High-Quality Axions and Higher-Form Symmetries

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Based on 2408.10295 with **Marius Kongsore**

See also [Reece, 2406.08543]

Part I: Introduction to Higher-Form Symmetries

Ordinary Symmetries

Let's start with a suggestive way of describing a familiar thing:

For a symmetry G with conserved current J^μ, integrate the time component over all space to get the charge Q.
(Think of space as a codimension-1 manifold that splits the spacetime into two pieces)

$$Q(\mathcal{M}_{d-1}) = \int_{\mathcal{M}_{d-1}} d^{d-1}x \, J^\mu(x) \hat{n}_\mu$$

Express G transformations by constructing a unitary operator out of Q: $U_g = e^{i\alpha Q(\mathcal{M}_{d-1})}$ $\qquad g = e^{i\alpha}$

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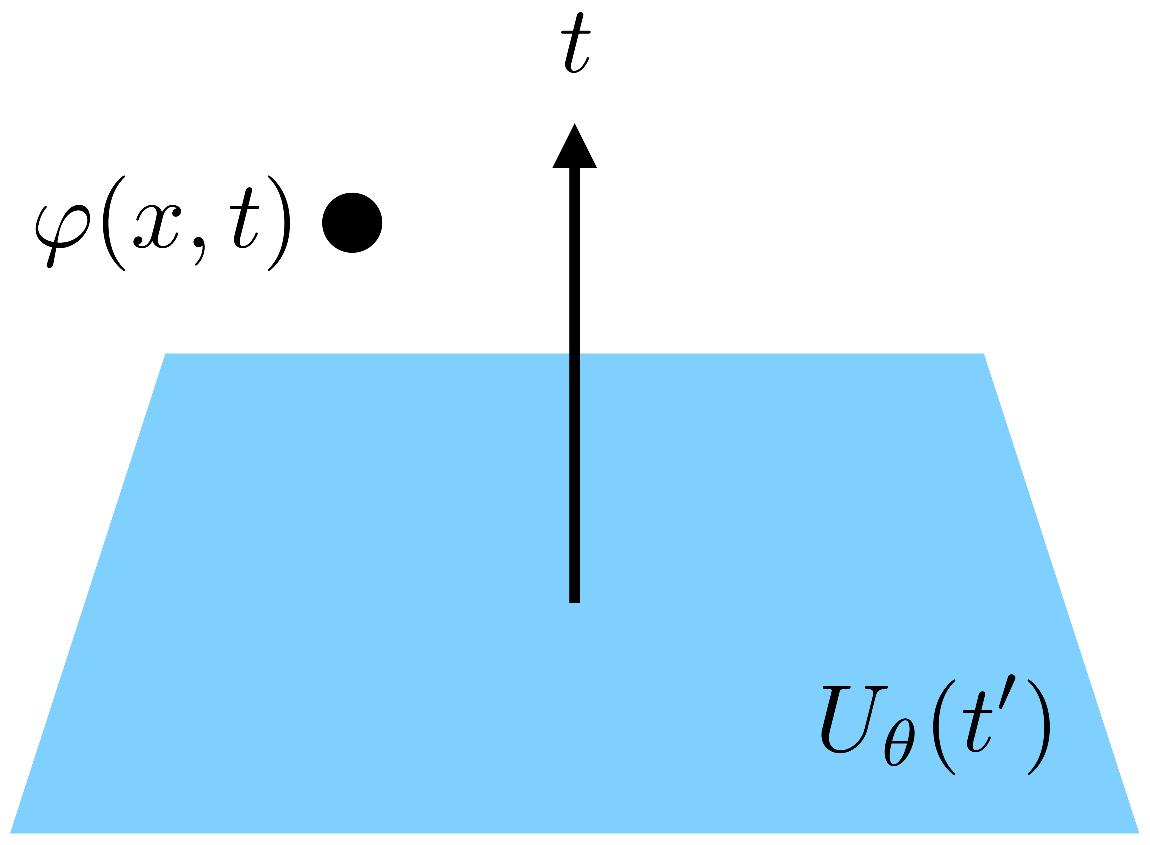
E.g. a 4d complex scalar φ with a U(1) symmetry

$$J^\mu = i(\varphi\partial^\mu\varphi^\dagger - \varphi^\dagger\partial^\mu\varphi) \qquad Q \sim \int d^3x\, J^0$$

The unitary $U_\theta = e^{i\theta Q}$ tells us that the field transforms as

$$\varphi(x) \to U_\theta \varphi(x) U_\theta^{-1} = e^{i\theta}\varphi(x)$$

or equivalently $\quad U_\theta\varphi(x) = [R(\theta)\cdot\varphi(x)]U_\theta$



Ordinary Symmetries

Now generalize by defining the "unitary" on an arbitrary (d-1)-dim manifold ("symmetry defect operator," SDO)

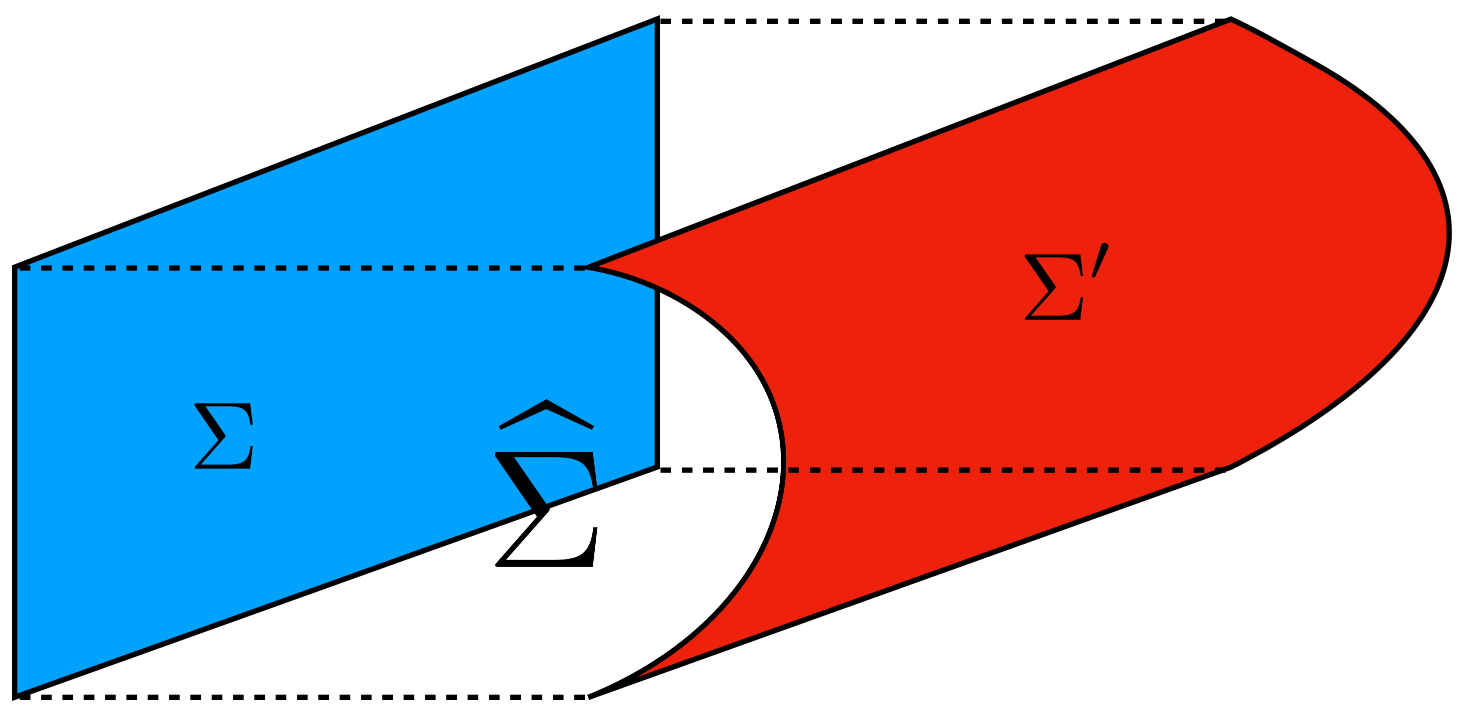
$$U_g(\Sigma_{d-1}) = \exp\left[i\alpha \int_\Sigma d^{d-1}x\, J^\mu(x)\hat{n}_\mu\right]$$

SDOs for conserved global symmetry currents obey nice properties:

- Obey the group multiplication law $U_{g_1}(\Sigma) \cdot U_{g_2}(\Sigma) = U_{g_1 g_2}(\Sigma)$
- Are topological, i.e. invt under smooth deformations

