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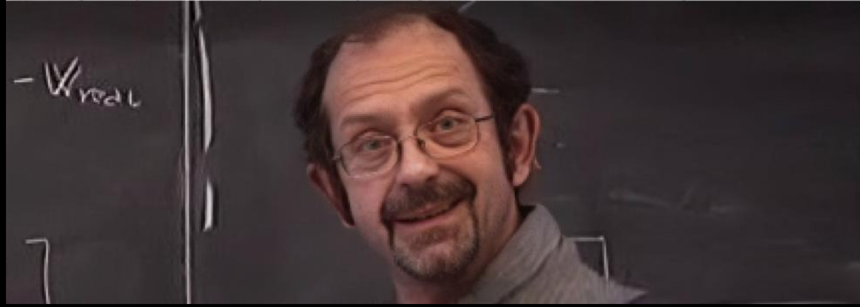

Stefano Catani and The Dipole Subtraction Algorithm

Mike Seymour

University of Manchester

Stefano Catani Memorial Symposium

Galileo Galilei Institute for Theoretical Physics

9 January 2025

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Stefano and me

- October 1989: First postdoc in Cambridge, PhD in Cambridge
- October 1991: CERN Fellow
- October 1992: First postdoc in Lund, started collaborating with Stefano and Yuri Dokshitzer
- February 1993: Catani, Dokshitzer, Seymour and Webber, Longitudinally invariant Kt clustering algorithms for hadron hadron collisions, Nucl. Phys. B 406 (1993) 187
- October 1993: Returned to Florence
- August 1994: QCD Summer Institute at Gran Sasso National Lab
  - Started work on Dipole Subtraction Algorithm

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Stefano and me

- January 1995: CERN Fellow – began regular visits to Florence
- February 1996: Catani and Seymour, The Dipole formalism for the calculation of QCD jet cross-sections at next-to-leading order, Phys. Lett. B 378 (1996) 287
- May 1996: Catani and Seymour, A General algorithm for calculating jet cross-sections in NLO, Nucl. Phys. B 485 (1997) 291
- October 1996: Catani, Seymour and Trócsányi, Regularization scheme independence and unitarity in QCD cross sections, Phys. Rev. D55 (1997) 6819
- January 1997: moved to Rutherford Appleton Lab theory group
- July 1997: Seymour, Jet shapes in hadron collisions: Higher orders, resummation and hadronization, Nucl. Phys. B 513 (1998) 269, strongly encouraged by Stefano

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Stefano and me

- October 1997: Keith Ellis became first person to use Catani–Seymour algorithm, discovered error in Initial-Initial kinematics
- November 1997: Catani and Seymour, A General algorithm for calculating jet cross-sections in NLO, Erratum Nucl. Phys. B 510 (1998) 503
- October 1998: Catani and Grazzini, … NNLO QCD …
- May 1999: Catani and Seymour, Corrections of O (alpha_S^2) to the forward backward asymmetry, JHEP 07 (1999) 023
- January 2002: Catani, Dittmaier, Seymour and Trócsányi, The Dipole formalism for NLO QCD calculations with massive partons, Nucl. Phys. B 627 (2002) 189

The Dipole Subtraction Algorithm

- Context
- The algorithm
  - Important details
  - Parton masses
- Some examples
- Catani–Seymour dipole showers
  - NLO matching
- Spurious singularities
  - Convergence factors

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The Dipole Subtraction Algorithm

- Context

$$\sigma_{LO} = \int_m \mathrm{d}\Phi^{(m)} \left|\mathcal{M}_m\right|^2 F_J^{(m)}(\Phi^{(m)})$$

$$\sigma_{NLO} = \sigma_R + \sigma_V\,,$$

$$\sigma_R = \int_{m+1} \mathrm{d}\Phi^{(m+1)} \left|\mathcal{M}_{m+1}\right|^2 F_J^{(m+1)}(\Phi^{(m+1)}),$$

$$\sigma_V = \int_m \mathrm{d}\Phi^{(m)} \left|\mathcal{M}_m\right|^2_{1-\mathrm{loop}} F_J^{(m)}(\Phi^{(m)}),$$

- σ_R and σ_V separately divergent – how to combine without knowing F_J ?

