# PMT Simulation

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**CYGNO Collaboration Meeting**

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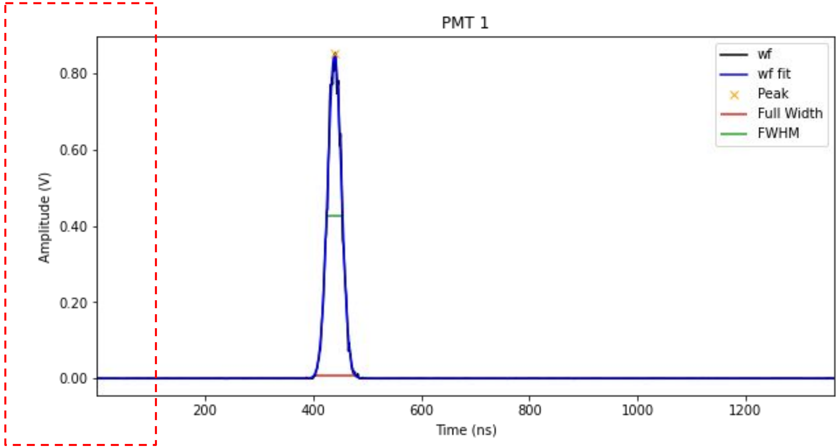
# Summary

1. Spectrum analysis
2. Noise characterization
3. SPE signal characterization
4. Digitization-PMT Simulation
5. Sim/data comparison
6. Conclusion

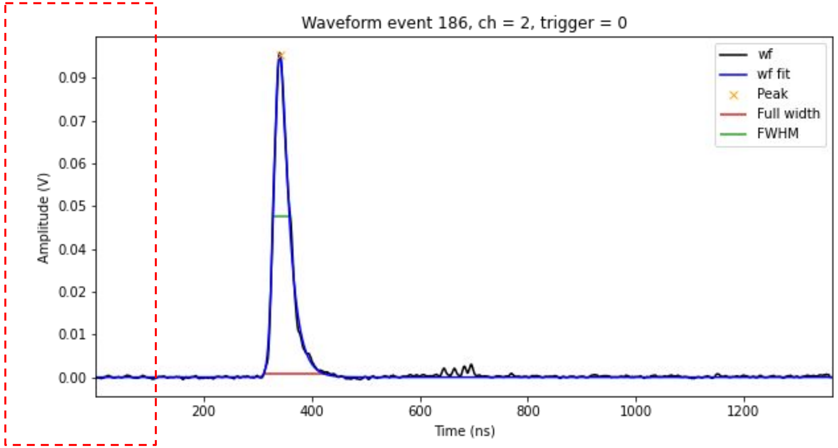
# Spectrum analysis

## Last year we faced a problem in the simulated waveforms height peak

### Simulation



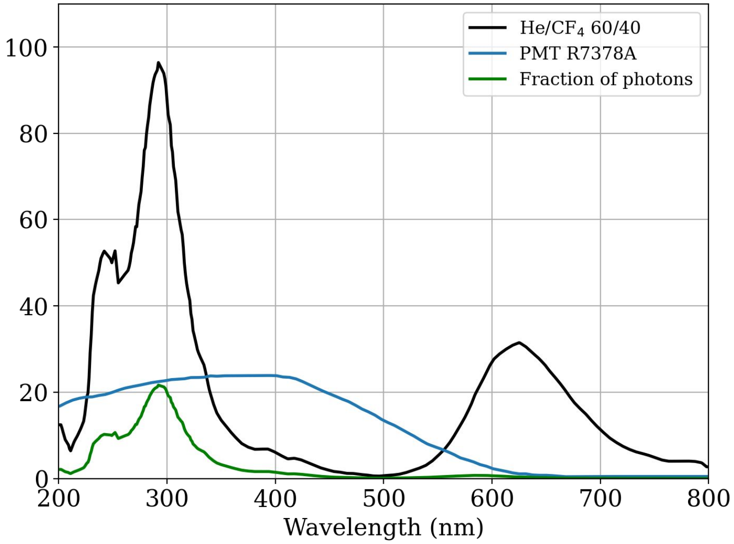

### Real data



# Spectrum analysis

**Last year we faced a problem in the simulated waveforms height peak**

- An analysis of the spectrum was done (**Light spectrum** + **PMT Quantum Efficiency**):




Fraction of photons:

$$\frac{Area_{green\_curve}}{Area_{black\_curve}} = 15\%$$

**Was not enough to fix the problem!**

↓

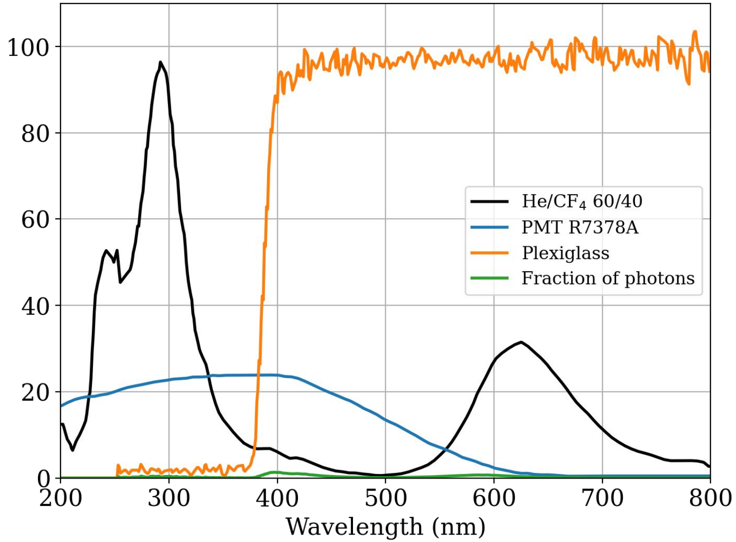
We were suggested to take a look at the

Glass Transmission Spectrum

# Spectrum analysis

## First update of the PMT Simulation of this year

**Light spectrum** + **PMT Quantum Efficiency** + **Glass Transmission**




Fraction of photons:

$$\frac{Area_{green\_curve}}{Area_{black\_curve}} = 1.36\%$$

Number of generated photons at GEM and possible photoelectrons reduced in **98.64%**

# Noise characterization

## Updates in noise simulation

| | Year | |
|---|---|---|
| | **2023** | **2024** |
| Fast digitizer | ✓ | ✓ |
| Slow digitizer | ✗ | ✓ |
| Method | Covariance Matrix | Power Spectral Density |
| Characterization | Single channel/PMT | All channels/PMTs |

# Noise characterization

**The new method of noise generation: Power Spectral Density**

- With this method it is possible to generate a noise with same **amplitude distribution** and preserve the **autocorrelation** of the real noises.

