Galileo Galilei Institute for Theoretical Physics

Stefano Catani Memorial Symposium

Florence, January 2025

F Hautmann

QCD at high energy: the resummation idea and Stefano's contributions

- Many thanks to Massimiliano, Roberto, Leandro, Daniel, Prasanna, Giancarlo, German for this Symposium.

- The subject of this talk is Stefano's work on high-energy limit in QCD and small-x physics:

  - Factorization theorem and resummation

  - Exclusive evolution equation (CCFM equation)

KEY IDEAS

- Resummation

  recast the high-s (multi-Regge) problem as an infinite tower of logarithmic terms to be summed to all orders in perturbation theory

- Matching

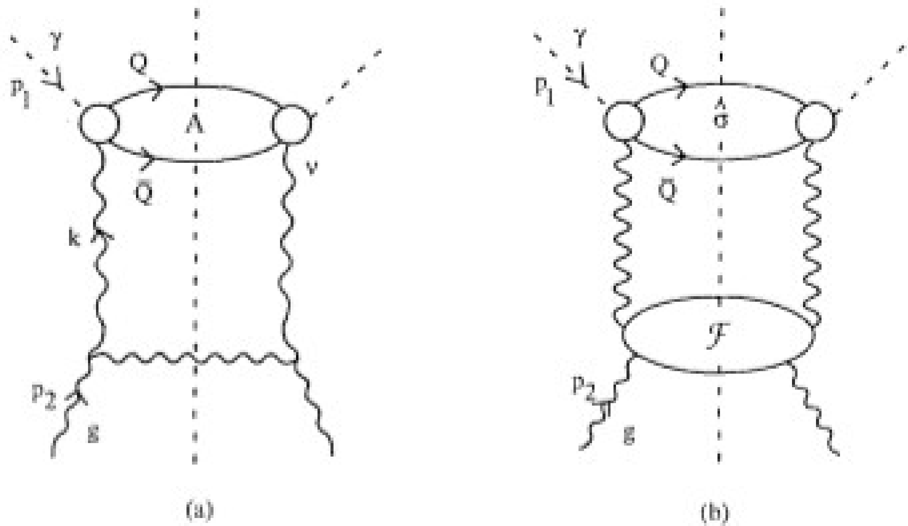
  what is the role of sub-asymptotic terms – look for methods to uniformly include finite orders, to be matched on to resummation

- Soft-Gluon Coherence

  what do multiple soft-gluon emission and exchange look like in the high-energy limit – how does this influence QCD evolution

Resummation of high-energy logarithmic corrections to heavy flavor production

Ex.: heavy flavor leptoproduction for $s \gg M^2 \gg \Lambda^2_{\rm QCD}$



$$\gamma + h \to Q + \bar{Q} + X$$

Catani, Ciafaloni, H [1990, 1991]

$$4M^2\, \sigma(x, M^2) = \int d^2\mathbf{k}_\perp \int_x^1 \frac{dz}{z}\, \hat{\sigma}_{\gamma g}(x/z, \mathbf{k}_\perp^2/M^2, \alpha_{\rm S}(M^2))\, \mathcal{A}_{g/h}(z, \mathbf{k}_\perp)$$

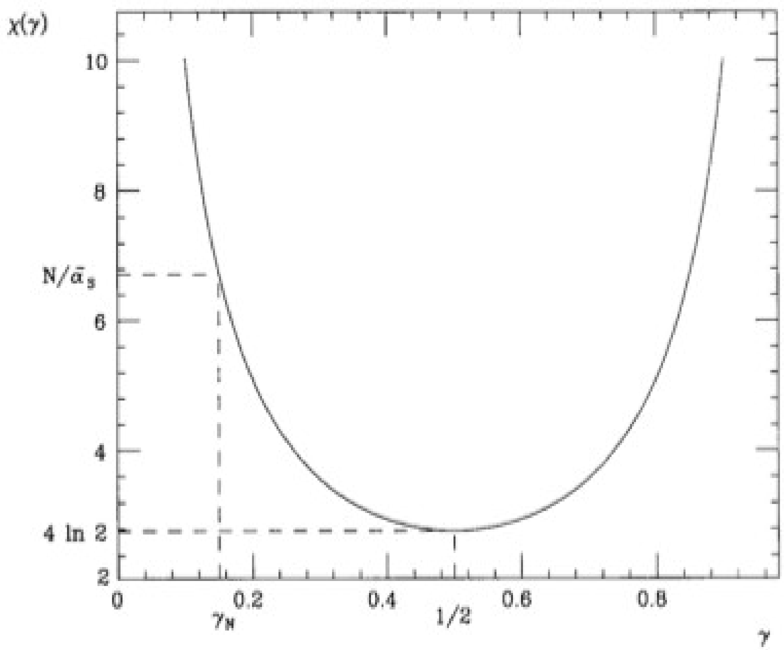
where the gluon distribution is given by

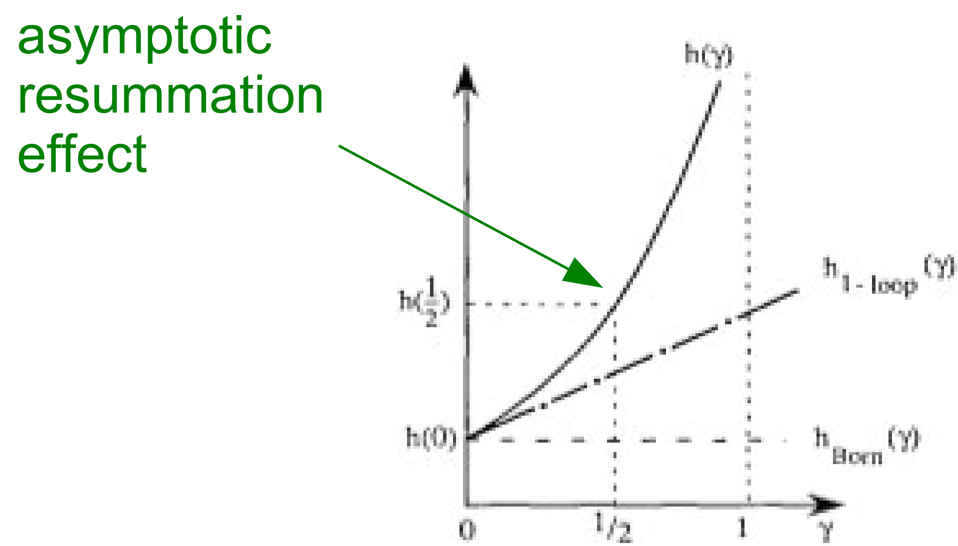
$$\mathcal{A}_{g/h}(x, \mathbf{k}_\perp) \sim \frac{1}{2\pi}\, e^{-\lambda\, \ln x}\, (\mathbf{k}_\perp^2)^{\gamma - 1} \quad , \quad \lambda \to 4\, C_A\, \frac{\alpha_{\rm S}}{\pi}\, \ln 2, \quad \gamma \to \frac{1}{2}$$

Resummation of high energy logarithms (cont'd)

$$\Rightarrow 4M^2\sigma(x, M^2) \sim x^{-\lambda} \, (M^2)^{\frac{1}{2}} \;\; h(1/2) \; ,$$

$$\text{where} \;\; h(1/2) = \frac{1}{2}\int_0^\infty \frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2}\left(\frac{\mathbf{k}_\perp^2}{M^2}\right)^{\frac{1}{2}} \int_0^1 \frac{dx}{x} \;\; \hat{\sigma}_{\gamma g}(x, \mathbf{k}_\perp^2/M^2, \alpha_S)$$





