Hubble friction and string cosmology

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based on:

Curvature-induced moduli stabilization [hep-th/2407.21104]

- in string compactifications, can Hubble friction help moduli stabilization, and, if so, to what extent?

1. Context: moduli stabilization

Moduli stabilization, pt. 1

- a very generic prediction of string theory is that compactifications come with a plethora of scalar fields
  - closed strings: dilaton and radion, (essentially) ubiquitously, and generally volumes and shapes of internal cycles
  - open-string scalars controlling the position of branes
  - also general: Kaluza-Klein reductions of higher-dimensional fields and further brane degrees of freedom

  note: the field-space metric might be extremely involved

- to prevent them from mediating long-range forces, all such scalars need to be stabilized and sufficiently heavy

for a review, see e.g. Cicoli, Conlon, Maharana, Parameswaran, Quevedo, Zavala [hep-th/2303.04819]
McAllister, Quevedo [hep-th/2310.20559]

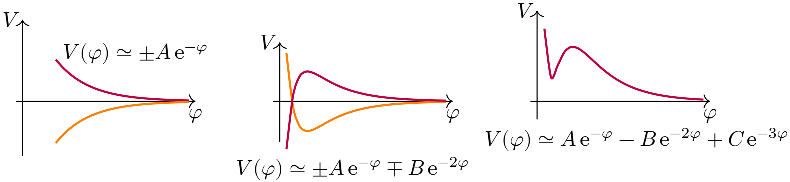
see also Cicoli, De Alwis, Maharana, Muia, Quevedo [hep-th/1808.08967]

- a typical strategy is to focus on solutions where stabilizing potentials are generated by effects such as internal geometric curvature and background fluxes

for reviews, see e.g. Graña [hep-th/0509003]
Van Riet, Zoccarato [hep-th/2305.01722]

- however, all string-theoretic potentials are generated dynamically, with perturbative expansions that also control the validity of the EFT itself
  - ▶ Dine-Seiberg problem: hard to find weakly-coupled vacua

Dine, Seiberg [*Phys.Lett.B 162 (1985) 299-302*]



- possibly further obstacles: e.g. no-scale structure of type-IIB on CY_3/O, i.e. Kähler multiplet $\rho = \theta + \mathrm{i}\,(\mathrm{vol}_{\mathrm{E},6})^{2/3}$ is a flat direction in the 4d EFT

$$\begin{cases} \kappa_4^2 K = -3\,\ln\left[-\mathrm{i}(\rho - \overline{\rho})\right] \\ \kappa_4^3 W = W_0 \end{cases} \quad \rightarrow \quad V_{\mathrm{F}} = 0$$

Giddings, Kachru, Polchinski [hep-th/0105097]

Moduli stabilization: different perspectives

- proposals for 4d vacua with Minkowski or de Sitter (dS) minima require a delicate interplay of (non)perturbative effects, such that the fields are stabilized in between the bulk and the asymptotic regions
  - e.g. KKLT-scenario: non-perturbative superpotential, then anti-D3-brane uplift

$$\begin{cases} \kappa_4^2 K = -3\,\ln\left[-\mathrm{i}(\rho-\overline{\rho})\right] \\ \kappa_4^3 W = W_0 + A\,\mathrm{e}^{a\mathrm{i}\rho} \end{cases}$$

Kachru, Kallosh, Linde, Trivedi [hep-th/0301240]
see recent progress in McAllister, Moritz, Nally, Schachner [hep-th/2406.13751]

  - LVS-scenario: use of perturbative α'-corrections

Balasubramanian, Berglund, Conlon, Quevedo [hep-th/0502058]

  - just perturbative terms also argued to be enough for a dS minimum

see e.g. summary in Cicoli, Hughes, Kamal, Marino, Quevedo, Ramos-Hamud, Villa [hep-th/2410.00097]

- yet, arguments based on the EFT-completion into a full quantum-gravity framework, and circumstantial evidence, might lead one to hypothesize the absence of potentials able to stabilize all moduli in asymptotic regions of moduli space as a general fact, if the potential is positive: for order-1 constants c and c', V is argued to fulfill

$$\frac{\partial V}{\kappa_d V} \gtrsim |c| \qquad \vee \qquad \frac{\partial^2 V}{\kappa_d^2 V} \lesssim -|c'|$$

Obied, Ooguri, Spodyneiko, Vafa [hep-th/1806.05506]
Ooguri, Palti, Shiu, Vafa [hep-th/1810.05506]

for a review of the criticisms to string-theoretic dS constructions,
see e.g. Danielsson, Van Riet [hep-th/1804.01120]

An alternative framework for moduli stabilization?

