Fast and accurate parameter estimation of high-redshift sources with the Einstein Telescope

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Fast and accurate parameter estimation

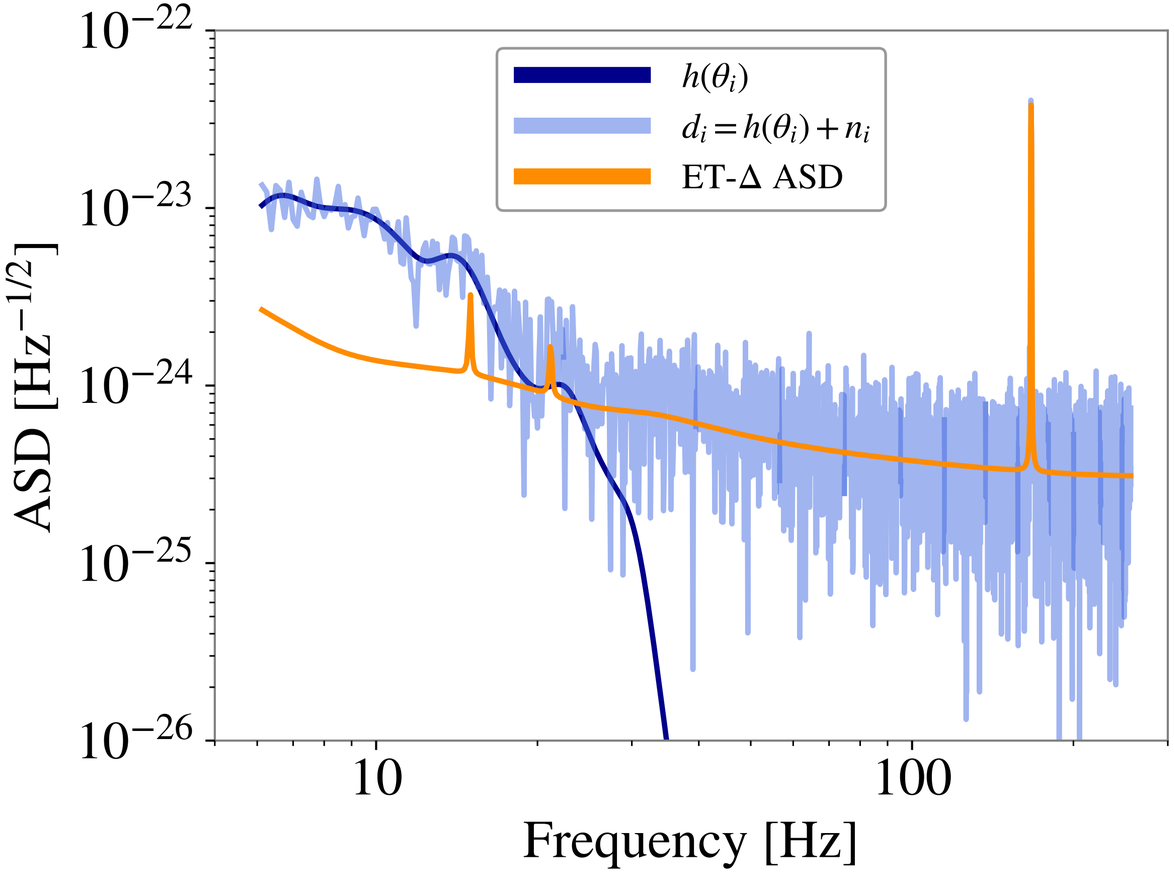
of high-redshift sources with the Einstein Telescope

4 1

Parameter estimation

$$p(\theta \mid d) = \frac{\mathscr{L}(d \mid \theta)\pi(\theta)}{\mathscr{Z}(d)}$$

$$\theta = \{\mathscr{M}_{\mathrm{d}},\ q,\ d_{\mathrm{L}},\ \mathrm{ra},\ \mathrm{dec},\ \theta_{\mathrm{JN}},\ \psi,\ \phi,\ t_{\mathrm{geocent}},\ \chi_1,\ \chi_2\}$$



Ref. Thrane et al. 2019, Christensen et al. 2022

Parameter estimation

θ through **stochastic sampling** (e.g. nested sampling).
This requires tens of millions of likelihood evaluations …

$$p(\theta|d) = \frac{\mathcal{L}(d|\theta)\pi(\theta)}{\mathcal{Z}(d)}$$

… and evaluating likelihood takes time

➡️ **Next generations detectors will observe $\sim 10^5$ sources per year**
➡️ **Need of new methods for fast parameter estimation**

Ref. Thrane et al. 2019, Speagle et al. 2019, Williams et al. 2021, Romero-Shaw et al. 2020, Wong et al. 2023, Papalini et al. 2025

Likelihood-free inference

Sampling parameters from the prior $\theta \sim \pi(\theta)$
and data from the likelihood $d \sim \mathscr{L}(d \mid \theta)$ is fast

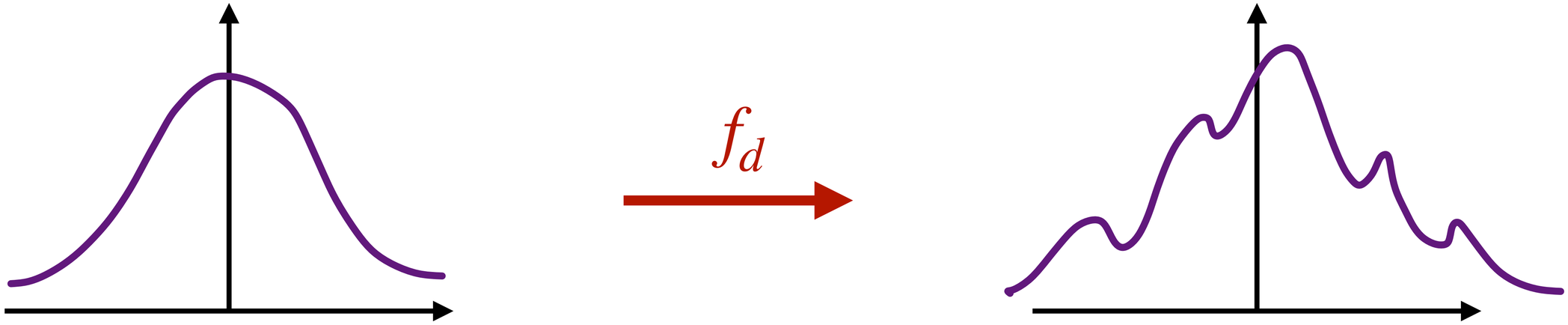
$$p(\theta \mid d) = \frac{\mathscr{L}(d \mid \theta)\pi(\theta)}{p(d)}$$

Using (θ, d) to construct with **deep learning** an estimator $q(\theta \mid d)$ of $p(\theta \mid d)$

Ref. Lueckmann et al. 2017, Greenberg et al. 2019, Cranmer et al. 2020, Chua et al. 2020, Dax et al. 2023

Normalising flows

Many advantages: they represent a complicated distribution q using a series of change of variables $f_d : u \to \theta$



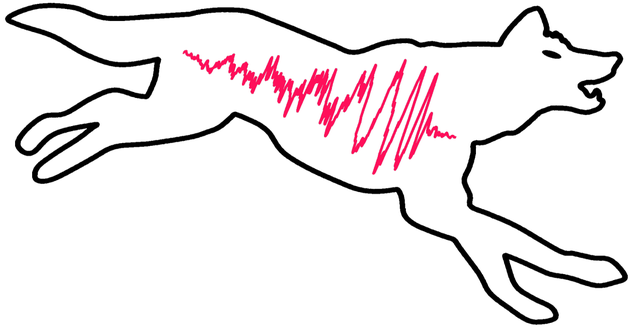
$\mathcal{N}^D(0,1)$

$$q(\theta \mid d) = \mathcal{N}^D(0,1)(f_d^{-1}(\theta)) \mid \det J_{f_d}^{-1} \mid$$

Rapidly evaluated and sampled from

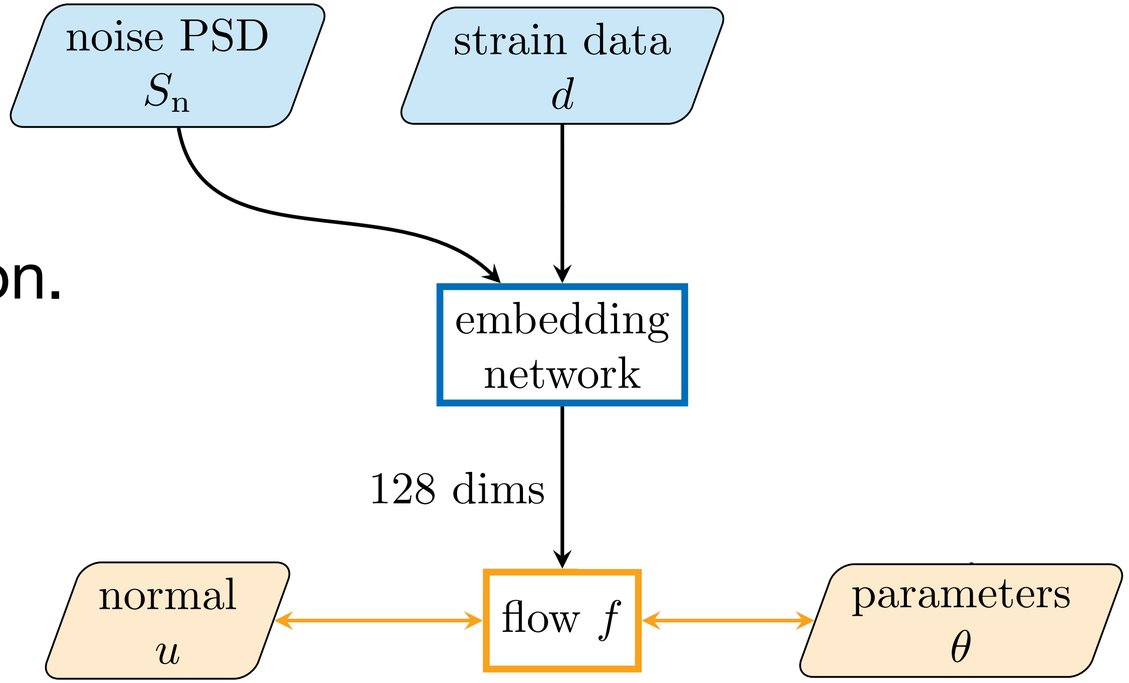
Ref. Kobyzev et al. 2019, Durkan et al. 2019, Polanska et al. 2024

Dingo



Dingo implements neural posterior estimation.

Training in **days**, inference in **minutes**

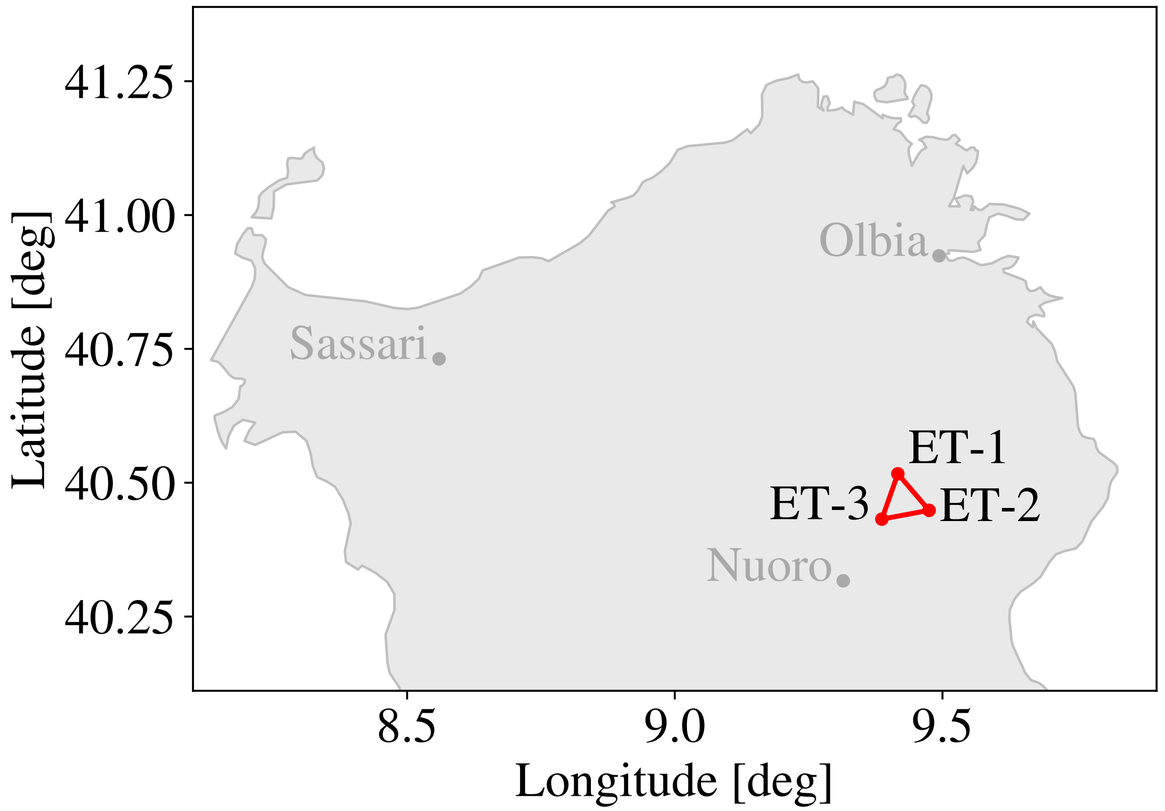


Adapted from Dax et al. 2023

Ref. Green et al. 2020, Green et al. 2021, Dax et al. 2021, Dax et al. 2022, Wildberger et al. 2022, Dax et al. 2024

