Workshop 3X60

Milano, 17-18th Oct.

A consistent description of the isoscalar giant monopole resonances in spherical nuclei

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2022/11, Milano

2023/11, Lanzhou

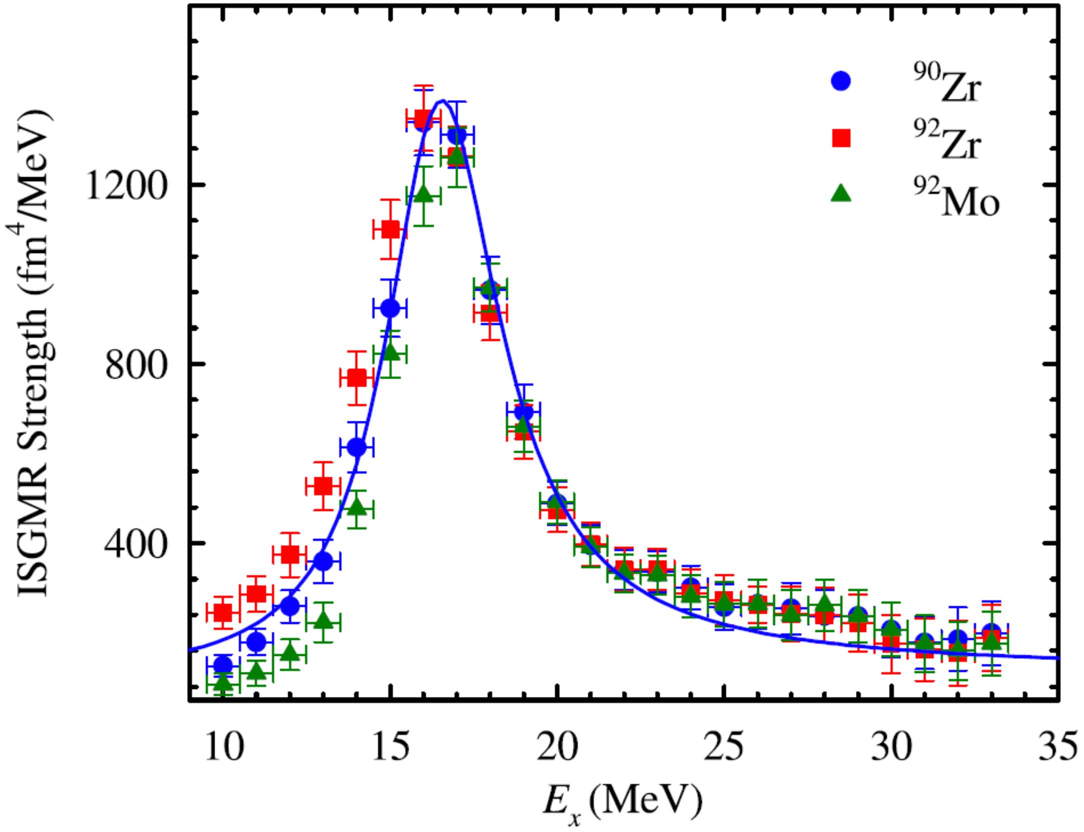
2024/10, Milano

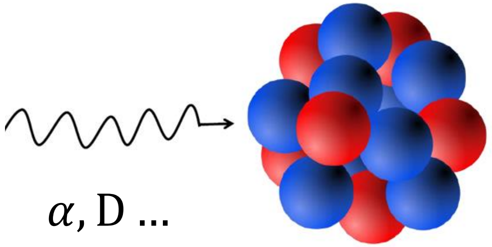
Nuclear giant monopole resonance

- Nuclear isoscalar giant monopole resonance (ISGMR) is one of the most studied GR modes, which was firstly observed in 1977.

Harakeh, et al., PRL 38, 676 (1977); Youngblood, et al., PRL 39, 1188 (1977).

- Experimental methods:





ISGMR can be measured by inelastic α (also deuterons…) scattering at small angle, with a multipole-decomposition analysis (MDA).

$$\frac{d^2\sigma^{exp}\,(\theta_{c.m.},E_x)}{d\Omega dE} = \sum a_L(E_x)\,\frac{d^2\,\sigma_L^{DWBA}\,(\theta_{c.m.},E_x)}{d\Omega\, dE}$$

Gupta, et al., PLB 760, 482 (2016).