- in this talk, we will explore an alternative framework, namely a dynamical mechanism where
    1. all the fields are stabilized, despite a vanishing scalar potential
    2. the geometry is fixed into Minkowski spacetime

- we will mention extensively “moduli stabilization”:
    - by “moduli”, we mean (pseudo)scalar fields with no potential
    - by “stabilization”, we mean that the moduli have zero kinetic energy

2. Curvature-induced moduli stabilization

Minimal setup

A. n-dimensional field-space metric

$$\mathrm{d}s_n^2 = G_{ab}(\varphi)\,\mathrm{d}\varphi^a \mathrm{d}\varphi^b$$

where the coordinates are moduli φ^a, for $a = 1, \dots, n$

B. d-dimensional FLRW-metric with hyperbolic slicing

$$d\tilde{s}_{1,d-1}^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1 + r^2/\ell^2} + r^2 \mathrm{d}\Omega_{d-2}^2\right]$$

where

- the curvature of the spatial slice is $k = -1/\ell^2$
- $a = a(t)$ is the scale factor, with Hubble parameter $H = \dot{a}/a$

- we might as well add a set of homogeneous and isotropic fluids with energy densities ρ^α and pressures p^α, for $\alpha = 1, \dots, b$, with constant equation of state

$$p^\alpha = w^\alpha \rho^\alpha$$

e.g. radiation: $w = 1/(d-1)$; matter: $w = 0$; curvature anisotropies: $w = 1$

Complete field equations

- one can write the complete field equations:
  - cosmological Klein-Gordon equations for the moduli

$$\ddot{\varphi}^a + \Gamma^a{}_{bc}\dot{\varphi}^b\dot{\varphi}^c + (d-1)H\dot{\varphi}^a = 0$$

  - continuity equations for the fluids

$$\dot{\rho}^\alpha + (d-1)H(p^\alpha + \rho^\alpha) = 0$$

  - Friedmann equation for the scale factor

$$H^2 = \frac{2\kappa_d^2}{(d-1)(d-2)}\left[\frac{1}{2}\,G_{ab}\dot{\varphi}^a\dot{\varphi}^b + \sum_{\alpha=1}^{b}\rho^\alpha\right] + \frac{1}{\ell^2 a^2}$$

also implied:

  - acceleration equation

$$\dot{H} = -\frac{\kappa_d^2}{d-2}\left[G_{ab}\dot{\varphi}^a\dot{\varphi}^b + \sum_{\alpha=1}^{b}(p^\alpha + \rho^\alpha)\right] - \frac{1}{\ell^2 a^2}$$

▶ without additional fluids ($\rho^\alpha = 0$), a formal analytic solution exists, but for our purposes it is sufficient to just focus on trivial physical observations

original full solution in Bergshoeff, Chemissany, Ploegh, Trigiante, Van Riet [hep-th/0806.2310]
plus alternative derivation in the paper

Basic argument for the stabilization, pt. 1

looking closely at the field equations, one immediately realizes that

- the set of moduli acts as a perfectly-stiff fluid, i.e. a fluid with $p_\varphi = \rho_\varphi$:

$$w_\varphi = \frac{p_\varphi}{\rho_\varphi} = \frac{\frac{1}{2}\dot{\varphi}_a \dot{\varphi}^a - \not{V}}{\frac{1}{2}\dot{\varphi}_a \dot{\varphi}^a + \not{V}} = 1$$

- the terms depending on $k = -1/\ell^2$ in the Friedmann equations can be treated as an on-shell fluid with

$$w_k = -\frac{d-3}{d-1}$$

note:

  - the curvature fluid saturates the strong energy condition (SEC), namely the bound for a fluid to fulfill the inequality $w \geq w_{\mathrm{SEC}} = -\dfrac{d-3}{d-1}$
  - many known fluids fulfill the SEC, i.e. they are such that $w > w_{\mathrm{SEC}}$, such as matter, radiation, curvature perturbations, and a stiff fluid

