# A bit of the QRPA story

- From RPA to QRPA with Skyrme EDF: self-consistency
- Incompressibility of nuclear matter
- Astro application: electron capture with pn-QRPA
- Temperature in QRPA

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Multifaceted aspects  of collaborative research on nuclear strucutre at UNIMI and INFN-MI
17-18 oct. 2024 Physics Department, Milano

# A bit of the QRPA story:

## Working with Gianluca

- 10 articles together
- Started with QRPA extension during my PhD (around 1998)
- Gianluca was postdoc in Orsay a few years before
- Work efficiently + good humor

Organisers: « The talk should be accessible also for the youngest members of our group that may not know all such achievements and could benefit, in this way, from your talk. »

# Outline = selected common topics

- From RPA to QRPA with Skyrme EDF: self-consistency
- Incompressibility of nuclear matter
- Astro application: electron capture with pn-QRPA
- Temperature in QRPA

N.B. : of course, this is just a small part of Gianluca's scientific activity (but I have only 20 min !)

# What is the RPA ?

- D. Pines and D. Bohm, Phys. Rev. 85 (1952) 338: collective oscillation of dense electron gas.

They use the Fourier transform of the density and neglect the $\exp[i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{x}_i]$ term

> These terms tend to average out to zero, since there are a very large number of particles distributed very nearly in random positions. As a first approximation, we neglect such terms. This procedure we call the random phase approximation.

- In the case of nuclei, the RPA corresponds to take into account the effect of the residual interaction as a perturbation of the mean-field h:

$$h[\rho+\delta\rho] \simeq h[\rho] + \frac{\partial h[\rho]}{\partial\rho}\delta\rho \quad \text{with} \quad \frac{\partial h[\rho]}{\partial\rho} = \boxed{\frac{\partial^2 E[\rho]}{\partial\rho^2} \equiv V_{res}}$$

- E[ρ] is the Energy Density Functional (EDF).

A proper description can be reached when a unique EDF gives both h and $V_{res}$

- Orsay: Skyrme EDF tradition since D. Vautherin

# What is the QRPA ?

- The RPA equations in matricial form:

$$\begin{pmatrix} A & B \\ B* & A \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \hbar\omega \begin{pmatrix} X^\nu \\ -Y^\nu \end{pmatrix} \xrightarrow{\text{X,Y}} \delta\rho$$

with $$A_{minj} = (\epsilon_m - \epsilon_i)\delta_{mn}\delta_{ij} + \frac{\partial^2 E[\rho]}{\partial\rho_{im}\partial\rho_{nj}}$$

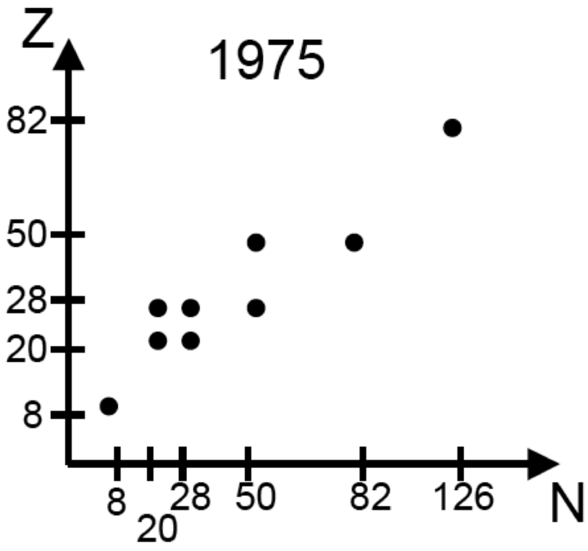
$$B_{minj} = \frac{\partial^2 E[\rho]}{\partial\rho_{im}\partial\rho_{jn}}$$

- Pairing among nucleons ⟶ quasiparticle=superposition of a particle and a hole

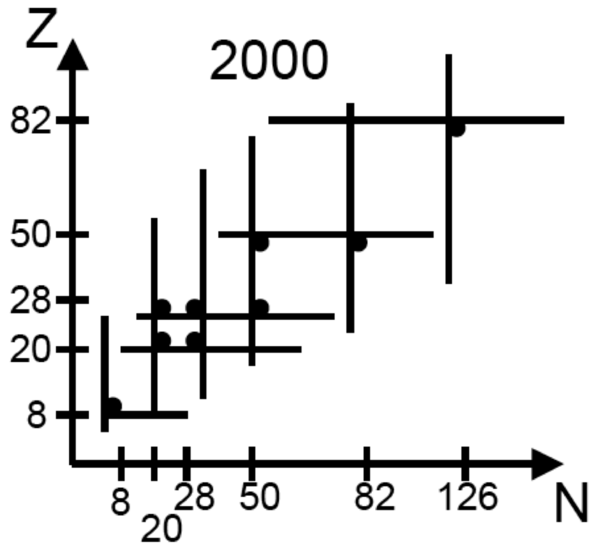
- QRPA equations: same form than the RPA ones but A and B contain additionnal terms due to particle-particle and hole-hole configurations, in addition to the particle-hole ones