Einstein Telescope

I trained Dingo using the **HFLF-cryo ASD** with **ET-Δ** configuration placed in Sardinia

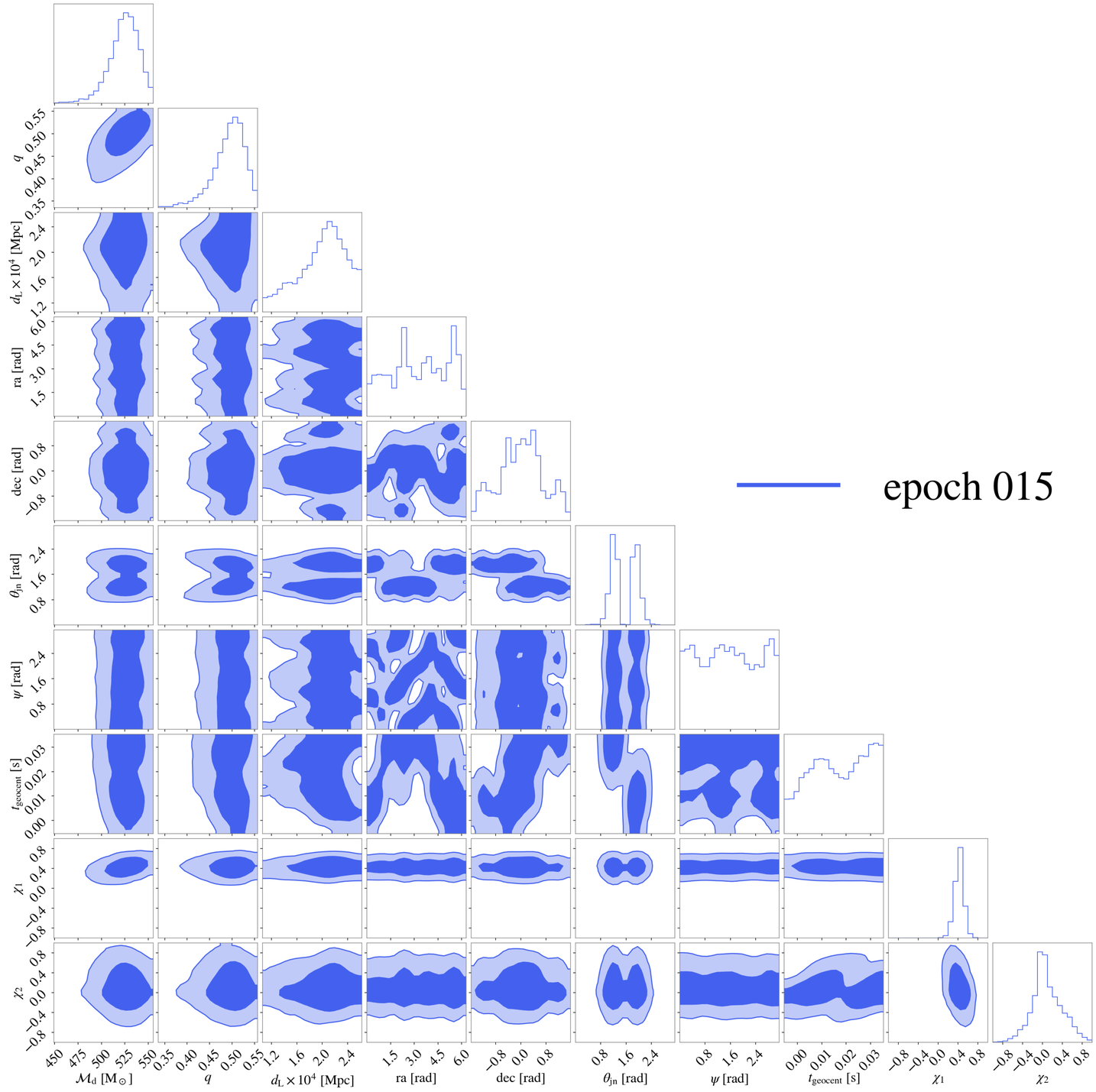


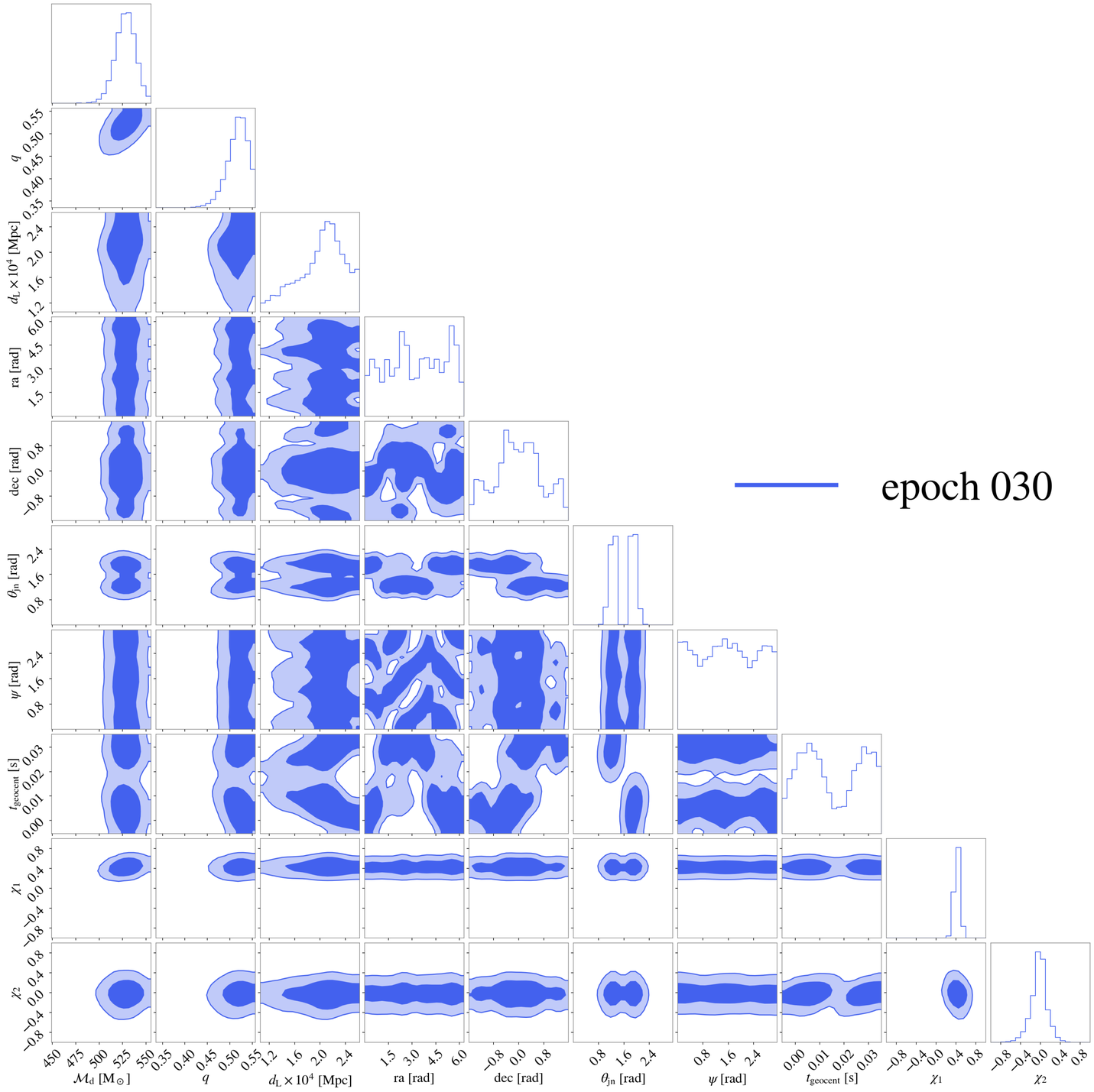
high-redshift sources

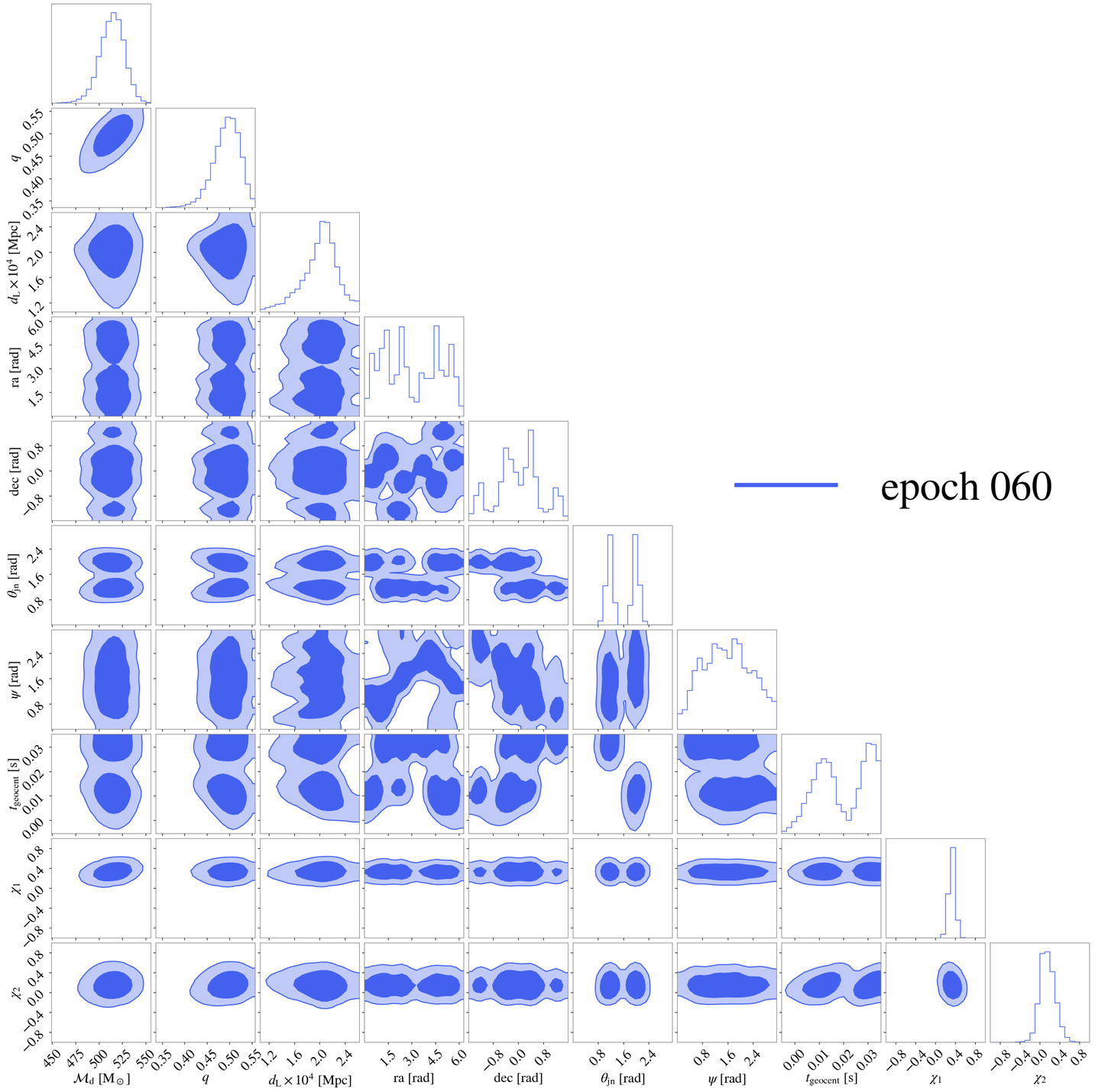
```
'chirp_mass': UniformInComponentsChirpMass(minimum=40, maximum=1100)

'luminosity_distance': UniformSourceFrame(minimum=5_000.0, maximum=500_000.0)
```