- Investigated at HERA / LHC

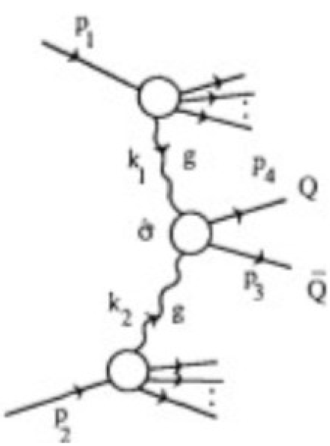
- Will be relevant for HL-LHC / FCC

Mapping on to short-distance factorization:

$$m_H^2\,\sigma = \sum_{a,b}\int dx_1\;dx_2\;f_a(x_1,\mu)\;C(\alpha_s(\mu_R), x/(x_1x_2), m_H/\mu)\;f_b(x_2,\mu)$$

- High-energy logarithms in hard-scattering function...

$$\begin{aligned} C(\alpha_s, x, m_H/\mu) \;&=\; c^{(0)}(x) + \frac{\alpha_s}{\pi}\left[c^{(1)}(x) + \overline{c}^{(1)}(x)\;L\right] \\ &+\; \left(\frac{\alpha_s}{\pi}\right)^2\left[c^{(2)}(x) + \overline{c}^{(2)}(x)\;L + \overline{\overline{c}}^{(2)}(x)\;L^2\right] + \cdots \end{aligned}$$



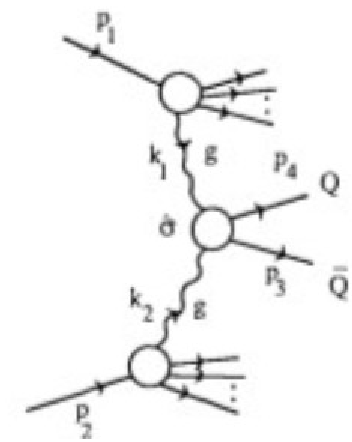
$$L = \ln(m_H^2/\mu^2),\; x = m_H^2/s$$

- ...as well as in the evolution of parton density functions

Hard-scattering coefficient functions

Moments of hard-scattering function:

$$C_N(\alpha_s, m_H/\mu) = \int_0^1 dx x^{N-1} C(\alpha_s, x, m_H/\mu)$$



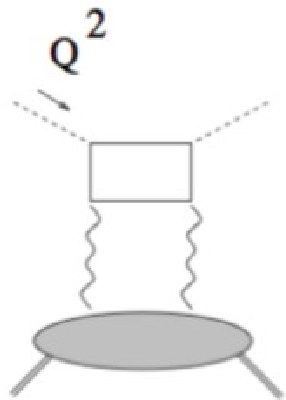
- $N \to 0$ (small-x) behavior to all orders in α_s

$$C_N(\alpha_s, m_H/\mu) = \underbrace{R_N^2(\gamma_N)\ (m_H^2/\mu^2)^{2\gamma_N}}_{RG\ \ factors}\ \underbrace{h_N(\gamma_N, \gamma_N)}_{finite\ \ part}$$

where

$$\gamma_N = \overline{\alpha}_s/N + 2\,\zeta(3)\ (\overline{\alpha}_s/N)^4\ +\ \ldots \qquad \text{(BFKL)}$$

QCD flavor-singlet evolution



Catani, H [1993, 1994]

$$\frac{d}{d\ln\mu^2}\begin{pmatrix}\Sigma\\ G\end{pmatrix} = \begin{pmatrix}P_{qq} & P_{qg}\\ P_{gq} & P_{gg}\end{pmatrix}\otimes\begin{pmatrix}\Sigma\\ G\end{pmatrix}$$

$$P_{gg} = \underbrace{\sum_{k=1}^{\infty} a_k \alpha_s^k x^{-1}\ln^{k-1} x}_{L(x)} + (b_0\alpha_s + \underbrace{\sum_{k=1}^{\infty} b_k \alpha_s \alpha_s^k x^{-1}\ln^{k-1} x}_{NL(x)}) + \dots$$

$$P_{qg} = c_0\alpha_s + \underbrace{\sum_{k=1}^{\infty} c_k \alpha_s \alpha_s^k x^{-1}\ln^{k-1} x}_{NL(x)} + \dots$$

k = 2: reobtained by full 3-loop calculation [Moch, Vermaseren, Vogt: 2004]

k = 3: ongoing 4-loop calculations [2022 - 2025] Moch et al; Falcioni et al; Gehrmann et al

Current efforts toward N3LO accurate phenomenology

N3LO PDFs

McGowan, Cridge, Harland-Lang, Thorne [*EPJC83 (2023) 185*, *arXiv:2207.04739*]

4-loop approximations constructed from x → 0 (and x → 1) asymptotics

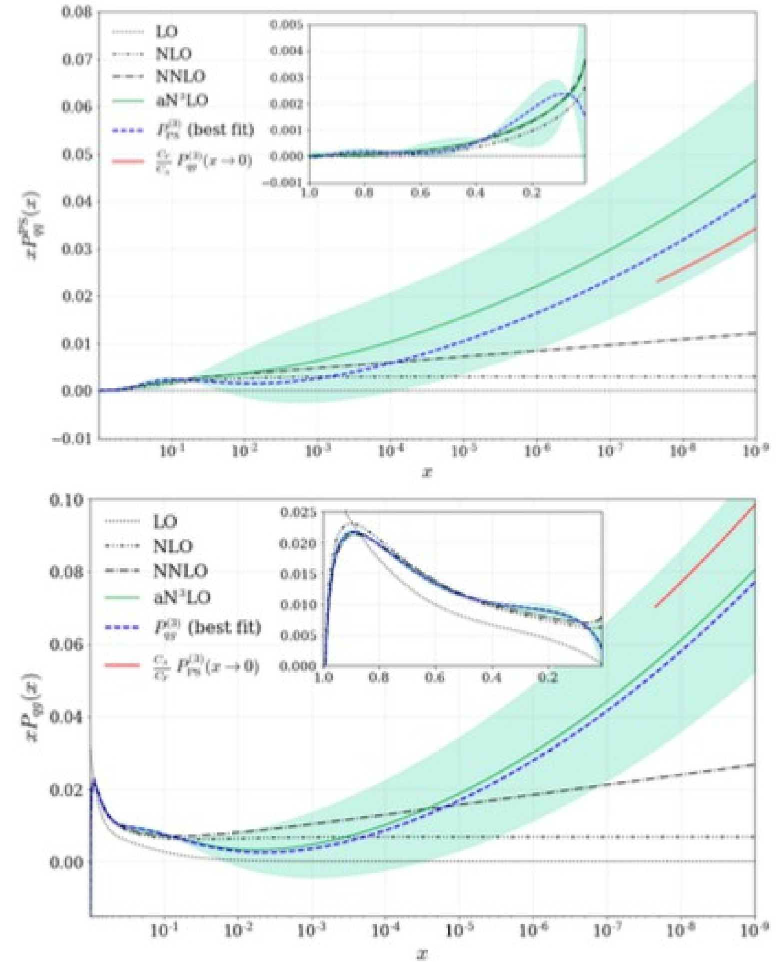
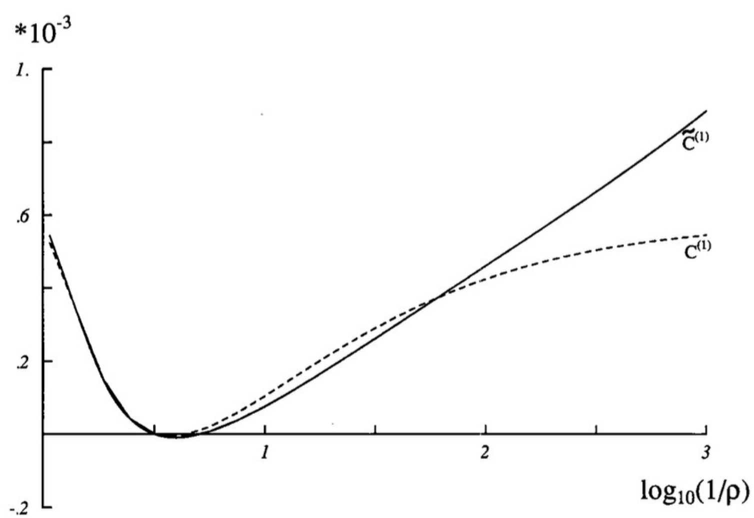