The Fourier transform of the autocorrelation $R_x(\tau)$ is called Power Spectral Density $S_x(f)$:

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-2\pi i f \tau} \, d\tau$$

# Noise characterization

## The new method of noise generation: Power Spectral Density

### Steps of a single noise generation

1. Get the Average Power Spectral Density of the real noise dataset
2. Introduce a phase spectrum from a uniform distribution between −π and +π
3. Create the noise spectrum from Steps 1 and 2
4. Perform an IFFT to obtain the time-domain noise series

The noise generation for the **slow digitizer** using the old method would need a covariance matrix with size **(4000, 4000)** for **each PMT.**

- Size around **128 MB** for each cov. matrix
- Four cov matrices would be needed

$$f > 0 \Rightarrow N(f) = \sqrt{\frac{\mathrm{PSD}_{\mathrm{noise}}(f)}{2}} \times e^{j\varphi(f)}$$

$$f < 0 \Rightarrow N(f) = \sqrt{\frac{\mathrm{PSD}_{\mathrm{noise}}(-f)}{2}} \times e^{-j\varphi(f)}$$

$$f = 0 \Rightarrow N(0) = DC$$

$$n(t) = \mathrm{Re}\left\{\mathrm{IFT}\left[N(f)\right]\right\}$$

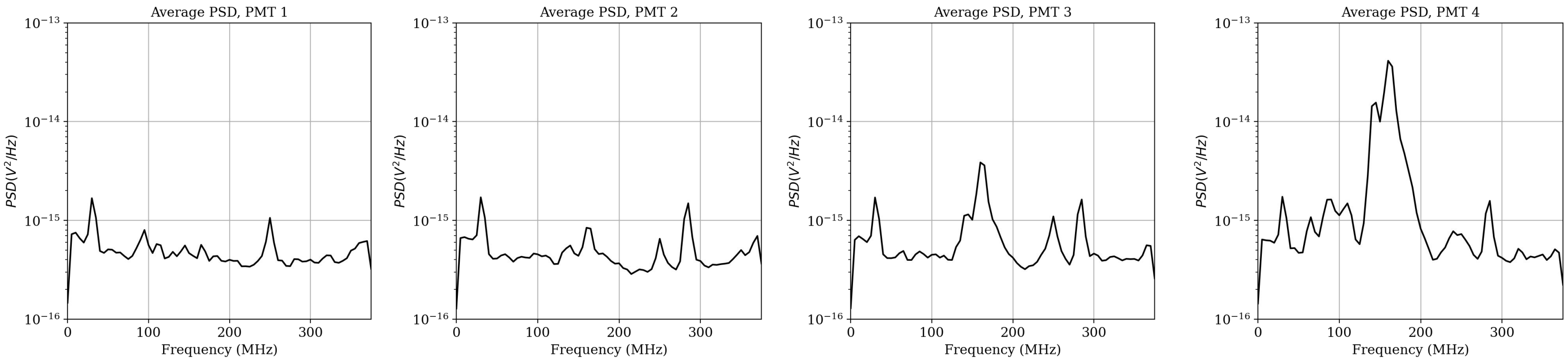
L. Smaini, *RF Analog Impairments Modeling for Communication Systems Simulation: Application to OFDM-based Transceivers.* Marvell Switzerland.

