic AdS_{D+1}	Boundary CFT_D
$\partial X_{D+1}^{(j)} = S_\beta^1 \times S^{D-1}$	$S_\beta^1 \times S^{D-1}$
Codim-2 Brane $\widetilde{\mathcal{U}}$	Codim-1 Symm $\mathcal{U}$

$$\frac{\sum_j e^{-I_j[\widetilde{\mathcal{U}}]}}{\sum_j e^{-I_j}} = \langle \mathcal{U} \rangle_\beta = \frac{\mathrm{Tr}(e^{-\beta H}\mathcal{U})}{\mathrm{Tr}(e^{-\beta H})}$$

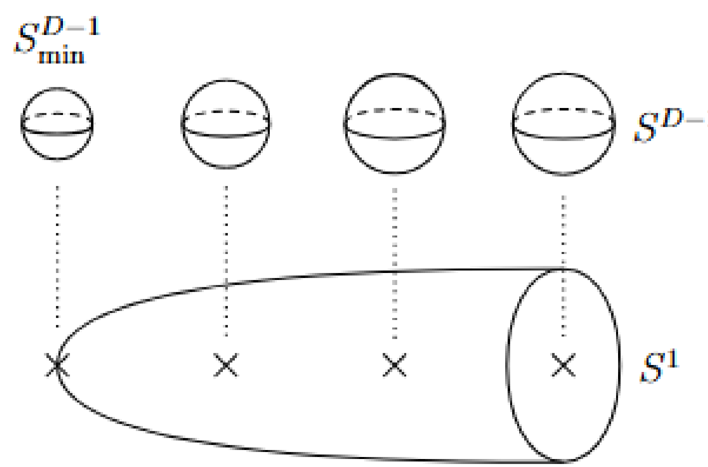
Gravitational Saddles

Hawking Page '82
Witten '98

Thermal AdS:
$S^1 \to \mathrm{Cone}(S^{D-1})$



AdS-Schwarzschild:
$S^{D-1} \to \mathrm{Cone}(S^1)$



Gravitational Saddles

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Thermal AdS:

$S^1 \to \mathrm{Cone}(S^{D-1})$

AdS-Schwarzschild:

$S^{D-1} \to \mathrm{Cone}(S^1)$

$$\frac{e^{-I_{\rm bh}}}{e^{-I_{\rm th}}} = \exp\left(\mathcal{S}_{\rm BH} \times \frac{r_+^2 - \ell_{\rm AdS}^2}{D r_+^2 + (D-2)\ell_{\rm AdS}^2}\right)$$

On-Shell Action + $\widetilde{\mathcal{U}}$

$$\frac{\sum_j e^{-I_j[\widetilde{\mathcal{U}}]}}{\sum_j e^{-I_j}} = \langle \mathcal{U} \rangle_\beta = \frac{\mathrm{Tr}(e^{-\beta H}\mathcal{U})}{\mathrm{Tr}(e^{-\beta H})}$$

Wrap Brane on minimal cycle of X_j;

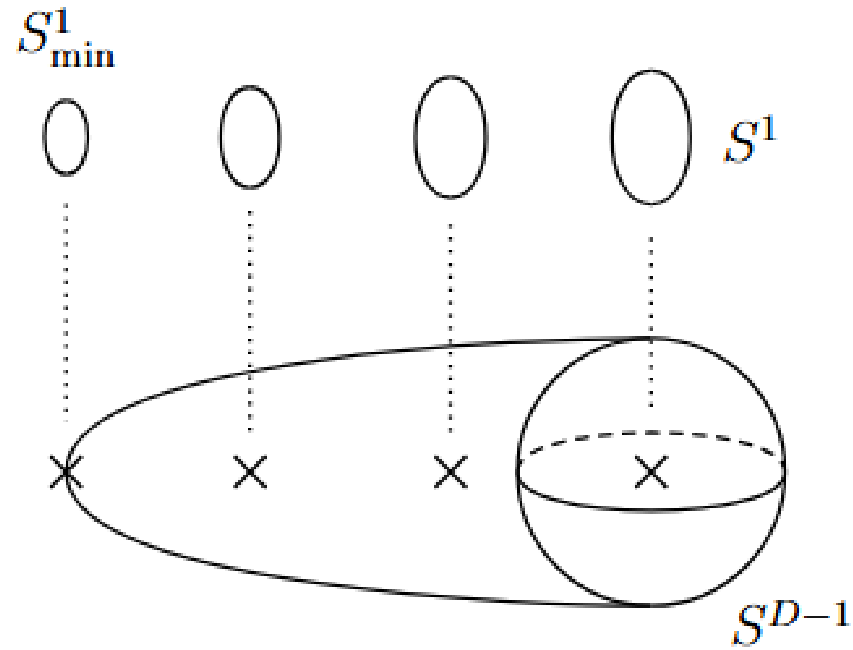
$\partial X^{(j)} = S^1_\beta \times S^{D-1}$