Phase Space Slicing

$$\sigma_R \approx \int_{m+1} \mathrm{d}\Phi^{(m+1)} \left|\mathcal{M}_{m+1}\right|^2 \Theta(y - y_{cut})\, F_J^{(m+1)}(\Phi^{(m+1)})$$
$$+ \int_{m+1} \mathrm{d}\Phi^{(m+1)} \left|\widetilde{\mathcal{M}}_{m+1}\right|^2 \Theta(y_{cut} - y)\, F_J^{(m)}(\Phi^{(m)})$$

- Fabricius, Kramer, Schierholz & Schmitt, 1981
- Kramer & Lampe, 1989
- Baer, Ohnemus & Owens, 1990
- Klasen & Kramer, 1995, Mirkes & Zeppenfeld, 1995
- Giele & Glover, 1992, & Kosower, 1993

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Subtraction Algorithms

$$\sigma_R - \sigma_S = \int_{m+1} \mathrm{d}\Phi^{(m+1)} \left\{ \left|\mathcal{M}_{m+1}\right|^2 F_J^{(m+1)}(\Phi^{(m+1)}) - \left|\widetilde{\mathcal{M}}_{m+1}\right|^2 F_J^{(m)}(\widetilde{\Phi}^{(m)}(\Phi^{(m+1)})) \right\},$$

$$\sigma_V + \sigma_S = \int_{m} \mathrm{d}\Phi^{(m)} \left\{ \left|\mathcal{M}_{m}\right|^2_{1-\mathrm{loop}} + \int \mathrm{d}\Phi^{(1)} \left|\widetilde{\mathcal{M}}_{m+1}\right|^2 \right\} F_J^{(m)}(\Phi^{(m)}).$$

- K.Ellis, Ross & Terrano, 1981
- S.Ellis, Kunszt & Soper, 1989 & 1992
- Kunszt & Nason, EVENT program 1989
- Graudenz, 1995
- Frixione, Kunszt & Signer, 1996

The Dipole Subtraction Algorithm

$$\sigma_R - \sigma_S = \int_{m+1} \mathrm{d}\Phi^{(m+1)} \Bigg\{ \left|\mathcal{M}_{m+1}\right|^2 F_J^{(m+1)}(\Phi^{(m+1)}) - \sum_{ij,k} \left|\mathcal{M}_{m+1}^{A(ij,k)}\right|^2 F_J^{(m)}(\widetilde{\Phi}_{ij,k}^{(m)}(\Phi^{(m+1)})) \Bigg\},$$

$$\sigma_V + \sigma_S = \int_m \mathrm{d}\Phi^{(m)} \Bigg\{ \left|\mathcal{M}_m\right|^2_{1-\mathrm{loop}} + \int \mathrm{d}\Phi^{(1)} \sum_{i,k} \left|\mathcal{M}_{m+1}^{A(i,k)}\right|^2 \Bigg\} F_J^{(m)}(\Phi^{(m)}).$$

- General subtraction matrix element calculated from soft and collinear structure of QCD matrix elements
- Exact factorization of emission phase space

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Important Details

1. "Dipole" structure is partitioned into emitter and spectator
    - Both final state:

$$\frac{p_i p_k}{(p_i p_j)(p_k p_j)} = \frac{p_i p_k}{(p_i p_j)(p_i p_j + p_k p_j)} + \frac{p_i p_k}{(p_k p_j)(p_k p_j + p_i p_j)} \quad \text{Final-state parton } p_k$$

    - Final state – initial state

$$\frac{p_i p_a}{(p_i p_j)(p_a p_j)} = \frac{p_i p_a}{(p_i p_j)(p_i p_j + p_a p_j)} + \frac{p_i p_a}{(p_a p_j)(p_a p_j + p_i p_j)} \quad \text{Initial-state parton } p_a$$

    - Disobeys naïve crossing
- Allows soft–collinear to be combined with collinear

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Important Details

2. Colour correlations

$$\left|\mathcal{M}_{m+1}^{A(ij,k)}\right|^2 = \frac{-1}{2p_ip_j}\,{}_m\langle\widetilde{\Phi}^{(m)}|\frac{\boldsymbol{T}_k\cdot\boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2}\boldsymbol{V}_{ij,k}|\widetilde{\Phi}^{(m)}\rangle_m$$

- $\boldsymbol{T}_i$ = colour-charge matrix in representation of parton *i*
- Physical matrix elements are colourless vectors in colour space
  - (conservation of colour)