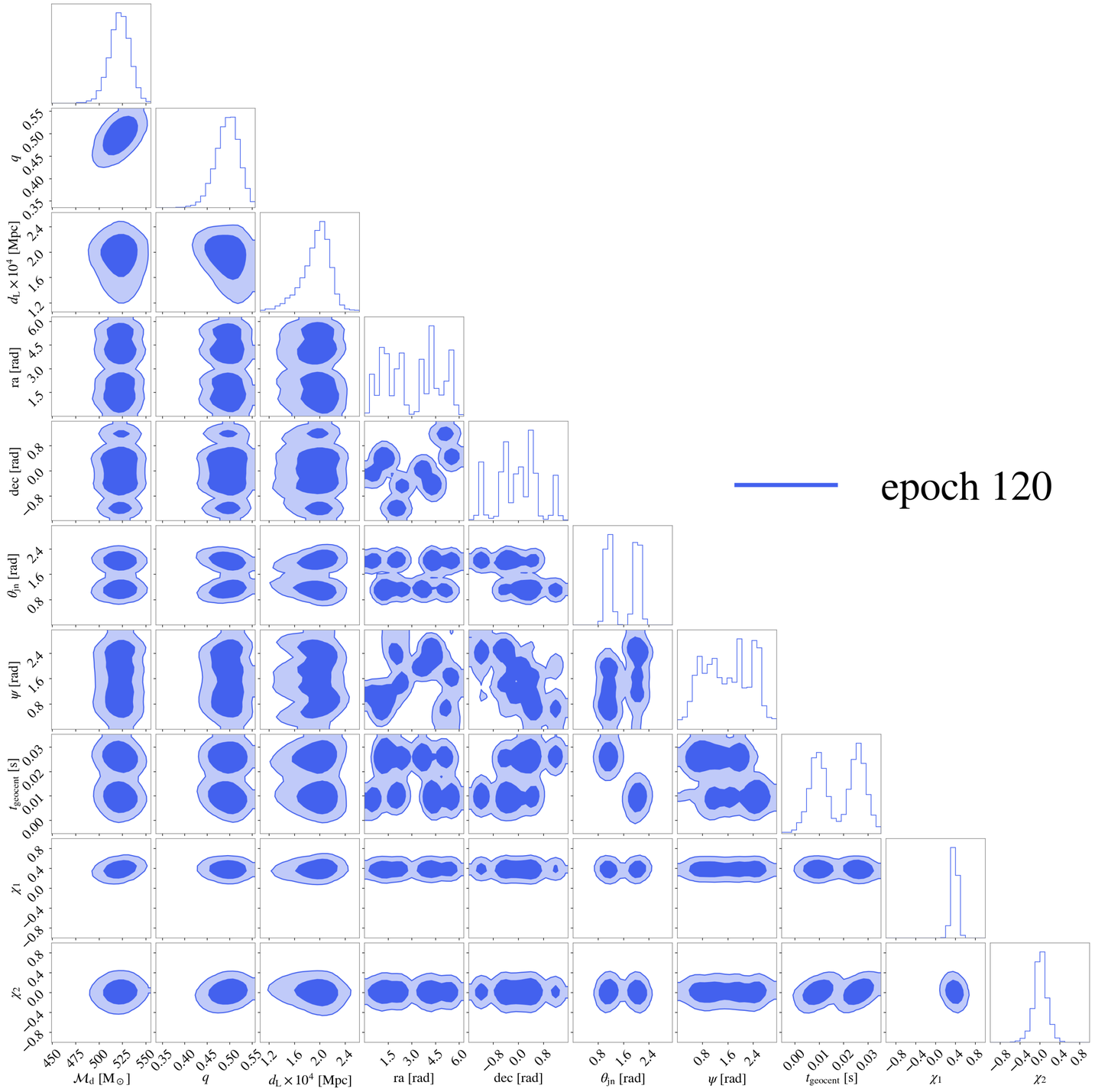
with $f \in [6,\ 256]$ Hz, `df = 1/8` Hz, waveform approximant = `IMRPhenomXPHM`

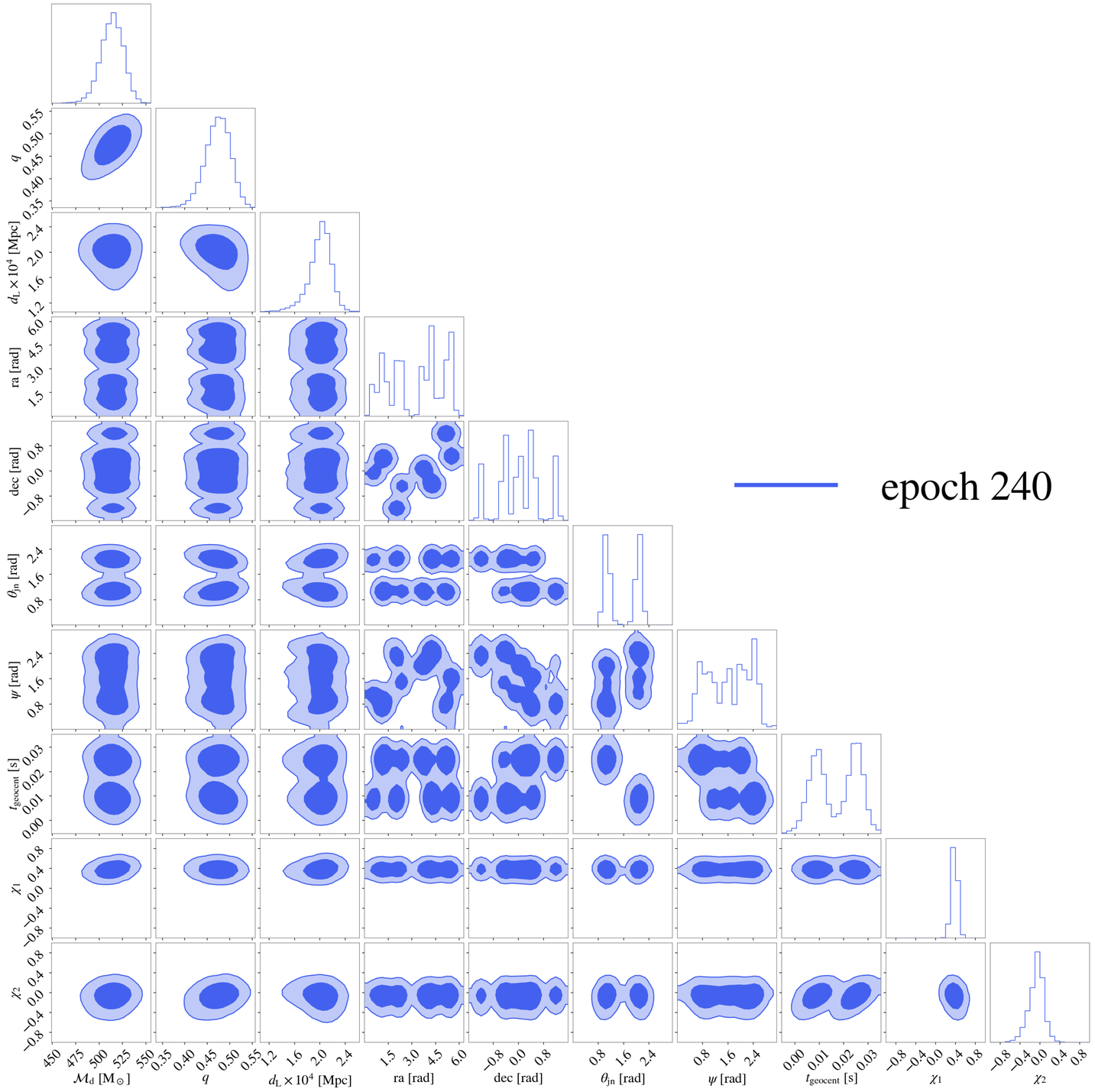
Ref. Pratten et al. 2021

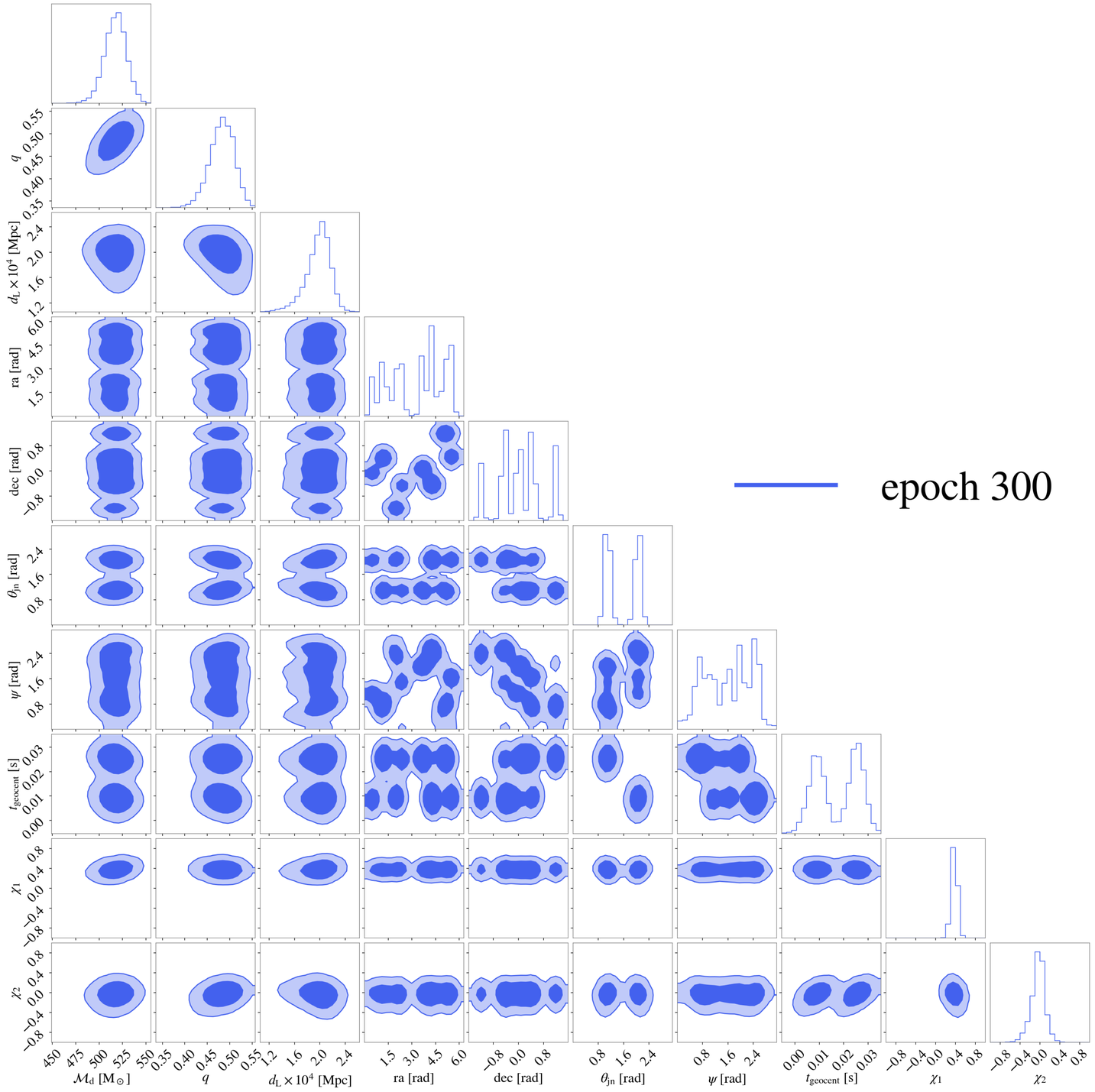

epoch 015
$\mathcal{M}_d$ [M$_\odot$]
q
$d_L \times 10^4$ [Mpc]
ra [rad]
dec [rad]
θ_{jn} [rad]
ψ [rad]
$t_{geocent}$ [s]
χ_1
χ_2


epoch 030
$\mathcal{M}_d$ [M$_\odot$]
q
$d_L \times 10^4$ [Mpc]
ra [rad]
dec [rad]
θ_{jn} [rad]
ψ [rad]
$t_{geocent}$ [s]
χ_1
χ_2


epoch 060
$\mathcal{M}_d$ [M$_\odot$]
q
$d_L \times 10^4$ [Mpc]
ra [rad]
dec [rad]
θ_{jn} [rad]
ψ [rad]
$t_{geocent}$ [s]
χ_1
χ_2


epoch 120
$\mathcal{M}_d$ [M$_\odot$]
q
$d_L \times 10^4$ [Mpc]
ra [rad]
dec [rad]
θ_{jn} [rad]
ψ [rad]
$t_{geocent}$ [s]
χ_1
χ_2


epoch 240
$\mathcal{M}_{\rm d}$ [M$_\odot$]
q
$d_{\rm L} \times 10^4$ [Mpc]
ra [rad]
dec [rad]
$\theta_{\rm jn}$ [rad]
ψ [rad]
$t_{\rm geocent}$ [s]
χ_1
χ_2

