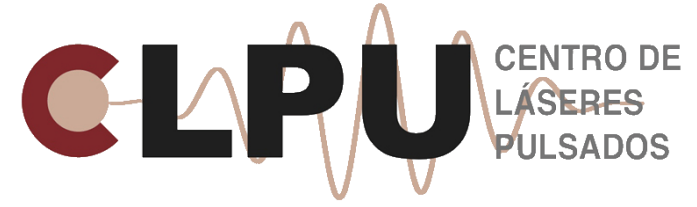




SAPIENZA
UNIVERSITÀ DI ROMA



Study and characterization of betatron radiation source from Laser Wakefield Accelerator

Daniele Francescone

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XXXVII PhD School in Accelerator Physics

University of Rome "Sapienza"

Supervisors:
Prof. Enrica Chiadroni
Dr. Giancarlo Gatti

Outline

- Introduction
 - Eupraxia/Eupas
- Theory
 - Plasma acceleration
 - Betatron radiation
 - Equation of motion
 - Calculated radiated spectrum
- Simulation
 - PIC simulations
- Experiment
 - Centro de Laseres Pulsados Ultracortos CLPU
 - Experimental setup
 - Measurements
 - Phase Contrast Imaging

Introduction

Eupraxia

- First plasma accelerator based user facility

EuAPS

- EuPRAXIA Advanced Photon Sources project

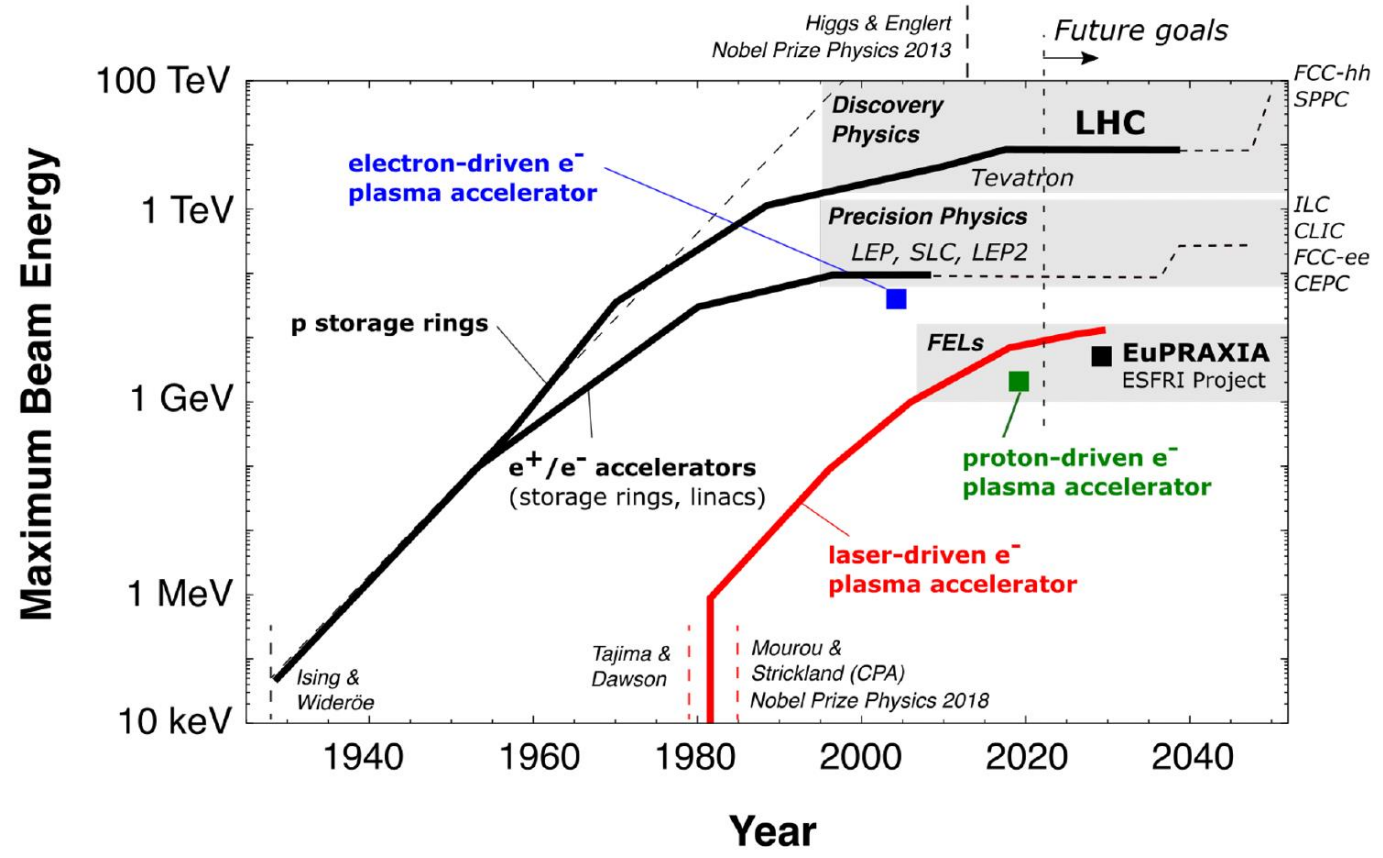
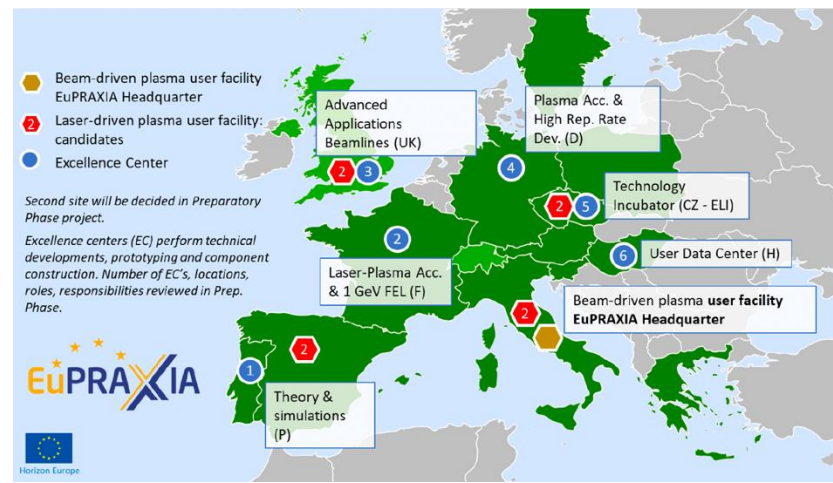
Collaboration

- INFN
- CNR
- University of Tor Vergata

- Objective: development a compact photon sources to drive plasma accelerators and the setup of ultra-compact, high performing X ray and particle sources for users

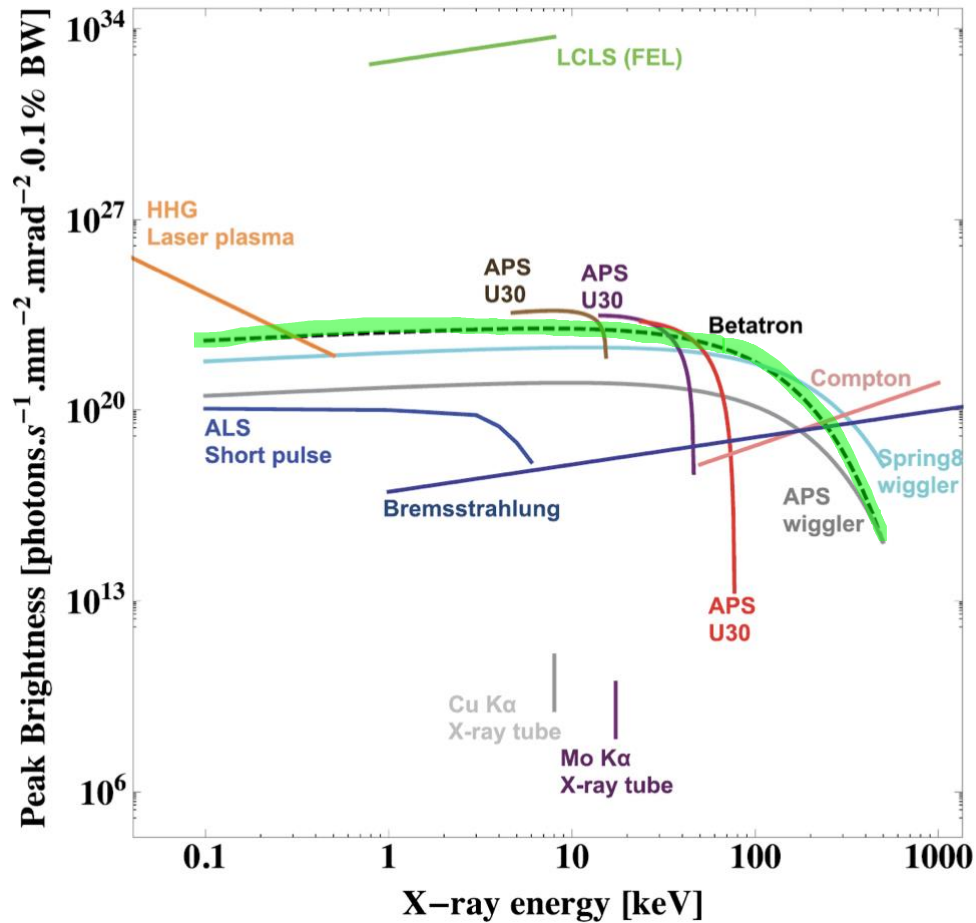


Betatron Radiation

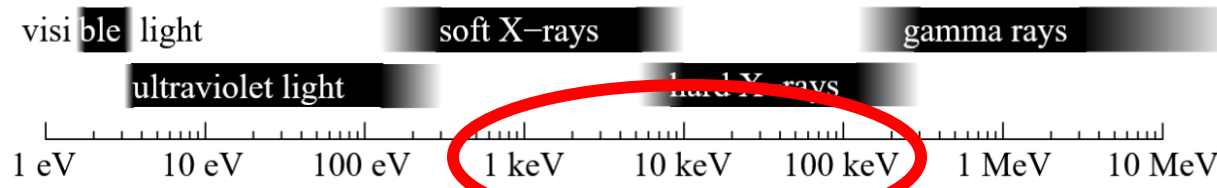


Why Betatron Radiation?

Applications



[Albert F and Thomas A G R 2016 Plasma Phys. Control. Fusion](#)



Imaging

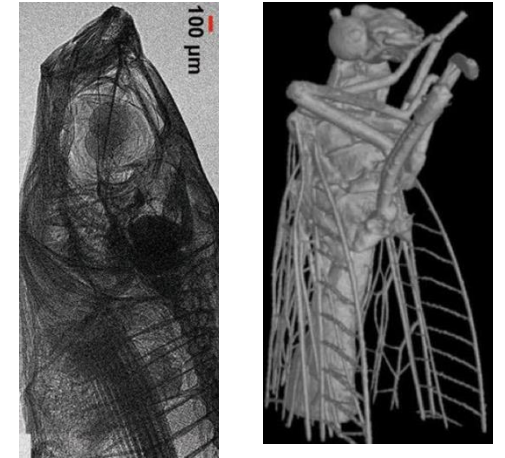
- Phase contrast imaging PCI
- PCI edge illumination (PCI-EI)
- PCI Beam tracking (PCI-BT)

Spectroscopy

- X-ray absorption XAS
- XANES
- EXASF

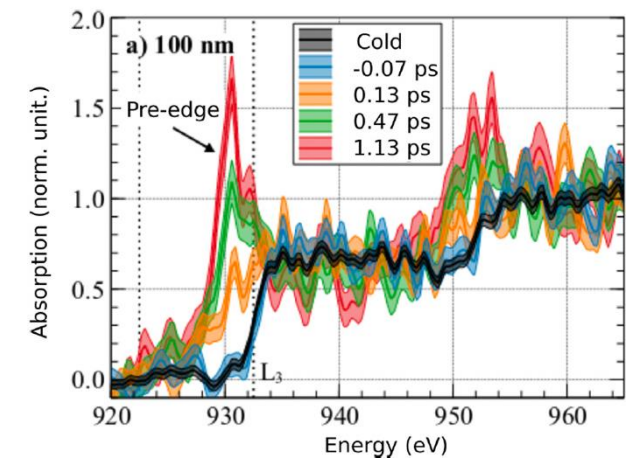
Ultrafast Studies

- Pump-probe experiment



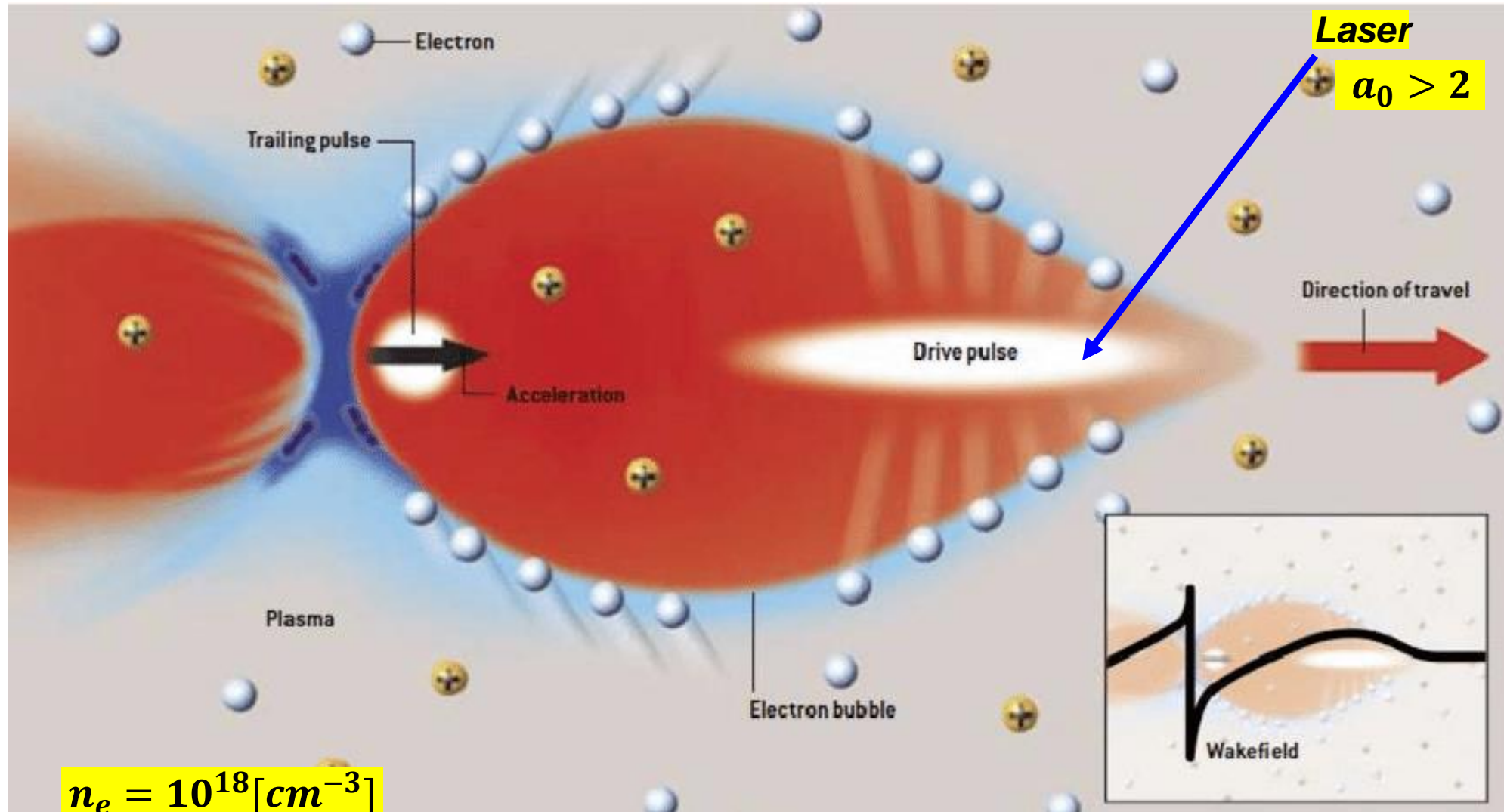
[Wenz et al. Nature communications 2015](#)

[Bo Guo et al. Scientific reports 2019](#)