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Microscopic theoretical methods

- Time-dependent Hartree(-Fock) theory. *Lalazissis, Ring, EPJA 55, 229 (2019).*
- Self-consistent quasiparticle random-phase approximation (QRPA) theory:
  - Based on Skyrme density functional theory
    Terasaki, et al., PRC 71, 034310 (2005).
  - Based on Gogny density functional theory
    Giambrone, et al., NPA 726, 3 (2003).
  - Based on relativistic density functional theory
    Paar, et al., PRC 67, 034312 (2003).

GRs: Energy; Width ✗

- Generalized time-dependent Generator coordinate method. *B. Li, et al., PRC 108, 014321 (2023).*
- Beyond QRPA approximation:
  - Nonrelativistic second RPA theory (SRPA)
    Gambacurta, et al., PRC 81, 024307 (2010); M.J. Yang, et al., PRC 103, 054308 (2021).
  - Relativistic quasiparticle-vibration coupling theory (QPVC).
    Litvinova, et al., PRC 78, 014312 (2008).
  - Nonrelativistic quasiparticle-vibration coupling theory.
    Z.Z. Li, et al, PRL 131, 082501 (2023).

Nuclear equation of state and giant resonances

- Nuclear equation of state

Oertel, et al., RMP 89, 015007 (2017);
Roca-Maza, Paar, PPNP 101, 96 (2018).

$$E(\rho,\delta) = E(\rho,\delta=0) + S(\rho)\,\delta^2$$

Symmetric NM energy

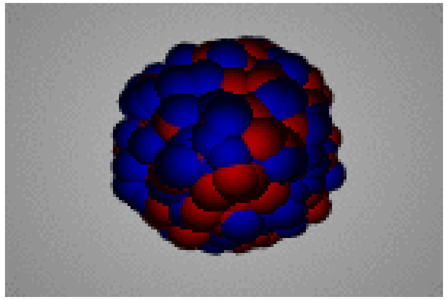
$$\delta = (\rho_n - \rho_P)/\rho$$

$$E(\rho,\delta=0) = E_0 + \frac{1}{2} K_\infty x^2 + \ \ldots$$

$$x = (\rho - \rho_0)/3\rho_0$$

Incompressibility coefficients

- The isoscalar giant monopole resonance provides a good way to constrain K_∞

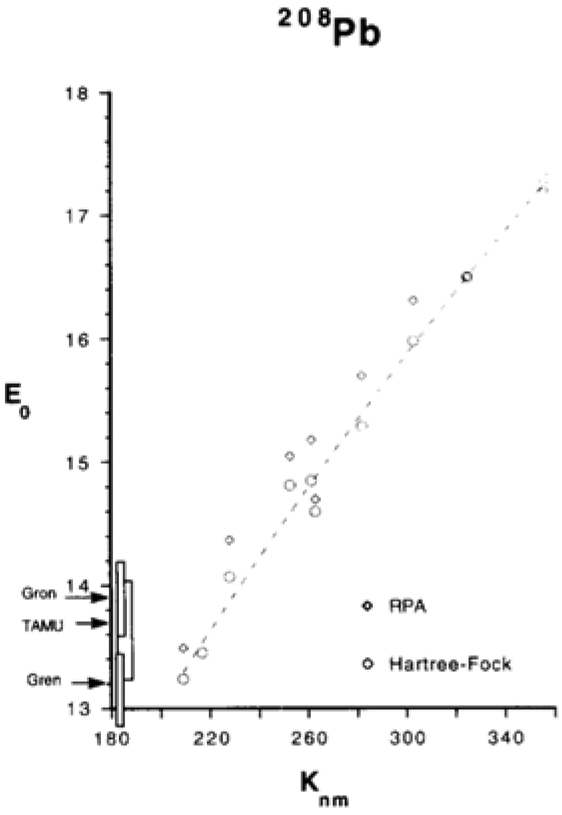


$$E^{\mathrm{ISGMR}} = \sqrt{\frac{\hbar^2 A K_A}{m\langle r^2\rangle}}$$

K_A: Incompressibility in finite nucleus

The linear correlation: $K_A = aK_\infty + b$

$$E^{\mathrm{ISGMR}} = a'\sqrt{K_\infty} + b'$$



Blaizot et al., NPA 591, 435 (1995).

Constraints on K_∞ from ISGMR experiments

- More than 40 isotopes were measured at TAMU, RCNP, iThemba LABS (recently).