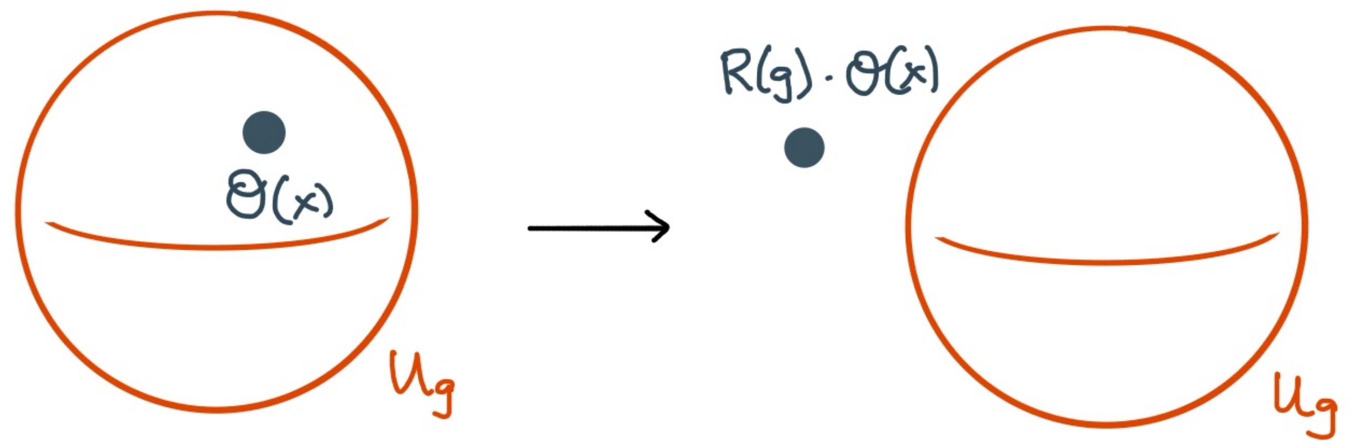
$$U_g(\Sigma_{d-1})U_{g^{-1}}(\Sigma'_{d-1}) = \exp\left[i\alpha \int_{\widehat{\Sigma}} \partial_\mu J^\mu(x)\, d^d x\right]$$

$$U_g(\Sigma) \cdot U_{g^{-1}}(\Sigma') = \mathbb{I} \to U_{g^{-1}}(\Sigma') \cong U_{g^{-1}}(\Sigma)$$



- Induce symmetry transformations on charged operators

$$U_g(\Sigma)\mathcal{O}_R(x) = R(g) \cdot \mathcal{O}_R(x)U_g(\Sigma')$$



Higher-form symmetries

"Define" the symmetry of a QFT by the collection of SDOs and their relations.

[Gaiotto, Kapustin, Seiberg, Willett '14]: use the same prescription to define generalized symmetries with (p+1)-index conserved currents acting on p-dimensional charged objects.

$$J^{\mu_1 \cdots \mu_{p+1}} \qquad\qquad \partial_{\mu_1} J^{\langle \mu_1 \cdots \mu_{p+1} \rangle} = 0$$

SDOs are defined on codimension-(p+1) surfaces $\Sigma_{(d-p-1)}$

$$U_g(\Sigma_{d-p-1}) = e^{i\alpha \int_\Sigma J_{d-p-1}}$$

(Ensures SDOs always have well-defined linking with charged objects. Note that local operators are charged only for ordinary p=0-form symmetries.)

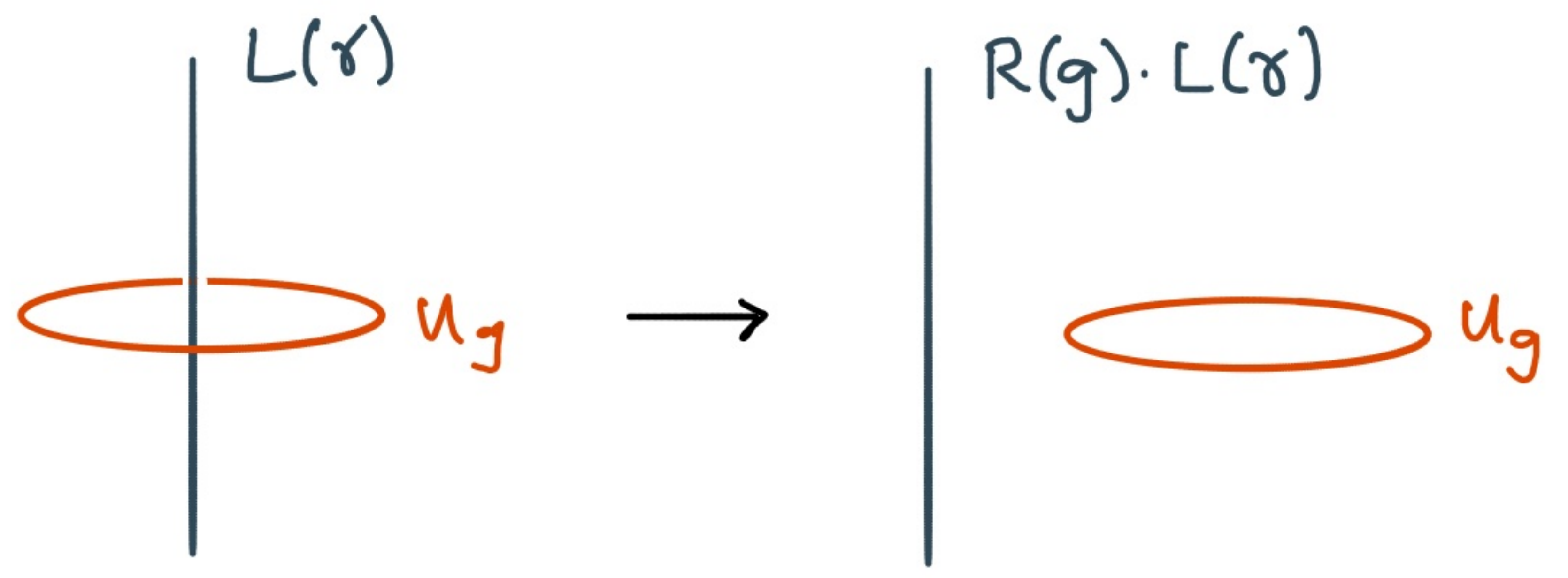
At this point differential forms are essentially obligatory. Conserved (p+1)-index current is co-closed, so convenient to define SDO via closed dual (d-p-1)-form

$$\partial_\mu J_{p+1}^{\mu_1 \cdots \mu_{p+1}} = 0 \quad \Longleftrightarrow \quad dJ_{d-p-1} = 0$$

Higher-form symmetries

For example: p=1-form symmetry in d=3.

Conserved 2-form current, charged objects are 1-dimensional (line operators), SDOs defined on circles.



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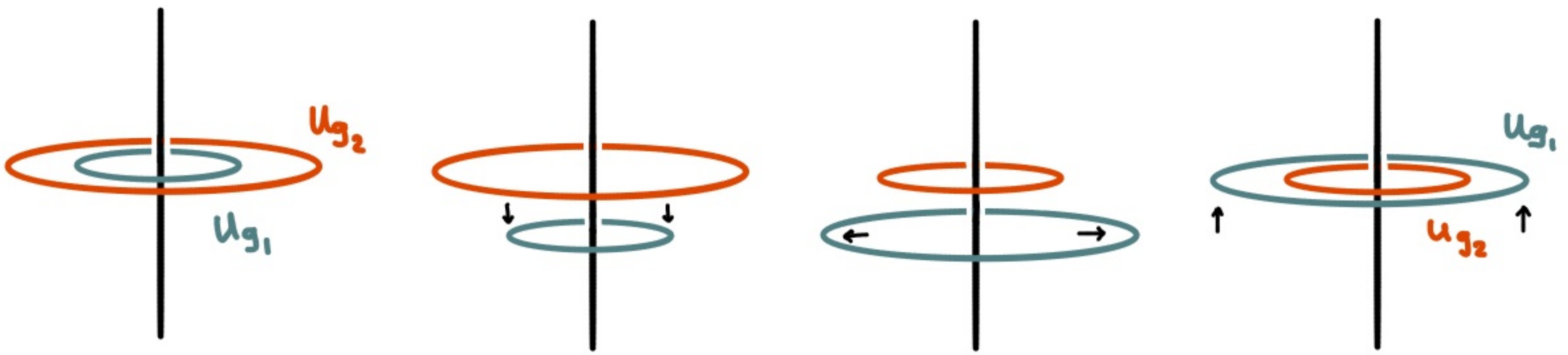
p-form global symmetries necessarily abelian for p>0:



Examples in d=4

Maxwell electrodynamics
(no charged matter)

1. "Electric" 1-form symmetry $J^{\mu\nu}(x) \propto F^{\mu\nu}(x)$

Conserved by EOM: $\partial_\mu J^{\mu\nu}(x) = 0$

Charged objects: Wilson lines $W_q(\gamma) = e^{iq\int_\gamma A}$

2. "Magnetic" 1-form symmetry $\tilde{J}^{\mu\nu}(x) \propto \tilde{F}^{\mu\nu}(x)$