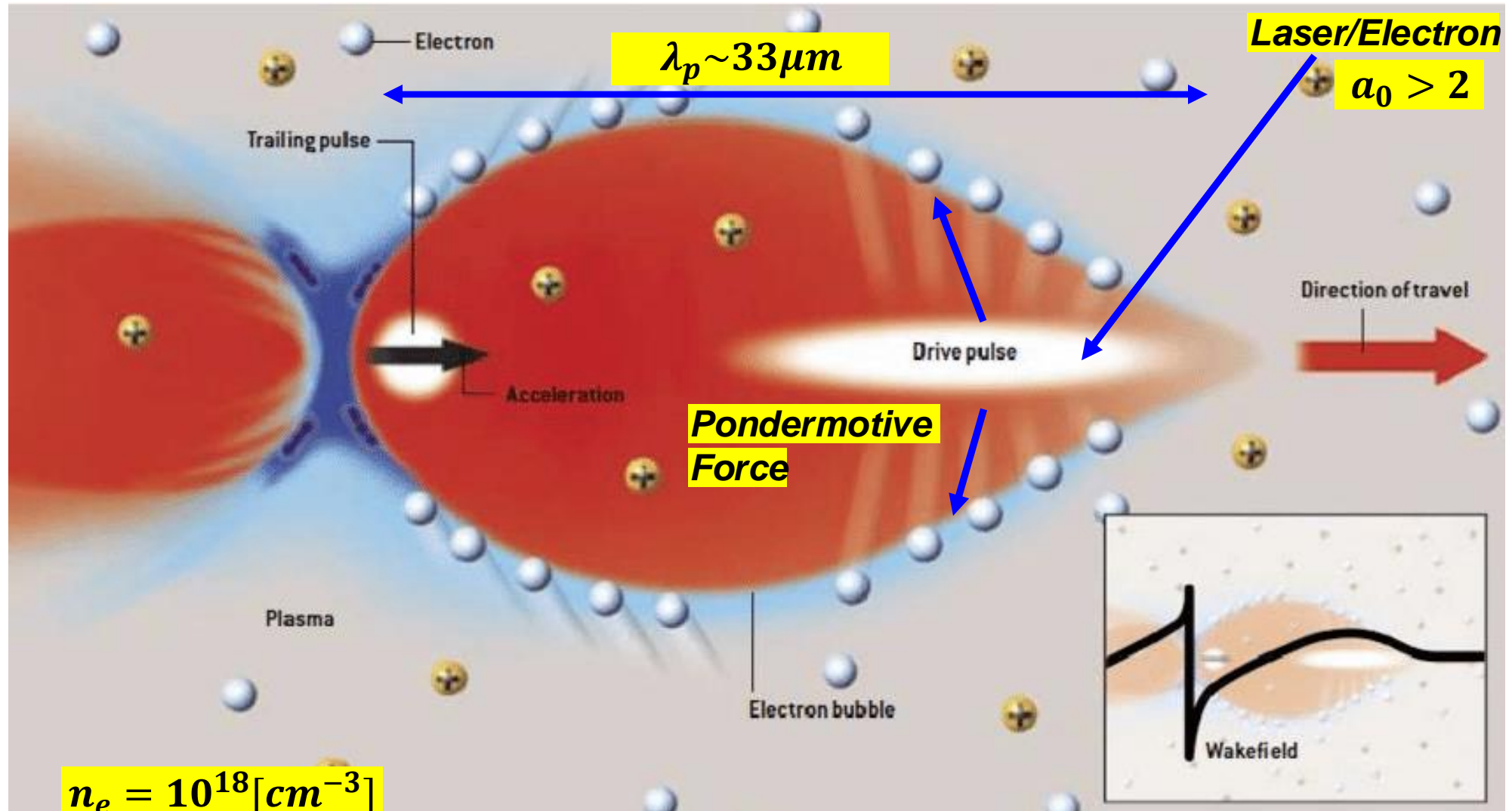
[F. Dorchies et al. Structural dynamics 2023](#)

Plasma acceleration



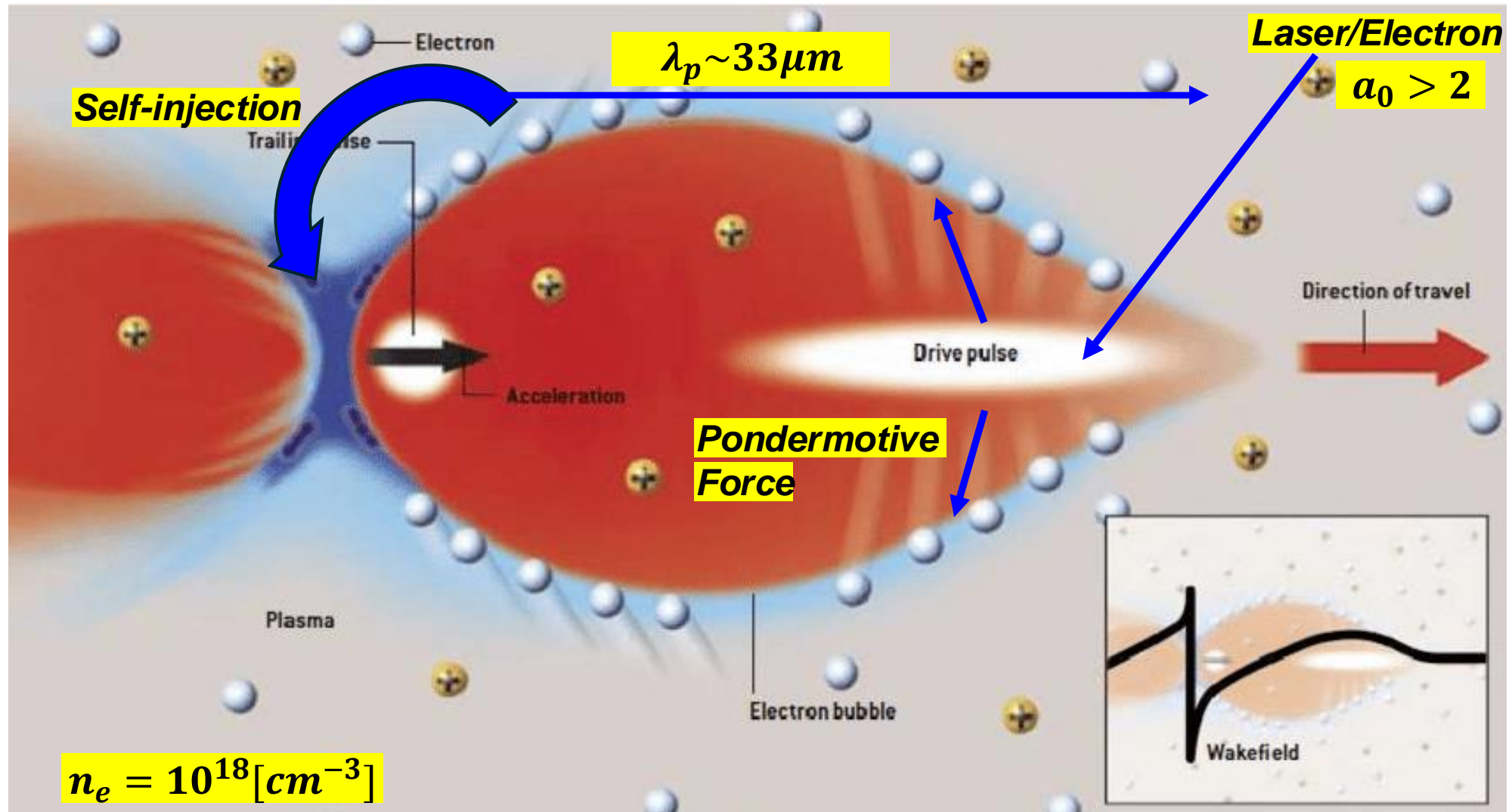
[Credits: C. Joshi, UCLA](#)

Plasma acceleration



Credits: C. Joshi, UCLA

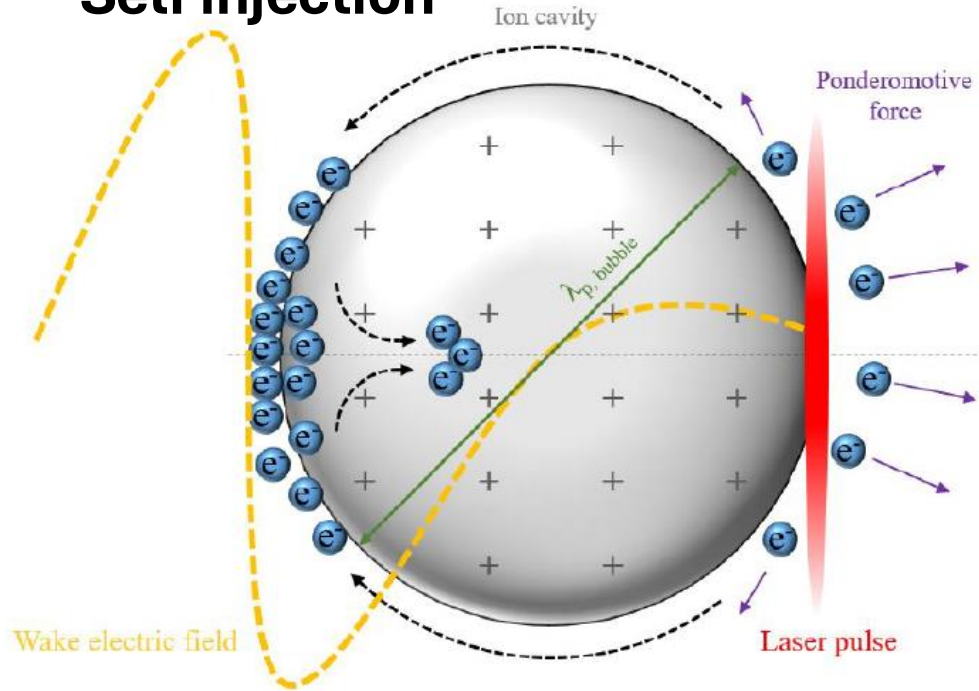
Plasma acceleration



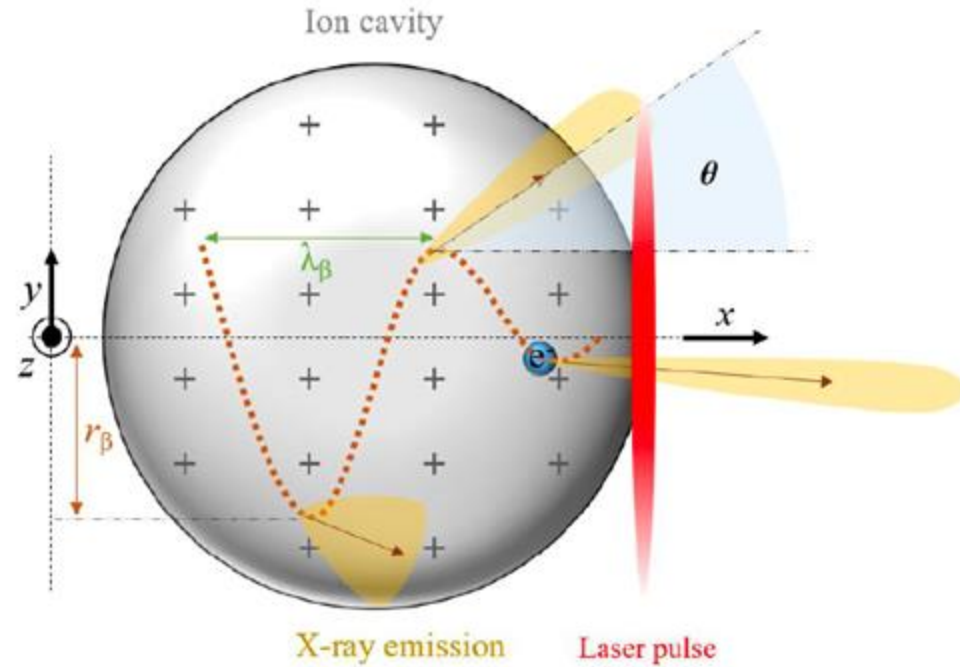
Credits: C. Joshi, UCLA

Strong longitudinal and transverse force

Self injection



Electron Oscillation in plasma bubble



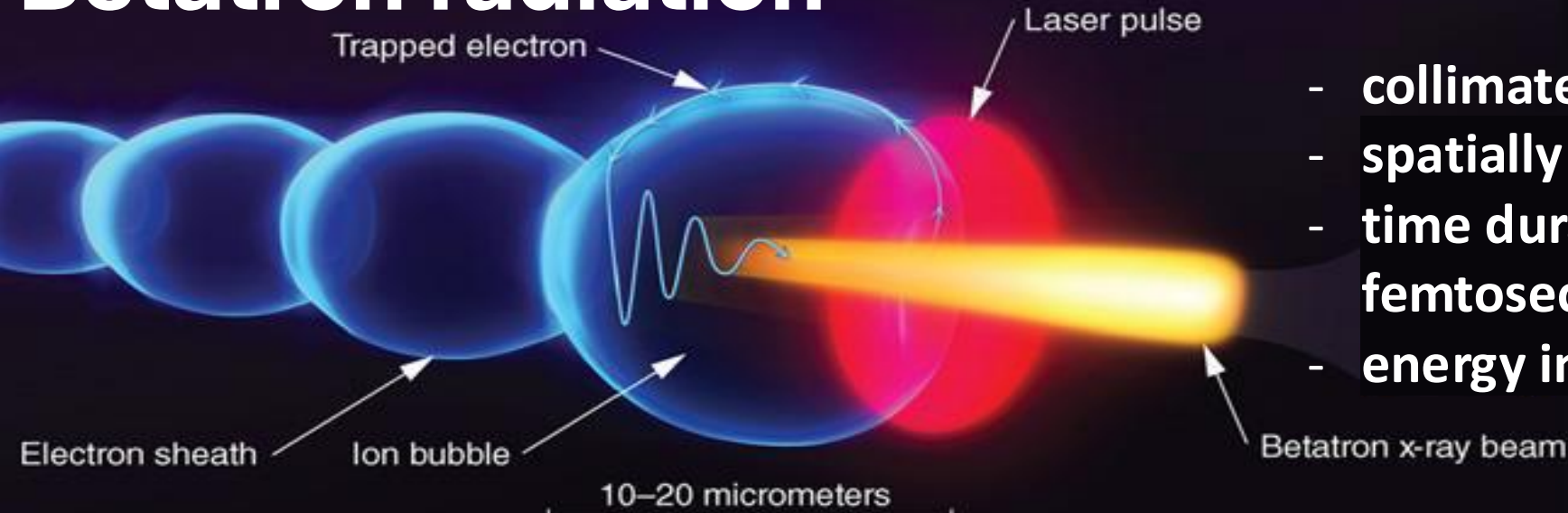
Strong longitudinal and transverse force

$$E_z [\text{V/m}] = \frac{m_e c \omega_p \sqrt{a_0}}{e}$$

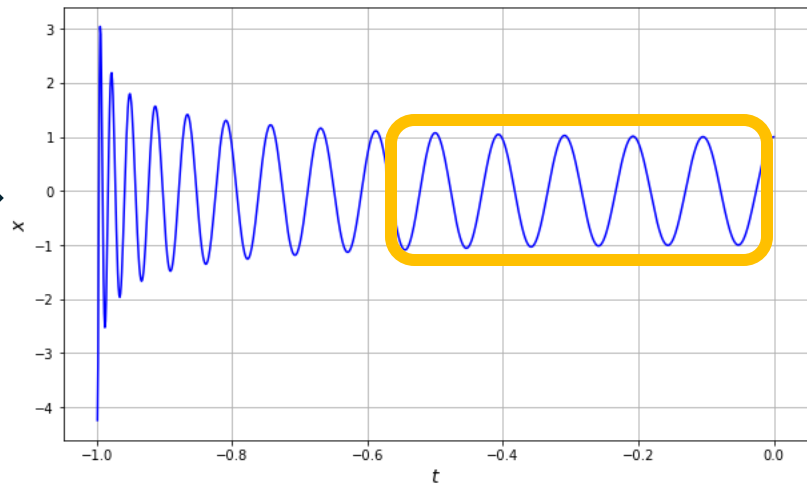
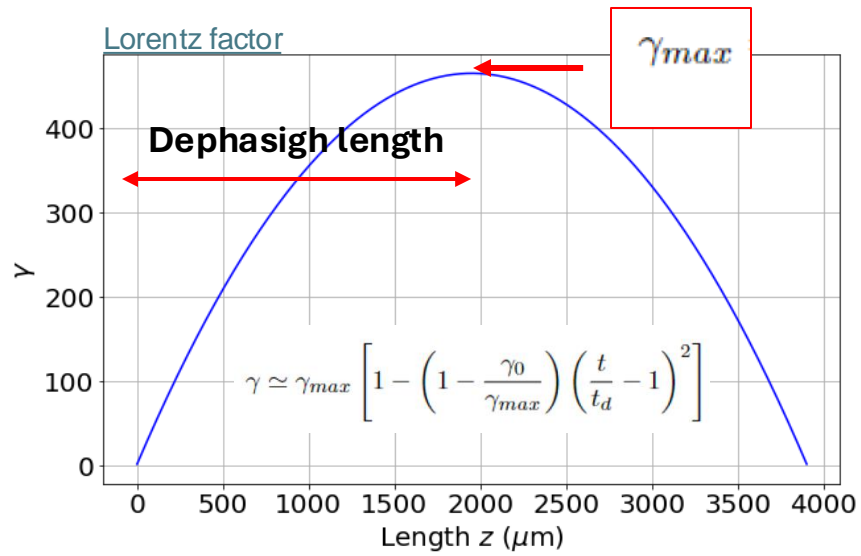
$$E_r [\text{V/m}] = \frac{m_e \omega_p^2 r}{2e}$$

$\approx \text{GV/m}$

Betatron radiation



- collimated beam of X-ray
- spatially coherent
- time duration in the femtosecond range
- energy in the keV range



The period and amplitude of betatron oscillations stabilize near the maximum energy gain