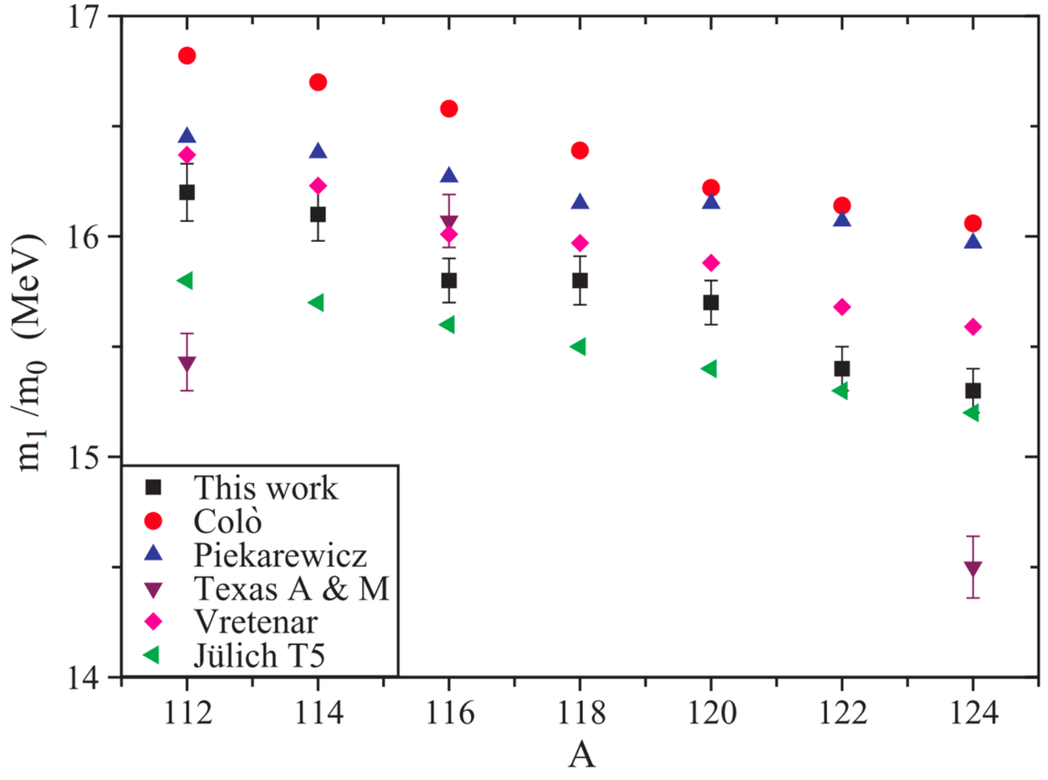
From ^{208}Pb and ^{90}Zr: $K_\infty = 240 \pm 20$ MeV

Reviewed by : Garg and Colò, PPNP 101, 55 (2018).

- In even-A $^{112-124}$Sn, GMR energy is overestimated by about 1 MeV by the same models which reproduce the GMR energy well in ^{208}Pb.

$$E^{\text{ISGMR}} = a'\sqrt{K_\infty} + b' \Rightarrow \frac{\delta K_\infty}{K_\infty} = \frac{2\delta E^{\text{ISGMR}}}{E^{\text{ISGMR}}}$$

$$\delta K_\infty = \frac{2\delta E^{\text{ISGMR}}}{E^{\text{ISGMR}}} \times K_\infty \simeq \frac{2 \times 1\text{MeV}}{15\text{ MeV}} \times 240\text{MeV} = 32\text{ MeV}$$

