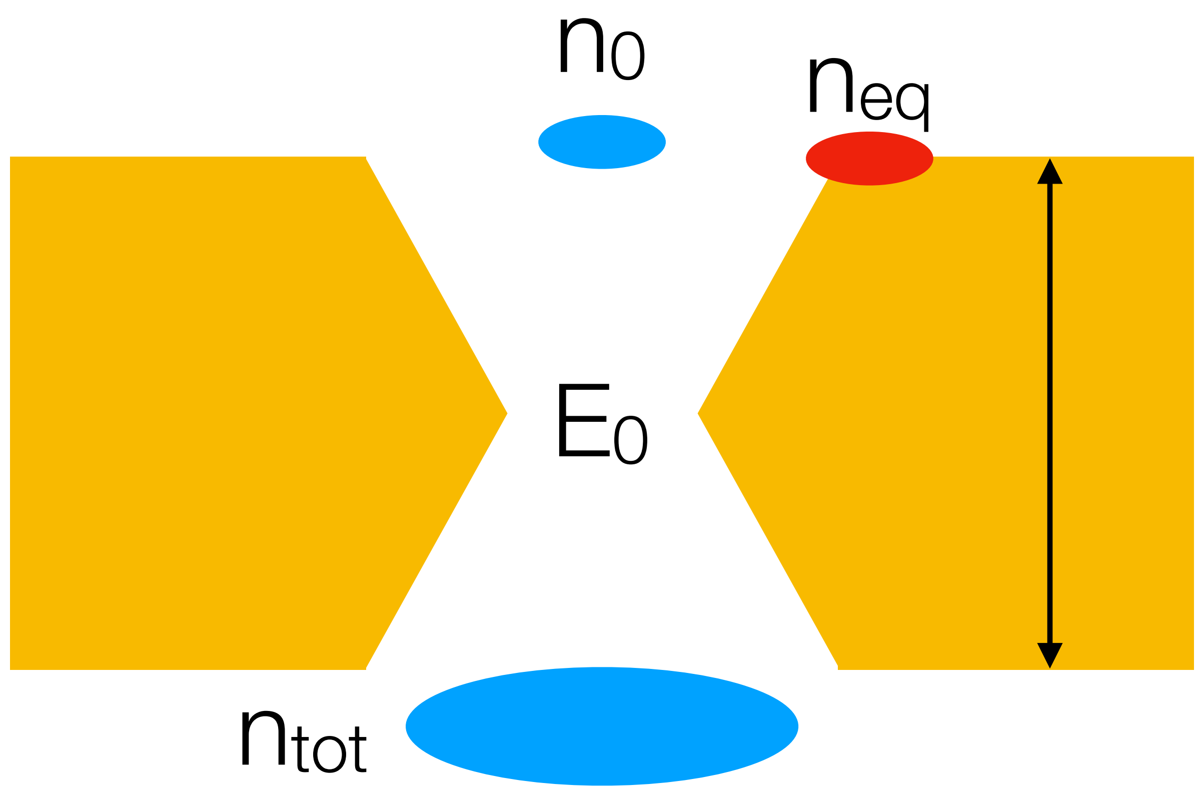
Saturation of the GEM gain

A simple model

Let's suppose that during the development of the avalanche within the gem multiplication channels a significant amount of electrons and positive ions are produced.

Under the effect of the electric field present in the channel, these slowly migrate toward the lower potential plane of the GEM, tending to partially shield the field itself.

If n_0 is the number of electrons entering a GEM channel and $E_0 \propto V_{GEM}/d$ the electric field in it:



d is the GEM thickness

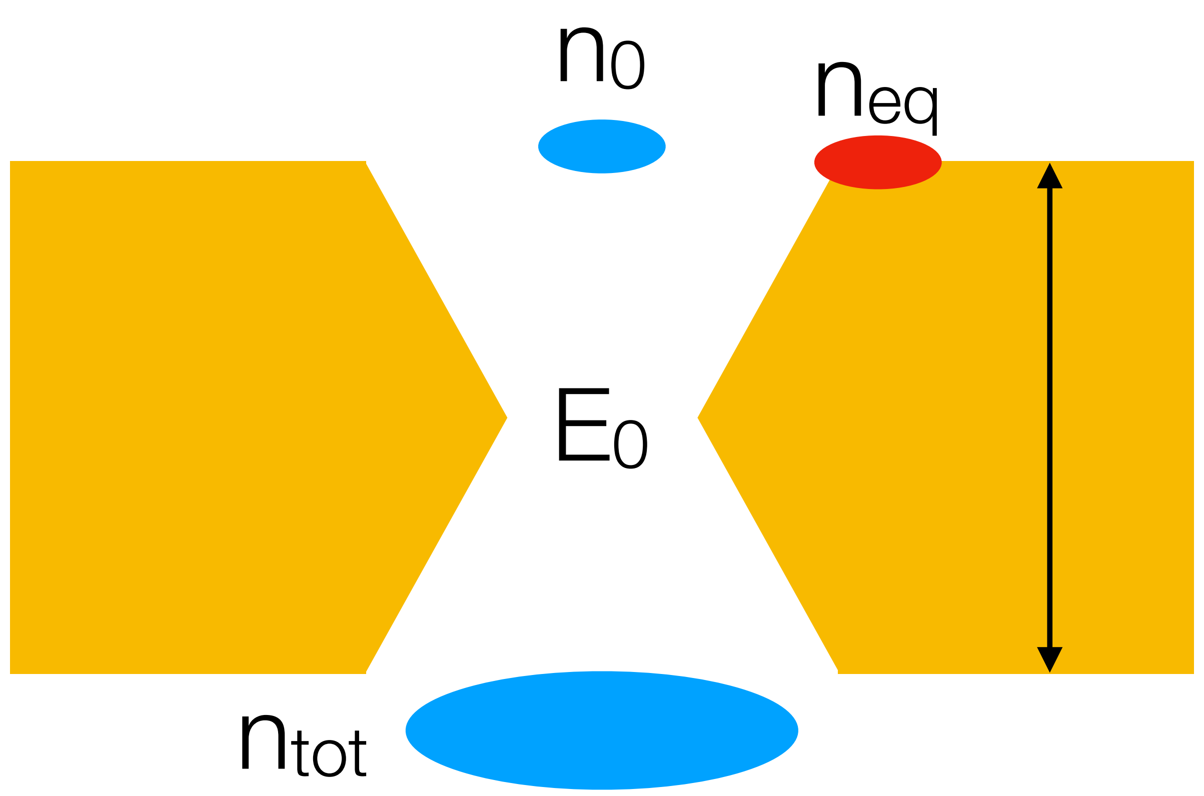
V_{GEM} is the voltage drop between GEM sides

Multiplication is described by a modified Townsend equation $$\frac{dn}{ds} = \alpha E_0(1 - \beta n)n$$

where β is can be interpreted as the inverse of the number of charges $\beta = 1/n_{eq}$ present on the GEM border of the channel and needed to produce E_0 in it ($\beta \propto 1/V_{GEM}$);