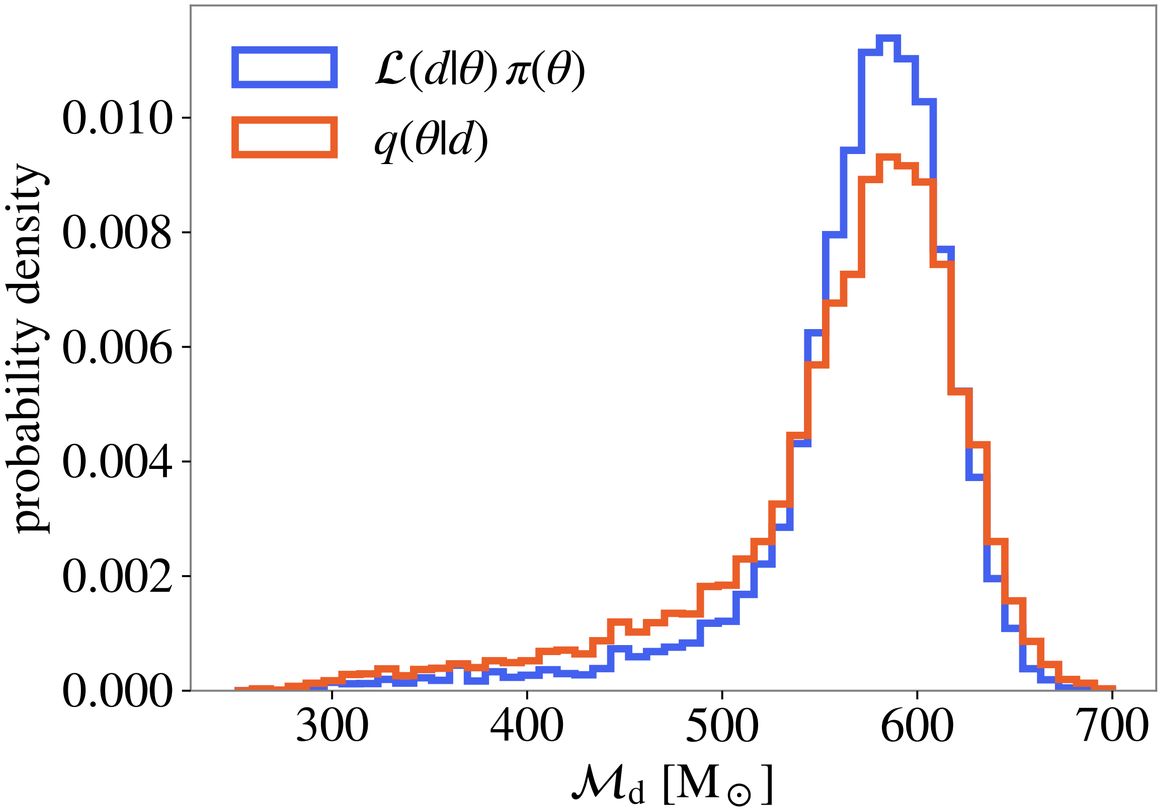

epoch 300
$\mathcal{M}_d$ [M$_\odot$]
q
$d_L \times 10^4$ [Mpc]
ra [rad]
dec [rad]
θ_{jn} [rad]
ψ [rad]
$t_{geocent}$ [s]
χ_1
χ_2

