Models for neutrino masses and mixings and their phenomenological implications

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Plan of the talk

- Revisiting the data
  - relevant features of masses and mixings
- Patterns of neutrino mass matrices
  - Tri- and Bi- maximal mixings
  - the impact of large θ_{13}
  - how to build a successful model: the role of additional symmetries
  - models with no built-in hierarchy
- Phenomenological implications
  - neutrinoless double-beta decay
  - rare decays of charged leptons

Revisiting the data

excursus: the neutrinos mixing matrix U_{PMNS}

- mass ν_i and interaction ν_α eigenstates are different objects:

$$\nu_\alpha = U^\nu_{\alpha i}\,\nu_i$$

- we also need to take into account the rotations of the charged leptons:

$$\ell_\alpha = U^\ell_{\alpha i}\,\ell_i$$

- from the lagrangian in the interaction basis to the mass basis:

$$\mathcal{L} \sim \bar{\ell}_\alpha\,\gamma_\mu\,\nu_\alpha\,W^\mu \to \underbrace{(U^\ell_{\alpha i})^\dagger\,U^\nu_{\alpha j}}_{U_{PMNS}}\,\bar{\ell}_i\,\gamma_\mu\,\nu_j\,W^\mu$$

- usual assumption: U_{PMNS} is a 3×3 unitary mixing matrix (CKM-like), with three angles θ_{ij} and one (possible) CP phase δ

$$U_{PMNS} = R_{23}(\theta_{23})R_{13}(\theta_{13},\delta)R_{12}(\theta_{12})$$

Revisiting the data

observation: masses of neutrinos

- from oscillation experiments only mass differences

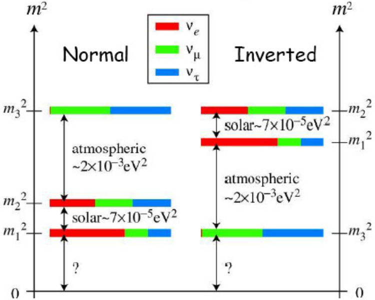
$$\Delta m^2_{21} \sim 8 \times 10^{-5}\, eV^2 \qquad \Delta m^2_{31} \sim 2.5 \times 10^{-3}\, eV^2$$

$$\sqrt{r} = \sqrt{\Delta m^2_{21}/\Delta m^2_{31}} \sim \lambda = 0.22$$

(a moderate hierarchy in the neutrino masses)

- the mass ordering is still unknown
- if absolute masses $m_i >> \Delta m^2$ $\rightarrow$ degenerate spectrum

in the 3-neutrino picture



Revisiting the data

observation: masses of neutrinos W. Rodejohann, Int.J.Mod.Phys. E20 (2011) 1833-1930

- what is the status on absolute masses ?
  - direct limits
    from experiments in which the non-zero neutrino mass influences the energy distribution of electrons in beta decays close to the kinematical endpoint of the spectrum
    $$m_i < 2.3\, eV \;@90\%\, CL$$
  - $0\nu\beta\beta$ decays
    controversial signal published by a part of the collaboration from the Heidelberg-Moscow experiment
    $$|m_{ee}| < 0.21 - 0.53\, eV \;@90\%\, CL$$
  - cosmology
    effects of neutrinos in cosmic structure formation:
    $$\Sigma m_i < 0.45 - 1.50\, eV \;@90\%\, CL$$

How to generate non-vanishing ν masses

- a Dirac mass term
  - right-handed neutrinos ν_R must be included in the standard picture lepton number L is conserved

weak isospin	ν_L	ν_R	$H = (h^+, h^0)$
I	$1/2$	0	$1/2$
I_3	$1/2$	0	$(+1/2, -1/2)$

$$\mathcal{L}_D = m_D\, \bar{\psi}_L\, \tilde{H}\, \nu_R + hc$$

with $\tilde{H} = -i\,(H^\dagger \tau_2)^T$

- Majorana mass terms
  - if lepton number is not conserved

$|\Delta I| = 1 \to \nu_L^T\, \nu_L$ we need two Higgs doublets

$$\mathcal{L}_{\nu_L} = \frac{m_L}{\Lambda}\, \psi_L^T\, \tilde{H}\, \tilde{H}\, \psi_L + hc$$

$|\Delta I| = 0 \to \nu_R^T\, \nu_R$ directly compatible with $SU(2) \times U(1)$

$$\mathcal{L}_{\nu_R} = M_R\, \nu_R^T\, \nu_R$$

How to generate non-vanishing ν masses

- the total lagrangian reads

$$\mathcal{L}_m = m_D\,\bar{\psi}_L\,\tilde{H}\,\nu_R + \frac{m_L}{\Lambda}\,\psi_L^T\,\tilde{H}\,\tilde{H}\,\psi_L + M_R\,\nu_R^T\,\nu_R$$

after electroweak simmetry breaking:

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \langle \tilde{H} \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\Downarrow$$

$$\mathcal{L}_m \sim \begin{pmatrix} m_L\,v^2/\Lambda & m_D \\ m_D^T & M_R \end{pmatrix} \text{ in the } (\nu_L, \nu_R) \text{ basis}$$

see-saw mechanism: $m_\nu = -m_D^T\,M_R^{-1}\,m_D \longrightarrow m_\nu \sim \dfrac{m_D^2}{M_R}$

for m_D ~100 GeV and $m_\nu \sim \sqrt{\Delta m_{31}^2}$ ~0.05 eV

$M_R \sim 10^{14} - 10^{15} GeV$ probe into GUT !