Betatron frequency $\omega_{\beta} = \frac{\omega_p}{\sqrt{2\gamma}}$

Trajectories $x(t) \simeq x_0 \cos(\omega_{\beta}t + \phi_x)$

Strength parameter K

Strength parameter

$$K = \frac{eB\lambda_u}{2\pi mc}$$

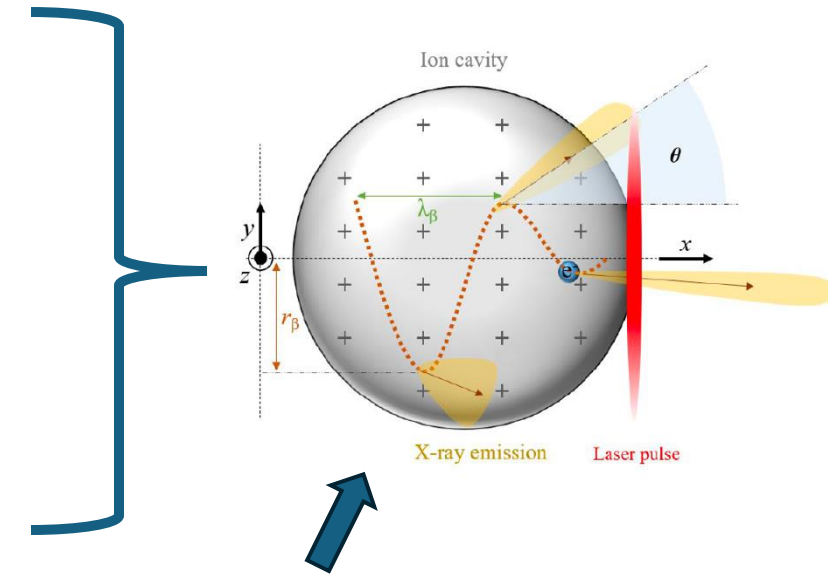
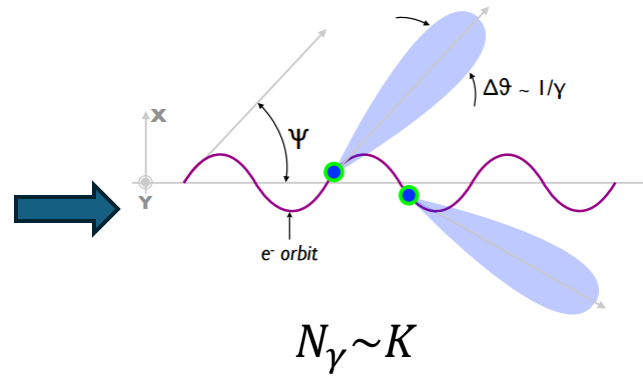
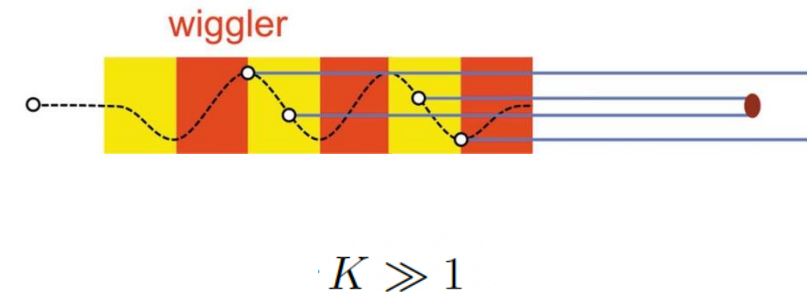
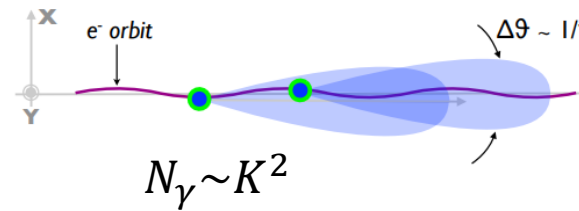
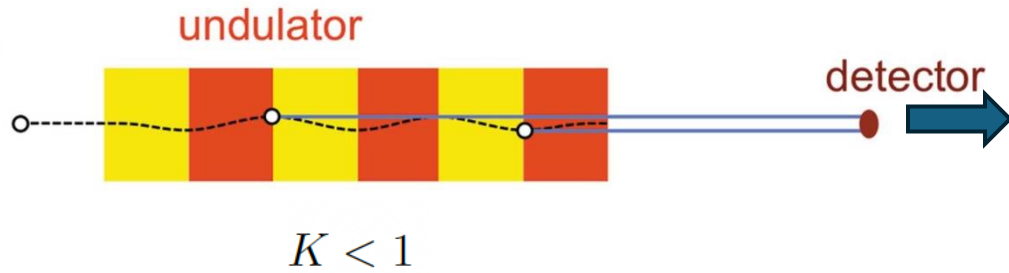
Betatron

Strength parameter

$$K_\beta = \gamma r \omega_\beta / c$$

Oscillation amplitude
Lorentz factor

Plasma frequency

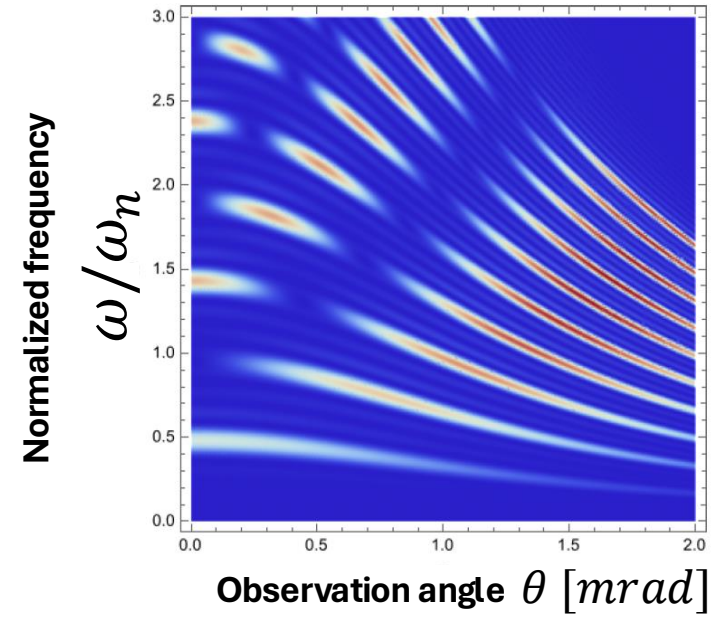


Wiggler + Undulator

Irradiated spectrum

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{16\pi^3 \epsilon_0 c} \left| \int_{-\infty}^{\infty} \frac{\mathbf{n} \times [(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \mathbf{n} \cdot \boldsymbol{\beta})^2} e^{i\omega(t - \mathbf{n} \cdot \mathbf{r}/c)} dt \right|^2$$

Spectral lines vs angle

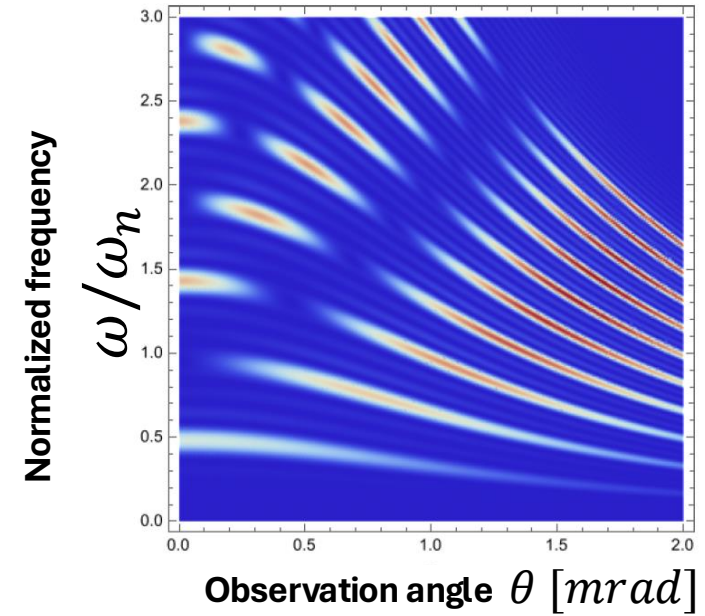


Irradiated spectrum

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{16\pi^3 \epsilon_0 c} \left| \int_{-\infty}^{\infty} \frac{\mathbf{n} \times [(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \mathbf{n} \cdot \boldsymbol{\beta})^2} e^{i\omega(t - \mathbf{n} \cdot \mathbf{r}/c)} dt \right|^2$$

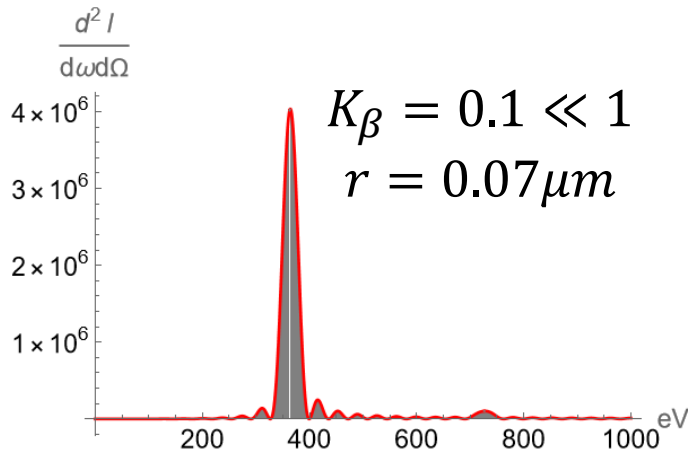
From Liénard–Wiechert potential

Spectral lines vs angle

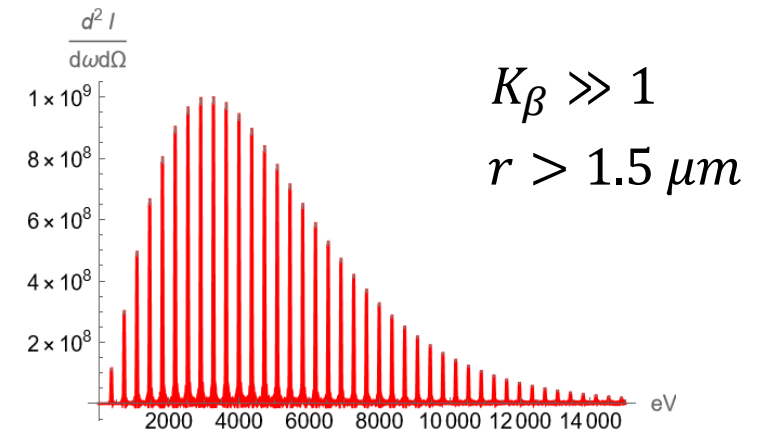
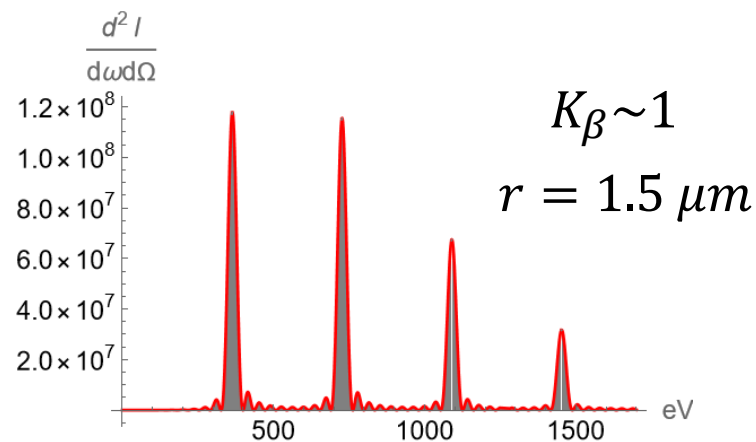


Spectral lines on axis

$$\theta = 0$$



Undulator-like



Wiggler like

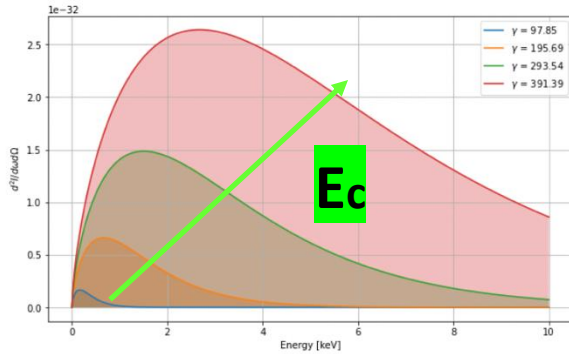


Asymptotic limit spectrum

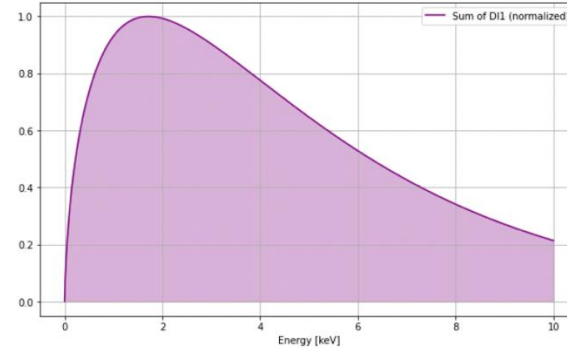
$$\left. \frac{d^2 I}{d\omega d\Omega} \right|_{\theta=0} \simeq N_{\beta} \frac{3e^2}{2\pi^3 \epsilon_0 c} \gamma^2 \zeta^2 \mathcal{K}_{2/3}^2(\zeta)$$

How does the spectrum change for:

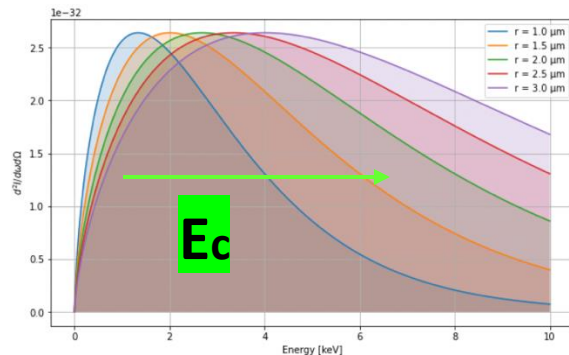
Different energies



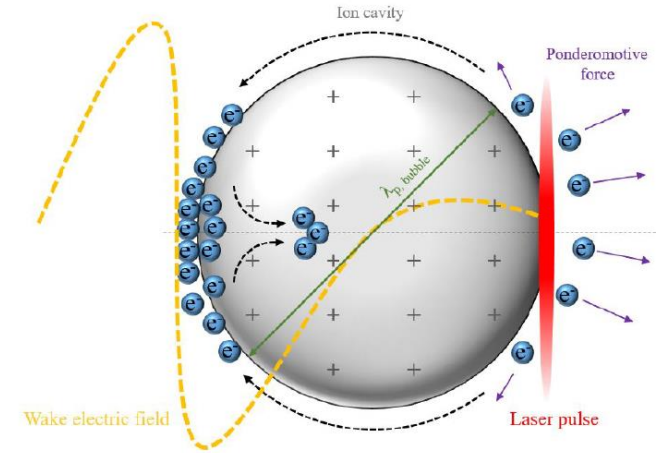
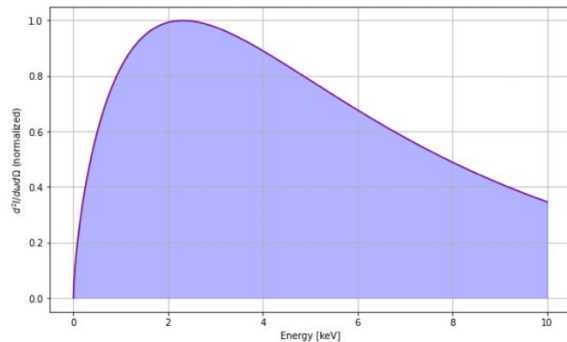
Sum of contribution



Different radius



Sum of contribution



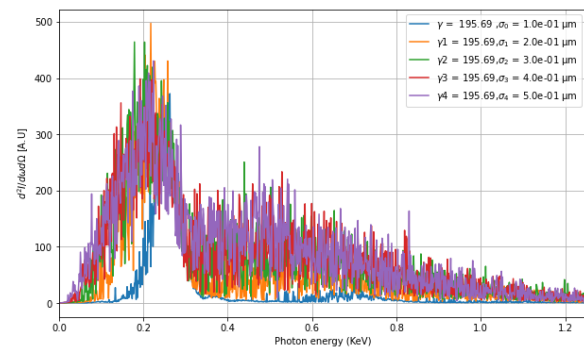
The radiation at the detector is the sum of different contribution:

- Radiation of Electron with different energies
- Radiation of electron with different amplitude of oscillation

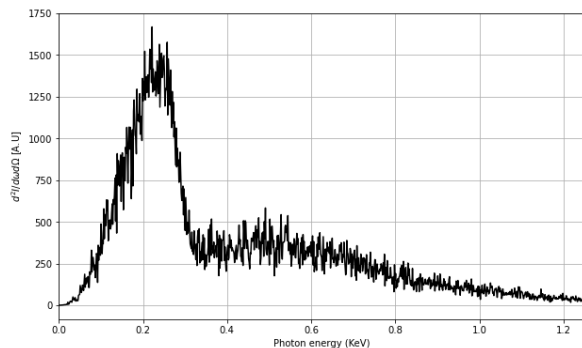
Ec = critical energy

Study of the spectra (no asymptotic limit)