# Noise characterization

## The new method of noise generation: Power Spectral Density

***Fast digitizer:*** 1024 samples, 750 MS/s

Main differences in **PMT 3** and **PMT 4**



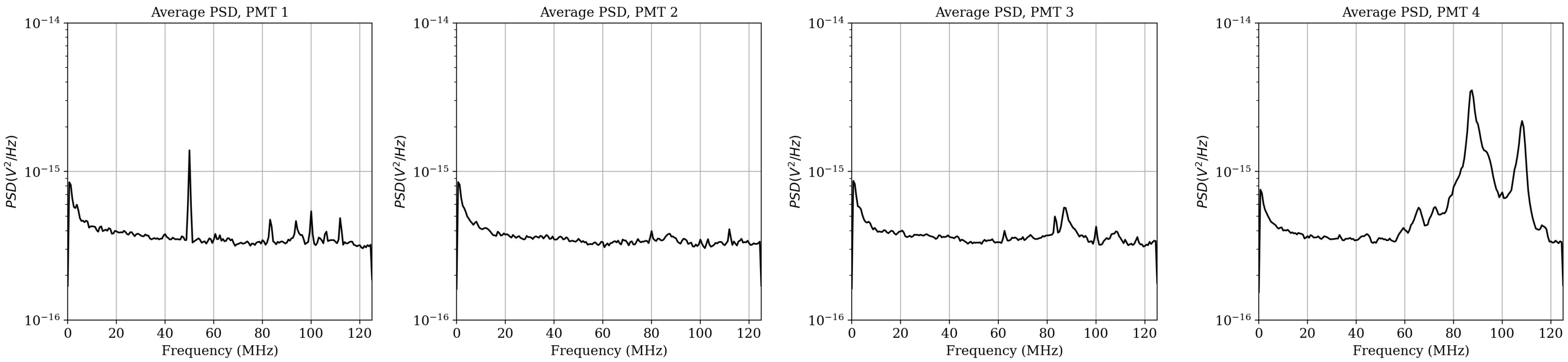

***Average Power Spectral Density of the real noise dataset for each PMT/channel***

# Noise characterization

## The new method of noise generation: Power Spectral Density

***Slow digitizer:*** 4000 samples, 250 MS/s

Main differences in **PMT 3** and **PMT 4**

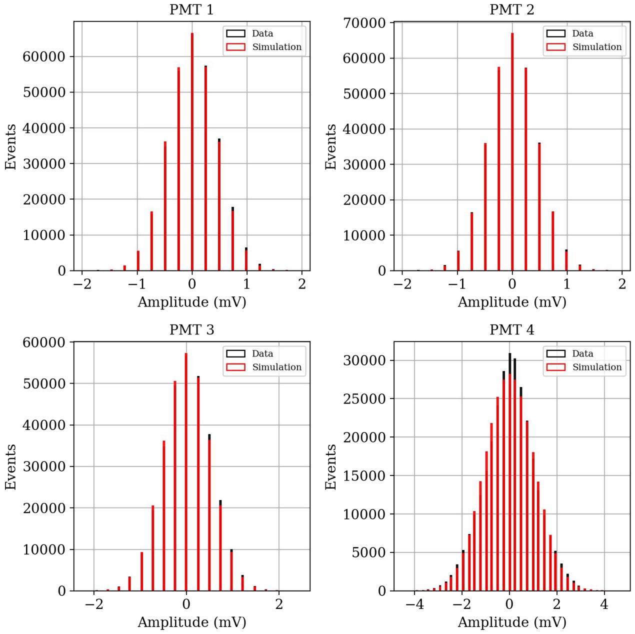



***Average Power Spectral Density of the real noise dataset for each PMT/channel***

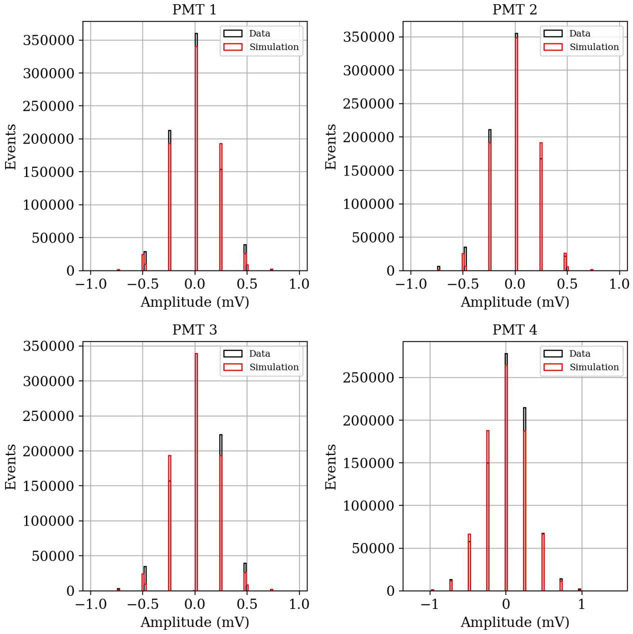
# Noise characterization

## Amplitude distribution comparison

***Fast digitizer***




***Slow digitizer***

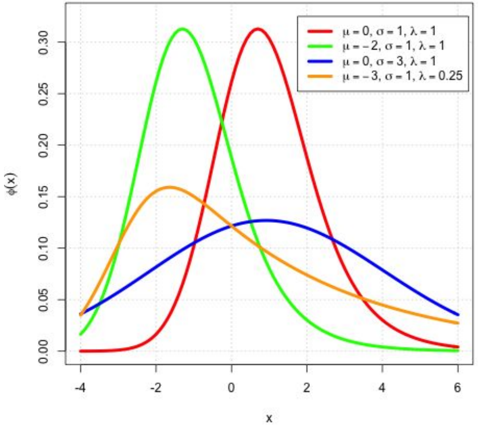


# SPE signal characterization

## Proposed model for the SPE signal

### Exponentially modified gaussian PDF

$$f(x;\mu;\sigma;\lambda) = \frac{\lambda}{2}\exp(\frac{\lambda}{2}(2\mu + \lambda\sigma^2 - 2x))\ \mathrm{erfc}(\frac{\mu + \lambda\sigma^2 - x}{\sqrt{2}\sigma})$$




$\mu$ = mean (arrival times)

$\sigma$ = standard deviation (width)

$\lambda$ = exponential decay rate

**Rise time** -> Gaussian part

**Fall time** -> Gaussian + Exponential part

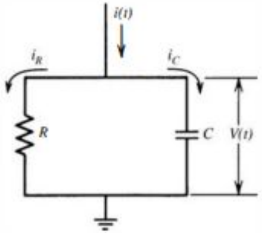
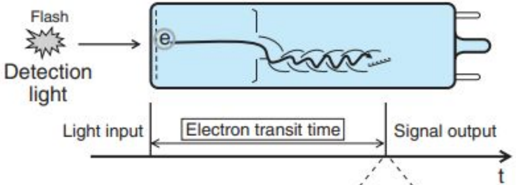
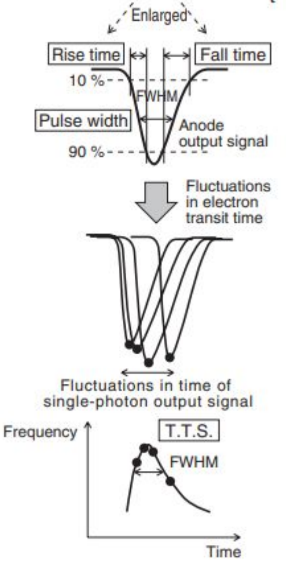



**Figure 9.20** Simple parallel *RC* circuit representing a PM tube anode circuit.

Knoll, G. F. (2010). Radiation Detection and Measurement (4th ed.)



# SPE signal characterization

## Proposed model for the SPE signal

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$$f(x;\mu;\sigma;\lambda) = \frac{\lambda}{2}\exp(\frac{\lambda}{2}(2\mu+\lambda\sigma^2-2x))\,\mathrm{erfc}(\frac{\mu+\lambda\sigma^2-x}{\sqrt{2}\sigma})$$

pdf --> Area = 1

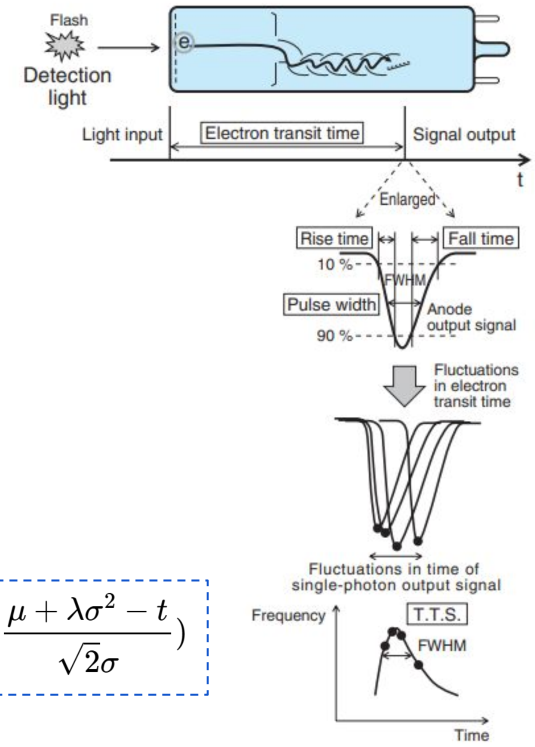
$$\mathrm{pdf} = \frac{v(t)}{\int_{t_0}^{t_1} v(t)\,dt} \qquad v(t) = \mathrm{pdf}\cdot\int_{t_0}^{t_1} v(t)\,dt$$

**From definition of electric current:**

$$I = \frac{V}{R} = \frac{dQ}{dt} \qquad Q = \frac{1}{R}\int_{t_0}^{t_1} v(t)\,dt$$

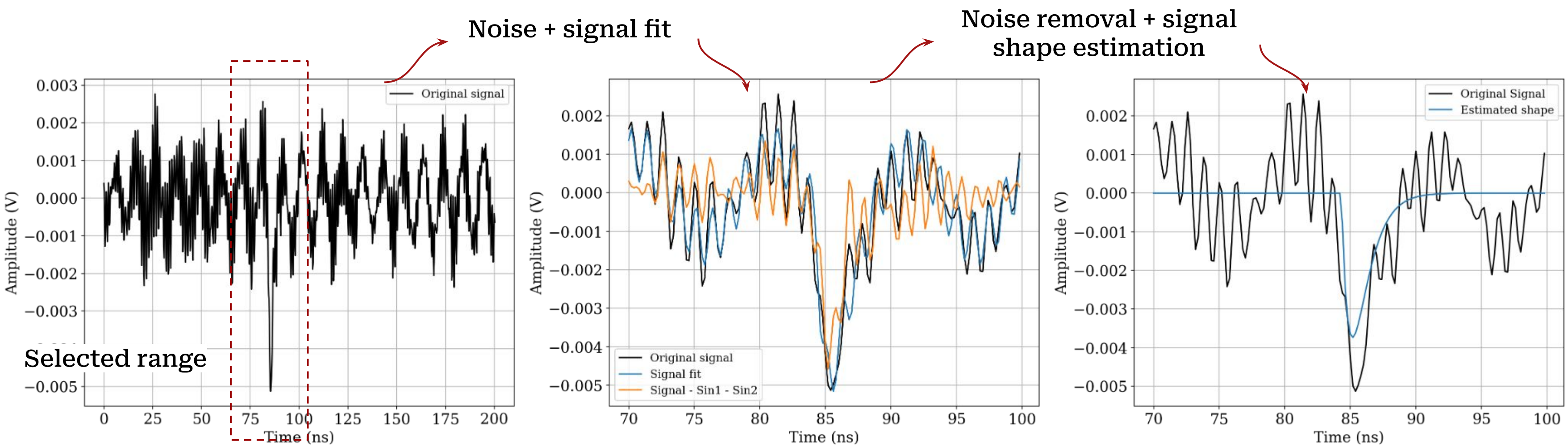
### SPE signal model

$$v(t) = QR\,\frac{\lambda}{2}\exp(\frac{\lambda}{2}(2\mu+\lambda\sigma^2-2t))\,\mathrm{erfc}(\frac{\mu+\lambda\sigma^2-t}{\sqrt{2}\sigma})$$