Figure 3: Perturbative expansions up to aN^{3}LO for the quark singlet splitting functions P_{qq}^{PS} (top) and P_{qg} (bottom) including any corresponding allowed $\pm 1\sigma$ variation (shaded green region). The best fit values (blue dashed line) display the predictions for each function determined from a global PDF fit.

Sub-asymptotic Effects: Matching Resummation and Finite Orders

Catani, Ciafaloni, H [1992]

- Devise procedure for subtractive matching at next-to-leading order.
- Insight from energy-momentum conservation (going from N = 0 to N = 1 moment).



Resummed coefficient function $\tilde{C}^{(1)}$ for bottom quark production ($M = 5$ GeV, $\Lambda = 260$ MeV) after performing the matching with the one-loop result $C^{(1)}$.

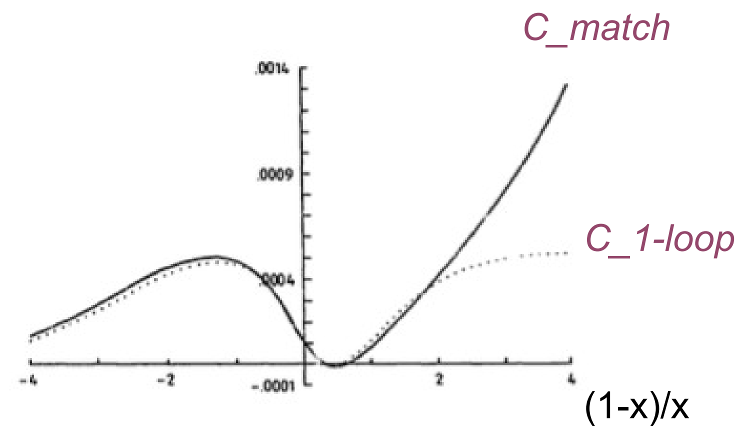


Fig. 9. Resummed coefficient function for bottom quark production ($M = 5$ GeV, $\Lambda = 210$ MeV) after performing the matching with the one-loop result (γg channel).

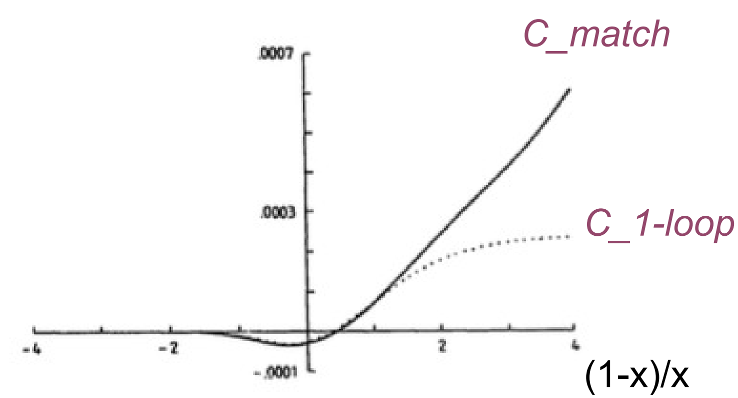
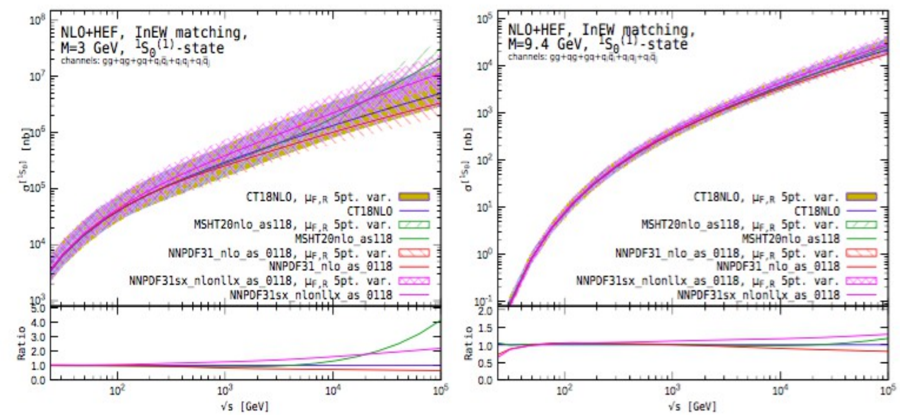


Fig. 10. Resummed coefficient function for bottom quark production ($M = 5$ GeV, $\Lambda = 210$ MeV) after performing the matching with the one-loop result (γq channel).

Applications of matching and high-energy resummation: quarkonium production

J.-P. Lansberg, M. Nefedov, and M. A. Ozcelik, *Matching next-to-leading-order and high-energy-resummed calculations of heavy-quarkonium-hadroproduction cross sections*, *JHEP* **05** (2022) 083, [2112.06789].

J.-P. Lansberg, M. Nefedov, and M. A. Ozcelik, *Curing the high-energy perturbative instability of vector-quarkonium-photoproduction cross sections at order $\alpha\alpha_s^3$ with high-energy factorisation, Eur. Phys. J. C* **84** (2024), no. 4 351, [2306.02425].



