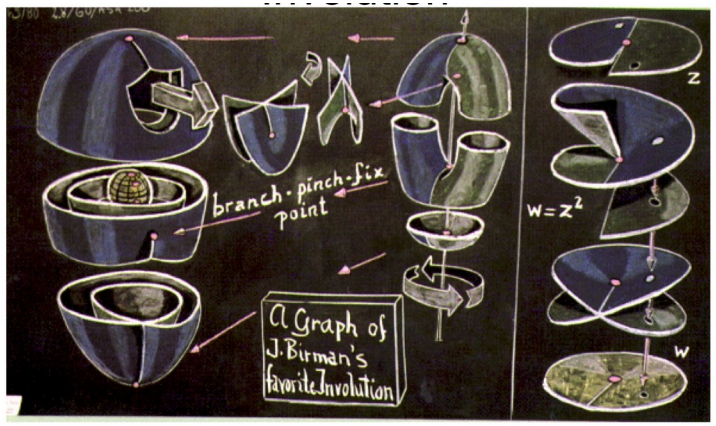
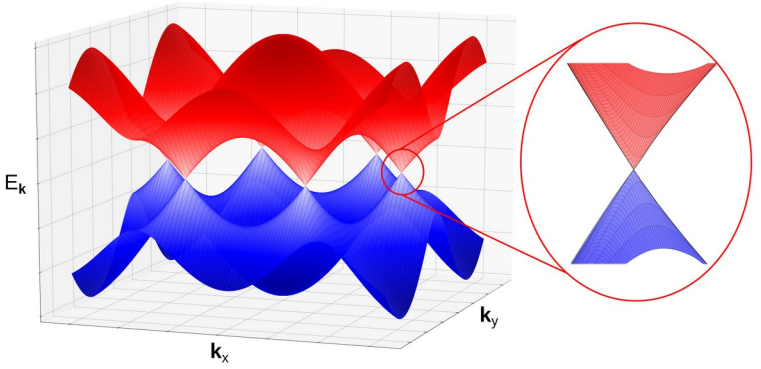
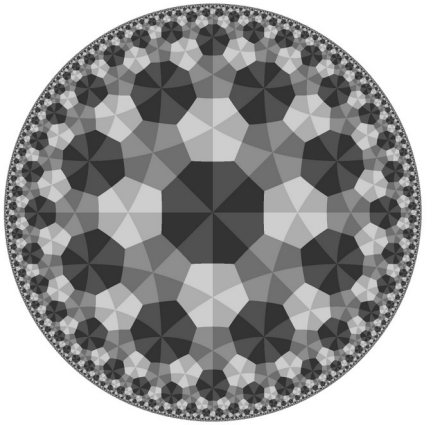
Hyperbolic Band Theory
Higgs Bundles
Supersymmetric Field Theory







Strings & Geometry 2025
ICTP Trieste

Steven Rayan
quanTA Centre / Math & Stats, USask
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April 7, 2025

Prairies Economic Development Canada | Développement économique Canada pour les Prairies
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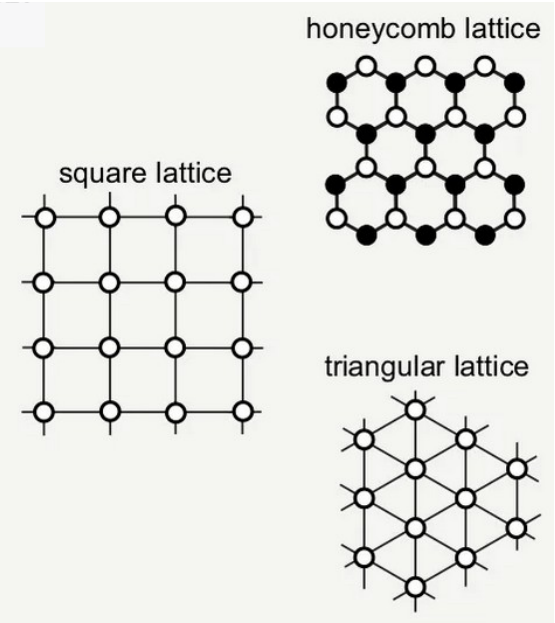
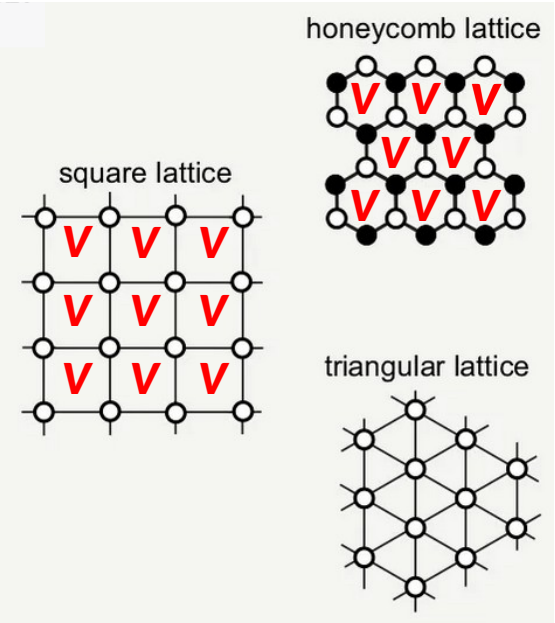
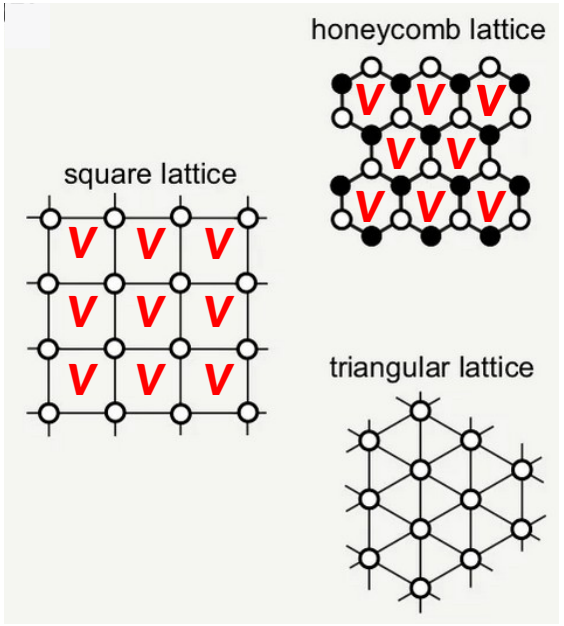
Ordinary Bloch-Floquet Theory

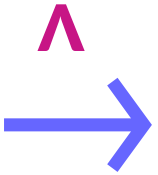


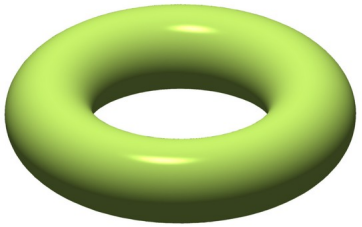
Ordinary Bloch-Floquet Theory



Ordinary Bloch-Floquet Theory

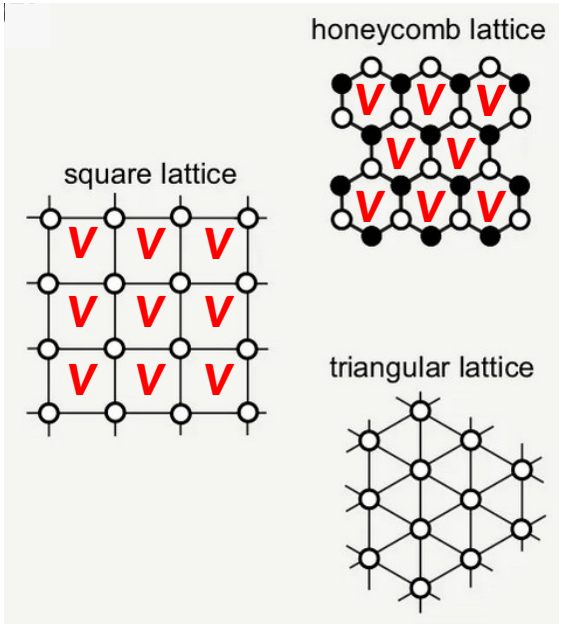


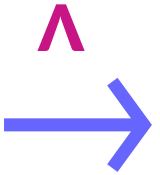


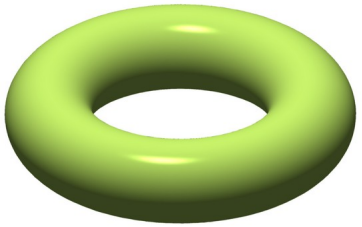


position

Ordinary Bloch-Floquet Theory







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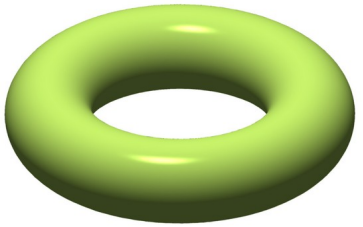
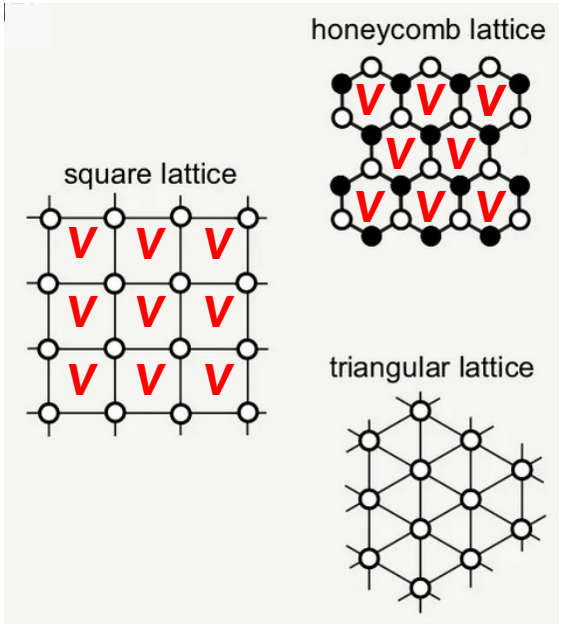
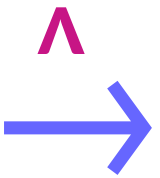
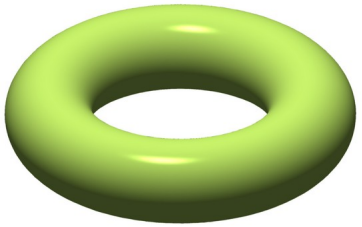


momentum

Ordinary Bloch-Floquet Theory







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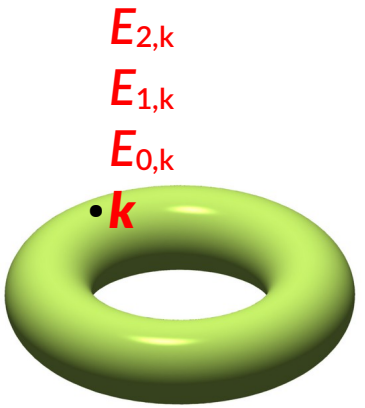
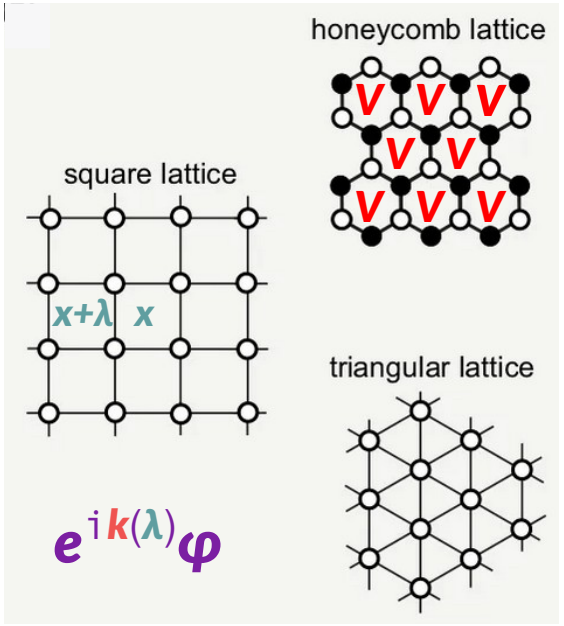


