

Tensor force, Equation of states and Particle-vibration coupling



Ligang Cao
Beijing Normal University

Multifaceted aspects of
collaborative research on nuclear structure
at UNIMI and INFN-MI



UNIVERSITÀ
DEGLI STUDI
DI MILANO



Celebrating F.Camera's, G.Colò's and S.Leoni's 60th birthday

Equation of states and Particle-vibration coupling

What are they talking about?



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Gets to know (2004) and meet (2007) Gianluca



**LNS Skyrme
effective
interaction**

**Euro-Asian
Link**

**Exoct 07 at
Catania**



As postdoc, works with Gianluca 2008

1. Skyrme tensor force and its applications
2. Equation of states of nuclear matter
3. Single-particle states from particle-vibration coupling

Totally published 18 papers, 14 papers in the journals and 4 papers contributed the proceedings of the conferences.

2017!

1 Skyrme Tensor force and its applications

$$V_{tensor} = \frac{T}{2} \left\{ [(\sigma_1 \cdot k')(\sigma_2 \cdot k') - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k'^2] \delta(r_1 - r_2) \right. \\ \left. + \delta(r_1 - r_2) [(\sigma_1 \cdot k)(\sigma_2 \cdot k) - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k^2] \right\} \\ + U [(\sigma_1 \cdot k')\delta(r_1 - r_2)(\sigma_2 \cdot k) - \frac{1}{3}(\sigma_1 \cdot \sigma_2)\delta(r_1 - r_2)(k' \cdot k)]$$

The energy density functional for central exchange and tensor part:

$$H_{sg} = -\frac{1}{16} (t_1 x_1 + t_2 x_2) J^2 + \frac{1}{16} (t_1 - t_2) [J_p^2 + J_n^2]$$

$$J_q(r)$$

$$H_{tensor} = \frac{5}{24} (T + U) J_n J_p + \frac{5}{24} U (J_n^2 + J_p^2)$$

is the spin density

$$\Delta H = \frac{1}{2} \alpha (J_n^2 + J_p^2) + \beta J_n J_p$$

$$\alpha = \alpha_c + \alpha_T, \beta = \beta_c + \beta_T$$

$$\alpha_c = \frac{1}{8} (t_1 - t_2) - \frac{1}{8} (t_1 x_1 + t_2 x_2); \beta_c = -\frac{1}{8} (t_1 x_1 + t_2 x_2)$$

$$\alpha_T = \frac{5}{12} U; \beta_T = \frac{5}{24} (T + U)$$



SLy5
G. Colo, et. al.
PLB646(2007)227

$$\alpha_c = 80.2 \text{ MeVfm}^5$$

$$\beta_c = -48.9 \text{ MeVfm}^5$$

$$\alpha_T = -170 \text{ MeVfm}^5$$

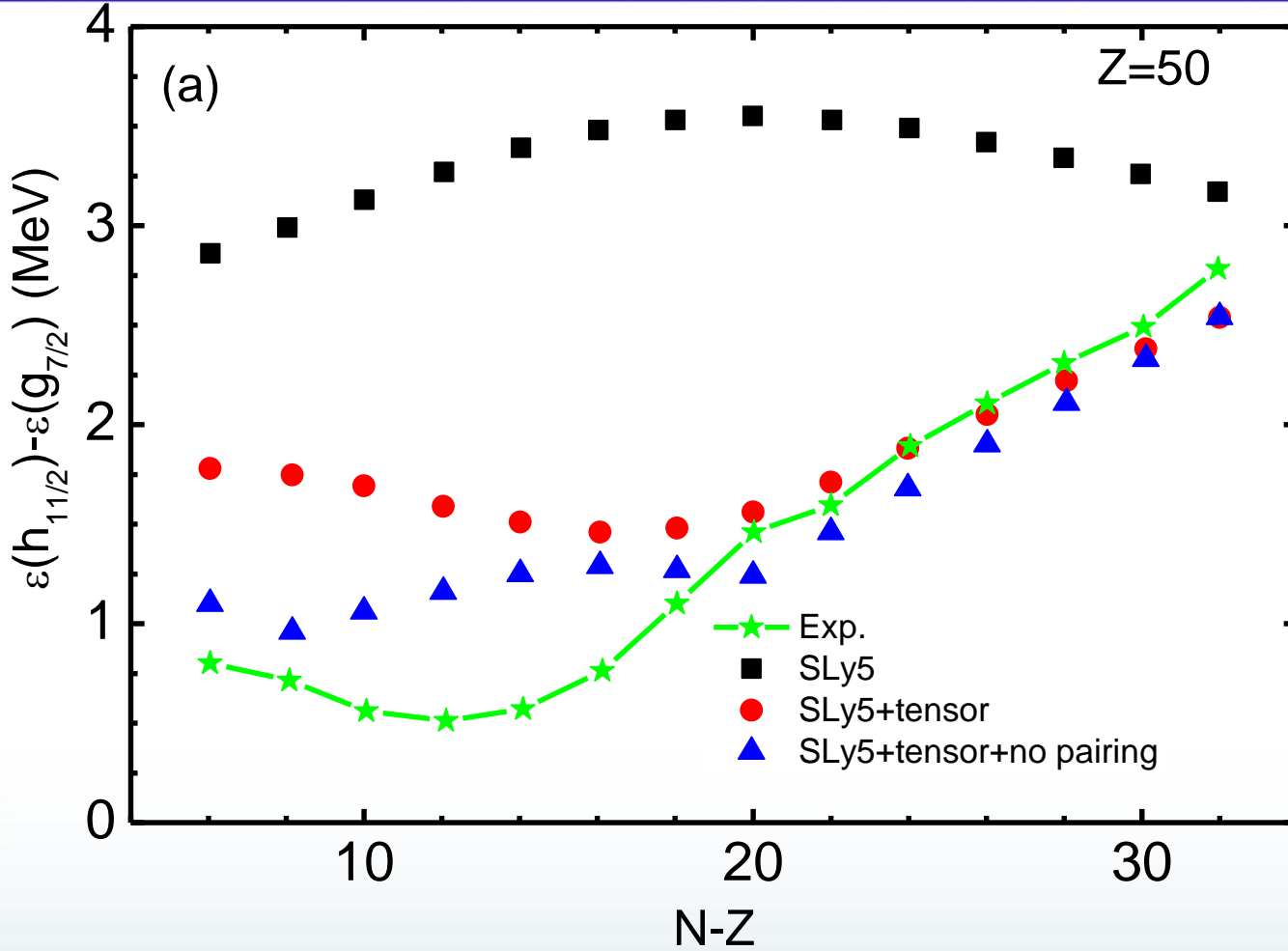
$$\beta_T = 100 \text{ MeVfm}^5$$

$$\alpha = -89.8 \text{ MeVfm}^5$$

$$\beta = 51.1 \text{ MeVfm}^5$$

$$T = 888.0 \text{ MeVfm}^5$$

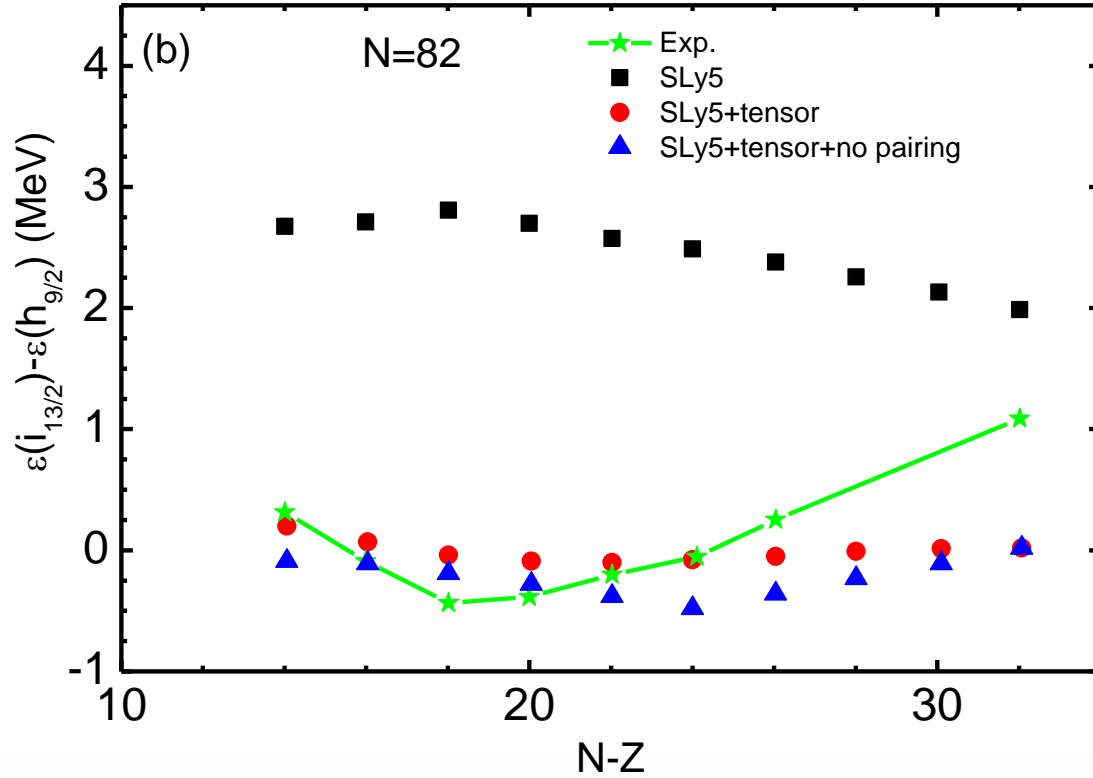
$$U = -408.0 \text{ MeVfm}^5$$



^{120}Sn neutron spin-saturated state
 $J_n = 0$

$$U_{s.o.}^p = \frac{W_0}{2r} \left(2 \frac{d\rho_p}{dr} + \frac{d\rho_n}{dr} \right) + \left(\alpha \frac{J_p}{r} + \beta \frac{J_n}{r} \right)$$

$$\alpha = -89.8 \text{ MeVfm}^5 < 0 \quad J_p > 0 \quad U_{s.o.}^p \uparrow \quad \varepsilon_{h11/2} \downarrow \quad \varepsilon_{g7/2} \uparrow$$



SLy5

G. Colo, et. al.

PLB646(2007)227

$$\alpha_c = 80.2 \text{ MeVfm}^5$$

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$$U = -408.0 \text{ MeVfm}^5$$

$$U_{s.o.}^n = \frac{W_0}{2r} \left(2 \frac{d\rho_n}{dr} + \frac{d\rho_p}{dr} \right) + \left(\alpha \frac{J_n}{r} + \beta \frac{J_p}{r} \right)$$

$$\alpha = -89.8 \text{ MeVfm}^5 < 0 \quad J_n > 0 \quad \alpha \frac{J_n}{r} < 0$$

$$\beta = 51.1 \text{ MeVfm}^5 > 0 \quad J_p > 0 \quad \beta \frac{J_p}{r} > 0 \quad U_{s.o.}^n \uparrow \quad \varepsilon_{i13/2} \downarrow \quad \varepsilon_{h9/2} \uparrow$$

¹³²Sn

We included Skyrme tensor force into RPA to investigate its effects on excited states

PHYSICAL REVIEW C **80**, 064304 (2009)

Effects of the tensor force on the multipole response in finite nuclei

PHYSICAL REVIEW C **81**, 044302 (2010)

Spin and spin-isospin instabilities and Landau parameters of Skyrme interactions with tensor correlations

Li-Gang Cao (曹李刚),^{1,2,3,4} Gianluca Colò,^{3,4} and Hiroyuki Sagawa⁵

¹*Institute of Modern Physics, Chinese Academy of Science, Lanzhou 730000, People's Republic of China*

PHYSICAL REVIEW C **83**, 034324 (2011)

Effects of tensor correlations on low-lying collective states in finite nuclei

Li-Gang Cao (曹李刚),^{1,2,3} H. Sagawa,² and G. Colò^{4,5}

¹*Institute of Modern Physics, Chinese Academy of Science, Lanzhou 730000, People's Republic of China*

²*Center for Mathematics and Physics, University of Aizu, Aizu-Wakamatsu, Fukushima 965-8560, Japan*