Thermal AdS

$$S^1 \to \mathrm{Cone}(S^{D-1})$$

$$\mathrm{Vol}(\gamma_{\mathrm{min}}) \to 0$$

$$\Rightarrow e^{-I_{\mathrm{th}}[\widetilde{\mathcal{U}}]} \simeq e^{-I_{\mathrm{th}}}$$

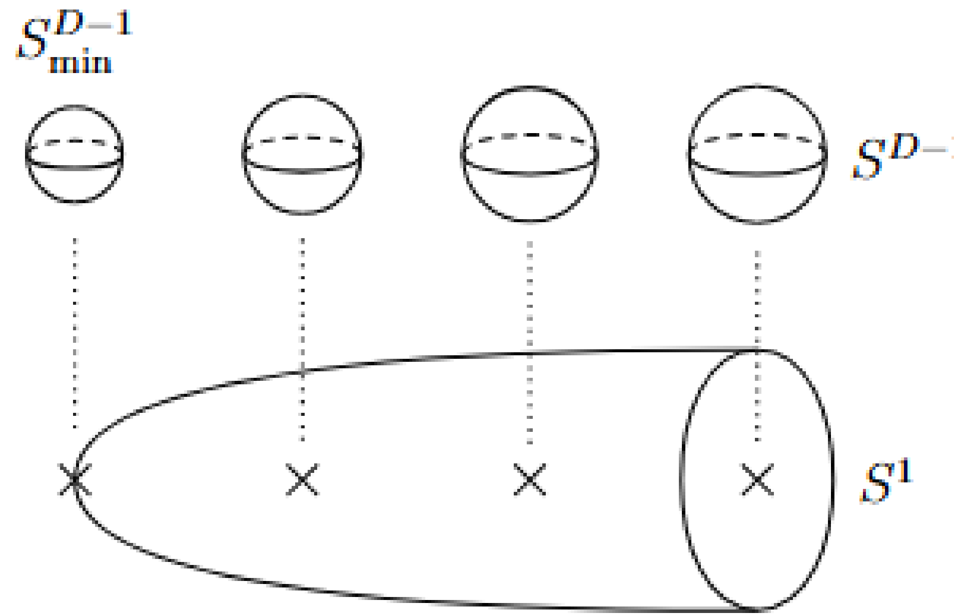


AdS-Schwarzschild + $\widetilde{\mathcal{U}}$

$S^{D-1} \to \text{Cone}(S^1)$

$\Rightarrow \widetilde{\mathcal{U}}$ on outer horizon

$\text{Vol}(\gamma_{\text{min}}) = \text{Area}(\text{Hor})$



On-Shell Action + $\widetilde{\mathcal{U}}$

$$\frac{\sum_j e^{-I_j[\widetilde{\mathcal{U}}]}}{\sum_j e^{-I_j}} = \langle \mathcal{U} \rangle_\beta = \frac{\mathrm{Tr}(e^{-\beta H}\mathcal{U})}{\mathrm{Tr}(e^{-\beta H})}$$

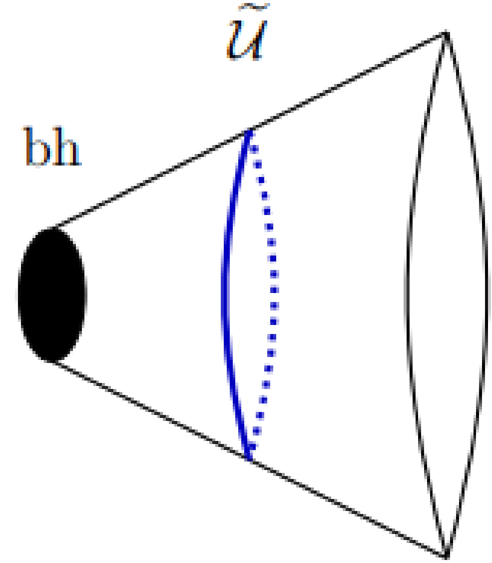
$$\frac{e^{-I_{\mathrm{th}}} + e^{-I_{\mathrm{bh}}[\widetilde{\mathcal{U}}]}}{e^{-I_{\mathrm{th}}} + e^{-I_{\mathrm{bh}}}} = \langle \mathcal{U} \rangle_\beta = \frac{\mathrm{Tr}(e^{-\beta H}\mathcal{U})}{\mathrm{Tr}(e^{-\beta H})}$$