momentum

Ordinary Bloch-Floquet Theory



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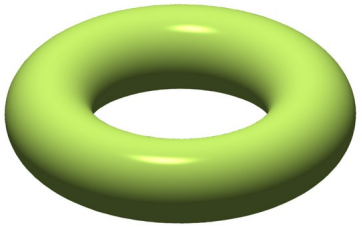
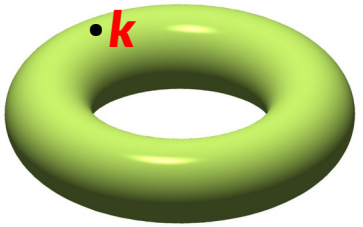


position

$E_{2,k}$
$E_{1,k}$
$E_{0,k}$
k



momentum

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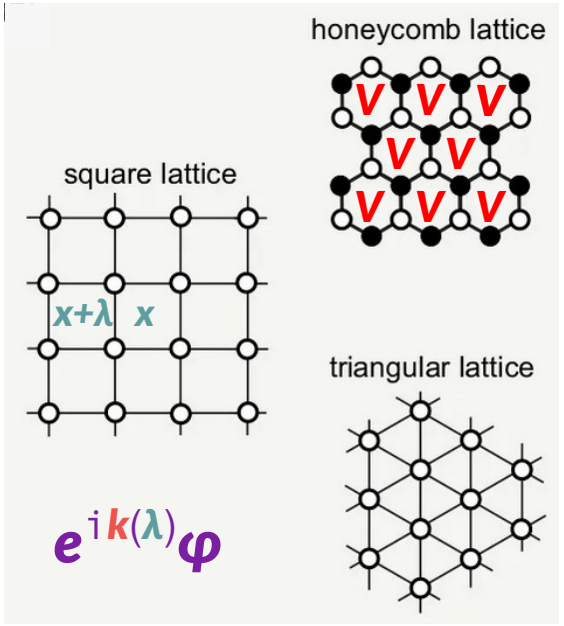

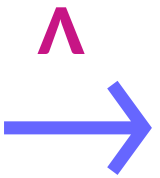

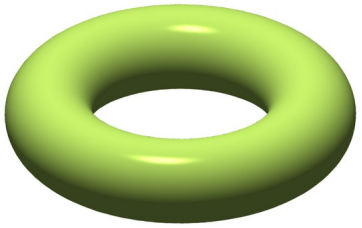

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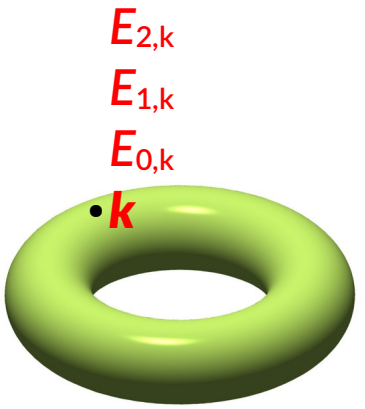

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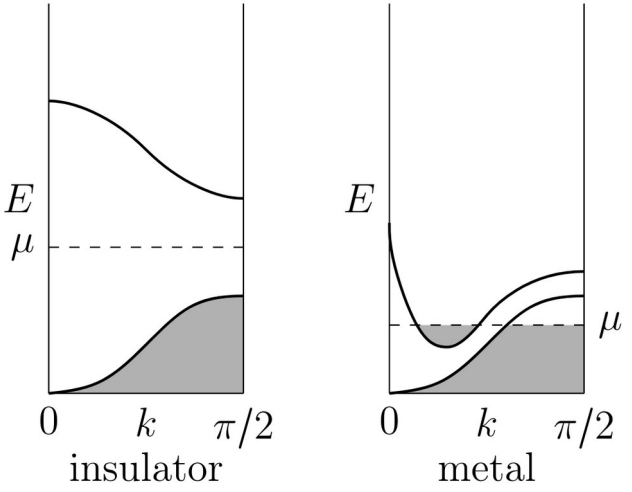

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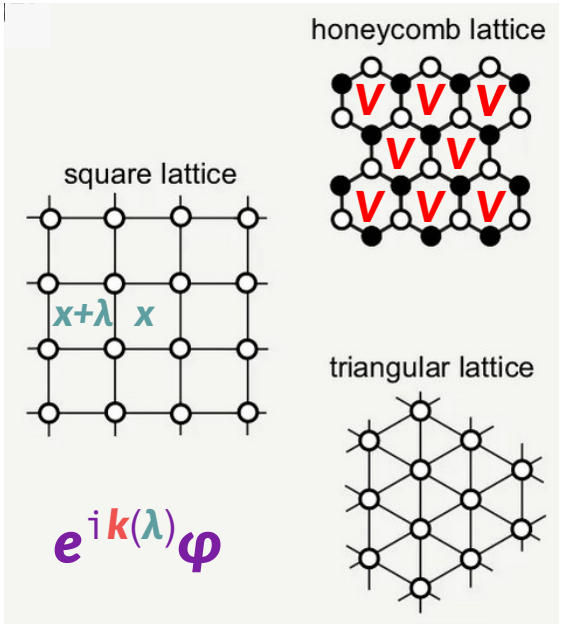
Ordinary Bloch-Floquet Theory



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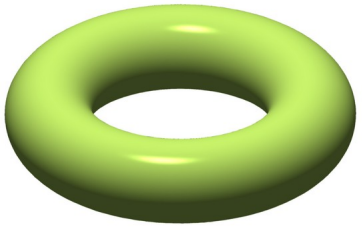
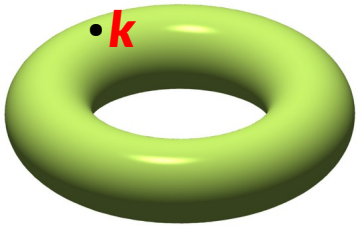
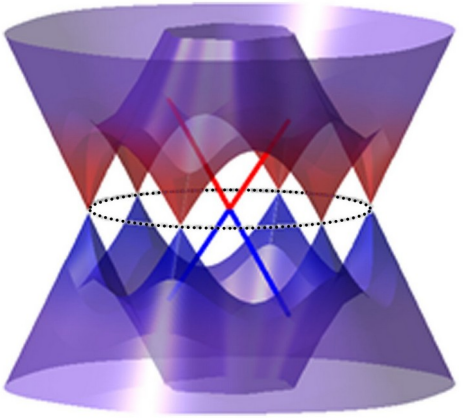


position

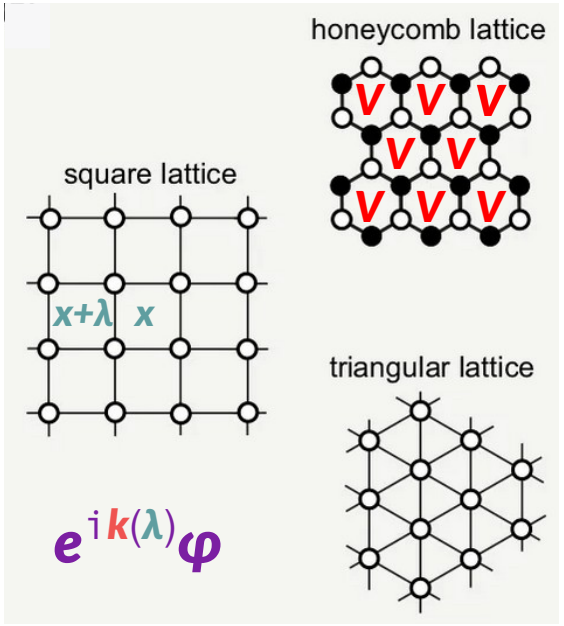
$E_{2,k}$
$E_{1,k}$
$E_{0,k}$
k



momentum



Ordinary Bloch-Floquet Theory



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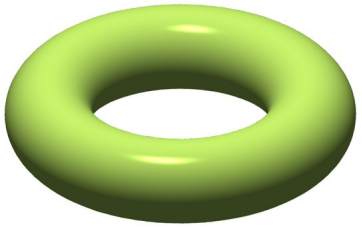

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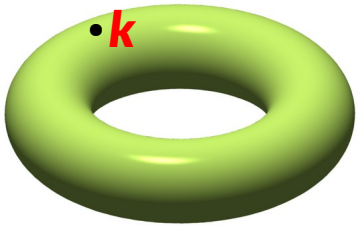
$E_{1,k}$
$E_{0,k}$
k



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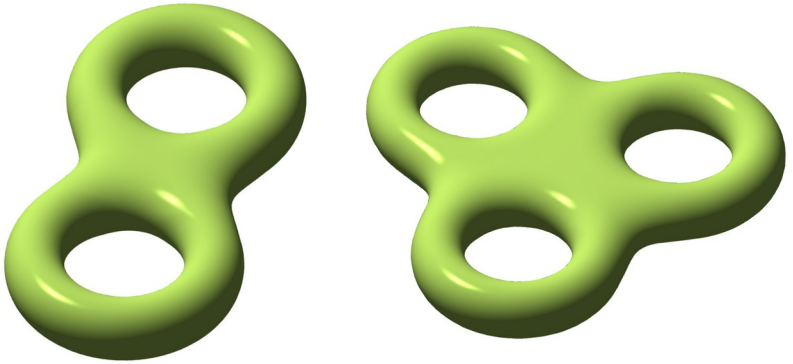

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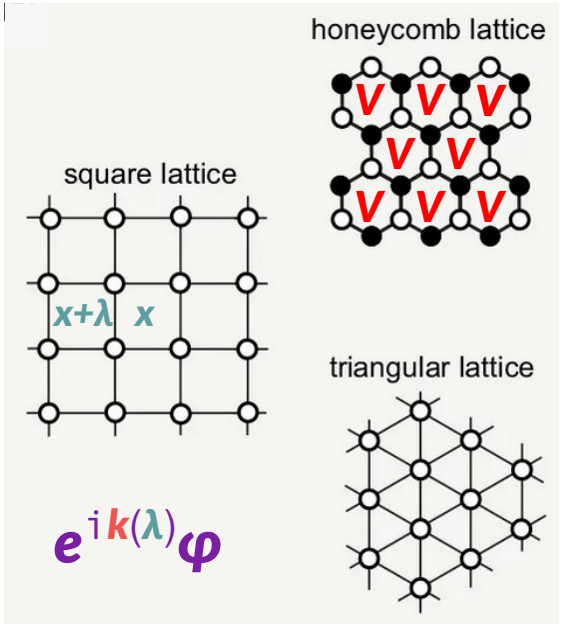

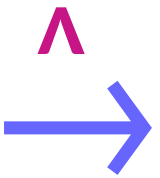

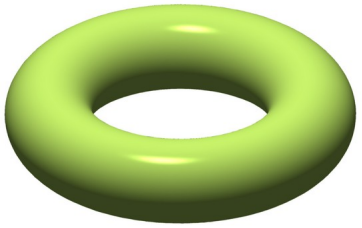

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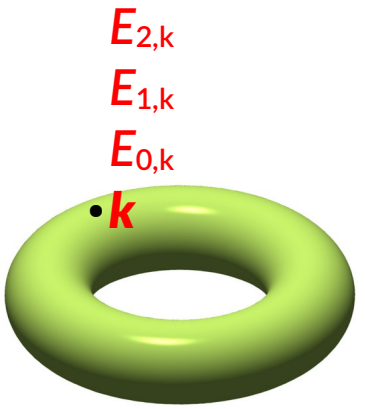