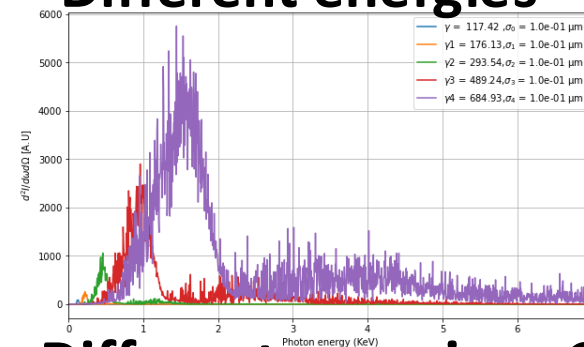
Different radius



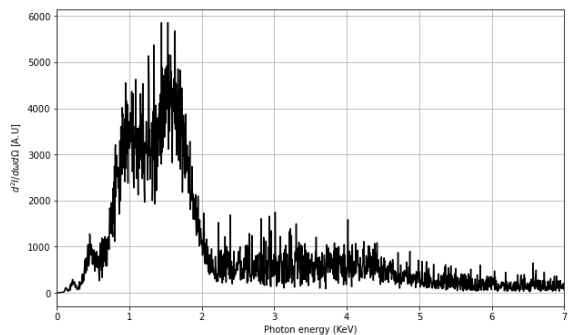
Sum of contribution



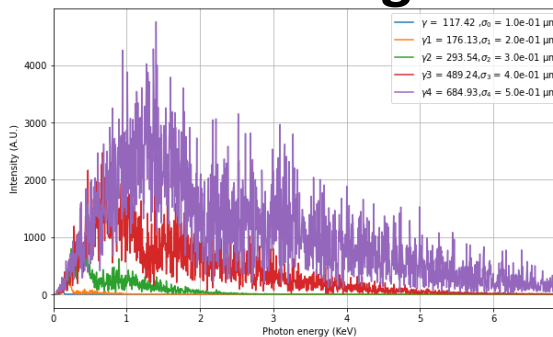
Different energies



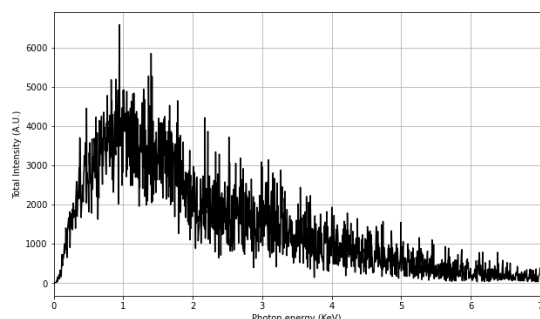
Sum of contribution



Different energies+ Gaussian bunch



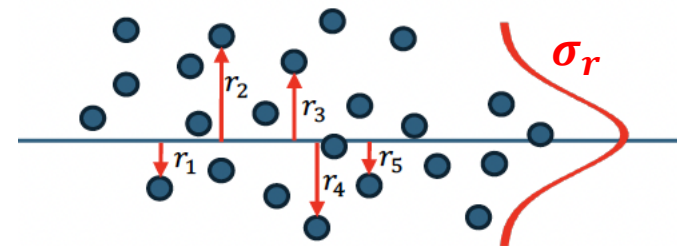
Sum of contribution



$$\frac{d^2 I}{d\omega d\Omega}(\theta = 0) = \sum_{n=1}^{\infty} \frac{e^2}{\pi \epsilon_0 c} \frac{\omega}{\omega_n} \frac{\gamma^2 N_{\beta}^2 F_n R_n}{1 + K_{\beta}^2/2},$$



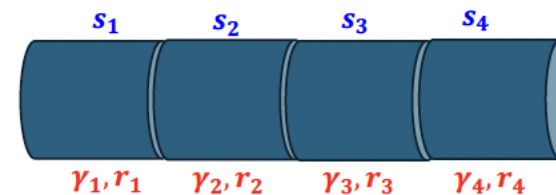
Considering a Gaussian bunch around the axis



$$\left. \frac{d^2 I}{d\omega d\Omega} \right|_{\theta=0} = \sum_{r=1}^{\infty} f(r) \sum_{n=1}^{\infty} \frac{e^2 \omega}{\pi \epsilon_0 c \omega_n(r)} \frac{\gamma^2 N_{\beta}^2 F_n(r) R_n(r)}{1 + K_{\beta}(r)^2/2}$$



Considering a different values of energies



$$\left. \frac{d^2 I}{d\omega d\Omega} \right|_{\theta=0} = \sum_{s=1}^N \sum_{r=1}^{\infty} f(r) \sum_{n=1}^{\infty} \frac{e^2 \omega}{\pi \epsilon_0 c \omega_n(r)} \frac{\gamma^2 N_{\beta}^2 F_n(r) R_n(r)}{1 + K_{\beta}(r)^2/2}$$

Particle in cell simulations

Simulations model the interaction between charged particles and electromagnetic fields, commonly used in plasma physics and accelerator studies

FBPIC



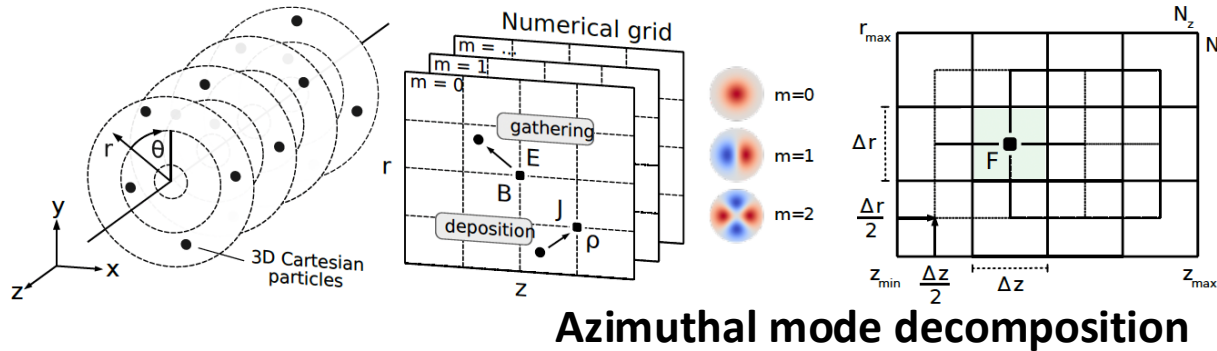
Trajectory



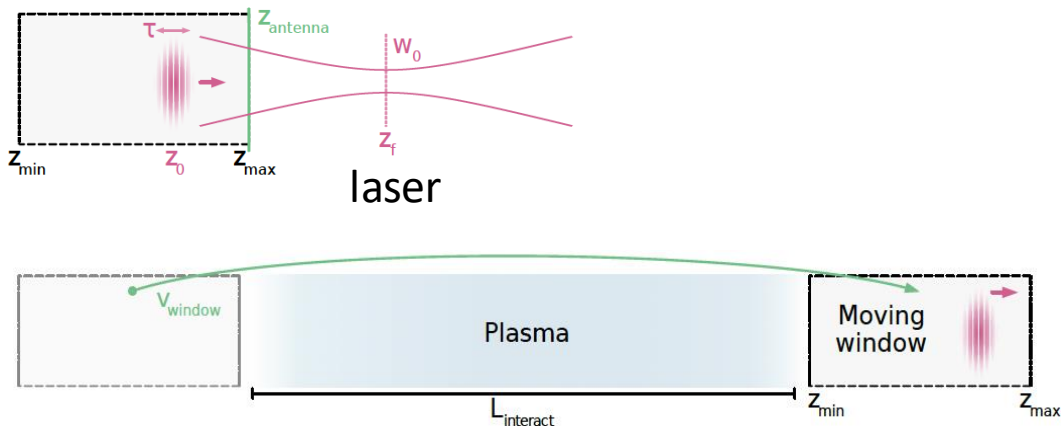
Synchrad



Radiation



Initialization of laser parameters
 Initialization of simulation box parameters
 Initialization of initial particle parameters

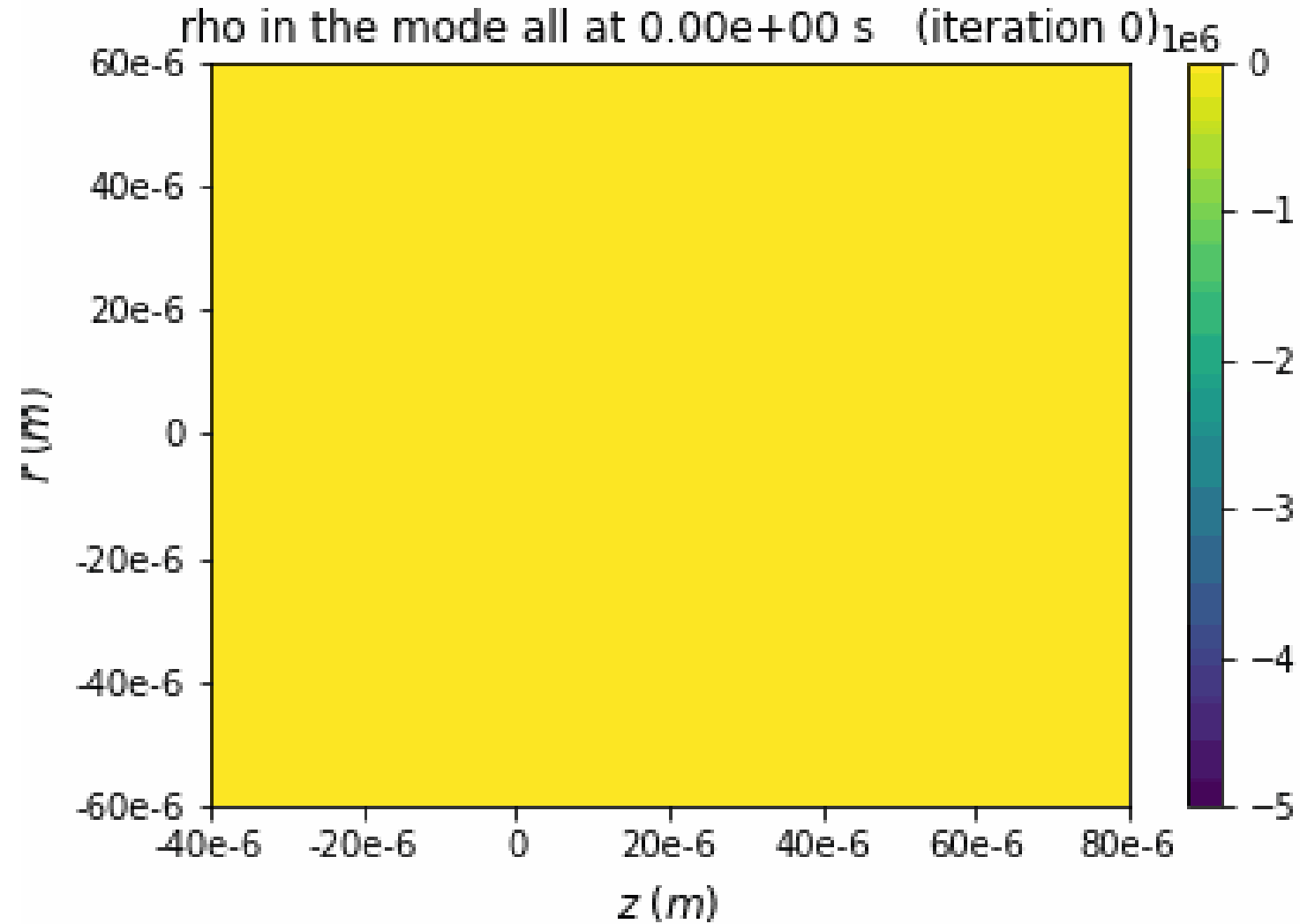


Parameter	Value
a_0	1.6
w_0	$24.2 \mu\text{m}$
τ	26 fs
z_0	$15 \mu\text{m}$

Parameter	Value
N_z	1024
z_{\max}	$80 \mu\text{m}$
z_{\min}	$-40 \mu\text{m}$
N_r	256
r_{\max}	$60 \mu\text{m}$
N_m	2
dt	$(z_{\max} - z_{\min})/N_z/c$

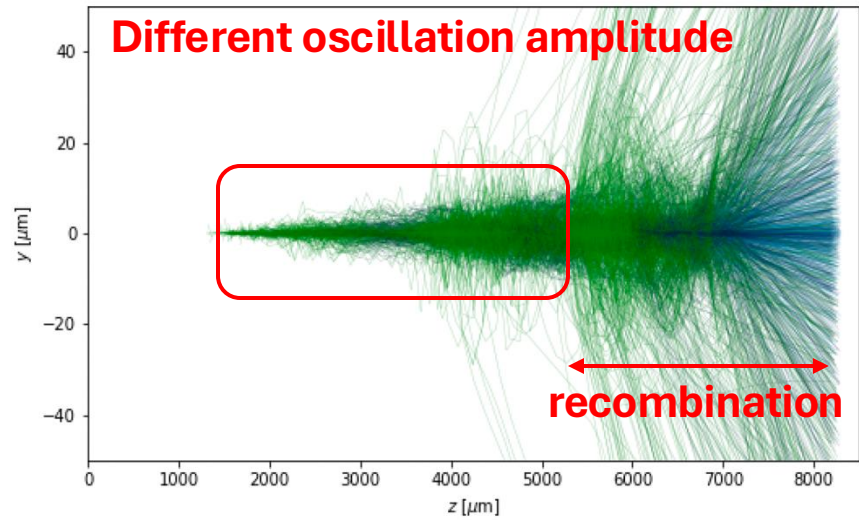
FBPIC-simulation

Electron density evolution: **Bubble formation – Self-injection – electron oscillation - recombination**