$$\left(\sum_i^m \boldsymbol{T}_i\right)|\widetilde{\Phi}^{(m)}\rangle_m = 0$$

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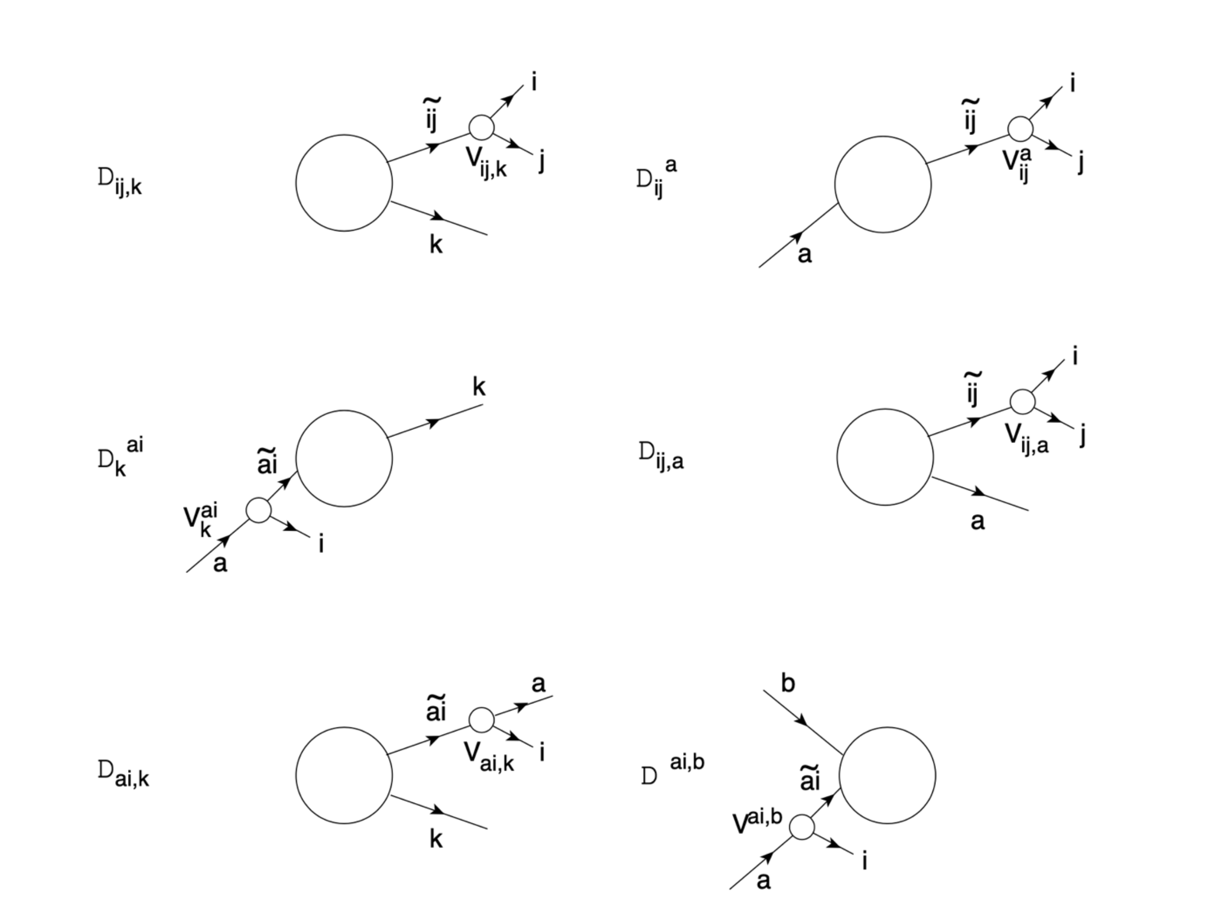
Important Details

3. Exact phase space factorization
    - physical parton configurations cover full phase space – smooth subtraction cross section
    - modified kinematics for each identified parton direction



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Important Details

4. All integration constants and functions are simple and universal

$$-\frac{1}{2}\sum_b \int_0^1 \mathrm{d}z\,((z(1-z))^{-\epsilon}\hat{P}_{ab}(z;\epsilon) = 2C_a\frac{1}{\epsilon} + \gamma_a + \left(K_a - \frac{\pi^2}{6}C_a\right)\epsilon + \mathcal{O}(\epsilon^2)$$

- $\gamma_g = \beta_0,\ K_g =$ CMW scheme parameter
- ISR and FSR distribution function convolutions equal

$$\overline{K}^{ab}(x) = \delta^{ab}\left[C_a\left(\frac{2}{1-x}\ln\frac{1-x}{x}\right)_+ - \delta(1-x)\left(\gamma_a + K_a - \frac{5}{6}\pi^2 C_a\right)\right]$$

$$+ P^{ab}_{\mathrm{reg}}(x)\ln\frac{1-x}{x} + \hat{P}'_{ab}(x)$$

$$\hat{P}_{ab}(x;\epsilon) = \delta_{ab}C_a\frac{2}{1-x} + P^{ab}_{\mathrm{reg}}(x) + \epsilon\hat{P}'_{ab}(x) + \mathcal{O}(\epsilon^2)$$

Important Details

5. Observable calculated for every subtraction configuration

$$\sigma_R - \sigma_S = \int_{m+1} \mathrm{d}\Phi^{(m+1)} \Bigg\{ \left|\mathcal{M}_{m+1}\right|^2 F_J^{(m+1)}(\Phi^{(m+1)}) - \sum_{ij,k} \left|\mathcal{M}_{m+1}^{A(ij,k)}\right|^2 F_J^{(m)}(\widetilde{\Phi}_{ij,k}^{(m)}(\Phi^{(m+1)})) \Bigg\},$$