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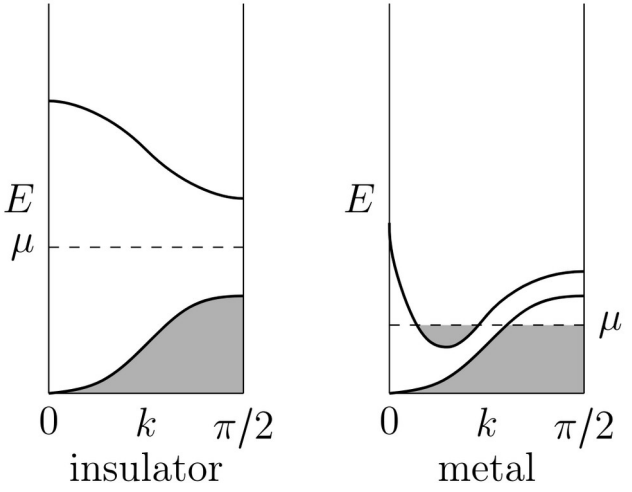

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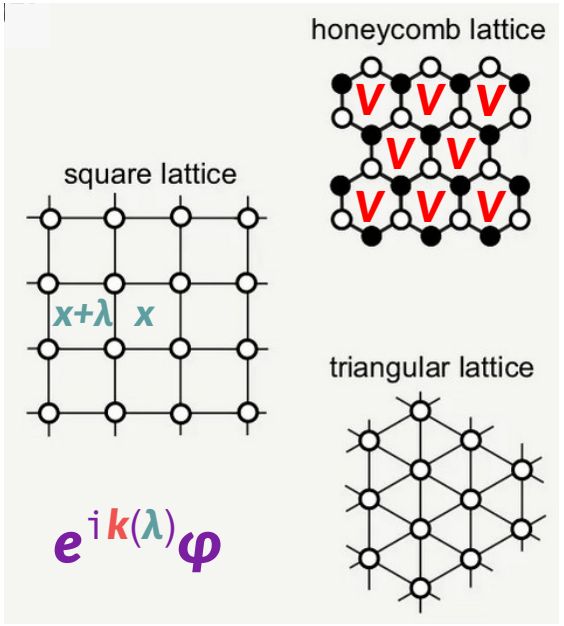
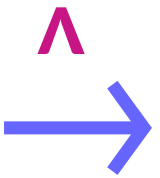
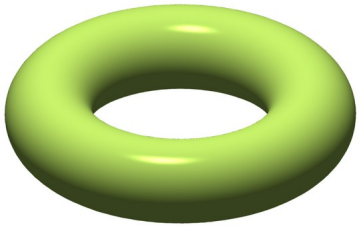
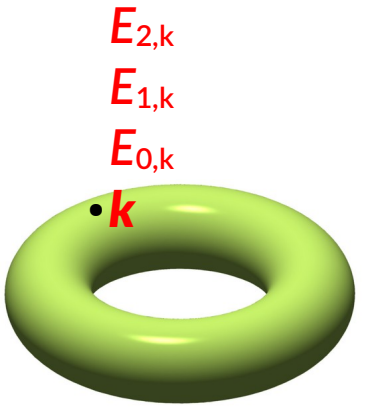
Ordinary Bloch-Floquet Theory

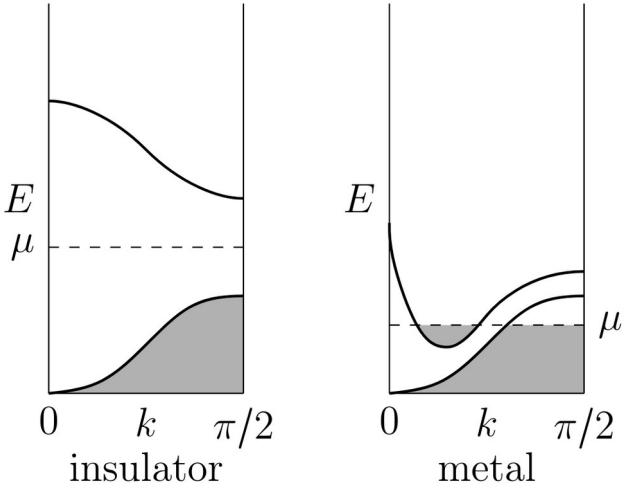


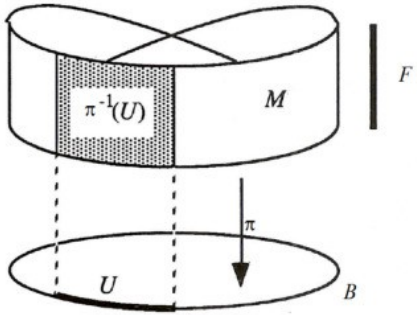






position **momentum**





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Band Theory	Geometry
Position space	Elliptic curve
Momentum space	Jacobian
Position-momentum duality	Abel-Jacobi map
Band structure	Branched cover of Jacobian
Band invariants	Genus, Chern classes

Hyperbolic Bloch-Floquet Theory

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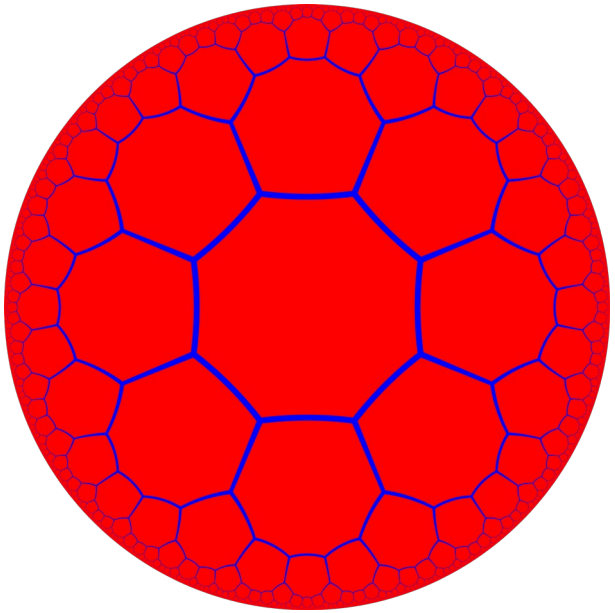
Hyperbolic Bloch-Floquet Theory



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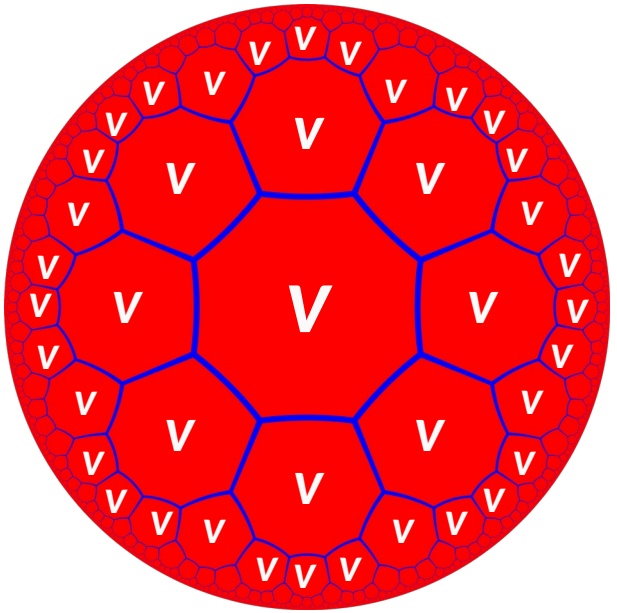
Hyperbolic Bloch-Floquet Theory



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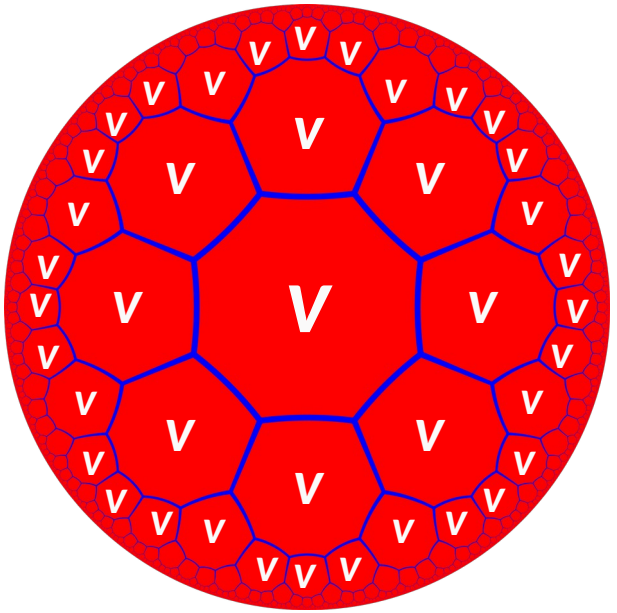
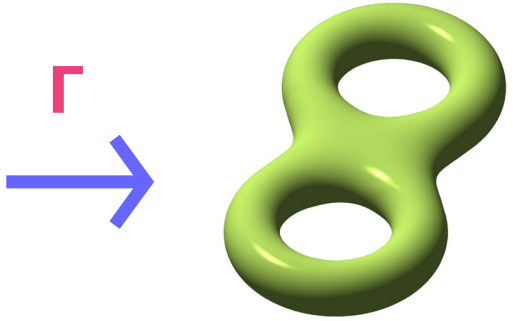
Hyperbolic Bloch-Floquet Theory

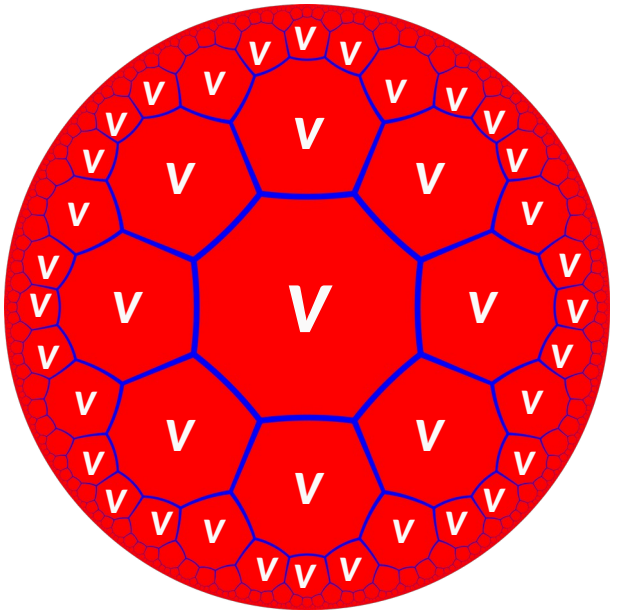


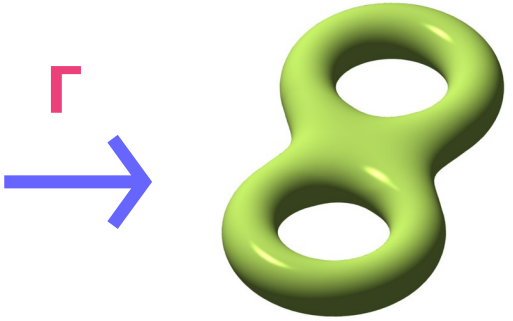


position

momentum

Hyperbolic Bloch-Floquet Theory





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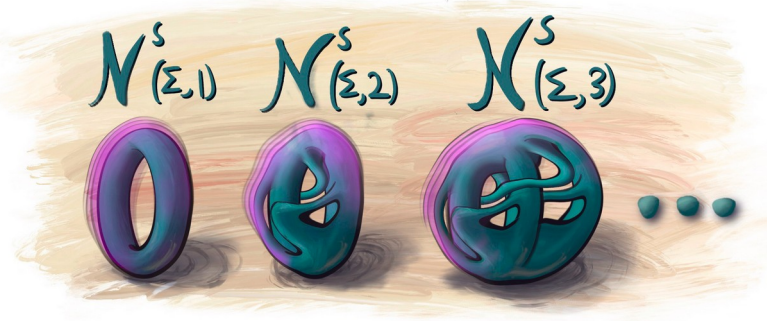