Importance sampling

$$w_i = \frac{\mathscr{L}(d\,|\,\theta)\pi(\theta)}{q(\theta\,|\,d)}$$

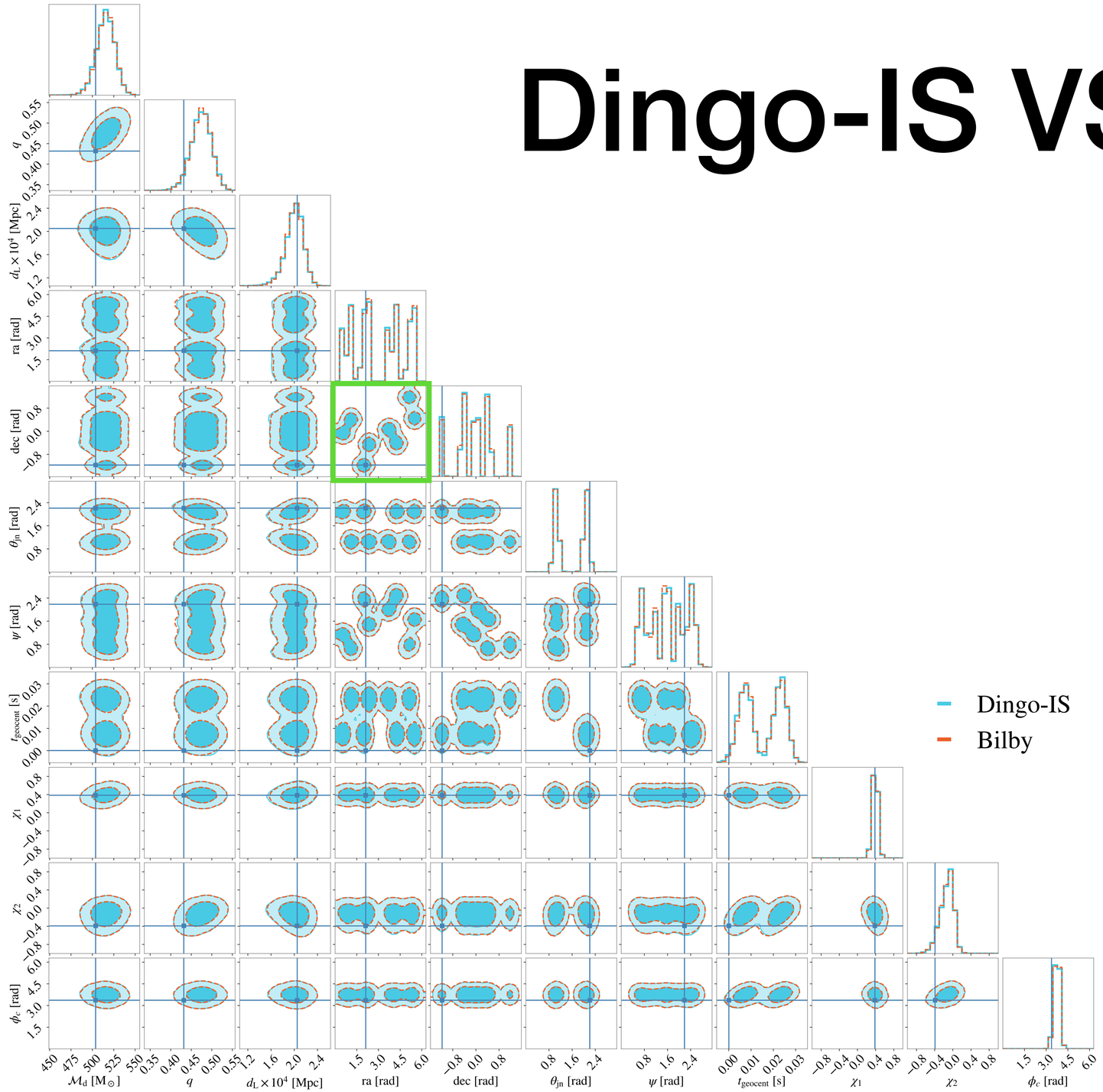
Target

Dingo proposal



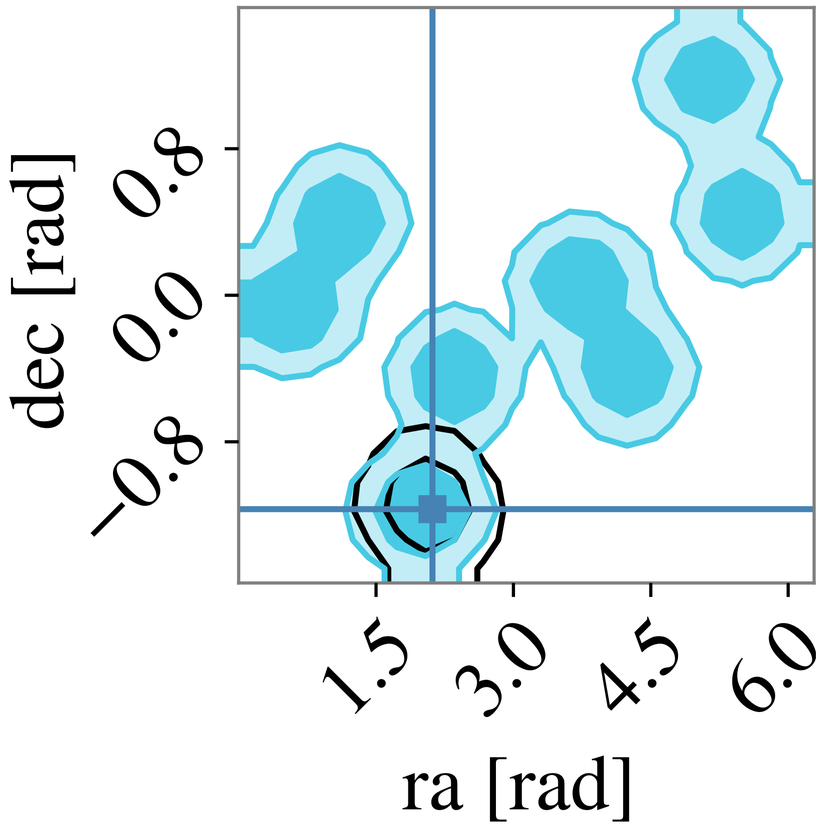
Ref. *Todkar et al. 2010*, *Owen 2013*

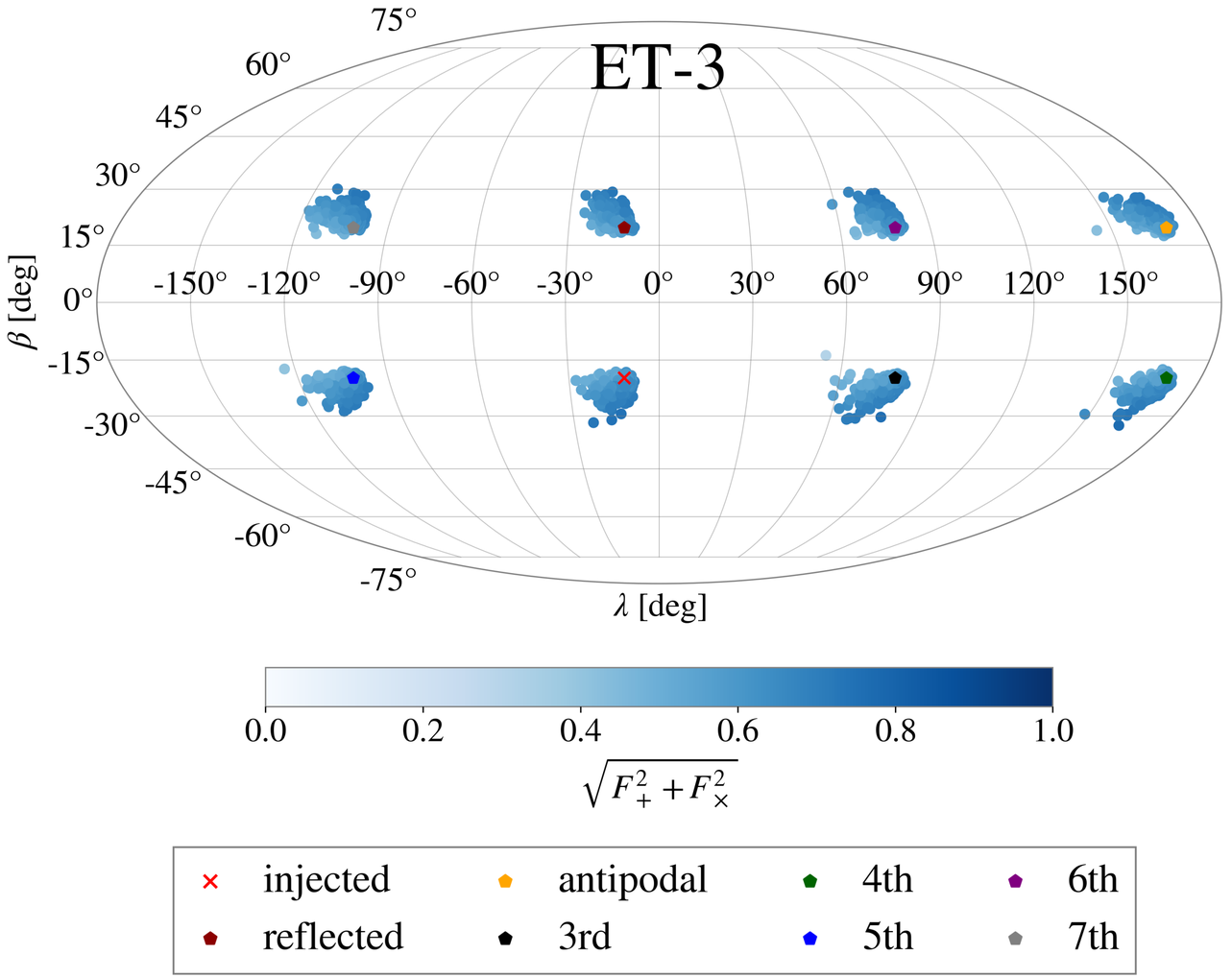
Dingo-IS VS Bilby



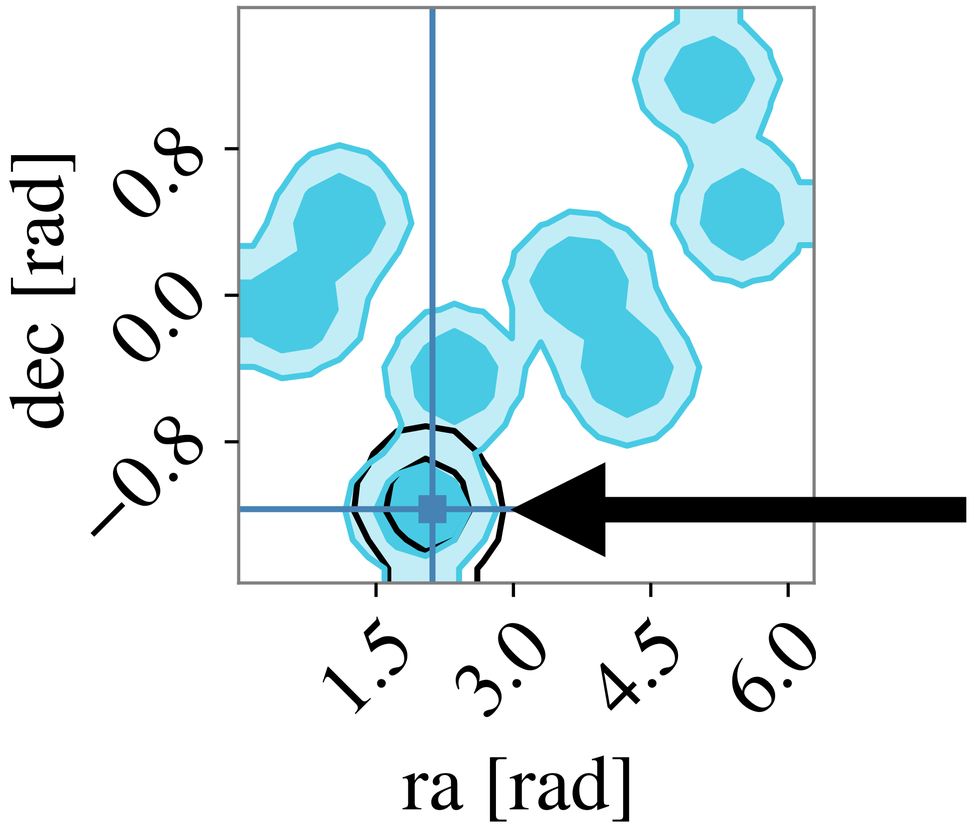
Ref. Romero-Shaw et al. 2020

Sky modes


dec [rad]
0.8
0.0
−0.8
1.5
3.0
4.5
6.0
ra [rad]


ET-3
75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
β [deg]
-150°
-120°
-90°
-60°
-30°
0°
30°
60°
90°
120°
150°
λ [deg]
0.0
0.2
0.4
0.6
0.8
1.0
$\sqrt{F_+^2 + F_\times^2}$
injected
reflected
antipodal
3rd
4th
5th
6th
7th

Sky modes

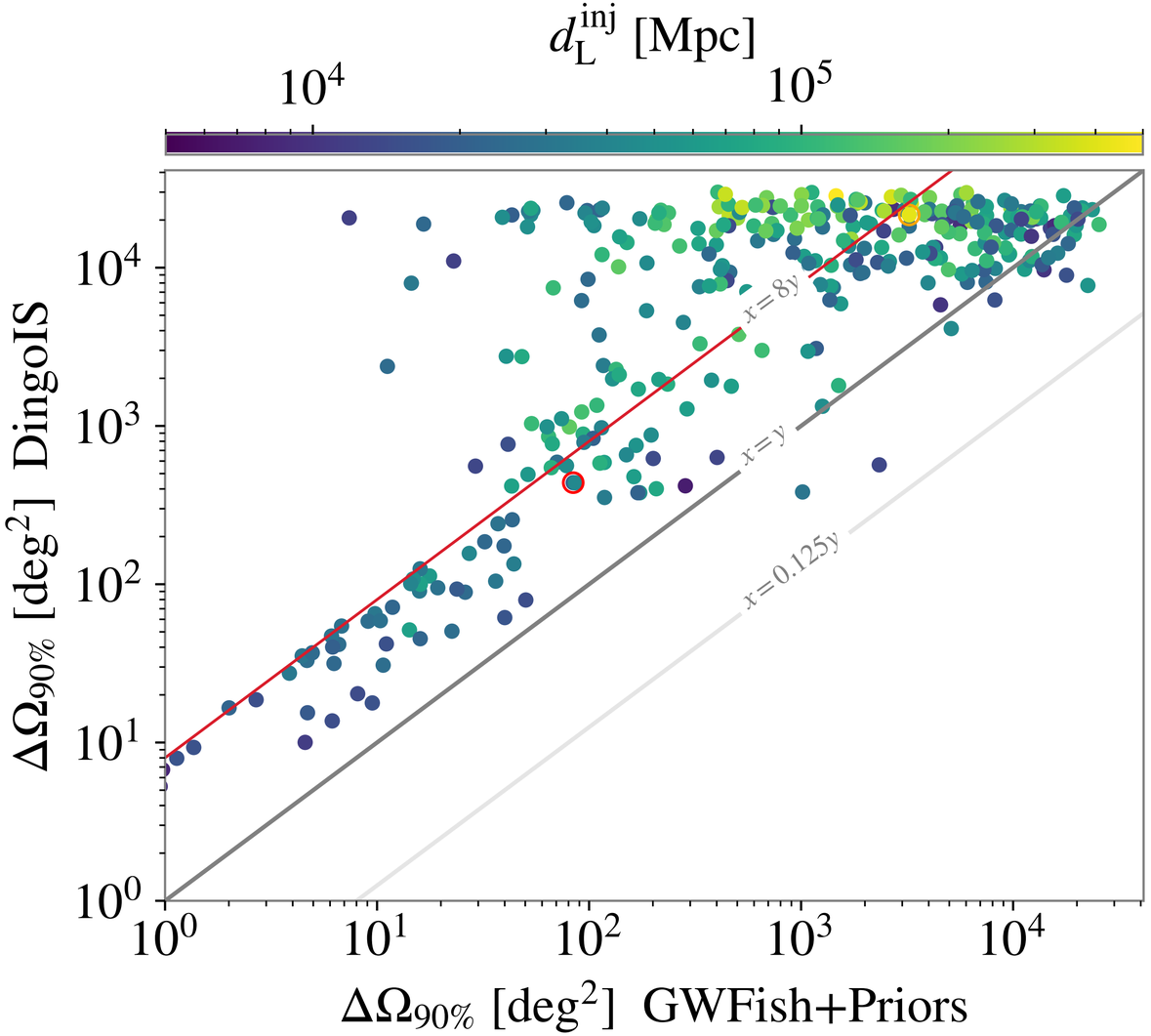


PRIORS
GWFISH

$\mathscr{L}(d\,|\,\theta)$ is approximated as a **multivariate Gaussian distribution** where the inverse of the covariance matrix is the **Fisher information matrix**

Ref. Dupletsa et al. 2025

Sky localisation



Ref. Santoliquido et al. 2025

Contributions

Fast and accurate parameter estimation for high-redshift sources

Look at multimodalities in sky localisation

This approach thrives where Fisher matrix approximation is less reliable

arXiv: 2504.21087

Backup slides

Other priors

```
‘mass_ratio’: UniformInComponentsMassRatio(minimum=0.125, maximum=1.0)

'dec': Cosine(minimum=-π/2, maximum=π/2)

'ra': Uniform(minimum=0, maximum=2π)

'theta_jn': Sine(minimum=0.0, maximum=π)

'psi': Uniform(minimum=0, maximum=π)

'chi_1': AlignedSpin(a_prior=Uniform(minimum=0, maximum=0.9))

'chi_2': AlignedSpin(a_prior=Uniform(minimum=0, maximum=0.9))

'phase': Uniform(minimum=0, maximum=2π)
```

The loss function

Kullback–Leibler divergence

$$D_{\mathrm{KL}}[p(\theta,s)\,||\,q(\theta,s)] = \int \mathrm{d}s\; p(s)\left[\int \mathrm{d}\theta\; p(\theta\,|\,s)\log\frac{p(\theta\,|\,s)}{q(\theta\,|\,s)}\right]$$

Bayes' theorem

$$= \int \mathrm{d}s\; p(s)\left[\int \mathrm{d}\theta\; \frac{p(s\,|\,\theta)p(\theta)}{p(s)}\log\frac{p(\theta\,|\,s)}{q(\theta\,|\,s)}\right]$$

This term is not affected by the neural network

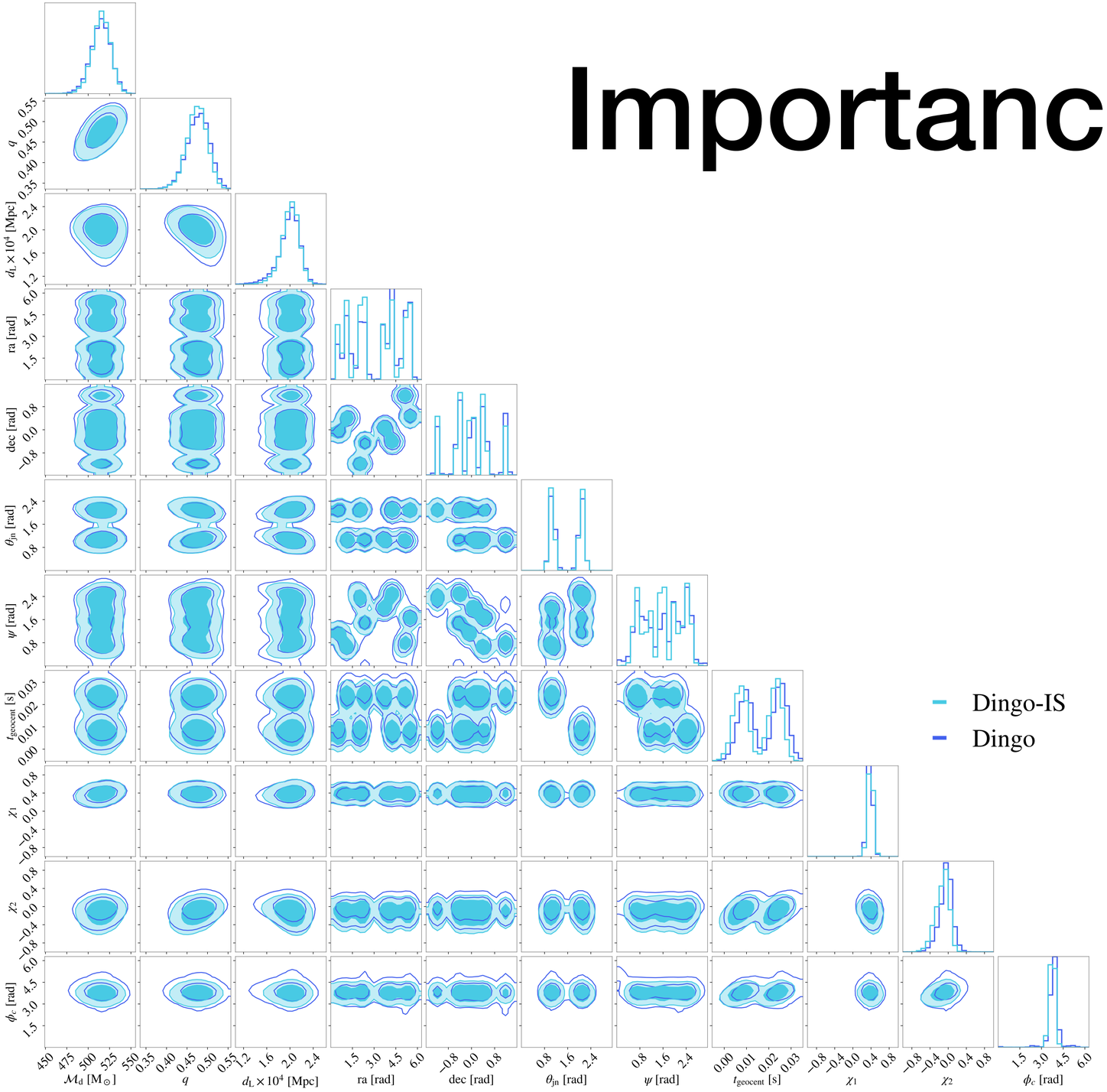
$$= \int \mathrm{d}s\; p(s)\left[\int \mathrm{d}\theta\; \frac{1}{p(s)}\left[p(\theta)p(s\,|\,\theta)\log p(\theta\,|\,s) - p(\theta)p(s\,|\,\theta)\log q(\theta\,|\,s)\right]\right]$$