# SPE signal characterization

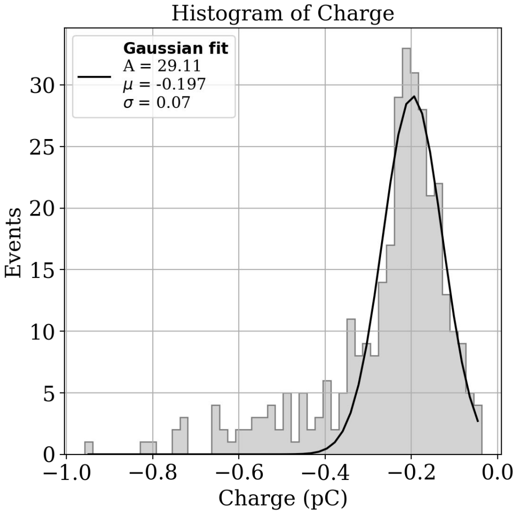
## Characterization procedure



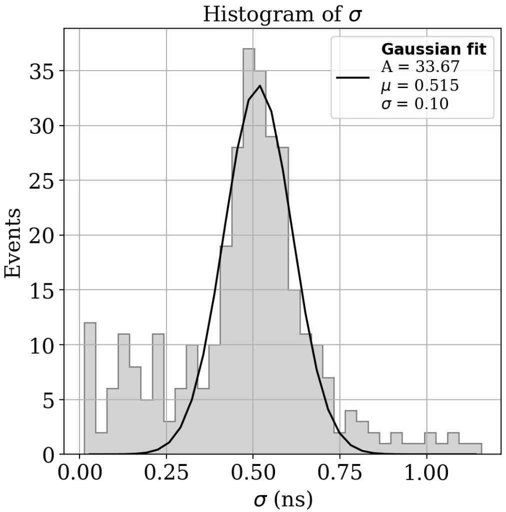

$$v(t) = QR\,\frac{\lambda}{2}\exp(\frac{\lambda}{2}(2\mu + \lambda\sigma^2 - 2t))\ \mathrm{erfc}(\frac{\mu + \lambda\sigma^2 - t}{\sqrt{2}\sigma})$$ → SPE Fit function

# SPE signal characterization

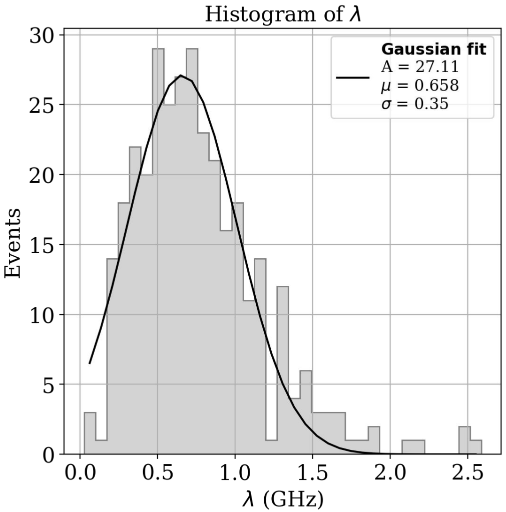
## Histograms




$$Q = (-0.197 \pm 0.069)\ pC$$




$$\sigma = (0.516 \pm 0.010)\ ns$$




$$\lambda = (0.658 \pm 0.355)\ GHz$$

# SPE signal characterization

## Histograms

Since we estimated the pulse shape parameters, we can change the Gain in the simulation to match with the PMTs calibration

→

PMT 1: **772 V**
PMT 2: **772 V**
PMT 3: **800 V**
PMT 4: **772 V**

We also know:

$$Q = G \cdot N \cdot (-e)$$

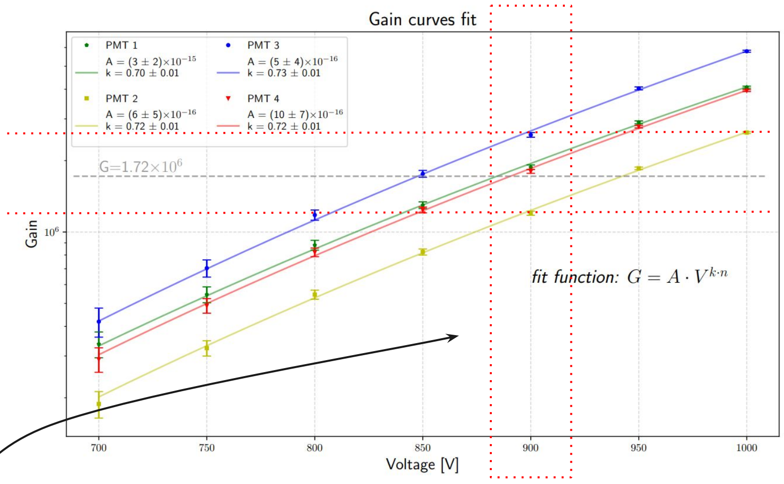
$$N = 1$$

$$e = 1.6 \times 10^{-19}\ C$$

Replacing Q by the found SPE charge:

$$-0.197 \times 10^{-12} = G \cdot 1 \cdot (-1.6 \times 10^{-19})$$

$$G = 1.23 \times 10^{6}$$



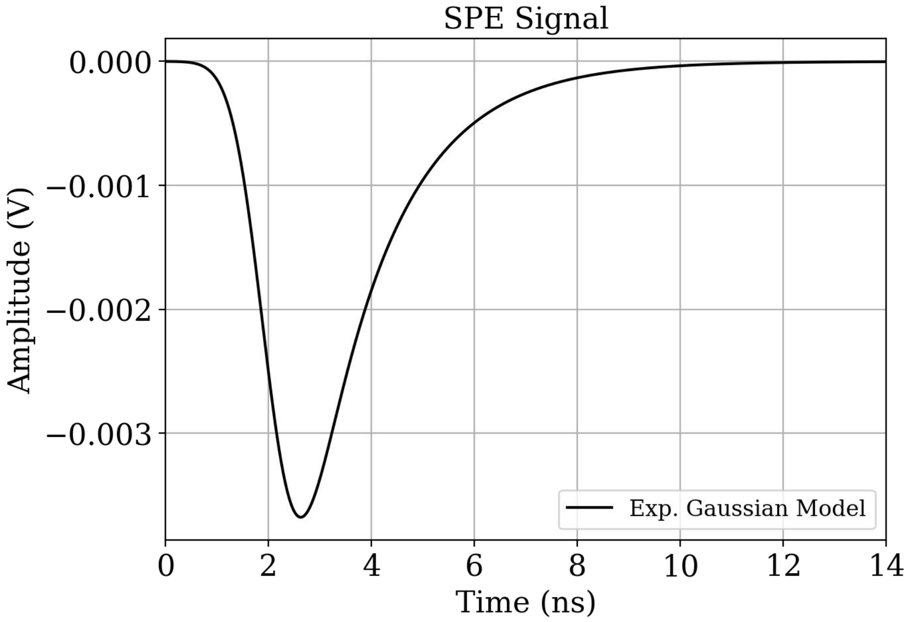

Martini, P. and Migliorati, D. and Muco, G. and Zappaterra, L., Physics Laboratory II report, CYGNO LAB GROUP, July 2022.