Which features are important?

observation: mixings of neutrinos

- from oscillation experiments

$$\sin^2\theta_{12} \sim 0.3 \qquad \sin^2\theta_{23} \sim 0.52$$
$$\sin^2\theta_{13} \sim 0.014 \qquad \delta_{\mathsf{CP}} \in [0,\, 360]$$

to explain the data, we can adopt two different points of view

#1: the measured values can be explained in a dynamical way

#2: the values are just numerical coincidences, no fundamental reasons behind them

Let us assume #1

The Tri-Bimaximal mixing approximation *Harrison, P.F. et al. Phys.Lett. B530 (2002) 167*

observation: one can consider a good starting point the following values of the mixing angles (particularly true before the T2K data)

$$\sin^2\theta_{12} = \frac{1}{3} \qquad \sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0$$

this implies the followwing structure of U_{PMNS}

$$U_{PMNS} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -1/\sqrt{6} & \sqrt{1/3} & -\sqrt{1/2} \\ -1/\sqrt{6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

related to θ_{13}

related to maximal 2-3 mixing

from unitarity

related to θ_{12}

- a very special pattern of the mixing matrix

The Tri-Bimaximal mixing approximation

- if we work in the basis where the charged leptons are diagonal

$$U_{PMNS} = U_\ell^\dagger U_\nu = U_\nu = U_{TBM}$$

$$\Downarrow$$

$$I$$

- since U_ν diagonalizes the neutrino mass matrix, we get:

$$m_\nu \;=\; U_{TBM}^\star \, \text{diag}(\underbrace{m_1, m_2, m_3}_{\text{neutrino masses}}) \, U_{TBM}^\dagger = \begin{pmatrix} x & y & y \\ y & x-v & y+v \\ y & y+v & x-v \end{pmatrix}$$

- x, y and v are complex parameters
- the mass matrix is independent on the neutrino masses
- mass matrix invariant under interchange of the 2^{nd} and 3^{rd} rows and columns (the $\mu - \tau$-symmetry: $\theta_{13} = 0$, $\theta_{23} = \pi/4$)

to get the TBM, the flavour model must produce m_ν as above

Beyond the Tri-Bimaximal mixing approximation

- of course, this cannot be the end of the story
  - we have to generate $\theta_{13} \neq 0$
  - we should include quarks in the picture

$$\Downarrow$$

corrections to this pattern must be included

two different sources of corrections (from higher dimensional operators)

corrections from m_ν

$$m_\nu \rightarrow m_\nu + \delta m_\nu$$
$$U_\nu = U_{TBM} + \delta U_\nu$$

corrections from m_ℓ

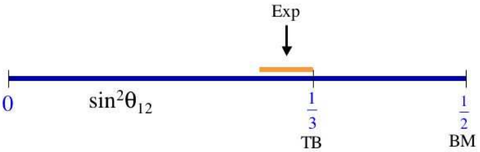
$$m_\ell \rightarrow m_\ell + \delta m_\ell$$
$$U_\ell = I + \delta U_\ell$$

$$U_{PMNS} = (I + \delta U_\ell)^\dagger \, (U_{TBM} + \delta U_\nu) \neq U_{TBM}$$

Beyond the Tri-Bimaximal mixing approximation

Altarelli&Feruglio Rev.Mod.Phys. 82 (2010) 2701-2729

- the size of the corrections depends on the solar angle, the best measured up to now



at the most $\mathcal{O}(\lambda^2)$

the general prediction is that all mixing angles (and masses) are corrected by terms of $\mathcal{O}(\lambda^2)$

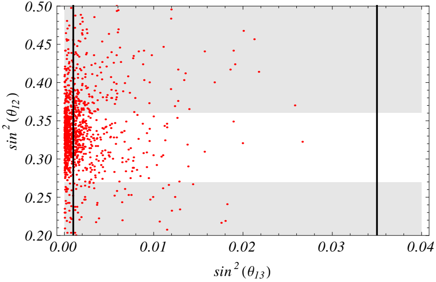
$$U_{PMNS} = U_{TBM} + \mathcal{O}(\lambda^2)$$

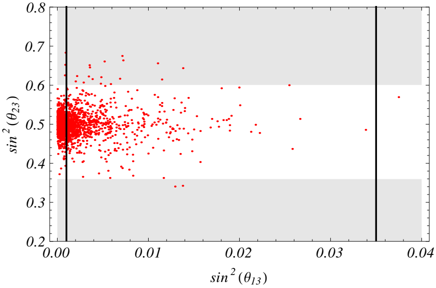
(notice that this is also true for quarks: $V_{CKM} = I + \mathcal{O}(\lambda^2)$)

Beyond the Tri-Bimaximal mixing approximation

G.Altarelli&DM, J.Phys.G G36 (2009) 085005

tipical scatter plots in the $(\sin^2\theta_{13} - \sin^2\theta_{12})$ and $(\sin^2\theta_{13} - \sin^2\theta_{23})$ planes





- many of the points are close to their TBM values
- deviations are small, as expected

in particular, θ_{13} is marginally allowed in these schemes
but there exist *tricks* to generate large θ_{13} mantaining the solar angle in agreement with data $\Longrightarrow$ TBM is still alive !

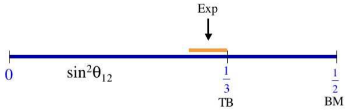
In a better shape now: the Bi-maximal mixing approximation

(particularly true after the T2K data)

$$\sin^2\theta_{12} = \frac{1}{2} \qquad \sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0$$

$$U_{BM} = \begin{pmatrix} \sqrt{1/2} & -\sqrt{1/2} & 0 \\ 1/2 & 1/2 & -\sqrt{1/2} \\ 1/2 & 1/2 & \sqrt{1/2} \end{pmatrix}$$

- now we have to require a large shift for θ_{12}, at the level of $\mathcal{O}(\lambda)$

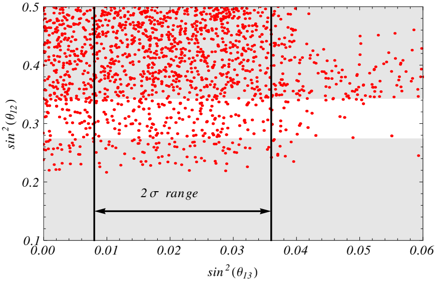


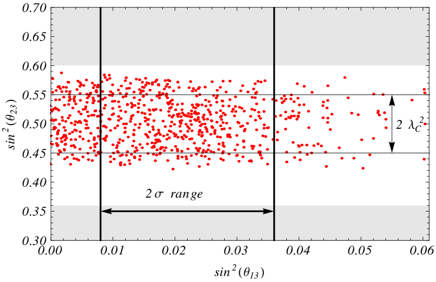
- this induces a shift of the same order in $\theta_{13} \Longrightarrow$ large reactor angles are easily obtained in this framework

In a better shape now: the Bi-maximal mixing approximation

example from Altarelli-Feruglio-Merlo, JHEP 0905 (2009) 020 and DM,JHEP 1110 (2011) 010

- the pattern of corrections is such as to leave θ_{23} almost maximal





- the models also provide a good description of quarks:

$$V_{CKM} = \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}$$

How to build flavour models *C.Hagedorn, talk at FLASY2011*

- Three families of elementary particles observed
- Strong hierarchy among charged fermions
- Mass hierarchy in the ν sector is much milder, ordering and m_0 unknown
- Only λ is sizable in the quark sector
- Special lepton mixing pattern could be realized? Which one?
- No excessive flavor violation observed, all in accordance with Standard Model

$$\Downarrow$$

Necessity of Constraints on Couplings y^u_{ij}, y^l_{ij}, etc.