momentum

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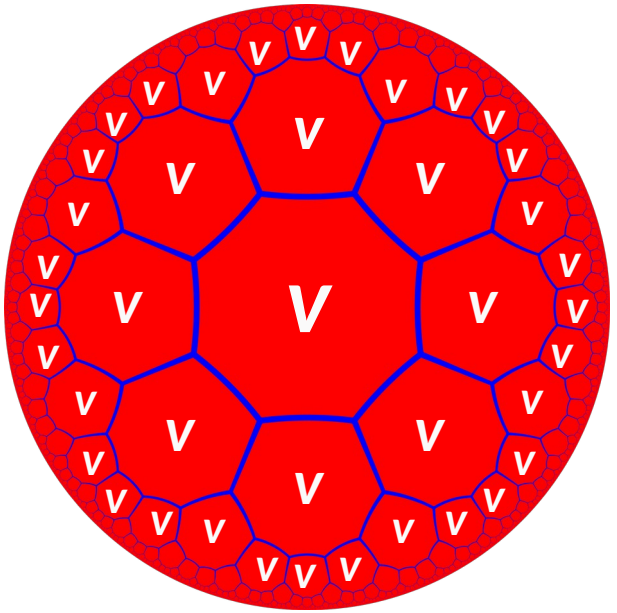
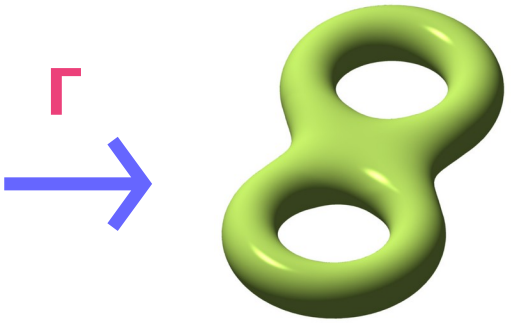
Hyperbolic Bloch-Floquet Theory





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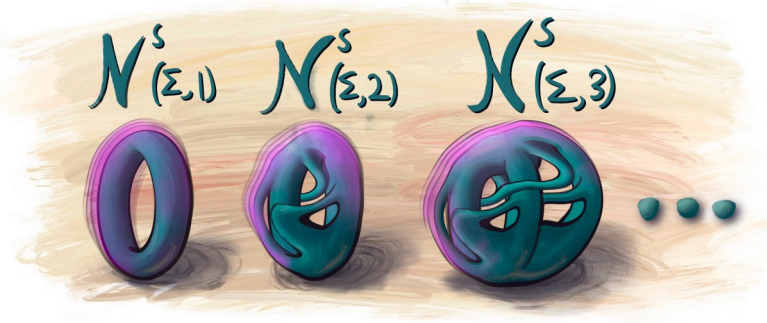
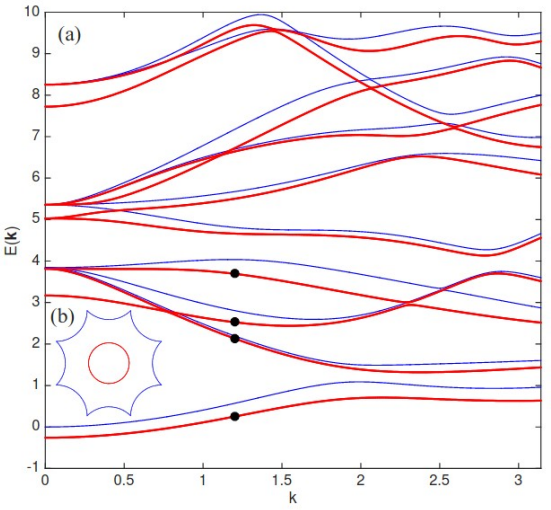
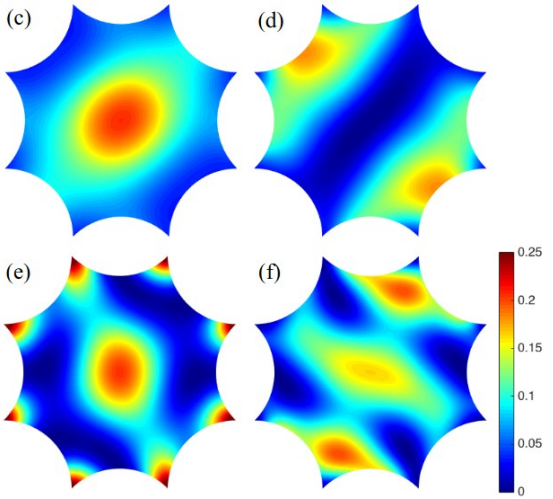


momentum





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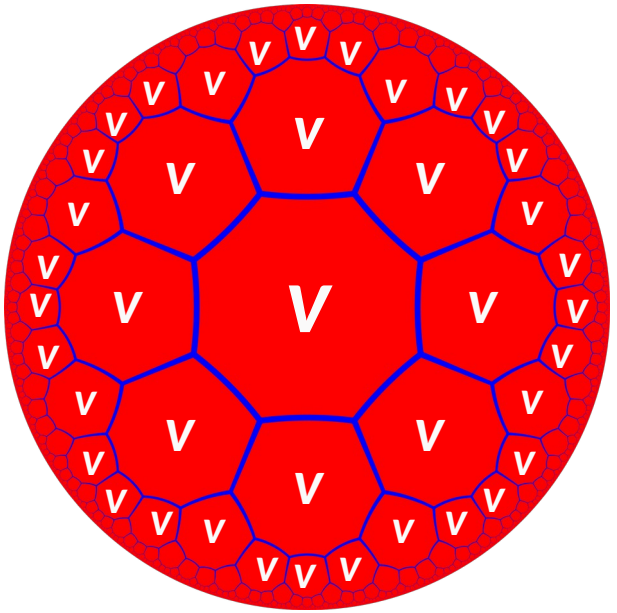
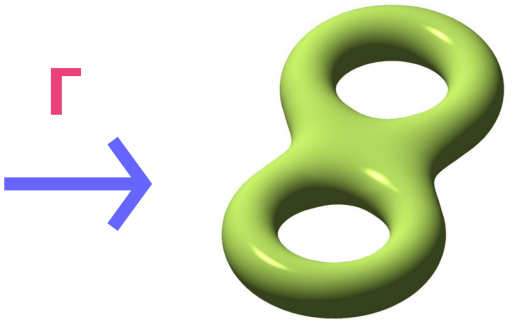
Hyperbolic Bloch-Floquet Theory





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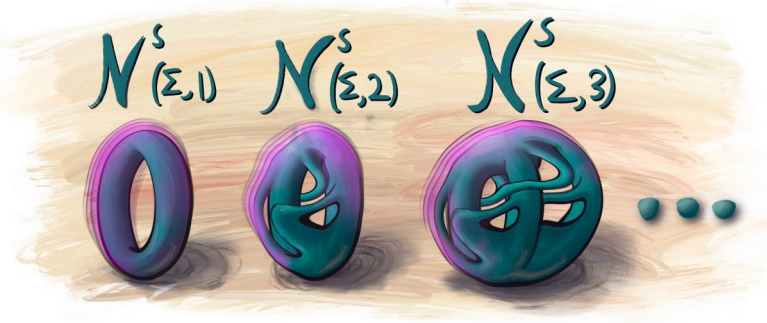
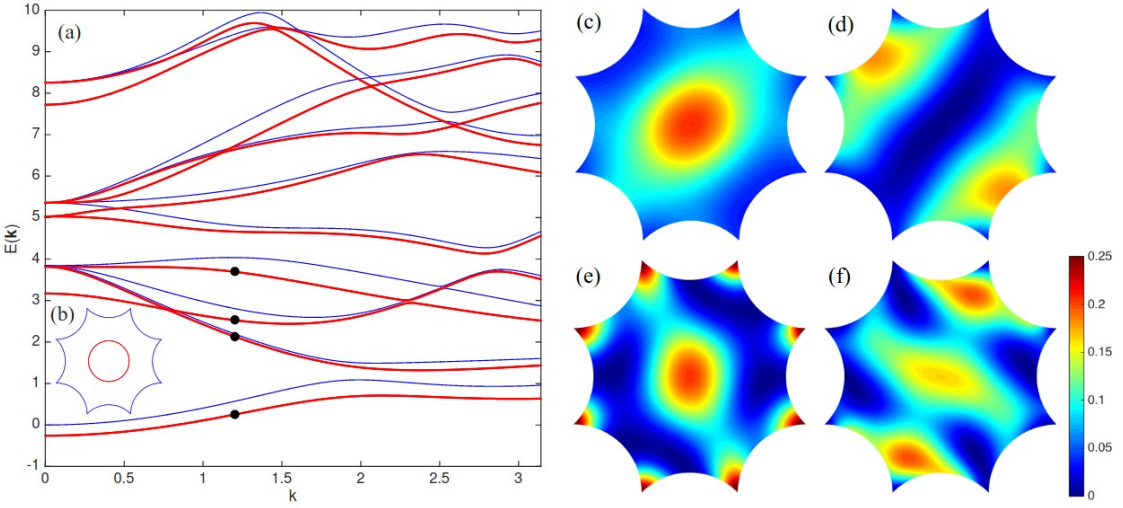
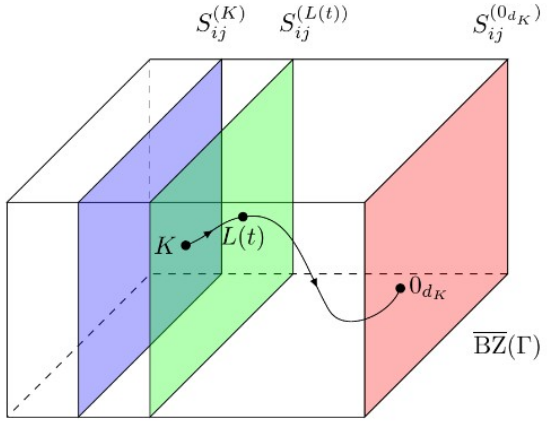


momentum





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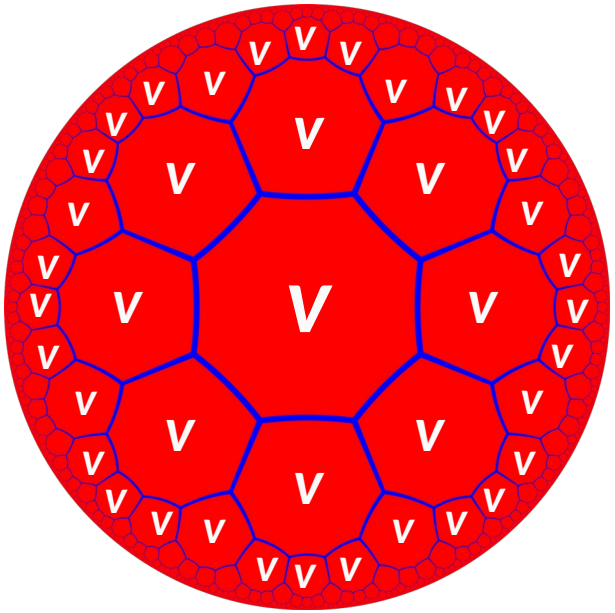
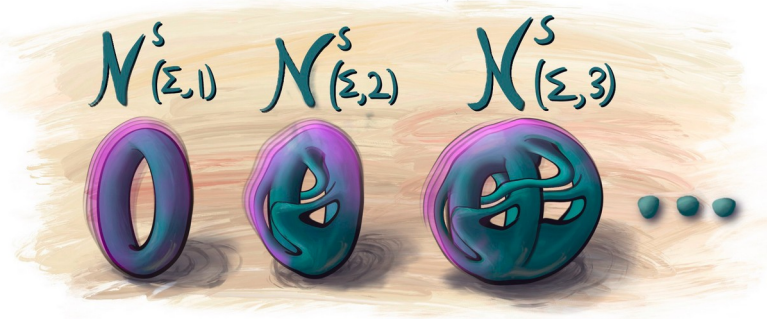
Hyperbolic Bloch-Floquet Theory