A simple model

$$\frac{dn}{ds} = \alpha E_0(1 - \beta n)n$$



d is the GEM thickness

V_{GEM} is the voltage drop between GEM sides

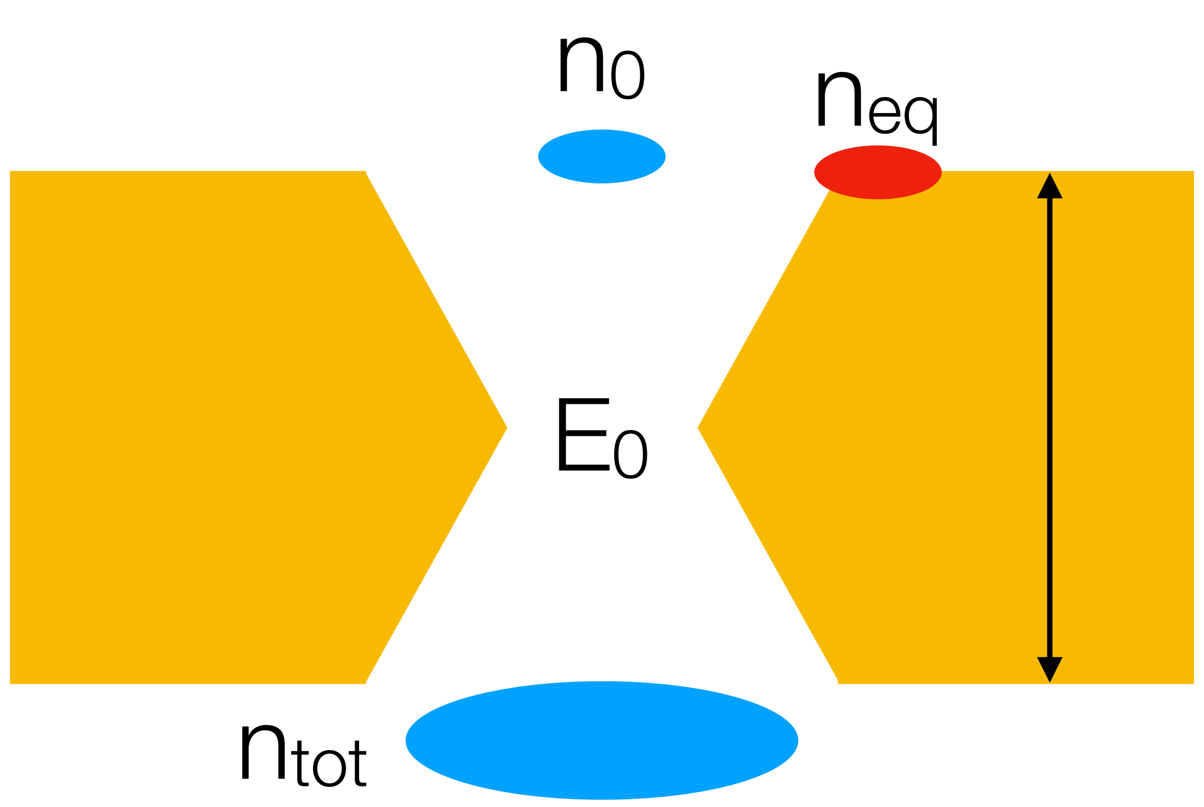
$$\int_{n_0}^{n_{tot}} \frac{dn}{(1 - \beta n)n} = \int_0^d \alpha E_0 ds$$

$$G = \frac{e^{\alpha V_{GEM}}}{1 + \beta n_0(e^{\alpha V_{GEM}} - 1)}$$

where $G=n_{tot}/n_0$ is the average gain of the single channel, while $g = e^{\alpha V_{GEM}}$ can be interpreted as the non-saturated gain one can obtain when the screening effect β is negligible

A simple model

$$\frac{dn}{ds} = \alpha E_0(1 - \beta n)n$$



d is the GEM thickness

V_{GEM} is the voltage drop between GEM sides

In fact we know that for $V_{GEM} << V_{thr}$, avalanches do not happen and the model doesn't work

We can then write the not-saturated gain as

$$g = e^{\alpha(V_{GEM} - V_{thr})} = ce^{\alpha V_{GEM}}$$

with $V_{thr} = -\dfrac{ln(c)}{\alpha}$

$$G = \frac{ce^{\alpha V_{GEM}}}{1 + \beta n_0(ce^{\alpha V_{GEM}} - 1)}$$

The meaning of n_0

To fix ideas let us now assume that after a drift over a path z drift and the multiplication process in the first 2 GEMs (GEM#1 and GEM#2) the electron cloud has a distribution in space describable as a Gaussian in 3 dimensions all with RMS equal to σ:

the total volume will then be approximately proportional to σ^3 and the amount of charge collected by each channel will decrease as $1/\sigma^3$

In the last GEM, the amount of charge collected by each channel n_0:

- Increases with the primary ionisation in the gas n_e;
- decreases as $1/\sigma^3$;
- increases as the product of the gains of G_1 and G_2

$$n_0 \propto n_e G_1 G_2 / \sigma^3$$

The meaning of n_0

$$G = \frac{ce^{\alpha V_{GEM}}}{1 + \beta n_0 (ce^{\alpha V_{GEM}} - 1)}$$

If n_0 is the number of electrons entering a GEM channel

$$n_0 = \frac{n_e G_1 G_2}{4/3\pi(3\sigma[\mu m])^3 \times 5 \cdot 10^{-5}/\mu m \times 1/50\mu m} = \frac{n_e G_1 G_2 \cdot 10^4}{\sigma^3}$$