# A brief history of (Q)RPA with EDF

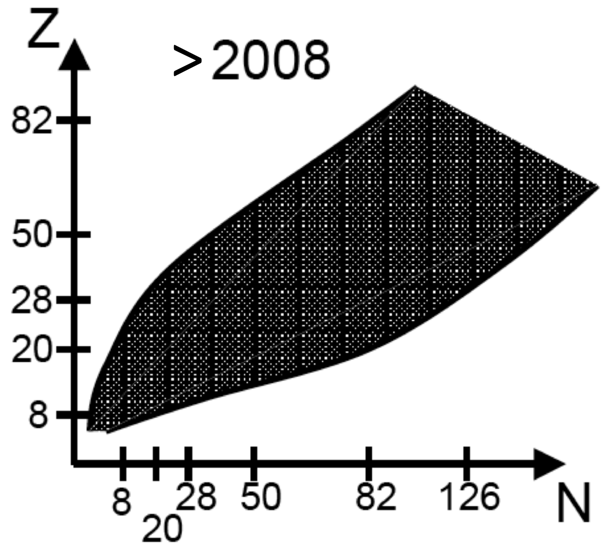
RPA

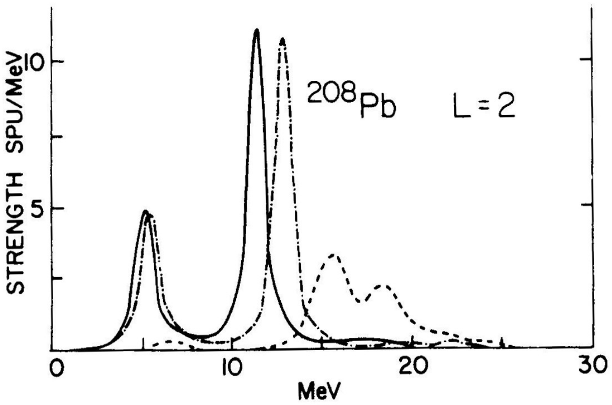



QRPA

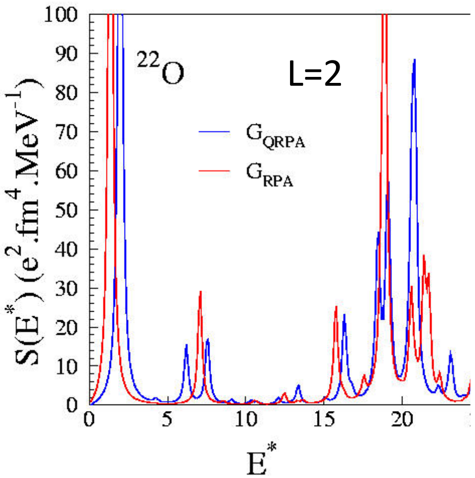



Deformed-QRPA

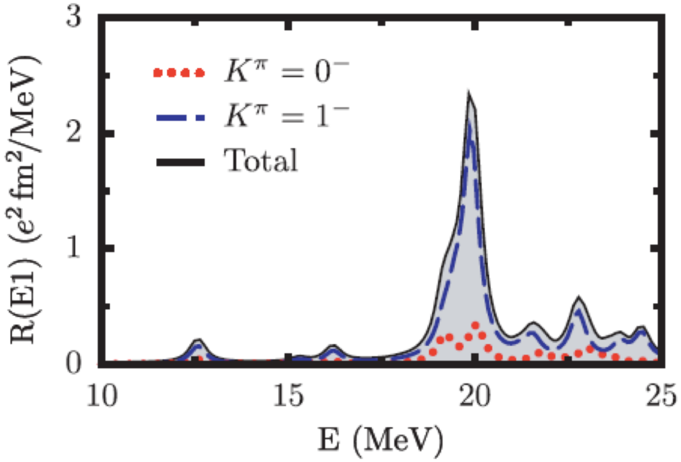



G.F. Bertsch et al., PRC 18, 125 (1975)




E. Khan, G. Colò et al. (experimentalist), PLB 490, 45 (2000)




D. Pena Arteaga et al., PRC 77, 034317 (2008)

# From RPA to QRPA

ELSEVIER

Nuclear Physics A 706 (2002) 61–84

NUCLEAR PHYSICS A

www.elsevier.com/locate/npe

## Folding model analysis of elastic and inelastic proton scattering on sulfur isotopes

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Self-consistent QRPA opened many applications for QRPA for nuclear structure, and also reactions or astrophysics applications