Neglecting constant values

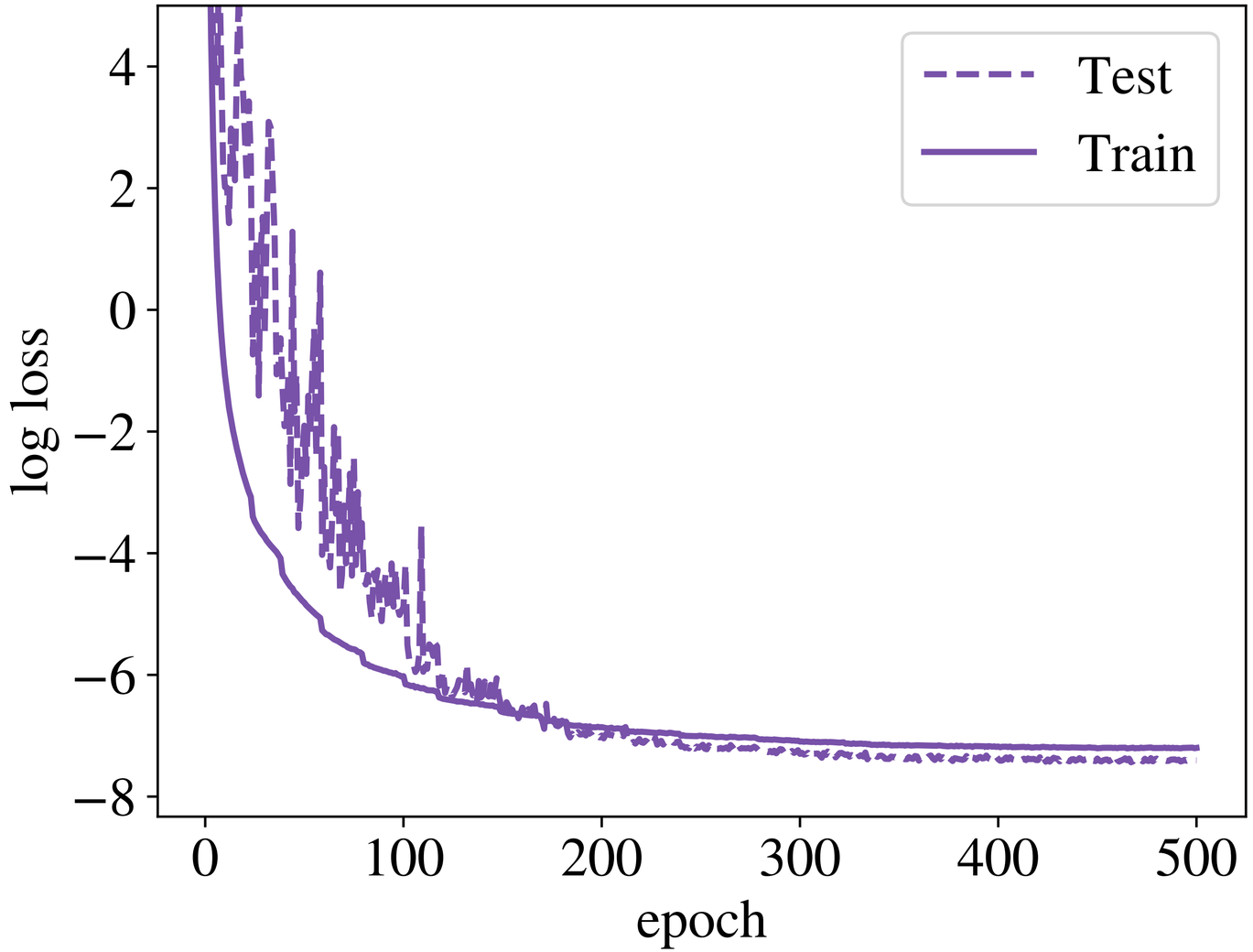
$$\propto -\int \mathrm{d}s\; \mathrm{d}\theta\; p(\theta)p(s\,|\,\theta)\log q(\theta\,|\,s) = -\frac{1}{N_S}\sum_{i=1}^{N_S}\log q(\theta_i, s_i) = \mathbb{E}_{p(\theta)}\mathbb{E}_{p(s|\theta)}[-\log q(\theta\,|\,d)]$$

Ref. Dax et al. 2022, Dax et al. 2023

Importance sampling



Training



Ref. Santoliquido et al. 2025

Antenna amplitude pattern functions

$$F_{+}^{i}(\beta,\lambda,\psi) = -\frac{\sqrt{3}}{4}[(1+\cos^2\beta)\sin 2\lambda\cos 2\psi + 2\cos\beta\cos 2\lambda\sin 2\psi],$$

$$F_{\times}^{i}(\beta,\lambda,\psi) = +\frac{\sqrt{3}}{4}[(1+\cos^2\beta)\sin 2\lambda\sin 2\psi - 2\cos\beta\cos 2\lambda\cos 2\psi],$$

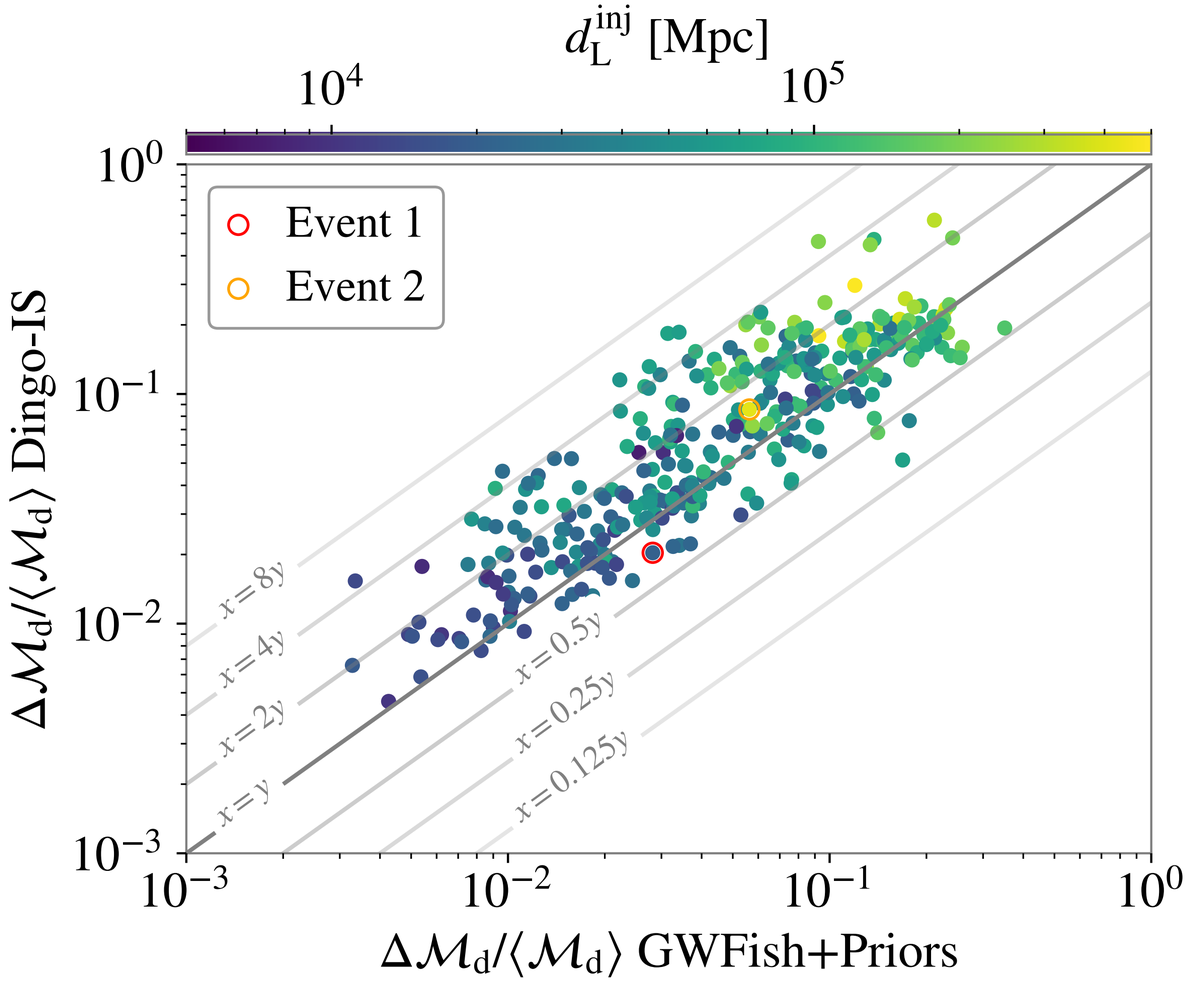
Ref. Regimbau et al. 2012, Vishal et al. 2020, Sylvain et al. 2021

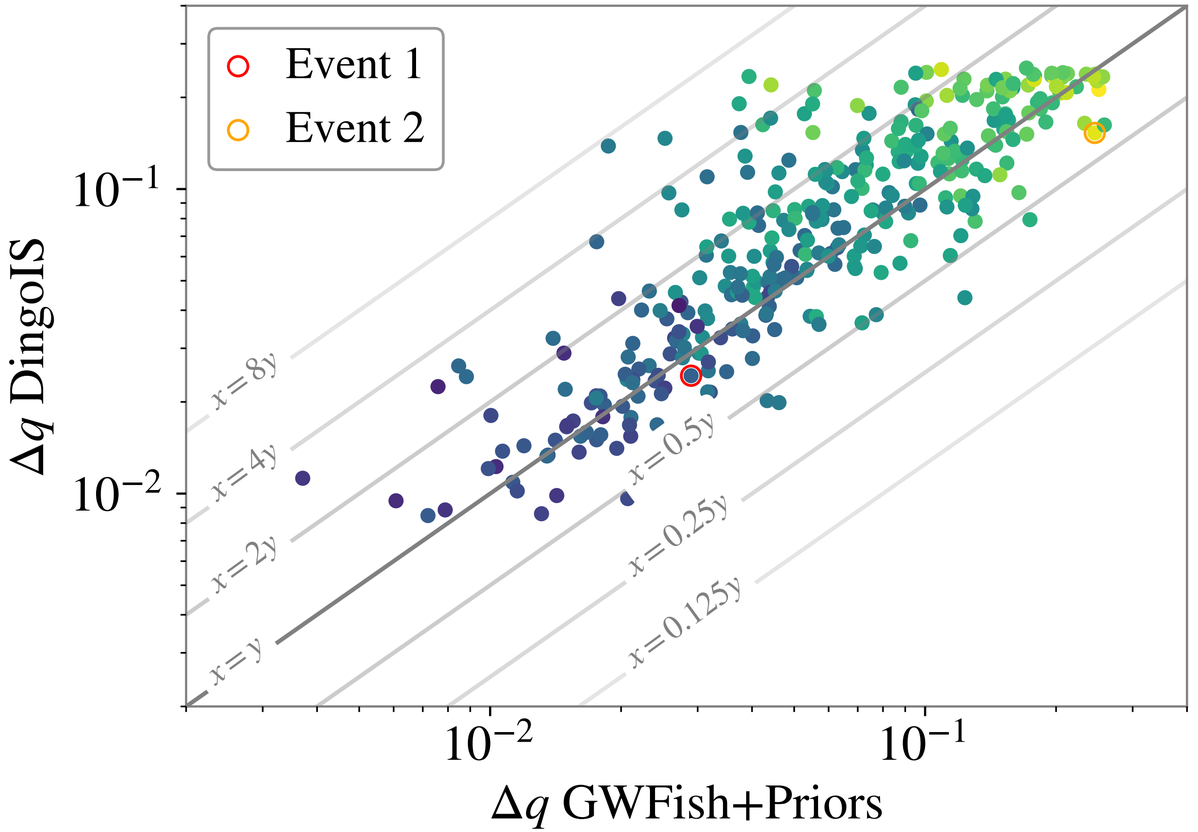
Antenna power pattern function

$$\sqrt{(F_+^i)^2 + (F_\times^i)^2} = \sqrt{\frac{3}{12}\left[(1+\cos^2\beta)^2 \sin^2 2\lambda + \cos^2\beta \cos^2 2\lambda\right]}$$