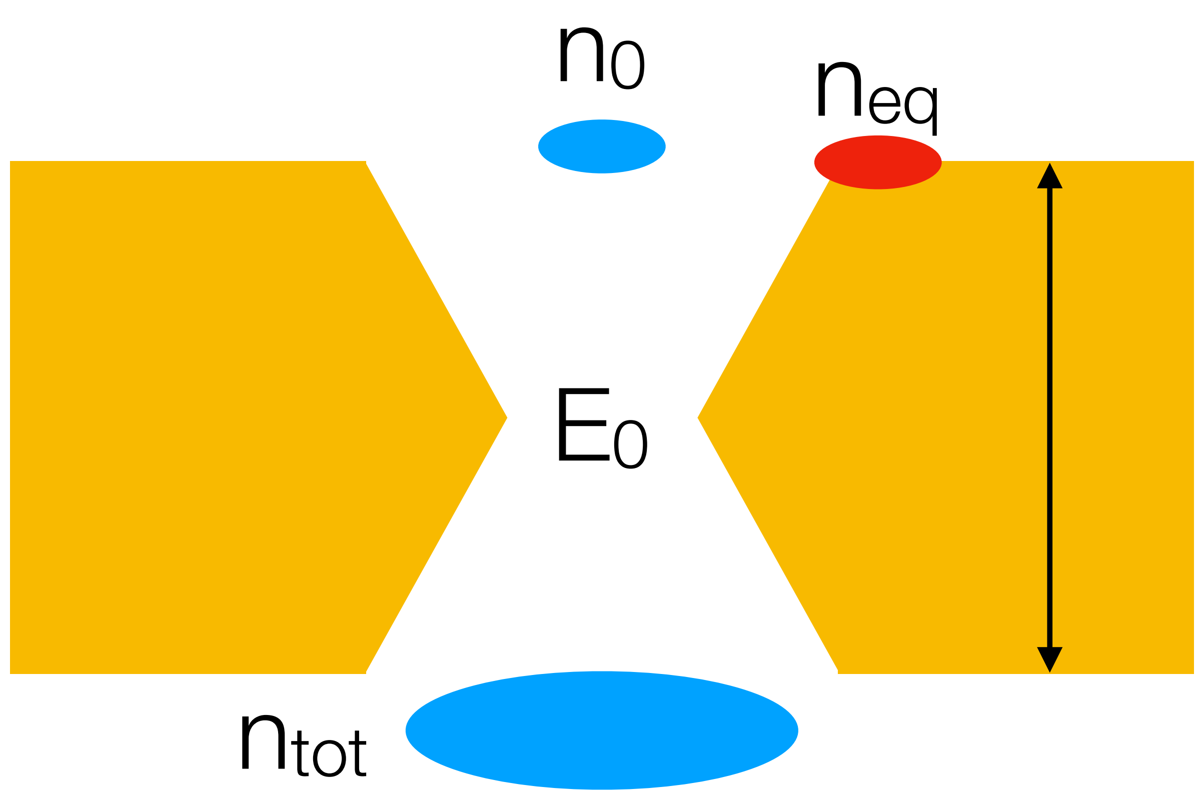
Number of electrons entering the channel

Volume of the sphere containing the electrons exiting from GEM2

channels surface density

GEM channel thickness

The meaning of β



d is the GEM thickness

V_{GEM} is the voltage drop between GEM sides

If one writes β as $1/n_{eq}$ then

- if $n_0 \ll n_{eq} \rightarrow \beta n_0 \simeq 0$ negligible screen effect;
- if $n_0 \simeq n_{eq} \rightarrow \beta n_0 \simeq 1$ i.e. total screen effect;

$$G = \frac{e^{\alpha V_{GEM}}}{1 + \beta n_0 (e^{\alpha V_{GEM}} - 1)}$$

therefore n_{eq} can be interpreted as the number of charges present on the GEM border of the channel and needed to produce E_0 in it

Since the total charge accumulated on the GEM is $Q_{GEM} = C_{GEM} \cdot V_{GEM} \rightarrow \beta \propto 1/V_{GEM}$

A simple model

Let's suppose that only in GEM#3 we have non linear gain because of the larger amount of charges.

$$G_{tot} = \frac{G_1 G_2 c e^{\alpha V_{GEM}}}{1 + \beta n_0 (c e^{\alpha V_{GEM}} - 1)} = \frac{G_1 G_2 G_3}{1 + (p_1/V_{GEM}) G_1 G_2/\sigma^3 \ (G_3 - 1)}$$

If $G_1 = G_2 = G_3 = ce^{\alpha V} = p_0$

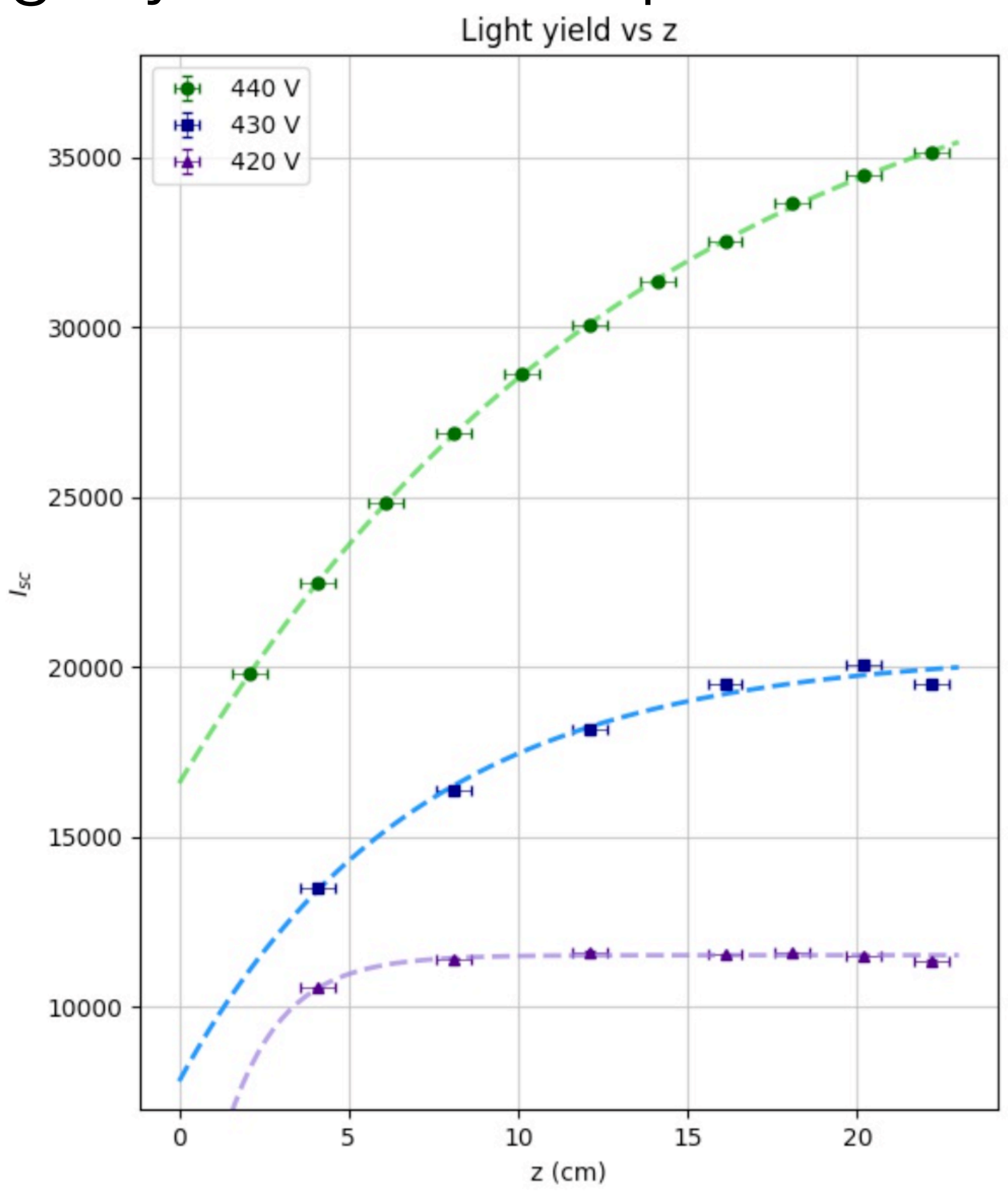
$$G_{tot} = \frac{p_0^3}{1 + (p_1/V_{GEM}) p_0^2/\sigma_3 (p_0 - 1)} = \frac{p_0^3 \sigma^3}{\sigma^3 + (p_1/V_{GEM}) p_0^2 (p_0 - 1)}$$