# Description of reaction data from QRPA

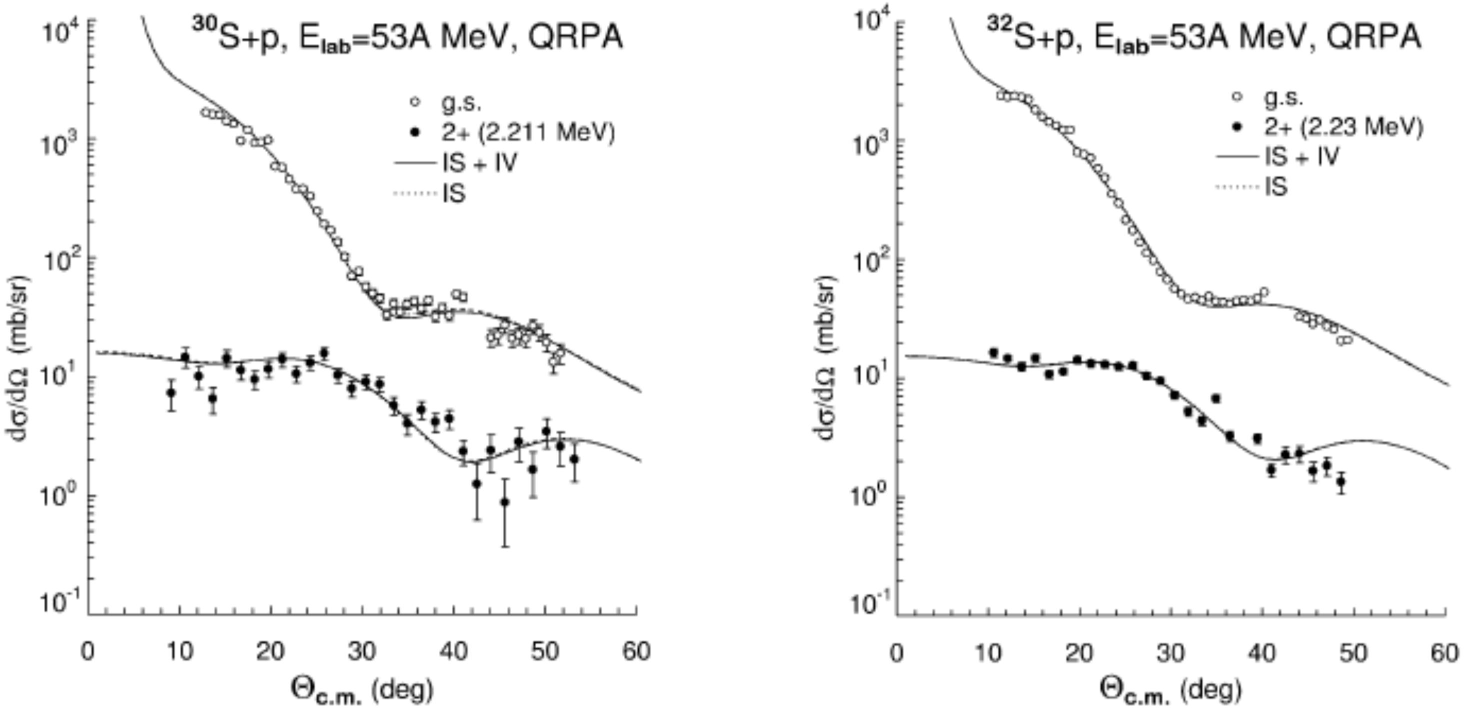
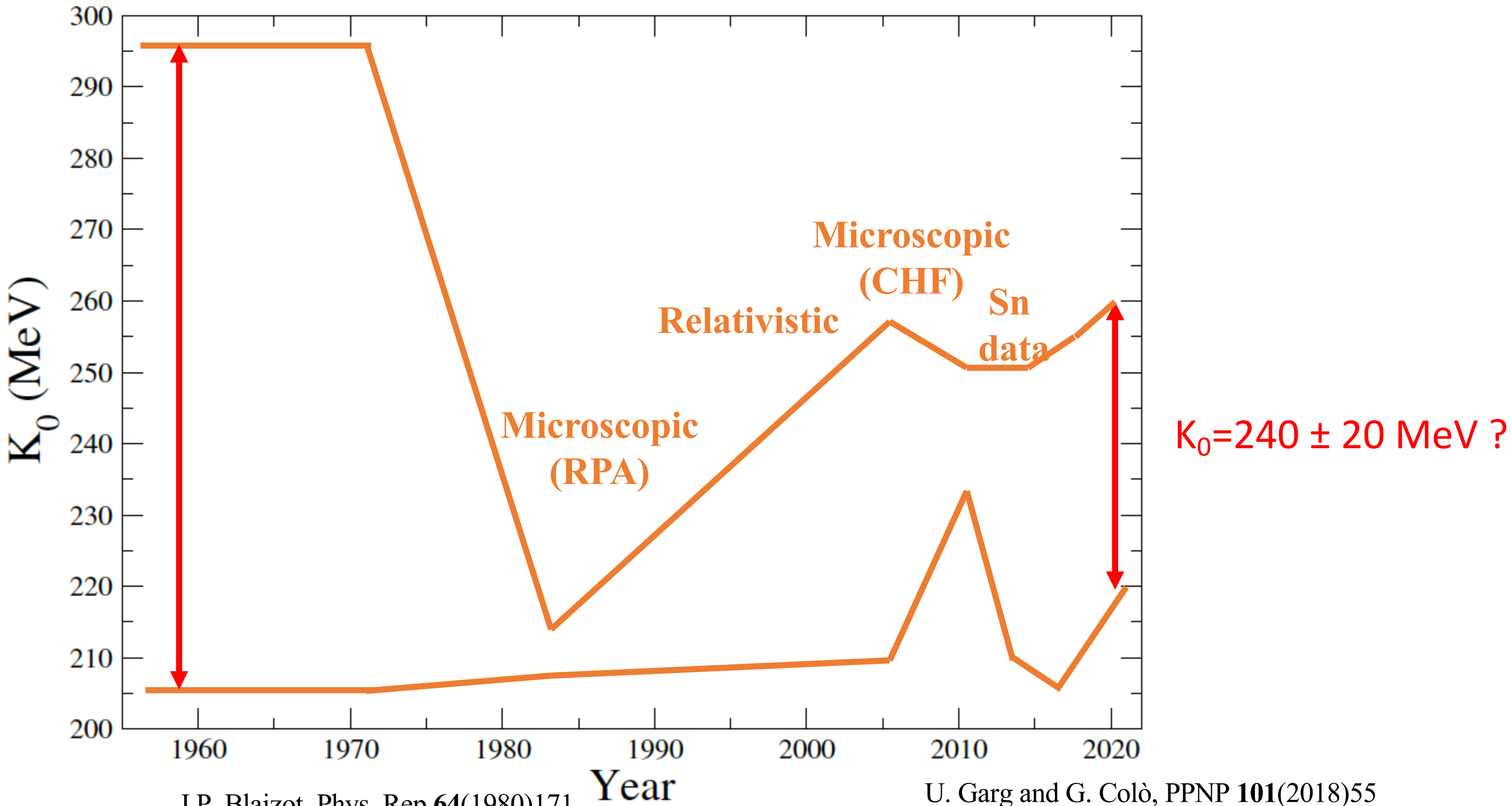