Basic argument for the stabilization, pt. 2

- as we all know, for fluids with constant w-parameter, the continuity equation $\dot{\rho} + (d-1)H(1+w)\rho = 0$ is solved as

$$\rho = \rho_0 \left(\frac{a_0}{a} \right)^{(1+w)(d-1)}$$

- hence, because $w_\varphi > w_k = w_{\mathrm{SEC}}$, at late-enough times we have

$$\frac{\rho_\varphi}{\rho_k} \overset{t \to \infty}{\to} 0$$

  and the same holds for all other SEC-fulfilling fluids

- the energy scale of the theory is set by the Hubble parameter:

$$H^2 = \frac{2\kappa_d^2}{(d-1)(d-2)} \left[\rho_\varphi + \rho_k + \sum_{\alpha=1}^{b} \rho^\alpha \right] \overset{t \to \infty}{\sim} \frac{2\kappa_d^2 \rho_k}{(d-1)(d-2)}$$

▶ at late-enough times, the total scalar kinetic energy flows to zero:

$$\frac{1}{2} \frac{\dot{\varphi}_a \dot{\varphi}^a}{H^2} = \frac{\rho_\varphi}{H^2} \overset{t \to \infty}{\to} 0$$

note:

the full solution also shows that the total field displacement is finite

Stabilization into the Milne universe

- at late times, the only component in the universe is the curvature fluid:

$$H^2 = \frac{1}{\ell^2 a^2}$$
$$-\dot{H} = \frac{1}{\ell^2 a^2}$$

- the solution is the Milne universe, i.e. $k = -\frac{1}{\ell^2}$ and $a(t) = \frac{t}{\ell}$:

$$\begin{aligned} d\tilde{s}^2_{1,d-1} &= -\mathrm{d}t^2 + t^2\left[\frac{\mathrm{d}r^2}{\ell^2 + r^2} + \frac{r^2}{\ell^2}\mathrm{d}\Omega^2_{d-2}\right] \\ &= -\mathrm{d}t^2 + t^2\left[\mathrm{d}\chi^2 + \sinh^2\chi\,\mathrm{d}\Omega^2_{d-2}\right] \end{aligned}$$

with $\chi(r) = \mathrm{artanh}\left[\frac{r}{\sqrt{r^2+\ell^2}}\right]$

▶ the Milne universe is just the interior of a future lightcone in Minkowski spacetime, with inconvenient coordinates!

Stabilization into Minkowski spacetime

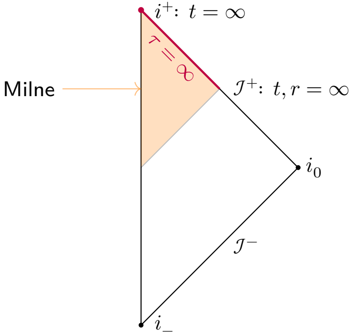
- by the coordinate transformation

$$\tau = t\cosh\chi,$$
$$\rho = t\sinh\chi$$

we see immediately that the Milne universe is a patch of Minkowski spacetime:

$$\underbrace{d\tilde{s}^2_{1,d-1} = -\mathrm{d}t^2 + t^2\left[\mathrm{d}\chi^2 + \sinh^2\chi\,\mathrm{d}\Omega^2_{d-2}\right]}_{\text{Milne}} = \underbrace{-\,\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \rho^2\,\mathrm{d}\Omega^2_{d-2} = ds^2_{\mathbb{M}^{1,d-1}}}_{\text{Minkowski}}$$

- $t = \infty$ implies $\tau = \infty$ at all radial distances!



3. Relationship with other results in the literature

Non-generality of the Milne/Minkowski solution

- a generic linear scale factor does not imply a Minkowski universe!
  - for $a = \alpha\,\frac{t}{\ell}$, with $k = -\frac{1}{\ell^2}$, the Ricci scalar is $\tilde{R} = \frac{(d-1)(d-2)}{t^2}\left(1 - \frac{1}{\alpha^2}\right)$
- this is an expected scenario in string compactifications
  - for a single scalar ϕ with a potential $V = \Lambda\,\mathrm{e}^{-\kappa_d \gamma \phi}$, if $\gamma^2 \geq 4/(d-2)$, one can argue that the late-time solution has the form

$$a(t) = \frac{1}{\sqrt{1 - \frac{4}{(d-2)\gamma^2}}}\,\frac{t}{\ell}$$

Marconnet, Tsimpis [hep-th/2210.10813]
Andriot, Tsimpis, Wrase [hep-th/2309.03938]