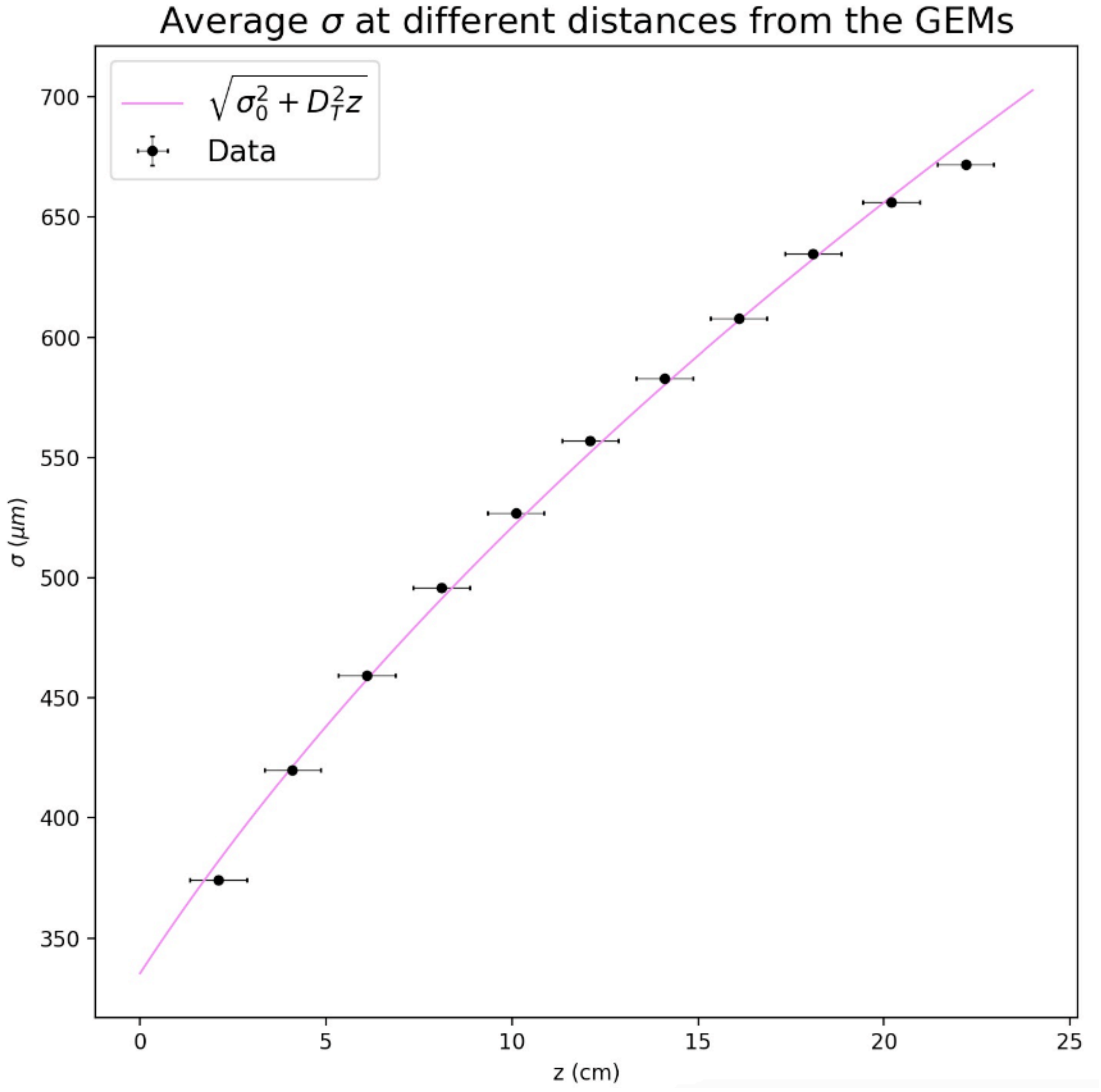
- We can try to fit this last function on the data expecting:
  - p_0 to be the not-saturated gain of the three GEMS;
  - p_1 is constant

A simple model

- From the GIN data we can evaluate the electron gain in 3 different V_{GEM} setup (440, 430 and 420) and the behavior of σ

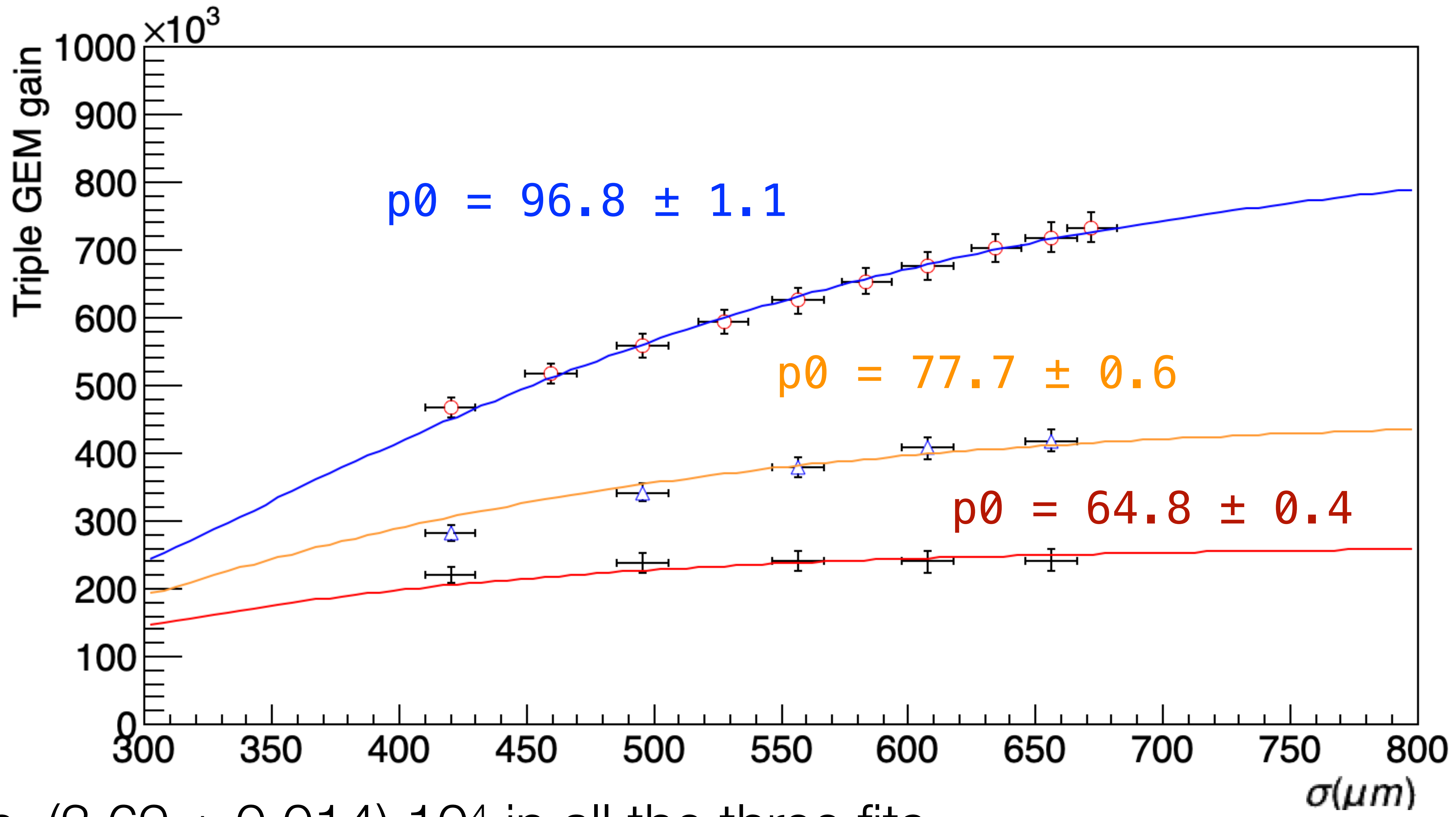
- We can start from the light yield for ^{55}Fe spots