Necessity of Flavor Symmetry G_F ?!

How to build flavour models

- Yukawa couplings

$$(y_\nu)_{ij}\, L_i\, \tilde{H}\, \nu^c_j \qquad (y_M)_{ij}\, \nu^c_i\, \nu^c_j \quad \text{with} \quad y_{\nu,M} \in \mathbb{C}$$

- Enforce invariance under G_F
  $\hookrightarrow$ Constraints on y_{ij}
  $\hookrightarrow$ Extension of scalar sector needed to break the symmetry

multi-Higgs doublets	or	flavon fields (gauge singlets)
$y_{ij,k}\, L_i\, \tilde{H}_k\, \nu^c_j$	or	$y_{ij,k}\, L_i\, \tilde{H}\, \nu^c_j \left(\frac{\phi_k}{(M,\Lambda)}\right)$

but in both cases alignment is needed

- many possible choices for G_F
  abelian, non-abelian, continuous, discrete, local, global, spontaneously broken, commuting with the gauge group or not, broken at low or high energies...

How to build flavour models

G_F invariant theory

residual symmetry in the neutrino sector $\boxed{G_\nu} \Longrightarrow \boxed{U_\nu}$

residual symmetry in the charged lepton sector $\boxed{G_\ell} \Longrightarrow \boxed{U_\ell}$

$$\begin{aligned} \text{if } U_\nu &\neq U_\ell \\ U_{PMNS} &= U_\ell^\dagger U_\nu \end{aligned}$$

- choices for G_ν, G_ℓ dictated by the wanted pattern for U_{PMNS}
- permutation groups like A_4 and S_4 suitable for TBM
- the breakings of the generators G_ν and G_ℓ usually generate NLO corrections

Models with no built-in angles

Let us assume #2: the TBM (BM or whatever) does not play any role

- class of models with generally large θ_{23} and small $\theta_{13} \sim \mathcal{O}(\lambda, \lambda^2)$
- hierarchies in masses and mixings due to different flavour quantum numbers

$$\Downarrow$$

easy to implement using an abelian $U(1)$ symmetry Froggatt-Nielsen,NPB147, 277 (1979)

- suppose some fields transform as $\psi \to e^{i\, q_\psi}\, \psi$
- a generic mass term is

$$\bar{\psi}_L\, m\, \psi_R\, H \to e^{i\,(-q_L + q_R + q_H)}\, \bar{\psi}_L\, m\, \psi_R\, H$$

- invariant if $-q_L + q_R + q_H = 0$
- $U(1)$ is broken by the vev of flavon fields θ, so that

$$\bar{\psi}_L\, m\, \psi_R\, H \left(\frac{\theta}{\Lambda}\right)^k \text{with} \left(\frac{\theta}{\Lambda}\right) \sim \mathcal{O}(\lambda, \lambda^2)$$

- the suppression factor depends on the index k

Models with no built-in angles

- in this way it is possible to build models that reproduce the salient features of ν masses and mixings (in $SU(5)$ inspired models also quarks are well described) Altarelli-Feruglio-Masina JHEP **0301**, 035 (2003)
- different situations described by matrices with similar structure

$$m_\nu = \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \ (\mathtt{A,SA,H}) \quad , \qquad m_\nu = \begin{pmatrix} \epsilon^2 & 1 & 1 \\ 1 & \eta^2 & \eta^2 \\ 1 & \eta^2 & \eta^2 \end{pmatrix} \ (\mathtt{IH}) \quad ,$$

- A= anarchical models SA= semi-anarchical H= hierarchical IH= inverted hierarchy
- for H models, r and θ_{23} related to Det of the $2-3$ subsector

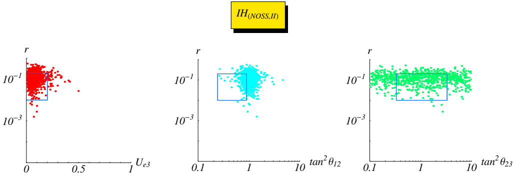
Model	par	$Det(23)$	r	U_{e3}	$\tan^2\theta_{12}$	$\tan^2\theta_{23}$
A	$\epsilon=1$	O(1)	O(1)	O(1)	O(1)	O(1)
SA	$\epsilon=\lambda$	O(1)	O(d_{23}^2)	O(λ)	O(λ^2/d_{23}^2)	O(1)
$\mathtt{H}_{2\theta}$	$\epsilon=\lambda^2$	O(λ^2)	O(λ^4)	O(λ^2)	O(1)	O(1)
$\mathtt{H}_{\theta}$	$\epsilon=\lambda^2$	0	O(λ^6)	O(λ^2)	O(1)	O(1)
IH	$\epsilon=\eta=\lambda$	O(λ^4)	O(λ^2)	O(λ^2)	1+O(λ^2)	O(1)

Models with no built-in angles

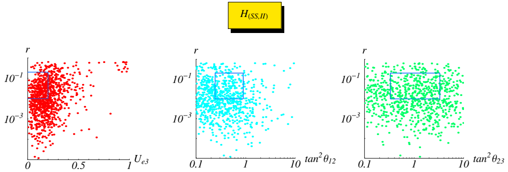
Altarelli-Feruglio-Masina JHEP **0301**, *035 (2003)*

$\mathcal{O}(1)$ parameters extracted in the interval $[0.5, 2]$

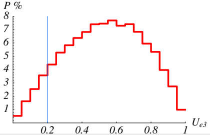
inverted hierarchy



normal hierarchy



anarchy



a good option for a relatively large θ_{13} !