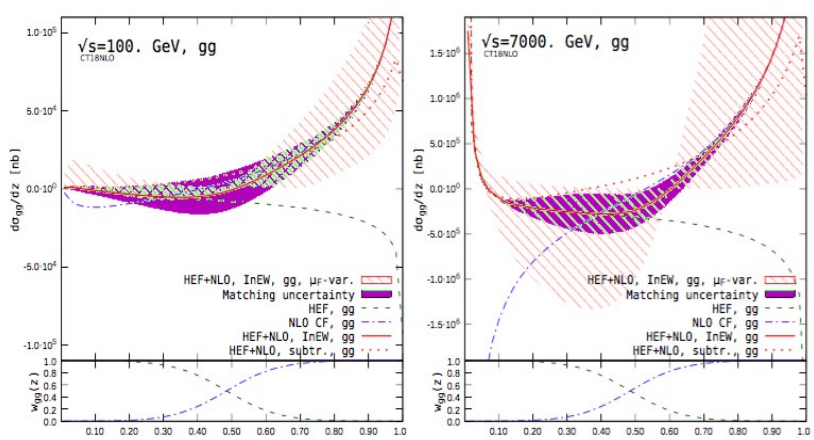


Figure 6. Matching plots for the *gg*-channel contribution to the ${}^1S_0^{[1]}$-state hadroproduction with $M = 3$ GeV. The solid curve depicts the *z*-integrand of eqn. (3.2), the dashed curve the HEF contribution (without the InEW weight), the dash-dotted curve the NLO CF contribution (without the InEW weight), and the dotted red line the integrand of eqn. (3.1), i.e. the result of the subtractive matching prescription for comparison. The plots of the InEW weights are shown in the bottom inset, while the matching uncertainty (3.4) is shown as the solid band. The LDME $\langle\mathcal{O}[{}^1S_0^{[1]}]\rangle$ was set to 1 GeV^3.

D Boer et al., Physics case for quarkonium studies at the Electron Ion Collider, arXiv:2409.03691

Soft-gluon coherence

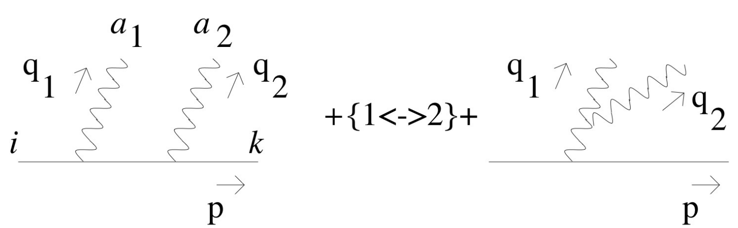
The CCFM approach: what does soft-gluon coherence look like in the high-energy (multi-Regge) kinematics

- Eikonal insertion currents:

Catani, Fiorani, Marchesini [1990]
Ciafaloni [1988]

$$|M_{n+1}^{a_1 \dots a_n a}(p_1, p_n, q)\rangle = \mathbf{J}^a |M_n^{a_1 \dots a_n}(p_1, p_n)\rangle \ , \quad \mathbf{J}^{a\mu} = \sum_i \mathbf{Q}_i^a \frac{p_i^\mu}{p_i \cdot q}$$

- Color coherence effect:

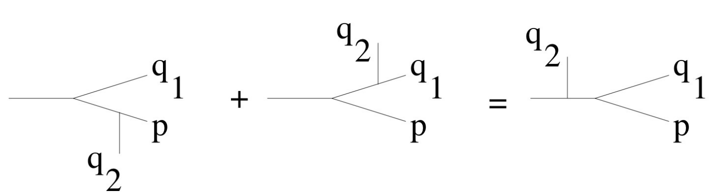


(a)

$q_2^0 \ll q_1^0$

$$\mathcal{M}_{ki}^{a_1 a_2} = g_s^2 \, \langle a_1 \ k | \, \mathbf{J}_2 \cdot \varepsilon_2 \, | a' \ i' \rangle \, \langle \ i' \ | \mathbf{J}_1 \cdot \varepsilon_1 | \ i \ \rangle$$

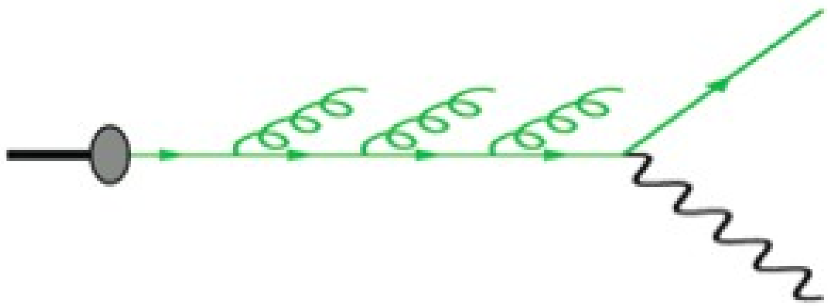
$$= g_s^2 \, \frac{p \cdot \varepsilon_1}{p \cdot q_1} \left(\frac{p \cdot \varepsilon_2}{p \cdot q_2} \, t^{a_2} t^{a_1} + \frac{q_1 \cdot \varepsilon_2}{q_1 \cdot q_2} \, [t^{a_1}, t^{a_2}] \right)_{ki}$$



(b)

- small angle: bremsstrahlung cones
- large angle ($\theta_{pq_2} \gg \theta_{pq_1}$): sees total charge $\mathbf{Q}_p + \mathbf{Q}_{q_1}$

CCFM: coherence in the high-energy limit



- Longitudinal momenta exchanged along the initial-state decay chain become small: x << 1
- Internal emissions non-negligible
- Yet, current is factorizable at high energy!

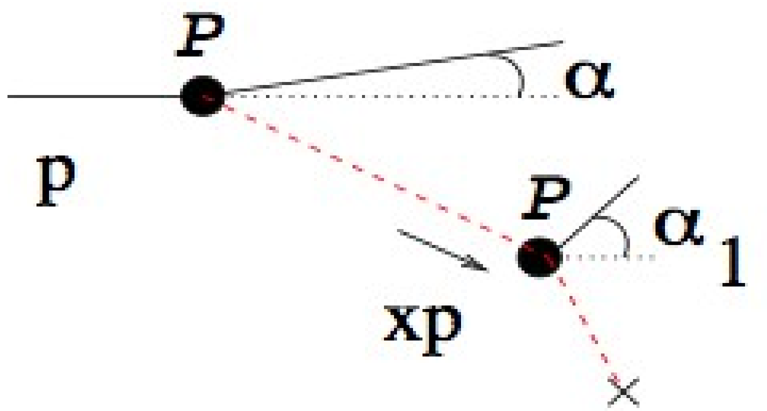
$$
\begin{aligned}
|M^{(n+1)}(k,p)|^2 \quad = \quad & \{[M^{(n)}(k+q,p)]^\dagger \, [\mathbf{J}^{(R)}]^2 \, M^{(n)}(k+q,p) \\
- \quad & [M^{(n)}(k,p)]^\dagger \, [\mathbf{J}^{(V)}]^2 \, M^{(n)}(k,p)\} \quad . \quad \text{BUT...} \triangleright
\end{aligned}
$$

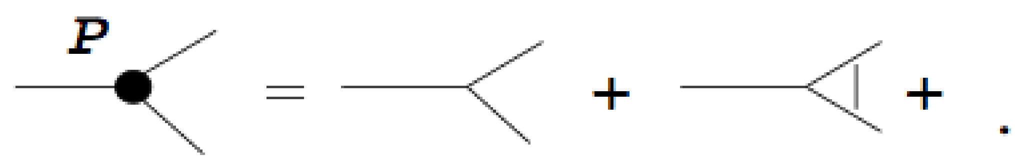
> J depends on transverse momentum transmitted down the chain
→ TMD (or unintegrated) matrix elements and
parton density functions
(→ see present-day "TMDs")

> virtual corrections imply, besides Sudakov form factors,
modified branching probabilities
(→ "parton-shower" MC)