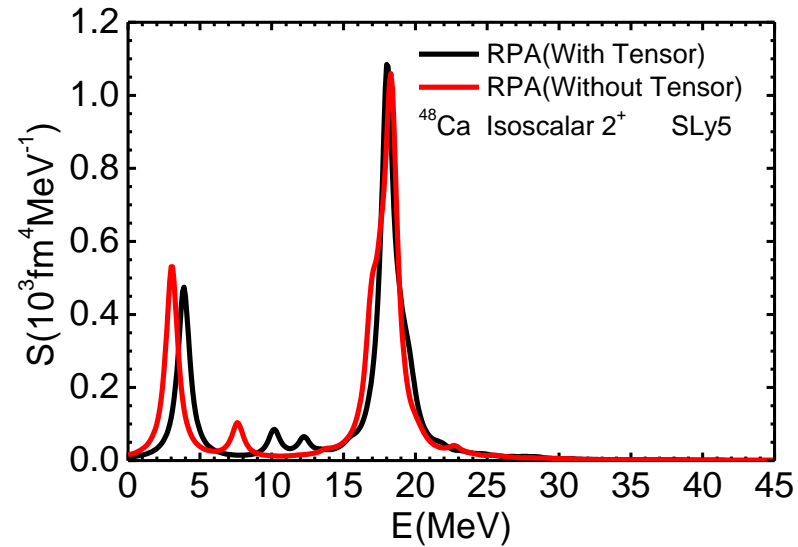
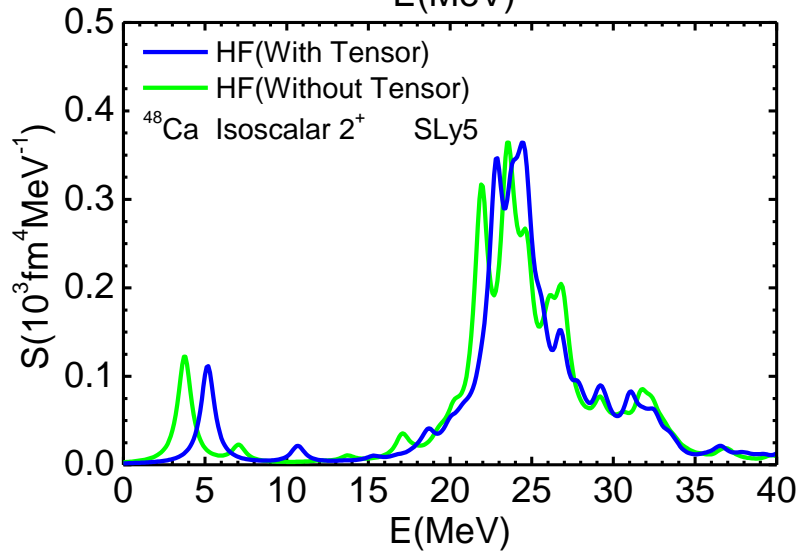
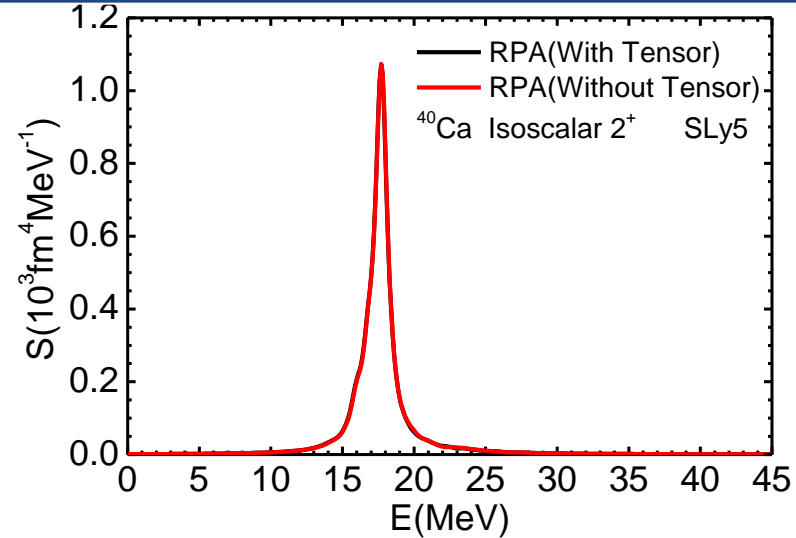
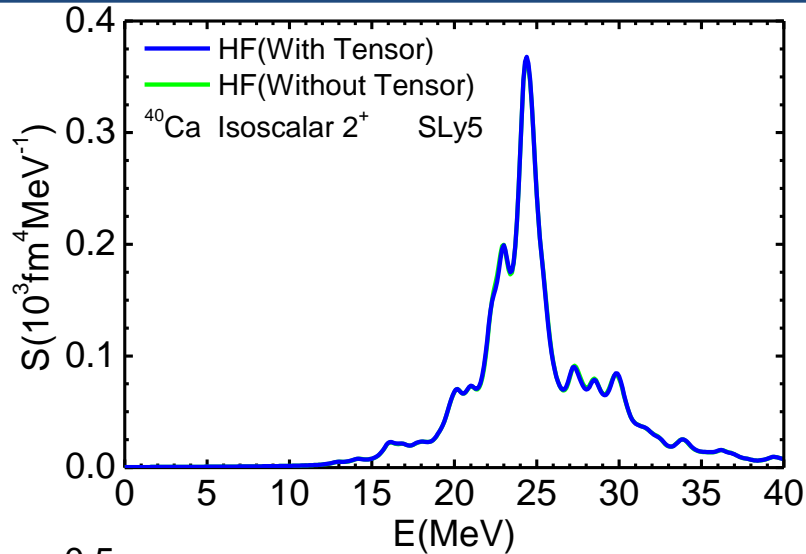
³*Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator of Lanzhou, Lanzhou 730000, People's Republic of China*

⁴*Dipartimento di Fisica, Università degli Studi di Milano, via Celoria 16, I-20133 Milano, Italy*

⁵*Istituto Nazionale di Fisica Nucleare (INFN), Sez. di Milano, via Celoria 16, I-20133 Milano, Italy*

(Received 4 January 2011; revised manuscript received 17 February 2011; published 29 March 2011)

We present a systematic analysis of the effects induced by tensor correlations on low-lying collective states of magic nuclei, by using the fully self-consistent random phase approximation (RPA) model with Skyrme interactions. The role of the tensor correlations is analyzed in detail in the case of quadrupole (2^+) and octupole (3^-) low-lying collective states in ^{208}Pb . The example of ^{40}Ca is also discussed, as well as the case of magnetic dipole states (1^+).



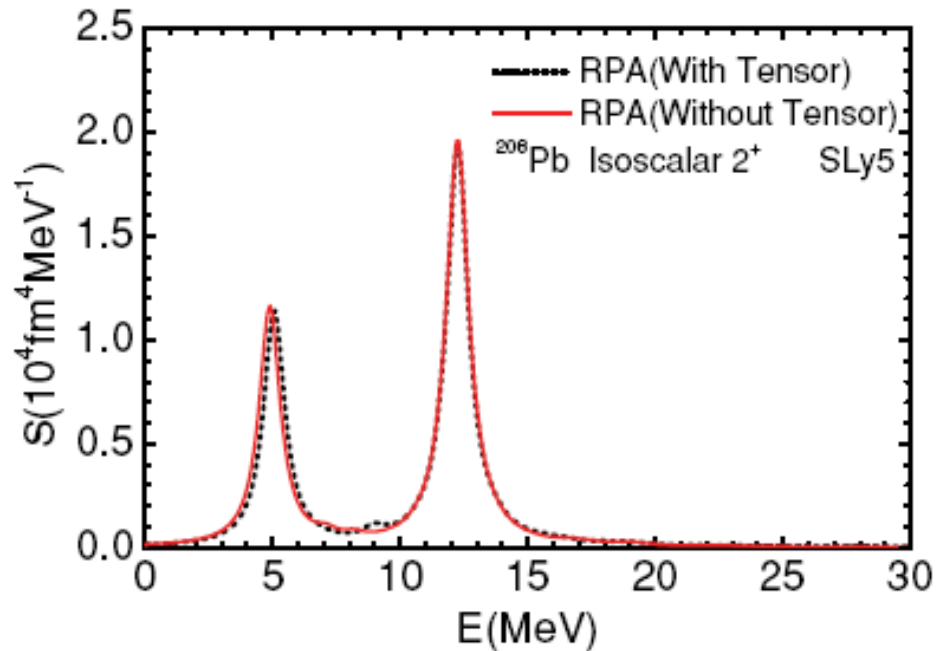
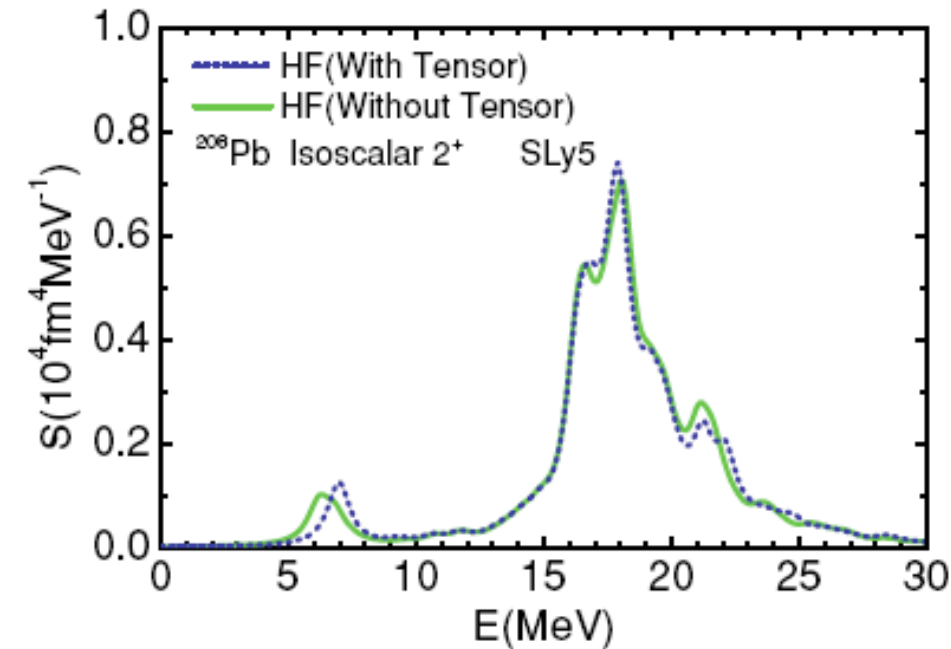
$$\Delta E_{RPA} \approx \Delta E_{HF} + \langle V_{tensor} \rangle$$