JJH Hübner Murdia '25

AdS-Schwarzschild + $\widetilde{\mathcal{U}}$: Probe Approximation

$$I_{\mathrm{bh}}[\widetilde{\mathcal{U}}] - I_{\mathrm{bh}} = \frac{\chi}{8\pi G_N} \times \mathrm{Area}$$

$$= \frac{\chi}{2\pi} \times \mathcal{S}_{\mathrm{BH}}$$



Deficit Angle: $0 \leq \chi < 2\pi$

Bekenstein-Hawking Entropy: $\mathcal{S}_{\mathrm{BH}}$

Examples

Symmetries of $\mathcal{N} = 4$ SYM

- Duality / Triality Defects

  Constant τ_{IIB} $[p, q]$ 7-branes @ infinity

  JJH Hübner Torres Zhang Yu ’22

- R-symmetries

  KK monopoles from metric isometries

  Cvetič JJH Hübner Murdia ’25

Example:
Duality / Triality Defects
4D $\mathcal{N} = 4$ $\mathfrak{su}(N)$ SYM

Duality Defect: CFT_4 Picture

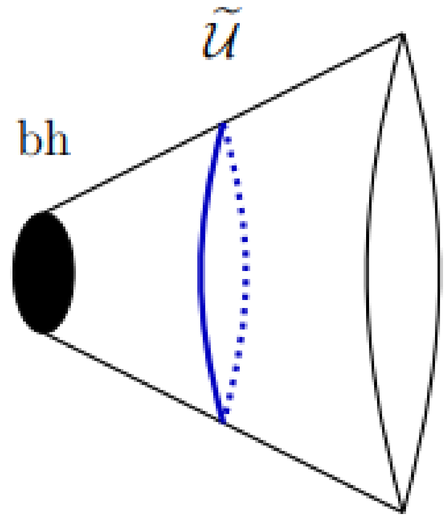
Global Form: $G = (SU(N)/\mathbb{Z}_K)_m$

$$\tau_{\mathrm{IIB}} = \tau_{\mathrm{YM}} = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$$

Kaidi Ohmori Zheng '21; Choi Cordova Hsin Lam Shao '21;
JJH Hübner Torres Yu Zhang '22 + Many...

G	$G' = G$
$\tau = i$	$\tau' = -1/\tau = i$

Gravity Picture



Const. $\tau_{\rm IIB}$ 7-brane on $S^3 \times S^5 \subset \mathrm{AdS}_5 \times S^5$

$$\exp\left(-\frac{\chi}{2\pi} \times \mathcal{S}_{\rm BH}\right) \simeq \langle \mathcal{U} \rangle_\beta$$

JJH Hübner Torres Yu Zhang '22
JJH Hübner Murdia '25

Gravity Picture

τ_{IIB}	Lie Algebra	Kodaira Fiber	7-branes	χ
$\exp(2\pi i/6)$	$\mathfrak{e}_8$	II^*	A^7BC^2	$10 \times \frac{2\pi}{12}$
$\exp(2\pi i/4)$	$\mathfrak{e}_7$	III^*	A^6BC^2	$9 \times \frac{2\pi}{12}$
$\exp(2\pi i/6)$	$\mathfrak{e}_6$	IV^*	A^5BC^2	$8 \times \frac{2\pi}{12}$
$\exp(2\pi i/6)$	$\mathfrak{su}_3$	IV	A^3C	$4 \times \frac{2\pi}{12}$
$\exp(2\pi i/4)$	$\mathfrak{su}_2$	III	A^2C	$3 \times \frac{2\pi}{12}$
$\exp(2\pi i/6)$	$\mathfrak{su}_1$	II	AC	$2 \times \frac{2\pi}{12}$

Const. τ_{IIB} 7-brane on $S^3 \times S^5 \subset \text{AdS}_5 \times S^5$

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JJH Hübner Torres Yu Zhang '22
JJH Hübner Murdia '25

Example:
R-Symmetries
4D $\mathcal{N} = 4$ $\mathfrak{su}(N)$ SYM

JJH Hübner Murdia '25

JJH Hübner Murdia Cvetič '25

$\mathrm{Isom}(S^5)$ & R-Symmetries

10D Picture: $\mathrm{AdS}_5 \times S^5$

Focus on Cartan: $U(1)^3 \subset SO(6)$

CFT_4: $\mathcal{U}_\alpha = \exp(2\pi i(\alpha_1 Q_1 + \alpha_2 Q_2 + \alpha_3 Q_3))$