Conserved by Bianchi identity: $\partial_\mu \tilde{J}^{\mu\nu}(x) = 0$

Charged objects: 't Hooft lines

Periodic scalar (axion)

$$a \sim a + 2\pi f$$

1. 0-form symmetry (shift)

$$J_\mu \propto \partial_\mu a$$

2. 2-form symmetry

$$J_{\mu\nu\rho} \sim \epsilon_{\mu\nu\rho\sigma}\partial^\sigma a$$

Charged objects: axion string worldsheets

Yang-Mills
(No matter charged under center)

1-form center symmetry

Charged objects: Wilson loops

$$W_R(\gamma) = \mathrm{Tr}_R \exp\left(i \oint_\gamma T^a_R A^a_\mu dx^\mu\right)$$

Spontaneous higher-form symmetry breaking

p-form symmetries can be spontaneously broken, giving p-form goldstone bosons

0-form symmetry: charged operator acquires an expectation value

$$\lim_{|x-y|\to\infty} \langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \langle \mathcal{O}(x)\rangle\langle \mathcal{O}(y)\rangle \neq 0$$

"depends only on the points on which the operators live"

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p-form symmetry: spontaneously broken if charged operator defined on a surface Γ_p acquires an expectation value that depends only on Γ_p

$$\langle W_q(\Gamma_p)\rangle \sim e^{-F(\Gamma_p)}$$

$$\lim_{\mathrm{Vol}(\Gamma_p)\to\infty} \mathrm{Re}\frac{F(\Gamma_p)}{\mathrm{Vol}(\Gamma_p)} = \begin{cases} \text{finite} & \text{SSB} \\ \infty & \text{unbroken} \end{cases}$$

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For example: in 4d Maxwell theory, Wilson loops have perimeter law scaling in the Coulomb phase, electric 1-form symmetry is spontaneously broken.

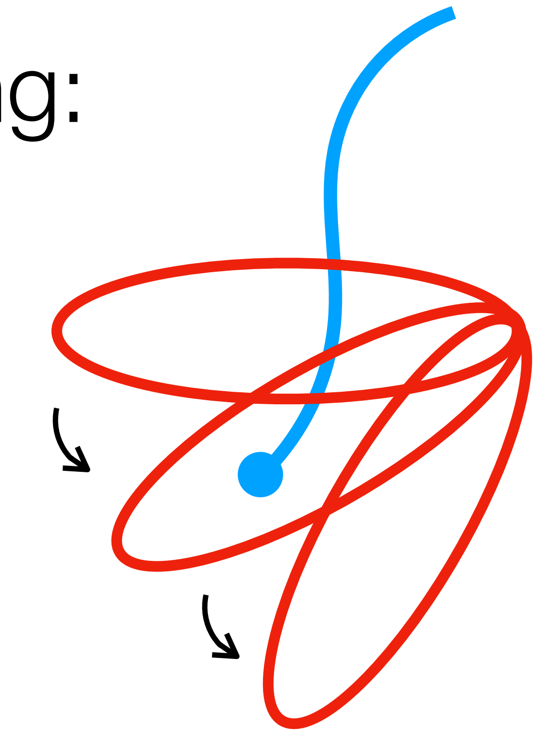
$$\langle W\rangle \sim e^{-g^2 P}$$

⇒ The photon is the goldstone boson of the spontaneously broken 1-form symmetry [Kovner & Rosenstein '91]

Explicit higher-form symmetry breaking

What about explicit breaking? *Charged matter explicitly breaks higher-form symmetries.*

For example, electric 1-form symmetry of U(1) electrodynamics. Two ways of seeing:



- Current conservation violated by matter current $\partial_\mu F^{\mu\nu} \propto J^\nu$, SDO not topological.
- Dynamical charged matter makes Wilson lines endable, SDOs no longer link lines.

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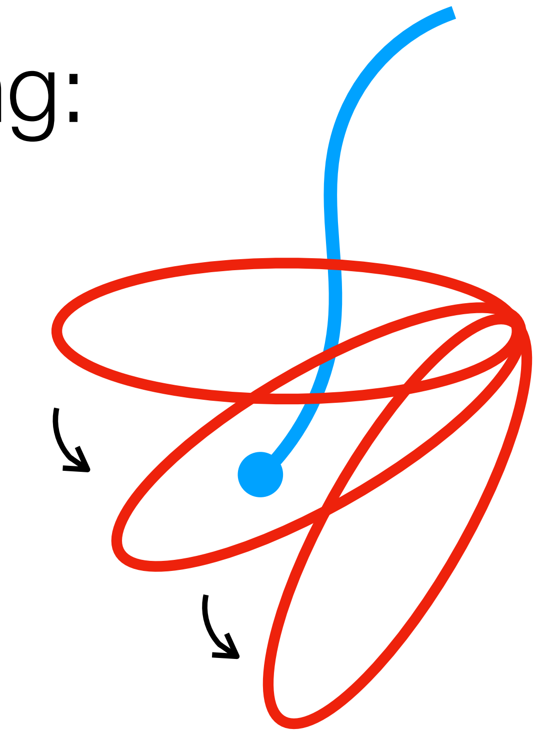

Symmetry is broken above the mass threshold of charged matter, but *exact* in the local Lagrangian density* below it.
This is very different from explicit breaking of 0-form symmetries….

$$\mathcal{L} \sim -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\not{D}\Psi + m\bar{\Psi}\Psi$$

m

$$\mathcal{L} \sim -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + cF^4 + \ldots$$

Although EFT below threshold contains irrelevant operators that change the EOM, there is still a conserved 2-form current [Cordova, Ohmori, Rudelius '22]

$$\partial_\mu F^{\mu\nu} \sim \partial_\mu \left(cF^{\mu\nu}F^2 + \ldots\right) \qquad J^{\mu\nu} \sim F^{\mu\nu} + cF^{\mu\nu}F^2 + \ldots$$

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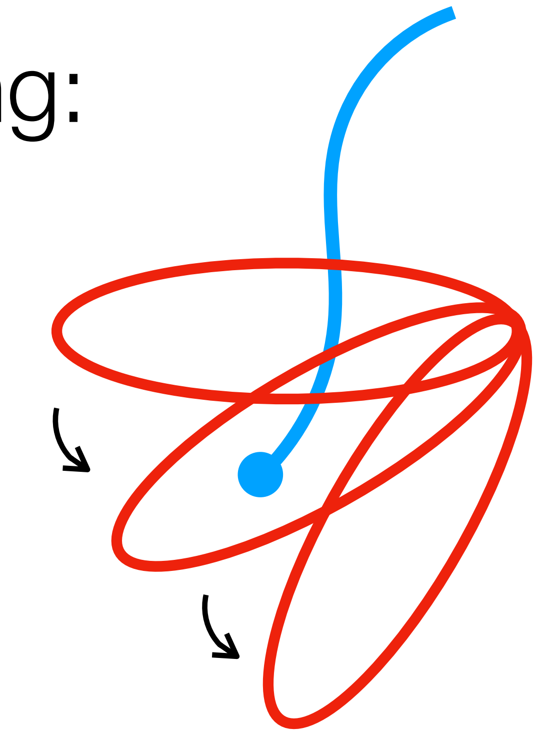