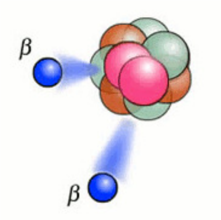
Neutrinoless double beta decay ($0\nu\beta\beta$)

for a review, see W. Rodejohann, Int. J. Mod.Phys. E **20**, *1833 (2011)*

- transition of a nucleus into a nucleus with proton number larger by two units

$$(A, Z) \rightarrow (A, Z+2) + 2e \qquad (0\nu\beta\beta)$$

- NOT allowed in the SM
- requires violation of the lepton number



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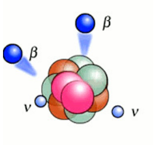


- $(A, Z) \rightarrow (A, Z+2) + 2e + 2\bar{\nu}_e$
- allowed in the SM
- second order weak transition

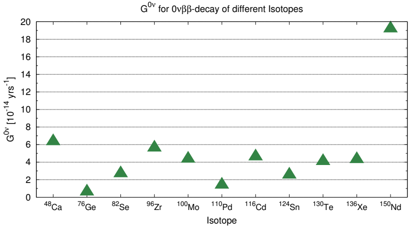
- Standard Interpretation: $0\nu\beta\beta$ mediated by light and massive Majorana neutrinos (the ones which oscillate); all other mechanisms are negligible

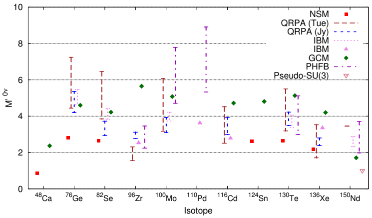
Neutrinoless double beta decay ($0\nu\beta\beta$)

The expression for the decay rate is

$$\Gamma^{0\nu} = G^{0\nu}(Q,Z)\,|\mathcal{M}^{0\nu}|^2 \frac{m_{eff}^2}{m_e^2}$$

- $G^{0\nu}(Q,Z)$ is the phase space factor
- m_{eff}^2 is a particle physics parameter in case of light neutrino exchange
- $\mathcal{M}^{0\nu}$ are the nuclear matrix elements





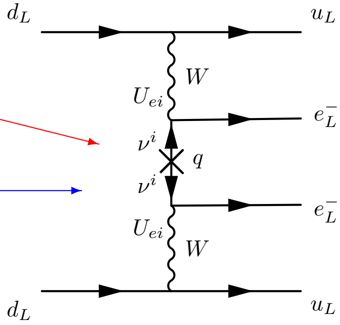
- large uncertainties from $\mathcal{M}^{0\nu}$

here we concentrate on m_{eff}

Neutrinoless double beta decay ($0\nu\beta\beta$)

RH $\bar{\nu}$ emitted

LH ν absorbed



$$A = \sum G_F^2\, U_{ei}^2\, \gamma_\mu\, \gamma_+ \frac{\not{q} + m_i}{q^2 - m_i^2}\, \gamma_\nu\, \gamma_- \simeq \sum G_F^2\, U_{ei}^2\, \frac{m_i}{q^2} \gamma_\mu\, \gamma_+\, \gamma_\nu$$

$\gamma_\pm = (1 \pm \gamma_5)/2$

The decay width is proportional to the square of the so-called effective mass

$$m_{eff} = \left| \sum U_{ei}^2\, m_i \right| = \left| m_1\, |U_{e1}|^2 + m_2\, |U_{e2}|^2\, e^{2i\alpha} + m_3\, |U_{e3}|^2\, e^{2i\beta} \right|$$

Neutrinoless double beta decay ($0\nu\beta\beta$)

- normal ordering

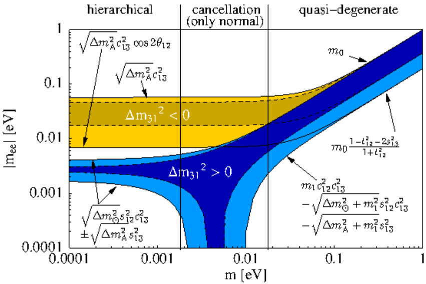
$$m_{eff}^{\rm nor} = \left| m_1\, c_{12}^2\, c_{13}^2 + \sqrt{m_1^2 + \Delta m^2_{sol}}\, s_{12}^2\, c_{13}^2\, e^{2i\alpha} + \sqrt{m_1^2 + \Delta m^2_{atm}}\, s_{13}^2\, e^{2i\beta} \right|$$

- inverted ordering

$$m_{eff}^{\rm inv} = \left| \sqrt{m_3^2 + \Delta m^2_{atm}}\, c_{12}^2\, c_{13}^2 + \sqrt{m_3^2 + \Delta m^2_{sol} + \Delta m^2_{atm}}\, s_{12}^2\, c_{13}^2\, e^{2i\alpha} + m_3\, s_{13}^2\, e^{2i\beta} \right|$$

- quasi degenerate

$$m_{eff}^{\rm QD} = m_0 \left| c_{12}^2\, c_{13}^2 + s_{12}^2\, c_{13}^2\, e^{2i\alpha} + s_{13}^2\, e^{2i\beta} \right|$$



Neutrinoless double beta decay ($0\nu\beta\beta$)

- in specific models there exist non trivial relations among ν masses
- this happens when the ν mass entries are related in non-trivial way (neutrino mass sum-rule); for example G.Altarelli&DM, J.Phys.G G36 (2009) 08500

$$m_\nu^{TBM} = \begin{pmatrix} x & y & y \\ y & x-v & y+v \\ y & y+v & x-v \end{pmatrix}; \qquad v = -\frac{(x-y)^2}{x-4y}$$

$$\Longrightarrow \quad \frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

- much more predictivity because of the reduced numbers of independent model parameters
  - the parameter space for $0\nu\beta\beta$ cannot be completely filled
  - one usually finds a lower value for the lightest neutrino mass

Neutrinoless double beta decay ($0\nu\beta\beta$)

L.Dorame, DM, S.Morisi, E.Peinado and J.W.F.Valle,arXiv:1111.5614 [hep-ph]

- a complete catalog of sum-rules :

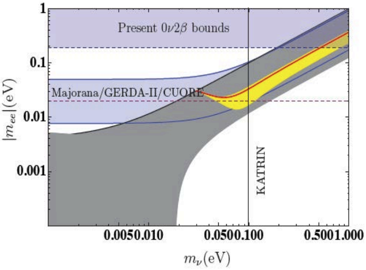
$$A)\quad \chi\, m_2^\nu + \xi\, m_3^\nu = m_1^\nu$$