Exclusive evolution equation

$$\mathcal{G}(x, k_T, \mu) = \mathcal{G}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2}\, \Theta(\mu - zq)$$
$$\times \underbrace{\Delta(\mu, zq)}_{\text{Sudakov}} \; \underbrace{\mathcal{P}(z, q, k_T)}_{\text{unintegr. splitting}} \; \mathcal{G}\left(\frac{x}{z}, k_T + (1-z)q, q\right)$$





Ciafaloni
Catani-Fiorani-Marchesini

- Returns the Balitski-Fadin-Kuraev-Lipatov (BFKL) anomalous dimension in the inclusive limit for $x \to 0$
- Incorporates angular ordering and soft-gluon radiation with leading logarithmic accuracy to all orders in the strong coupling for $x \to 1$

Evolution codes and Monte Carlo event generators based on CCFM equation

- SMALLX (Marchesini and Webber 1991, 1992)
- CASCADE (Jung and Salam 2001; Jung et al. 2010; Baranov et al. 2021)
- LDCMC (Gustafson, Lonnblad and Miu, 2002; Lonnblad and Sjodahl, 2005)
- uPDFevolv (Jung, Taheri-Monfared and H, 2014; Jung et al., 2024)
- DIPSY (Gustafson, Lonnblad et al. 2011, 2015)

PB TMD: an evolution equation developing from CCFM but with full collinear-emission kernels.

- Like CCFM, it is based on TMD densities and angular ordering.
- Unlike CCFM, it does not include BFKL corrections.
  However, these may be restored via TMD splittings.

Jung, Lelek, Radescu, Zlebcik, H: JHEP 01 (2018) 070

PB TMD: an evolution equation developing from CCFM

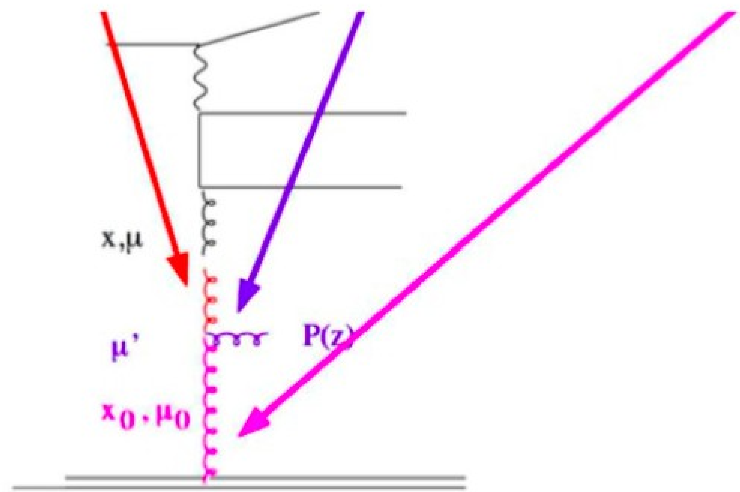
Jung, Lelek, Radescu, Zlebcik, H: JHEP 01 (2018) 070

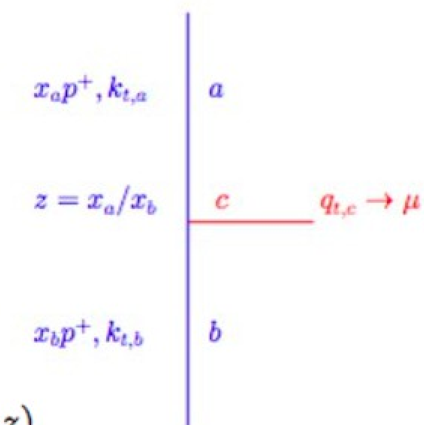
$$\mathcal{A}_a(x,\mathbf{k},\mu^2) = \Delta_a(\mu^2)\,\mathcal{A}_a(x,\mathbf{k},\mu_0^2) + \sum_b \int \frac{d^2\mathbf{q}'}{\pi \mathbf{q}'^2}\,\frac{\Delta_a(\mu^2)}{\Delta_a(\mathbf{q}'^2)}\,\Theta(\mu^2 - \mathbf{q}'^2)\,\Theta(\mathbf{q}'^2 - \mu_0^2)$$

$$\times \int_x^{z_M} \frac{dz}{z}\, P_{ab}^{(R)}(\alpha_s, z)\, \mathcal{A}_b\left(\frac{x}{z}, \mathbf{k} + (1-z)\mathbf{q}', \mathbf{q}'^2\right) \quad ,$$

↖ NB: angular ordering

- solvable by iterative MC technique





$$\mu = |\mathbf{q}_c|/(1-z)$$

where

$$\Delta_a(z_M, \mu^2, \mu_0^2) = \exp\left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\, z\, P_{ba}^{(R)}(\alpha_s, z) \right) \quad , \qquad P_{ba}^{(R)}(\alpha_s, z) = \delta_{ba} k_b(\alpha_s)\, \frac{1}{1-z} + R_{ba}(\alpha_s, z)$$

$$k_b(\alpha_s) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n k_b^{(n-1)}, \quad R_{ba}(\alpha_s, z) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n R_{ba}^{(n-1)}(z)$$

Integrated PB-TMD with angular ordering:

$$\widetilde{f}_a(x,\mu^2) = \Delta_a(\mu^2,\mu_0^2)\widetilde{f}_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_x^1 dz$$

$$\times \quad \Theta(1 - q_0/\mu' - z) \, \frac{\Delta_a(\mu^2,\mu_0^2)}{\Delta_a(\mu'^2,\mu_0^2)} \, P_{ab}^R\left(z, \alpha_s\left((1-z)^2\mu'^2\right)\right) \widetilde{f}_b\left(\frac{x}{z},\mu'^2\right)$$

Keersmaekers, Lelek, van Kampen & H, Nucl. Phys. B949 (2019) 114795

- after integration over kT, shown to coincide with "CMW" result for coherent branching

[Catani-Marchesini-Webber, Nucl. Phys. B349 (1991) 635]