Fig. 1. Elastic and inelastic $^{30,32}S + p$ scattering data at $E/A = 53$ MeV [21] in comparison with the DWBA cross sections given by the elastic and inelastic potentials folded with the HF + BCS ground-state and QRPA transition densities, respectively. The cross sections given by the isoscalar potentials alone are plotted as dotted curves.

# Incompressibility

- Long story together, lots of discussion (and probably mutual refering ?), pairing effect
- Relativistic vs non-relativistic, soft vs stiff




J.P. Blaizot, Phys. Rep.**64**(1980)171

U. Garg and G. Colò, PPNP **101**(2018)55

Z.Y. Ma et al, NPA**686**(2001)173

G. Colò et al, PRC**70**(2004)024307

E. Khan PRC**80**(2009)011307(R)

J.Li et al, PRC**78**(2008)064303

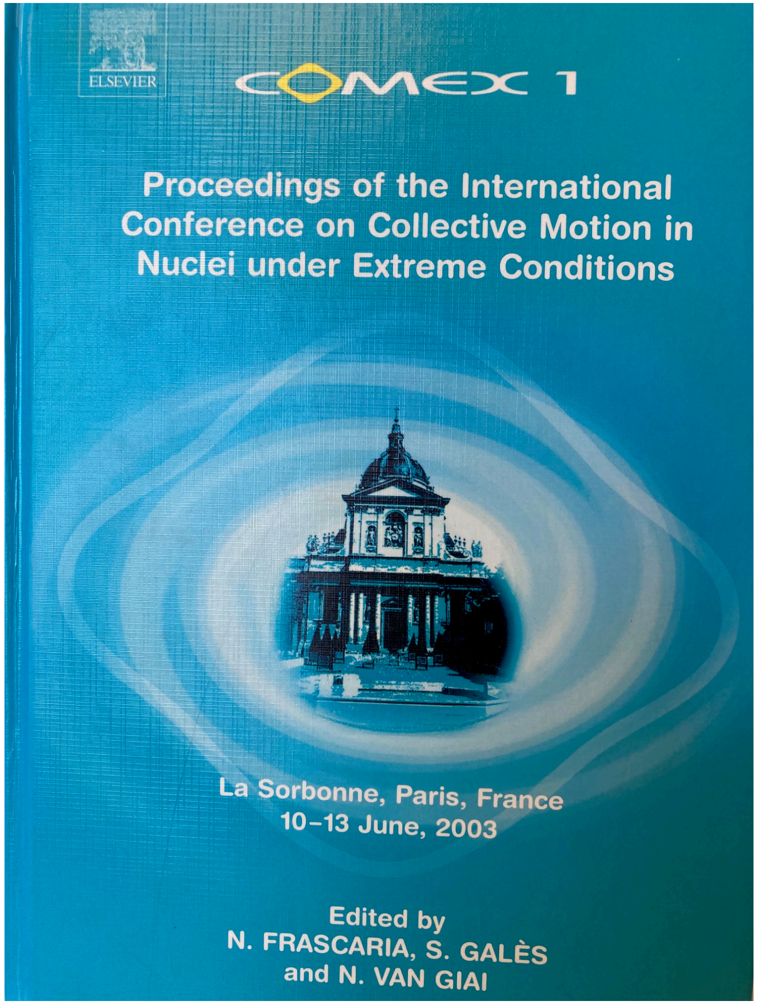
# Gianluca in first Comex conferences

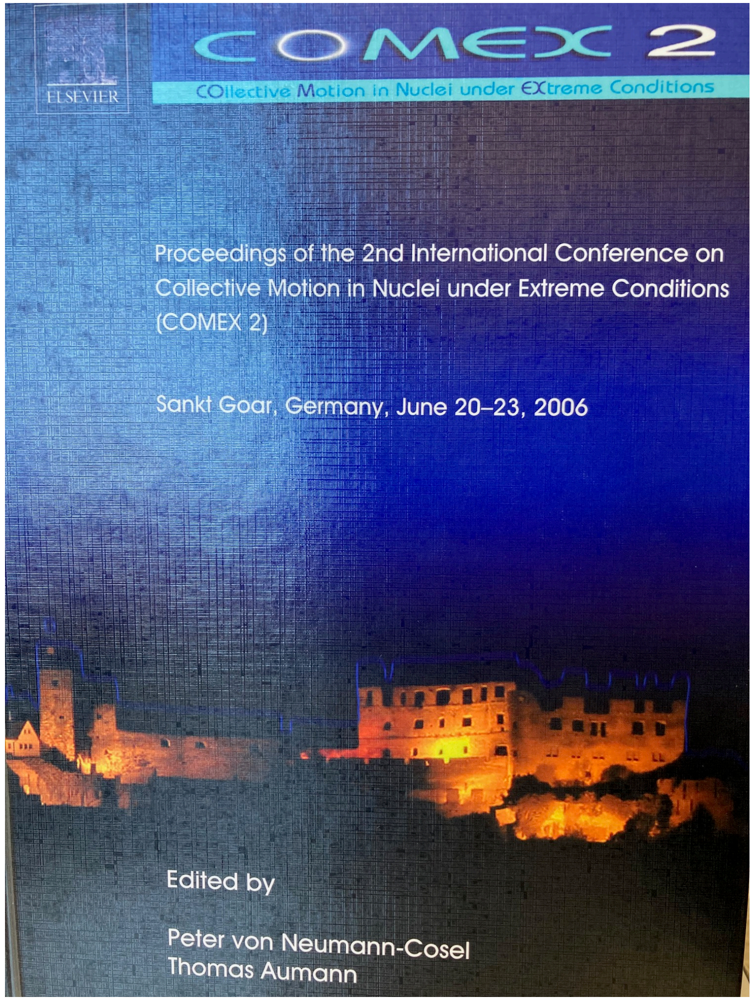
- Comex 1 (2003): Theoretical understanding of nuclear incompressibility: where do we stand ?

Relativistic vs non-relativistic

- Comex 2 (2006): What can we learn from recent non-relativistic calculations ?

Incompressibility, symmetry energy, correlations beyond mean field



# Toward a Unified Description of Isoscalar Giant Monopole Resonances in a Self-Consistent Quasiparticle-Vibration Coupling Approach

Z. Z. Li (李征征),[1,2,3] Y. F. Niu (牛一斐),[1,2,*] and G. Colò[3,4,†]
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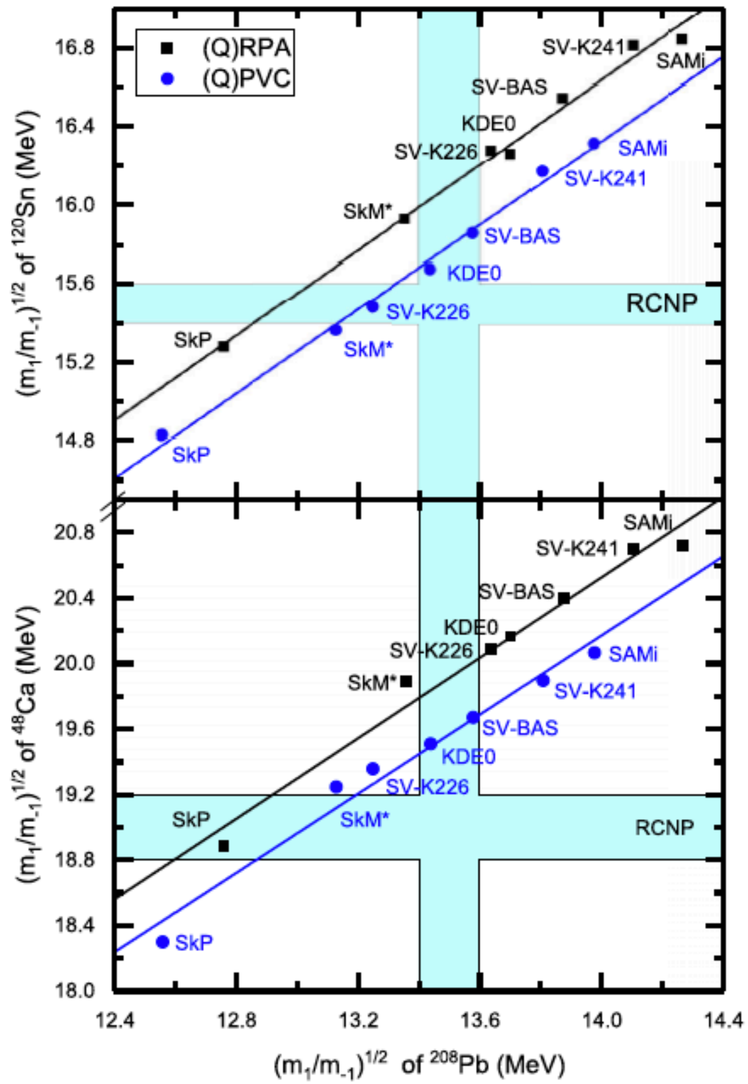