SPE Dataset: .../PMT-Test-270922/BA1642_single_photoelectron —> C4--1642-**900V**-A3

# SPE signal characterization

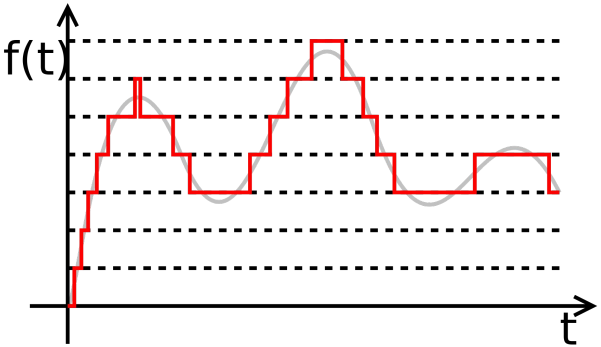
## Single photoelectron signal model

PMT R7378A
900 V of supply voltage



# Waveforms quantization

## Until this year we were not applying quantization to the simulated waveforms




Amplitude discretization of a continuous signal

| Digitizer | Module | # Samples | Sample Rate | ADC |
|---|---|---|---|---|
| Fast | CAEN v1742 | 1024 | 750 MS/s | 12 bit |
| Slow | CAEN v1720 | 4000 | 250 MS/s | 12 bit |

$$Resolution = \frac{1}{2^{12}} = \frac{1}{4096} V$$

# Waveforms quantization

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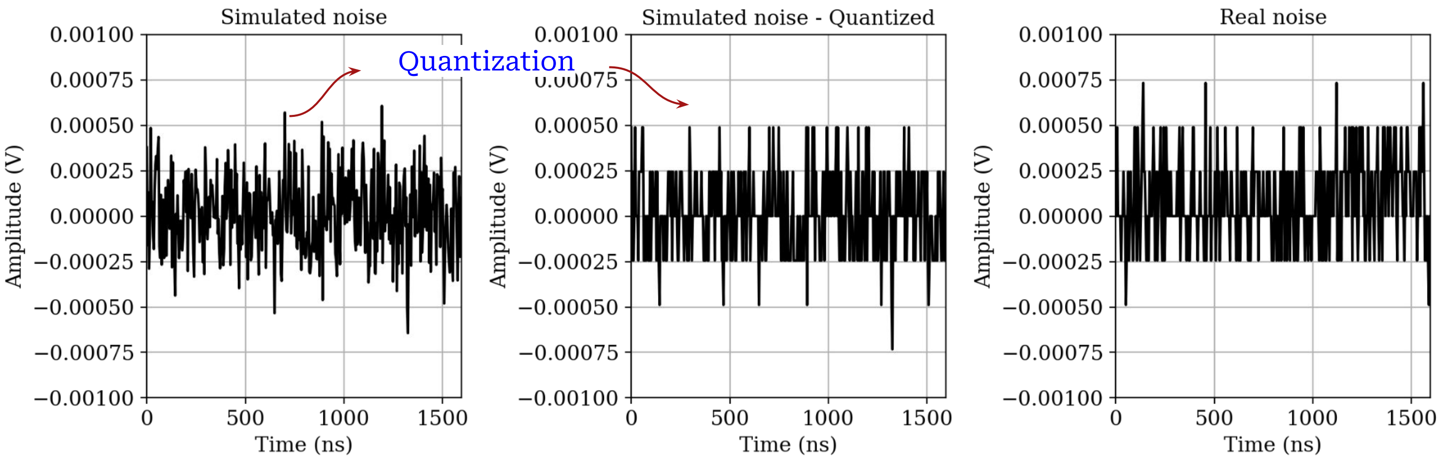
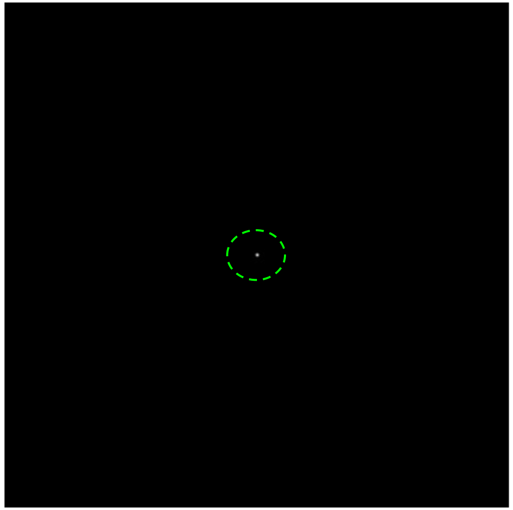