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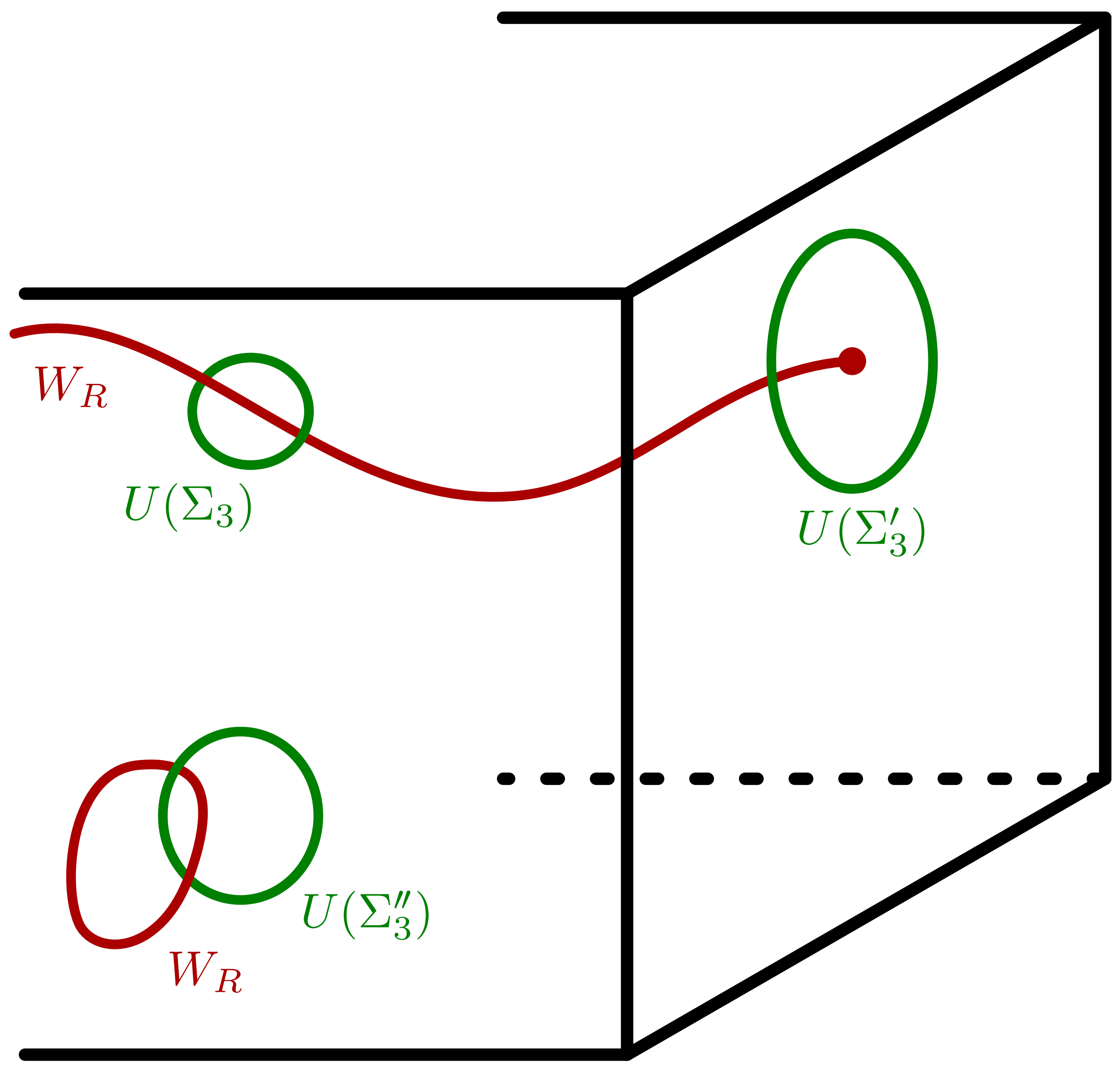
**However, breaking is apparent in effective theory of Wilson lines in the IR.*

Explicit higher-form symmetry breaking

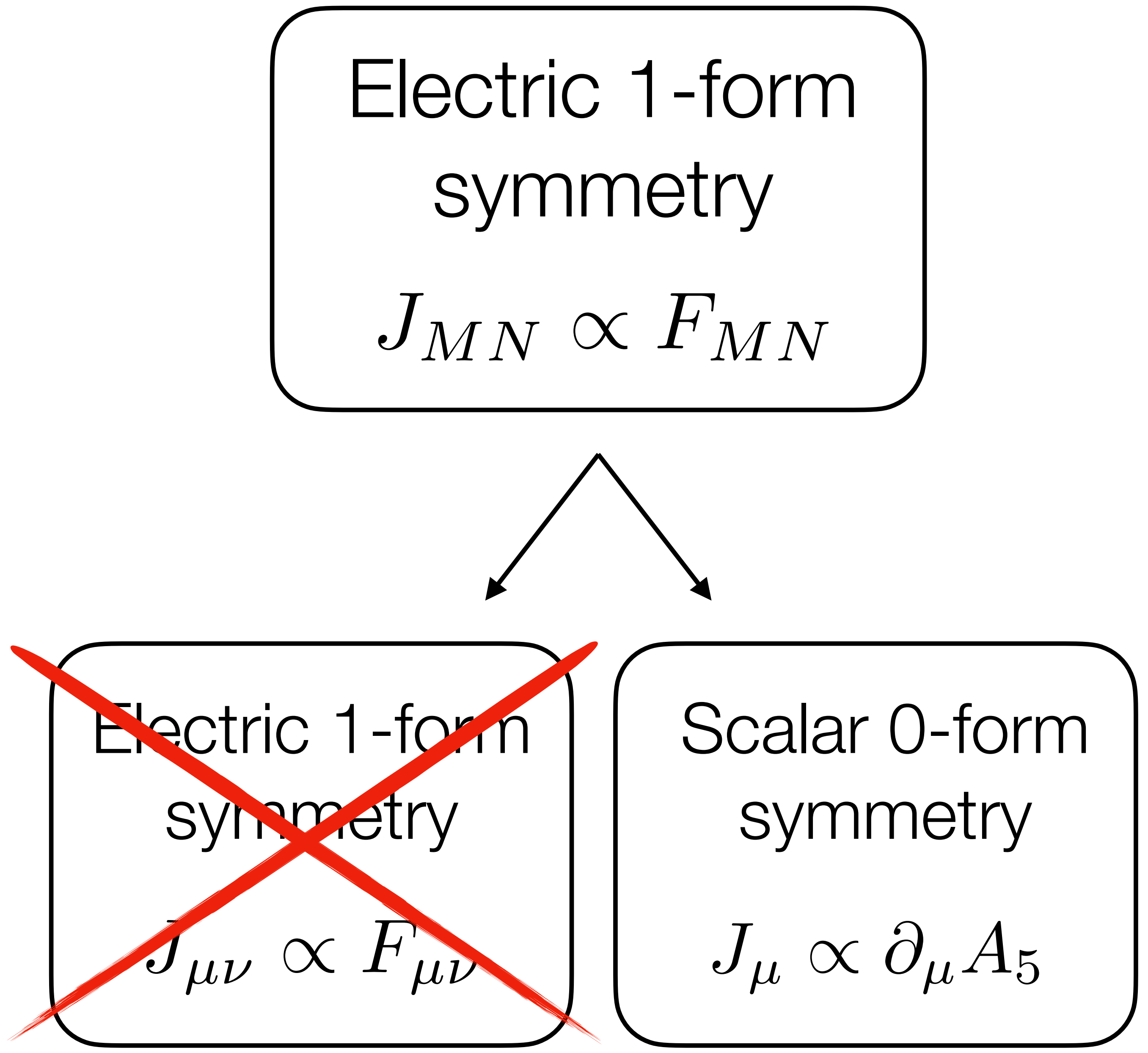
Higher-form symmetries decompose under compactification and can be further broken via boundary conditions

For example: 5d U(1) on R^4 x S^1/Z_2 $\quad A_M \to (A_\mu, A_5)$



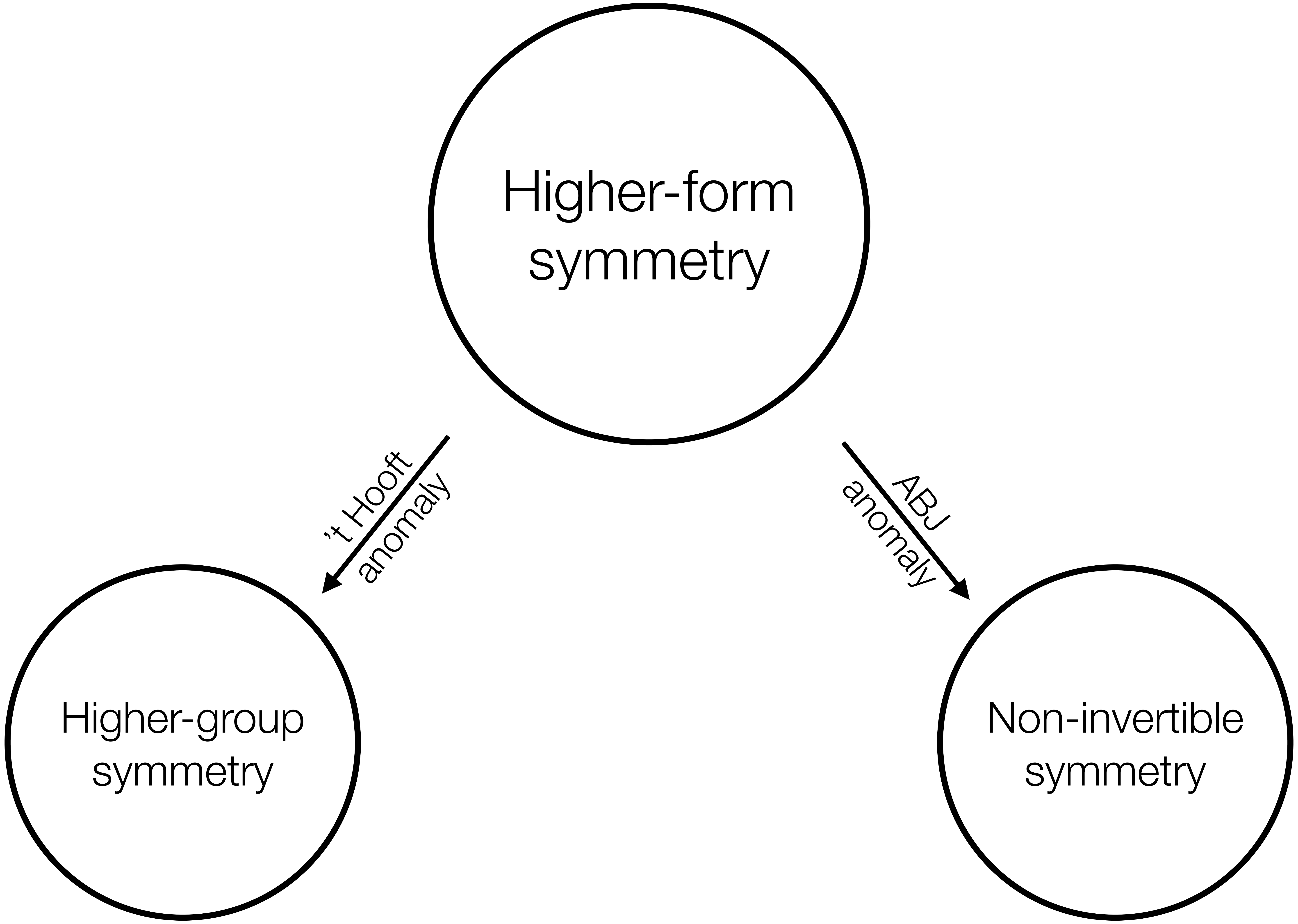
Dirichlet BCs for A_μ (i.e. odd parity) explicitly break 1-form symmetry on boundaries, Wilson lines can now end there.