- The electron gain is evaluated by taking into account 0.07 γ/e, 150 n_e and $\Omega = 9.2\text{x}10^{-4}$

A simple model - 2 steps fit



We can fit the behavior

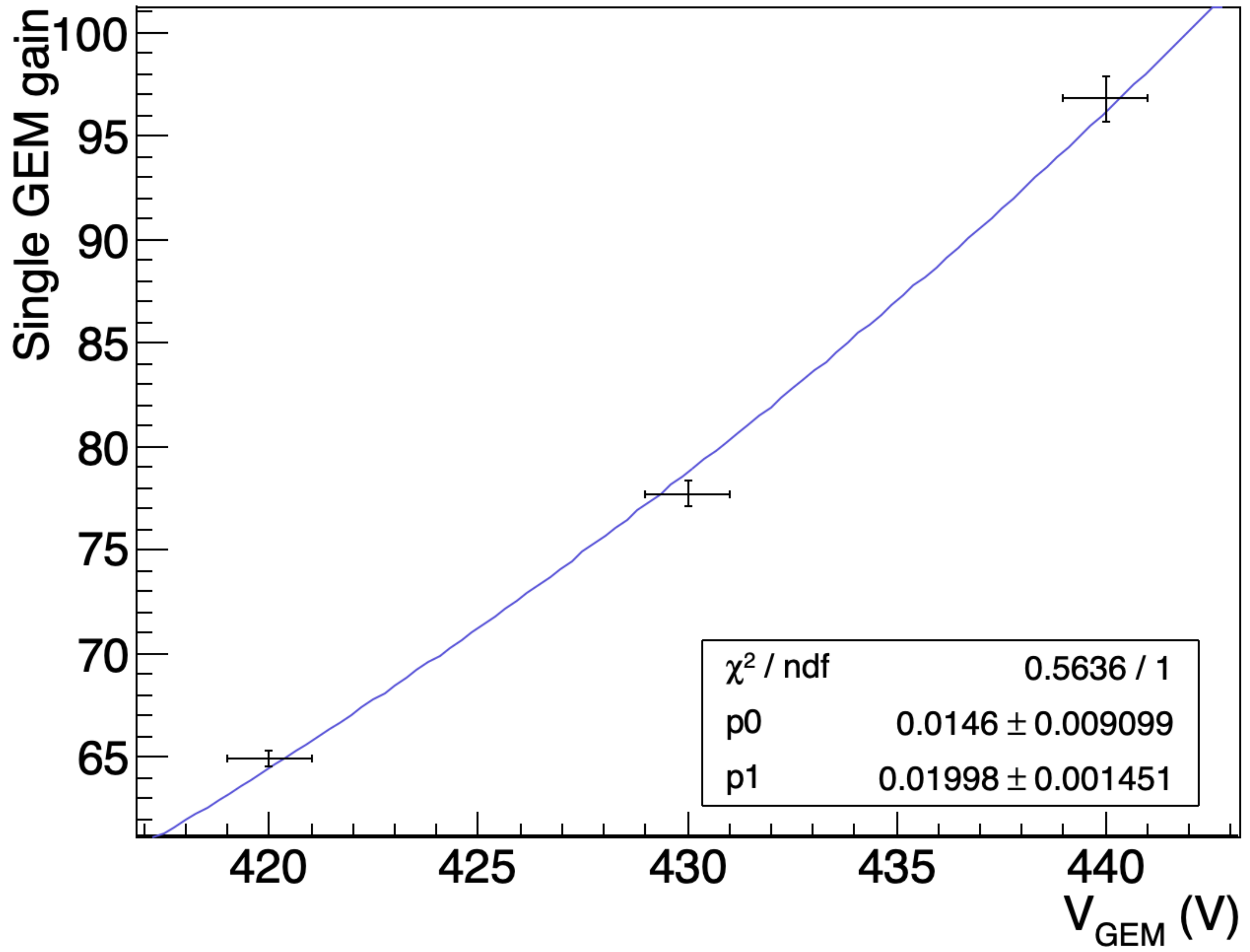
$$G_{tot} = \frac{p_0^3\sigma^3}{\sigma^3 + (p_1/V_{GEM})p_0^2(p_0 - 1)}$$

Where p_0 is the single GEM non-saturated gain;