  - this indeed reduces to a Milne solution in the limit $\gamma \to \infty$

- scalar fields tend to have a "tracker behavior": their w-parameter is not constant and it tracks down the lowest w-value of other fluids in the theory, at late times

Copeland, Liddle, Wands [gr-qc/9711068]

- it is the scalar potential that induces the tracker behavior: with no potential, a set of scalars is instead a purely kinating fluid, with constant parameter $w_\varphi = 1$

Other examples of diluted kination energy

- it is of course not surprising that fluids (help) freeze the scalar kinetic energy relatively to the Hubble scale
  - example: see recent studies of kination phases in early moduli-driven stringy cosmologies, where radiation terminates the kination epoch

    Conlon, Revello [hep-th/2207.00567]
    Apers, Conlon, Copeland, Mosny, Revello [hep-th/2401.04064]
  - Hubble damping generally helps overcome the overshoot problem

    Brustein, Steinhardt [hep-th/9212049]
  - a similar mechanism has been exploited extensively in the past

    Barreiro, de Carlos, Copeland [hep-th/9805005]
    Huey, Steinhardt, Ovrut, Waldram [hep-th/0001112]
    Brustein, de Alwis, Martens [hep-th/0408160]
    Conlon, Kallosh, Linde, Quevedo [hep-th/0806.0809]

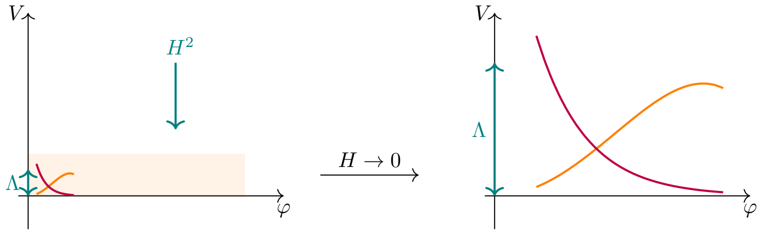
▶ the combination of negative spatial FLRW-curvature and a vanishing potential is special because it gives vanishing kinetic energy in Minkowski spacetime

Beyond moduli stabilization?

- the scalars have no Lagrangian mass and thus they might mediate long-range forces

- the stabilization mechanism is unstable to the presence of scalar potentials: as the Hubble energy asymptotes to zero, any scalar potentials (that one may have consistently neglected at higher energies) become relevant, changing the dynamics

  - if such potentials are irrelevant at least initially, the dynamics that we described might at least serve as an explanation as to why the field kinetic energy is small, and thus also circumvent the discussion on long-range forces



  - this might be of use in constructions that require some degree of fine tuning of the initial conditions to construct realizations of dark energy, i.e. $\dot{\varphi}^2(t_0) = 0$

e.g. Gomes, Hardy, Parameswaran [hep-ph/2311.08888]
Casas, Montero, Ruiz [hep-th/2406.07614]

An example: GKP compactifications

- Giddings-Kachru-Polchinski (GKP) backgrounds:
  type-IIB on a Calabi-Yau orientifold CY_3/O with RR-3- and NSNS-3-form fluxes

  Giddings, Kachru, Polchinski [hep-th/0105097]

  - axio-dilaton $\tau = \tau(y)$, complex 3-form flux $G_3 = F_3 - \tau H_3$, self-dual 5-form flux $\tilde{F}_5 = (1 + \hat{*}_{10})\,\mathrm{d}\alpha \wedge \tilde{\mathrm{vol}}_{1,3}$ with $\alpha = \alpha(y)$, localized sources $(T_{4,6})$ and

$$d\hat{s}^2_{1,9} = \mathrm{e}^{2A(y)}\, d\tilde{s}^2_{1,3}(x) + \mathrm{e}^{-2A(y)}\, d\breve{s}^2_6(y)$$

  - 4d components of Einstein's equations and 5-form flux field equation combine into

$$\breve{\nabla}^2(\mathrm{e}^{4A} - \alpha) = \tilde{R}_4 + \frac{\mathrm{e}^{8A}}{4\,\mathrm{Im}\,\tau}\,[(\breve{*}_6 G_3) - \mathrm{i}G_3]\,\breve{\cdot}\,[(\breve{*}_6 \overline{G}_3) + \mathrm{i}\overline{G}_3] + \mathrm{e}^{-4A}\,\partial(\mathrm{e}^{4A} - \alpha)\,\breve{\cdot}\,\partial(\mathrm{e}^{4A} - \alpha) + T_{4,6}$$