Unsurprising from perspective of 4D EFT; odd parity for A_μ projects out zero mode, no massless U(1) in 4d.



[Aguilera Damia, Argurio, Garcia-Valdecasas '22]

Anomalous higher-form symmetry breaking



Higher-group symmetry

When higher-form symmetries mix due to ’t Hooft anomalies [Cordova, Dumitrescu, Intriligator ’18]

To study mixing, revisit trick from ’t Hooft anomalies: imagine weakly gauging p-form symmetries, i.e. couple (p+1)-form current to (p+1)-form background gauge field

$$\Delta S = \int B_{p+1} \wedge J_{d-p-1} \qquad\qquad \delta B_{p+1} = d\Lambda_p$$

Consider a theory with a U(1) 1-form symmetry and a U(1) 0-form symmetry.

If the symmetries decoupled, theory invariant under separate background gauge transformations

$$\delta A = d\lambda_0 \qquad\qquad \delta B = d\Lambda_1$$

If symmetries mix w/ a higher-group structure (“2-group”), only invariant under correlated transformations:

$$\delta A = d\lambda_0 \qquad\qquad \delta B = d\Lambda_1 - \frac{\kappa}{2\pi}\lambda_0 F$$

Field strength of A

Leads to emergence conjecture: if both symmetries emerge in the IR, the “bad” symmetry cannot emerge at a higher energy than the “good” symmetry. (Satisfied by anomaly inflow, monopole catalysis, etc.)

Axion Yang-Mills

An interesting example: Axion Yang-Mills [Brennan, Cordova '20]

$$S = \frac{1}{2}\int da \wedge *da + \frac{1}{g^2}\int \mathrm{tr}(G \wedge *G) - \frac{i}{8\pi^2 f}\int a\,\mathrm{tr}(G \wedge G)$$

3 global symmetries of interest:

- 0-form PQ symmetry $J_\mu \propto \partial_\mu a$
- 1-form $\mathbb{Z}_N$ center symmetry of SU(N)
- 2-form scalar symmetry $J_{\mu\nu\rho} \sim \epsilon_{\mu\nu\rho\sigma}\partial^\sigma a$

2-form and 1-form symmetry together have a 3-group structure

$\Rightarrow$ Emergence scales should satisfy $E_2 \gtrsim E_1$

✓ Satisfied by anomaly inflow, also by perturbative effects.

What else can this do for particle theory?

Part II: High-quality Axions and Higher-Form Symmetries

Axion Quality Problem

[Holman, Hsu, Kephart, Kolb, Watkins, Widrow ’92; Kamionkowski & March-Russell ’92; Barr & Seckel ’92; Lusignoli & Roncadelli ‘92]

For a conventional PQ axion, axion is the pNGB of a spontaneously broken $U(1)_{\rm PQ}$ global symmetry: $\Phi(x) \to f e^{ia(x)/f}$

By assumption, the only explicit breaking is from QCD via the anomaly: $V(a) \simeq -\Lambda_{\rm QCD}^4 \cos(a/f)$

Axion vev neutralizes QCD θ term at the minimum of the potential 🎉

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However, additional explicit violation of $U(1)_{\rm PQ}$ in the UV can spoil the solution:

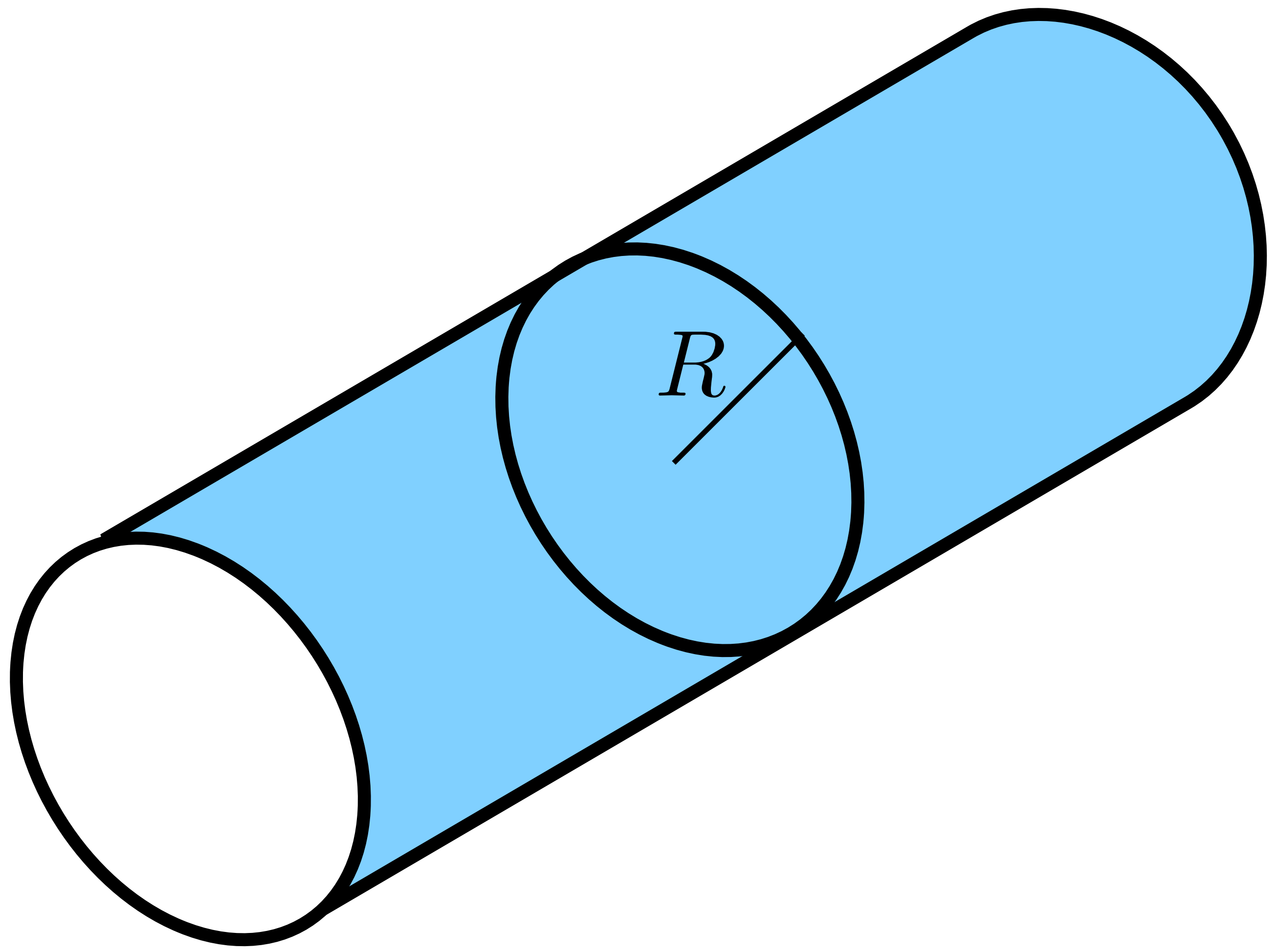
$$V_{\rm UV} \supset \frac{|c|e^{i\delta}}{\Lambda_{\rm UV}^n}|\Phi|^4\Phi^n \Rightarrow V(a) \supset \left(\frac{f}{\Lambda_{\rm UV}}\right)^n f^4 \cos(na/f + \delta)$$

Expect global symmetries broken by quantum gravity; for $f \sim 10^{9-11}\,{\rm GeV},\ \Lambda_{\rm UV} \sim M_{\rm Pl}$ problematic below $n \sim 6-8$

Not just a naive EFT expectation: wormhole contributions give rise to global symmetry breaking of the expected form
[Kallosh, Linde, Linde, Susskind '95; …]

Many model-building solutions: compositeness, discrete (gauge) symmetries, …
but significantly complicates the simplicity of the axion solution to the Strong CP problem.