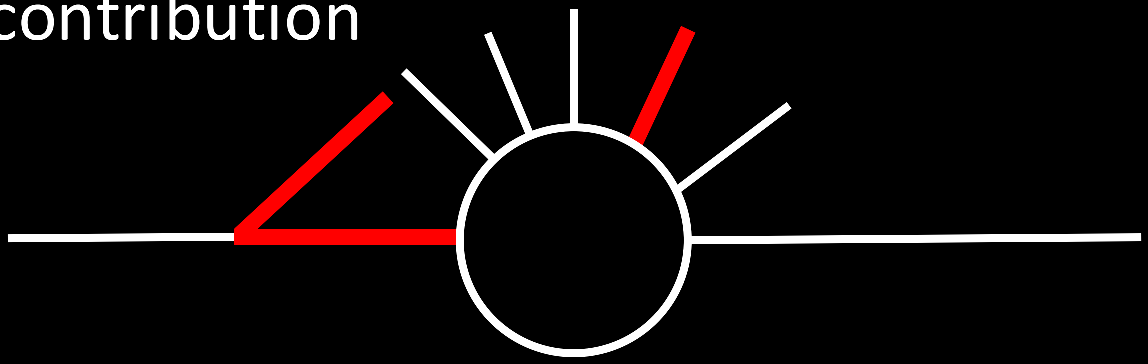
$$\sigma_V + \sigma_S = \int_m \mathrm{d}\Phi^{(m)} \Bigg\{ \left|\mathcal{M}_m\right|^2_{1-\mathrm{loop}} + \int \mathrm{d}\Phi^{(1)} \sum_{i,k} \left|\mathcal{M}_{m+1}^{A(i,k)}\right|^2 \Bigg\} F_J^{(m)}(\Phi^{(m)}).$$

- Nagy and Trócsánsyi 1997 – additional cutoff and partitioning

Parton Masses

- In principle straightforward
  - but lots of technical complications…
  - Catani, Dittmaier, Seymour and Trócsányi, 2002
- Key requirement: smooth mapping to massless algorithm
  - (when massive parton unidentified)

  →mass logarithms under analytical control
- One "missing" dipole contribution



- Stefano: no proof of factorization into universal PDFs

Some Examples

- EVENT2 – 2- and 3-jet observables in e^+e^-
  - e.g. Dasgupta, Fregoso, Marzani & Powling, 2013, extraction of log coefficients

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Some Examples

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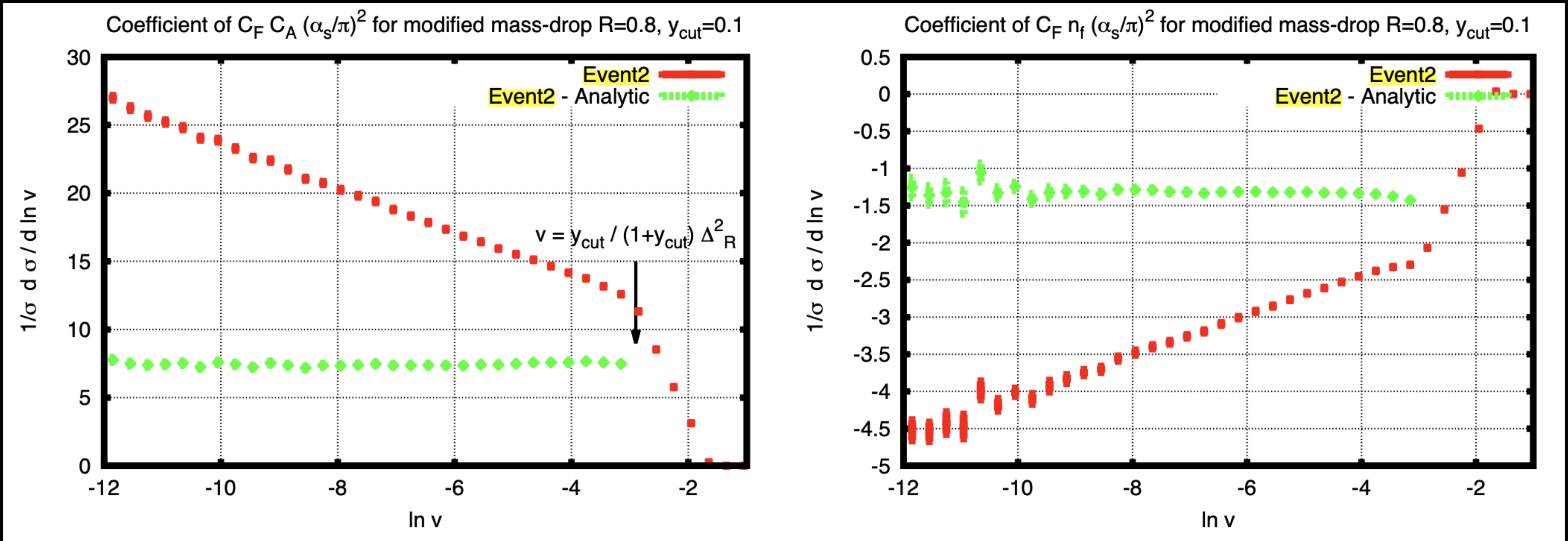