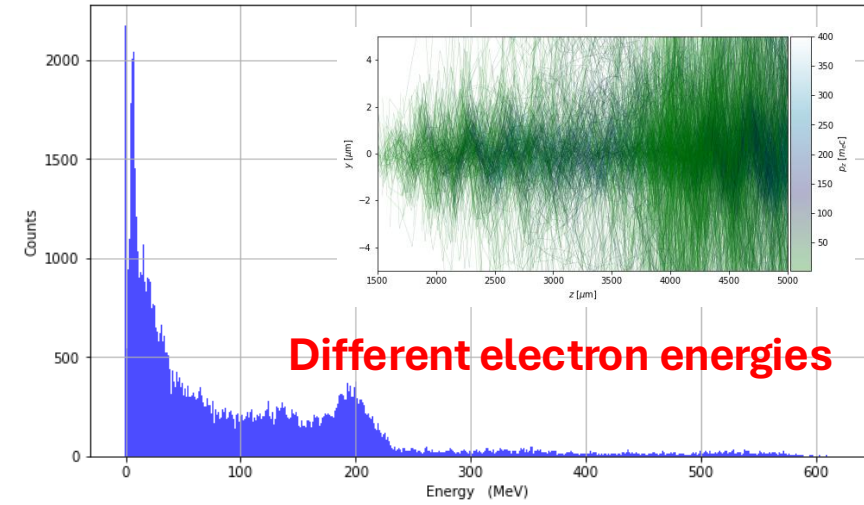


Result of simulations

Trajectories on PIC simulation

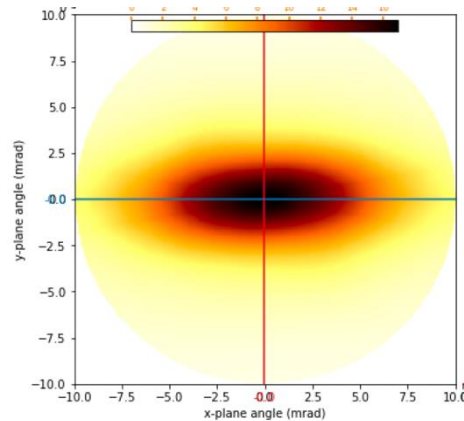


Energy distribution PIC

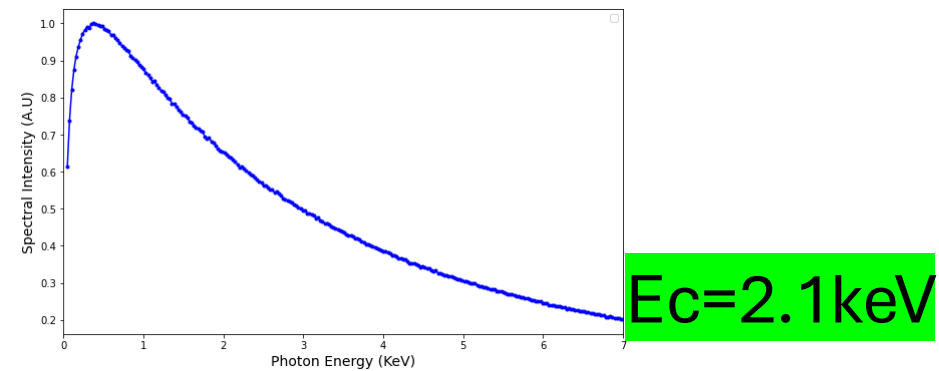


Synchrad

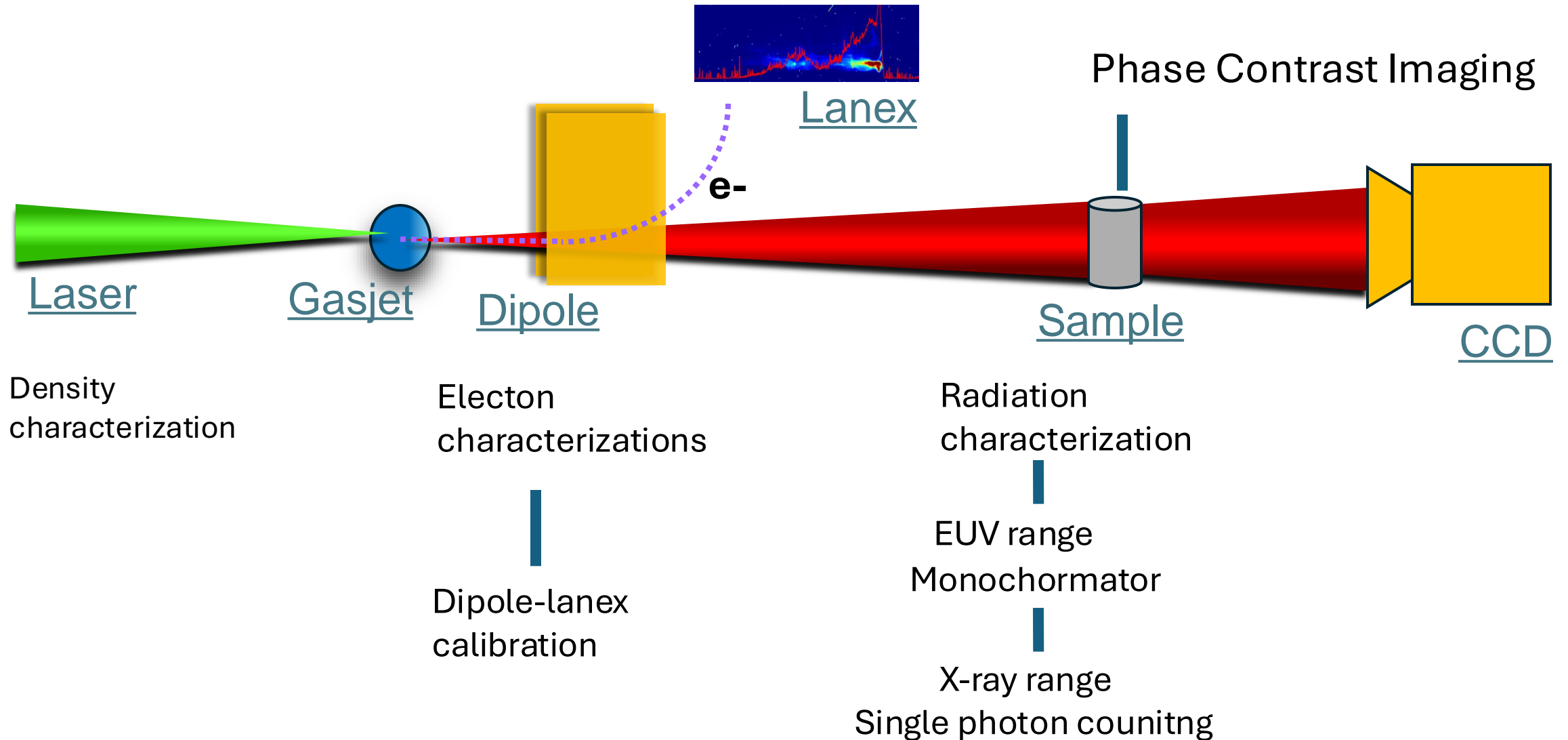
Photon spot



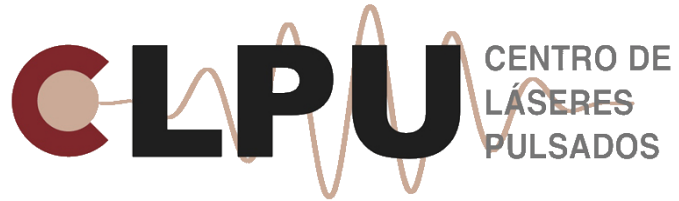
Photon Spectrum



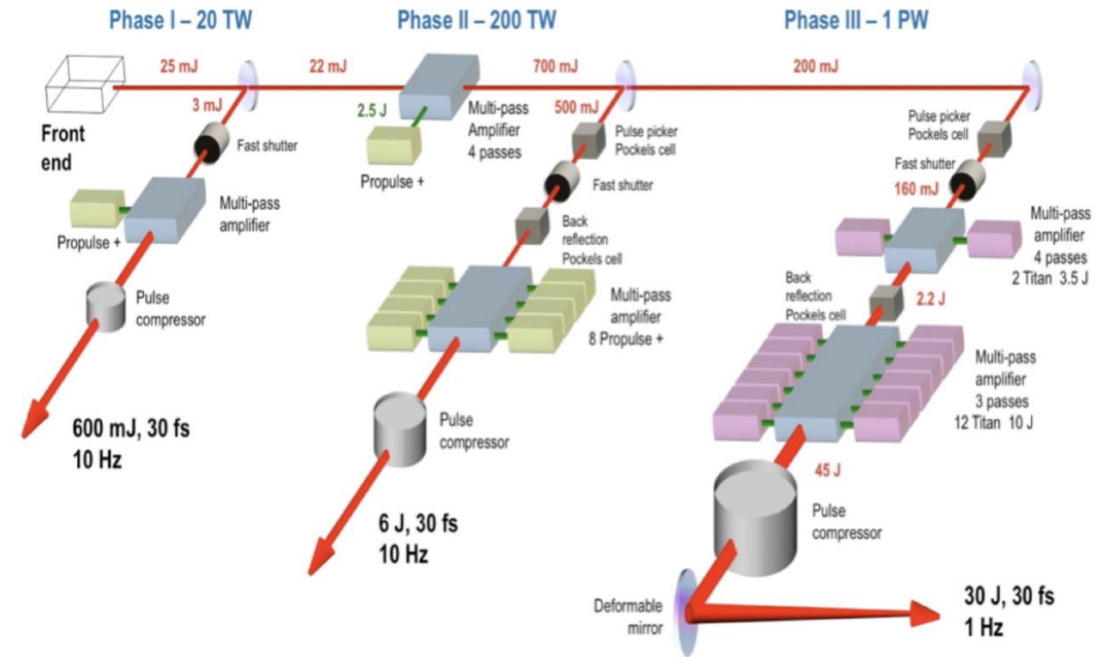
Experimental campaign: LWFA experiment and characterization of betatron radiation



Experimental campaign



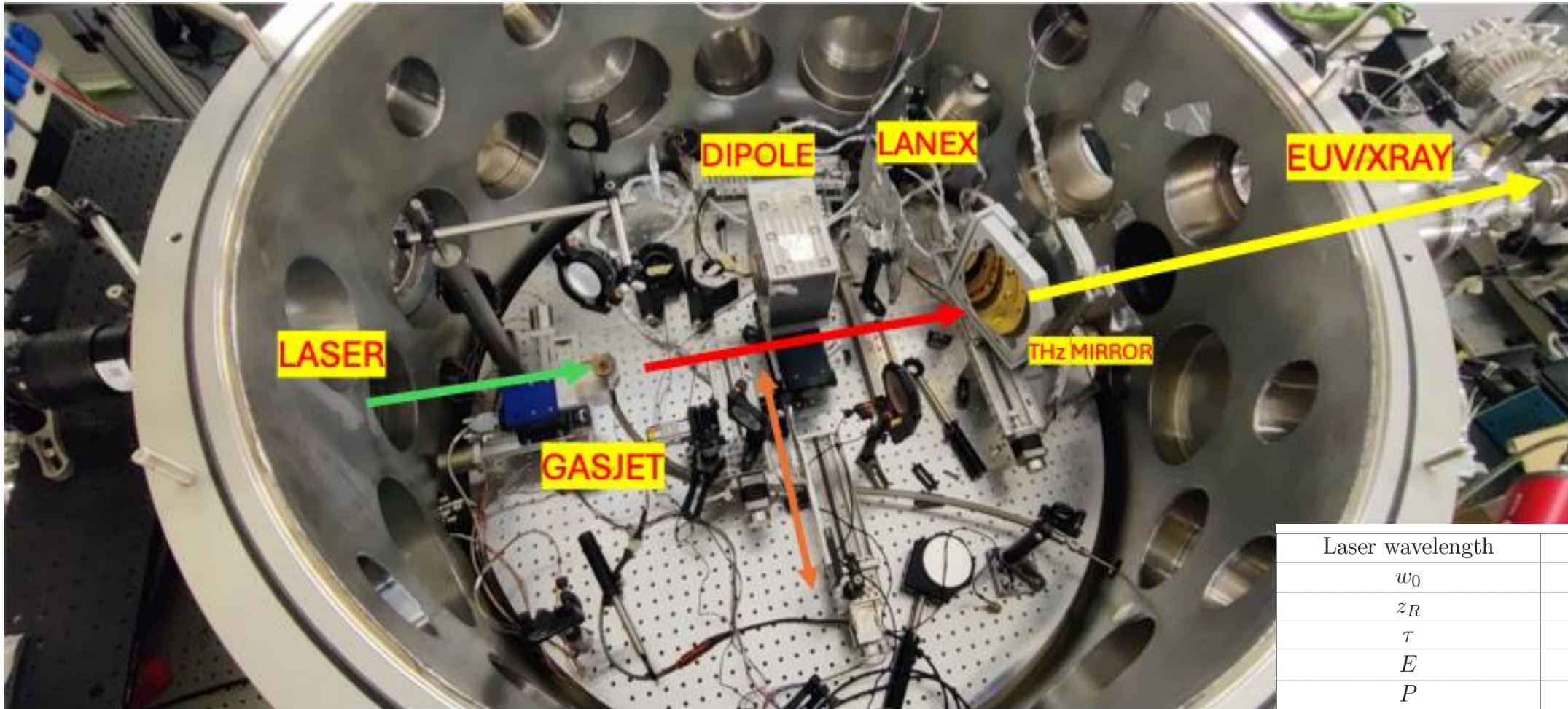
Laser system



Laser Vega II parameter used

	Peak Power	Energy/shot	pulse	Rep.Rate	Central @
VEGA II	~200 TW	~4 J	~30 fs	10 Hz	800 nm

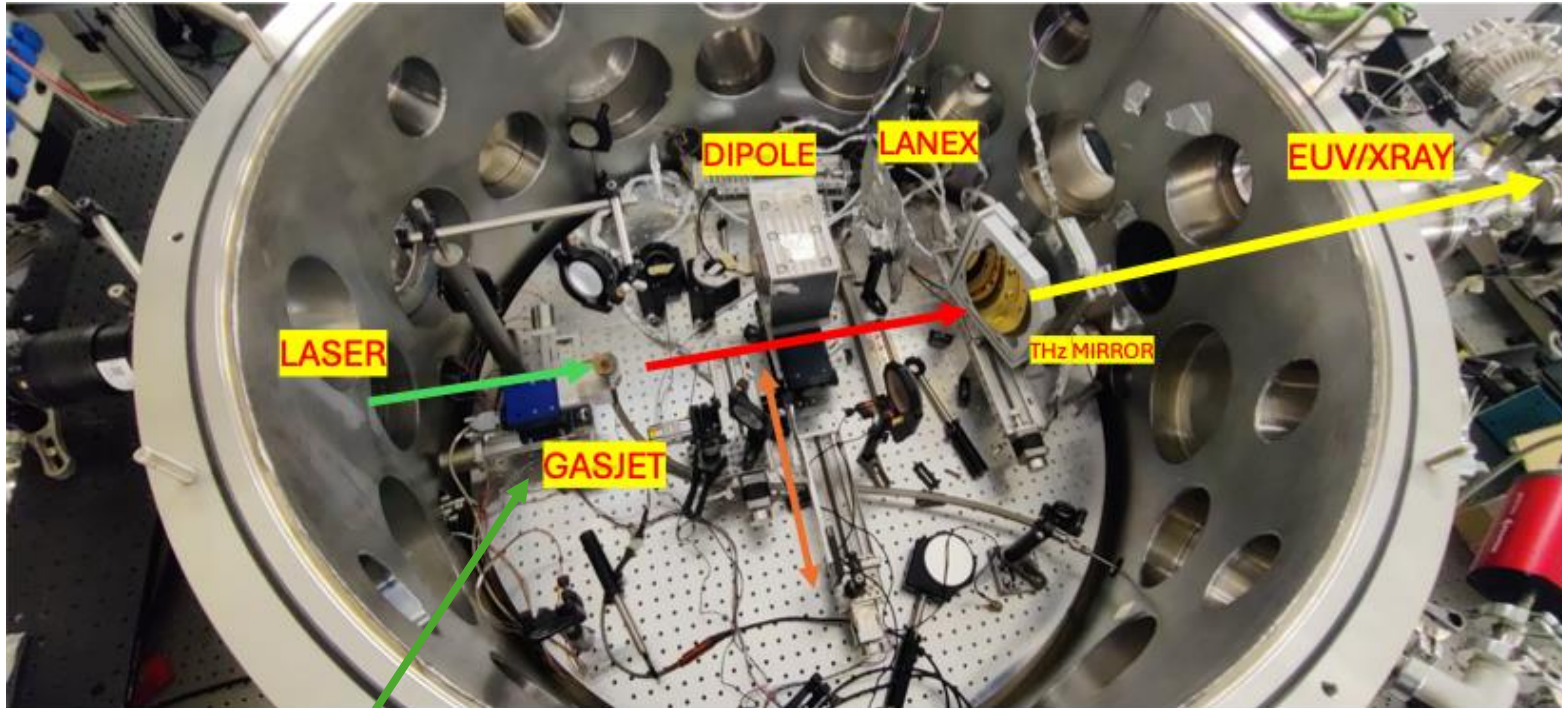
Vacuum chamber of the experiment



Laser wavelength	800 nm
w_0	$21.3 \pm 0.3 \mu\text{m}$
z_R	$1.8 \pm 0.1 \text{ mm}$
τ	$27 \pm 1 \text{ fs}$
E	$4.0 \pm 0.5 \text{ J}$
P	$139 \pm 21 \text{ TW}$
I_0	$(1.9 \pm 0.4) \times 10^{19} \text{ W/cm}^2$
a_0	3.0 ± 0.3
Plasma electron density	$3\text{-}4 (10^{18} \text{ cm}^{-3})$
gas pressure	30 bar

Parameter of the experiment

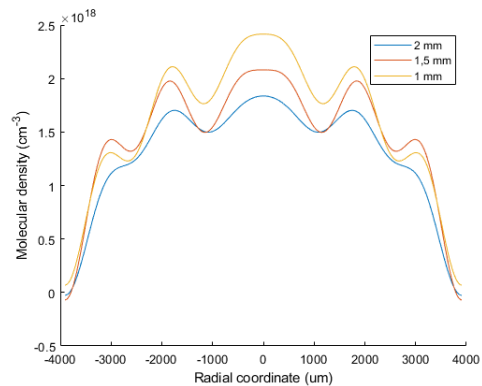
Vacuum chamber of the experiment



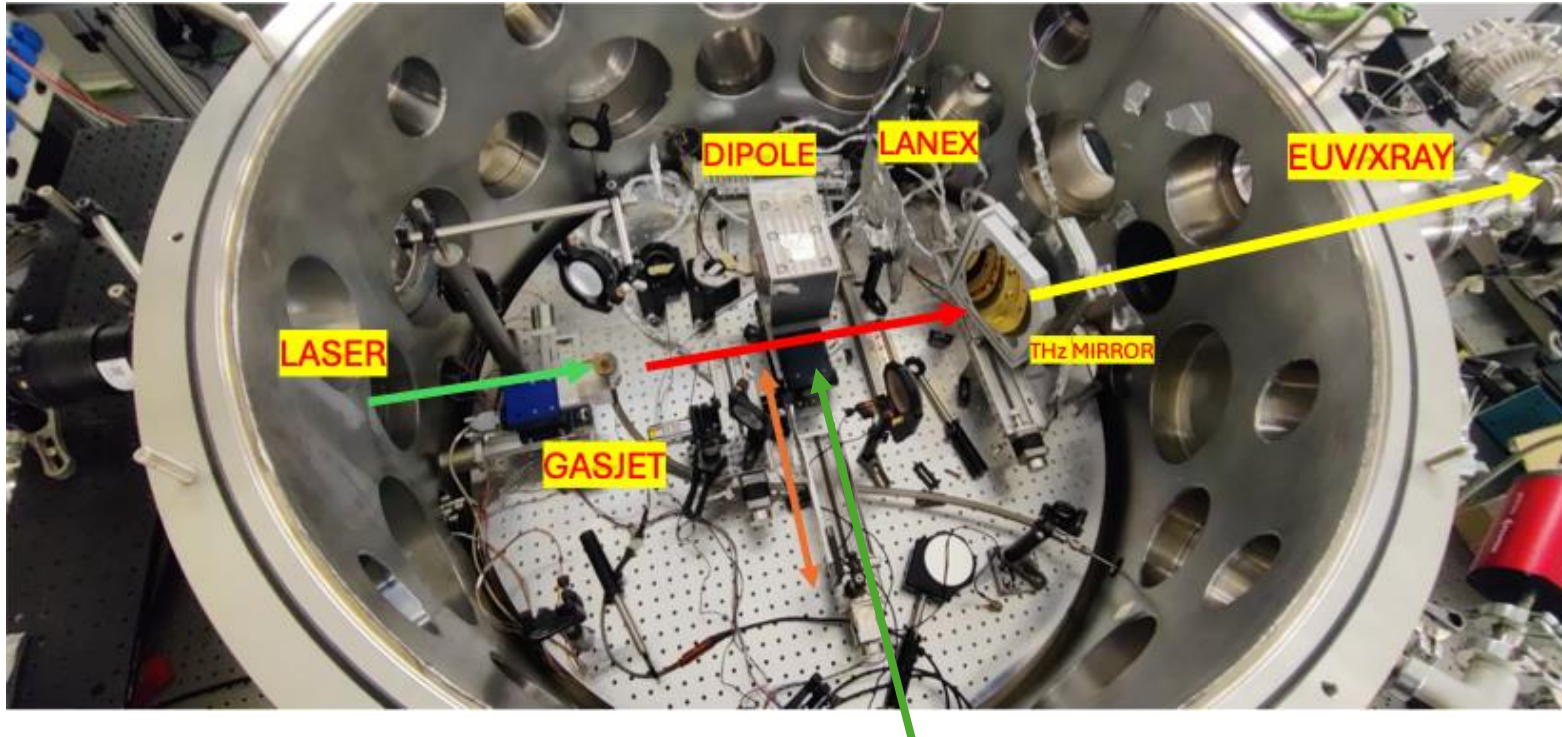
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Plasma electron density	3-4 (10^{18} cm^{-3})
gas pressure	30 bar