Extra-dimensional Axions



Axions also arise from the compactification of higher-dimensional theories.

Long history in string theory [Witten ’84; Choi & Kim ’85; Barr ’85; Dine & Seiberg ’86, …], but the essential physics can be understood in field theory. [Arkani-Hamed, Cheng, Creminelli, Randall ’03; Choi ’03; …]

Simplest toy model: U(1) field theory in 5D, one dimension compactified on S^1

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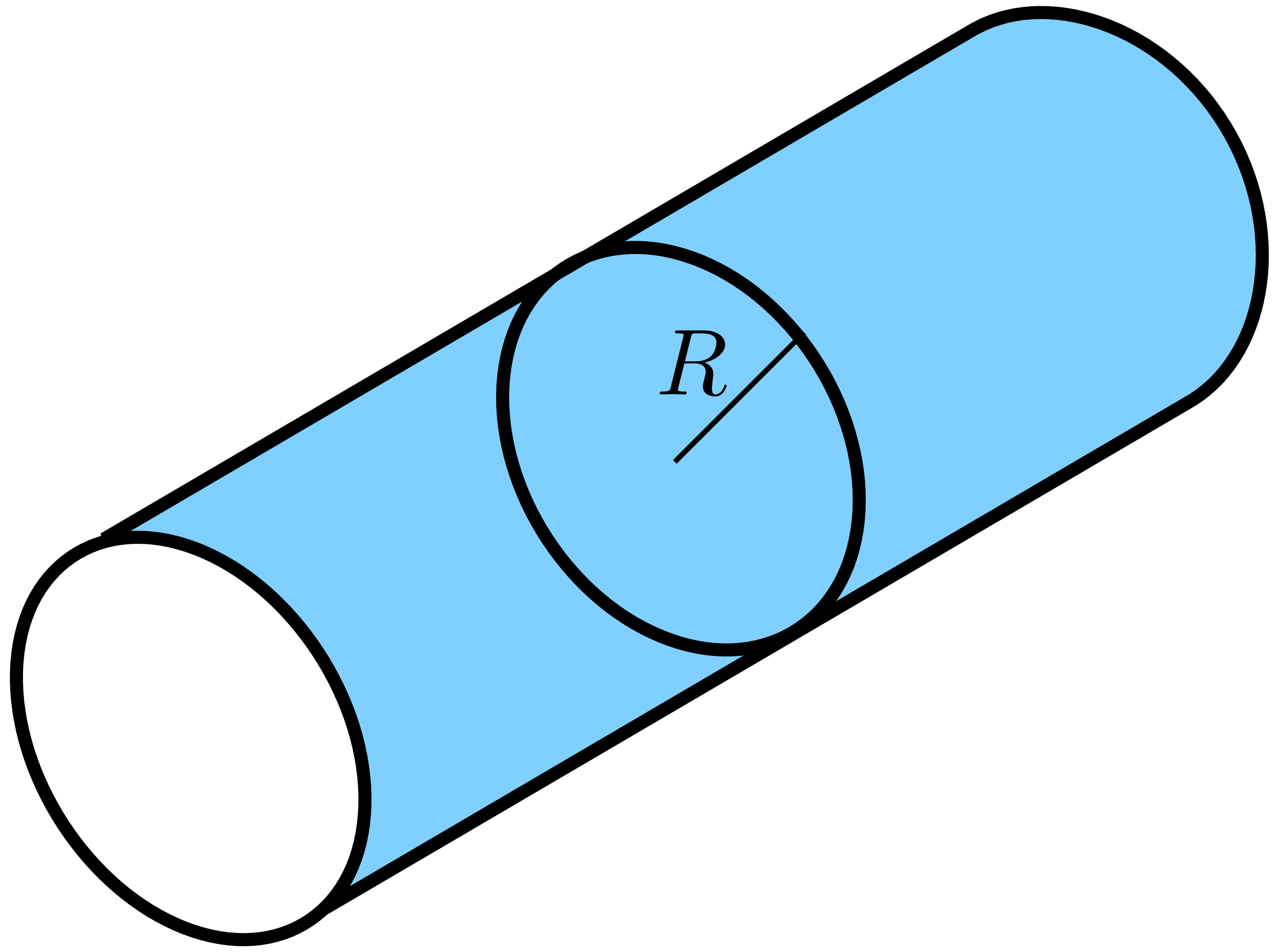

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U(1) gauge field

SU(N) gauge field ("QCD")

$$S^{5\mathrm{D}} = \int \left(-\frac{1}{2g_5^2} \mathrm{d}C \wedge \star \mathrm{d}C - \frac{1}{2e_5^2} G \wedge \star G + \frac{N}{8\pi^2} C \wedge \mathrm{Tr}\,[G \wedge G] \right)$$

$$[g_5] = [e_5] = -1/2$$

$$C_M \Rightarrow (C_\mu, C_5) \qquad \theta \equiv \int_{S^1} C$$

$$S^{4\mathrm{D}} = \int \left(-\frac{1}{2} f^2 \mathrm{d}\theta \wedge \star \mathrm{d}\theta - \frac{1}{2e_4^2} \mathrm{Tr}\,[\tilde{G} \wedge \star \tilde{G}] + \frac{N}{8\pi^2} \theta\, \mathrm{Tr}\,[\tilde{G} \wedge \tilde{G}] + \dots \right)$$

$$e_4^2 \equiv e_5^2/(2\pi R)$$

$$f^2 \equiv 1/g_5^2(2\pi R)$$

Extra-dimensional Quality

Extranatural Inflation

Nima Arkani-Hamed, Hsin-Chia Cheng, Paolo Creminelli and Lisa Randall
Jefferson Physical Laboratory,
Harvard University, Cambridge, MA 02138, USA

The extra component A_5 of an abelian gauge field propagating in the bulk cannot have a local potential, due to the higher dimensional gauge invariance; a shift symmetry protects it similarly to what happens to a four-dimensional PNGB.

Quantum gravity corrections to the potential (5) are negligible if the extra dimension is bigger than the Planck length, different from what is expected in a 4d PNGB model. Again locality in the extra space is the key feature; virtual black holes cannot spoil the gauge invariance and do not introduce a local potential for A_5, while non-local effects are exponentially suppressed by $\sim e^{-2\pi M_5 R}$, because the typical length scale of quantum gravity effects (the 5d Planck length M_5^{-1}) is much smaller than the size of the extra dimension.

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- Extra-dimensional gauge invariance doesn't protect against local effects that badly violate the axion shift symmetry, e.g. a Stueckelberg mass for the 5d U(1) gauge field.

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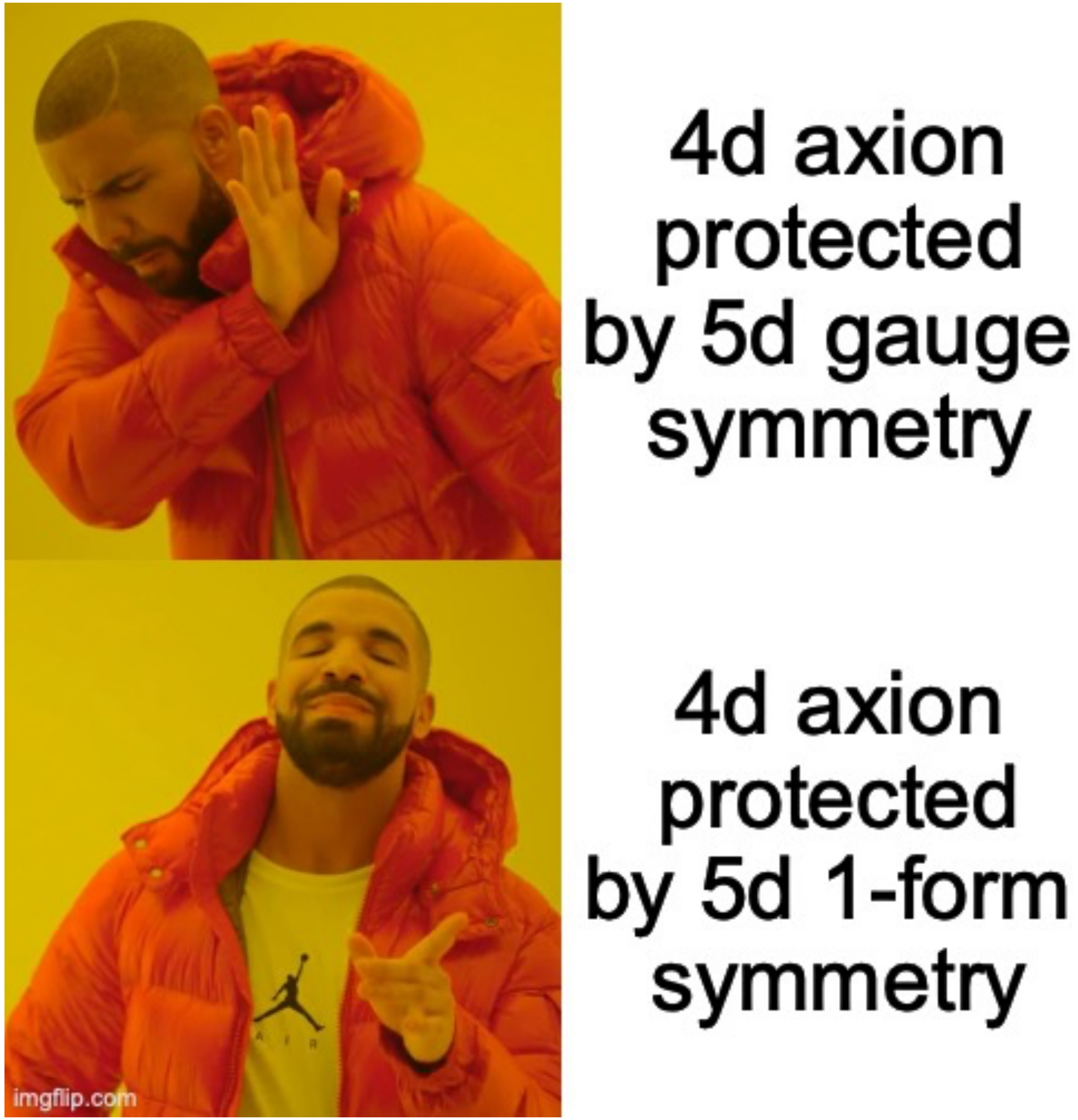
- Extra-dimensional gauge invariance doesn't protect against local effects that badly violate the axion shift symmetry, e.g. a Stueckelberg mass for the 5d U(1) gauge field.
- How can we systematically account for the whole host of non-local effects? What contributes?