position **momentum**

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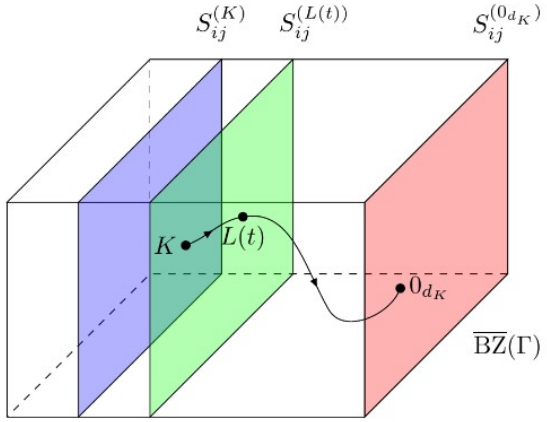
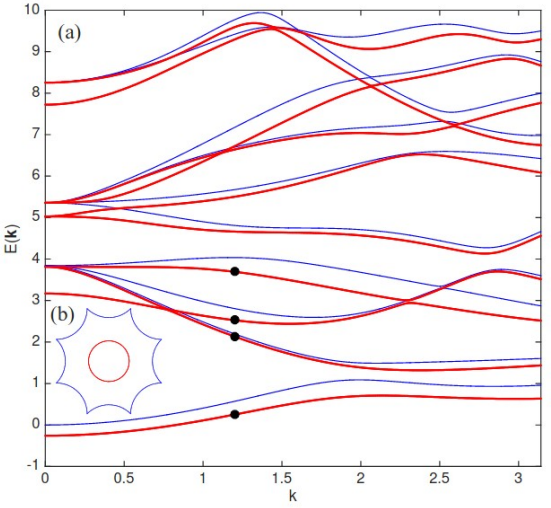
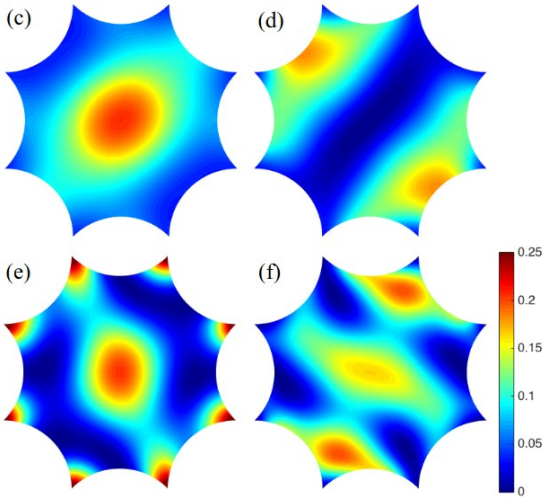
Maciejko, R (2022), *P.N.A.S.* 119, e2116869119
Boettcher, Gorshkov, Kollár, Maciejko, R, Thomale (2022), *Phys. Rev. B* 105, 125118
Nagy, R (2024), *Ann. Henri Poincaré* 25,1713—1732
Sun, Chen, Bzdušek, Maciejko (2024), *SciPost Phys.* 17.5.124







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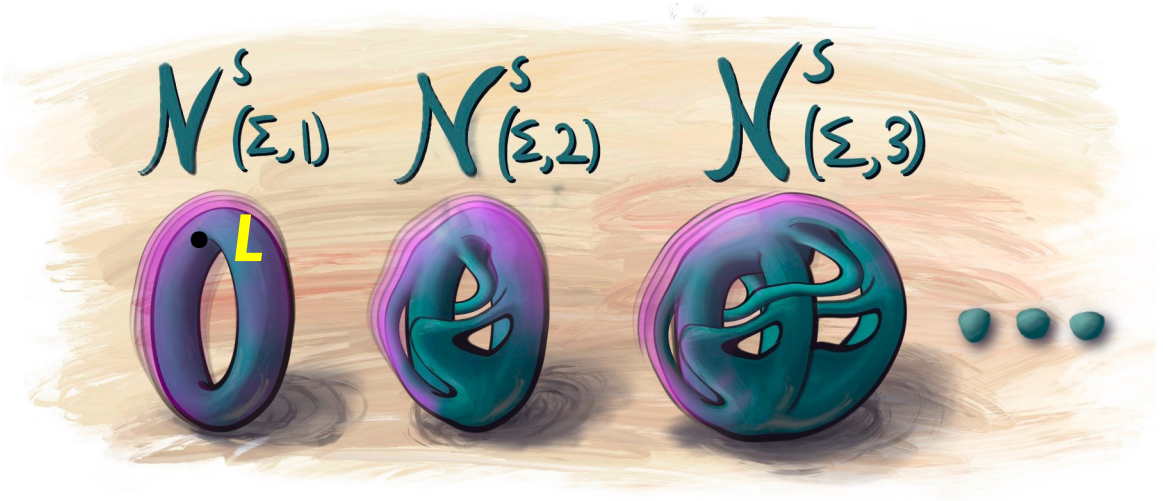
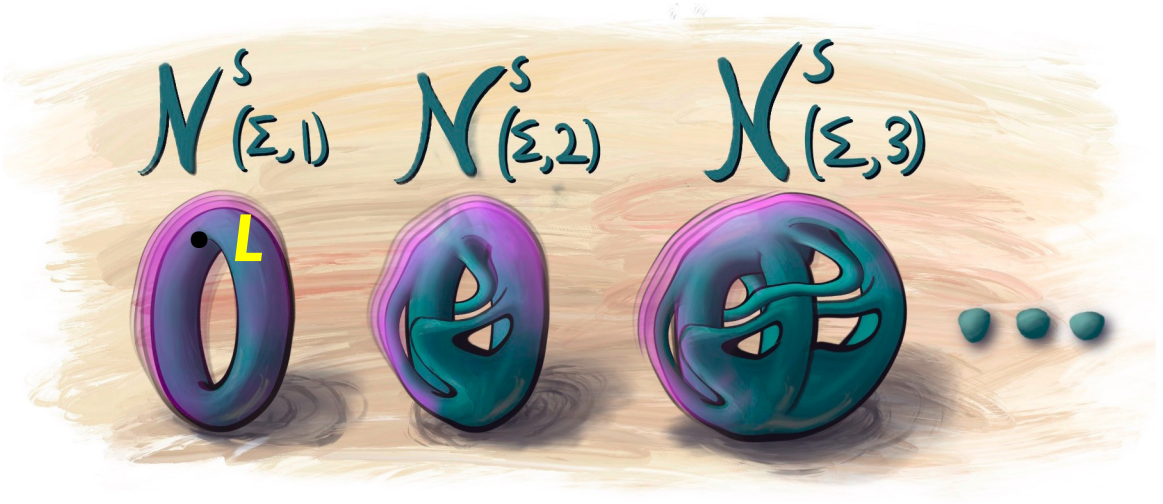
Higgs Bundles as Crystal Moduli



Position*

Brillouin Zone

Higgs Bundles as Crystal Moduli



Position*

Brillouin Zone

***Assume position space $\Sigma = \mathbb{H} / \Gamma$ is branched $r : 1$ cover in $\mathrm{Tot}(F) \to X$ for some curve X**

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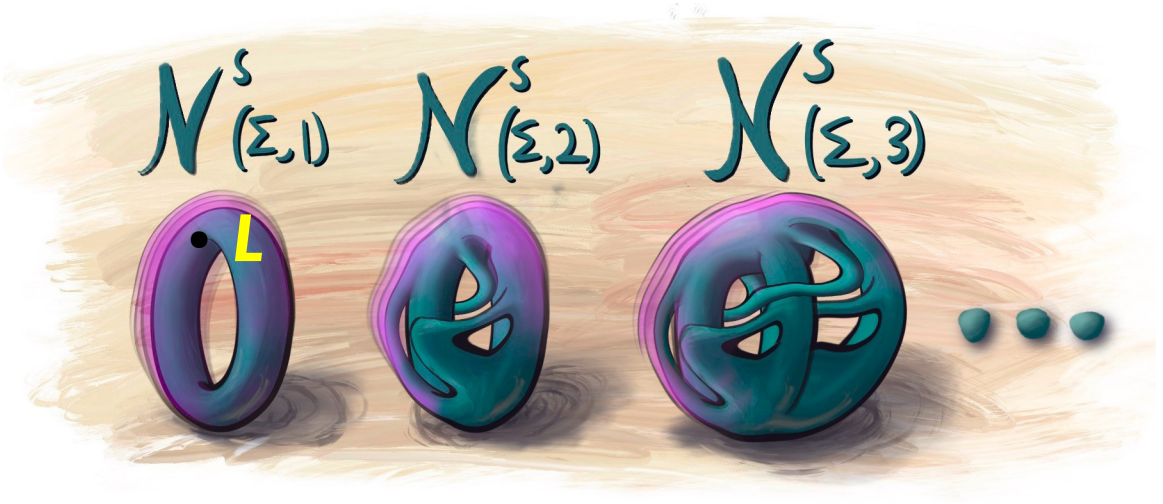
Higgs Bundles as Crystal Moduli



Position*

Brillouin Zone

***Assume position space $\Sigma = \mathbb{H} / \Gamma$ is branched $r:1$ cover in $\mathrm{Tot}(F) \to X$ for some curve X**

$\Rightarrow L \to \Sigma \approx (E,\theta) \to X$ via spectral correspondence, where E has rank r and $\theta : E \to E \otimes F$

Kienzle, R (2022), *Adv. Math.* 409, 108664

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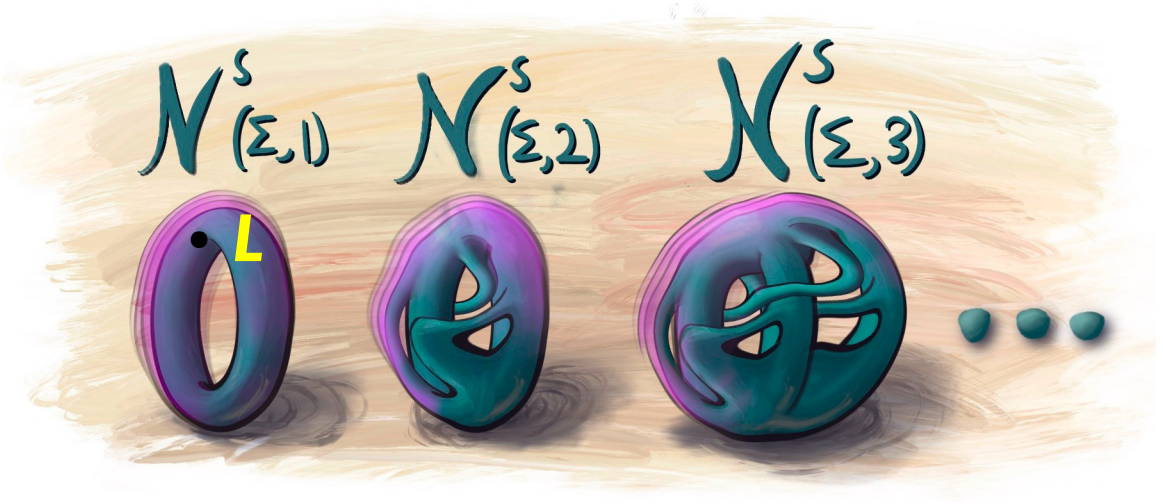
Higgs Bundles as Crystal Moduli