AdS_5: $\widetilde{\mathcal{U}}_\alpha = \text{KK monopole for } U(1)^3$

JJH Hübner Murdia Cvetič '25

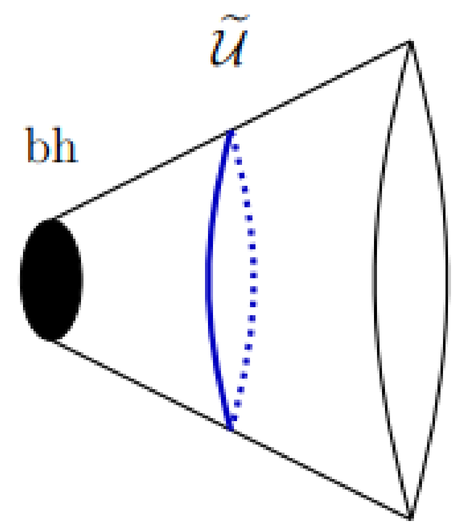
Tension of $\widetilde{\mathcal{U}}_\alpha$

Tension: $T = \frac{\chi}{8\pi G_N} = \frac{\alpha_1+\alpha_2+\alpha_3}{4G_N}$

Arav Gauntlett Jiao Roberts Rosen '24; Bomans Tranchedone '24

Cvetič JJH Hübner Murdia '25

$$e^{-(\alpha_1+\alpha_2+\alpha_3)\mathcal{S}_{\rm BH}} \simeq \langle \mathcal{U}_\alpha \rangle_\beta$$



Cross-Check:
5D BPS Black Holes

JJH Hübner Murdia ’25

Black Holes in AdS_5

Chong Cvetič Lu Pope '04;
Kunduri Lucietti Reall '05;
Hosseini Hristov Zaffaroni '17;
Cabo-Bizet Cassani Martelli Murthy '18;
Choi Kim Kim Nahmgoong '18;
Benini Milan '18;
....

R-charges: Q_1, Q_2, Q_3

Ang. Momenta: J_1, J_2

Entropy Function: $\Gamma_{\text{crit}} = \mathcal{S}_{\text{BH}}$

Sen '05
Hosseini Hristov Zaffaroni '17

$$\Gamma = I - 2\pi i\left(\sum_a Q_a\varphi_a + \sum_j J_j\omega_j\right)$$
$$-2\pi i\Lambda\left(\sum_a \varphi_a - \sum_j \omega_j + 1\right)$$

On-Shell Action: $I = -\pi i N^2 \frac{\varphi_1\varphi_2\varphi_3}{\omega_1\omega_2}$

CFT_4 Index

R-charges: Q_1, Q_2, Q_3
Ang. Momenta: J_1, J_2

Cabo-Bizet Cassani Martelli Murthy '18;
Choi Kim Kim Nahmgoong '18;
Benini Milan '18;
....

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \exp\left(2\pi i \sum_a \alpha_a Q_a - (\alpha_1 + \alpha_2 + \alpha_3)\, \mathcal{S}_{\text{BH}} \right)$$

JJH Hübner Murdia '25

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \frac{\text{Tr}\Big((-1)^F e^{-\beta \{\mathcal{Q}, \overline{\mathcal{Q}}\}} p^{J_1 + \frac{1}{2}R} q^{J_2 + \frac{1}{2}R} y_1^{q_1} y_2^{q_2} \mathcal{U}_\alpha \Big)}{\text{Tr}\Big((-1)^F e^{-\beta \{\mathcal{Q}, \overline{\mathcal{Q}}\}} p^{J_1 + \frac{1}{2}R} q^{J_2 + \frac{1}{2}R} y_1^{q_1} y_2^{q_2} \Big)}$$

CFT$_4$ Index

R-charges: Q_1, Q_2, Q_3

Ang. Momenta: J_1, J_2

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \exp\left(2\pi i \sum_a \alpha_a Q_a - (\alpha_1 + \alpha_2 + \alpha_3)\, \mathcal{S}_{\text{BH}} \right)$$