h/t M. Reece, TASI lectures [2304.08512]

Higher-Form Symmetry to the Rescue

The free 5d U(1) has an electric one-form symmetry and a magnetic two-form global symmetry.

The axion real number shift symmetry is embedded in the electric one-form symmetry.
This symmetry is difficult to break, protecting the axion potential from dangerous corrections from UV physics.

5d

Electric 1-form symmetry
$J_{MN} \propto F_{MN}$

Magnetic 2-form symmetry
$\tilde{J}_{MNR} \propto \tilde{F}_{MNR}$

1/R

4d

Electric 1-form symmetry
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Scalar 0-form symmetry
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Magnetic 1-form symmetry
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Breaking the Symmetry

Free Maxwell theory in 5d: $J_e = \frac{1}{g_5^2}\star \mathrm{d}C\,, \quad J_m = \frac{1}{2\pi}\mathrm{d}C$

Conserved via EOM

$$\mathrm{d}J_e \propto \mathrm{d}\star\mathrm{d}C = 0$$

Conserved via Bianchi

$$\mathrm{d}J_m \propto \mathrm{d}\mathrm{d}C = 0$$

Can break the electric 1-form symmetry directly with charged matter, anomalies, or gauging the magnetic symmetry.
Axion can also be eaten by gauging the electric symmetry.

Symmetry Modification	Current Equation	Remnant Symmetry	Potential
Electric Matter	$\mathrm{d}J_e = j_{\mathrm{matter}}$	$\mathbb{Z}_q^{(1)}$	$V_\theta \simeq -\frac{(m_{5\mathrm{D}}R)^2}{(2\pi R)^4} e^{-2\pi R m_{5\mathrm{D}}} \cos(q\theta)$
Magnetic Gauging	$\mathrm{d}J_e = \frac{M}{2\pi}\mathrm{d}K$	$\mathbb{Z}_M^{(1)}$	$m_\theta = \frac{M}{2\pi}\, g_4\, e_{K,4}$
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Electric 1-form symmetry no longer conserved in presence of matter: $\mathrm{d}J_e = j_{\mathrm{matter}}$

What's the consequence for the axion? Gives effective potential via worldline instantons.
Heuristic: theory has distinct topological sectors in which charged particle winds around extra dimension a different number of times.

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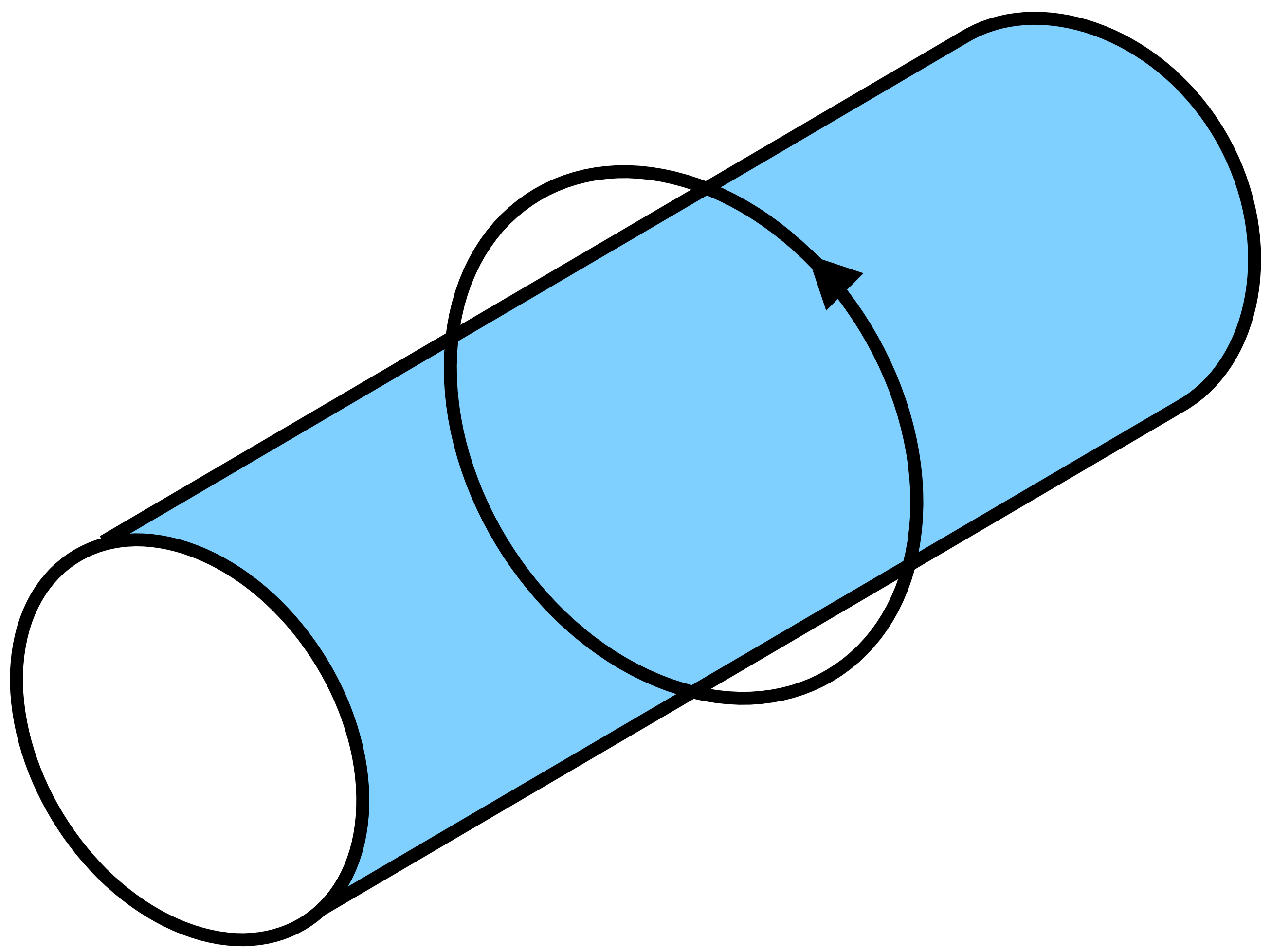

Action from winding ω times: $S_E = m_{5\mathrm{D}} \int_\gamma \mathrm{d}\tau + iq \int_\gamma C \simeq 2\pi R \omega m_{5D} + iq\omega\theta$

Effective potential: $V(\theta) \propto \sum_{\omega \in \mathbb{N}} e^{-S_E(\omega)} + e^{-S_E(-\omega)} \propto \sum_{\omega \in \mathbb{N}} e^{-2\pi R \omega m_{5\mathrm{D}}} \cos(\omega q \theta)$

Actual calculation: $V(\theta) = -2\pi R \sum_{n=1}^{\infty} \frac{2 m_{5\mathrm{D}}^5}{(2\pi)^{5/2}} \frac{K_{5/2}(2\pi R m_{5\mathrm{D}} n)}{(2\pi R m_{5\mathrm{D}} n)^{5/2}} \cos(nq\theta)$