Ref. Regimbau et al. 2012, Vishal et al. 2020, Sylvain et al. 2021

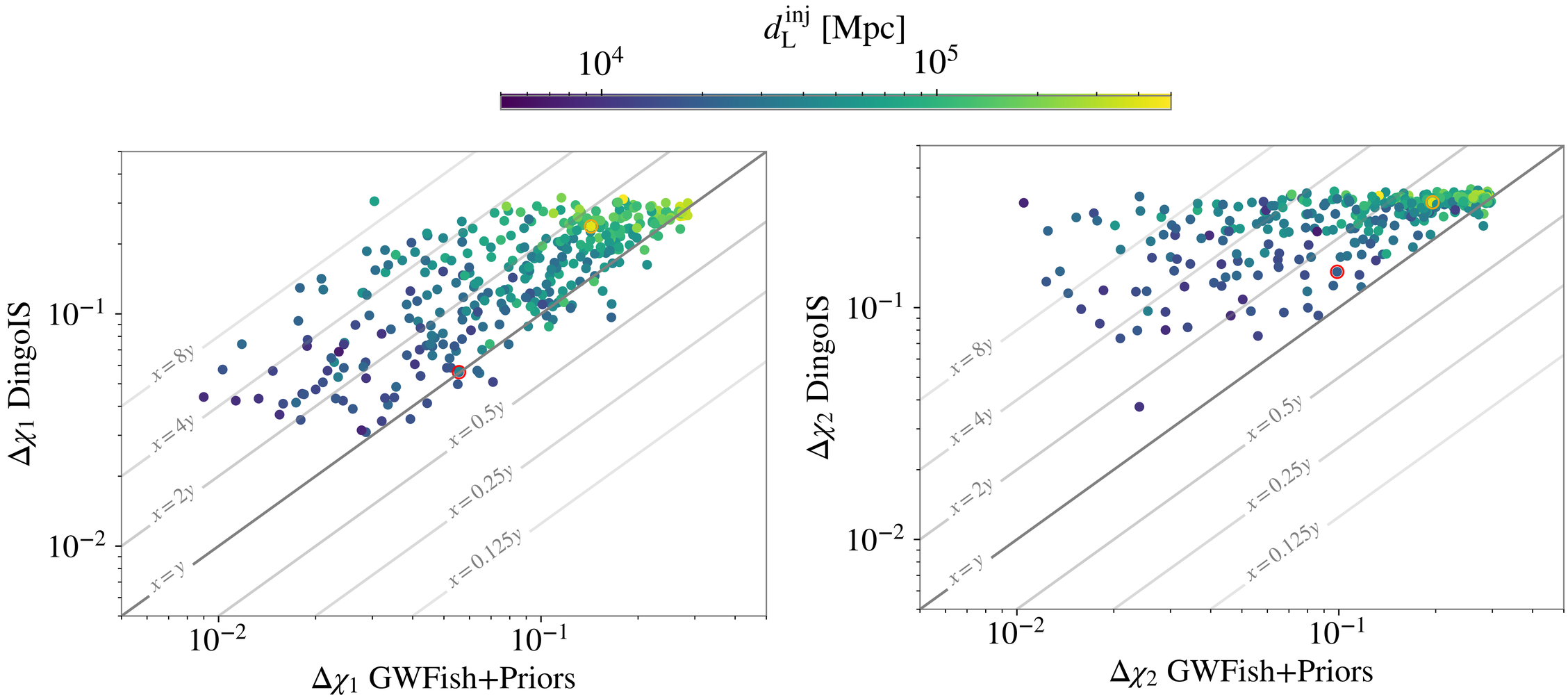
Masses





Ref. [Santoliquido et al. 2025]

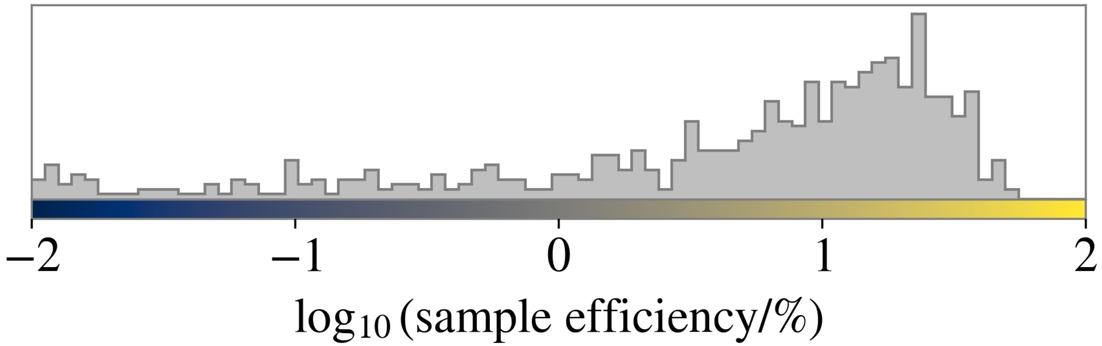
Aligned spins

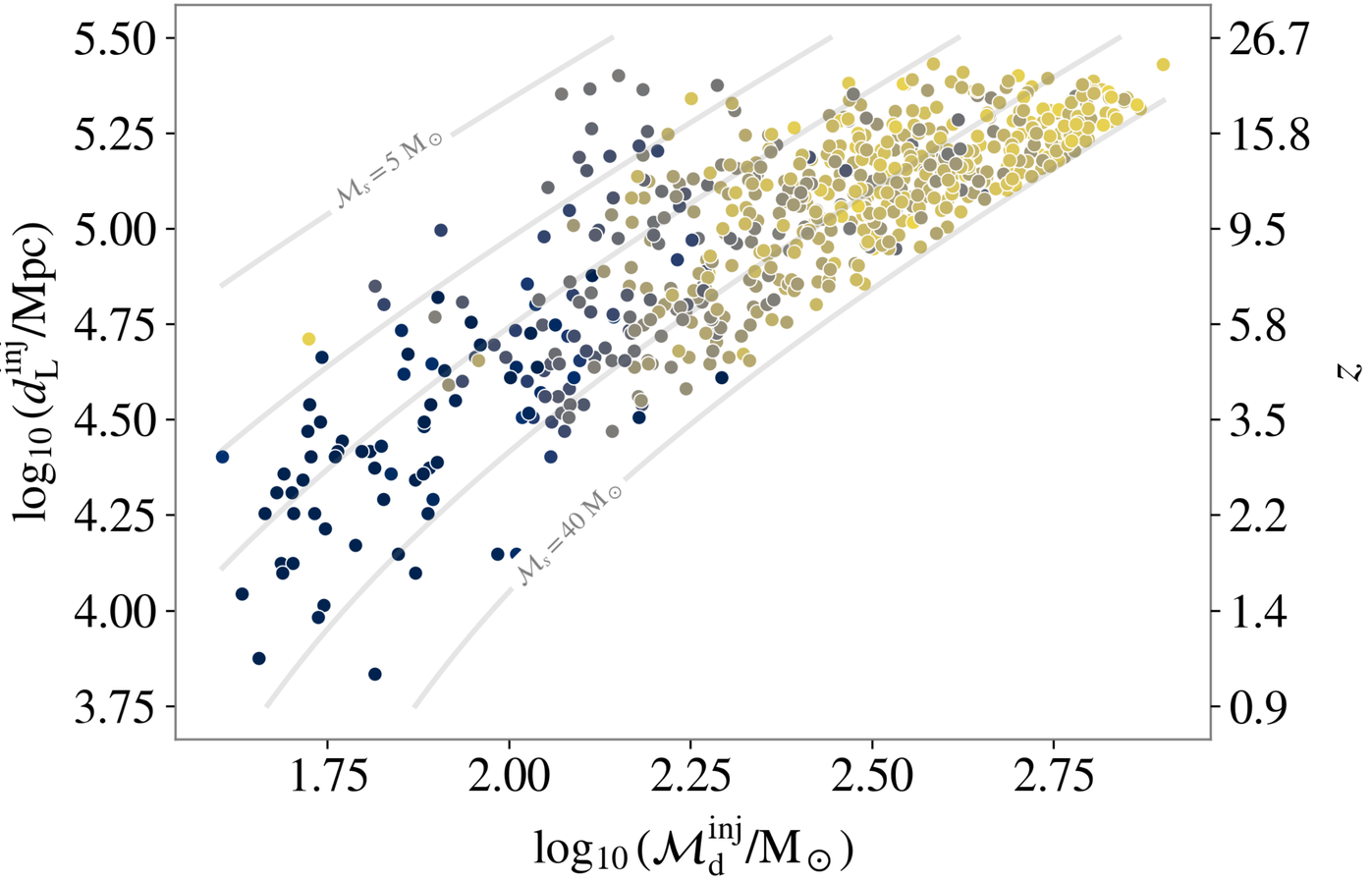


Ref. Santoliquido et al. 2025

Astrophysical population

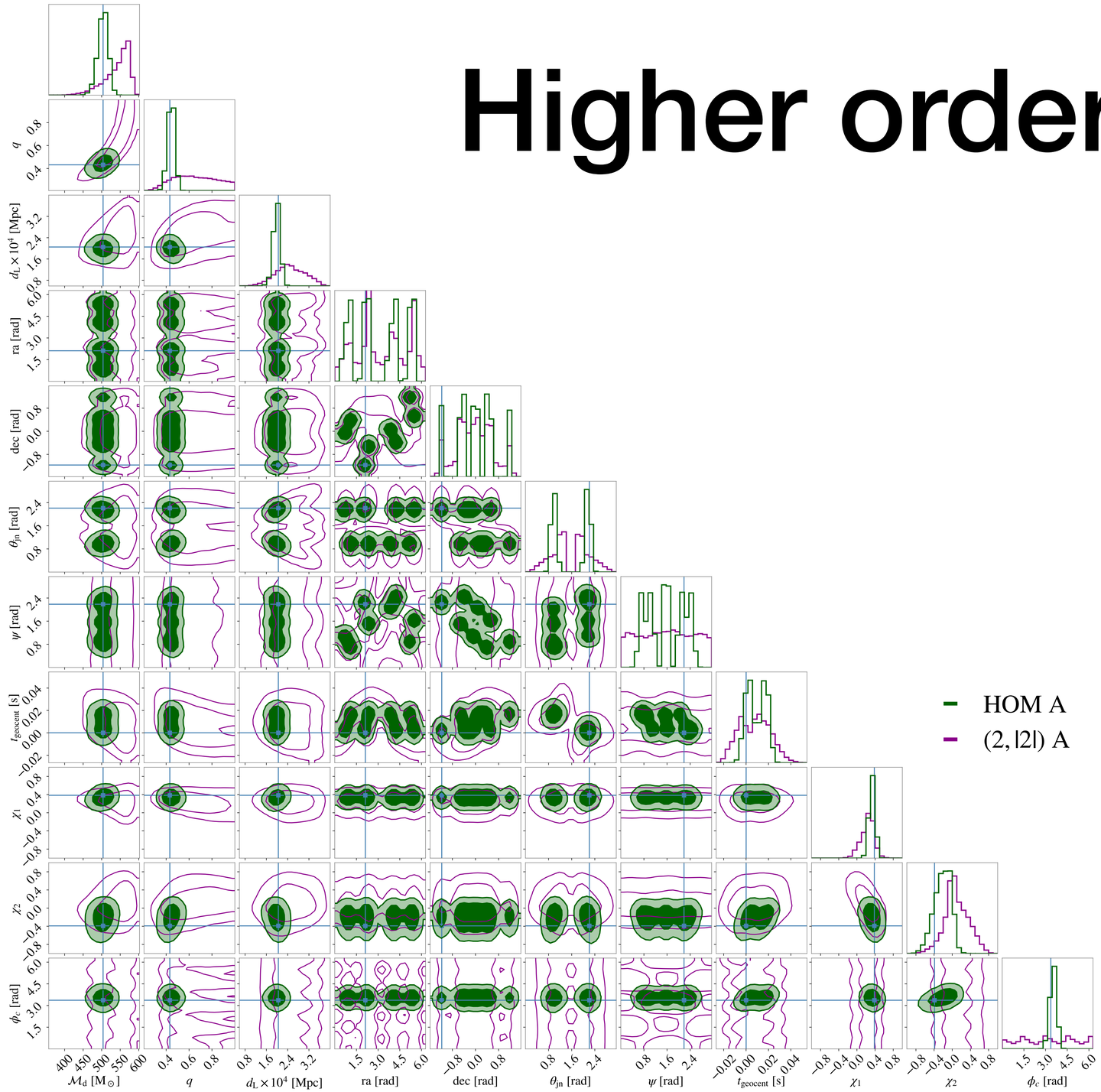
Binary black holes formed from Population III stars