  - in 4d Minkowski spacetime, with $\tilde{R}_4 = 0$, we must have

$$\breve{*}_6 G_3 = \mathrm{i}G_3$$
$$\mathrm{e}^{4A} = \alpha$$

- in the 4d EFT, fluxes stabilize the axio-dilaton and the complex-structure moduli, but the Kähler multiplet $\rho = \theta + \mathrm{i}\,(\mathrm{vol}_{\mathrm{E},6})^{2/3}$ is a flat direction: the mechanism we showed stabilizes it dynamically, while solving the 10d equations of motion

4. Recap and future perspectives

Recap and future perspectives, pt. 1

- we described a generic framework where all the moduli are dynamically stabilized at constant finite values, into a Minkowski geometry
  - independently of the moduli-space curvature and of any compactification details
  - the stabilization mechanism takes place in the asymptotic future
  - throughout the evolution, the kinetic energy of the fields becomes parametrically smaller and smaller compared to the energy set by the Hubble scale, in an associated formulation in terms of an FLRW-cosmology

Recap and future perspectives, pt. 2

- the setup for the result is classical field theory:
  - the final state is a Minkowski vacuum with all the scalars fixed at a finite value
  - however, such scalars have no Lagrangian mass, hence their fluctuations might mediate long-range forces and imply time-dependent fundamental constants
  - yet, the mechanism might still help in other contexts, such as in explaining vanishing initial-time kinetic energy

- we have not addressed yet the cosmological moduli problem

Banks, Kaplan, Nelson [hep-ph/9308292]
de Carlos, Casas, Quevedo, Roulet [hep-ph/9308325]

- this scenario circumvents the claim that compactifications of 10d supergravity down to 4d Minkowski spacetime always include a massless scalar

Andriot, Horer, Marconnet [hep-th/2204.05327]

  - Minkowski vacua with no running moduli have also been obtained in non-geometric backgrounds that are mirror duals of rigid Calabi-Yau manifolds

Rajaguru, Sengupta, Wrase [hep-th/2407.16756]
Becker, Brady, Graña, Morros, Sengupta, You [hep-th/2407.16758]

Thank you!

5. Backup material

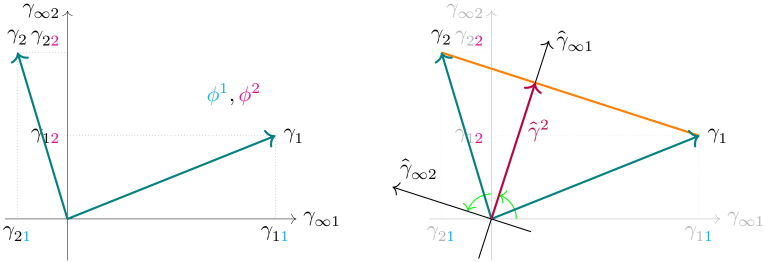
- the coupling of a scalar theory to the FLRW-background might seem to dramatically complicate the equations, but actually it makes the study of the asymptotics easier because it induces universal attractor behaviors, independently of the initial conditions
- for instance, this at the core of the intuition of the proof of scaling cosmologies as universal attractors of multi-field multi-exponential potentials

- for brevity, we set $\kappa_d = 1$ from now on

- consider a pair of canonical scalars ϕ^1, ϕ^2 with a potential

$$V = \Lambda_1\, \mathrm{e}^{-\gamma_{11}\phi^1 - \gamma_{12}\phi^2} + \Lambda_2\, \mathrm{e}^{-\gamma_{21}\phi^1 - \gamma_{22}\phi^2}$$

- after a field-basis rotation, we find $V = \left[\Lambda_1\, \mathrm{e}^{|\hat{\gamma}_{12}|\hat{\phi}^2} + \Lambda_2\, \mathrm{e}^{-|\hat{\gamma}_{22}|\hat{\phi}^2}\right] \mathrm{e}^{-\hat{\gamma}\hat{\phi}^1}$