$\Sigma \to X$



Brillouin Zone

Consider all deformations of the real space crystal that preserve the property of ramifying *r* : 1 over X

Kienzle, R (2022), *Adv. Math.* 409, 108664

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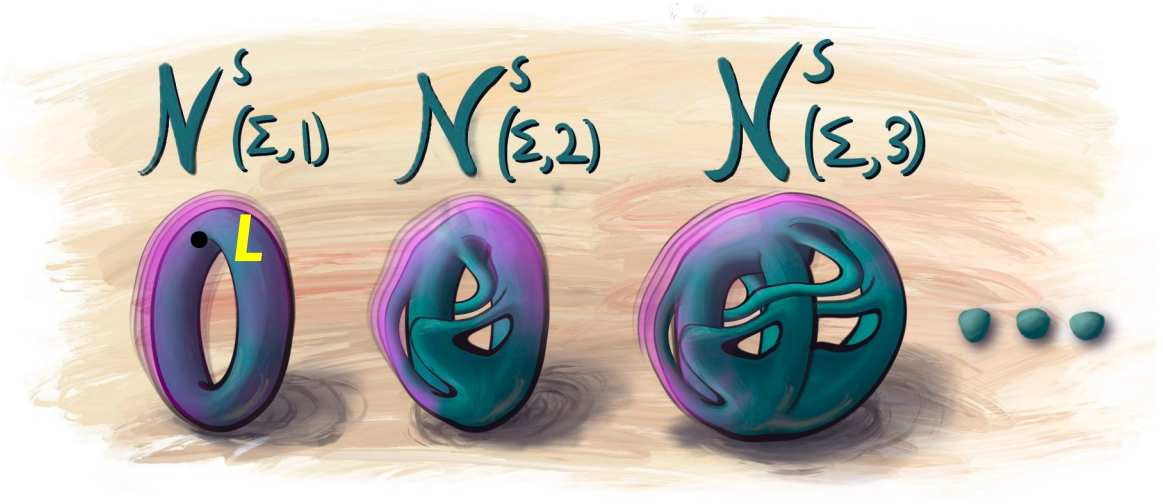
Higgs Bundles as Crystal Moduli



$\Sigma \to X$

Brillouin Zone

Consider all deformations of the real space crystal that preserve the property of ramifying $r : 1$ over X

⇒ Hitchin moduli space parametrizes all 2D hyperbolic crystals up to abelian momenta

Kienzle, R (2022), *Adv. Math.* 409, 108664

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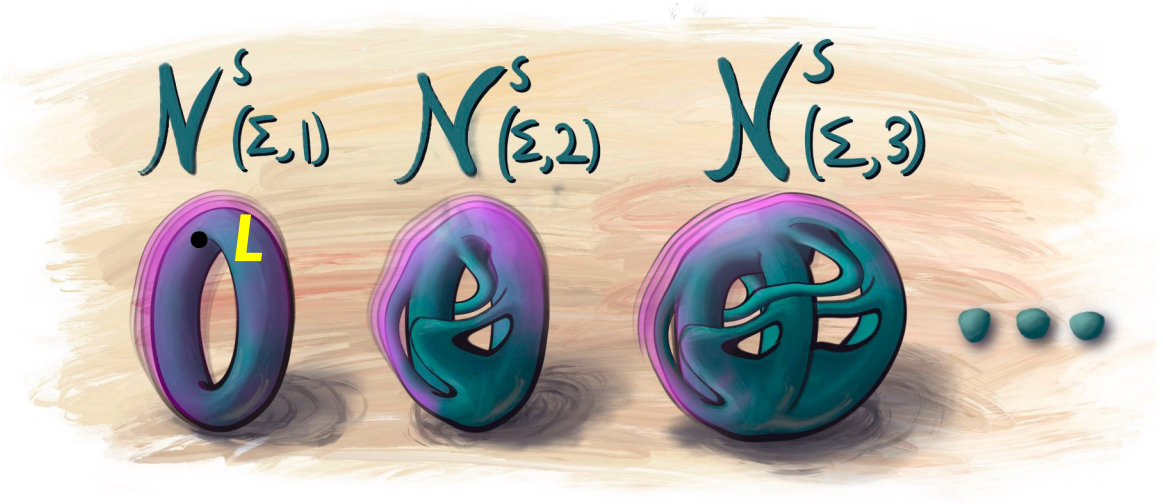
Higgs Bundles as Crystal Moduli



$\Sigma \to X$

Brillouin Zone

Consider all deformations of the real space crystal that preserve the property of ramifying $r : 1$ over X

⇒ Hitchin moduli space parametrizes all 2D hyperbolic crystals

Kienzle, R (2022), *Adv. Math.* 409, 108664
Banerjee, R (2023), arXiv: 2310.02413

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N=2 SUSY Yang-Mills	Hitchin Systems	Hyperbolic Band Theory
Moduli of vacua	Hitchin base	Hyperbolic crystallography
Seiberg-Witten curve	Spectral curve	Hyperbolic crystal
Lattice of EM charges	Regular Hitchin fiber	Abelian Brillouin zone
Semiclassical vacua	Large Higgs field limit	Thermodynamic limit

N=2 SUSY Yang-Mills	Hitchin Systems	Hyperbolic Band Theory
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Lattice of EM charges	Regular Hitchin fiber	Abelian Brillouin zone
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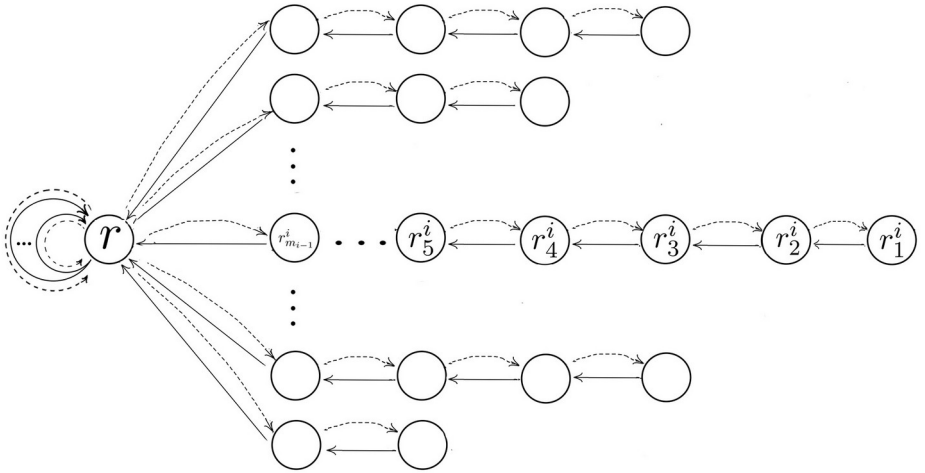
Kienzle, R (2022), *Adv. Math.* 409, 108664

Some Questions

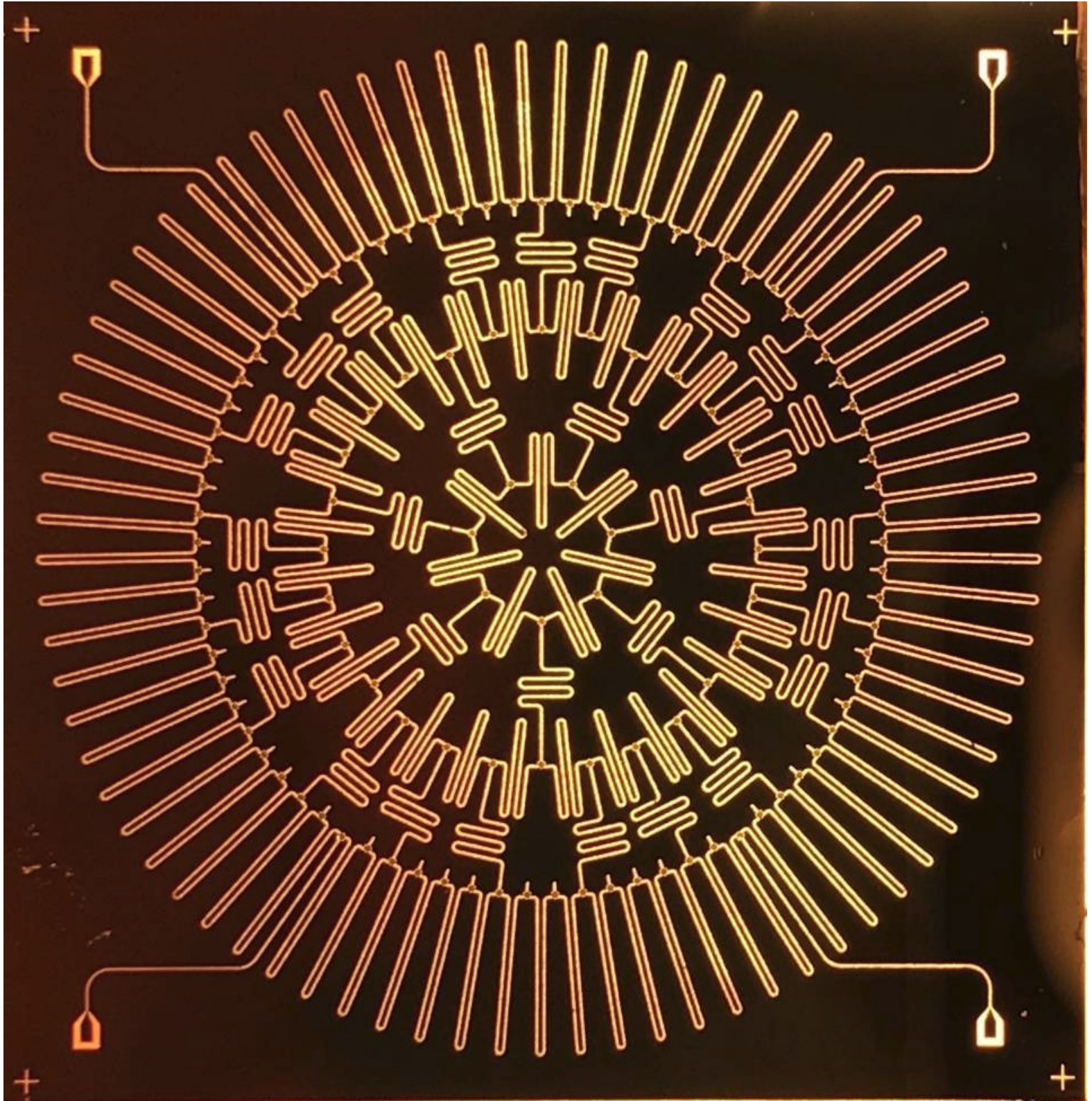
- Nakajima quiver variety for comet-shaped quivers with *g* loops and *n* arms embeds (*I*-compatibly) into a meromorphic Hitchin system on genus *g* curve with *n* simple poles



- Provides a simple moduli space of hyperelliptic hyperbolic crystals (with potential wells at the punctures) whose underlying *E* is trivial
- What is the role of the associated quiver gauge theory and quiver dualities in the band theory?
- Geometric engineering of QFTs ↔ band gap engineering?

More Questions

- How can we physically access such matter?