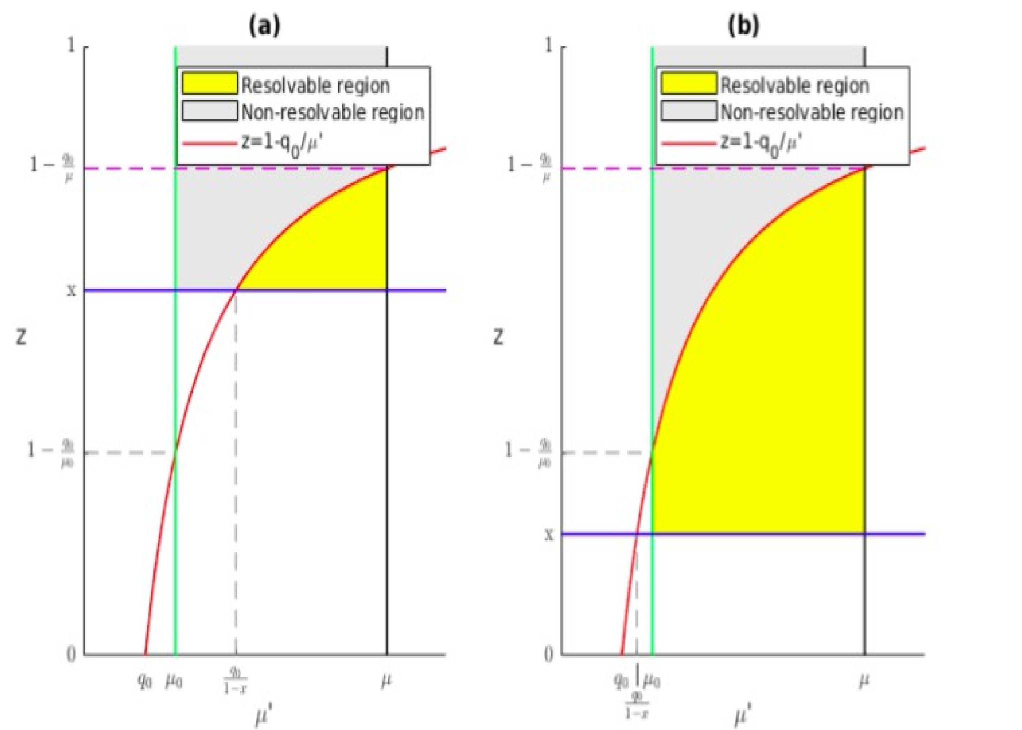


Figure 2: The angular ordering condition $z_M(\mu') = 1 - q_0/\mu'$ with the resolvable and non-resolvable emission regions in the (μ', z) plane: a) the case $1 > x \geq 1 - q_0/\mu_0$; b) the case $1 - q_0/\mu_0 > x > 0$.

Extension of branching algorithm to TMD splitting functions

Hentschinski et al., Phys. Lett. B 833 (2022) 137276 [arXiv:2205.15873]

- formulates new TMD branching including off-shell (TMD) splitting functions defined from high-energy (Regge) limit of partonic decay amplitudes

- relevant for future developments at the highest energy frontier, where small-x phase space opens up and high-energy resummations are called for

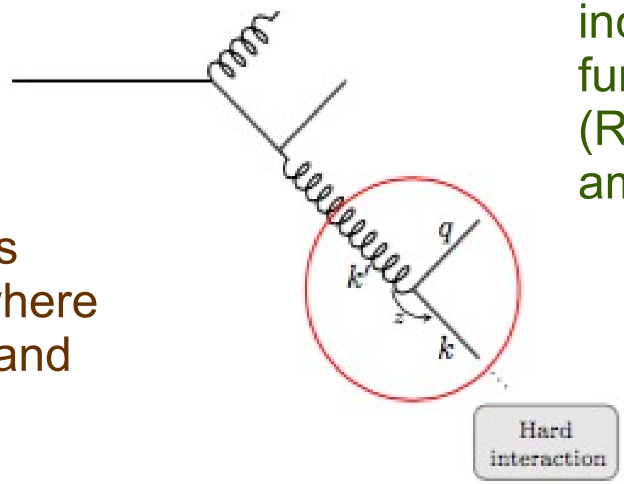


Figure 1: *Spacelike parton cascade.*

The splitting probability for the off-shell gluon to quark splitting process in Fig. 1 can be defined and computed by high-energy factorization [20] as a function of the strong coupling α_s, lightcone momentum transfer z, and transverse momenta $k'_\perp$ and $\tilde{q}_\perp$. Its explicit expression is given by

Catani, H [1994]

$$P_{qg}(\alpha_s, z, k'_\perp, \tilde{q}_\perp) = \frac{\alpha_s T_F}{2\pi} \frac{\tilde{q}_\perp^2 z(1-z)}{(\tilde{q}_\perp^2 + z(1-z)k'^2_\perp)^2} \left[\frac{\tilde{q}_\perp^2}{z(1-z)} + 4(1-2z)\tilde{q}_\perp \cdot k'_\perp - 4\frac{(\tilde{q}_\perp \cdot k'_\perp)^2}{k'^2_\perp} + 4z(1-z)k'^2_\perp \right], \quad (3)$$

$$\tilde{q}_\perp = k_\perp - zk'_\perp$$

- TMD splitting probabilities now computed for all flavor channels

Extension of branching algorithm to TMD splitting functions

Hentschinski et al., Phys. Lett. B 833 (2022) 137276 [arXiv:2205.15873]

- Momentum sum rule check (for 3 different boundary conditions on the strong coupling and the soft-gluon resolution scale)

	Full Result		
μ^2 (GeV^2)	$\alpha_s(\mu^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.000	1.000	1.000
10	0.999	0.999	0.999
10^2	0.997	0.996	0.997
10^3	0.994	0.992	0.994
10^4	0.991	0.987	0.991
10^5	0.984	0.978	0.983
	TMD-Resolvable		
μ^2 (GeV^2)	$\alpha_s(\mu^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.029	1.038	1.000
10	1.087	1.139	1.007
10^2	1.156	1.304	1.045
10^3	1.195	1.413	1.091
10^4	1.219	1.478	1.129
10^5	1.229	1.507	1.148
	Collinear Kernels		
μ^2 (GeV^2)	$\alpha_s(\mu^2)$ fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.000	1.000	1.000
10	0.999	0.999	0.999
10^2	0.997	0.997	0.997
10^3	0.995	0.993	0.995
10^4	0.992	0.989	0.992
10^5	0.986	0.981	0.984

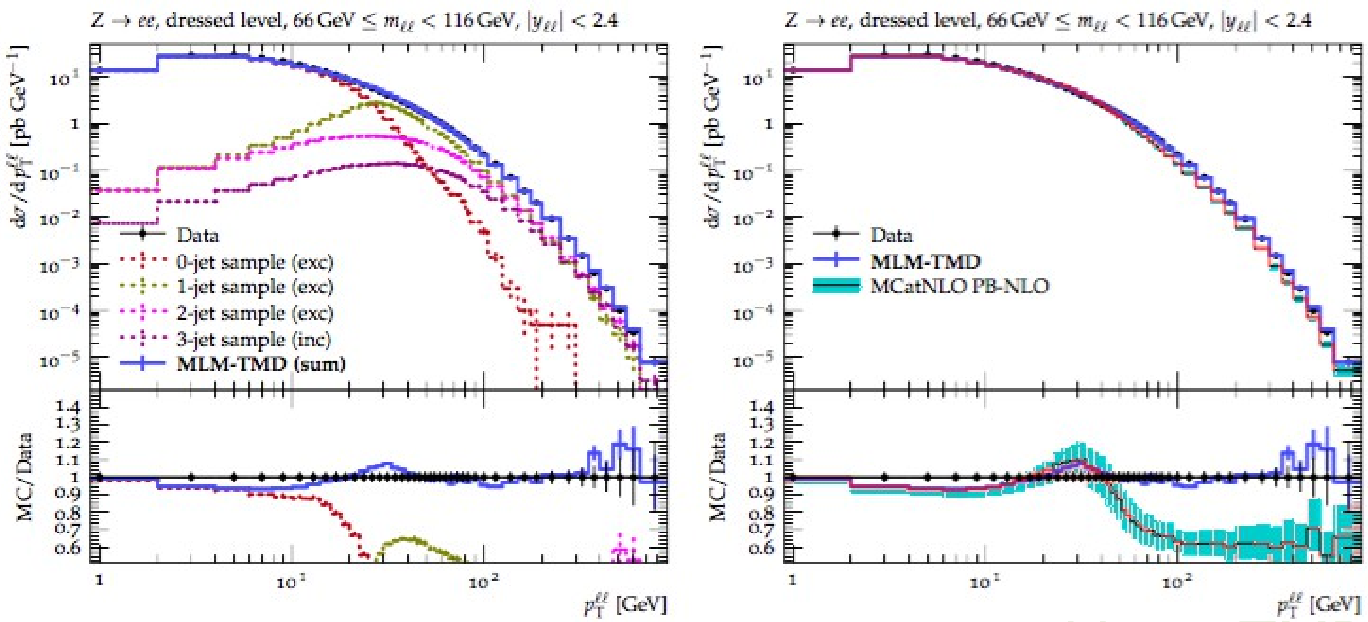
- Modified Sudakov form factor from angular-averaged TMD splittings