Some Examples

- EVENT2 – 2- and 3-jet observables in e^+e^-
  - e.g. Dasgupta, Fregoso, Marzani & Powling, 2013, extraction of log coefficients
- DISENT – 1+1- and 2+1-jet observables in DIS
  - D. Graudenz comparison, 1997

Some Examples

- EVENT2 – 2- and 3-jet observables in e^+e^-
  - e.g. Dasgupta, Fregoso, Marzani & Powling, 2013, extraction of log coefficients
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  - D. Graudenz comparison, 1997

	leading order			next-to-leading order		
bin	DISASTER++	MEPJET	DISENT	DISASTER++	MEPJET	DISENT
1	50.6± 0.12	50.7± 0.13	50.7± 0.15	58.6± 1.29	72.9± 1.56	54.7± 2.1
2	27.1± 0.05	27.1± 0.16	27.0± 0.07	36.4± 0.57	40.0± 0.84	34.9± 1.0
3	8.51± 0.02	8.51± 0.02	8.52± 0.02	13.8± 0.35	13.3± 0.43	13.9± 0.2
4	49.8± 0.10	49.7± 0.05	49.6± 0.14	41.2± 0.55	47.2± 0.91	41.9± 0.38
5	29.0± 0.05	29.0± 0.03	28.8± 0.07	27.3± 0.52	28.2± 0.42	26.4± 0.19
6	9.09± 0.01	9.07± 0.01	9.04± 0.02	9.58± 0.06	9.16± 0.15	9.54± 0.06
7	30.6± 0.08	30.5± 0.04	30.5± 0.12	32.0± 0.34	36.3± 0.59	32.4± 0.52

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Some Examples

- EVENT2 – 2- and 3-jet observables in e^+e^-
  - e.g. Dasgupta, Fregoso, Marzani & Powling, 2013, extraction of log coefficients
- DISENT – 1+1- and 2+1-jet observables in DIS
  - D. Graudenz comparison, 1997
- MCFM – Campbell & K.Ellis, 1998 –
  - general framework for Monte Carlo for FeMtobarn processes
  - ~ 200 processes – ~ 7000 citations!
- NLOJET++ – Nagy & Trócsányi, 1998 –
  - 2-jet and 3-jet cross sections in hadron–hadron collisions, 4-jet in e^+e^-
- Herwig/Matchbox & Sherpa
  - full automation for arbitrary processes

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Some Examples

- Corrections of $\mathcal{O}(\alpha_s^2)$ to the forward backward asymmetry
  - Catani & Seymour, 1999
  - Early example of "NNLO local subtraction" method

$$A_{FB}(e^+e^- \to b\bar{b}) = \frac{\sigma(\theta > 90^\circ) - \sigma(\theta < 90^\circ)}{\sigma(\theta > 90^\circ) + \sigma(\theta < 90^\circ)}$$

$$\equiv \frac{\sigma_A}{\sigma_S} = \frac{\sigma_A^{(0)} + \sigma_A^{(1)R} + \sigma_A^{(1)V} + \sigma_A^{(2)VV} + \sigma_A^{(2)RV} + \sigma_A^{(2)RR}}{\sigma_S^{(0)} + \sigma_S^{(1)R} + \sigma_S^{(1)V} + \sigma_S^{(2)VV} + \sigma_S^{(2)RV} + \sigma_S^{(2)RR}}$$

$$= \frac{\sigma_A^{(0)}}{\sigma_S^{(0)}}\left[1 + \left(1 - \frac{\sigma_S^{(1)}}{\sigma_S^{(0)}}\right)\left(\frac{\sigma_A^{(1)R}}{\sigma_A^{(0)}} - \frac{\sigma_S^{(1)R}}{\sigma_S^{(0)}}\right) + \frac{\sigma_A^{(2)RV}}{\sigma_A^{(0)}} - \frac{\sigma_S^{(2)RV}}{\sigma_S^{(0)}} + \frac{\sigma_A^{(2)RR}}{\sigma_A^{(0)}} - \frac{\sigma_S^{(2)RR}}{\sigma_S^{(0)}}\right]$$

One-loop V and two-loop VV cancel in dim reg

NLO integral calculated with standard dipole subtraction method

(+ extra complication from 4b final states)

Some Examples

- Corrections of $\mathcal{O}(\alpha_s^2)$ to the forward backward asymmetry
  - Catani & Seymour, 1999
  - Early example of “NNLO local subtraction” method
  - Were able to resolve long-standing discrepancy between Altarelli & Lampe and Ravindram & van Neerven
    - (neither was correct! But RvN only by numerically-insignificant 4b term)
  - And able to use any axis definition, e.g. thrust axis, like experiments
  - Removed 1% uncertainty from global EW fits
    - (remaining uncertainty estimated ~0.5%)
    - (final LEP/SLC uncertainty ~2%)

Some Examples

- Energy-energy correlation in e^+e^-
  - Calculated to NLO by 10 different groups! With many discrepancies
  - Dixon, Luo, Shtabovenko, Yang & Zhu, 2018 – analytical result ("remarkably simple"!)