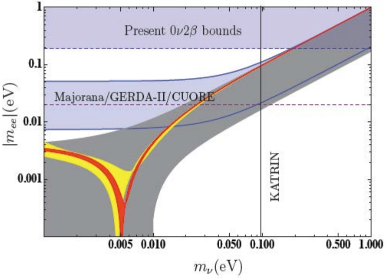
$$B)\quad \frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} = \frac{1}{m_1^\nu}$$

$$C)\quad \chi\sqrt{m_2^\nu} + \xi\sqrt{m_3^\nu} = \sqrt{m_1^\nu}$$

$$D)\quad \frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}} \qquad \chi, \xi = \text{ integer numbers}$$

type C) with $(\chi, \xi) = (3, 3)$ and $(\chi, \xi) = (2, 1)$





Neutrinoless double beta decay ($0\nu\beta\beta$)

- a complete catalog of sum-rules:

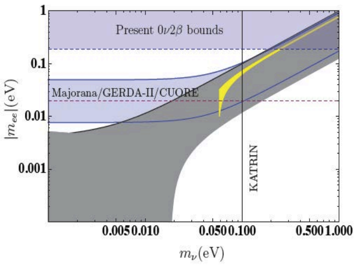
$$A) \quad \chi\, m_2^\nu + \xi\, m_3^\nu = m_1^\nu$$

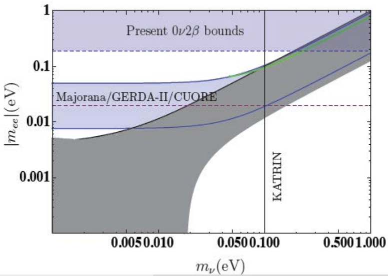
$$B) \quad \frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} = \frac{1}{m_1^\nu}$$

$$C) \quad \chi\sqrt{m_2^\nu} + \xi\sqrt{m_3^\nu} = \sqrt{m_1^\nu}$$

$$D) \quad \frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}} \qquad \chi, \xi = \text{ integer numbers}$$

type A) with $(\chi, \xi) = (3, 3)$



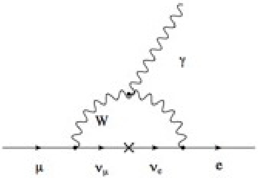


Rare decays of leptons

- The existence of the flavour neutrino mixing implies that the individual lepton charge L_ℓ , $\ell = e, \mu, \tau$, are not conserved and processes like $\mu \to e\gamma$, $\mu \to eee$, $\tau \to e\gamma$, $\tau \to \mu\gamma$ etc should take place

Process	Present Bounds	Expected Future Bounds
BR($\mu \to e, \gamma$)	2.4×10^{-12}	$\mathcal{O}(10^{-13} - 10^{-14})$
BR($\mu \to e, e, e$)	1.1×10^{-12}	$\mathcal{O}(10^{-13} - 10^{-14})$
BR($\mu \to e$ in Nuclei)	1.1×10^{-12}	$\mathcal{O}(10^{-13} - 10^{-14})$
BR($\tau \to e, \gamma$)	3.3×10^{-8}	$\mathcal{O}(10^{-8})$
BR($\tau \to e, e, e$)	2.7×10^{-8}	$\mathcal{O}(10^{-8})$
BR($\tau \to \mu, \gamma$)	6.8×10^{-8}	$\mathcal{O}(10^{-9})$

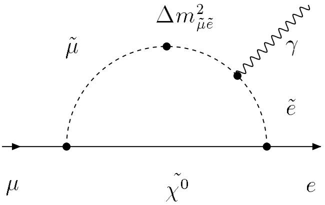
http://pdglive.lbl.gov



$$BR(\mu \to e\gamma) \sim 2.2 \times 10^{-4} \, |U_{ej} m_j^2 U_{\mu j}^*|^2 \sim 10^{-48}$$

Rare decays of leptons

- theories involving new degrees of freedom can give rates much larger
  - this is the case of SUSY theories where, however, the soft supersymmetry breaking lagrangians contain a large number of flavour violating couplings
  - nonzero off-diagonal matrix elements in the slepton mass matrix introduce LFV



$$BR(\ell_i \to \ell_j \gamma) \sim \frac{\alpha^3\, |\Delta \tilde{m}^2_{ij}|^2}{G_F^2 m_S^8} \tan^2 \beta$$

m_S=typical mass of superparticles

Rare decays of leptons

- a possible way to make things easier:
  - particular classes of soft lagrangians resulting from models that break SUSY in a flavour-blind manner

$$(m^2_{\tilde{L}})_{ij} = (m^2_{\tilde{e}})_{ij} = (m^2_{\tilde{\nu}})_{ij} = \delta_{ij} m_0^2 \qquad \text{charged slepton and sneutrino masses}$$
$$(A_\nu)_{ij} = A_0 (Y_\nu)_{ij} \qquad (A_e)_{ij} = A_0 (Y_e)_{ij} \qquad \text{trilinear couplings}$$

$$\begin{cases} m_0 & \text{universal scalar masses} \\ A_0 = a_0\, m_0 = \mathcal{O}(1) \times m_0 & \text{trilinear scalar couplings} \\ m_{1/2} & \text{universal gaugino mass} \end{cases}$$

- flavour violation solely due to a mechanism generating neutrino masses and mixings (see-saw in our case)
- in the presence of heavy right-handed neutrinos, the slepton mass matrices feel the flavour violation present in the neutrino Dirac Yukawa couplings (what we previously called m_D)

Rare decays of leptons

- in fact, terms like $Y_\nu^{ij}\, N_i\, L_j\, H$ and the SUSY-breaking $A_\nu^{ij}\, \tilde{N}_i\, \tilde{L}_j\, H$ induce off-diagonal elements of $m^2_{\tilde{L}}$ through the radiative corrections

$$\Downarrow$$

$$(\Delta m^2_{\tilde{L}})_{ij} \sim -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y_\nu^\dagger Y_\nu)_{ji} \log\frac{M_X}{M_R}$$

$$\frac{B(\ell_i \to \ell_j + \gamma)}{B(\ell_i \to \ell_j + \nu_i + \bar{\nu}_j)} \approx B_0(m_0, m_{1/2}) \left|\sum_k (\hat{Y}_\nu^\dagger)_{ik} \log\left(\frac{M_X}{M_k}\right)(\hat{Y}_\nu)_{kj}\right|^2 \tan^2\beta$$

where $$B_0(m_0, m_{1/2}) \approx \frac{\alpha_{em}^3}{G_F^2 m_S^8}\left|\frac{(3+a_0^2)m_0^2}{8\pi^2}\right|^2$$

$$m_S^8 = \approx 0.5 m_0^2\, m_{1/2}^2\, (m_0^2 + 0.6\, m_{1/2}^2)^2$$

Rare decays of leptons

- general remark (using the previous formula):

$$\frac{BR(\tau\to\mu\gamma)}{BR(\mu\to e\gamma)} \sim BR(\tau\to\mu\nu_\mu\nu_\tau)\,\frac{|\cdots|^2_{32}}{|\cdots|^2_{21}}$$

$$\frac{BR(\tau\to e\gamma)}{BR(\mu\to e\gamma)} \sim BR(\tau\to e\nu_e\nu_\tau)\,\frac{|\cdots|^2_{31}}{|\cdots|^2_{21}}$$