- in particular, we can prove that the universal attractor solution reads

$$\hat{\phi}^1(t) = \hat{\phi}^1_0 + \frac{2}{\hat{\gamma}} \ln \frac{t}{t_0}$$

$$\hat{\phi}^2(t) = \hat{\phi}^2_0$$

$$H(t) = \frac{4}{d-2} \frac{1}{\hat{\gamma}^2} \frac{1}{t}$$

Hubble friction and scaling solutions, pt. 2

- this can be generalized to arbitrary numbers of scalars ϕ^a with the arbitrary multi-field multi-exponential potential

$$V = \sum_{i=1}^{m} \Lambda_i \, \mathrm{e}^{-\gamma_{ia}\phi^a}$$

Shiu, FT, Tran [hep-th/2306.07327]
Shiu, FT, Tran [hep-th/2406.17030]

note: under some assumptions on the couplings, these solutions have been known for a long time as linearly-stable exact solutions;
see e.g. Collinucci, Nielsen, Van Riet [hep-th/0407047]
Hartong, Ploegh, Van Riet, Westra [gr-qc/0602077]

- noticeably, thanks to Hubble friction, both the kinetic energy and the scalar potential scale in the same parametric way with time

$$\frac{1}{2}\,\dot{\phi}_a\dot{\phi}^a = \frac{1}{2t^2}\,(d-2)p$$
$$V = \frac{1}{2t^2}\,\big[(d-1)\,p-1\big]\,(d-2)\,p$$

with

$$H = \frac{p}{t} = \frac{4}{d-2}\,\frac{1}{\hat{\gamma}^2}\,\frac{1}{t}$$

A new notion of field-space distance?, pt. 1

- according to the distance conjecture, moving across the moduli space over an arbitrarily large geodesic distance

$$\Delta = \int_\Gamma \mathrm{d}\sigma \sqrt{G_{ab} \frac{\mathrm{d}\varphi^a}{\mathrm{d}\sigma} \frac{\mathrm{d}\varphi^b}{\mathrm{d}\sigma}} = \int_\Gamma \mathrm{d}\sigma \sqrt{2T}$$

a tower of states with mass gaps m becomes exponentially light as

$$m(\Delta) = m(0)\, \mathrm{e}^{-\alpha\Delta}$$

Ooguri, Vafa [hep-th/0605264]

- in asymptotic limits corresponding to weak coupling and/or large volume, the potential is dominated by classical effects such as background fluxes, brane tensions and internal curvatures: all such potentials are exponential in the dilaton and the radions

for general arguments, see e.g. Ooguri, Palti, Shiu, Vafa [hep-th/1810.05506]
Hebecker, Wrase [hep-th/1810.08182]

same goes for complex-structure moduli: see e.g. Grimm, Li, Valenzuela [hep-th/1910.09549]

  - such theories evolve towards the field-space asymptotics in such a way that

$$\sqrt{2T + 2V} = \sqrt{(d-1)p}\,\sqrt{2T}$$

  - evidence for axions not to change this qualitative behavior

see e.g. Sonner, Townsend [hep-th/0608068]
Cicoli, Dibitetto, Pedro [hep-th/2002.02695]
Revello [hep-th/2311.12429]
Shiu, FT, Tran [hep-th/2406.17030]

▶ maybe this hints at a generalized notion of field-space distance?

A new notion of field-space distance?, pt. 2

- if we consider

$$\Delta = \int_{\mathrm{C}} \mathrm{d}\sigma \, \sqrt{2T + 2V}$$

Debusschere, FT, Van Riet [hep-th/2407.03715]

along the asymptotic on-shell trajectory C, then

  - the distance Δ automatically reduces to the standard one for a vanishing potential
  - in many string compactifications, the conclusions in the literature do not qualitatively change, such as the refined dS conjecture
  - can be tested in simple examples, e.g. circle reductions of known AdS vacua

see also the alternative proposal in Mohseni, Montero, Vafa, Valenzuela [hep-th/2407.02705]

see also Schimmrigk [hep-th/1810.11699]
Basile, Montella [hep-th/2309.04519]

- this also allows one to incorporate other ideas of UV-complete quantum theories of gravity into EFTs where the potential does not tend to zero, e.g. ekpyrosis
  - ekpyrosis EFTs are expected to break down not at the Planck scale $m_{\mathrm{P}} = 1$, but at the species scale $\Lambda_s < 1$
  - this is automatic with the generalized notion of Δ: with negative exponential potentials one also finds the same scaling-type behavior