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Some Examples

- Energy-energy correlation in e^+e^-
  - Calculated to NLO by 10 different groups! With many discrepancies
  - Dixon, Luo, Shtabovenko, Yang & Zhu, 2018 – analytical result ("remarkably simple"!)

$$
\begin{aligned}
B_{\mathrm{lc}} = &+ \frac{122400z^7 - 244800z^6 + 157060z^5 - 31000z^4 + 2064z^3 + 72305z^2 - 143577z + 63298}{1440(1-z)z^4} \\
&- \frac{-244800z^9 + 673200z^8 - 667280z^7 + 283140z^6 - 48122z^5 + 2716z^4 - 6201z^3 + 11309z^2 - 9329z + 3007}{720(1-z)z^5} g_1^{(1)} \\
&- \frac{244800z^8 - 550800z^7 + 422480z^6 - 126900z^5 + 13052z^4 - 336z^3 + 17261z^2 - 38295z + 19938}{720(1-z)z^4} g_2^{(1)} \\
&+ \frac{4z^7 + 10z^6 - 17z^5 + 25z^4 - 96z^3 + 296z^2 - 211z + 87}{24(1-z)z^5} g_1^{(2)} \\
&+ \frac{-40800z^8 + 61200z^7 - 28480z^6 + 4040z^5 - 320z^4 - 160z^3 + 1126z^2 - 4726z + 3323}{120z^5} g_2^{(2)} \\
&- \frac{1-11z}{48z^{7/2}} g_3^{(2)} - \frac{120z^6 + 60z^5 + 160z^4 - 2246z^3 + 8812z^2 - 10159z + 4193}{120(1-z)z^5} g_4^{(2)} \\
&- 2\left(85z^4 - 170z^3 + 116z^2 - 31z + 3\right) g_1^{(3)} + \frac{-4z^3 + 18z^2 - 21z + 5}{6(1-z)z^5} g_2^{(3)} + \frac{z^2+1}{12(1-z)} g_3^{(3)}, \qquad (9)
\end{aligned}
$$

where the $g_m^{(n)}$ are pure functions of uniform transcendental weight n. Their explicit definitions are

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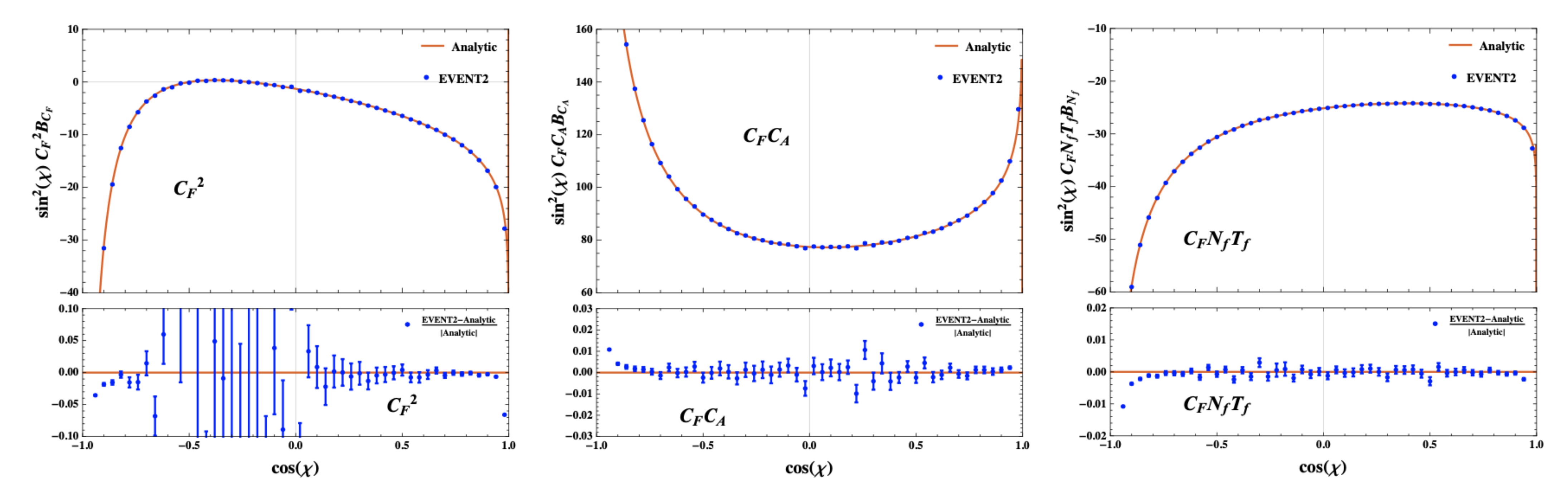
Some Examples