PHYSICAL REVIEW C **82**, 024322 (2010)

# Effect of pairing correlations on incompressibility and symmetry energy in nuclear matter and finite nuclei

E. Khan,[1] J. Margueron,[1] G. Colò,[2] K. Hagino,[3] and H. Sagawa[4]
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[3]*Department of Physics, Tohoku University, 980-8578 Sendai, Japan*
[4]*Center for Mathematics and Physics, University of Aizu, Aizu-Wakamatsu, 965-8580 Fukushima, Japan*

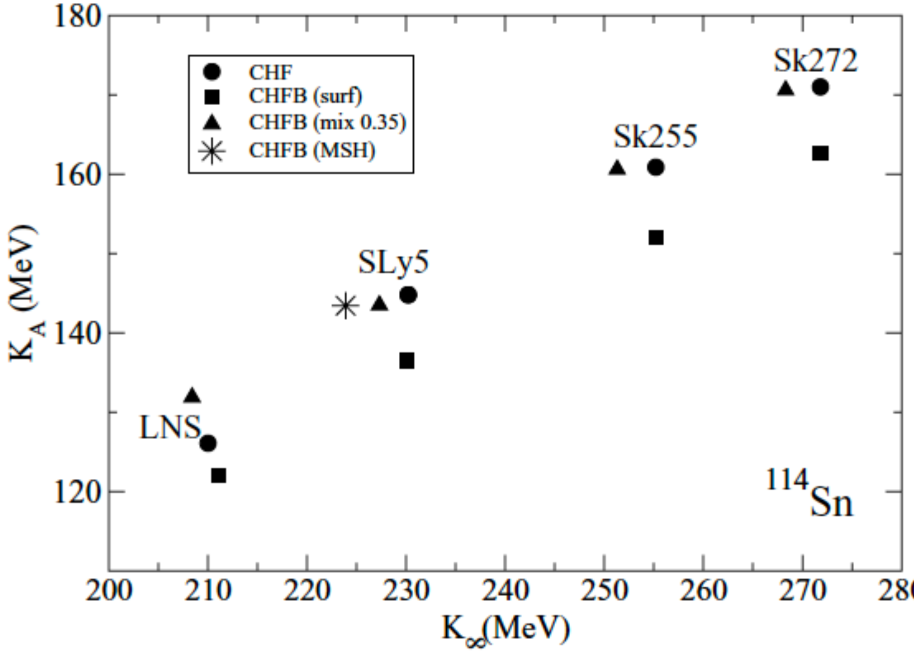
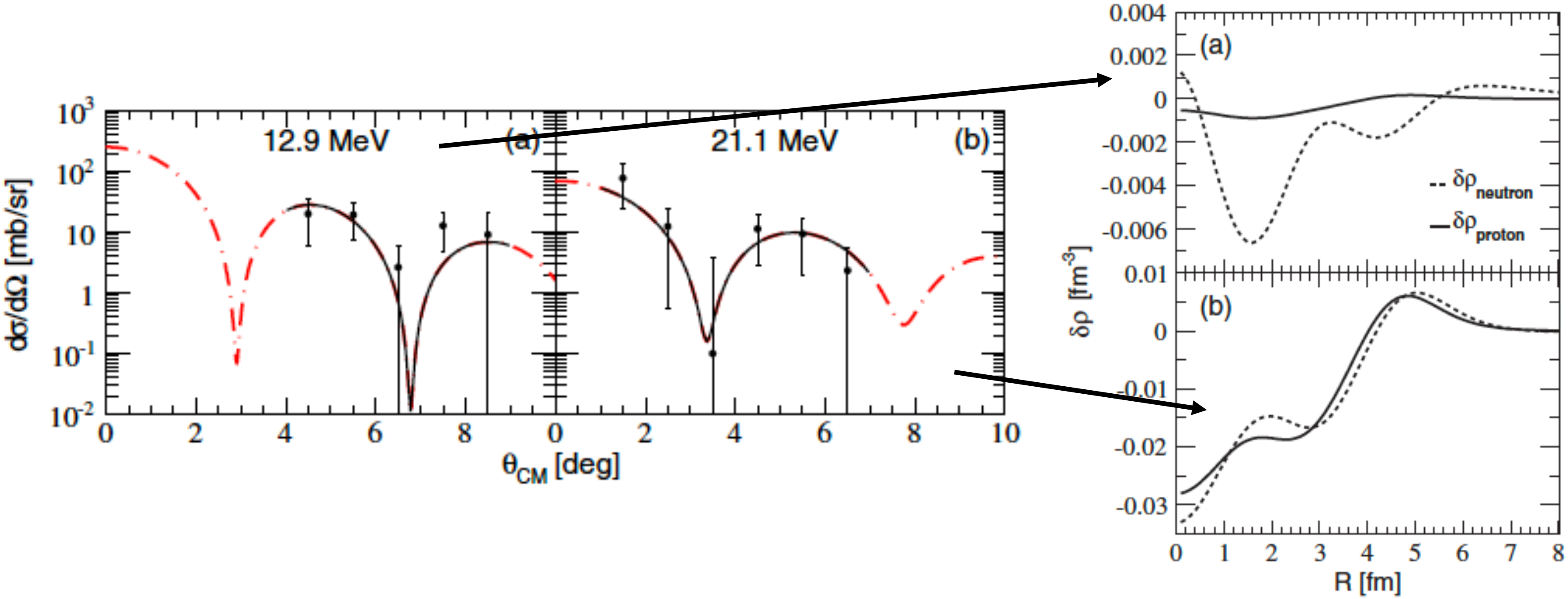



FIG. 5. $K_\infty$ vs $K_A$ for $^{114}$Sn obtained by the CHF and the CHFB method with surface-type and mixed-type pairing interactions for several Skyrme interactions.