- $\hat{Y}_\nu$ is proportional to the Dirac neutrino mass matrix ($m_D = \hat{Y}_\nu\, v_u$) and computed in the basis where the RH neutrino mass matrix is diagonal: $\hat{Y}_\nu = \hat{Y}_\nu(M_R)$

$$\hat{Y}_\nu = U_R^T\, Y_\nu\, U_\ell$$

  - U_R diagonalizes M_R
  - U_ℓ diagonalizes the charged lepton mass matrix

$\hookrightarrow$ many model-dependent quantities: the flavour symmetry may constrain U_R, U_ℓ and Y_ν !

An example (from A_4) G.Altarelli&DM, J.Phys.G G36 (2009) 085005

- leading order results
  - charged leptons are diagonal
  - the flavour symmetry constrains the couplings

$$w_\nu = y_\nu(\nu_1^c e + \nu_2^c \tau + \nu_3^c \mu)\, h_u \quad + \quad M\,(\nu_1^c\nu_1^c + \nu_2^c\nu_3^c + \nu_3^c\nu_2^c) + b\,(2\nu_i^c\nu_i^c - \nu_1^c\nu_2^c - \nu_1^c\nu_3^c - \nu_2^c\nu_3^c + i \leftrightarrow j)$$

$$m_D = y_\nu\, v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = v_u\, Y_\nu \qquad m_M = \begin{pmatrix} M+2b & -b & -b \\ -b & 2b & M-b \\ -b & M-b & 2b \end{pmatrix}$$

- neutrino masses from see-saw are $m_i = y_\nu^2 v_u^2 / M_i$
- $U_R = U_{TBM},\, U_\ell = I$: easy to evaluate $\hat{Y}_\nu$

An example (from A_4)

C.Hagedorn et al., JHEP **1002**, 047 (2010)

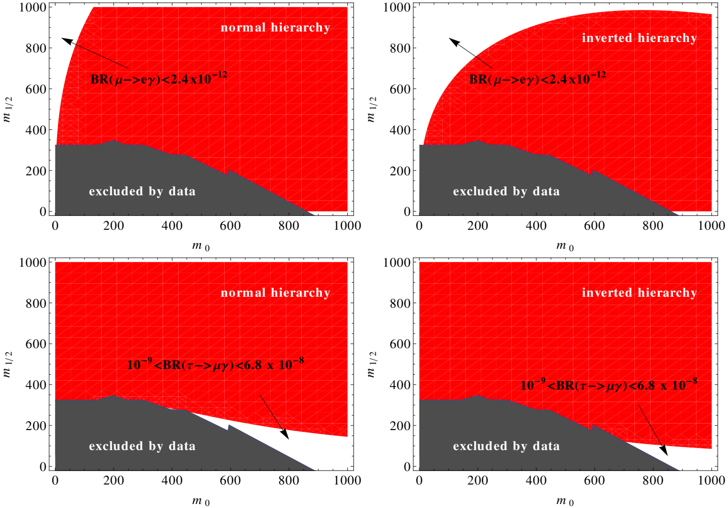
- we express the heavy neutrino masses in terms of the light ones
  - $BR(\mu \to e\gamma) \sim BR(\tau \to e\gamma) \propto \left| \frac{1}{3} y_\nu^2 \log\left(\frac{m_2}{m_1}\right) \right|^2$
  - $BR(\tau \to \mu\gamma) \propto \left| \frac{1}{3} y_\nu^2 \log\left(\frac{m_2}{m_1}\right) - \frac{1}{2} y_\nu^2 \log\left(\frac{m_3}{m_1}\right) \right|^2$
- we use the experimental Δm^2_{sol} and Δm^2_{atm}

	$BR(\mu, \tau \to e\gamma)$	$BR(\tau \to \mu\gamma)$
NH	$\left| \frac{1}{6} y_\nu^2 \log\left(1 + \frac{\Delta m^2_{sol}}{m_1^2}\right) \right|^2$	$\left| \frac{1}{4} y_\nu^2 \log\left(1 + \frac{\Delta m^2_{atm}}{m_1^2}\right) \right|^2$
IH	$\left| \frac{1}{6} y_\nu^2 \log\left(1 + \frac{\Delta m^2_{sol}}{m_3^2 + \Delta m^2_{atm}}\right) \right|^2$	$\left| \frac{1}{4} y_\nu^2 \log\left(1 + \frac{\Delta m^2_{atm}}{m_3^2}\right) \right|^2$

An example (from A_4)

- we include corrections to the previous matrices to fit the data in the ν sector

exclusion regions from P. Bechtle et al., arXiv:1105.5398 [hep-ph]



Conclusions

- neutrino physics has given this year many interesting new results
- among them, the relatively large θ_{13} must be incorporated into a consistent theoretical framework
- models exist which correctly take into account many of the features of ν mixing, going from a maximum of symmetry, with discrete non-abelian flavour groups, to the opposite extreme of anarchy
- not a clear and convincing scenario has been proposed for the understanding of fermion masses and mixings
- running and planned $0\nu\beta\beta$ and LFV experiments can lead to extremely important developments in the near future