- Azimuthal correlations from full TMD splitting functions

Z-BOSON TRANSVERSE MOMENTUM

[Bermudez Martinez, Mangano & H , PLB 822 (2021) 136700 [arXiv:2107.01224]]

with TMD multi-jet merging



without TMD merging

FIG. 3: Transverse momentum p_T spectrum of Z-bosons DY lepton pairs from Z-boson decays. Experimental measurements by ATLAS [39] at $\sqrt{s} = 8$ TeV are shown. Left: result of the MLM-TMD calculation and separate contributions from the different jet samples. All jet multiplicities are obtained in exclusive (exc) mode except for the highest multiplicity which is calculated in inclusive (inc) mode. Right: the curves show the MLM-TMD calculation and the NLO calculation of Ref. [29], based on inclusive Z production. The uncertainty band on the NLO calculation represents the TMD and scale uncertainties as discussed in [29].

An application of PB TMD evolution equation to jet event shapes at the LHC

A. Bermudez Martinez, M. Mangano, M. van Kampen, H: in progress

slide by
M. van Kampen

Discussion with Stefano, autumn 2022

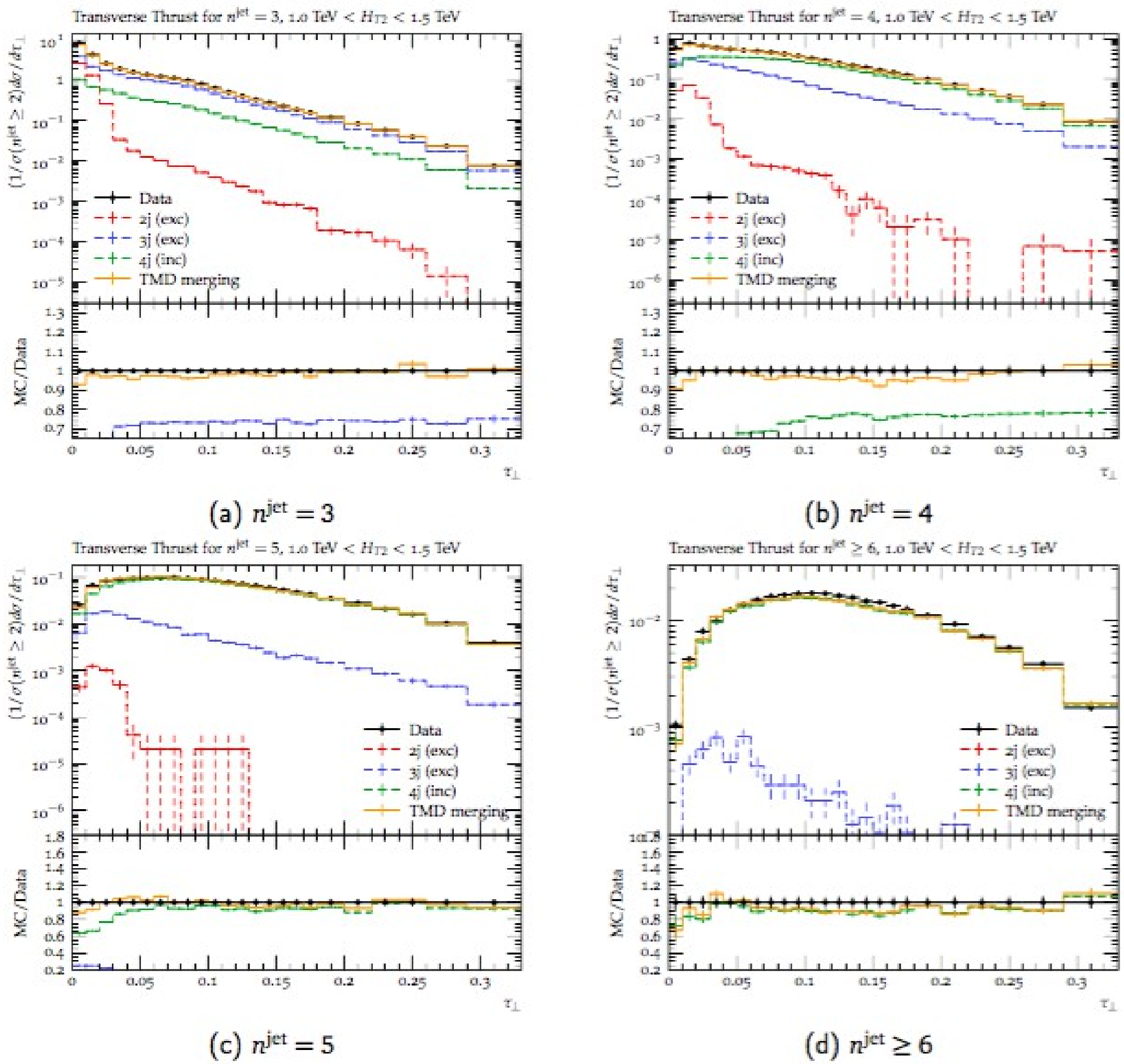


Fig. 3: Transverse thrust $\tau_\perp$ for different jet multiplicities

An application of PB TMD evolution equation to jet event shapes at the LHC

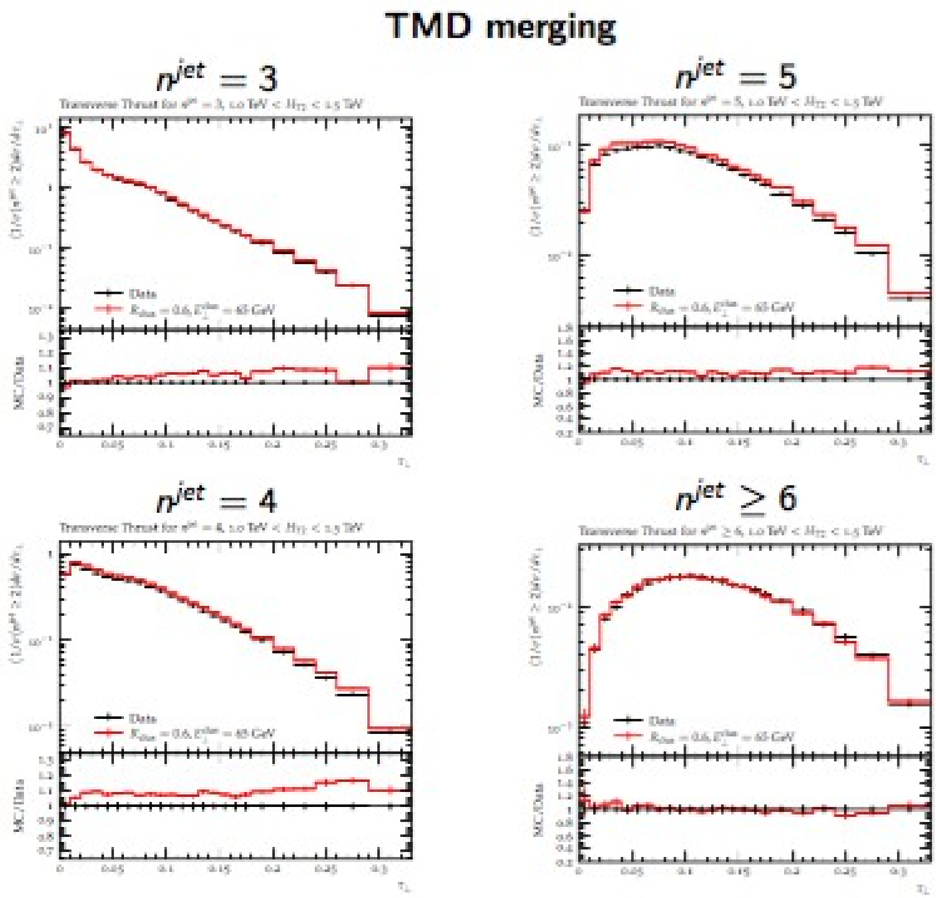
A. Bermudez Martinez, M. Mangano, M. van Kampen, H: in progress