neutron $f_{7/2} \rightarrow p_{3/2}$

$$\Delta E_{HF} = 1.41 \text{ MeV}$$

$$\Delta E_{RPA} = 0.83 \text{ MeV}$$

$$\langle V_{tensor} \rangle = -0.58 \text{ MeV}$$



$$(2f_{7/2} 1h_{11/2}^{-1})_{\pi}$$

$$(2g_{9/2} 1i_{13/2}^{-1})_{\nu}$$

$$\Delta E_{RPA} \approx \Delta E_{HF} + \langle V_{tensor} \rangle$$

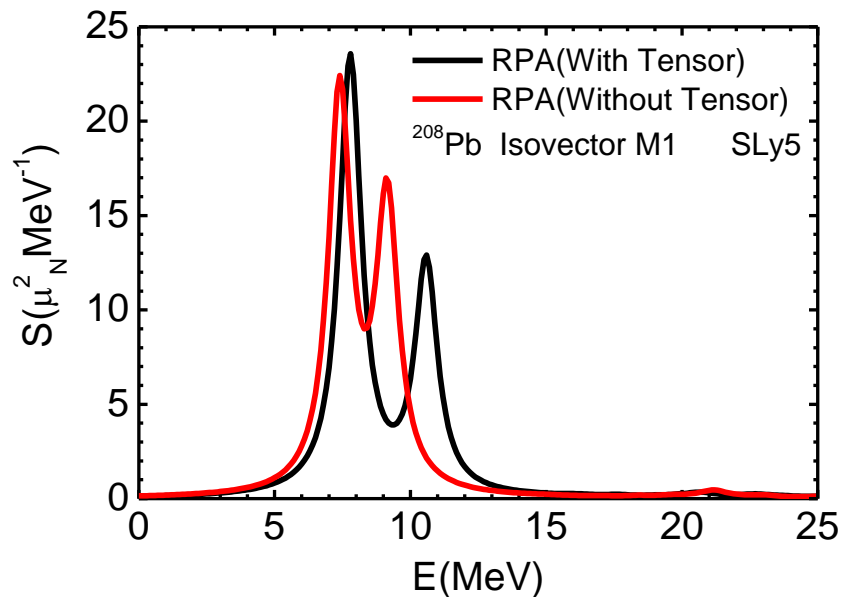
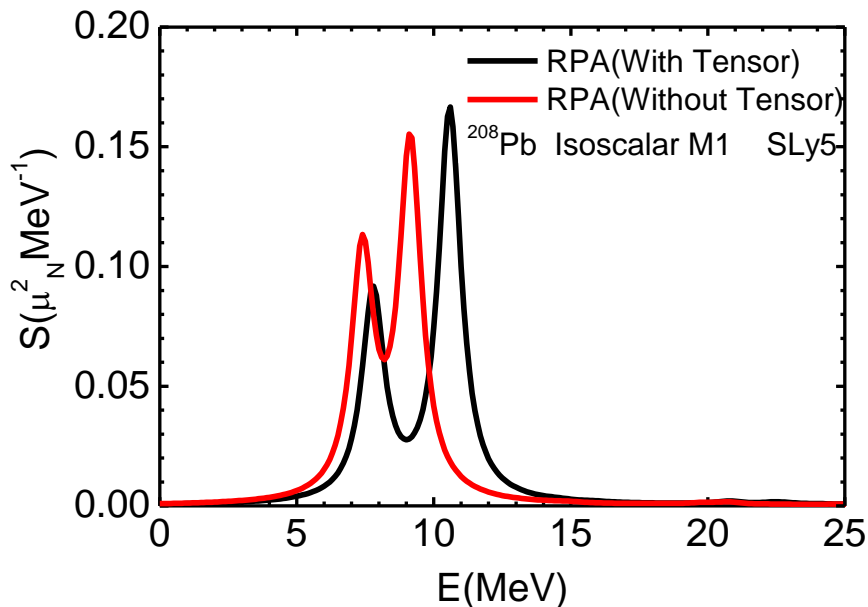
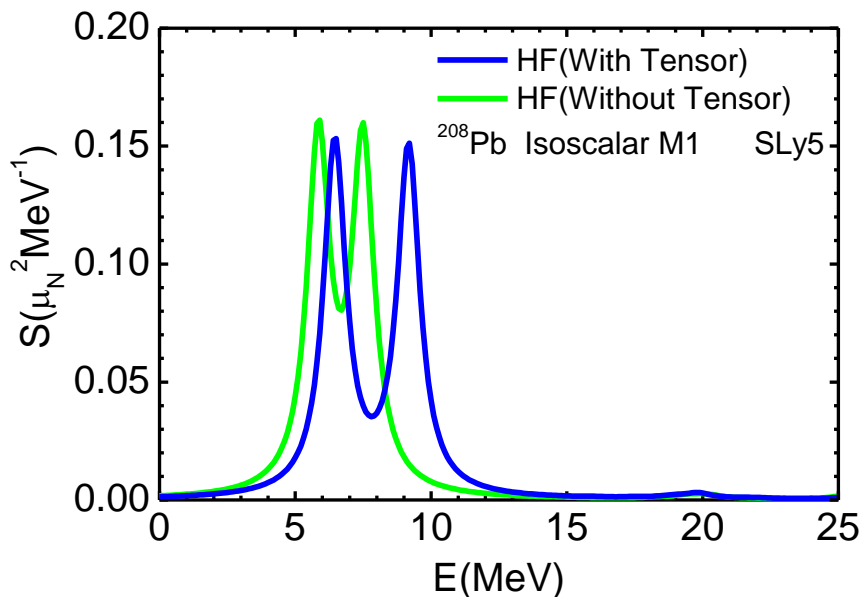
$$\Delta E_{HF} = 0.60 \text{MeV}$$

$$\Delta E_{RPA} = 0.17 \text{MeV}$$

$$\langle V_{tensor} \rangle = -0.43 \text{MeV}$$

M1 states:

Cao L. G. et.al., PRC 80, 064304(2009)



$$\Delta E_{RPA} \approx \Delta E_{HF} + \langle V_{tensor} \rangle$$

proton $h_{11/2} \rightarrow h_{9/2}$

$5.85\text{MeV} \rightarrow 6.46\text{MeV}$

$\Delta E_{HF} = 0.61\text{MeV}$

$7.39\text{MeV} \rightarrow 7.79\text{MeV}$

$\Delta E_{RPA} = 0.40\text{MeV}$

$\langle V_{tensor} \rangle = -0.21\text{MeV}$

neutron $i_{13/2} \rightarrow i_{11/2}$

$7.49\text{MeV} \rightarrow 9.17\text{MeV}$

$\Delta E_{HF} = 1.68\text{MeV}$


$9.14\text{MeV} \rightarrow 10.57\text{MeV}$

$\Delta E_{RPA} = 1.43\text{MeV}$

$\langle V_{tensor} \rangle = -0.25\text{MeV}$

PHYSICAL REVIEW C **102**, 034327 (2020)

Electric and magnetic dipole strength in $^{112,114,116,118,120,124}\text{Sn}$

S. Bassauer,^{1,*} P. von Neumann-Cosel ,^{1,†} P.-G. Reinhard,² A. Tamii,³ S. Adachi,³ C. A. Bertulani,⁴ P. Y. Chan,³ A. D'Alessio,¹ H. Fujioka,⁵ H. Fujita,³ Y. Fujita,³ G. Gey,³ M. Hilcker,¹ T. H. Hoang,³ A. Inoue,³ J. Isaak,^{1,3} C. Iwamoto,⁶ T. Klaus,¹ N. Kobayashi,³ Y. Maeda,⁷ M. Matsuda,⁸ N. Nakatsuka,¹ S. Noji,⁹ H. J. Ong,^{10,3} I. Ou,¹¹ N. Pietralla,¹ V. Yu. Ponomarev,¹ M. S. Reen,¹² A. Richter,¹ M. Singer,¹ G. Steinhilber,¹ T. Sudo,³ Y. Togano,¹³ M. Tsumura,¹⁴ Y. Watanabe,¹⁵ and V. Werner¹

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⁵*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

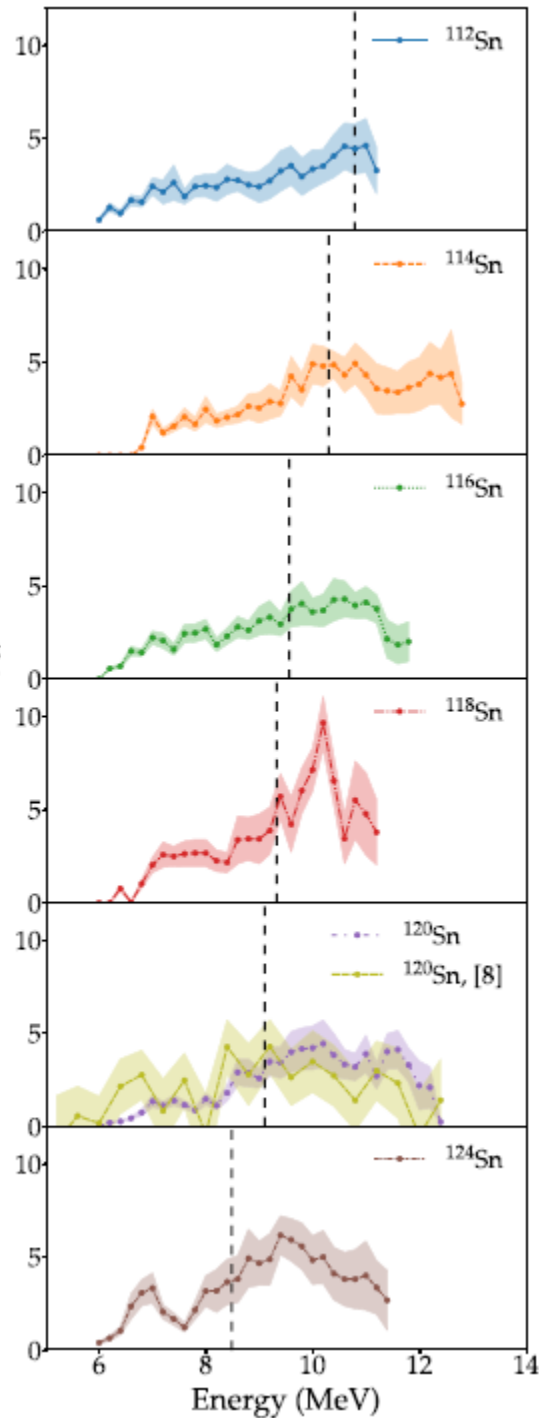
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PHYS

Electric and

$\frac{dB(M1)/dE}{dE} (\mu_N^2 \text{ MeV}^{-1})$



S. Bassauer,^{1,*} P. von Neumann-Cros
A. D'Alessio,¹ H. Fujioka,⁵ H. Fujita,³
T. Klaus,¹ N. Kobayashi,³ Y. Maeda,
V. Yu. Ponomarev,¹ M. S. R

- ¹Institut für Kernphysik
- ²Institut für Theoretische
- ³Research Center for
- ⁴Department of Physics and
- ⁵Department of
- ⁶RIKEN, Nishina Center for
- ⁷Department of

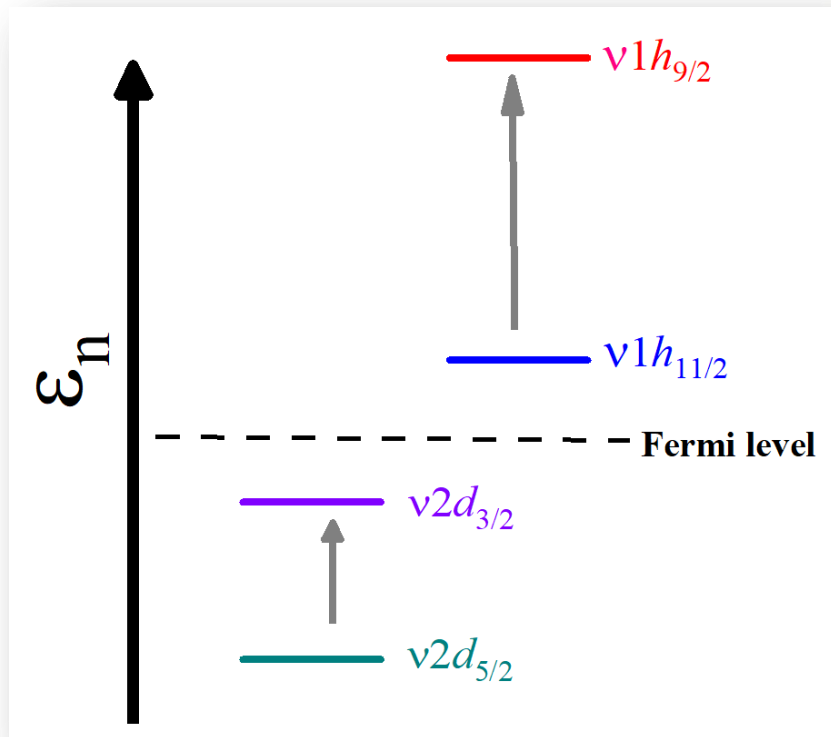
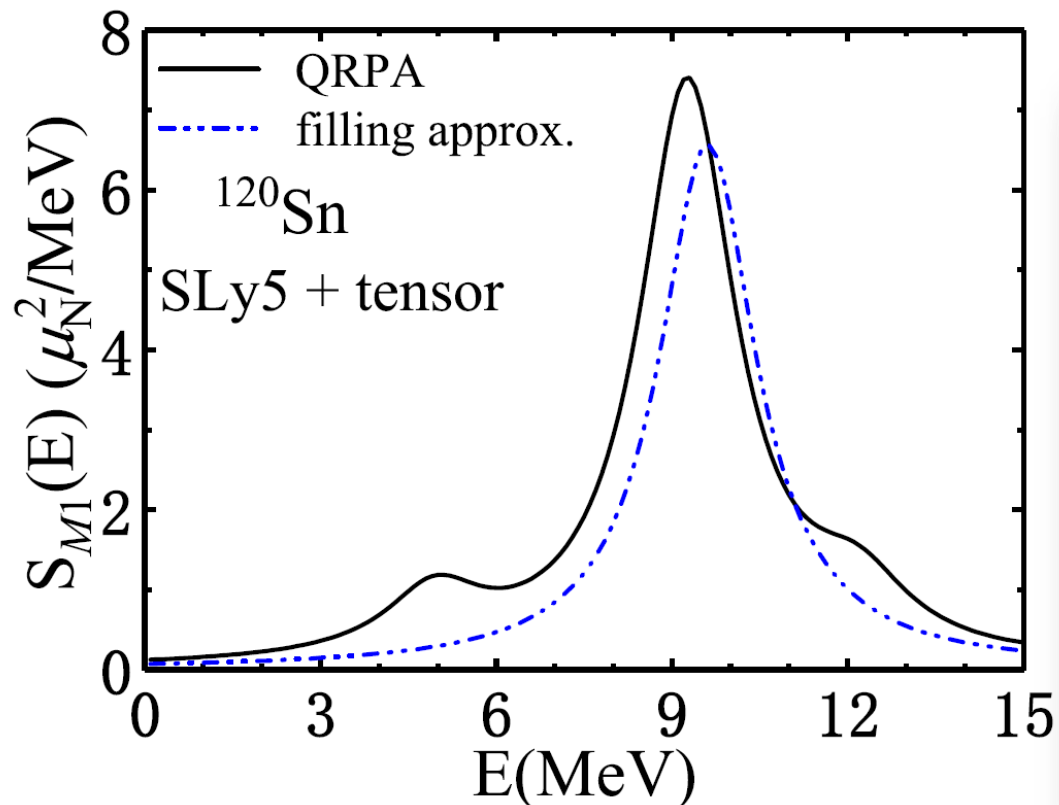
2020)

