ASYMPTOTIC COSMOLOGY AND THE DISTANCE CONJECTURE

WISPs in String Cosmology, Bologna, 24/10/24

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WiP with Thomas Grimm, Damian van de Heisteeg

INTRODUCTION

WISPs: *bridge between strings and phenomenology*

+moduli!

Direct:

DM, QCD axion, ALPs…

Indirect:

consequences for cosmology

Review: [Cicoli,Conlon,Maharana,Parameswaran,Quevedo,Zavala '23]

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This Talk: *cosmology of 1-modulus asymptotic limits*

Interplay with Swampland conjectures

Implications for Phenomenology

PLAN

1) Dynamics & the distance conjecture?

2) Classification of 1-modulus asymptotic cosmologies

3) Comments on phenomenology & outlook

PART 1

DYNAMICS & THE DISTANCE CONJECTURE

MOTIVATION – THE DISTANCE CONJECTURE

Swampland Distance Conjecture (SDC)

Infinite distance points in moduli space → *towers of light states*

[Ooguri, Vafa '06]
[Ooguri,Palti,Shiu,Vafa '19]

Invalidate EFT

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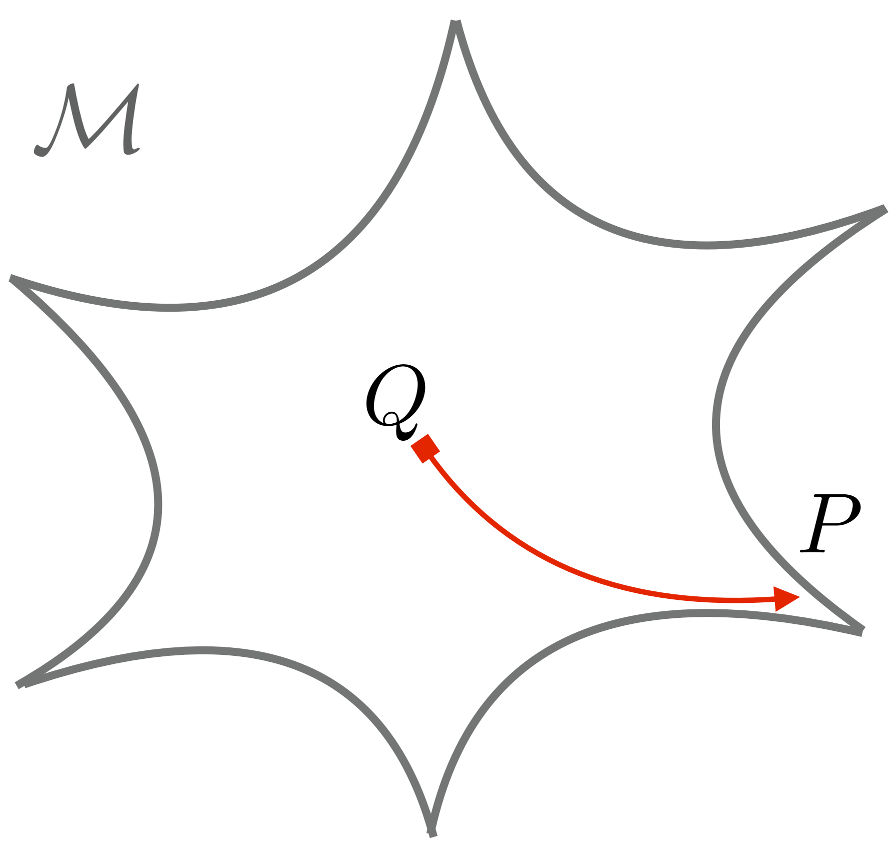