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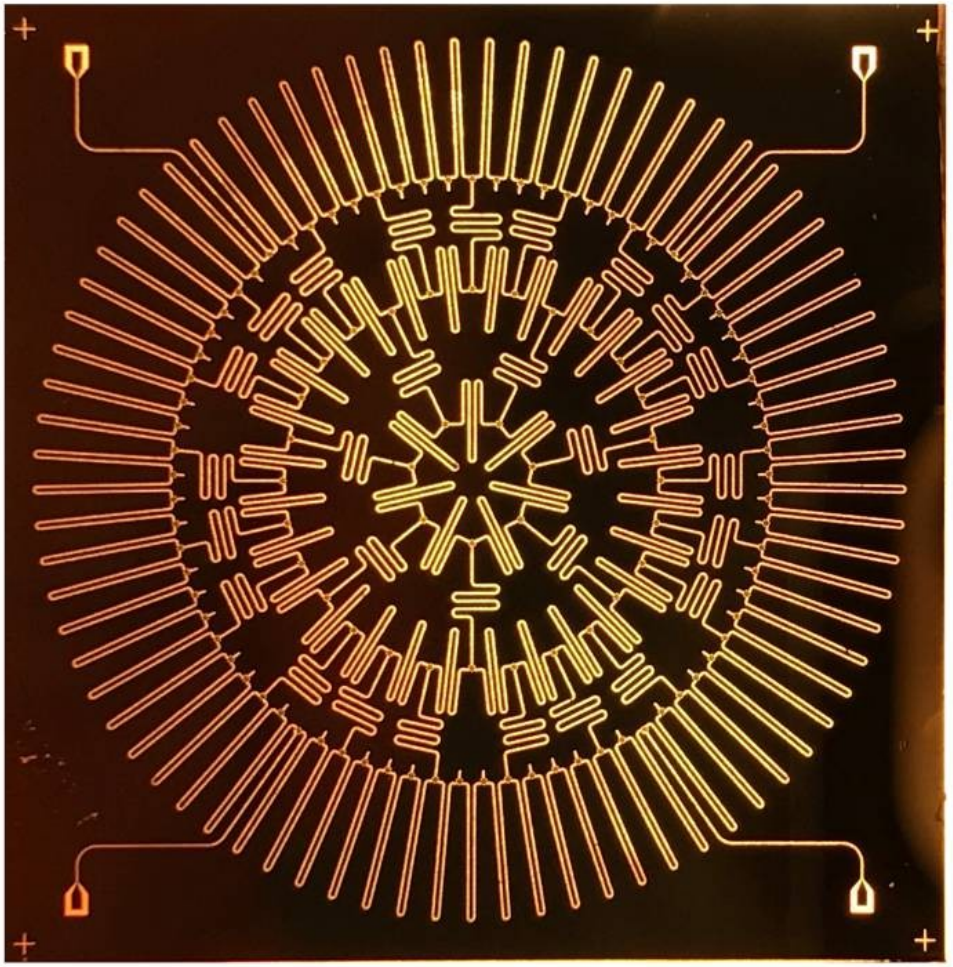
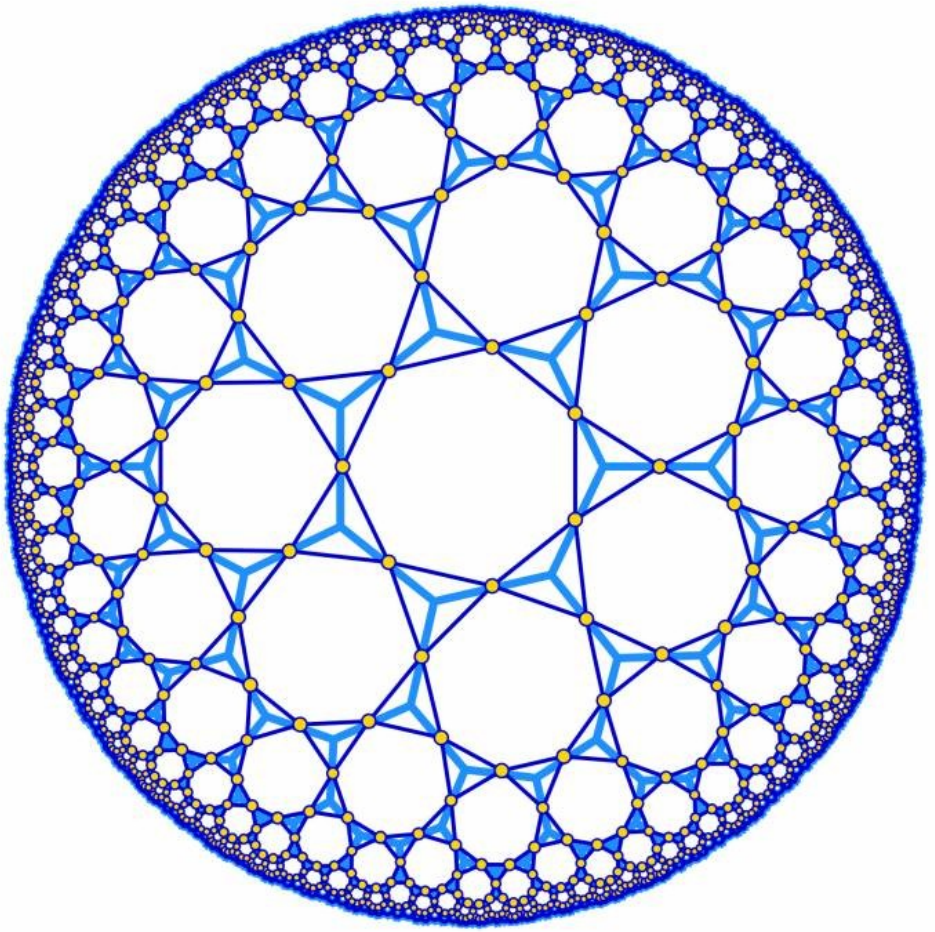
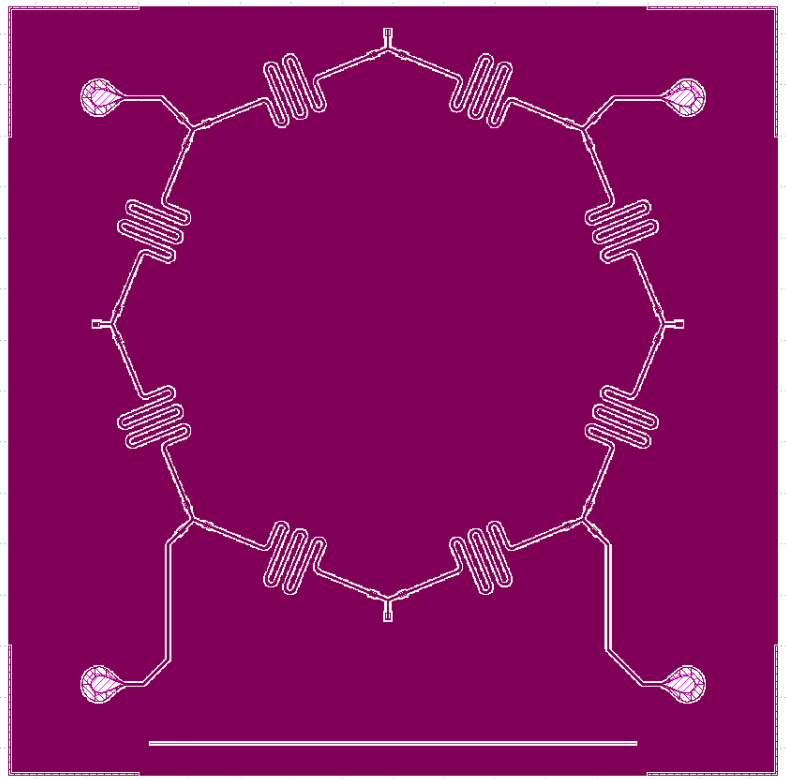
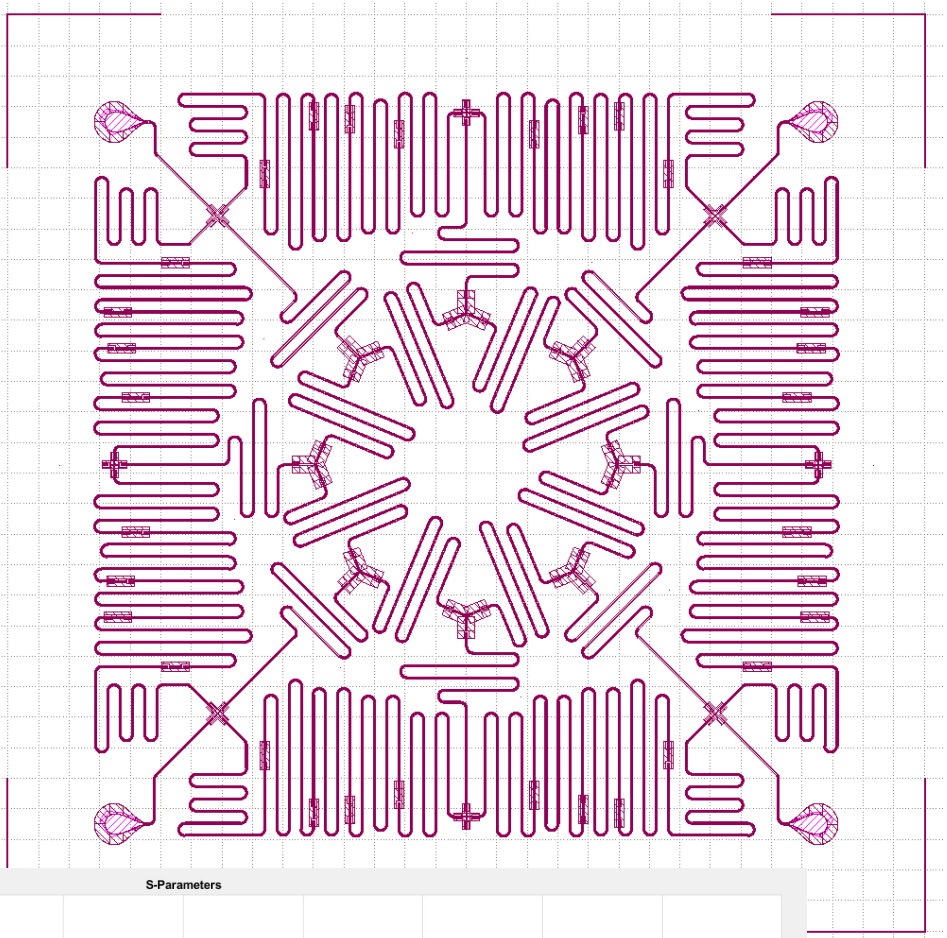
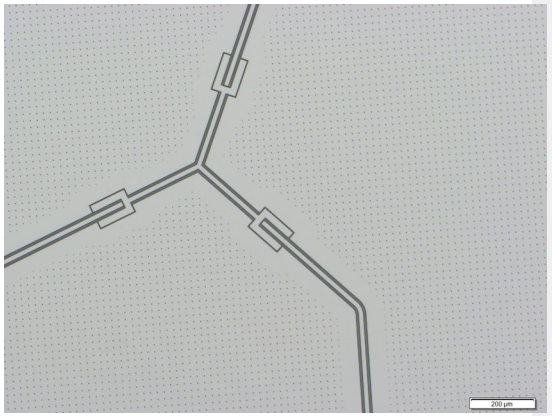