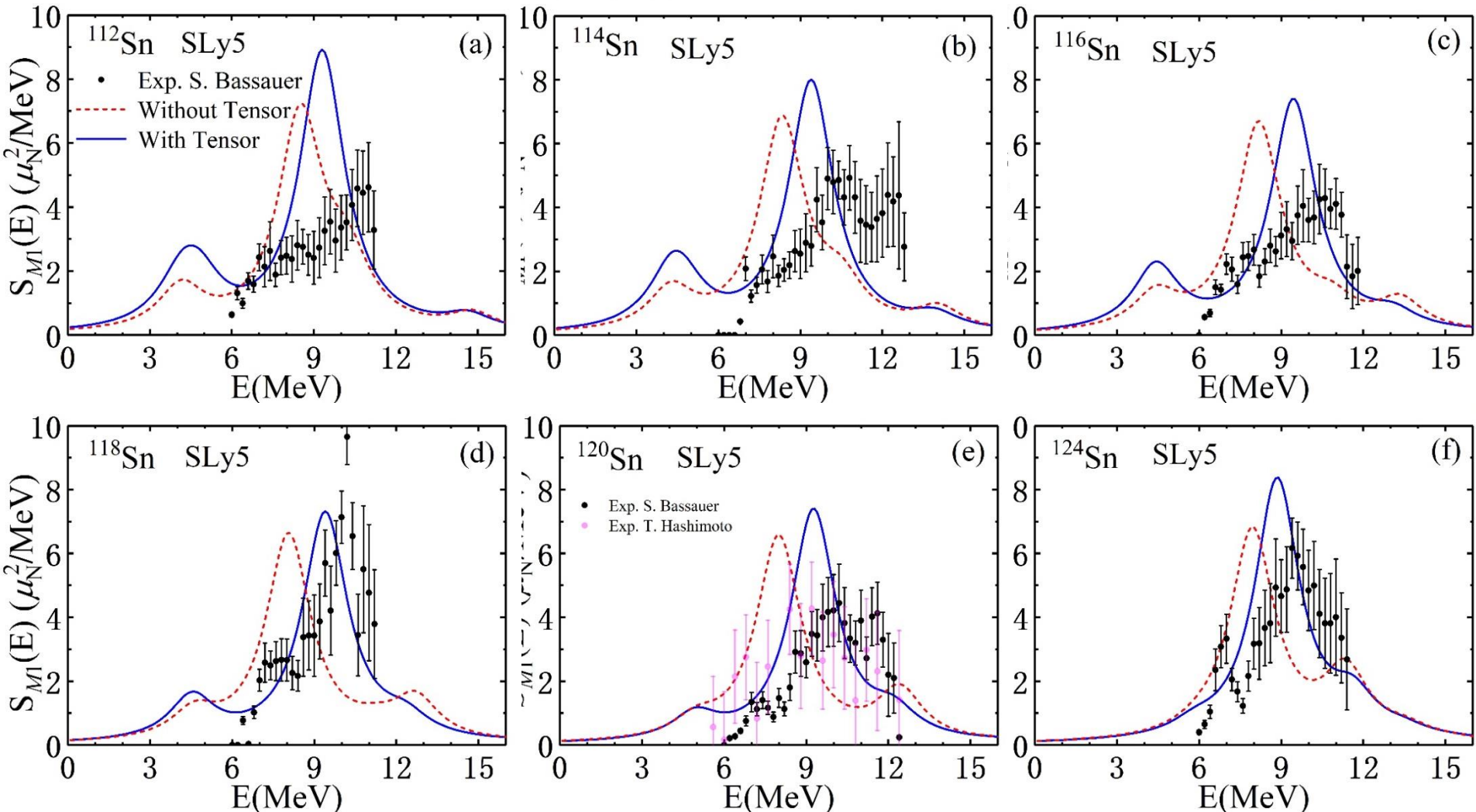
^{116,118,120,124}Sn

chi,³ C. A. Bertulani,⁴ P. Y. Chan,³
g,³ A. Inoue,³ J. Isaak,^{1,3} C. Iwamoto,⁶
I. J. Ong,^{10,3} I. Ou,¹¹ N. Pietralla,¹
er,¹ T. Sudo,³ Y. Togano,¹³
1

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- Erlangen, Germany
- Osaka 567-0047, Japan
- Commerce, Texas 75429, USA
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- 351-0198, Saitama, Japan
- 389-2192, Japan

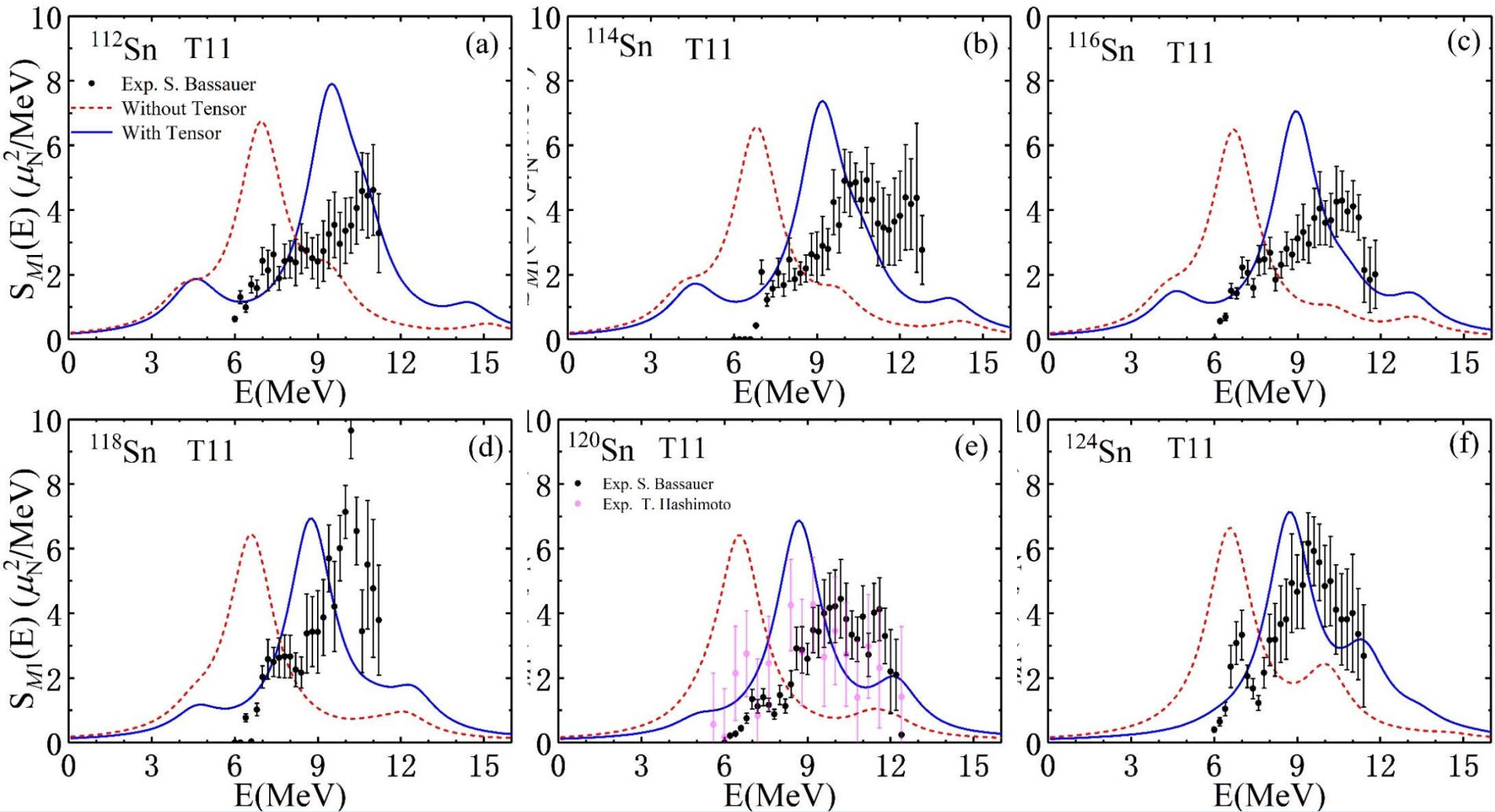
Pairing effect:





M1 states:

Sun S. et.al., PRC 109, 014312(2024)

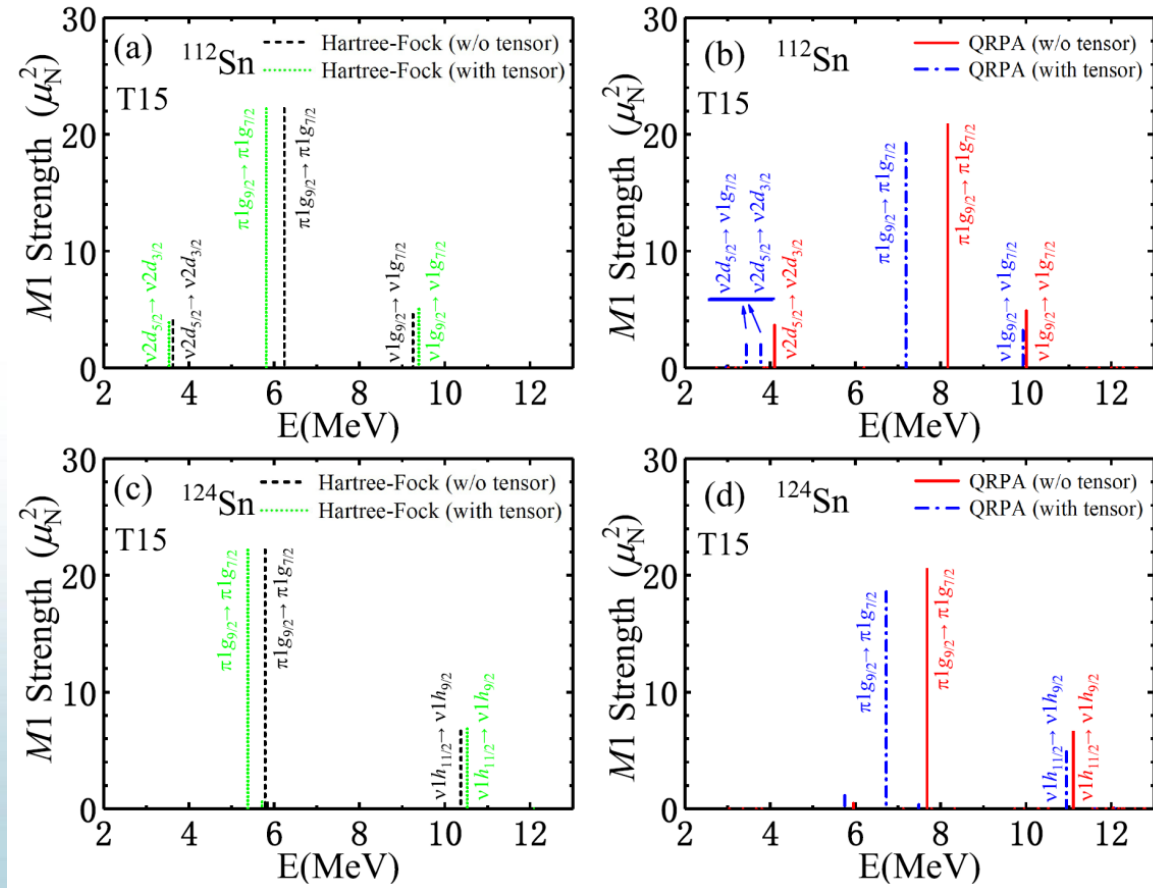


M1 states:

Sun S. et.al., PRC 109, 014312(2024)



	T	U	α	β	α_C	β_C	α_T	β_T
SLy5	888.0	-408.0	-89.8	51.1	80.2	-48.9	-170.0	100.0
T11	258.9	-342.8	-60.0	-60.0	82.8	-42.5	-142.8	-17.5
T15	-500.9	173.3	180.0	-60.0	107.8	8.3	72.2	-68.3



2. Equation of states of nuclear matter

$$\frac{E}{A}(\rho, I) \approx E_{SNM}(\rho) + S_2(\rho)I^2$$

$$E_{SNM}(\rho) = E_0 + \frac{K_0}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \frac{Q_0}{6} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^3 + O(4)$$

$$S_2(\rho) = E_{sym} + L \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \frac{K_{sym}}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \frac{Q_{sym}}{6} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^3 + O(4)$$

$$K_0 = 9\rho_0^2 \left. \frac{\partial^2 E_{SNM}(\rho)}{\partial \rho^2} \right|_{\rho=\rho_0}$$

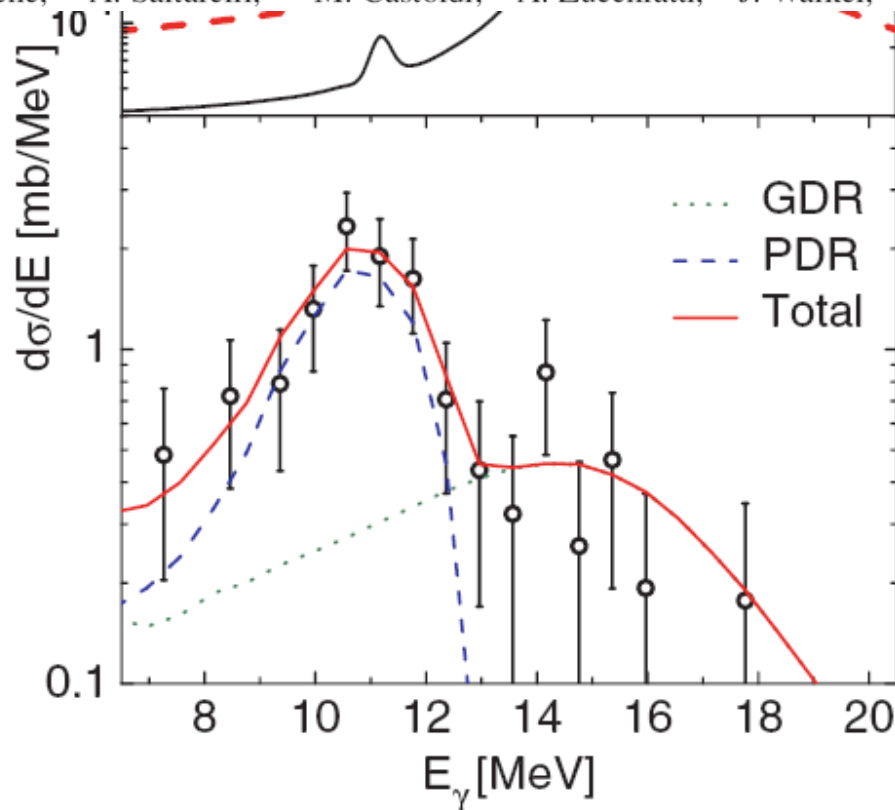
$$L = 3\rho_0 \left. \frac{\partial S_2(\rho)}{\partial \rho} \right|_{\rho=\rho_0}$$



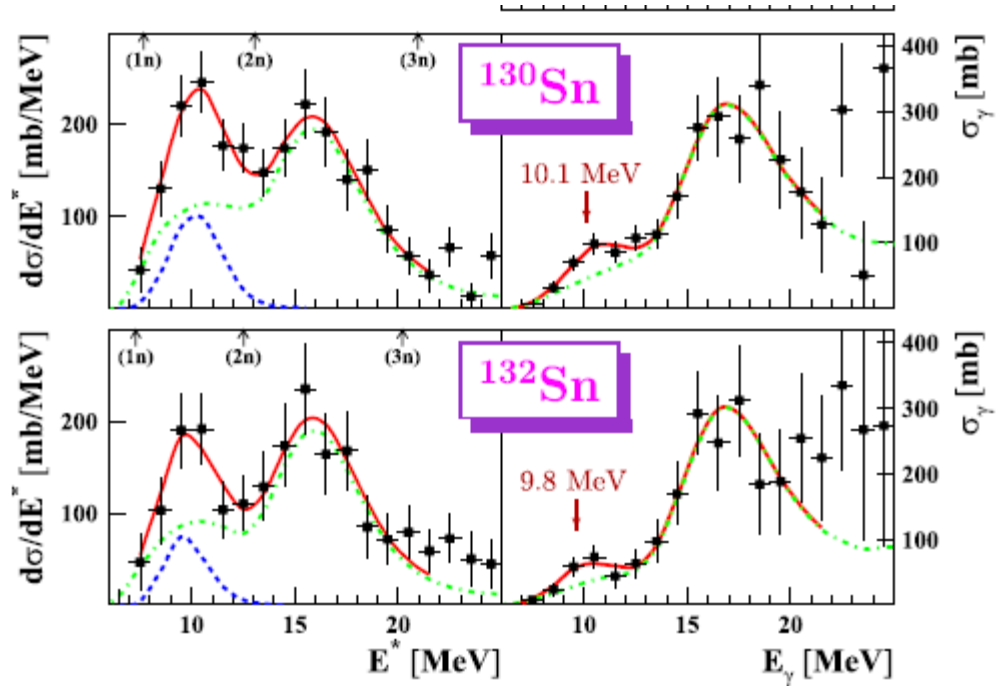
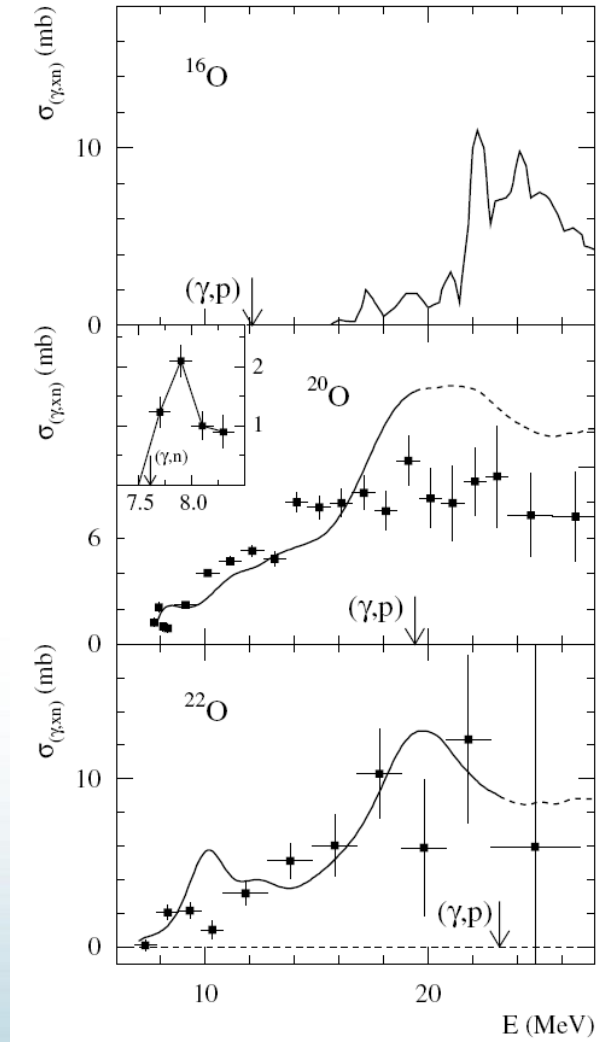
Nuclear structure
Heavy ion collision
Physics of neutron star

Search for the Pygmy Dipole Resonance in ^{68}Ni at 600 MeV/nucleon

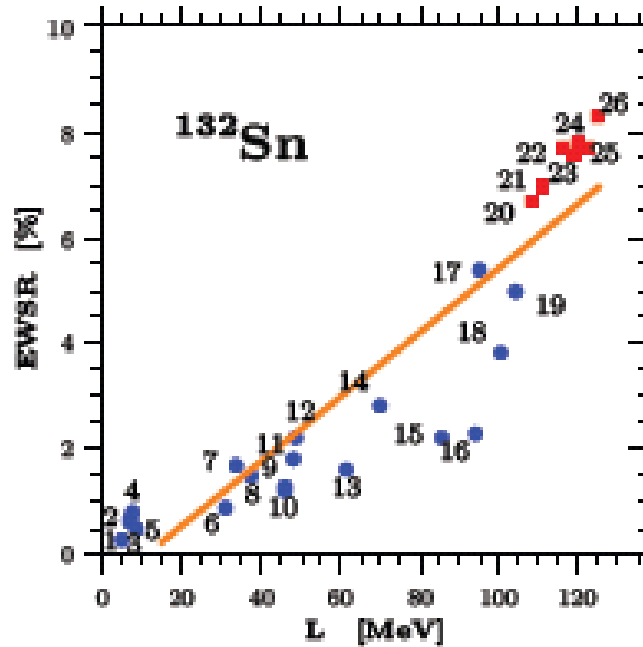
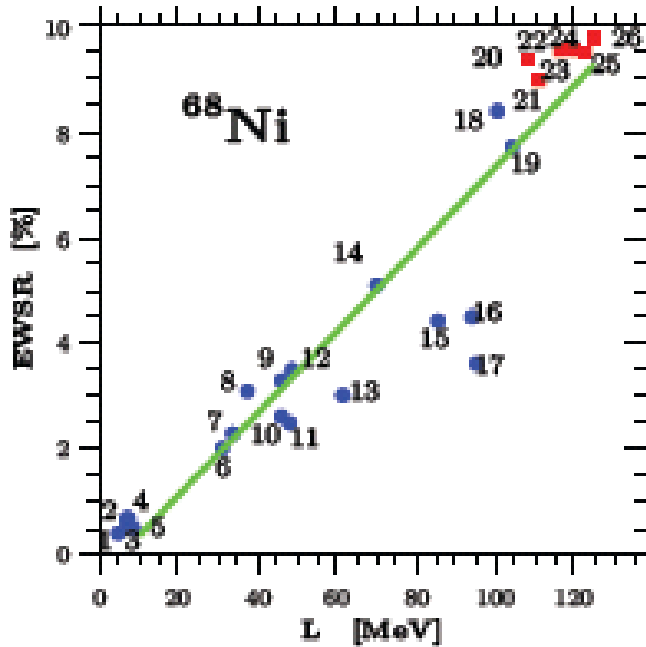
O. Wieland,¹ A. Bracco,^{1,2} F. Camera,^{1,2} G. Benzoni,¹ N. Blasi,¹ S. Brambilla,¹ F. C. L. Crespi,^{1,2} S. Leoni,^{1,2} B. Million,¹ R. Nicolini,^{1,2} A. Maj,³ P. Bednarczyk,³ J. Grebosz,³ M. Kmiecik,³ W. Meczynski,³ J. Styczen,³ T. Aumann,⁴ A. Banu,⁴ T. Beck,⁴ F. Becker,⁴ L. Caceres,^{4,*} P. Doornenbal,^{4,†} H. Emling,⁴ J. Gerl,⁴ H. Geissel,⁴ M. Gorska,⁴ O. Kavatsyuk,⁴ M. Kavatsyuk,⁴ I. Kojouharov,⁴ N. Kurz,⁴ R. Lozeva,⁴ N. Saito,⁴ T. Saito,⁴ H. Schaffner,⁴ H. J. Wollersheim,³ J. Jolie,⁵ P. Reiter,⁵ N. Warr,⁵ G. deAngelis,⁶ A. Gadea,⁶ D. Napoli,⁶ S. Lenzi,^{7,8} S. Lunardi,^{7,8} D. Balabanski,^{9,10} G. LoBianco,^{9,10} C. Petrache,^{9,‡} A. Saltarelli,^{9,10} M. Castoldi,¹¹ A. Zucchiatti,¹¹ J. Walker,¹² and A. Bürger^{13,§}