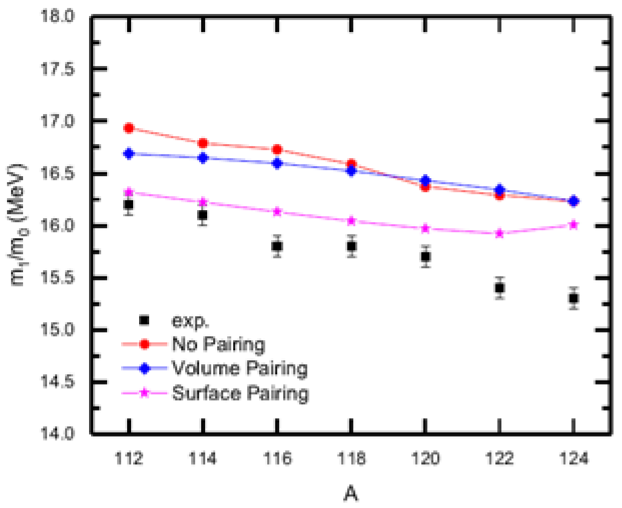


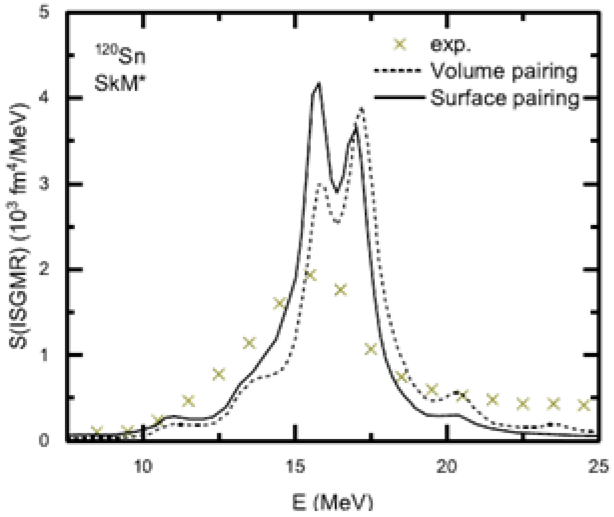
Why is the EOS for tin so soft ?

Piekarewicz, PRC 76, 031301R (2007).

Why is the EOS for tin so soft?

- In QRPA theory, the pairing effects are studied to explain this problem:





The surface pairing can partly reconcile the discrepancy.

J. Li, et al., PRC 78, 064304 (2008);
L.G. Cao, et al., PRC 86, 054313 (2012)…

However, there is no strong argument on which type of pairing force should be favored over others.

- Other attempts: "mutually enhanced magicity" …

Leaving the question of "softness" of the Sn nuclei an important "open problem" in nuclear structure theory.

Khan, PRC 80,057302 (2009); Garg and Colò, PPNP 101, 55 (2018).

we developed a fully self-consistent QRPA+QPVC theory based on Skyrme density functional theory, and try to understand the problem "Why is the EOS for tin so soft?".

QRPA + QPVC theory

- In QRPA + QPVC theory, the creation operator writes,

$$O_\nu^+ = \sum_{a<b} X_{ab}^{(\nu)} \alpha_a^+ \alpha_b^+ - Y_{ab}^{(\nu)} \alpha_b \alpha_a + \sum_{a<b,n} \boxed{\left(X_{ab,n}^{(\nu)} \alpha_a^+ \alpha_b^+ Q_n^+ - Y_{ab,n}^{(\nu)} Q_n \alpha_b \alpha_a \right)}$$

2qp Coupling with phonons

- From equation of motion, one gets the QPVC equations

2qp space

$$\begin{pmatrix} A_{ab,a'b'} & B_{ab,a'b'} & A_{ab,a'b'n} & 0 \\ -B^*_{ab,a'b'} & -A^*_{ab,a'b'} & 0 & -A^*_{ab,a'b'n} \\ A_{abn,a'b'} & 0 & A_{abn,a'b'n} & 0 \\ 0 & -A^*_{abn,a'b'} & 0 & -A^*_{abn,a'b'n'} \end{pmatrix} \begin{pmatrix} X_{a'b'} \\ Y_{a'b'} \\ X_{a'b'n'} \\ Y_{a'b'n'} \end{pmatrix} = \hbar\Omega_\nu \begin{pmatrix} X_{ab} \\ Y_{ab} \\ X_{abn} \\ Y_{abn} \end{pmatrix}$$

2qp ⊗ phonon space

$$\begin{aligned} A_{abn,a'b'} &= \langle 0 | Q_n \alpha_b \alpha_a, [H, \alpha_{a'}^+ \alpha_{b'}^+] | 0 \rangle \\ A_{ab,a'b'n'} &= \langle 0 | \alpha_b \alpha_a, [H, \alpha_{a'}^+ \alpha_{b'}^+ Q_{n'}^+] | 0 \rangle \\ A_{abn,a'b'n'} &= \langle 0 | Q_n \alpha_b \alpha_a, [H, \alpha_{a'}^+ \alpha_{b'}^+ Q_{n'}^+] | 0 \rangle \end{aligned}$$

H is consistent with the ground state.

It is difficult to solve QPVC equation directly.

QRPA + QPVC theory

- Projecting to the 2qp space, one gets the energy-dependent QPVC equation,

$$\begin{pmatrix} D + A_1^{\downarrow}(E) & -A_2^{\downarrow}(E) \\ A_3^{\downarrow}(E) & \bar{D} - A_4^{\downarrow}(E) \end{pmatrix} \begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix} = (\hbar\Omega_\nu - i\frac{\Gamma_\nu}{2}) \begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix}$$

$$A_1^{\downarrow}(E)_{nn'} = \sum X_{ab}^{(n)} X_{a'b'}^{(n')} W_{ab,a'b'}^{\downarrow}(E) + Y_{ab}^{(n)} Y_{a'b'}^{(n')} W_{ab,a'b'}^{\downarrow *}(-E)$$

$$W_{ab,a'b'}^{\downarrow}(E) = \sum_{a_1 < b_1, n} \frac{\langle ab|H|a_1 b_1, n\rangle \langle a_1 b_1, n|H|a'b'\rangle}{E - (E_n + E_{a_1} + E_{b_1}) + i\varepsilon}$$