A Minuit simultaneous fit was performed with 4 parameters: the three not saturated gains and a common p_1;

p_1 is $(3.69 \pm 0.014)\ 10^4$ in all the three fits

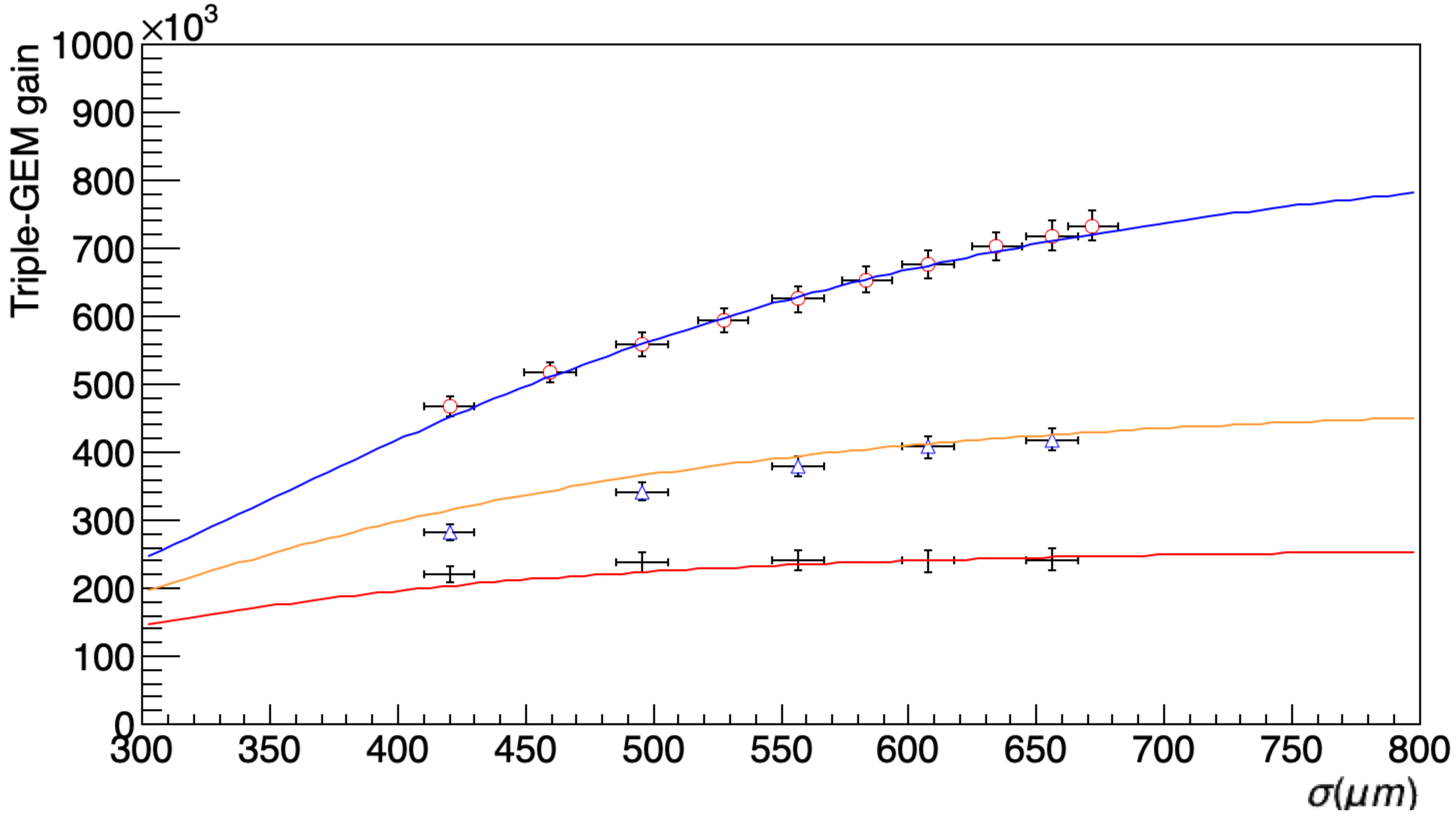
Identical results obtained with Minuit2



By fitting $\boldsymbol{p_0}$ vs V_{GEM} ($p_0 = ce^{\alpha V_{GEM}}$) we evaluate a negligible constant term and $\alpha = 0.020 \pm 0.001$

pValue = 0.67 x 0.48 = 0.32

A simple model - 1 step fit



We can fit the behavior

$$G_{tot} = \frac{p_0^3 \sigma^3}{\sigma^3 + (p_1/V_{GEM}) p_0^2 (p_0 - 1)}$$

Where p_0 is the single GEM non-saturated gain and can be expressed as:

$$p_0 = c e^{\alpha V_{GEM}}$$

A Minuit simultaneous fit was performed with 3 parameters:

- a normalisation $c = 0.013 \ \pm \ 0.001$

- $\alpha = 0.020 \ \pm \ 0.001$

- p_1 = $(3.62 \pm 0.014)\ 10^4$

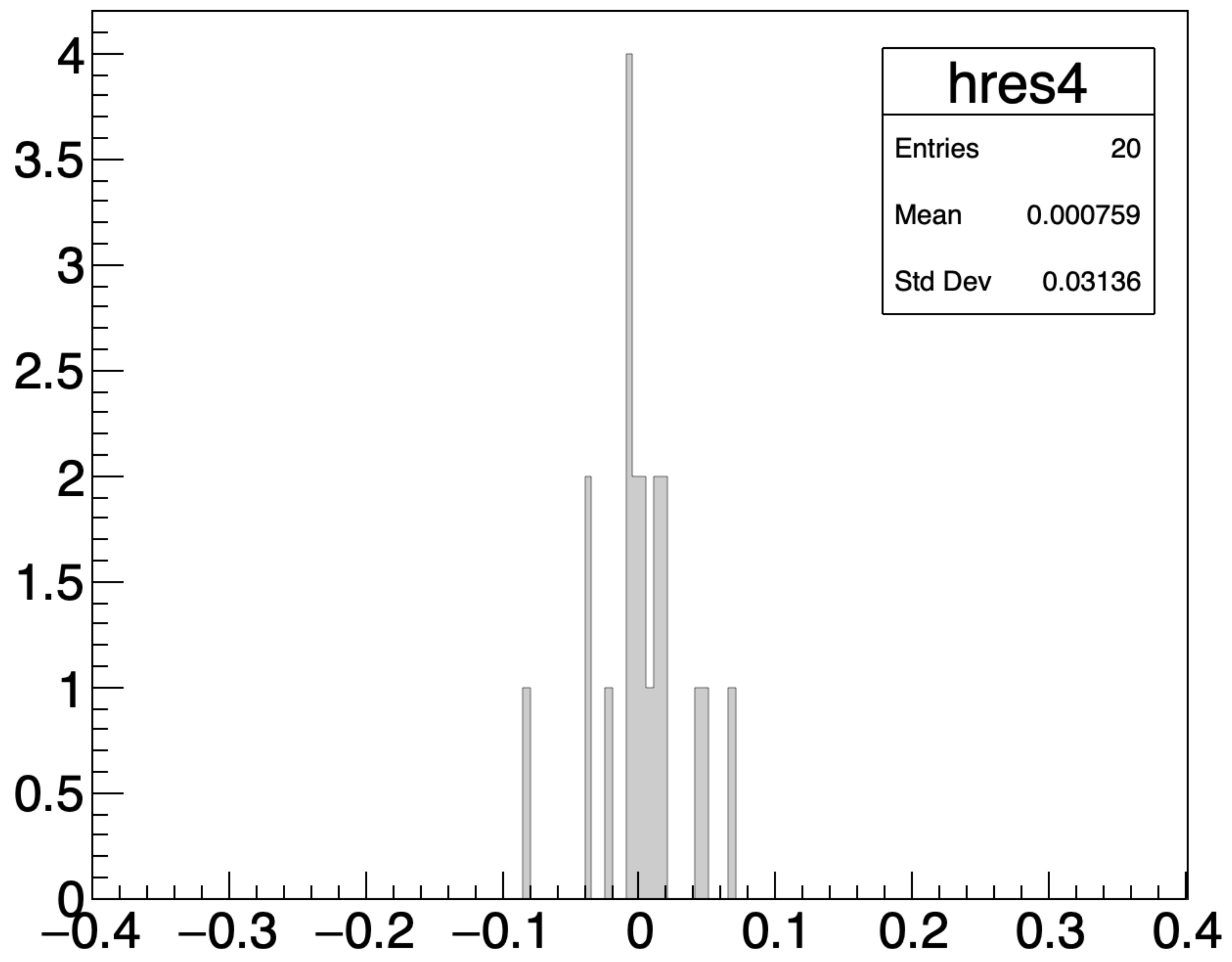
From the value of *c* one can evaluate

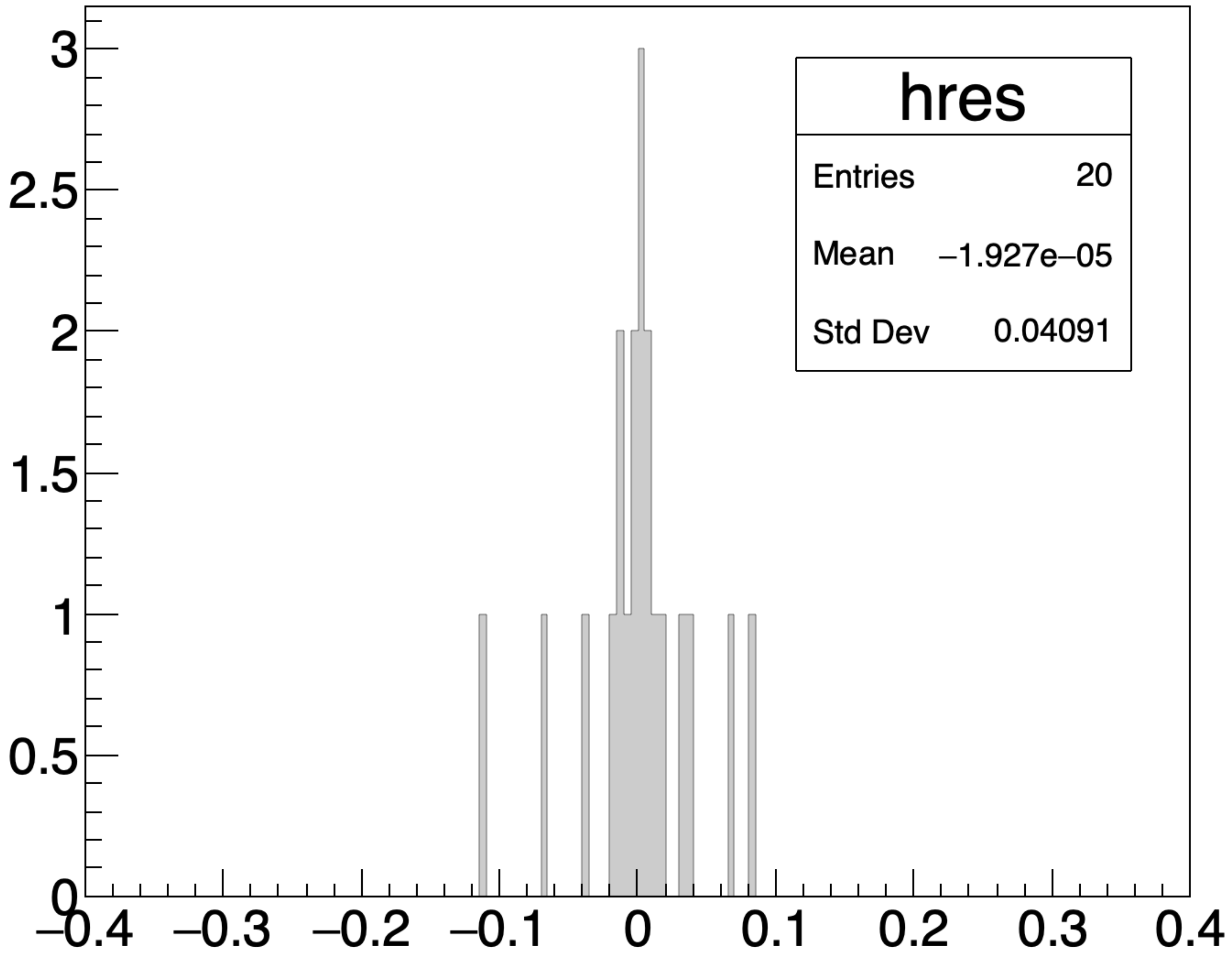
$$V_{thr} = -\frac{ln(c)}{\alpha} \simeq 200 \ V$$

pValue = 0.19

Fit comparison

To compare the fit “goodness” we can evaluate the distribution of the normalised residuals (data-fit)/data