The Pygmy dipole states were also found in $^{20,22}\text{O}$ and $^{130,132}\text{Sn}$

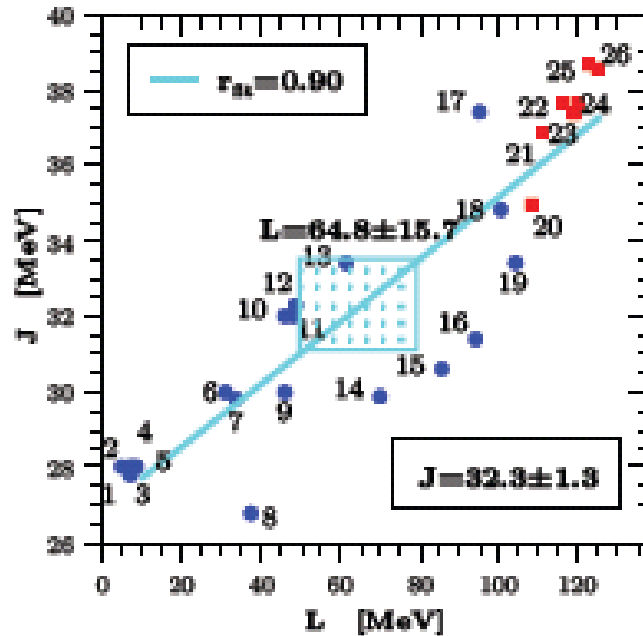
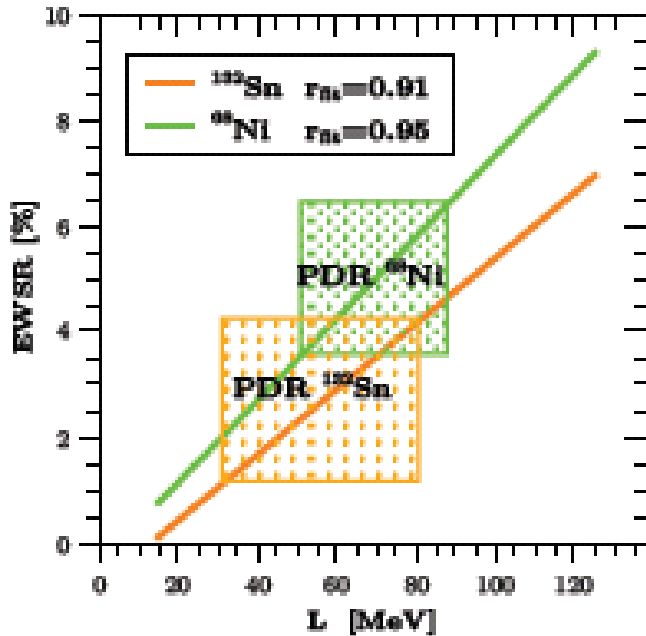


A. Leistenschneider, et al., Phys. Rev. Lett. 86, 5442 (2001)
P. Adrich, et al., Phys. Rev. Lett. 95, 132501 (2005).



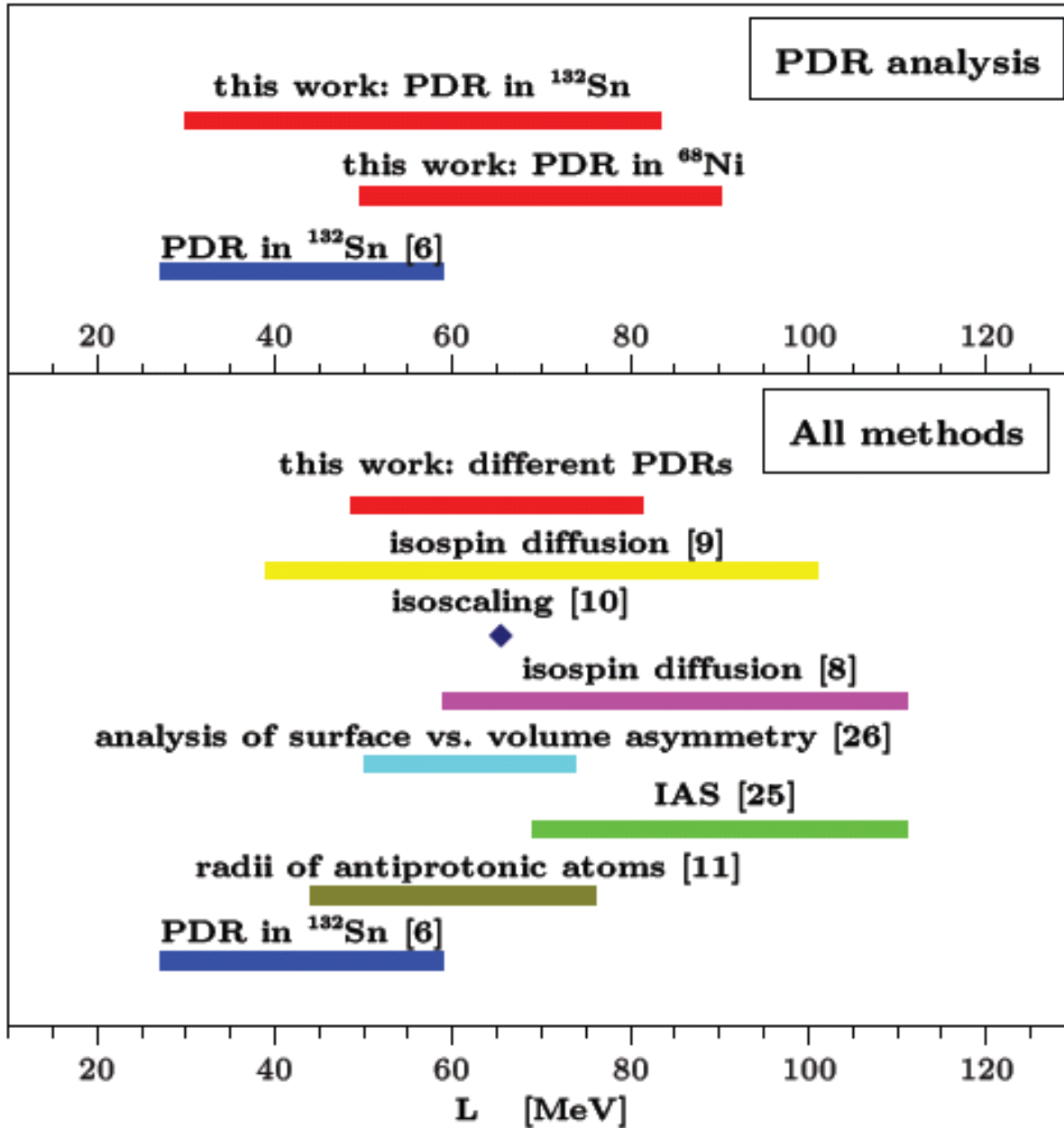
Ni68
50.3-89.4 MeV

Sn132
29.0-82.0 MeV



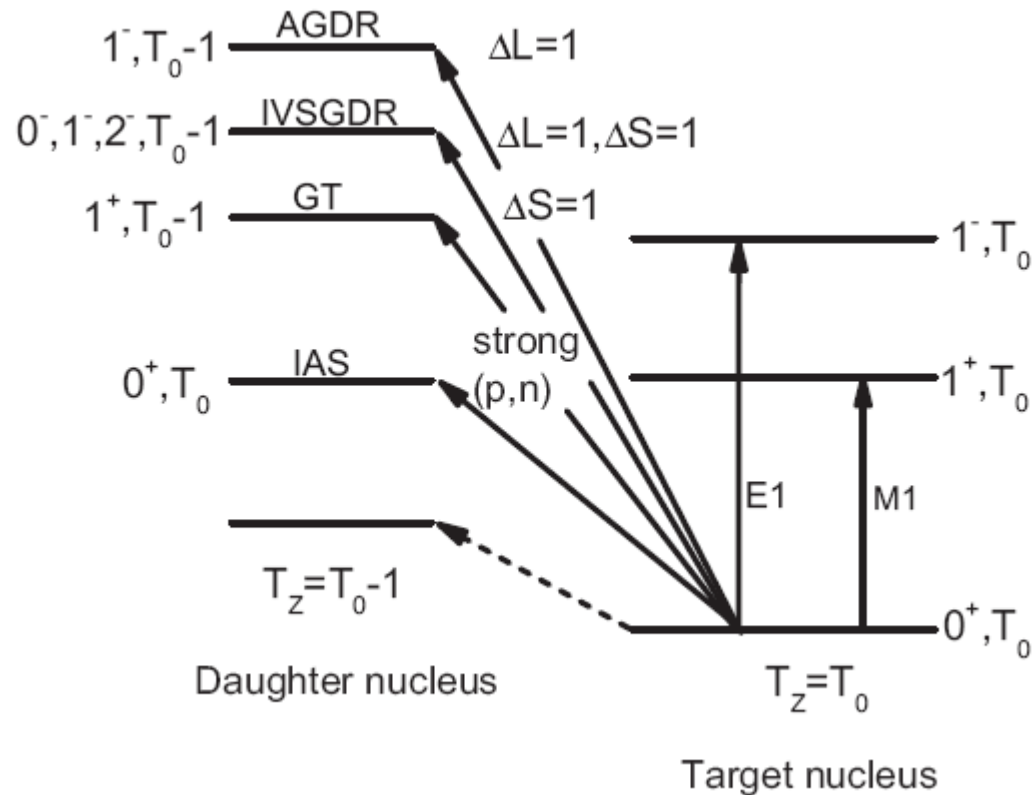
$L(\rho_0) = 64.8 \pm 15.7 \text{ MeV}$

$S(\rho_0) = 32.3 \pm 1.3 \text{ MeV}$



**Andrea Carbone
Gianluca Colo,
Angela Bracco, Li-
Gang Cao, Pier
Francesco
Bortignon, Franco
Camera, and
Oliver Wieland,
Phys. Rev. C 81,
041301(R) (2010).**