*Example of the quantization effect on a slow digitizer simulated noise*

# Digitization - PMT Simulation

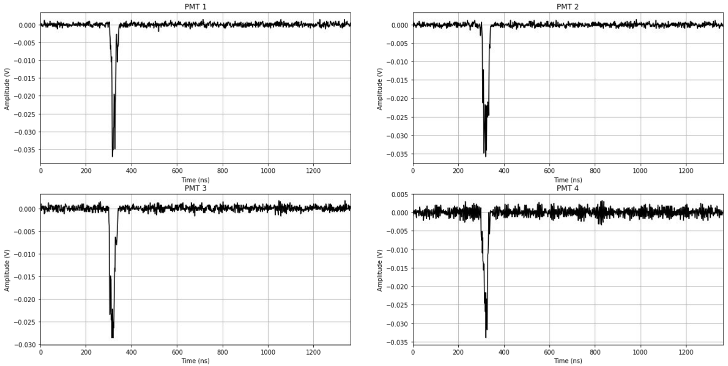
## Simulation example 1

### Image + PMT Waveforms

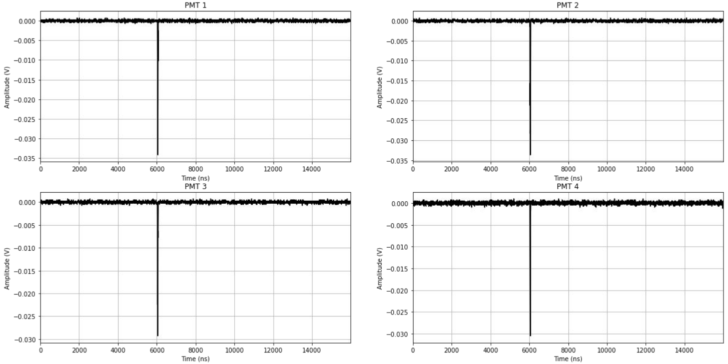


6 keV event simulation

***Fast digitizer***



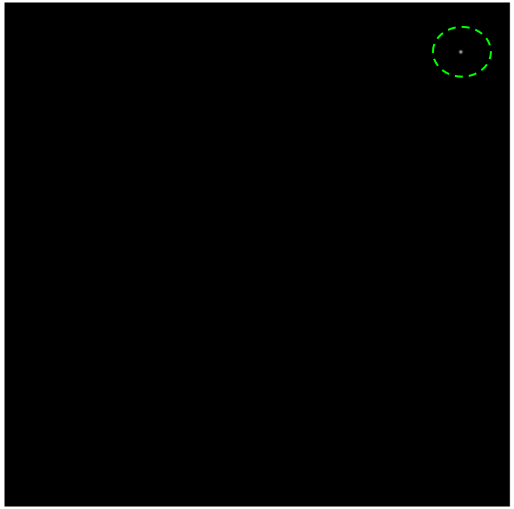

***Slow digitizer***



# Digitization - PMT Simulation

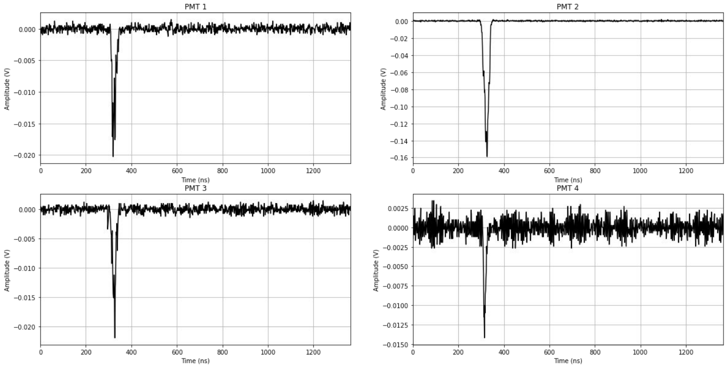
## Simulation example 2

### Image + PMT Waveforms

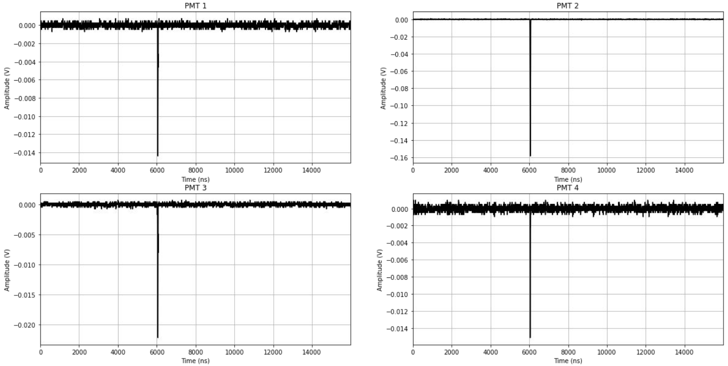


6 keV event simulation

***Fast digitizer***




***Slow digitizer***



# Sim/data comparison

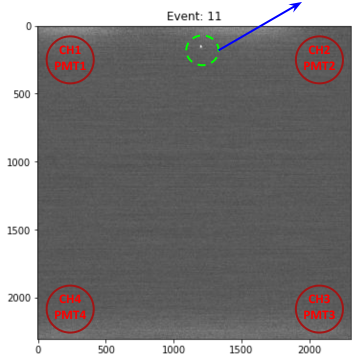
## 3D analysis (x, y, z) - 55Fe dataset

- Based on runs of 55Fe source
- Selected just events with one cluster
- Use PMT + camera reco for data analysis

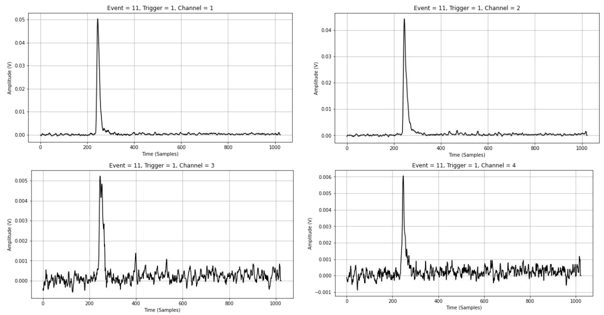
Image

Waveform

Get x-y position




Get data from waveforms analysis




- **Run 27316: Step 1:** 5.0 cm
- **Run 27317: Step 2:** 15.1 cm
- **Run 27318: Step 3:** 25.1 cm
- **Run 27319: Step 4:** 35.1 cm
- **Run 27320: Step 5:** 46.6 cm

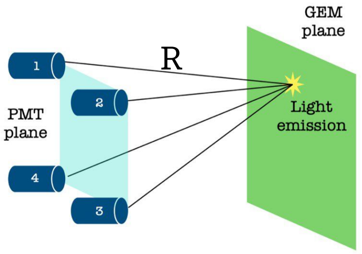
**55Fe runs - Daily calibration**

(runs with one cluster images)