$$m(P) = m(Q)e^{-\lambda d(P,Q)}$$

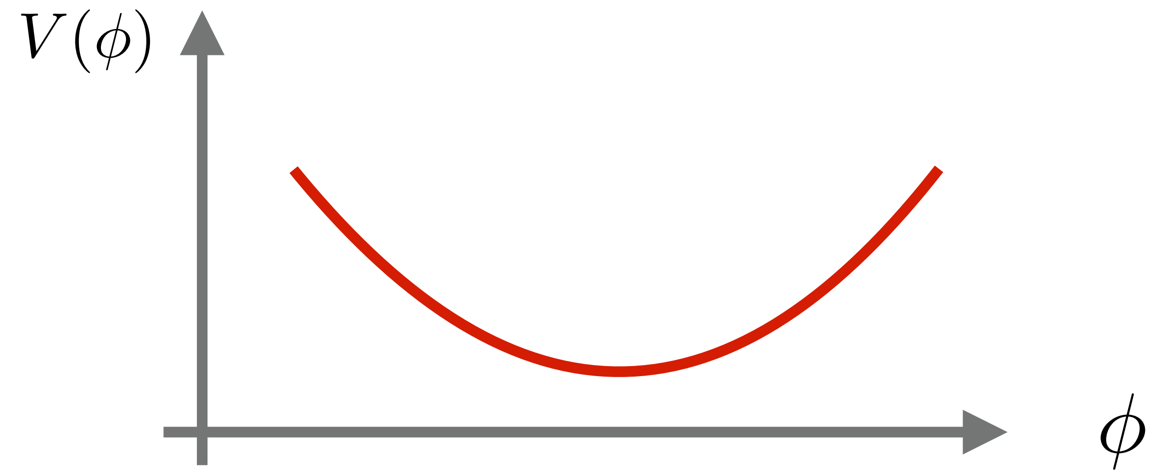
↓ *geodesic distance*

Best established for ***exact*** *moduli spaces*

SDC WITH A POTENTIAL

*Moduli **must** be stabilised*

SDC still thought to hold,
far less evidence



[Klaewer,Palti '16] [Calderon,Uranga,Valenzuela '20] + …

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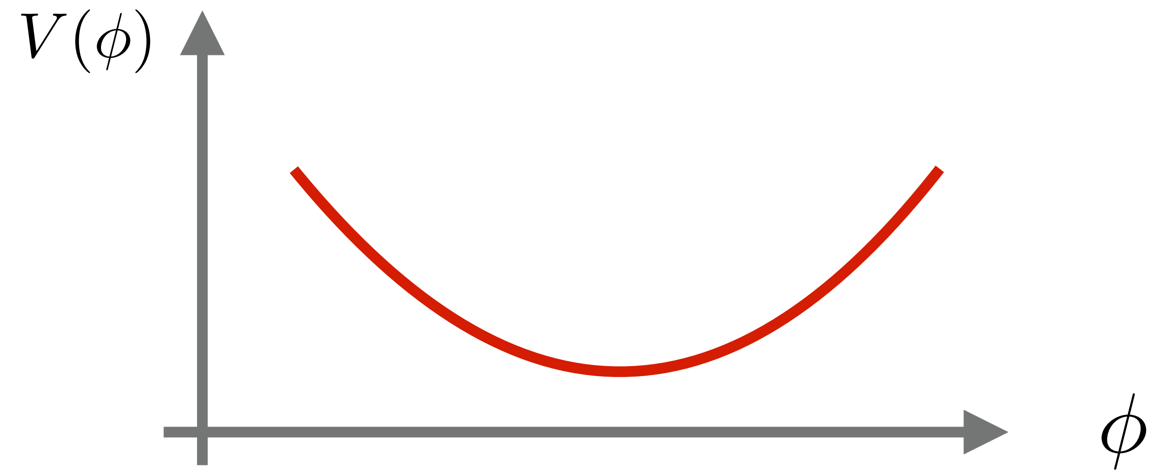