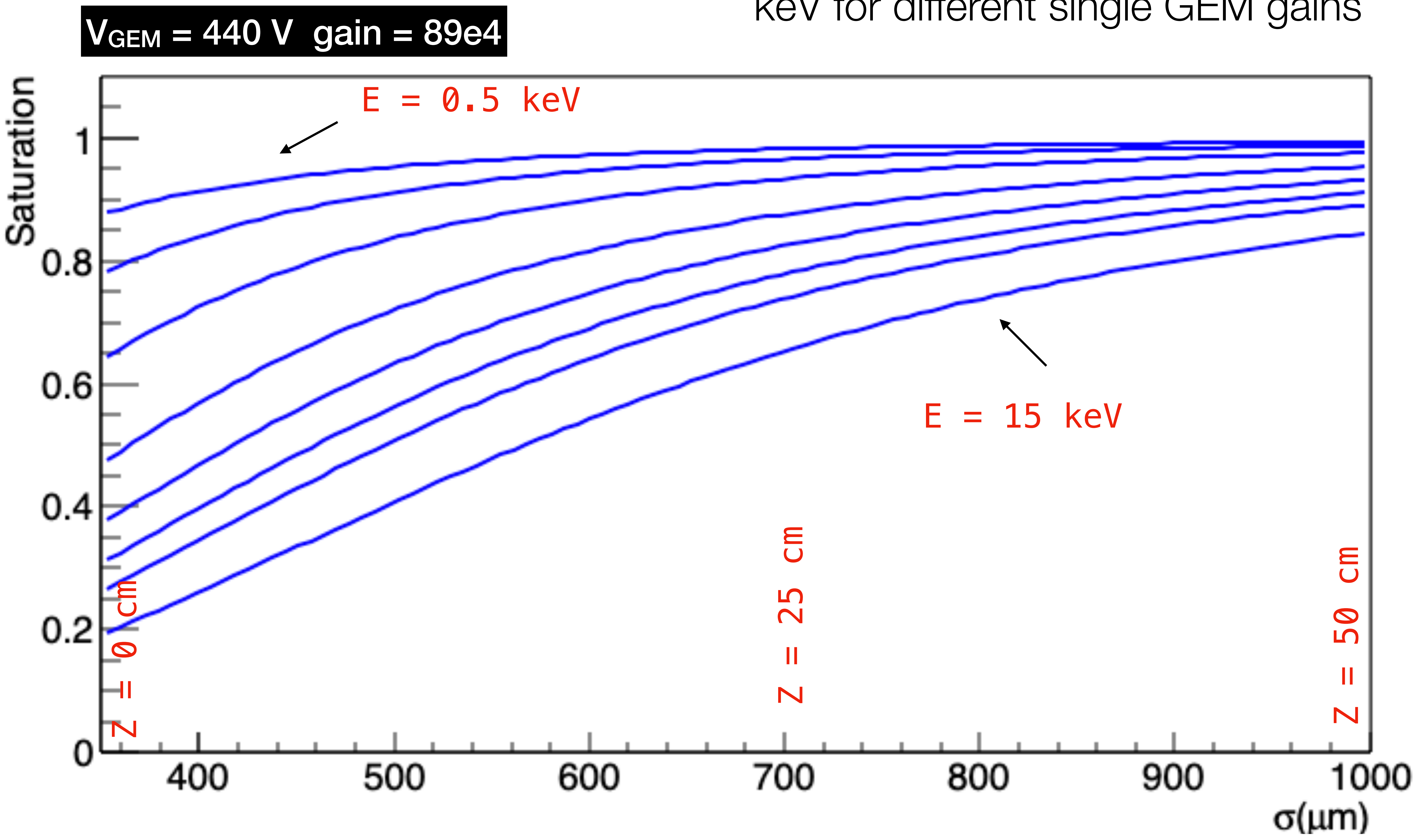


The 2-steps method provide a slightly narrower distribution;

In both cases RMS of residuals is 3-4%, while data span more than 300%

A simple model

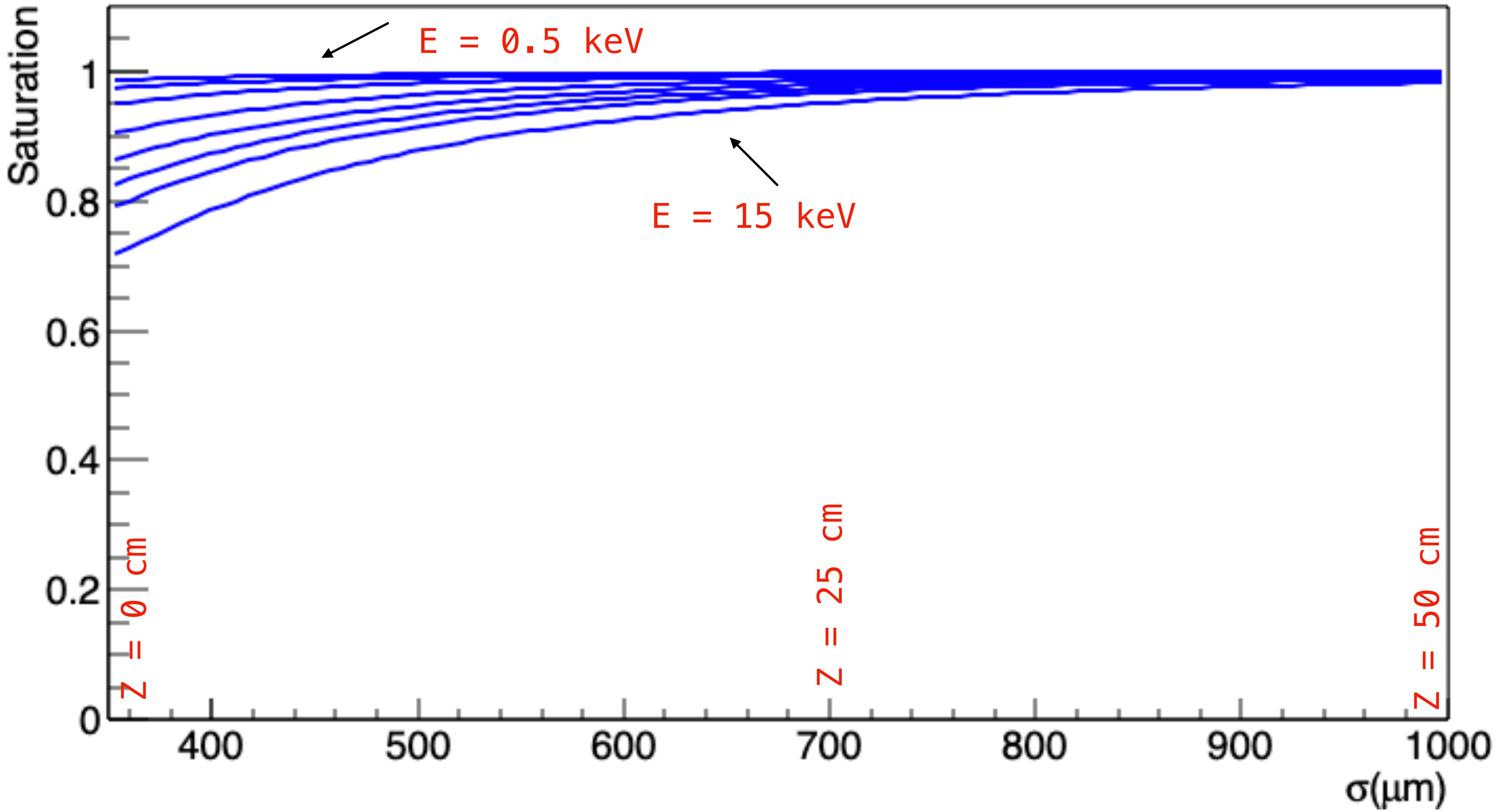
By using the fitted function and the fit results, one can evaluate the saturation as a function of energy released: 15, 10, 8, 6, 4, 2, 1, 0.5 keV for different single GEM gains



A simple model

By using the fitted function and the fit results, one can evaluate the saturation as a function of energy released: 15, 10, 8, 6, 4, 2, 1, 0.5 keV for different single GEM gains

V_{GEM} = 400 V gain = 7.8e4



Conclusion

The gain of the GEM depends on the number of secondaries created within the channels;

Therefore, it depends on:

- ionization density;
- diffusion;
- nominal gain of the channel;

A simple model based on a modified Townsend equation was developed;

It was tested against experimental results;

Even if only average behaviors were taken into account, the model can well describe the data, indicating values for the free parameters in agreement with the ones obtained in the digitization studies where a more accurate dependence on the spacial charge density is taken into account;