- Energy-energy correlation in e^+e^-
  - Calculated to NLO by 10 different groups! With many discrepancies
  - Dixon, Luo, Shtabovenko, Yang & Zhu, 2018 – analytical result compared to EVENT2



(a) B_{C_F} (b) B_{C_A} (c) B_{N_f}

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- Catani–Seymour dipole showers
  - NLO matching
- Spurious singularities
  - Convergence factors

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Catani–Seymour dipole showers

- Exact phase space factorization
- Partitioned dipole kinematics / local recoil
- Colour correlations → leading colour
  - Nagy & Soper, 2005 – (DEDUCTOR)
  - Dinsdale, Ternick & Weinzierl arXiv:0709.1026
  - Schumann & Krauss arXiv:0709.1027, Winter & Krauss arXiv:0712.3913 (SHERPA)
    - Drell-Yan recoil only from first emission
  - Plätzer & Gieseke, 2009 – (HERWIG)
    - modified recoil scheme for initial state partons
  - Höche & Prestel, 2015 – (DIRE – SHERPA & PYTHIA)
    - aiming for NLL accuracy
  - Höche (MCnet build-your-own-parton-shower tutorial), 2016 –

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Catani–Seymour dipole showers
– the next generation

- Dasgupta, Dreyer, Hamilton, Monni, Salam & Soyez, 2020 –
  - pointed out impossibility of reaching NLL accuracy with local recoil
  - PanScales (PanGlobal)
  - systematic studies of recoil and partitioning
  - first NNLL (final state) shower, 2024
- Forshaw, Holguin & Plätzer, 2020 –
  - explorations beyond leading colour
  - systematic studies of recoil and partitioning
- Herren, Höche, Krauss, Reichelt & Schoenherr, 2023 – (ALARIC – SHERPA)
- Seymour & Sule, 2024
  - GPU implementation (based on Höche tutorial) ~ 250 x faster than CPU

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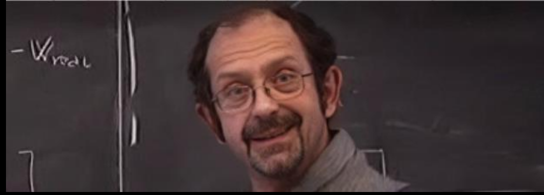

Subtractive NLO Matching

– MC@NLO and friends

$$\sigma_H = \int_{m+1} d\Phi^{(m+1)} \left\{ \left|\mathcal{M}_{m+1}\right|^2 - \left|\widetilde{\mathcal{M}}_{m+1}\right|^2 \right\} I_J^{(m+1)}(\Phi^{(m+1)}),$$

$$\sigma_S = \int_{m} d\Phi^{(m)} \left\{ \left|\mathcal{M}_{m}\right|^2 + \left|\mathcal{M}_{m}\right|^2_{1-\text{loop}} + \int d\Phi^{(1)} \left|\widetilde{\mathcal{M}}_{m+1}\right|^2 \right\} I_J^{(m)}(\Phi^{(m)}).$$

$$I_J^{(m+1)}(\Phi^{(m+1)}) = F_J^{(m+1)}(\Phi^{(m+1)}) + \mathcal{O}(\alpha_s)$$

$$I_J^{(m)}(\Phi^{(m)}) = F_J^{(m)}(\Phi^{(m)}) + \int d\Phi^{(1)} \frac{\left|\widetilde{\mathcal{M}}_{m+1}\right|^2}{\left|\mathcal{M}_{m}\right|^2} \left\{ F_J^{(m+1)}(\Phi^{(m+1)}) - F_J^{(m)}(\Phi^{(m)}) \right\} + \mathcal{O}(\alpha_s^2)$$

- Frixione & Webber, 2002
- Parton shower finite *m* and *m*+1 parton events without double-counting
- Using Catani–Seymour shower allows easy automation → Herwig & Sherpa

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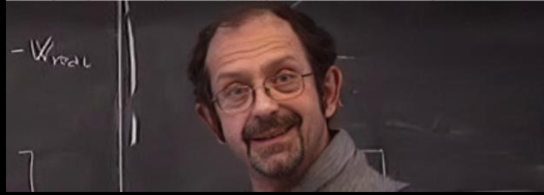
Spurious Singularities

$$\sigma_R - \sigma_S = \int_{m+1} \mathrm{d}\Phi^{(m+1)} \left\{ \left|\mathcal{M}_{m+1}\right|^2 F_J^{(m+1)}(\Phi^{(m+1)}) - \sum_{ij,k} \left|\mathcal{M}_{m+1}^{A(ij,k)}\right|^2 F_J^{(m)}(\widetilde{\Phi}_{ij,k}^{(m)}(\Phi^{(m+1)})) \right\},$$

$$\sigma_V + \sigma_S = \int_m \mathrm{d}\Phi^{(m)} \left\{ \left|\mathcal{M}_m\right|^2_{1-\text{loop}} + \int \mathrm{d}\Phi^{(1)} \sum_{i,k} \left|\mathcal{M}_{m+1}^{A(i,k)}\right|^2 \right\} F_J^{(m)}(\Phi^{(m)}).$$