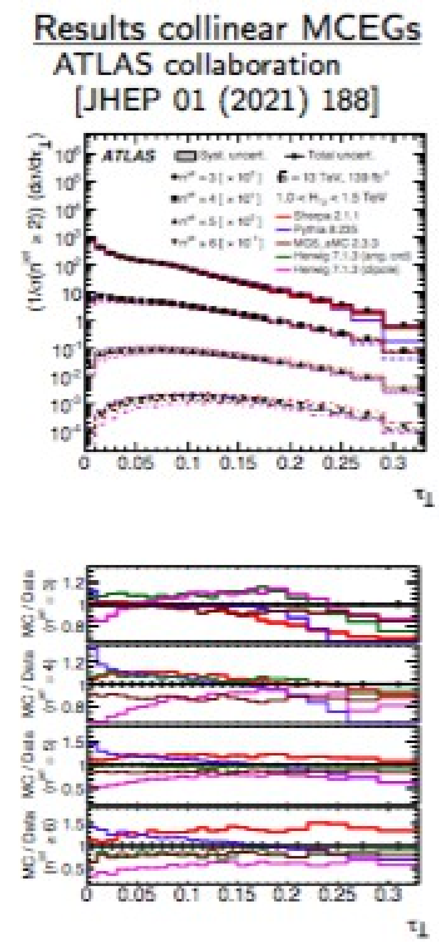
slide by M. van Kampen

Discussion with Stefano, autumn 2022

Jets at 13 TeV - Transverse thrust $\tau_\perp$ - $1.0 < H_{T2} < 1.5$

$$\tau_\perp = 1 - \frac{\sum_i |\bar{p}_{T,i} \cdot \hat{n}_T|}{\sum_i |\bar{p}_{T,i}|}$$





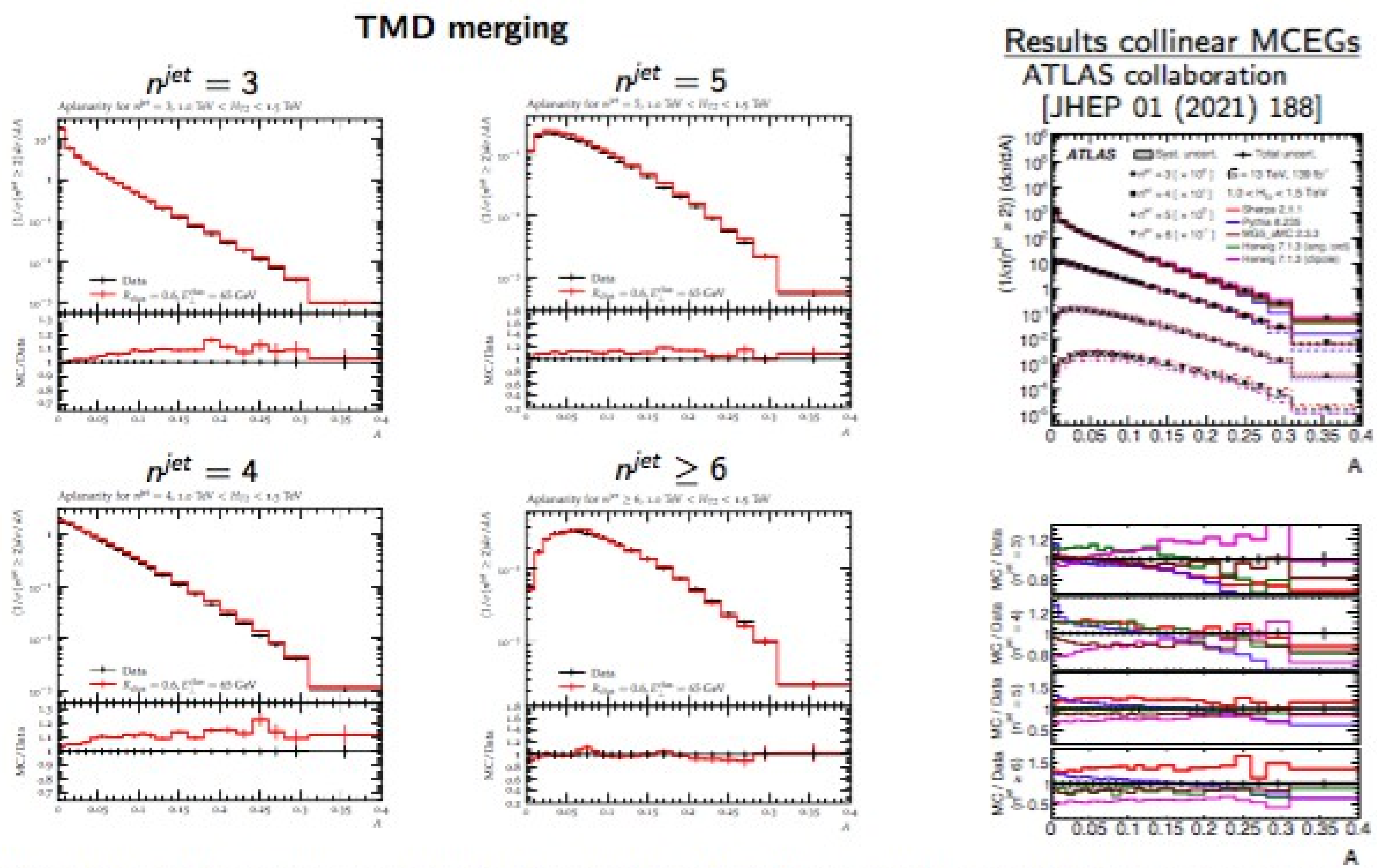
An application of PB TMD evolution equation to jet event shapes at the LHC

A. Bermudez Martinez, M. Mangano, M. van Kampen, H: in progress

slide by M. van Kampen

Discussion with Stefano, autumn 2022

Jets at 13 TeV - Aplanarity* A - $1.0 < H_{T2} < 1.5$



*Remark: These distributions are obtained by multiplication with a scaling factor of 1.376 as was applied in the ATLAS erratum: [JHEP 12 (2021) 053]

Thanks for your attention

Thank you Stefano,

for the physics

and the good times