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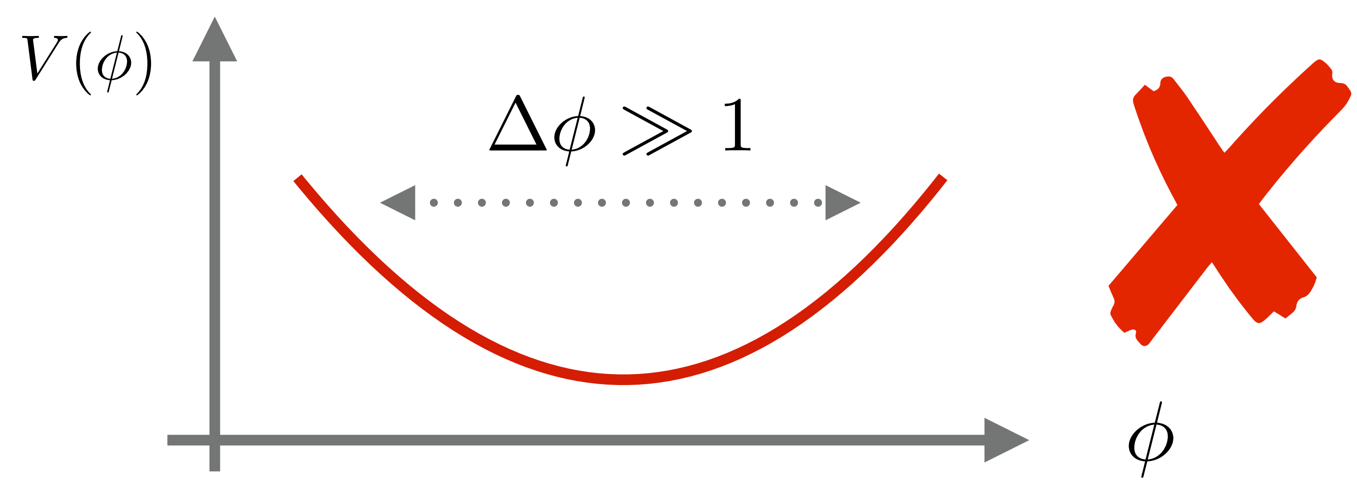
Typical application:
Rule out large field inflation



Compact directions (axions) important:
e.g. monodromy inflation

More recent:
generalised notion of distance, including V?

[Tonioni, Van Riet '24]
[Montero,Mohseini,Vafa,Valenzuela '24]

DYNAMICS

Related issue: SDC applies to ***adiabatic field variations***

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$V \neq 0$

(space)time dependence $\dot{\phi} \neq 0$

non-geodesic trajectories

$$\ddot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k + (d-1)H\dot{\phi}^i + \partial^i V = 0$$

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adiabatic: $\dot{\phi} \to 0$	*(space)time dependence* $\dot{\phi} \neq 0$
geodesic trajectories	*non-geodesic trajectories*

$$\ddot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k + (d-1)H\dot{\phi}^i + \partial^i V = 0$$

What becomes of the SDC in a cosmological setting?

Some (sparse) comments appear in [Conlon,FR '22][Tonioni,Tran,Shiu' 23][Tonioni,Van Riet '24]

A POSSIBLE GENERALISATION*

Question:

For trajectories approaching the boundary of moduli space, do towers of states become exponentially light in the dynamical distance ?

[Shiu,Landete '18] [Tonioni, Van Riet '24]