# Measurement of the Isoscalar Monopole Response in the Neutron-Rich Nucleus $^{68}$Ni

M. Vandebrouck,[1,2,*] J. Gibelin,[2] E. Khan,[1] N. L. Achouri,[2] H. Baba,[4] D. Beaumel,[1] Y. Blumenfeld,[1] M. Caamaño,[5] L. Càceres,[3] G. Colò,[6] F. Delaunay,[2] B. Fernandez-Dominguez,[5] U. Garg,[7] G. F. Grinyer,[3] M. N. Harakeh,[8,3] N. Kalantar-Nayestanaki,[8] N. Keeley,[9] W. Mittig,[10] J. Pancin,[3] R. Raabe,[11] T. Roger,[11,3] P. Roussel-Chomaz,[12] H. Savajols,[3] O. Sorlin,[3] C. Stodel,[3] D. Suzuki,[10,1] and J. C. Thomas[3]



# Electron capture rates in core collapse supernovae

PHYSICAL REVIEW C **80**, 055801 (2009)

**Calculation of stellar electron-capture cross sections on nuclei based on microscopic Skyrme functionals**

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G. Colò
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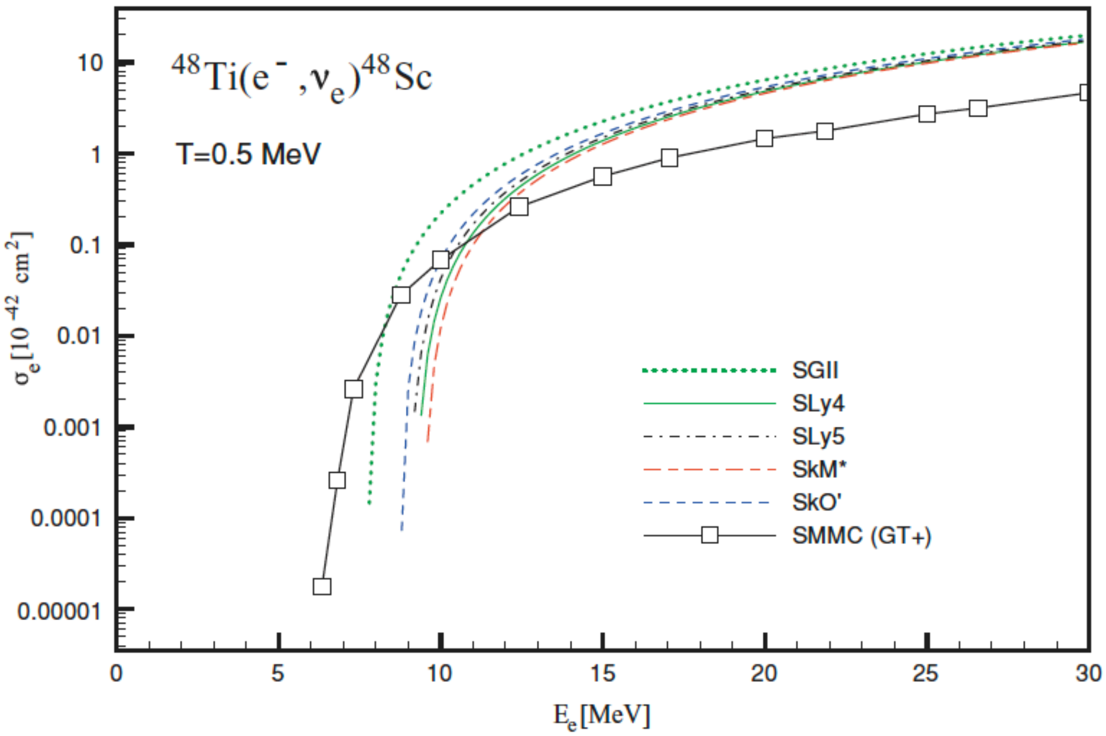
E. Khan and D. Vretenar
*Institut de Physique Nucléaire, IN2P3-CNRS/Université Paris-Sud, F-91406 Orsay, France*

- Gianluca makes some astro !
- Collaboration with Dario Vretenar
- Asked Gianluca for his expertise: pn QRPA with Skyrme



# Let’s add temperature

PHYSICAL REVIEW C **96**, 024303 (2017)