Anti-analog giant dipole resonance and the symmetry energy



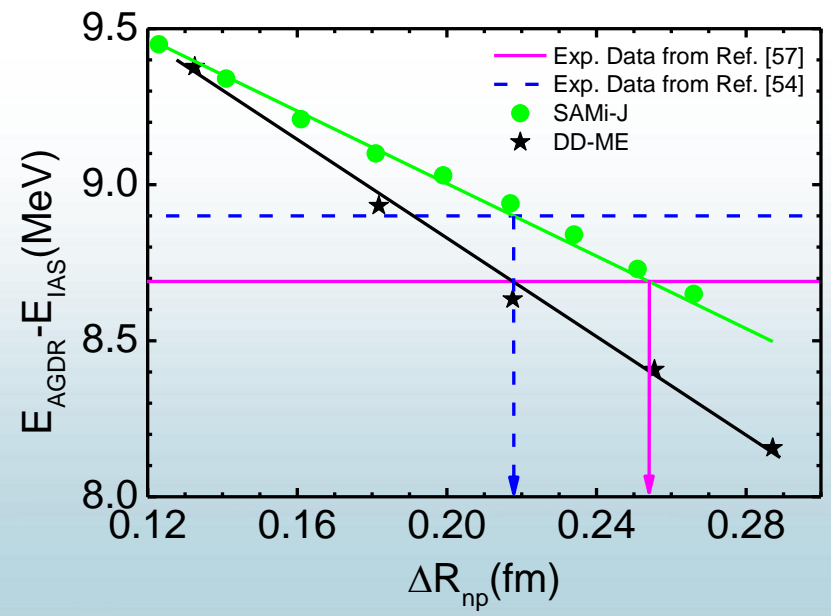
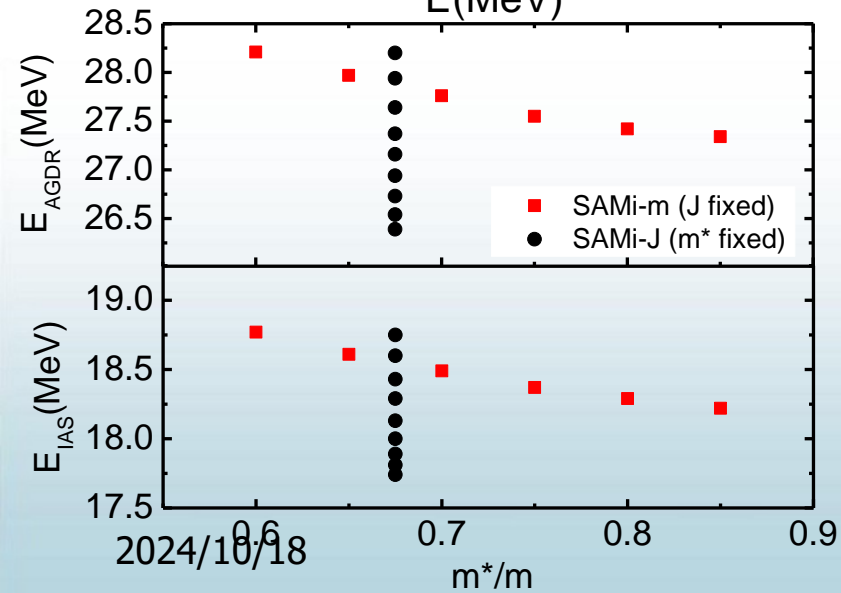
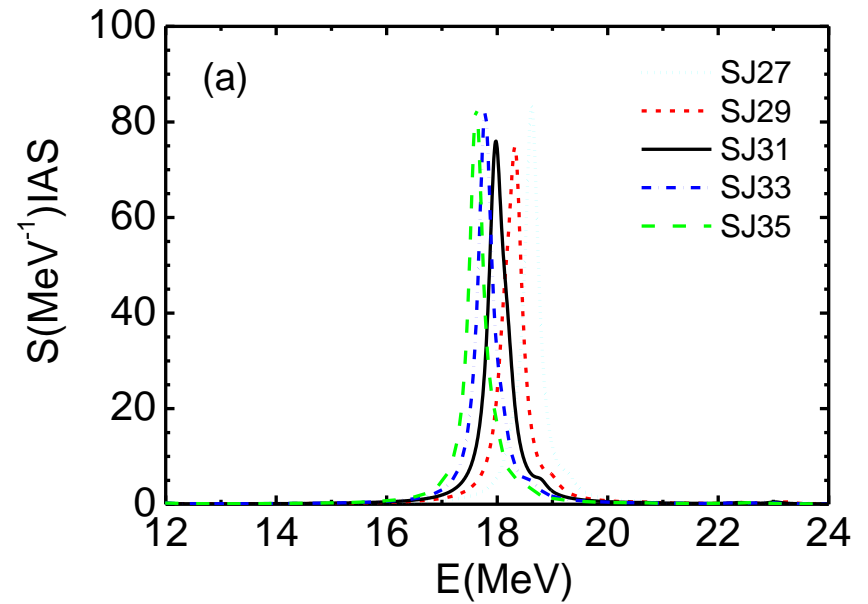
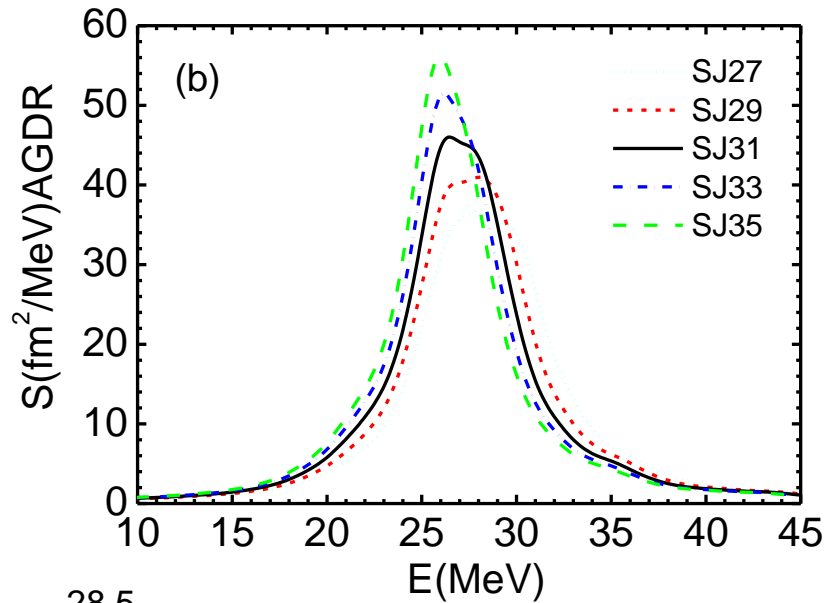
Spin-isospin GRs

- 1) Spin dipole, Yako, PRC74, 051303(R) (2006)

$$S_- - S_+ = \frac{9}{4\pi} (N \langle r^2 \rangle_n - Z \langle r^2 \rangle_p),$$

- 2) In Krmpotic's work, they claimed that the excitation energy of the AGDR is sensitive to the neutron skin thickness.

F. Krmpotic, K. Nakayama, and A. Pio Galeao, Nucl. Phys. A 399, 478 (1983).

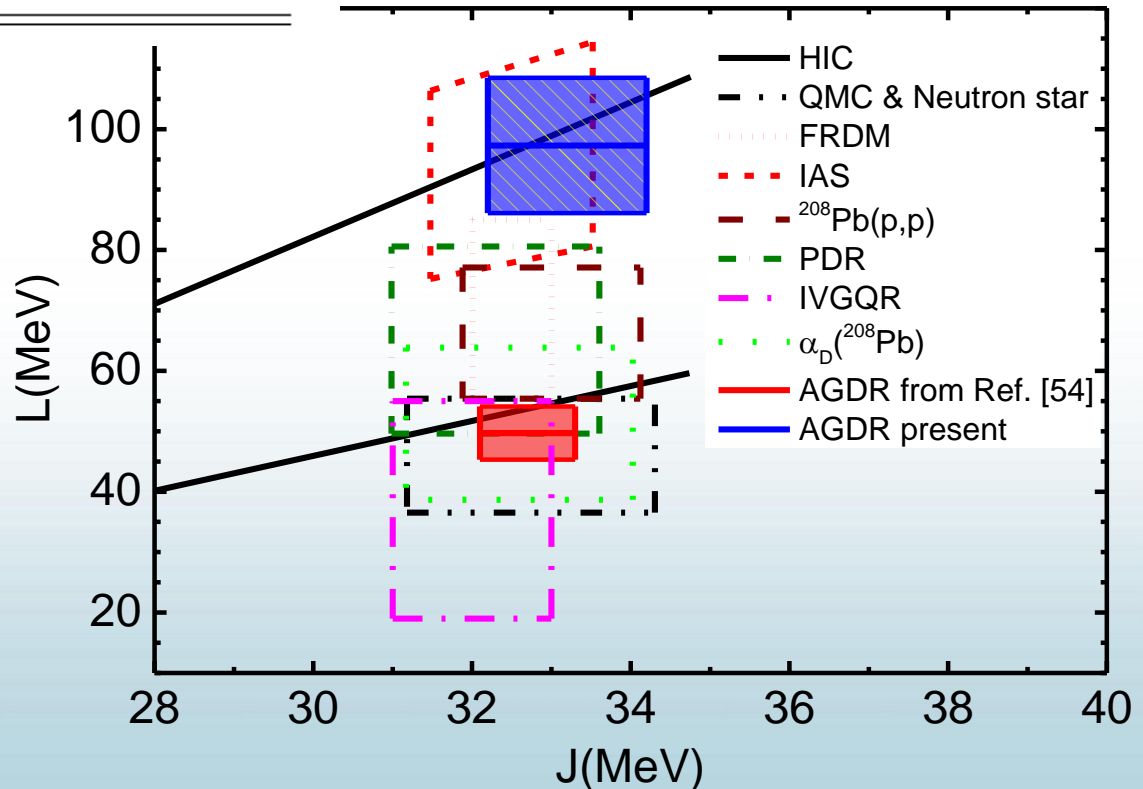


Method	Ref.	Date	ΔR_{np} (fm)
antiproton absorption	[31]	2001	0.180 ± 0.030
(α, α') IVGDR	[69]	2004	0.120 ± 0.070
PDR	[43]	2010	0.194 ± 0.024
(\vec{p}, \vec{p}')	[35]	2011	0.156 ± 0.025
α_D	[41]	2012	0.168 ± 0.022
parity violation	[29]	2012	0.330 ± 0.170
AGDR from Exp1	[57]	2013	0.216 ± 0.048
AGDR from Exp2	[54]	2013	0.190 ± 0.028
(γ, π^0)	[1]	2014	0.150 ± 0.030
AGDR from Exp1	present	2015	0.254 ± 0.062
AGDR from Exp2	present	2015	0.218 ± 0.015

$$R_{np} = 0.236 \pm 0.018 \text{ fm}$$

$$J = 33.2 \pm 1.0 \text{ MeV}$$

$$L = 97.3 \pm 11.2 \text{ MeV}$$

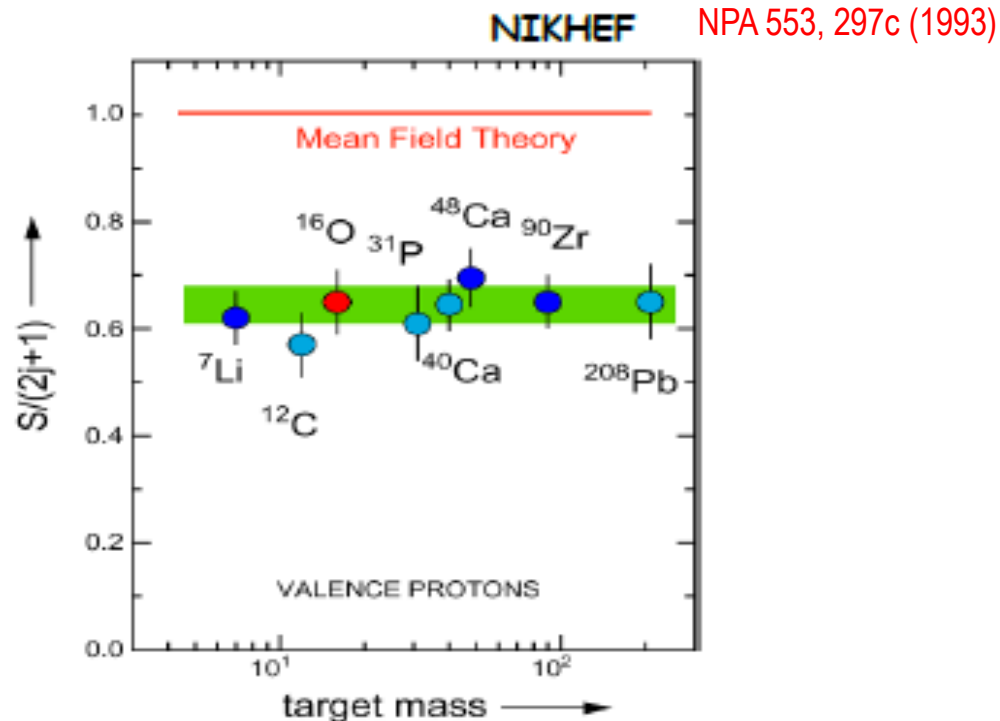


3. Single-particle states from particle-vibration coupling

limitations of EDFT

- Widths of GRs.
- Single-particle states and their spectroscopic factors

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = S^2 \left(\frac{d\sigma}{d\Omega}\right)_{\text{DWBA}}$$