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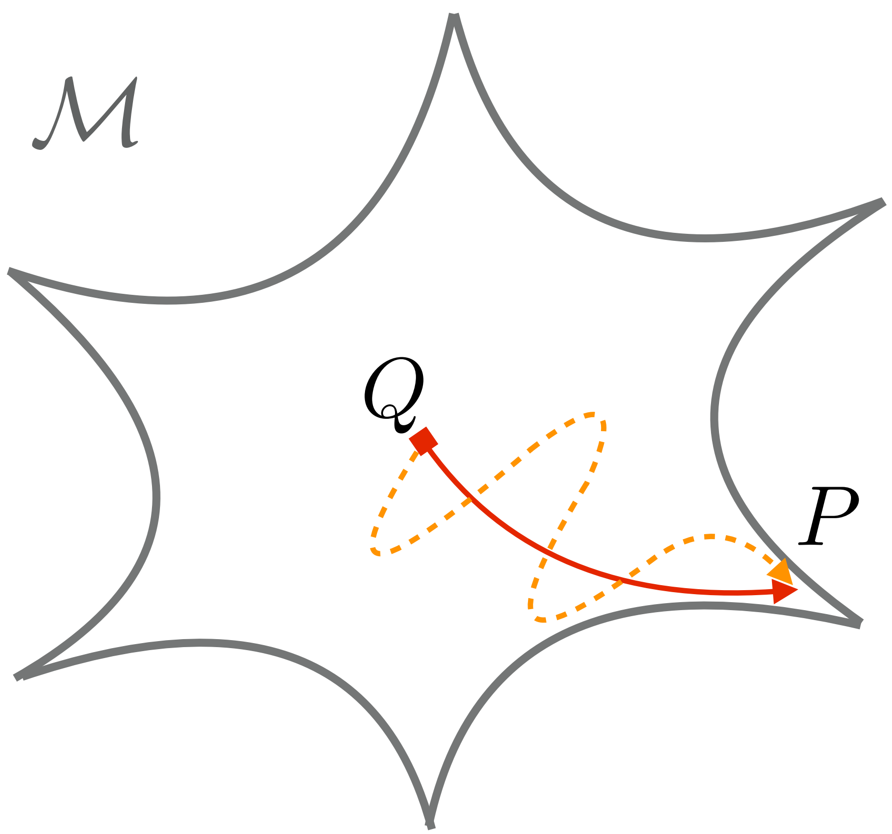

$$m(P) = m(Q)e^{-\lambda\Delta(P,Q)}$$

$$\Delta = \int_{t_1}^{t_2} \mathrm{d}\tau \sqrt{G_{I\bar{J}}\dot{\Phi}^I\dot{\bar{\Phi}}^{\bar{J}}}$$

along trajectory

From usual SDC, equivalent to relationship between length of trajectories and geodesics

PART 2

CLASSIFICATION OF 1-MODULUS ASYMPTOTIC COSMOLOGIES

SETTING

Cosmology of asymptotic limits in type IIB/F-theory flux compactifications

[See also Calderon-Infante,Ruiz,Valenzuela '22,FR '23]

$$S = \frac{M_{P,d}^2}{2} \int d^d x \sqrt{-g} \left\{ \mathcal{R} + \frac{1}{2} G_{I\bar{J}} \partial_\mu \Phi^I \partial^\mu \bar{\Phi}^{\bar{J}} + V(\Phi, \bar{\Phi}) \right\}$$

Complex Structure moduli

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Complex Structure moduli

EFTs classified with Asymptotic Hodge Theory

[Grana,Grimm,Herraez,Monnee,Plauschinn,Palti,Lanza,Li,Schlechter,Valenzuela,van de Heisteeg… '19-24]

$$V \sim \sum_{\ell \in \mathcal{E}} \left(\frac{s^1}{s^2} \right)^{\ell_1 - 4} \cdots \left(\frac{s^{\hat{n}-1}}{s^{\hat{n}}} \right)^{\ell_{\hat{n}-1} - 4} \left(s^{\hat{n}} \right)^{\ell_{\hat{n}} - 4} \| \rho_\ell (G_4, a_i) \|_\infty^2$$

Simple case: *single modulus* $\Phi = s + ia$

EQUATIONS OF MOTION

Solve coupled EOMs of scalar fields, on FLRW background

$$\ddot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k + (d-1)H\dot{\phi}^i + \partial^i V = 0$$

$$\frac{(d-1)(d-2)}{2}H^2 = \frac{1}{2}G_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - V\left(\phi^i\right)$$

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Polynomials

$$G_{ij} = \frac{C}{s^2}\delta_{ij} \qquad V(s,a) = \frac{1}{s^\lambda}\sum_{n=0}^{N}\frac{1}{s^n}P_n\left(\frac{a}{s}\right)$$

Hyperbolic metric — *Flux scalar potential, positive definite*

Complete classification for all 1-modulus limits

DYNAMICAL SYSTEM FORMULATION

Simpler case $$V(s,a) = \frac{P_n(w)}{s^\lambda}$$ $$\left(V(s,a) = \frac{1}{s^\lambda} \sum_{i=0}^{n} \frac{P_i(w)}{s^i} \right)$$