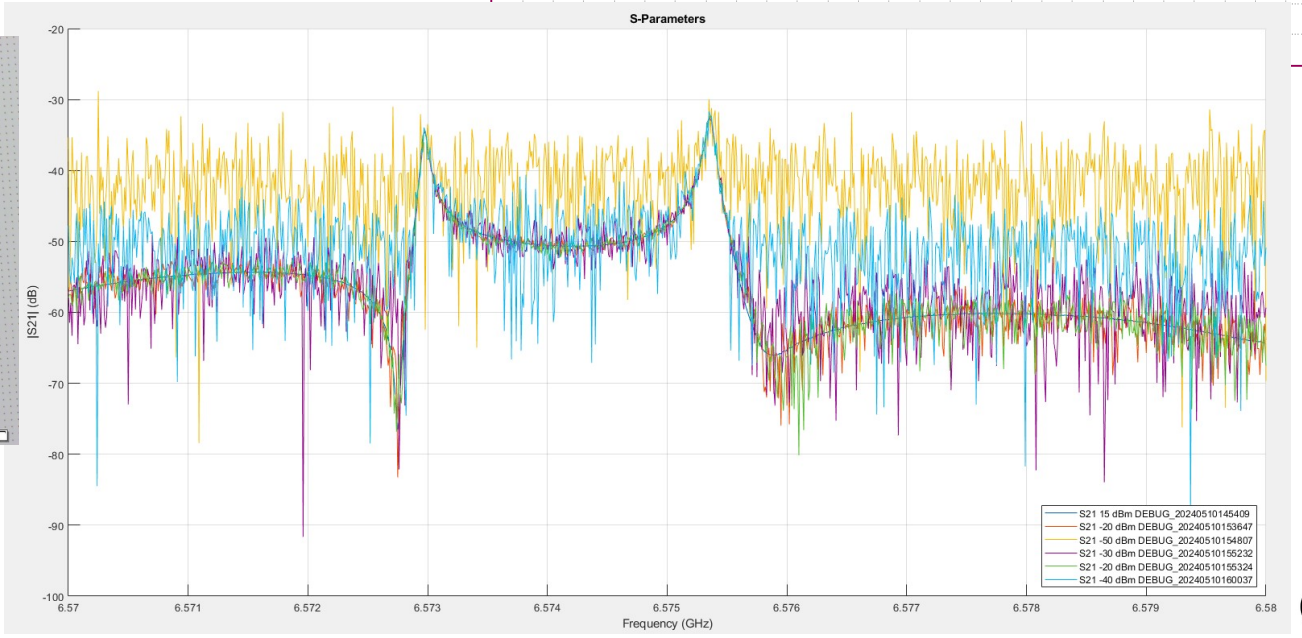


- Kollár, Fitzpatrick, Houck, Nature 571 (2019): 45–50 (Copyright Springer-Nature 2019)






200 µm


S-Parameters
|S21| (dB)
-20
-30
-40
-50
-60
-70
-80
-90
-100
6.57
6.571
6.572
6.573
6.574
6.575
6.576
6.577
6.578
6.579
6.58
Frequency (GHz)
S21 15 dBm DEBUG_20240510145409
S21 -20 dBm DEBUG_20240510153647
S21 -50 dBm DEBUG_20240510154807
S21 -30 dBm DEBUG_20240510155232
S21 -20 dBm DEBUG_20240510155324
S21 -40 dBm DEBUG_20240510160037

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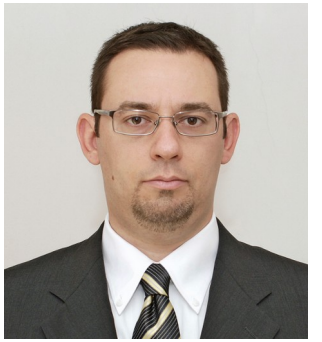

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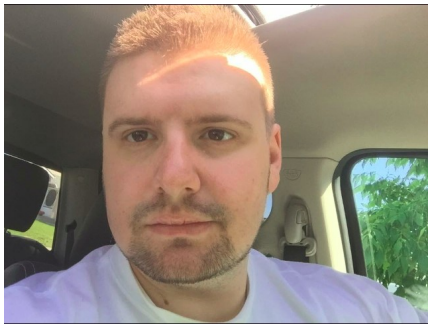

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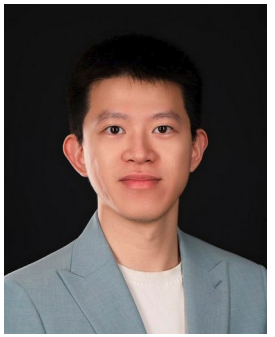

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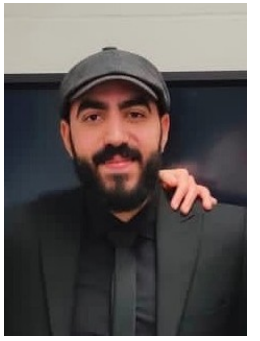

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Chris Mahadeo
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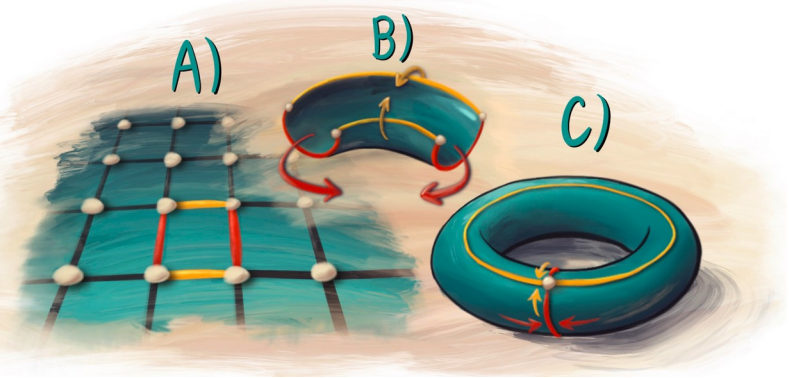
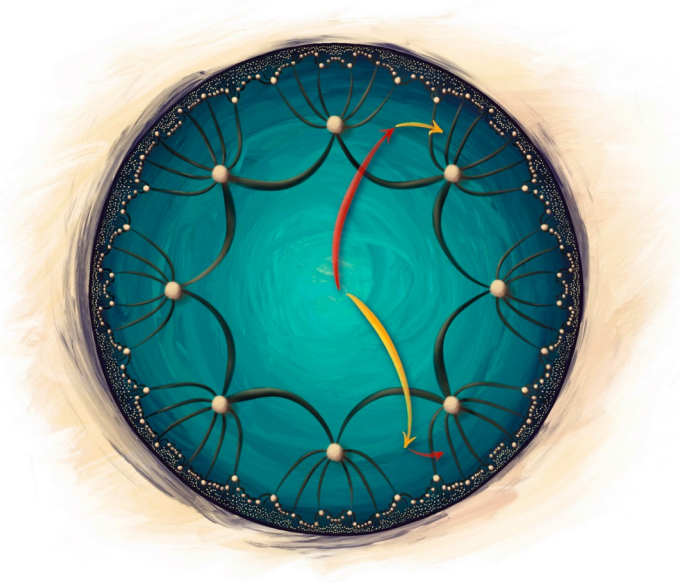
Collaborators





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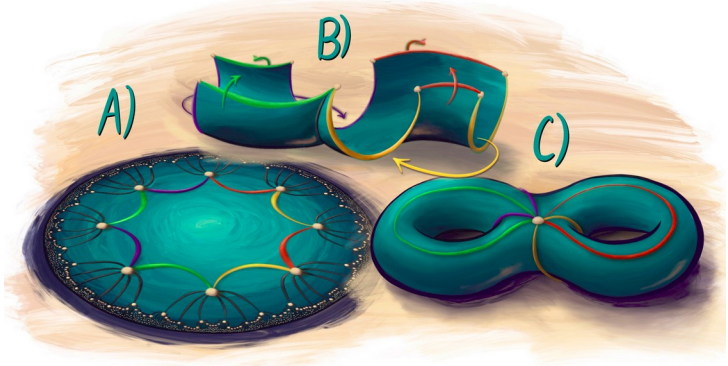

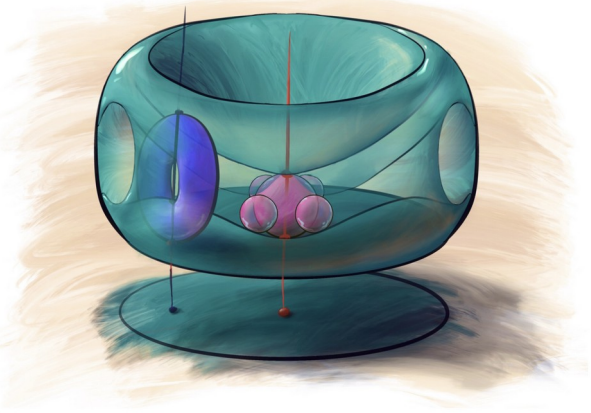

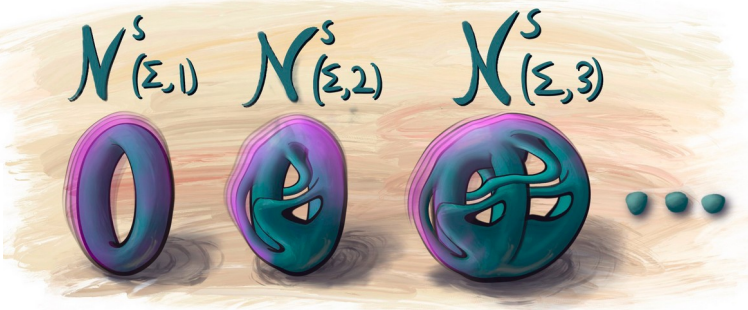

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Canadian Light Source | Centre canadien de rayonnement synchrotron

Digital Research Alliance of Canada | Alliance de recherche numérique du Canada

UK NATIONAL QUANTUM TECHNOLOGIES PROGRAMME

Prairies Economic Development Canada | Développement économique Canada pour les Prairies

Canada

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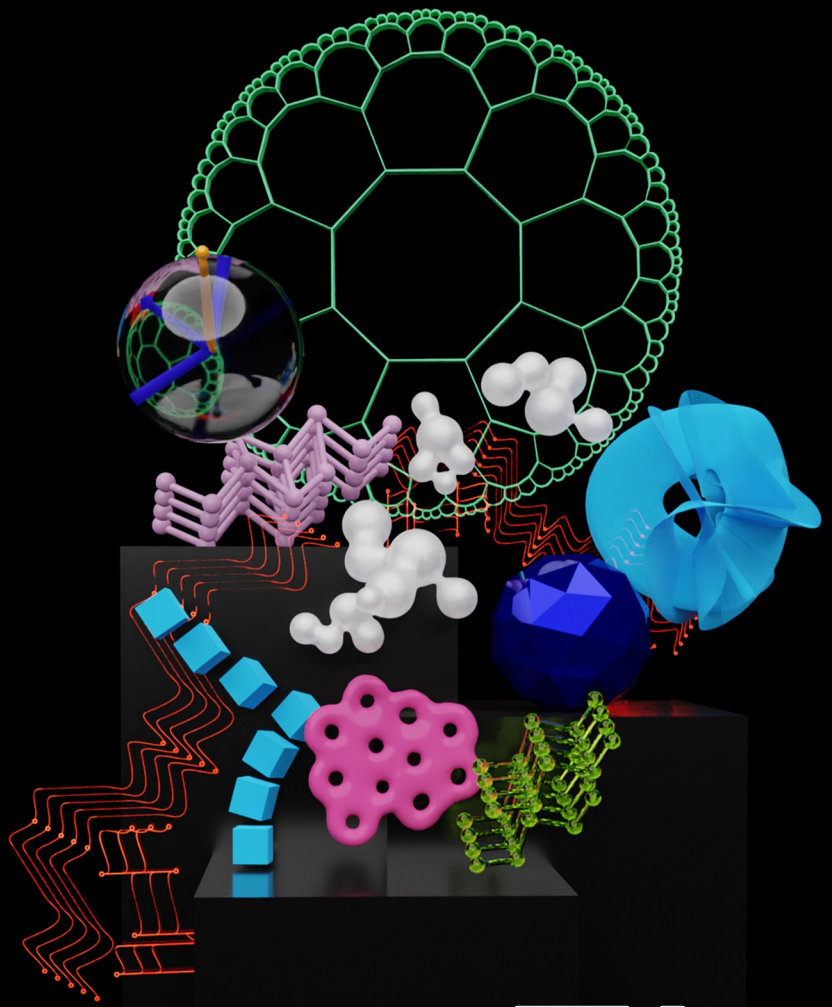

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Thank You