Gas density measurement



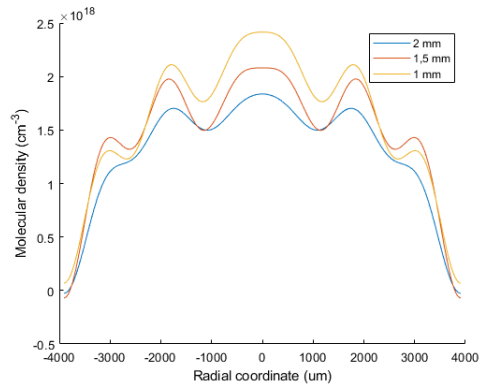
Vacuum chamber of the experiment



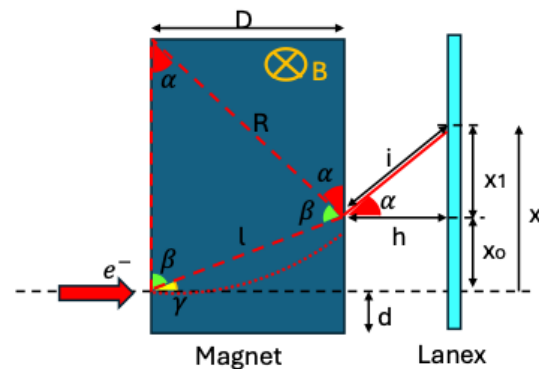
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Gas density measurement



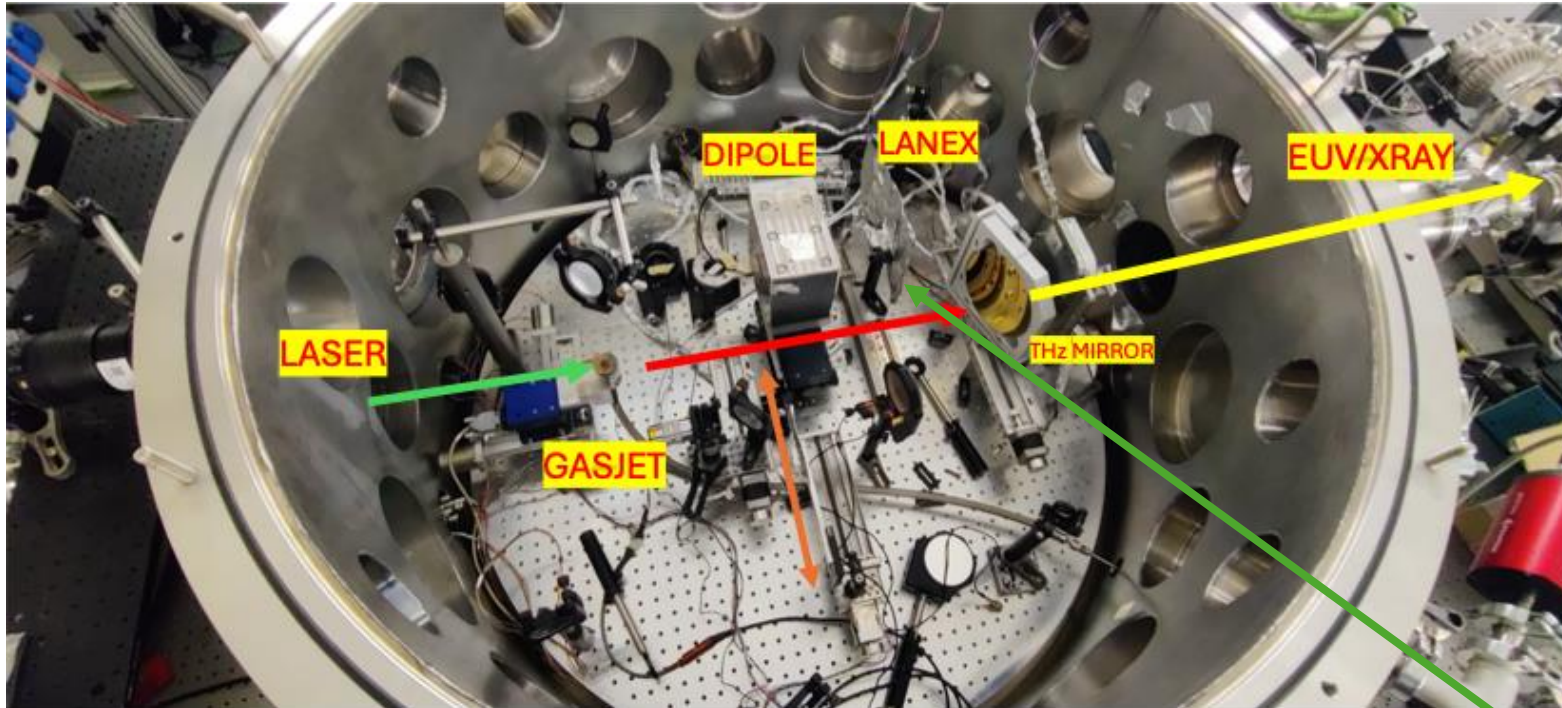
Dipole calibration for energy measurements



$$R = \frac{\gamma m_e \beta c}{eB}$$

$$p[\text{cm}] = 2R \sin^2\left(\frac{\alpha}{2}\right) + dtg(\alpha)$$

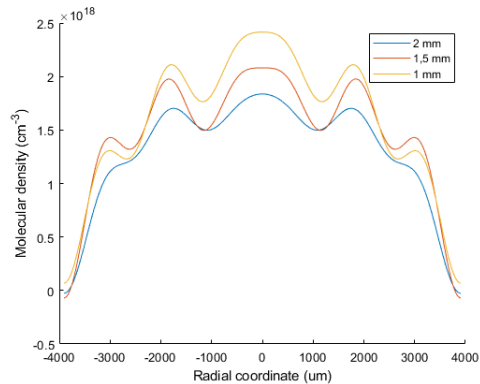
Vacuum chamber of the experiment



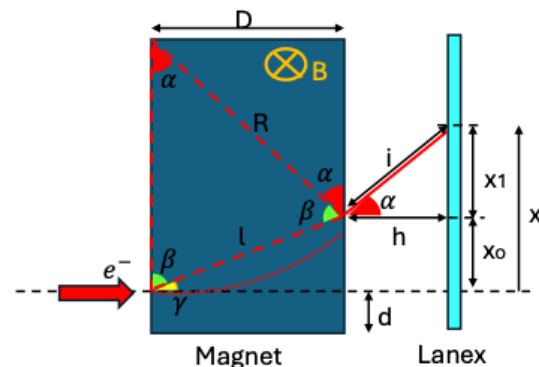
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Gas density measurement



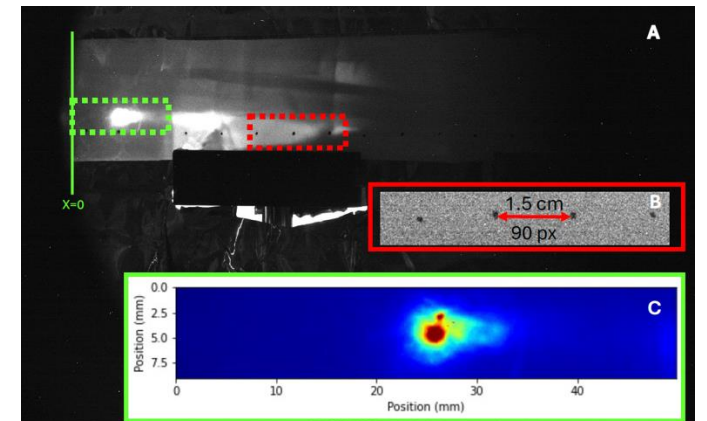
Dipole calibration for energy measurements



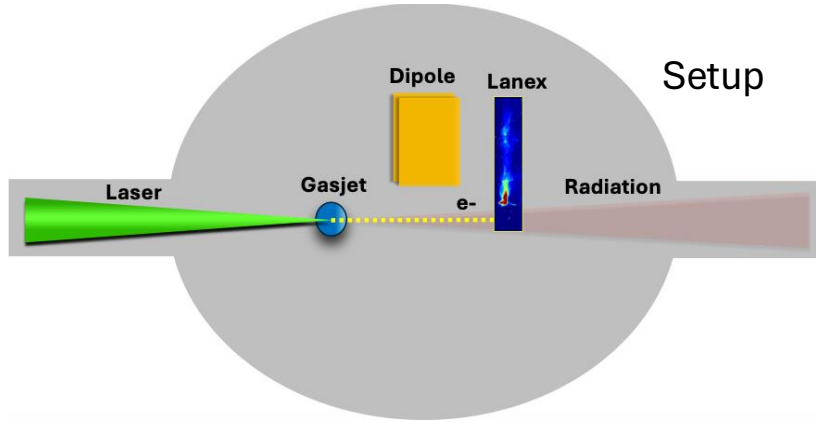
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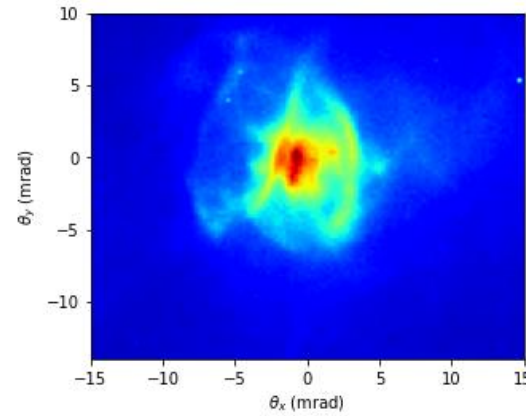
Lanex calibration and post-processing of images



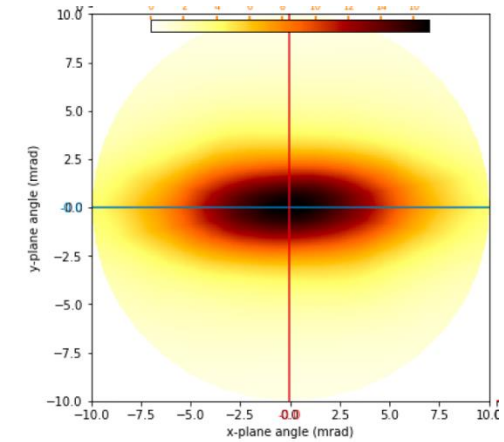
Electron spotsize measurements



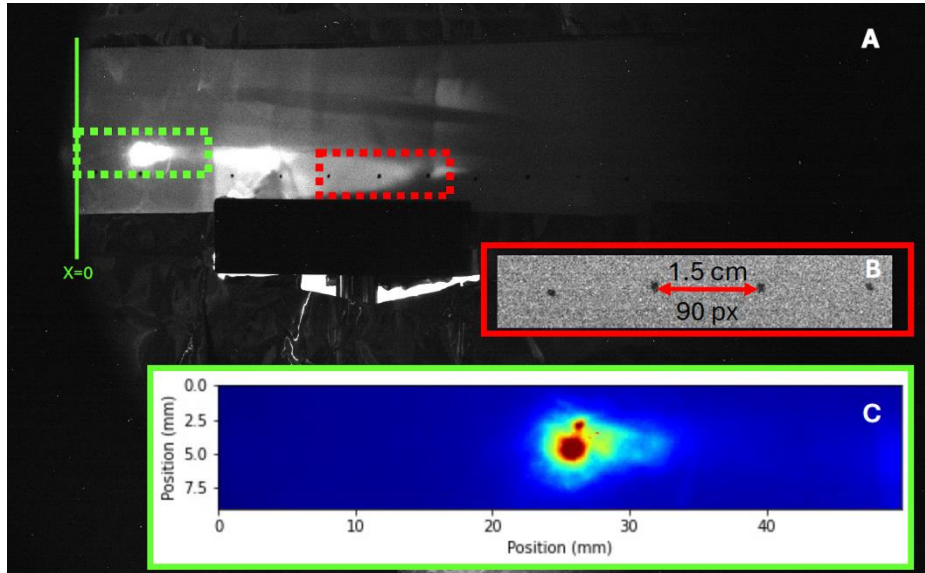
Measurements



Simulation



Lanex screen



Data

Shot n.	θ_x mrad	θ_y mrad	σ_x mrad	σ_y mrad
Shot 1	14.000	7.200	7.000	3.600
Shot 2	10.360	9.400	5.180	4.700
Shot 3	12.180	7.400	6.090	3.700
Shot 4	10.660	6.200	5.330	3.100
Shot 5	8.344	8.386	4.172	4.193
Shot 6	7.720	9.260	3.860	4.630
Shot 7	10.036	8.614	5.018	4.307
Shot 8	7.862	7.800	3.931	3.900
Shot 9	7.862	8.608	3.931	4.304
Shot 10	7.440	8.054	3.720	4.027
Mean	9.6464	8.0922	4.8232	4.0461

Divergence

$$\theta = \sqrt{\theta_x^2 + \theta_y^2} = \sqrt{9.64^2 + 8^2} = 12.52 \text{ mrad}$$

$$K = r_\beta \gamma k_\beta$$

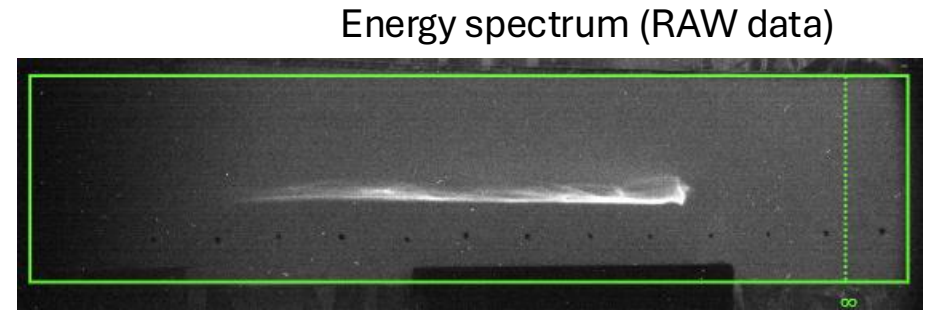
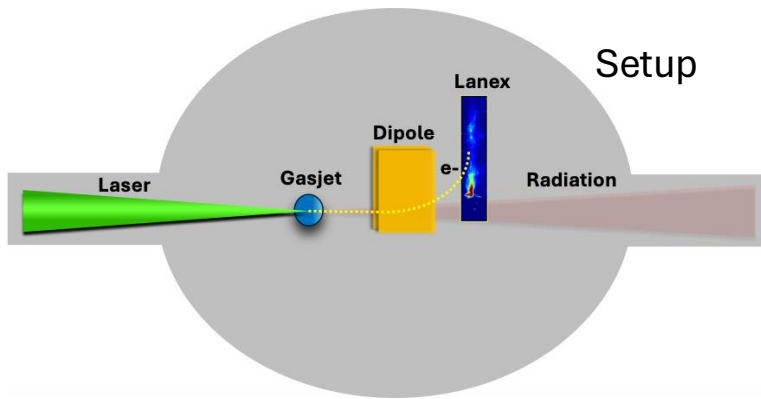
$$\theta = K / \gamma$$

$$r_\beta = \frac{\theta}{K}$$

We need the energy to calculate the amplitude of oscillation

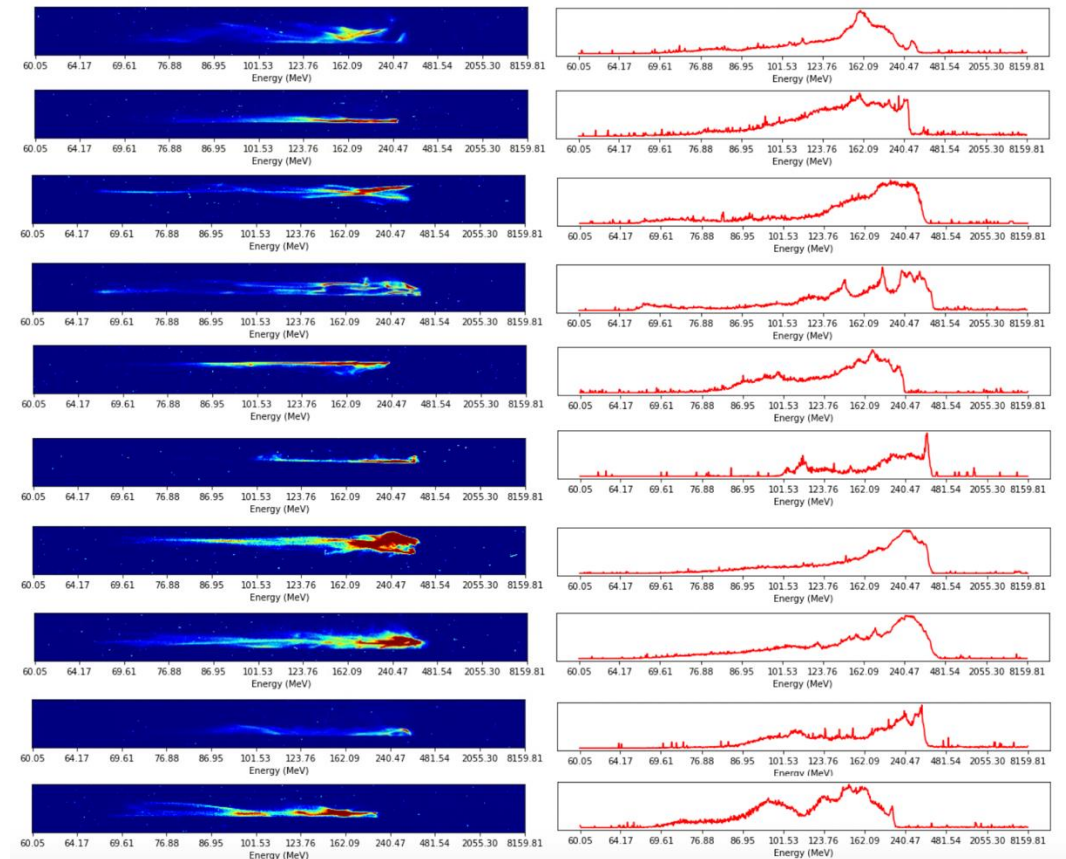
Amplitude of oscillation

Electron energy measurements



Shot n.	$\langle E \rangle$ (MeV)	σ_E (MeV)
Shot 1	160.47	59.58
Shot 2	229.88	78.49
Shot 3	184.2	56.32
Shot 4	194.73	77.17
Shot 5	144.29	42.37
Shot 6	215.23	120.18
Shot 7	197.82	68.48
Shot 8	189.16	74.42
Shot 9	193.17	109.68
Shot 10	157.83	308.23
Average	186.59	99.08