- Subtraction cross section matches all singularities of real cross section
- But not guaranteed not to introduce new ("spurious") singularities
- E.g. $e^+e^- \to 4$ partons with q and qbar back to back
- Not a problem in subtraction algorithm



Spurious Singularities

- But other applications separate them

$$\sigma_R - \sigma_S = \int_{m+1} \mathrm{d}\Phi^{(m+1)} \left\{ \left|\mathcal{M}_{m+1}\right|^2 - \sum_{ij,k} \left|\mathcal{M}_{m+1}^{A(ij,k)}\right|^2 \right\} F_J^{(m+1)}(\Phi^{(m+1)})$$

$$+ \int_{m+1} \mathrm{d}\Phi^{(m+1)} \sum_{ij,k} \left|\mathcal{M}_{m+1}^{A(ij,k)}\right|^2 \left\{ F_J^{(m+1)}(\Phi^{(m+1)}) - F_J^{(m)}(\widetilde{\Phi}_{ij,k}^{(m)}(\Phi^{(m+1)})) \right\}.$$

  - E.g. second line = dipole shower,
  - first line = subtractive matching H event

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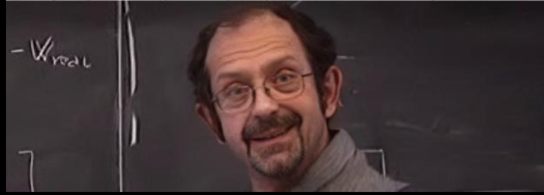

Convergence Factors

- Introduce factor $R^{(ij,k)}(\Phi^{(m+1)})$
  - → 1 in all singular limits
  - → 0 in all spurious singular limits

$$\sigma_R - \sigma_S = \int_{m+1} \mathrm{d}\Phi^{(m+1)} \left\{ \left|\mathcal{M}_{m+1}\right|^2 - \sum_{ij,k} R^{(ij,k)} \left|\mathcal{M}_{m+1}^{A(ij,k)}\right|^2 \right\} F_J^{(m+1)}(\Phi^{(m+1)})$$

$$+ \int_{m+1} \mathrm{d}\Phi^{(m+1)} \sum_{ij,k} R^{(ij,k)} \left|\mathcal{M}_{m+1}^{A(ij,k)}\right|^2 \left\{ F_J^{(m+1)}(\Phi^{(m+1)}) - F_J^{(m)}(\widetilde{\Phi}_{ij,k}^{(m)}(\Phi^{(m+1)})) \right\},$$

$$\sigma_V + \sigma_S = \int_m \mathrm{d}\Phi^{(m)} \left\{ \left|\mathcal{M}_m\right|^2_{1-\mathrm{loop}} + \int \mathrm{d}\Phi^{(1)} \sum_{i,k} \left|\mathcal{M}_{m+1}^{A(i,k)}\right|^2 \right.$$

$$\left. - \int \mathrm{d}\Phi^{(1)} \sum_{i,k} \left(1 - R^{(i,k)}\right) \left|\mathcal{M}_{m+1}^{A(i,k)}\right|^2 \right\} F_J^{(m)}(\Phi^{(m)}).$$

Finite analytical integral in *d* dimensions

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Convergence Factors

- E.g. $R^{(ij,k)} = \dfrac{1}{1 + \dfrac{p_\perp^2}{\mu^2}}$ where μ = scale of hard process
- E.g. coefficient of α_s^2 in $\langle 1 - \mathrm{thrust} \rangle$

	M4*F4-Msub*F3	Msub*(F4-F3)	(M4-Msub)*F4	3-parton	Total
Standard	14.68±0.05	95.24±5.98	–80.56±5.98	30.31±0.04	44.99±0.07
Conv. factor	43.13±0.06	32.39±0.06	10.75±0.05	1.85±0.05	44.99±0.07

- MHS & J. Whitehead, in preparation (started with Stefano in 1996)

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Summary

- The Dipole Subtraction Algorithm has been a major enabling development in QCD
  - Initiated and driven by Stefano
- 100s of NLO calculations, used in 1000s of experimental measurements
- Catani–Seymour dipole showers were a major surprise
  - Some advantages
  - But also some disadvantages
- Spurious Singularities can be cured with Convergence Factors (in preparation)
- Stefano's ultimate goal was the automation of NNLO

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Thank you!

- Thank you for listening, and to the organisers

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Thank you!

- Thank you for listening, and to the organisers

- And thank you especially to Stefano,
  for all the fun and inspiring collaboration
  and for such a deep and broad impact on our field