Backup slides

The role of non-abelian discrete symmetries: S_4

it is the groups of permutations of 4 object $\longrightarrow$ 24 elements, generated by S and T satisfying

$$S^4 = T^3 = 1, \quad ST^2S = T$$

The action of the generators S and T can be assigned as follows:

$$\begin{array}{ccc} (1234) & \to^S & (2341) \\ (1234) & \to^T & (2314) \end{array}$$

the 24 elements belong to 5 conjugate classes

$$\begin{aligned}
&\mathcal{C}_1 : 1 \\
&\mathcal{C}_2 : \ S^2 = (3412),\ TS^2T^2 = (4321),\ S^2TS^2T^2 = (2143) \\
&\mathcal{C}_3 : \ T,\ T^2 = (3124),\ S^2T = (1423),\ S^2T^2 = (2431),\ STST^2 = (4132) \\
&\quad STS = (4213),\ TS^2 = (4132),\ T^2S^2 = (1342) \\
&\mathcal{C}_4 : \ ST^2 = (1243),\ T^2S = (4231),\ TST = (1432) \\
&\quad TSTS^2 = (3214),\ STS^2 = (1324),\ S^2TS = (2134) \\
&\mathcal{C}_5 : \ S,\ TST^2 = (2413),\ ST = (3142),\ TS = (3421),\ S^3 = (4123),\ S^3T^2 = (4312)
\end{aligned}$$

The inequivalent irreducible representations of S_4 are 1_1, 1_2, 2 and 3. The one-dimensional unitary representations are given by:

$$\begin{array}{lll} 1_1 : & S = 1 & T = 1 \\ 1_2 : & S = -1 & T = 1 \end{array}$$

while the two-dimensional unitary representation, in a basis where the element T is diagonal, is given by:

$$T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \qquad S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

An example from A_4

Field	ν^c	ℓ	e^c	μ^c	τ^c	h_d	h_u	φ_T	ξ'	φ_S	ξ	φ_0^T	φ_0^S	ξ_0
A_4	3	3	1	1	1	1	1	3	1'	3	1	3	3	1
Z_4	-1	i	1	i	-1	1	i	i	i	1	1	-1	1	1
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	2	2	2

Additionally needed

- Z_4 symmetry to separate charged lepton and neutrino sector

$$w_\nu = y_\nu (\nu^c \ell)\, h_u + (M + a\,\xi)\, \nu^c \nu^c + b\, \nu^c \nu^c\, \varphi_S$$

$$m_D = y_\nu\, v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = v_u\, Y_\nu\ .$$

The other terms lead to the Majorana mass matrix:

$$m_M = \begin{pmatrix} M + a\,u + 2\,b\,v_S & -b\,v_S & -b\,v_S \\ -b\,v_S & 2\,b\,v_S & M + a\,u - b\,v_S \\ -b\,v_S & M + a\,u - b\,v_S & 2\,b\,v_S \end{pmatrix}$$

The role of non-abelian discrete symmetries: S_4

- it is the groups of permutations of 4 object $\longrightarrow$ 24 elements, generated by S and T satisfying

$$S^4 = T^3 = 1, \quad ST^2S = T$$

The action of the generators S and T can be assigned as follows:

$$\begin{array}{ccc} (1234) & \to^S & (2341) \\ (1234) & \to^T & (2314) \end{array}$$

the element $P_{23} = TSTS^2$

- irreducible representations are singlets, doublet and triplets

$$\begin{array}{lll} 1_1: & S = 1 & T = 1 \\ 1_2: & S = -1 & T = 1 \end{array} \qquad 2: T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$3: T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \; S = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}$$

$\omega = e^{2\pi i/3} = (-1 + \sqrt{3})/2$

Majorana neutrinos (I)

we start from the Dirac lagrangian:

$$\begin{aligned}\mathcal{L}_{\mathcal{D}} &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \\ &= i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R - m\bar{\psi}_L\psi_R - m\bar{\psi}_R\psi_L\end{aligned}$$

the equation of motion gives two equations decoupled in the limit $m = 0$:

$$\begin{aligned}i\gamma^\mu\partial_\mu\psi_L &= m\bar{\psi}_R \\ i\gamma^\mu\partial_\mu\psi_R &= m\bar{\psi}_L\end{aligned}$$

However, even in the massive case it is not mandatory to have a Dirac spinor with four degrees of freedom. With the Majorana condition:

$$\psi_R = \mathcal{C}\bar{\psi}_L{}^T$$

the previous equations are equivalent.

Majorana neutrinos (II)

Let us now write the Dirac spinor as $\psi = \psi_L + \mathcal{C}\bar{\psi_L}^T$; the Majorana fermion is self-conjugate

$$\psi^c = \mathcal{C}\bar{\psi}^T = \psi$$

Thus only neutral particles can be of Majorana type, as it could be the case of neutrinos. The mass terms now read:

$$\mathcal{M} = -\frac{1}{2}m(\bar{\psi_L}\psi_L^c + \bar{\psi_L^c}\psi_L)$$

Observing that $\bar{\psi_L^c} = -\psi_L^T\mathcal{C}^{-1}$ we get:

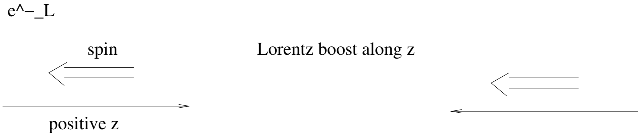
$$\mathcal{M} = -\frac{1}{2}m(\psi_L^T\mathcal{C}^{-1}\psi_L + h.c.)$$

Two-component spinor fields

- Let start from electrons: we have 4 different objects:

$$e^-_L \qquad e^-_R \qquad e^+_L \qquad e^+_R$$

- suppose an observer sees an e^-_L; a different observer moving in the same positive z-direction faster than e^-_L sees a right-handed object:



- since charge is conserved by Lorentz transformation, this object must be e^-_R
- for massive neutrinos, we only have ν_L and $\bar{\nu}_R$, distinguished by the lepton number L only
- if we allow for L-violation, ν_L and $\bar{\nu}_R$ are the boosted counterparts of one another! $\Longrightarrow$ Two-component spinor field

Which features are important?

observation: masses of the charged fermions C.Hagedorn, talk at FLASY2011

	Mass (at M_Z)	normalized to m_t
u	~ 1.7 MeV	λ^8
c	~ 0.62 GeV	λ^4
t	~ 171 GeV	1
		normalized to m_b
d	~ 3.0 MeV	λ^4
s	~ 54 MeV	λ^2
b	~ 2.87 GeV	1
		normalized to m_τ
e	~ 0.49 MeV	λ^{4-5}
μ	~ 102 MeV	λ^2
τ	~ 1.75 GeV	1

where $\lambda \sim 0.22$ is the Cabibbo angle

- large hierarchy in the *up* sector
- almost the same hierarchy in the *down* and *charged lepton* sectors

Which features are important?