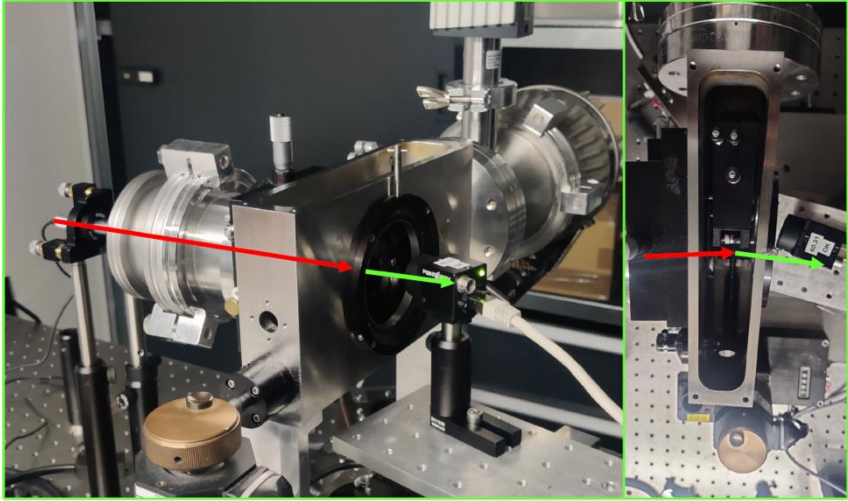
Energy spectrum (post processed data)



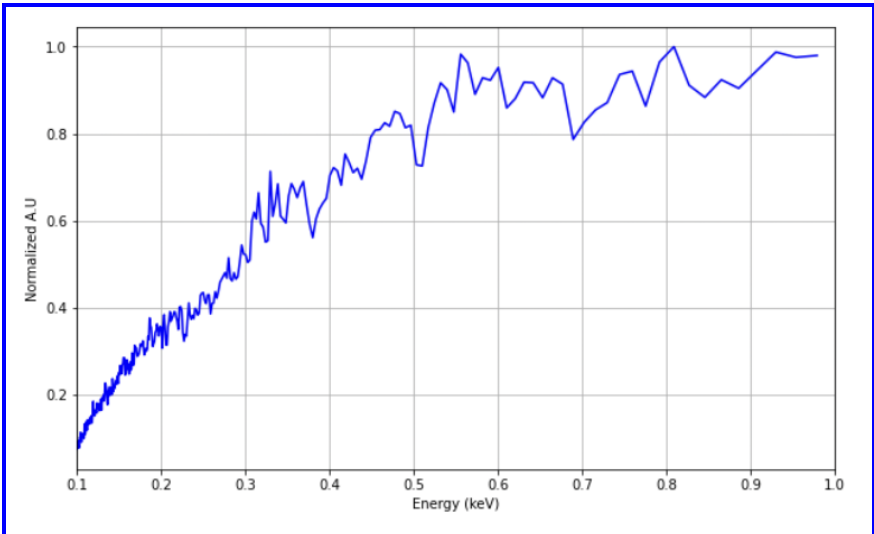
$$\langle \gamma \rangle = \frac{\langle E \rangle}{E_0} = 364.19 \quad \rightarrow \quad r_\beta = 0.88 \mu\text{m}$$

Spectra reconstruction

Using: Grazing Incidence Monochromator



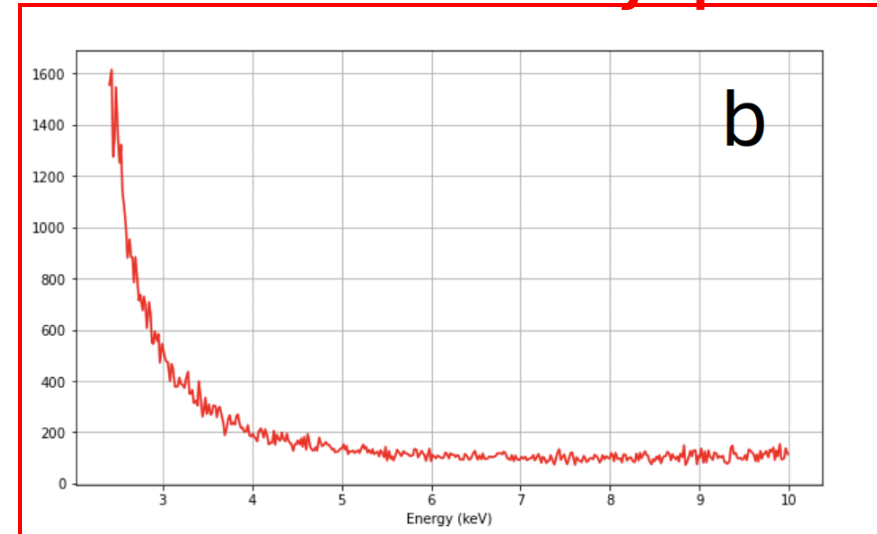
Measurements of EUV spectrum



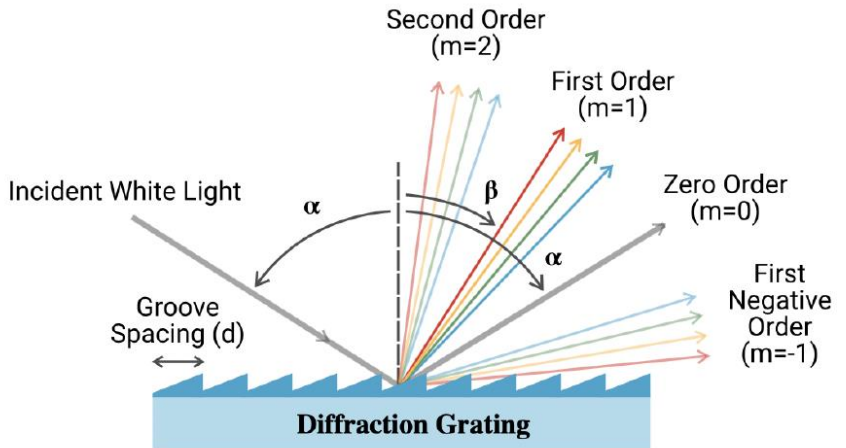
Using: Single photon counting



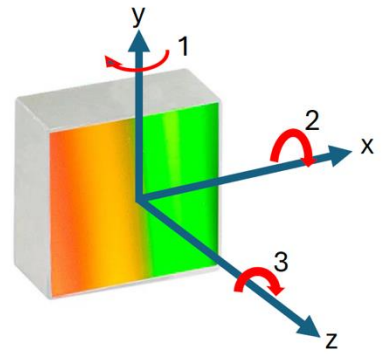
Measurements of X ray spectrum



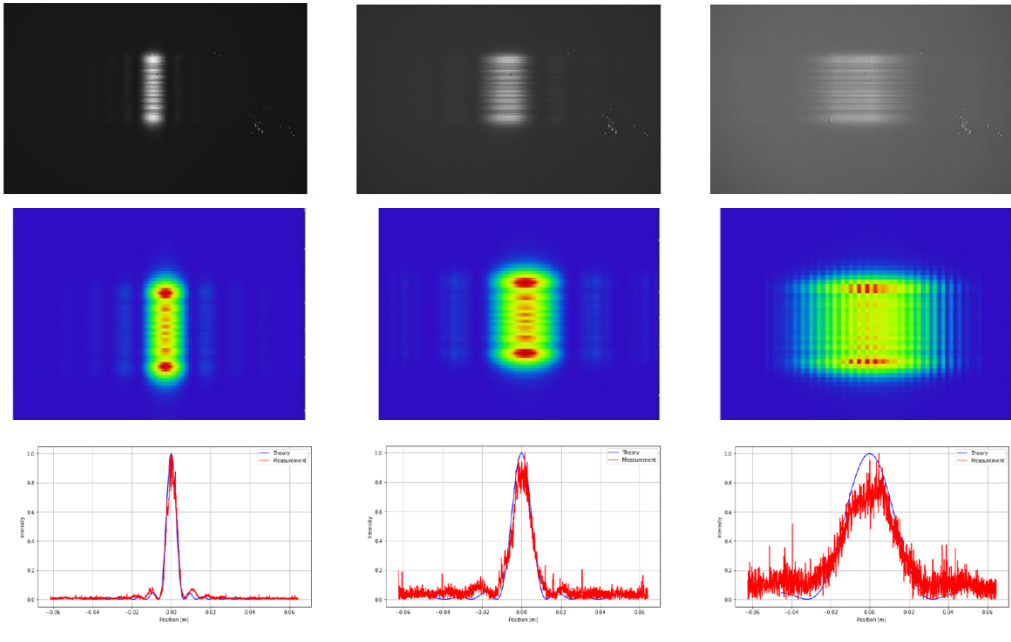
Grazing Incidence Monochromator



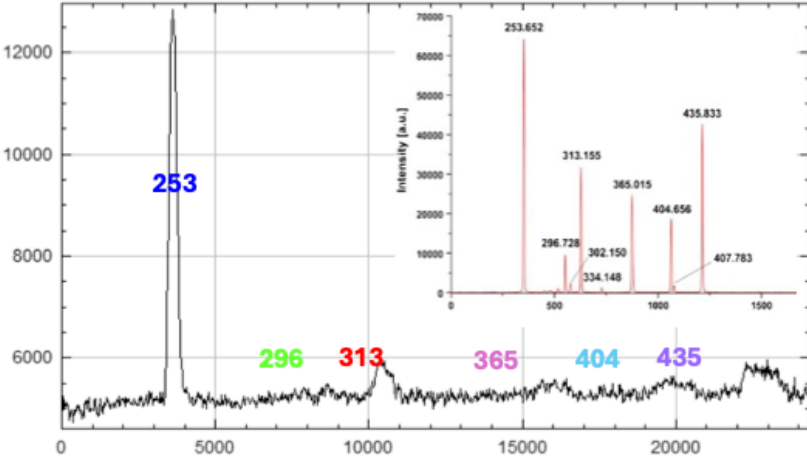
1) Alignment



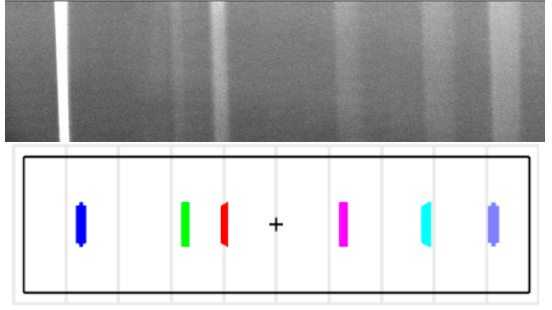
2) Slit measurement



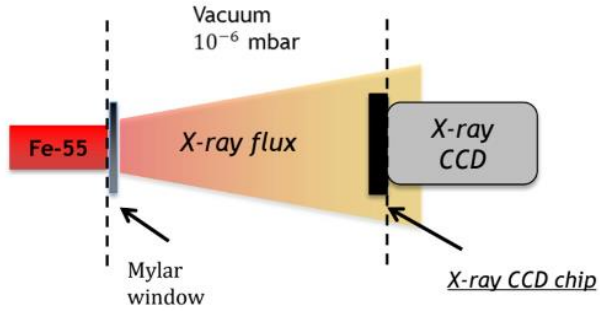
3) Calibration for EUV with mercury lamp and Zemax



Mercury line

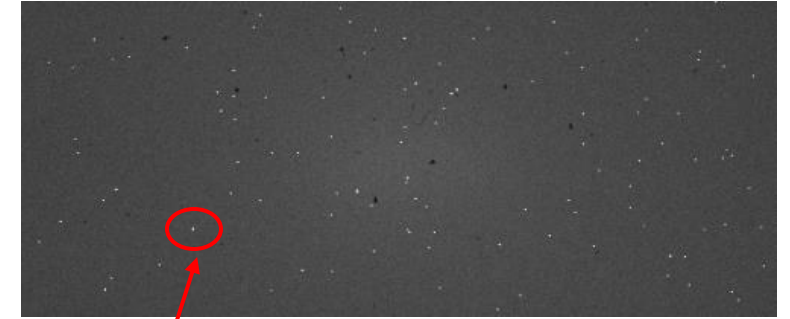


Single photon counting



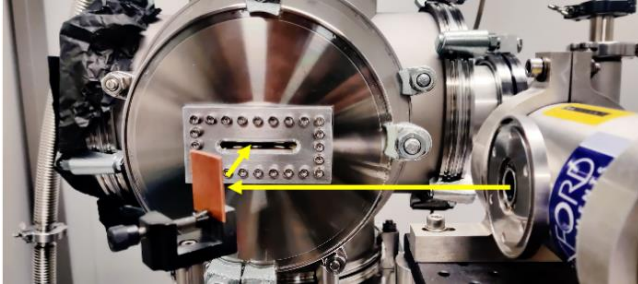
Photons directly hit the CCD camera.

The number of photons must be very low, and the pixels that will be hit will have a very low density

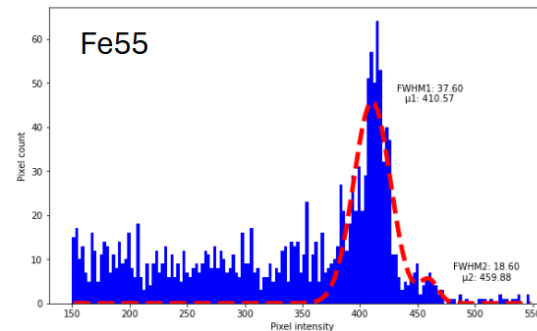
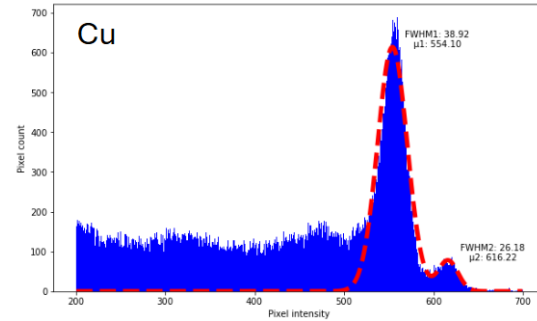