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$$x = \frac{\dot{s}}{\alpha H s} \quad y = \frac{\dot{a}}{\alpha H s} \quad w = \frac{a}{s}$$

x^2, y^2 *normalized kinetic terms*

$$\begin{cases} \frac{dx}{dN} = -\alpha y^2 - (1 - x^2 - y^2)\left[(d-1)x - \frac{\alpha}{2}\left(\lambda + \frac{w\partial_w P_n(w)}{P_n(w)}\right)\right] \\ \frac{dy}{dN} = \alpha x y - (1 - x^2 - y^2)\left[(d-1)y + \frac{\alpha}{2}\frac{\partial_w P_n(w)}{P_n(w)}\right] \\ \frac{dw}{dN} = \alpha(y - wx) \end{cases}$$

[Copeland,Liddle,Wands '97][Russo,Townsend '06-'19][(Brinkmann),Cicoli, Dibitetto,Pedro '20-'22][FR '23][Tonioni,Tran,Shiu '23-'24][Andriot,Parameswaran,Tsimpis,Wrase,Zavala '24]+…

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$$\Delta = \sqrt{C}\int_{s(P)}^{s(Q)} \frac{\mathrm{d}s}{s}\sqrt{1+\left(\frac{\mathrm{d}a}{\mathrm{d}s}\right)^2} \longrightarrow \frac{y^2}{x^2}$$

TECHNIQUES FOR DYNAMICAL SYSTEMS

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$\bar{x} \quad | \quad f(\bar{x}) = 0$ *Deduce local (in)stability from linearisation*

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Lyapunov-like theorems: ***Global***

Find Lyapunov function $\mathcal{L}(x(t))$ *s.t.* $\dot{\mathcal{L}}(x(t)) \leq 0$ *everywhere*

If $\mathcal{L}(x(t)) > 0, \mathcal{L}(\bar{x}) = 0$ → $x \to \bar{x}$

On compact set $x \to \left\{ y \quad | \quad \dot{\mathcal{L}}(y) = 0 \right\}$ *Not only points!*

Trivial example: $\dot{x} = -x, \quad \mathcal{L} = x^2 \quad \dot{\mathcal{L}} = 2x\dot{x} = -x^2 \leq 0$

SCHEMATIC CLASSIFICATION

Asymptotic behaviour → *Kination*

New variable: $T = x + yw \sim \frac{1}{H^2}\frac{s\dot{s} + a\dot{a}}{s^2}$

$$V(s,a) = \frac{P_n(w)}{s^\lambda} \longrightarrow T \to \frac{\alpha\lambda}{2(d-1)}$$

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Fixed point - easy

[(Brinkmann),Cicoli, Dibitetto,Pedro '20-'22][FR '23]

$w \to \bar{w} \quad | \quad P_0(w) = 0 \longrightarrow$ *New oscillating solutions*

A COUNTER-EXAMPLE?

"Growing" trajectories → *claim easy to show*

$$K = c \log s \qquad + \qquad V \sim \sum \frac{P_n(w)}{s^{\beta_n}}$$

Similar to axion backreaction, [Baume, Palti'16][Grimm,Li '20][Calderon-Infante,Uranga,Valenzuela '20]