## Multipole excitations in hot nuclei within the finite temperature quasiparticle random phase approximation framework

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# Let's add temperature

$$\begin{pmatrix} \widetilde{C} & \widetilde{a} & \widetilde{b} & \widetilde{D} \\ \widetilde{a}^{+} & \widetilde{A} & \widetilde{B} & \widetilde{b}^{T} \\ -\widetilde{b}^{+} & -\widetilde{B}^{*} & -\widetilde{A}^{*} & -\widetilde{a}^{T} \\ -\widetilde{D}^{*} & -\widetilde{b}^{*} & -\widetilde{a}^{*} & -\widetilde{C}^{*} \end{pmatrix} \begin{pmatrix} \widetilde{P} \\ \widetilde{X} \\ \widetilde{Y} \\ \widetilde{Q} \end{pmatrix} = E_{\nu} \begin{pmatrix} \widetilde{P} \\ \widetilde{X} \\ \widetilde{Y} \\ \widetilde{Q} \end{pmatrix}$$

$$\widetilde{A}_{abcd} = \sqrt{1-f_a-f_b}A'_{abcd}\sqrt{1-f_c-f_d} + (E_a+E_b)\delta_{ac}\delta_{bd},$$

$$\widetilde{B}_{abcd} = \sqrt{1-f_a-f_b}B_{abcd}\sqrt{1-f_c-f_d},$$

$$\widetilde{C}_{abcd} = \sqrt{f_b-f_a}C'_{abcd}\sqrt{f_d-f_c} + (E_a-E_b)\delta_{ac}\delta_{bd},$$

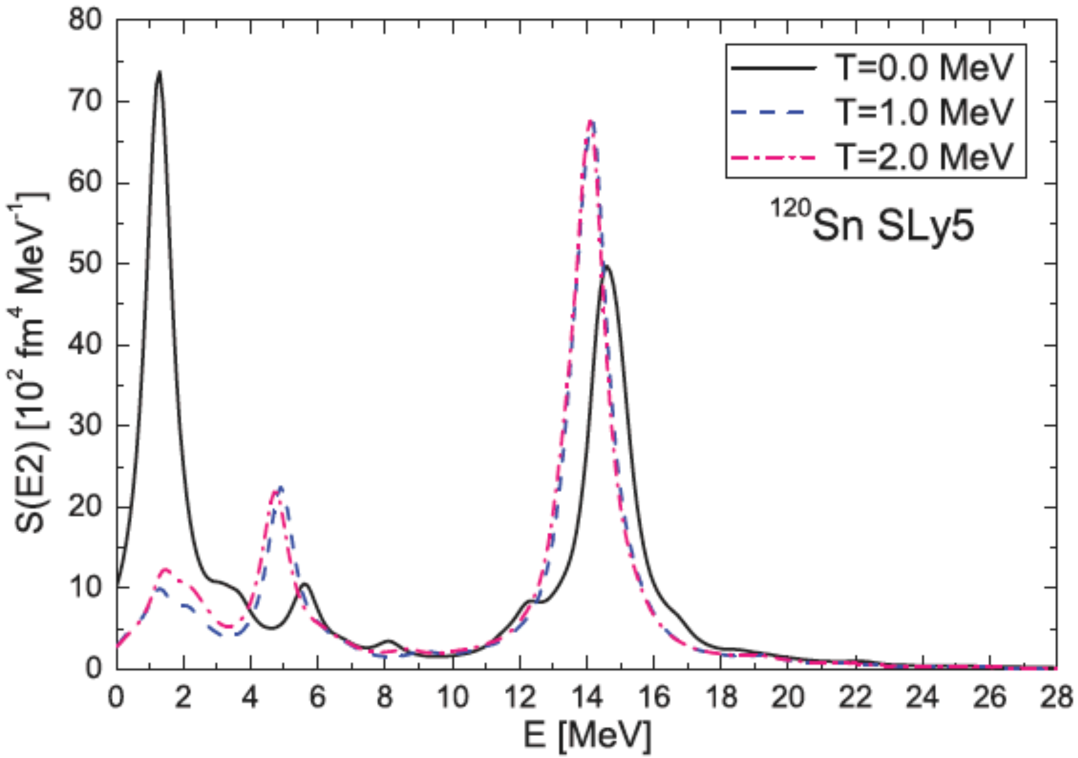
$$\widetilde{D}_{abcd} = \sqrt{f_b-f_a}D_{abcd}\sqrt{f_d-f_c},$$

$$\widetilde{a}_{abcd} = \sqrt{f_b-f_a}a_{abcd}\sqrt{1-f_c-f_d},$$

$$\widetilde{b}_{abcd} = \sqrt{f_b-f_a}b_{abcd}\sqrt{1-f_c-f_d},$$

$$\widetilde{a}^{+}_{abcd} = \widetilde{a}^{T}_{abcd} = \sqrt{f_d-f_c}a^{+}_{abcd}\sqrt{1-f_a-f_b},$$

$$\widetilde{b}^{T}_{abcd} = \widetilde{b}^{+}_{abcd} = \sqrt{f_d-f_c}b^{T}_{abcd}\sqrt{1-f_a-f_b},$$



# Temperature + charge exchange

PHYSICAL REVIEW C **101**, 044305 (2020)

## Gamow-Teller excitations at finite temperature: Competition between pairing and temperature effects

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# Temperature + charge exchange