Horizon Wrapping

$$\frac{\chi}{2\pi} \times \frac{\text{Area}}{4G_N}$$

CFT$_4$ Index

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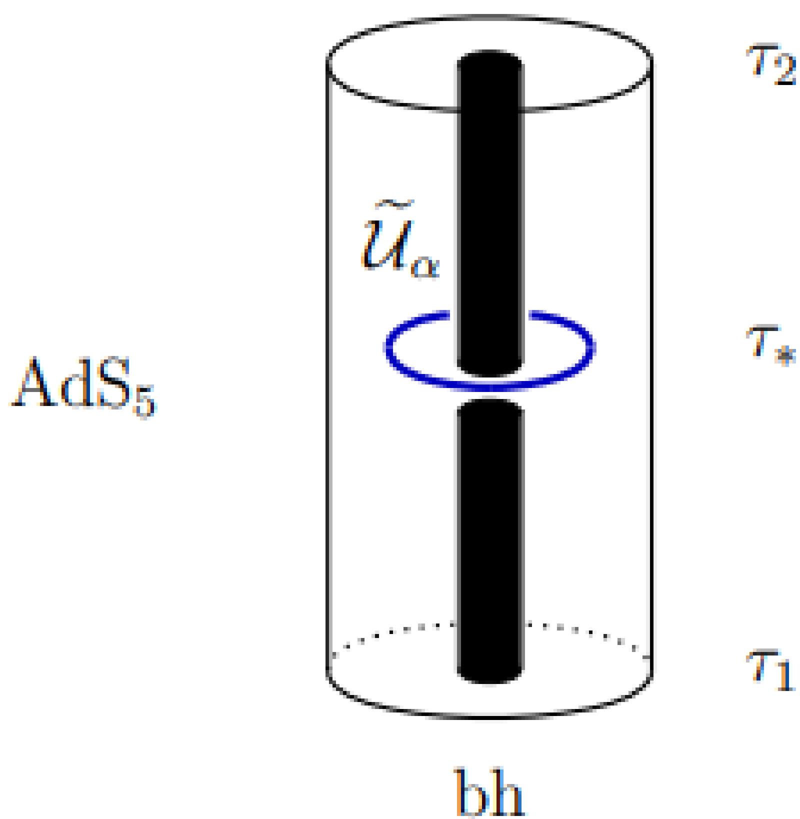
Topological Linking

Horizon Wrapping

$$\frac{\chi}{2\pi} \times \frac{\text{Area}}{4G_N}$$

Linking / Wrapping

$$\langle \mathcal{U}_\alpha \rangle_{\text{Index}} = \exp\left(2\pi i \sum_a \alpha_a Q_a - (\alpha_1 + \alpha_2 + \alpha_3)\, \mathcal{S}_{\text{BH}} \right)$$

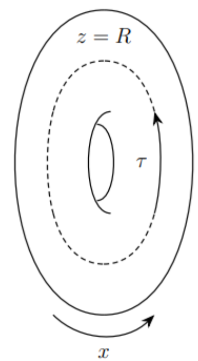


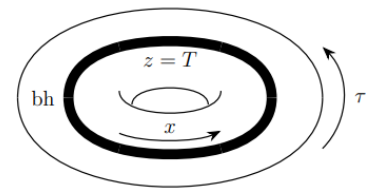
More Examples

Further Examples

JJH Hübner Murdia '25

- AdS$_3$ / CFT$_2$





- 4D $\mathcal{N} = 1$ SCFTs via D3's at Cone(SE_5)
  Symmetry Ops via wrapped Dp-branes

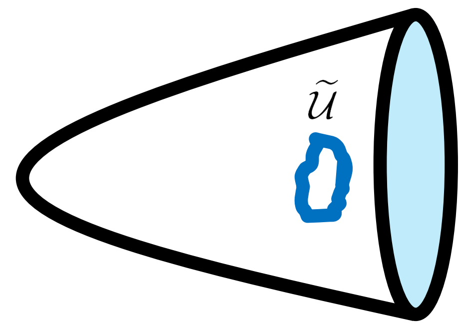
- R-Charged Black Holes

$$\langle \mathcal{U}_\alpha \rangle_\beta = \exp\left(2\pi i \sum_a \alpha_a Q_a - (\alpha_1 + \alpha_2 + \alpha_3)\, \mathcal{S}_{\rm BH} \right)$$

Summary / Future

What Was This Talk About?

- Symmetries $\Rightarrow$ Branes



- $\dfrac{\sum_j e^{-I_j[\tilde{\mathcal{U}}]}}{\sum_j e^{-I_j}} = \langle \mathcal{U} \rangle_\beta = \dfrac{\mathrm{Tr}(e^{-\beta H}\mathcal{U})}{\mathrm{Tr}(e^{-\beta H})}$

- Tensions & Thermal Expectation Values

Future Directions

- Other gravitational saddles?
- Phase Structure of R-Charged Black Holes?
- Lorentzian Signature?