# Sim/data comparison

## 3D analysis (x, y, z) - 55Fe dataset

### Results in function of the R distance (spot at GEM plane to PMT)




$$Z_{PMT} = 186\ mm$$

$$PMT_1:\ X_1 = 42\ mm,\ Y_1 = 312\ mm$$

$$PMT_2:\ X_2 = 312\ mm,\ Y_2 = 312\ mm$$

$$PMT_3:\ X_3 = 312\ mm,\ Y_3 = 42\ mm$$

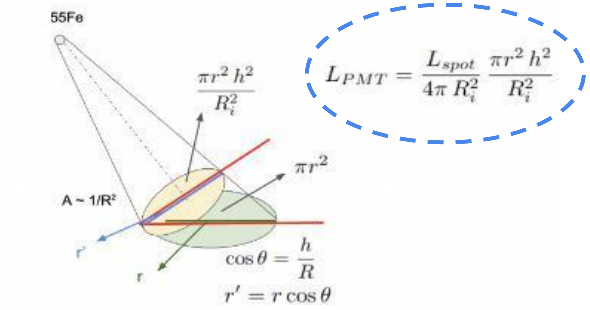
$$PMT_4:\ X_4 = 42\ mm,\ Y_4 = 42\ mm$$

$$R = \sqrt{(X_{PMT} - x_0)^2 + (Y_{PMT} - y_0)^2 + (Z_{PMT} - z_0)^2}$$

$$x_0, y_0 = spot\ position$$

$$z_0 = 0\ (GEM\ plane)$$

### Photon propagation method




Following with $\mathbf{R}^{-4}$

# Sim/data comparison

## 3D analysis (x, y, z) - 55Fe dataset

**Parameters to be verified**

**Peak**

$$max(v(t))$$

**Charge**

$$Q = \frac{1}{R} \int_{t_0}^{t_1} v(t)\, dt$$

$$RMS_{noise}$$

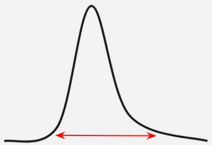
$$std(v(t)[0:100])$$

**Signal-to-noise ratio**

$$SNR = \frac{Peak}{RMS_{noise}}$$

**FWHM**

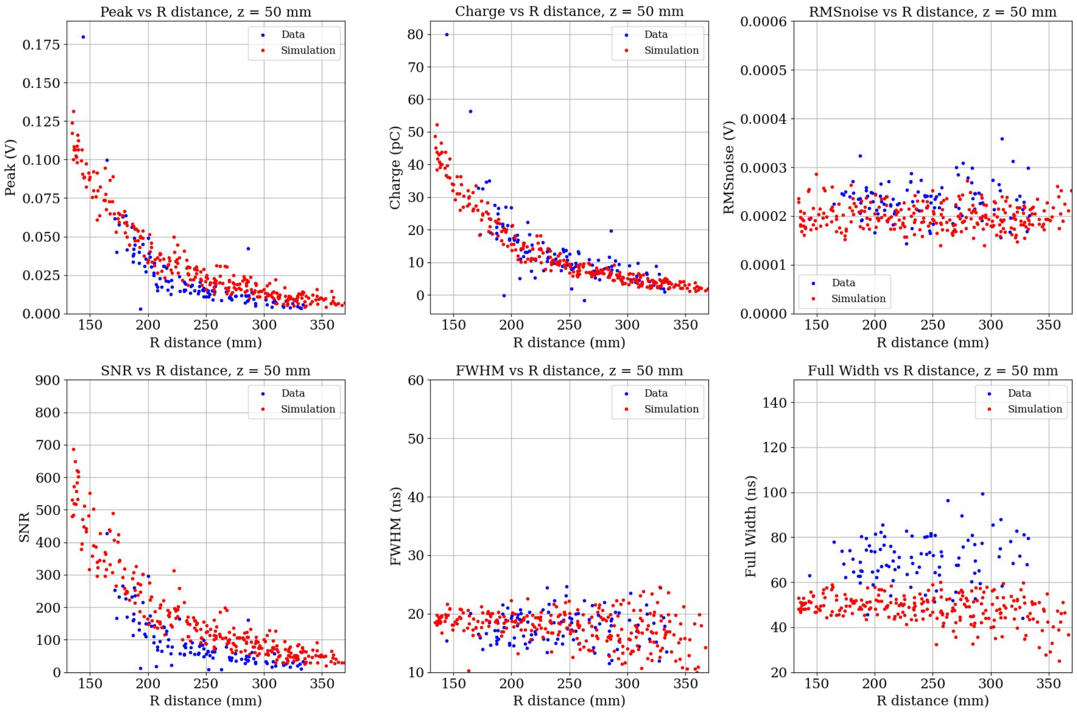
**Full width**



# Sim/data comparison

## 3D analysis (x, y, z) - 55Fe dataset

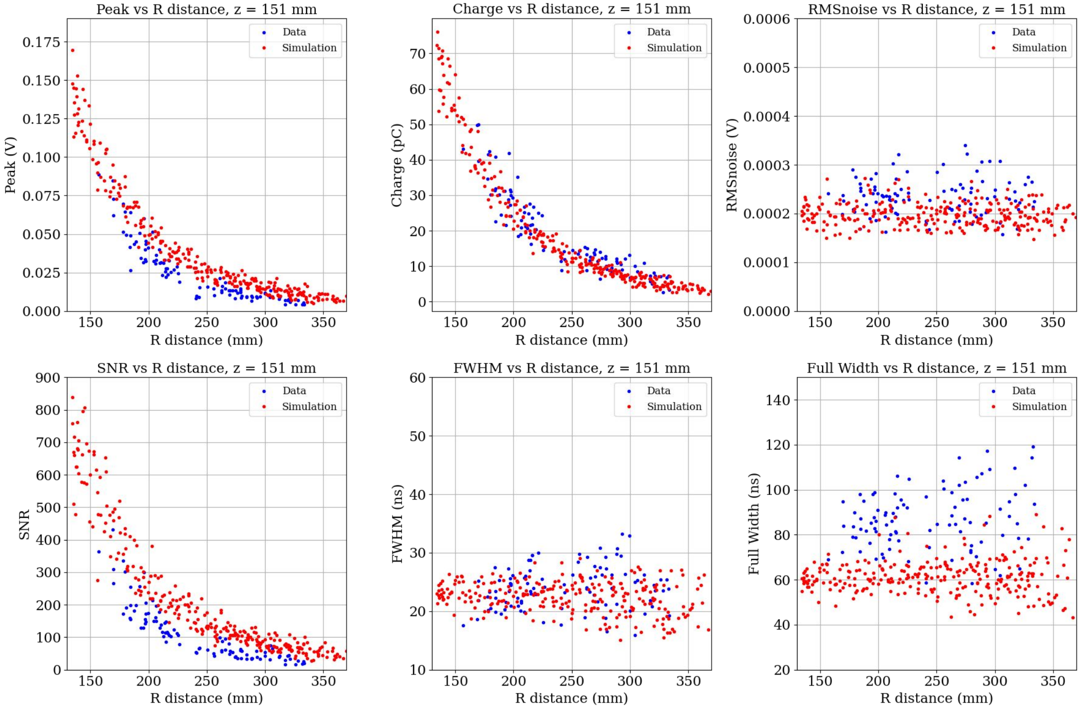
Scatter Plot - **Step 1 = 50 mm**



# Sim/data comparison

## 3D analysis (x, y, z) - 55Fe dataset

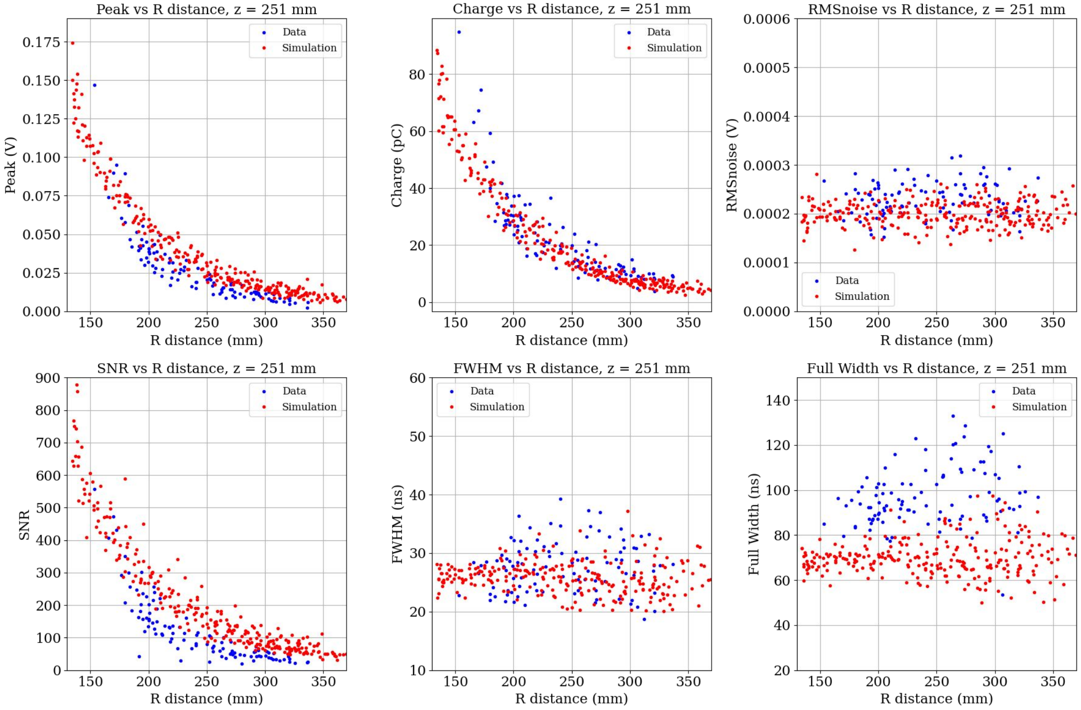