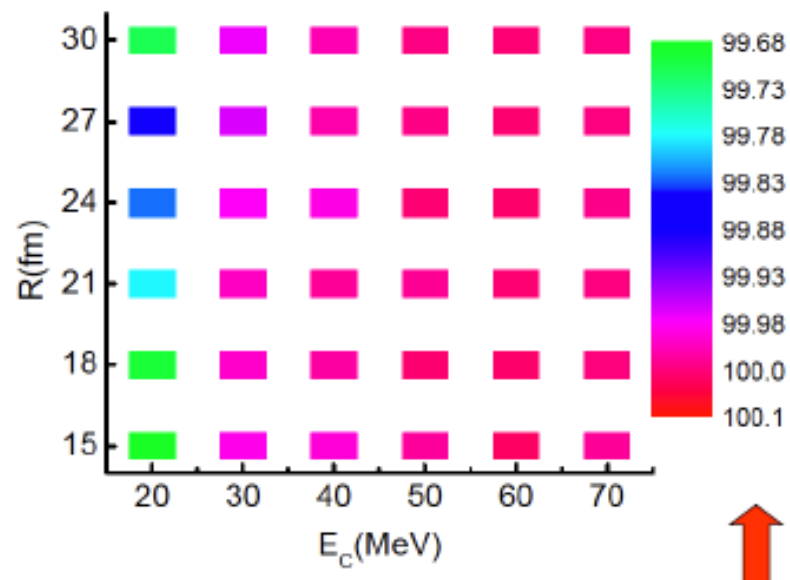
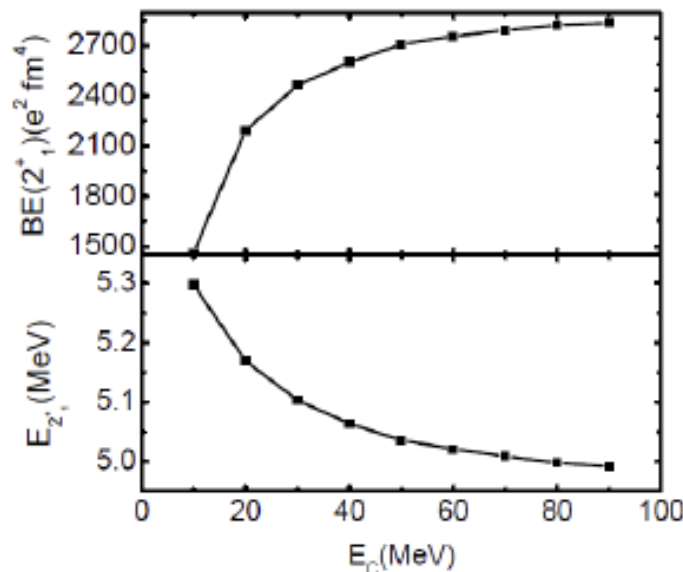
Our fully self-consistent implementation

The continuum is discretized. The basis must be large due to the zero-range character of the force. Parameters: R , E_C .

The energy-weighted sum rule should be equal to the double-commutator value: well fulfilled !

$$m_1(\hat{O}) = \sum_{\nu} E_{\nu} |\langle \nu | \hat{O} | \tilde{0} \rangle|^2 = \frac{1}{2} \langle 0 | [\hat{O}, [H, \hat{O}]] | 0 \rangle$$

^{208}Pb - SGII



G. Colò, L. Cao, N. Van Giai, L. Capelli
 Comp. Phys. Comm. 184, 142 (2013).

Percentages $m_1(\text{RPA})/m_1(\text{DC})$ [%]

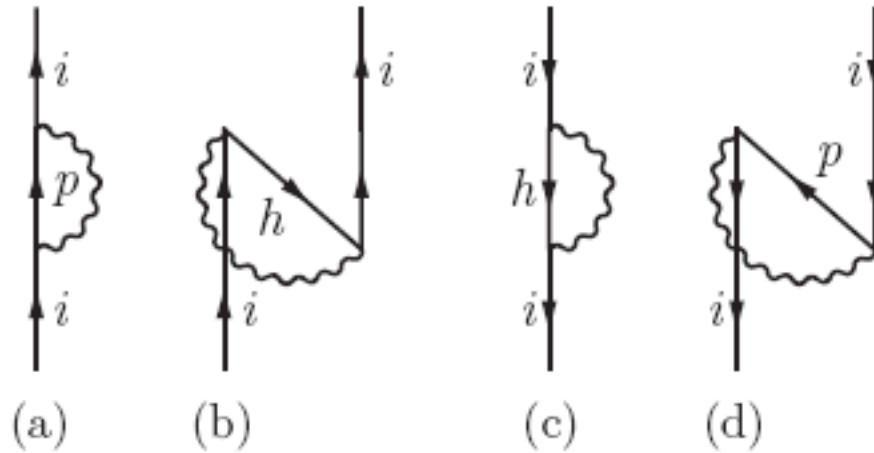
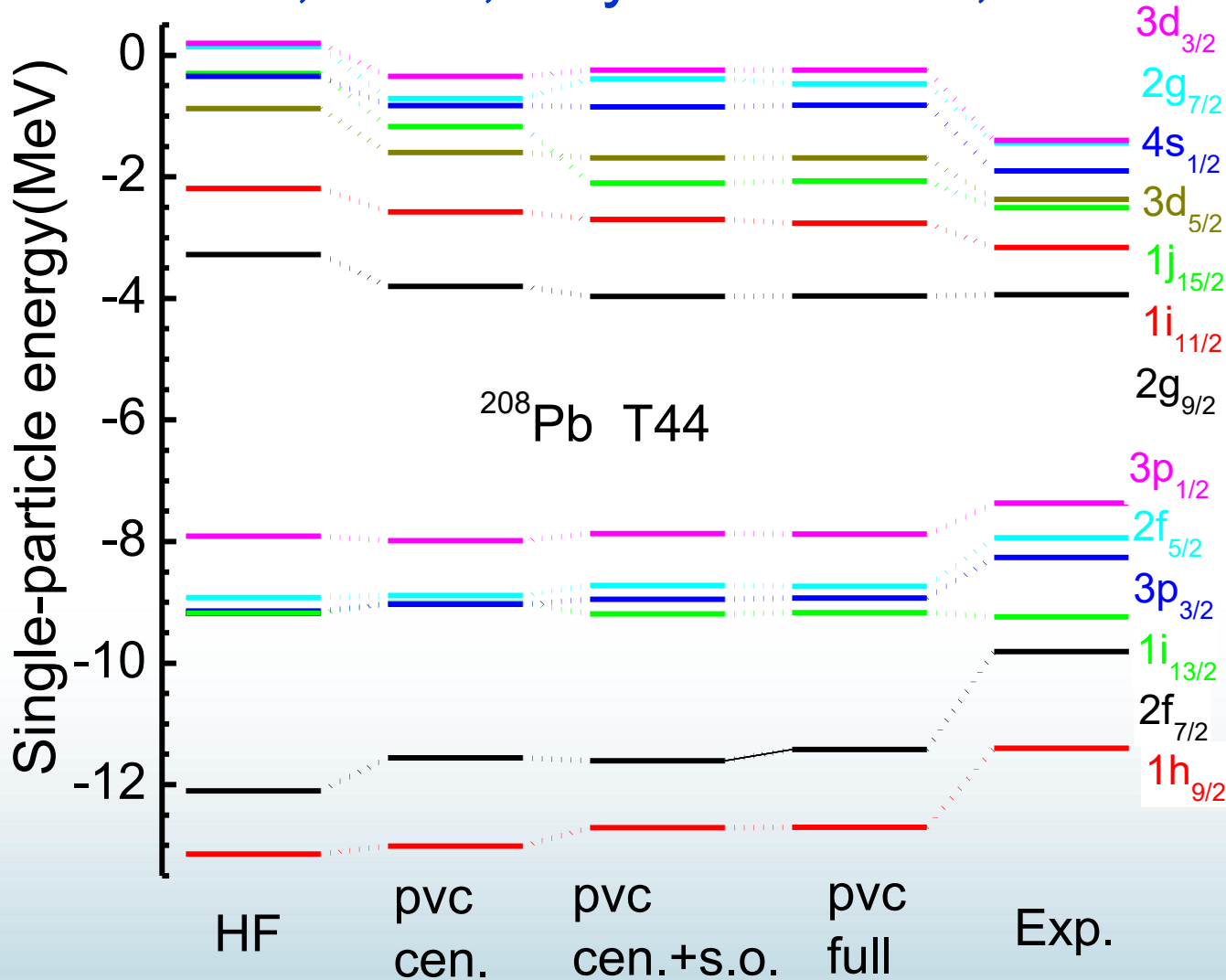


FIG. 1. The four diagrams associated with the single-nucleon self-energy. See the text for details.

$$\Sigma_i(\omega) = \frac{1}{2j_i + 1} \left(\sum_{nL, p > F} \frac{|\langle i || V || p, nL \rangle|^2}{\omega - \varepsilon_p - \omega_{nL} + i\eta} + \sum_{nL, h < F} \frac{|\langle i || V || h, nL \rangle|^2}{\omega - \varepsilon_h + \omega_{nL} - i\eta} \right),$$

Cao LG, et.al., Phys.Rev. C89, 044314 (2014)



The values of σ are 1.421, 1.002, 0.907, 0.873 for T44 .

Spectroscopic factor

$$S_{\alpha}^{\lambda} = \left(1 - \frac{\partial \Sigma_{\alpha}}{\partial \varepsilon} \right)_{\varepsilon = \varepsilon_{\alpha}^{\lambda}}^{-1}.$$

TABLE IV: The energies and spectroscopic factors of the single-particle states in ^{208}Pb in various approximations. The results are obtained by using SLy5 and T44 parameter sets. The experimental data are taken from Ref.[31, 32].

		HF	pvc central		pvc central+S.O.		pvc full		ε_i^{exp}	Spectroscopic factors	
		$\varepsilon^{(0)}$	$\Delta\varepsilon_i$	ε_i	$\Delta\varepsilon_i$	ε_i	$\Delta\varepsilon_i$	ε_i		S_i^{th}	S_i^{exp}
T44	$3d_{3/2}$	0.20	-0.55	-0.35	-0.44	-0.24	-0.44	-0.24	-1.40	0.895	1.09
	$2g_{7/2}$	0.14	-0.85	-0.71	-0.53	-0.39	-0.61	-0.47	-1.44	0.832	1.05
	$4s_{1/2}$	-0.35	-0.48	-0.83	-0.50	-0.85	-0.47	-0.82	-1.90	0.896	0.98
	$3d_{5/2}$	-0.88	-0.72	-1.60	-0.81	-1.69	-0.81	-1.69	-2.37	0.855	0.98
	$1j_{15/2}$	-0.30	-0.87	-1.17	-1.80	-2.10	-1.77	-2.07	-2.51	0.583	0.58
	$1i_{11/2}$	-2.19	-0.39	-2.58	-0.51	-2.70	-0.57	-2.76	-3.16	0.884	0.86
	$2g_{9/2}$	-3.28	-0.52	-3.80	-0.69	-3.97	-0.68	-3.96	-3.94	0.877	0.83
	$3p_{1/2}$	-7.91	-0.08	-7.99	0.04	-7.87	0.03	-7.88	-7.37	0.905	0.90
	$2f_{5/2}$	-8.92	0.03	-8.89	0.19	-8.72	0.18	-8.74	-7.94	0.888	0.60
	$3p_{3/2}$	-9.14	0.11	-9.03	0.19	-8.95	0.21	-8.93	-8.26	0.844	0.88
	$1i_{13/2}$	-9.18	0.17	-9.01	-0.01	-9.19	0.01	-9.17	-9.24	0.903	0.91
	$2f_{7/2}$	-12.10	0.54	-11.56	0.49	-11.61	0.68	-11.42	-9.81	0.580	0.95
	$1h_{9/2}$	-13.14	0.13	-13.01	0.43	-12.71	0.44	-12.70	-11.40	0.831	0.98



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Take this opportunity to give my deeply thanks to Gianluca, Franco and Silvia, happy 60th birthday !!!