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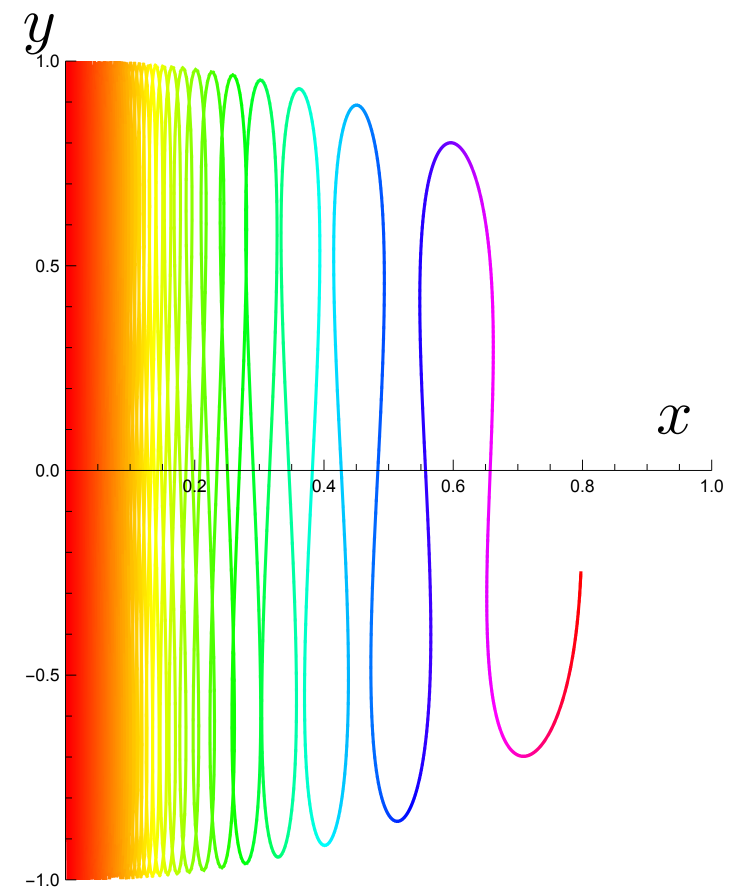
"Oscillating" trajectories

$$V(a,s) = \# f^2 \frac{a^2}{s^2}$$

(LCS point)

$x \to 0, w \to 0$ *Fixed segment*

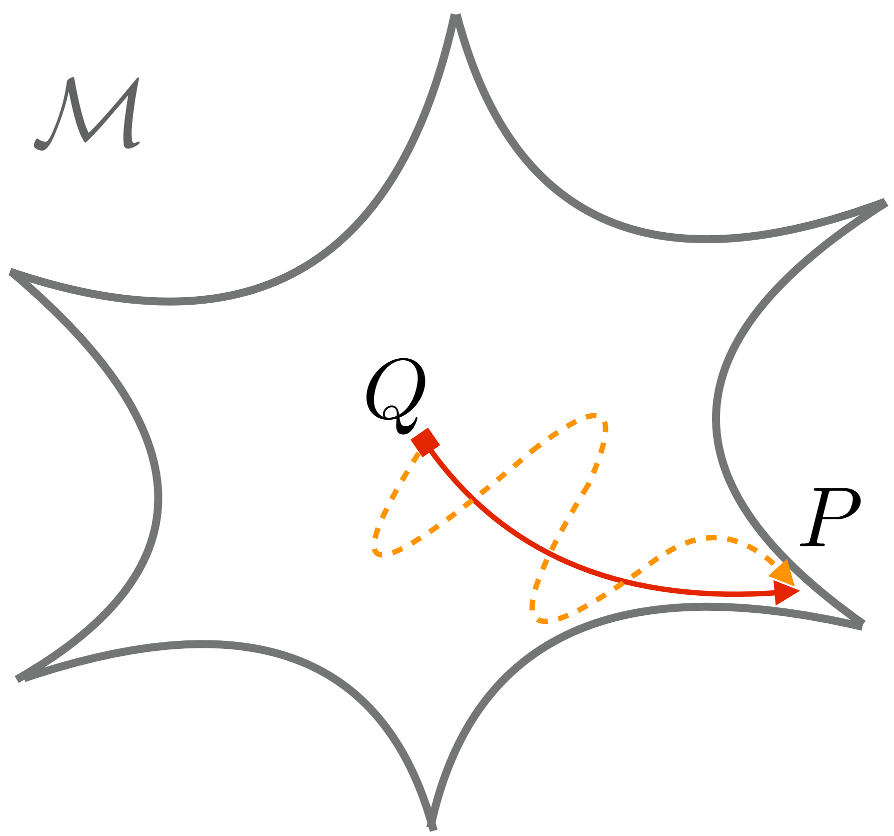


BUT corrections unavoidable & spoil behaviour

PART 3

COMMENTS ON PHENOMENOLOGY & OUTLOOK

ROLLING TOWARDS THE BOUNDARY



Abundance of small couplings etc

Maybe we live close to the boundary?