Scatter Plot - **Step 2 = 151 mm**



# Sim/data comparison

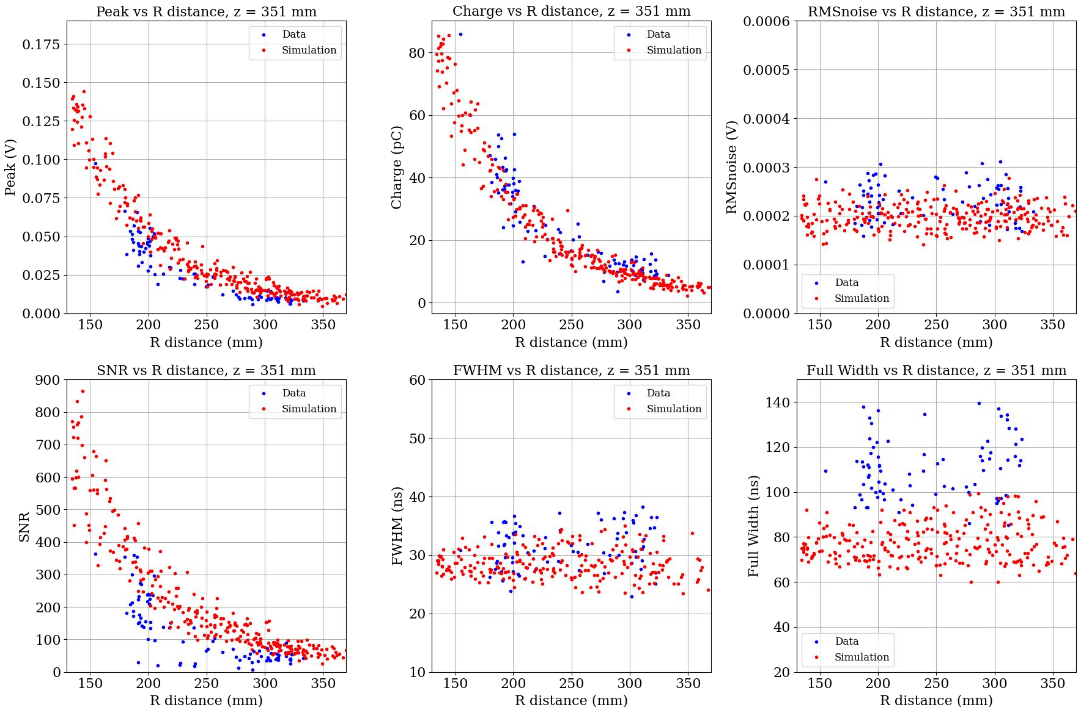
## 3D analysis (x, y, z) - 55Fe dataset

Scatter Plot - **Step 3 = 251 mm**



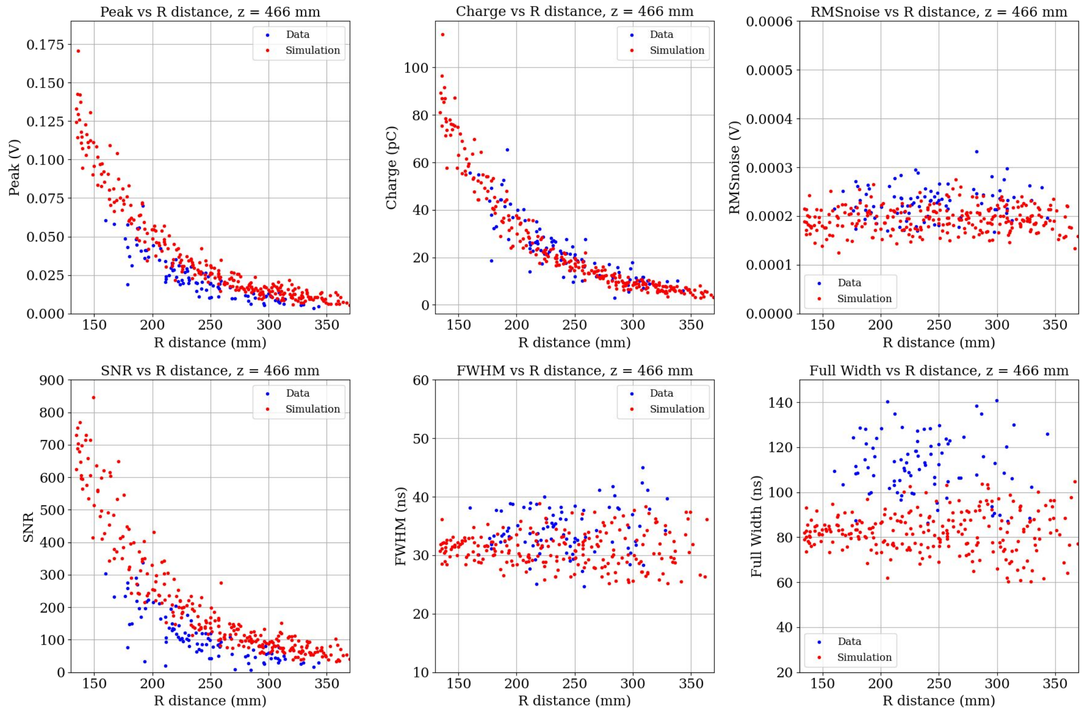
# Sim/data comparison

**3D analysis (x, y, z) - 55Fe dataset** Scatter Plot - **Step 4 = 351 mm**



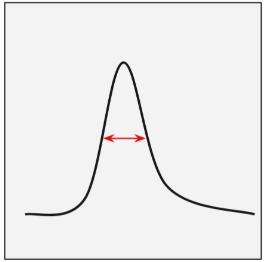
# Sim/data comparison

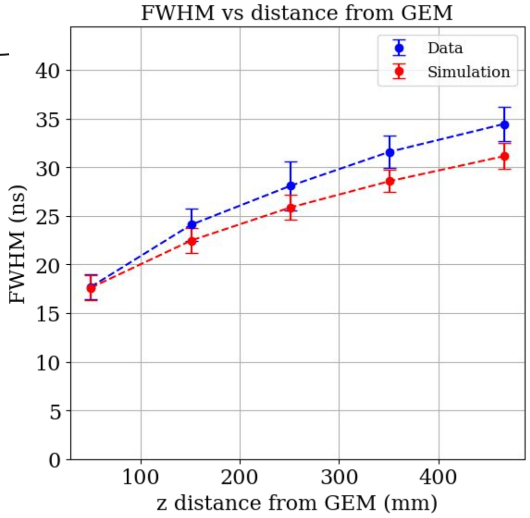
**3D analysis (x, y, z) - 55Fe dataset** Scatter Plot - **Step 5 = 466 mm**

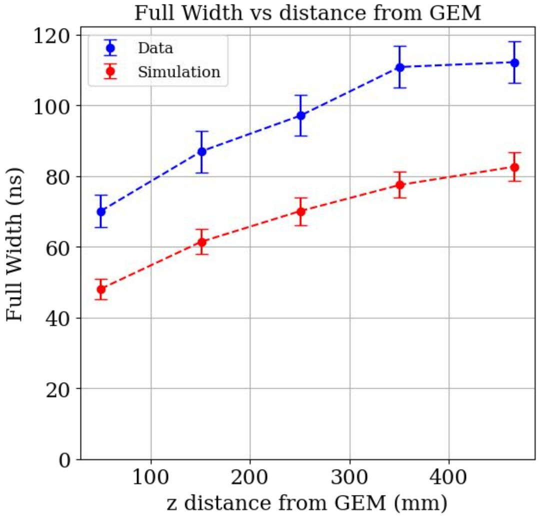


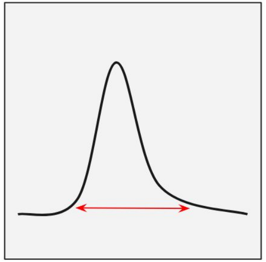
# Sim/data comparison

## 3D analysis (x, y, z) - 55Fe dataset





# Conclusion

## Completed tasks

- Analysis of spectrum (glass transmission, PMT quantum efficiency)
- Photon propagation
- PMT Signal characterization
  - SPE signal characterization
  - Noise characterization
    - Characterization for each PMT/channel
- Simulation for **fast** and **slow** digitizers
- Sim/data comparison
  - 55Fe dataset

## Future Plans

- Find exponential dispersion to fix full width
- Translate PMT Simulation code: Python → C++
- Simulate different tracks with different energies
  - 241-Am data
- Keep tuning the parameters of the simulation