Single pixel illuminated

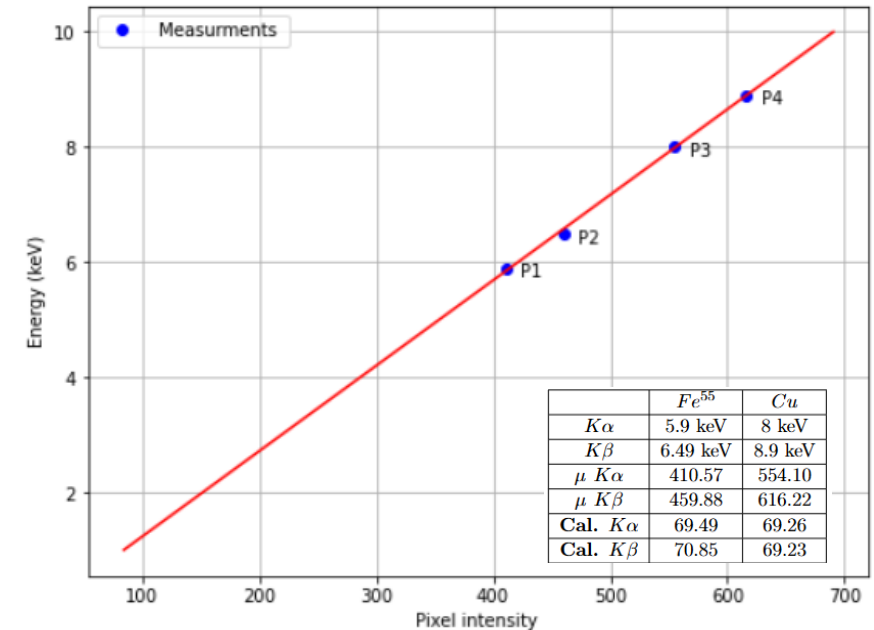
Calibration



Two sources were used:
Fe55 and Cu



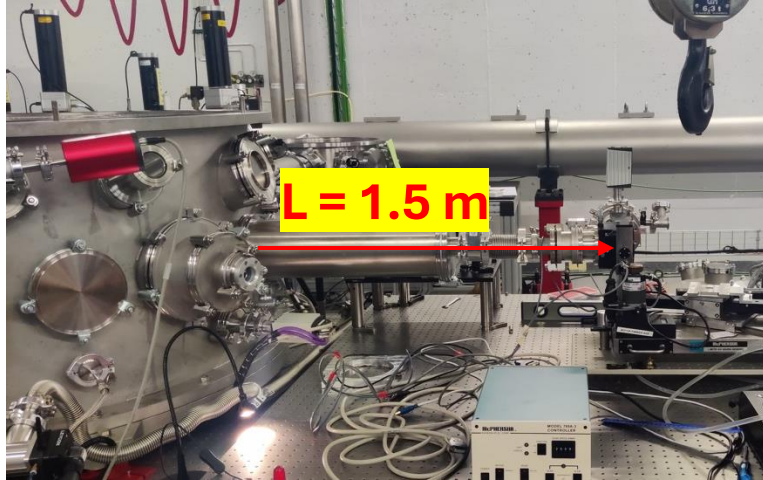
The intensity histogram is compared with the typical emission line



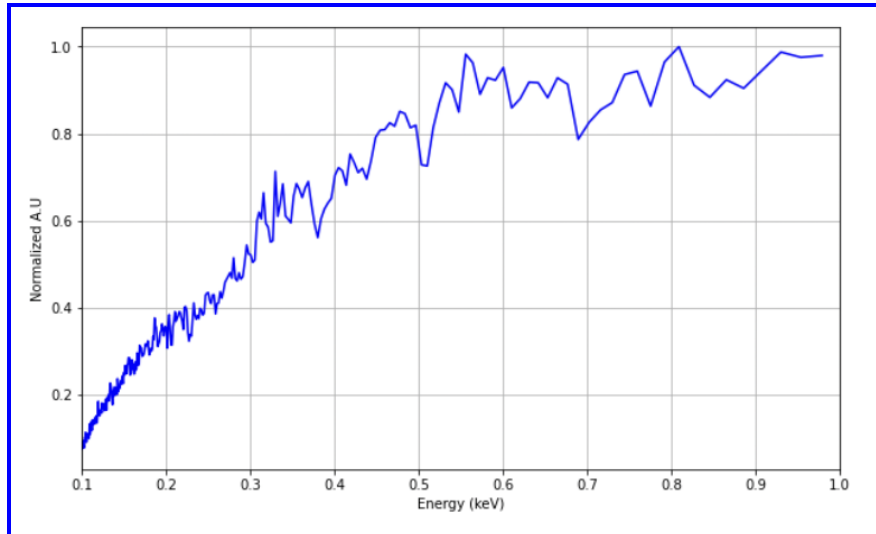
Calibration curve for the CCD camera

Spectra reconstruction

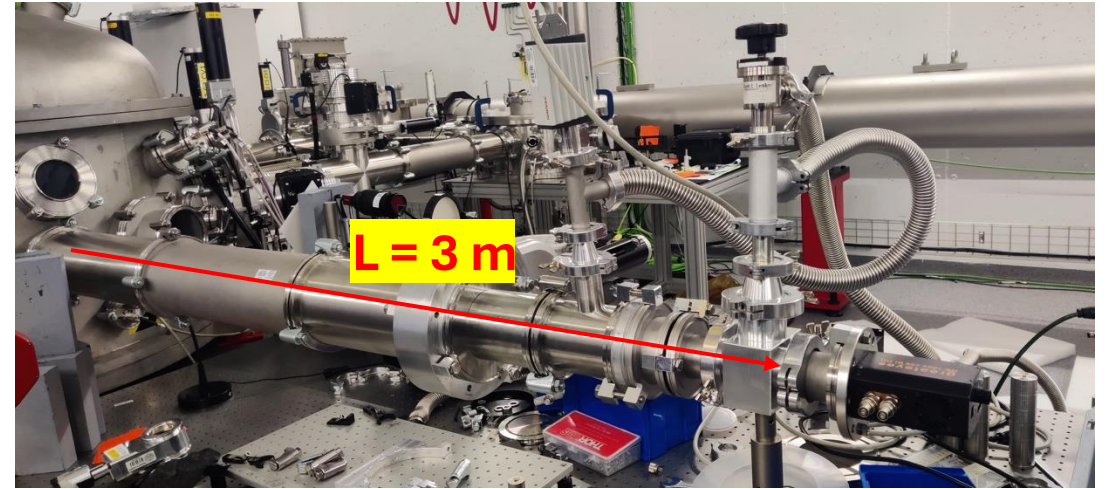
Plasma → Grating → Al filter → CCD



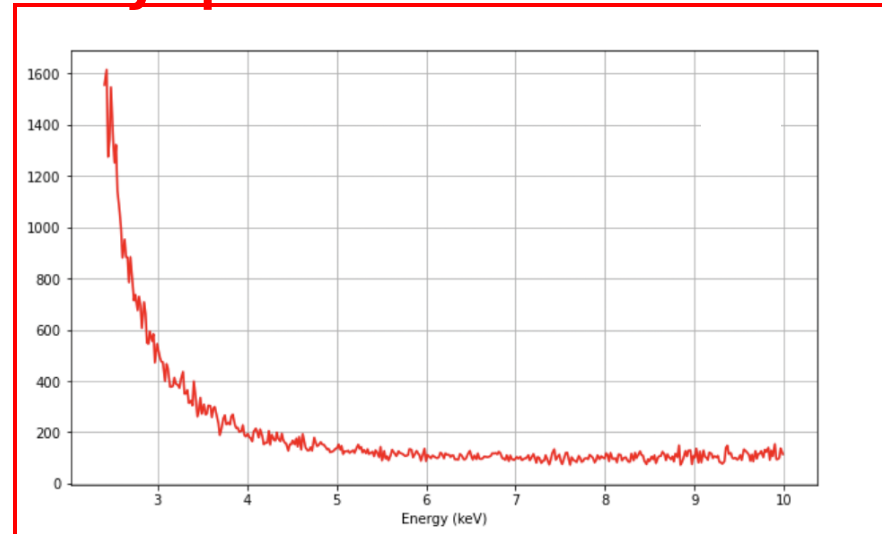
EUV spectrum



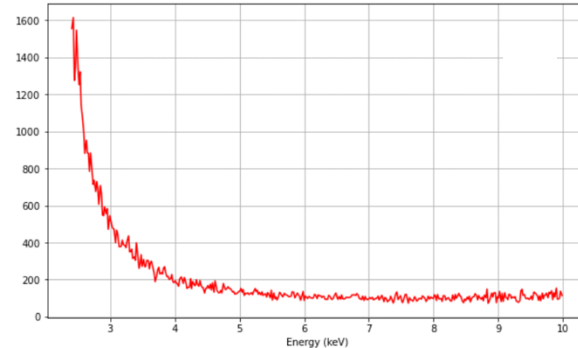
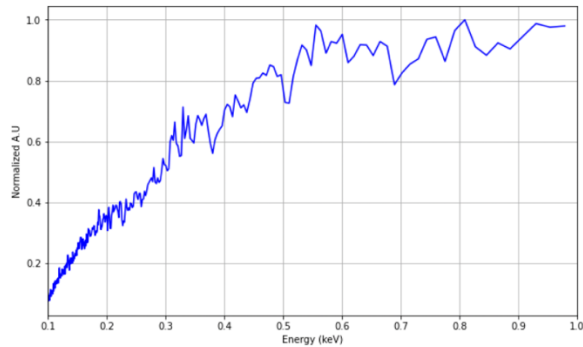
Plasma → Al filter → CCD



X ray spectrum



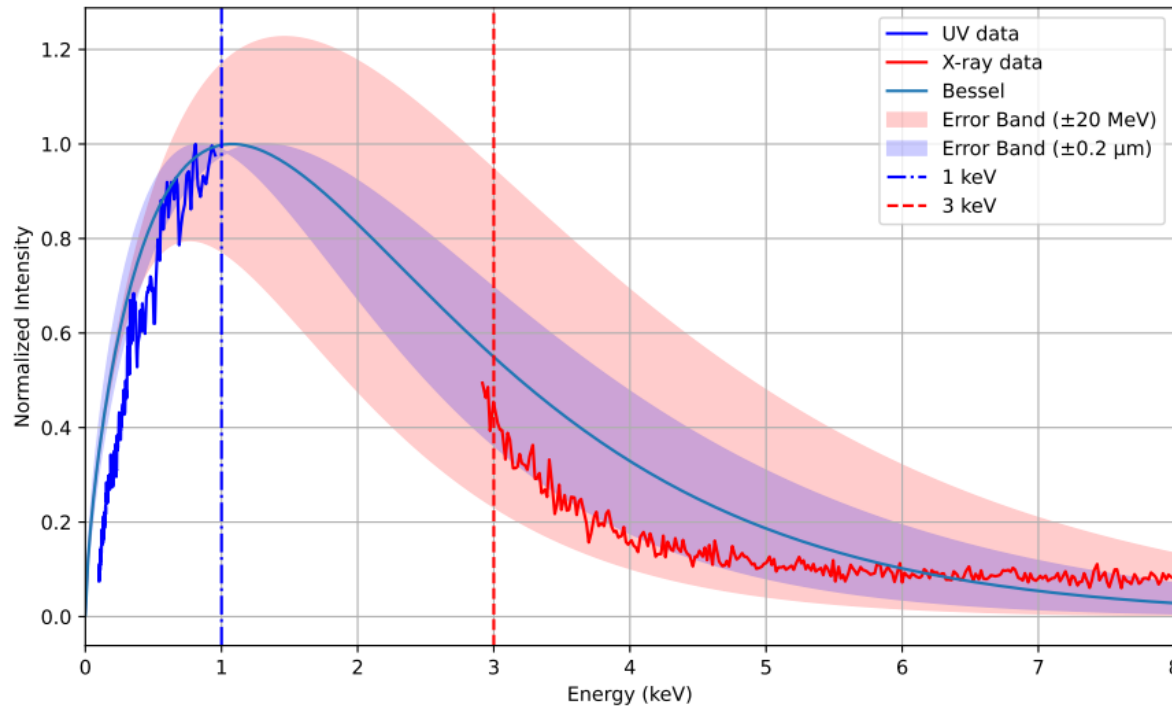
Spectra reconstruction EUV + X



Merging of the two part of the spectra using the asymptotic equation

$$\left. \frac{d^2 I}{d\omega d\Omega} \right|_{\theta=0} \simeq N_{\beta} \frac{3e^2}{2\pi^3 \epsilon_0 c} \gamma^2 \zeta^2 \mathcal{K}_{2/3}^2(\zeta)$$

Possible errors in the evaluation of the energy, the amplitude of oscillation or in the deconvolution



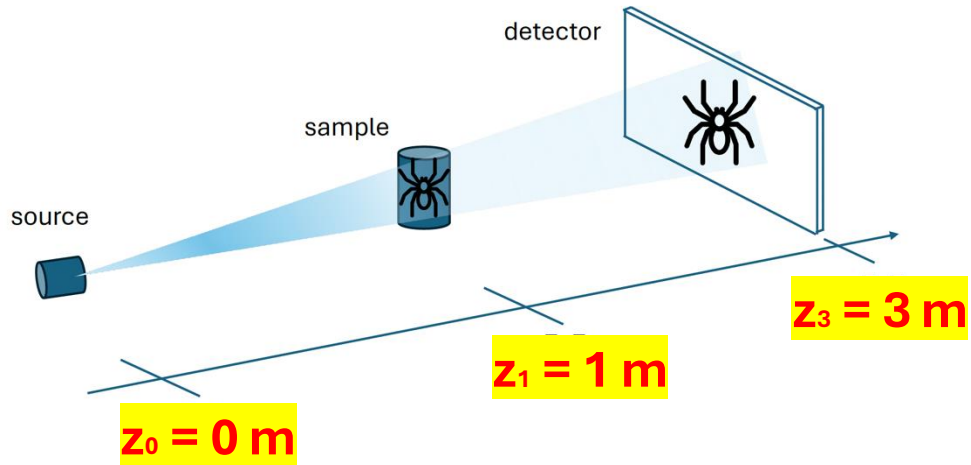
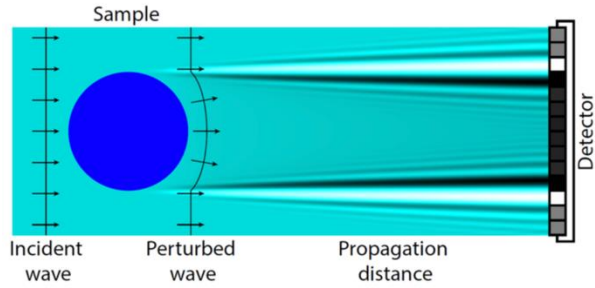
Parameter	value
n_e	$4 \times 10^{18} \text{ cm}^{-3}$
γ	364.19
θ	12.52 mrad
r_{β}	$0.88 \mu\text{m}$
E	$186.59 \pm 20 \text{ MeV}$
σ_E	$99.08 \pm \text{ MeV}$
E_c	2.3 keV

X-Ray phase contrast imaging (XPCI)

Propagation-Based

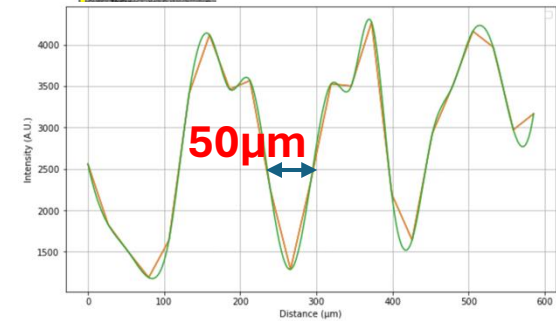
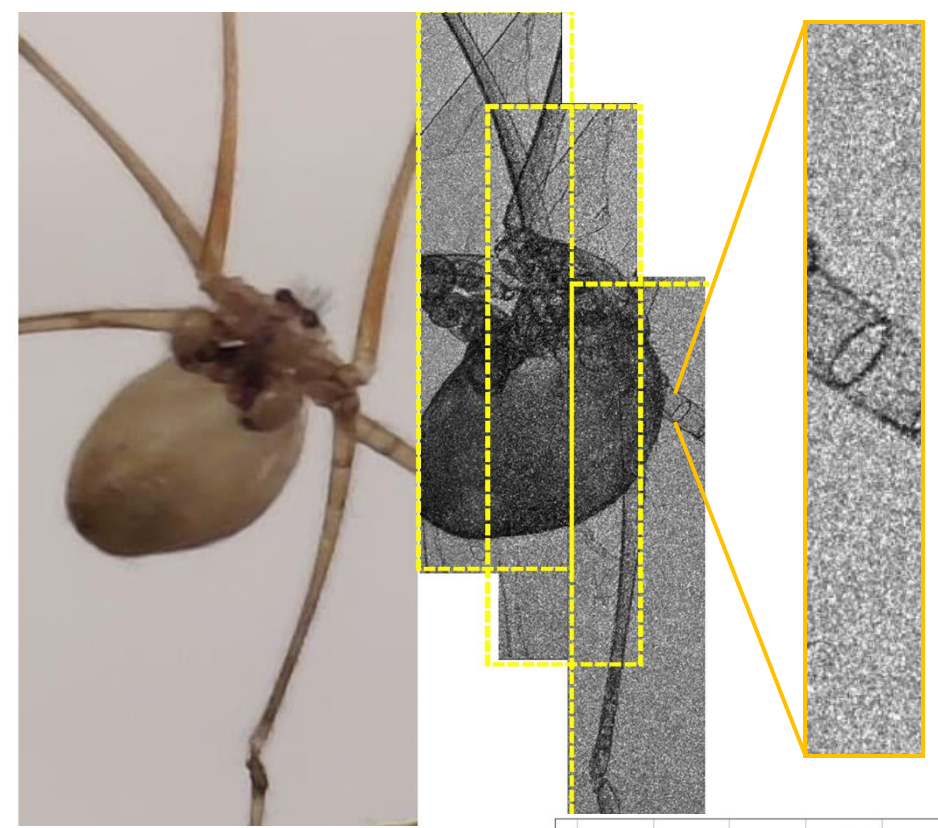
$$\eta = 1 - \delta + i\beta$$

XPCI $\delta \gg \beta$
at high x-ray energies



Magnification

$$M = \frac{z_3}{z_1} = 3$$



Conclusion

- Introduced the EuAPS project, outlining its objectives and significance.
- Provided an overview of the Laser Wakefield Acceleration (LWFA) theory.
- Explored the theory of betatron radiation, emphasizing key parameters such as electron density, electron energy, and oscillation radius, which define the betatron strength parameter.
- Analyzed how variations in electron energy and oscillation amplitude influence the shape of the radiation spectra.
- Presented simulations conducted with FBPIC and Synchrad.
- Described the experimental setup and results from the CLPU facility in Spain.