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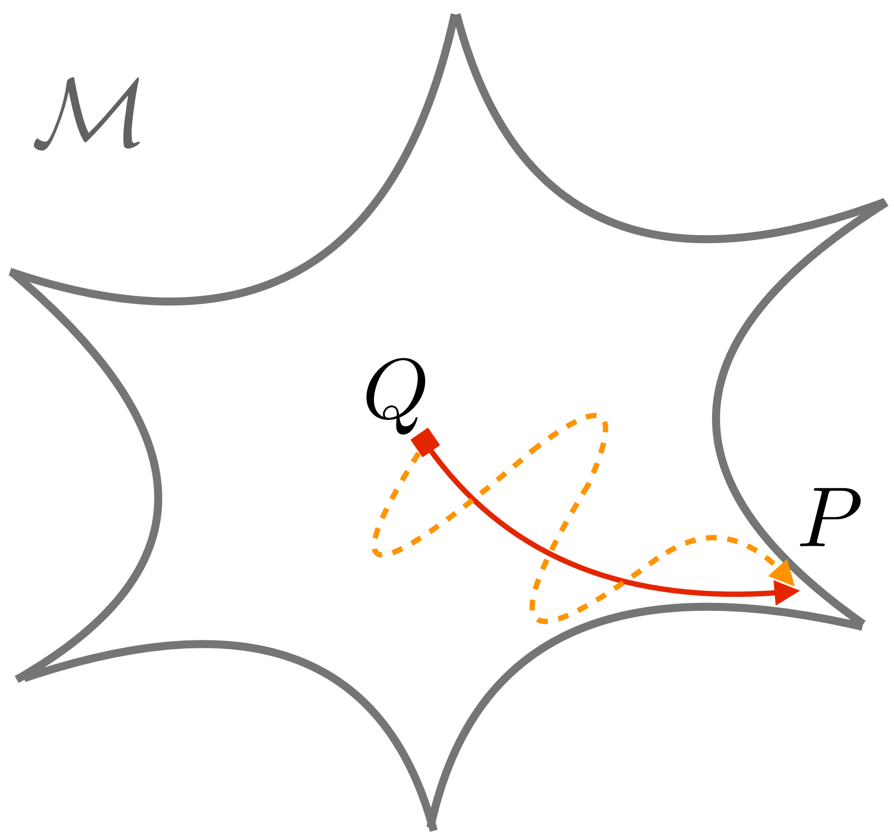

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See Fien Apers's talk

What are "typical" cosmologies?

No acc. expansion

- *Kination* — *Can even be (meta)stable?* *[Apers,Conlon,Copeland,Mosny,FR '22-'24]*
- *Trackers/scaling solutions*
- *Exotic "oscillating" solutions*

COMMENTS ON PHENO

Many caveats for ***realistic*** *scenarios:*

Kahler + spectator moduli, moduli stabilisation…

$$V = e^K K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} \longrightarrow V = \frac{\tilde{V}}{\mathcal{V}^3}$$

Runaway!

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General lessons?

SDC for traversed distance — *Hard to realise axion monodromy inflation close to boundary*

Don't forget the axions! — *Often set to zero, but can have qualitative effects on the dynamics*

Dynamical system techniques can be powerful !

OUTLOOK

Cosmology of 1-modulus asymptotic limits

Analytical results from dynamical system approach

Dynamical version of SDC?

Classification

$$d(P,Q) \longleftrightarrow \Delta(P,Q)$$

Acc. expansion?

Kination?

More examples: finite distance singularities?

Singular examples/Non-perturbative corrections?

Long term: "Dynamical" Swampland

THANK YOU FOR YOUR ATTENTION!