Gauging the magnetic 2-form symmetry

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Couple a 3-form gauge field K to the 3-form current of the magnetic 2-form symmetry

"5D BF theory"

$$S^{5\mathrm{D}} \supset \int \left(-\frac{1}{2e_K^2}\mathrm{d}K \wedge \star \mathrm{d}K + \frac{M}{2\pi} K \wedge \mathrm{d}C \right)$$

$[e_K] = 3/2$

$M \in \mathbb{Z}$

Modifies the EOM to $$\mathrm{d}J_e = \frac{M}{2\pi}\mathrm{d}K$$ Electric one-form symmetry is broken

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To identify IR consequences, integrate the EOM for K and plug it into EOM for C:

$$\mathrm{d}\star\mathrm{d}K = e_K^2 \frac{M}{2\pi}\mathrm{d}C \quad \Rightarrow \quad \mathrm{d}\star\mathrm{d}C = g_5^2 e_K^2 \frac{M^2}{4\pi^2} \star (C - \alpha)$$

C acquires a mass (eats dual of K). In terms of 4D axion field, $m_\theta = g_4 e_{K,4} M/2\pi$

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What about gauging the electric 1-form symmetry? Consistent coupling to 2-form gauge field $\boldsymbol{B}$:

$$S^{5D} = \int \left(-\frac{1}{2e_B^2}\mathrm{d}B \wedge \star\mathrm{d}B - \frac{1}{2g_5^2}(\mathrm{d}C - kB) \wedge \star(\mathrm{d}C - kB) \right)$$

preserves invariance under

$$C \to C + k\Lambda\,, \quad B \to B + \mathrm{d}\Lambda$$

Refined current $J'_e = \frac{1}{g_5^2}\star\mathrm{d}C - \frac{k}{g_5^2}\star B$

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$$\tilde{A}_\mu \sim B_{\mu 5}$$

$$e_{B,4}^2 \equiv e_B^2(2\pi R)$$

$$S^{4D} = \int \left(-\frac{1}{2e_{B,4}^2}\mathrm{d}\tilde{A} \wedge \star\mathrm{d}\tilde{A} - \frac{1}{2}f^2(\mathrm{d}\theta - k\tilde{A}) \wedge \star(\mathrm{d}\theta - k\tilde{A}) + \dots \right)$$

4d Stueckelberg theory where axion gets eaten,

$$m_{\tilde{A}} = k e_{B,4} f$$

Breaking by anomaly

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Axion coupling to QCD starts its life as a 5D Chern-Simons term,

$$S^{5\mathrm{D}} \supset \int \left(-\frac{1}{2e_5^2} G\wedge\star G + \frac{N}{8\pi^2} C\wedge \mathrm{Tr}\,[G\wedge G] \right)$$

which clearly breaks the electric 1-form symmetry:
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Reduced to four dimensions, gives the conventional axion potential

$$S^{4\mathrm{D}} \supset \int \left(-\frac{1}{2e_4^2}\mathrm{Tr}\,[\tilde{G}\wedge\star\tilde{G}] + \frac{N}{8\pi^2}\theta\,\mathrm{Tr}\,[\tilde{G}\wedge\tilde{G}] + \dots \right)$$

$$V(\theta) \simeq -\Lambda_{\mathrm{QCD}}^4 \cos(N\theta)$$

Magnetically charged matter

Related: anomaly terms involving additional U(1)'s w/ monopoles can break the electric 1-form symmetry.

Recall axion mass from monopoles in Axion-QED [Fan, Fraser, Reece, Stout 2105.09950]

In a theory with a monopole and a tower of dyons charged under a U(1), mass spectrum depends on θ angle:

$$m_n^2 = m_M^2 + m_\Delta^2 \left(n - \frac{\theta}{2\pi}\right)^2$$

[Jackiw]

If there is an axion coupled to $F\tilde{F}$, mass spectrum depends on the axion field.

Integrating out dyons gives a Coleman-Weinberg potential for the axion

$$V_{eff}(\theta) = -\sum_{\ell=1}^{\infty} \frac{m_\Delta^2 m_M^2}{32\pi^4 \ell^3} e^{-2\pi \ell m_M / m_\Delta} \cos(\ell\theta)\,(1 + \dots)$$

Key point (for us) is that the monopoles have dyonic excitations thanks to the theta term.

Magnetically charged matter

In 5D, if another U(1) couples to C via a Chern-Simons term, its monopole strings will have excitations that act like charged matter to break the electric 1-form symmetry.

$$S^{5\mathrm{D}} \supset \int \left(-\frac{1}{2e_A^2}\mathrm{d}A \wedge \star\mathrm{d}A + \frac{1}{8\pi^2} C \wedge \mathrm{d}A \wedge \mathrm{d}A\right)$$

$$\mathrm{d}J_e = \frac{1}{8\pi^2}\mathrm{d}A \wedge \mathrm{d}A$$

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5D monopoles are strings; their worldsheet theory takes the form

$$S^{\mathrm{Worldsheet}} = \int \left(-\frac{1}{2} \frac{T_M}{m_W^2} \mathrm{d}_A \sigma \wedge \star \mathrm{d}_A \sigma + \frac{1}{2\pi} C \wedge \mathrm{d}_A \sigma \right)$$

$\mathrm{d}_A \equiv \mathrm{d} + 2A$

Gives analogous effective potential to axion:

$$V(\theta) = -\sum_{\ell=1}^{\infty} \frac{R^2 T_M^2 m_W^2}{8\pi^2 \ell^3} e^{-4\pi^2 \ell R T_M / m_W} \cos(\ell\theta) \times \left(1 + \frac{3m_W}{4\pi^2 \ell R T_M} + \frac{3m_W^2}{16\pi^4 \ell^2 R^2 T_M^2} \right)$$

New effect revealed by the higher-form symmetry perspective.

Irrelevant Effects

Things you might have worried would break the electric 1-form symmetry, but do not:

Kinetic mixing between U(1)s

$$S^{5\mathrm{D}} \supset \int \left(-\frac{1}{2e_A^2}\mathrm{d}A \wedge \star\mathrm{d}A + \frac{\kappa}{g_5 e_A}\mathrm{d}C \wedge \star\mathrm{d}A \right)$$

Improved current still conserved

$$J_e' = \frac{1}{g_5^2}\star\mathrm{d}C - \frac{\kappa}{g_5 e_A}\star\mathrm{d}A$$

Local irrelevant operators

$$S^{5\mathrm{D}} = \int \left(-\frac{1}{2g_5^2}\mathrm{d}C \wedge \star\mathrm{d}C - \mathcal{O}\left[\frac{(\mathrm{d}C)^4}{\Lambda^4}\right] \right)$$

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Gravitational Expectations

Weak gravity conjectures parameterize the extra-dimensional axion quality problem

[Arkani-Hamed, Motl, Nicolis, Vafa '06; de la Fuente, Saraswat, Sundrum '14; Heidenreich, Reece, Rudelius '15; Cordova, Ohmori, Rudelius '22; …]

5D Electric WGC

There must be electrically charged matter satisfying

$$m_{5\mathrm{D}} \lesssim g_5 q M_{\mathrm{Pl},5\mathrm{D}}^{3/2}$$

Bounds contribution to axion potential:

$$\begin{aligned} S &= 2\pi m_{5\mathrm{D}} R \\ &\lesssim 2\pi q g_5 R M_{\mathrm{Pl},5\mathrm{D}}^{3/2} \\ &= q M_{\mathrm{Pl}}/f \end{aligned}$$

"Log of the 4D quality problem"

5D Magnetic WGC

The local electric description of the U(1) should break down by

$$\Lambda \lesssim g_5 (M_{\mathrm{Pl},5\mathrm{D}})^{3/2}$$

Given an ABJ anomaly and monopoles charged under the other U(1), bounds action contributing to axion potential:

$$S \propto \frac{R}{g_5^2} \lesssim M_{\mathrm{Pl}}^2 R^2$$

5D 2-form WGC

If the 1-form electric symmetry is gauged, there must be 1d objects whose tension satisfies

$$\frac{2}{3} T_2^2 \leq q_2^2 e_B^2 M_{\mathrm{Pl},5\mathrm{D}}^3$$

The mass of the gauge boson that eats the axion cannot be arbitrarily small compared to the strings,

$$m_{\tilde{A}} \gtrsim \frac{k}{q_2} \frac{T_2}{g_4 M_{\mathrm{Pl}}}$$

A compelling answer to the axion quality problem.

Conclusions

- Generalized symmetries offer a new paradigm for particle physics, although it remains to be seen what new things they can teach us in four dimensions.
- They are abundant in axion models: higher-form symmetries, higher-group symmetries, non-invertible symmetries…
- They bring clarity to the remarkably high quality of extra-dimensional axions. Suggests other phenomenological implications await exploration…

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- Solving the quality problem in this way has far-reaching implications for cosmology (e.g. [Benabou et al. 2312.08425; Lu et al. 2312.07650; …])

Thank you!