observation: masses and mixing of neutrinos

- it easy to compute the probability for a flavour conversion $\nu_\alpha \to \nu_\beta$:
  - temporal evolution of the state α:

$$i\,\frac{d\nu_\alpha}{dt} = H\nu_\alpha$$

  - projection on a definite flavour $\beta \longrightarrow$ amplitude of the conversion

$$A_{\alpha\beta} \;=\; \langle \nu_\alpha(t)|\nu_\beta\rangle$$

The neutrino oscillation probability

$$P_{\alpha\beta} = |A_{\alpha\beta}|^2 = \sum_{i,j} U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\exp\left(i\frac{m_j^2 - m_i^2}{2E}L\right)$$

E = neutrino energy, L = the baseline length, m_i = masses of the ith neutrino eigenstate

The role of non-abelian discrete symmetries

Let us assume we want to reproduce the TBM pattern
we observe that the neutrino mass matrix

$$m_\nu = \begin{pmatrix} x & y & y \\ y & x-v & y+v \\ y & y+v & x-v \end{pmatrix}$$

is invariant under S and P_{23}

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \; P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S^T\, m_\nu\, S = m_\nu \qquad P_{23}^T\, m_\nu\, P_{23} = m_\nu$$

technically speaking, m_ν has a $Z_2 \times Z_2$ symmetry

- a group containing S (and P_{23}) is a good candidate to reproduce m_ν

$\hookrightarrow$ permutation groups

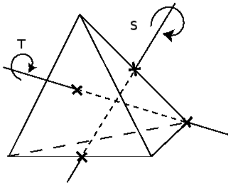
The role of non-abelian discrete symmetries: A_4

- it is the groups of even permutations of 4 object $\longrightarrow$ 12 elements, generated by S and T satisfying
  - $S^2 = T^3 = (ST)^3 = 1$

The action of the generators S and T can be assigned as follows:

$$\begin{array}{ccc} (1234) & \to^S & (4321) \\ (1234) & \to^T & (2314) \end{array}$$

- geometrically, it is the invariance group of a tetrahedron



- irreducible representations are singlets $(1, 1', 1'', S = 1)$ and one triplet 3

$$\begin{array}{ll} 1 : & T = 1 \\ 1' : & T = \omega = e^{2\pi i/3} \\ 1'' : & T = \omega^2 \end{array} \qquad 3 : T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \ S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

The role of non-abelian discrete symmetries: A_4

- we start from a gauge and A_4 invariant lagrangian

  we need to know how to build an A_4 singlet

$$1' \otimes 1' = 1'' \qquad 3 \otimes 3 = 1 + 1' + 1''...$$

- A_4 must be broken by the vacuum expectation values of scalar fields ϕ
- the breaking is along definite directions:
  - in the neutrino sector, we want to preserve S (invariance under P_{23} is obtained at the lagrangian level since P_{23} does not belong to A_4); for scalars in triplet representation:

$$\langle \phi_\nu \rangle \sim (1, 1, 1)$$

    so that:

$$S \langle \phi_\nu \rangle = \langle \phi_\nu \rangle$$

  - in the charged lepton sector, a diagonal mass matrix is such that $m_\ell^\dagger m_\ell = T^\dagger m_\ell^\dagger m_\ell T$ so that $\langle \phi_\ell \rangle \sim (1, 0, 0)$ and T is preserved

An example (from A_4) G.Altarelli&DM, J.Phys.G G36 (2009) 085005

- leading order results

  field assignment

$$\ell = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \\ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \\ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \end{pmatrix} \qquad \nu^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}^c$$

  - charged leptons are diagonal
  - ξ, φ_S are flavon fields with vevs $\langle \xi \rangle \neq 0$ and $\langle \varphi_S \rangle = v_S\,(1,1,1)$

$$w_\nu = y_\nu(\nu^c \ell)\, h_u + (M + a\,\xi)\,\nu^c\nu^c + b\,\nu^c\nu^c\,\varphi_S$$

$$m_D = y_\nu\, v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = v_u\, Y_\nu \qquad m_M = \begin{pmatrix} x+2z & -z & -z \\ -z & 2z & x-z \\ -z & x-z & 2z \end{pmatrix}$$

- neutrino masses from see-saw are $m_i = y_\nu^2 v_u^2 / M_i$
- $U_R = U_{TBM}$, $U_\ell = I$: easy to evaluate $\hat{Y}_\nu$

An example from A_4

C.Hagedorn et al., JHEP **1002**, 047 (2010)

we make use of the relation among light and heavy neutrino masses

$$\sum_k (\hat{Y}_\nu^\dagger)_{ik} \log\left(\frac{M_X}{M_k}\right)(\hat{Y}_\nu)_{kj} \to$$

$$(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ij} \log\left(\frac{m_1}{m^*}\right) + (\hat{Y}_\nu^\dagger)_{i2}(\hat{Y}_\nu)_{2j} \log\left(\frac{m_2}{m_1}\right) + (\hat{Y}_\nu^\dagger)_{i3}(\hat{Y}_\nu)_{3j} \log\left(\frac{m_3}{m_1}\right)$$

$$m^* = \frac{y_\nu^2 v_u^2}{M_X}$$

- $BR(\mu \to e\gamma) \sim BR(\tau \to e\gamma) \propto \left|\frac{1}{3} y_\nu^2 \log\left(\frac{m_2}{m_1}\right)\right|^2$
- $BR(\tau \to \mu\gamma) \propto \left|\frac{1}{3} y_\nu^2 \log\left(\frac{m_2}{m_1}\right) - \frac{1}{2} y_\nu^2 \log\left(\frac{m_3}{m_1}\right)\right|^2$

	$BR(\mu, \tau \to e\gamma)$	$BR(\tau \to \mu\gamma)$
NH	$\left\lvert\frac{1}{6} y_\nu^2 \log\left(1 + \frac{\Delta m^2_{sol}}{m_1^2}\right)\right\rvert^2$	$\left\lvert\frac{1}{4} y_\nu^2 \log\left(1 + \frac{\Delta m^2_{atm}}{m_1^2}\right)\right\rvert^2$
IH	$\left\lvert\frac{1}{6} y_\nu^2 \log\left(1 + \frac{\Delta m^2_{sol}}{m_3^2 + \Delta m^2_{atm}}\right)\right\rvert^2$	$\left\lvert\frac{1}{4} y_\nu^2 \log\left(1 + \frac{\Delta m^2_{atm}}{m_3^2}\right)\right\rvert^2$