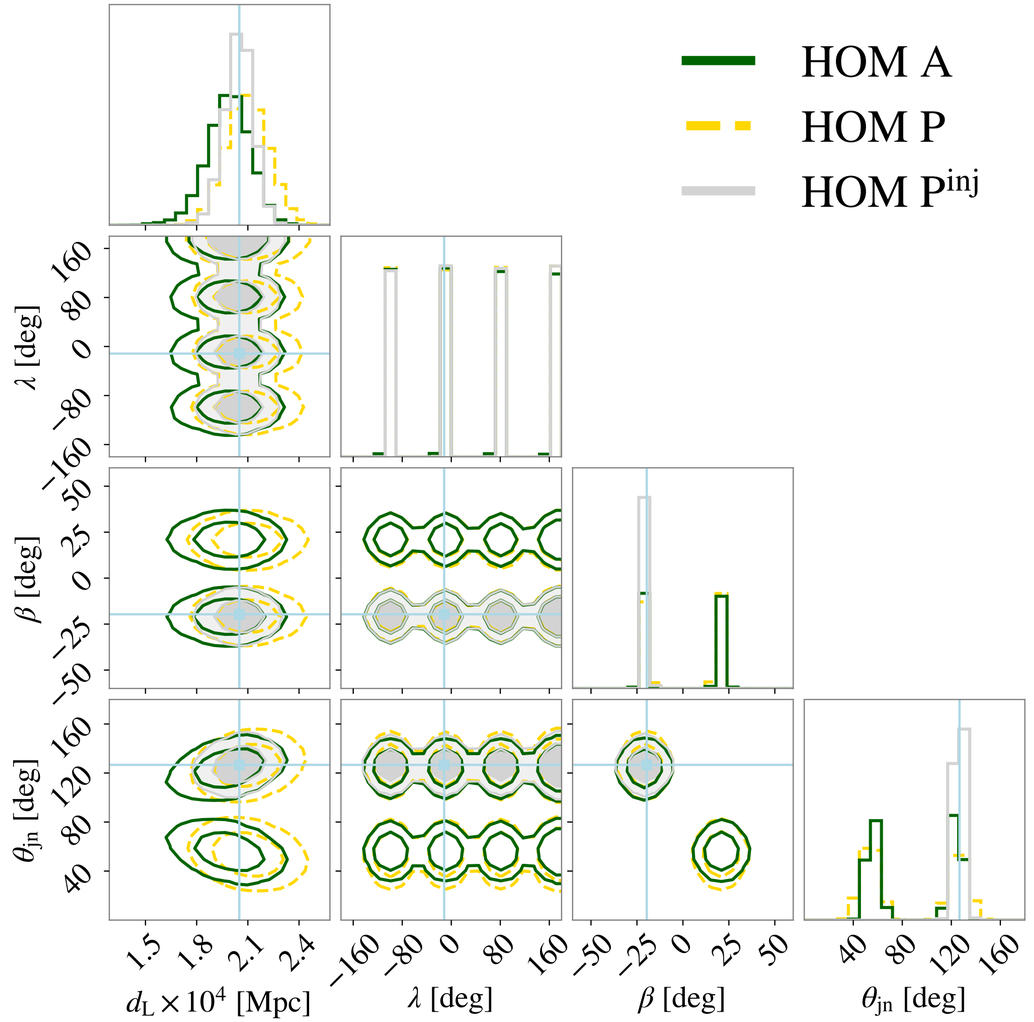
Ref. [Costa et al. 2023](), [Santoliquido et al. 2023](), [Santoliquido et al. 2024](), [Santoliquido et al. 2025]()

Higher order modes



Ref. Santoliquido et al. 2025

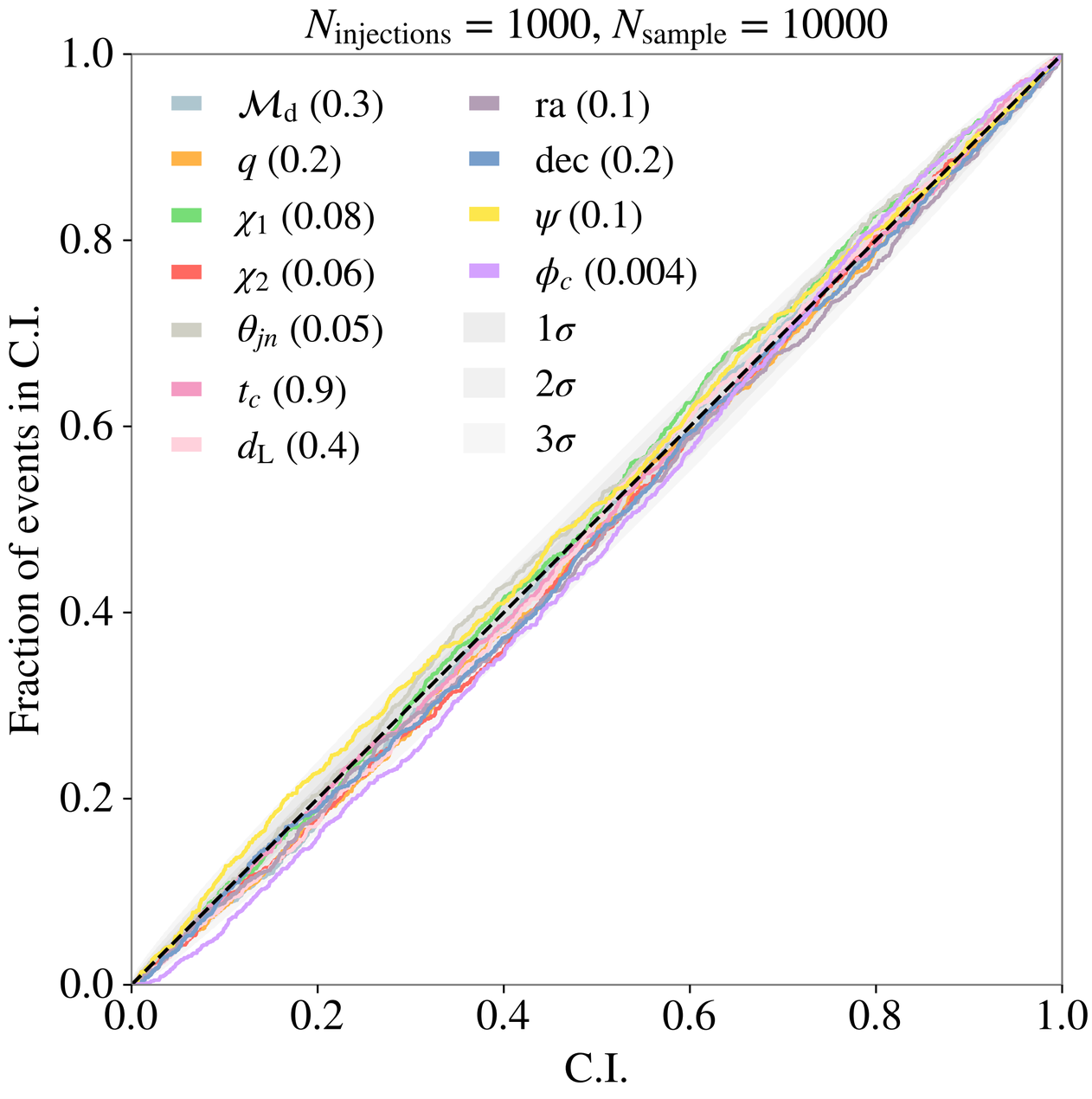
Precessing spins



Ref. Santoliquido et al. 2025

Validating results

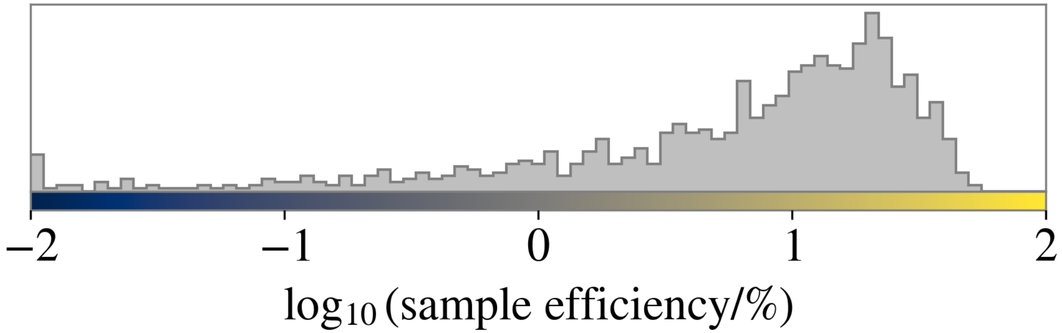
1000 random injections sampled from the priors ➡️

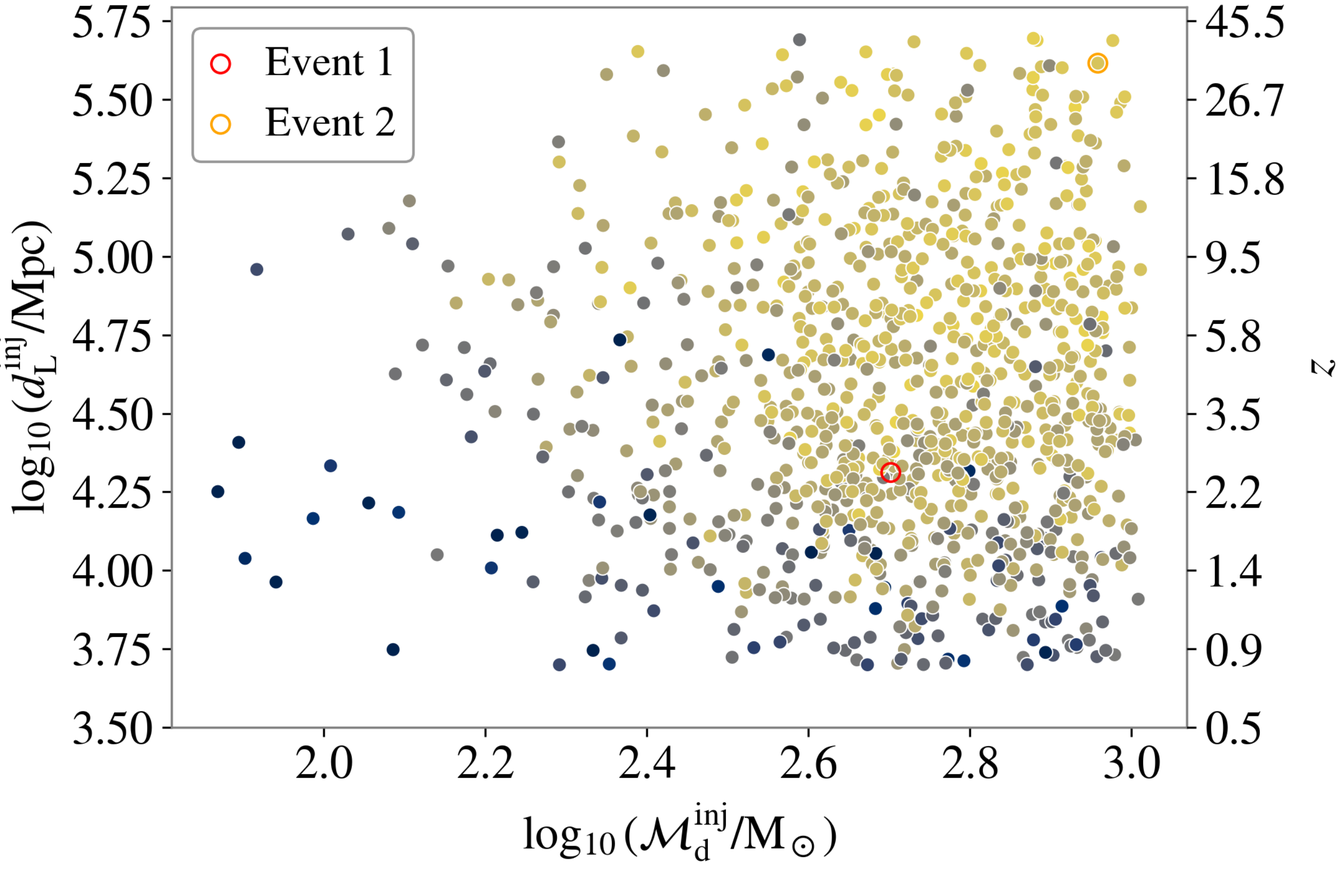


Ref. [Santoliquido et al. 2025]

Wide redshift range accuracy ➡️

85% of sources with sample efficiency > 1%





Sky modes in LISA

β_L, λ_L	injected sky location

Sky mode	*Full*	*Frozen*	*Low-f*	*Frozen low-f*
reflected: $-\beta_L$, λ_L	t-dep.	degen.	t-dep.	degen.
antipodal: $-\beta_L$, $\lambda_L + \pi$	f-dep.+$\Delta\Phi_R$	f-dep.	$\Delta\Phi_R$	degen.
β_L, $\lambda_L + \pi/2$	t-f-dep.	f-dep.	t-dep.	degen.
β_L, $\lambda_L + \pi$	t-f-dep.	f-dep.	t-dep.	degen.
β_L, $\lambda_L - \pi/2$	t-f-dep.	f-dep.	t-dep.	degen.
$-\beta_L$, $\lambda_L + \pi/2$	t-f-dep.	f-dep.	t-dep.	degen.
$-\beta_L$, $\lambda_L - \pi/2$	t-f-dep.	f-dep.	t-dep.	degen.

Ref. Vishal et al. 2020, Sylvain et al. 2021, Singh and Bulik 2021, Singh and Bulik 2022

Energy cost

- Hardware: **GPU** NVIDIA A100 80GB, **CPU** AMD EPYC 7513 32-core
- `Dingo-IS`:
  - Training: 970 kWh {1 GPU + 32 CPUs running for 6 days}
  - Inference: 45 kWh {8 CPUs, 1.7 minutes per source on average}
    - Total: **1015 kWh**
- `Bilby`:
  - Inference: **4000 kWh** {4 CPUs, ~5 hours per source on average}

Ref. Wouters et al. 2024, Hu et al. 2024