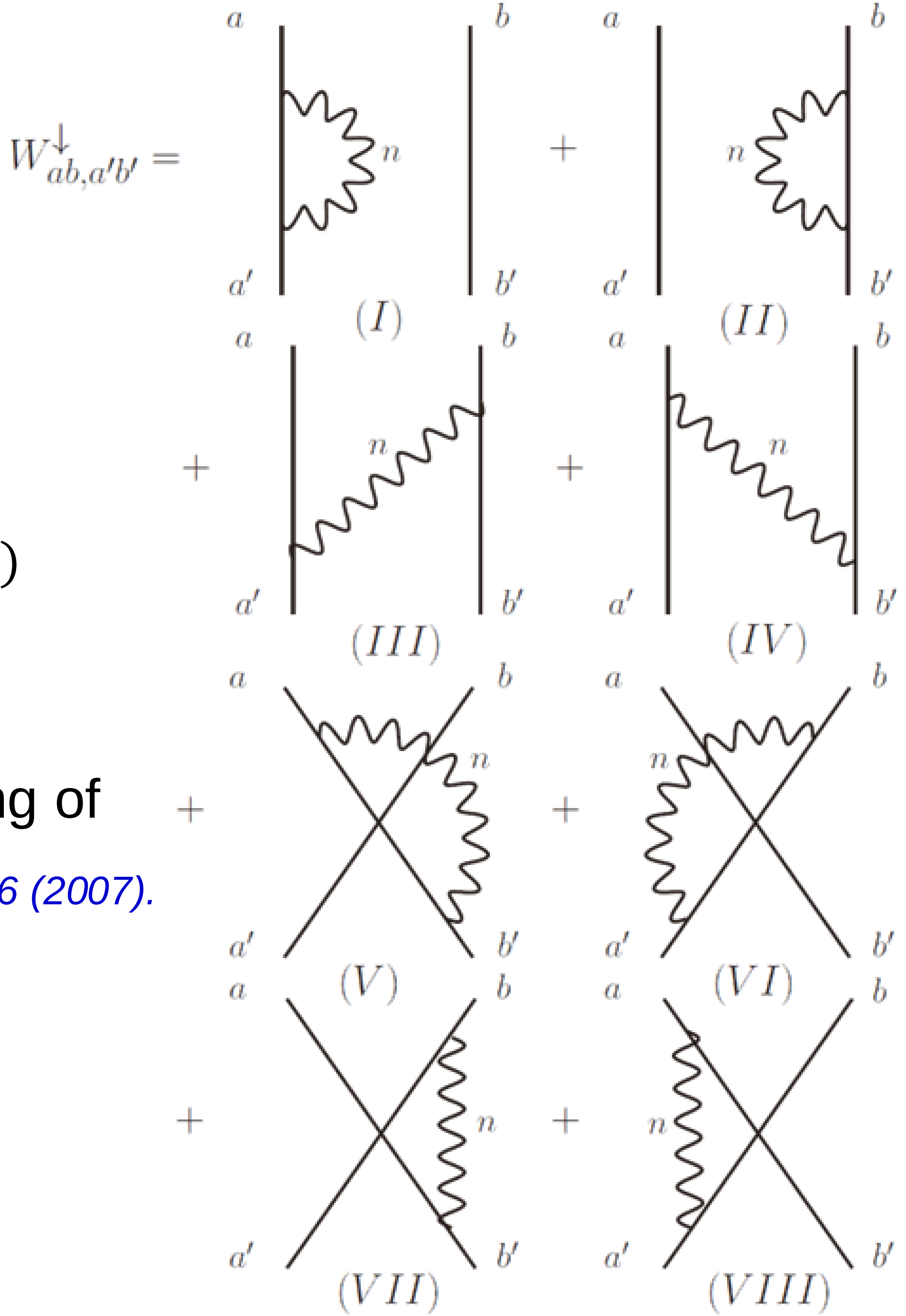
- Subtraction method is used to avoid the double counting of BMF effects,

Tselyaev, PRC 75, 024306 (2007).

$$W_{ab,a'b'}^{\downarrow}(E) \rightarrow W_{ab,a'b'}^{\downarrow}(E) - W_{ab,a'b'}^{\downarrow}(0)$$

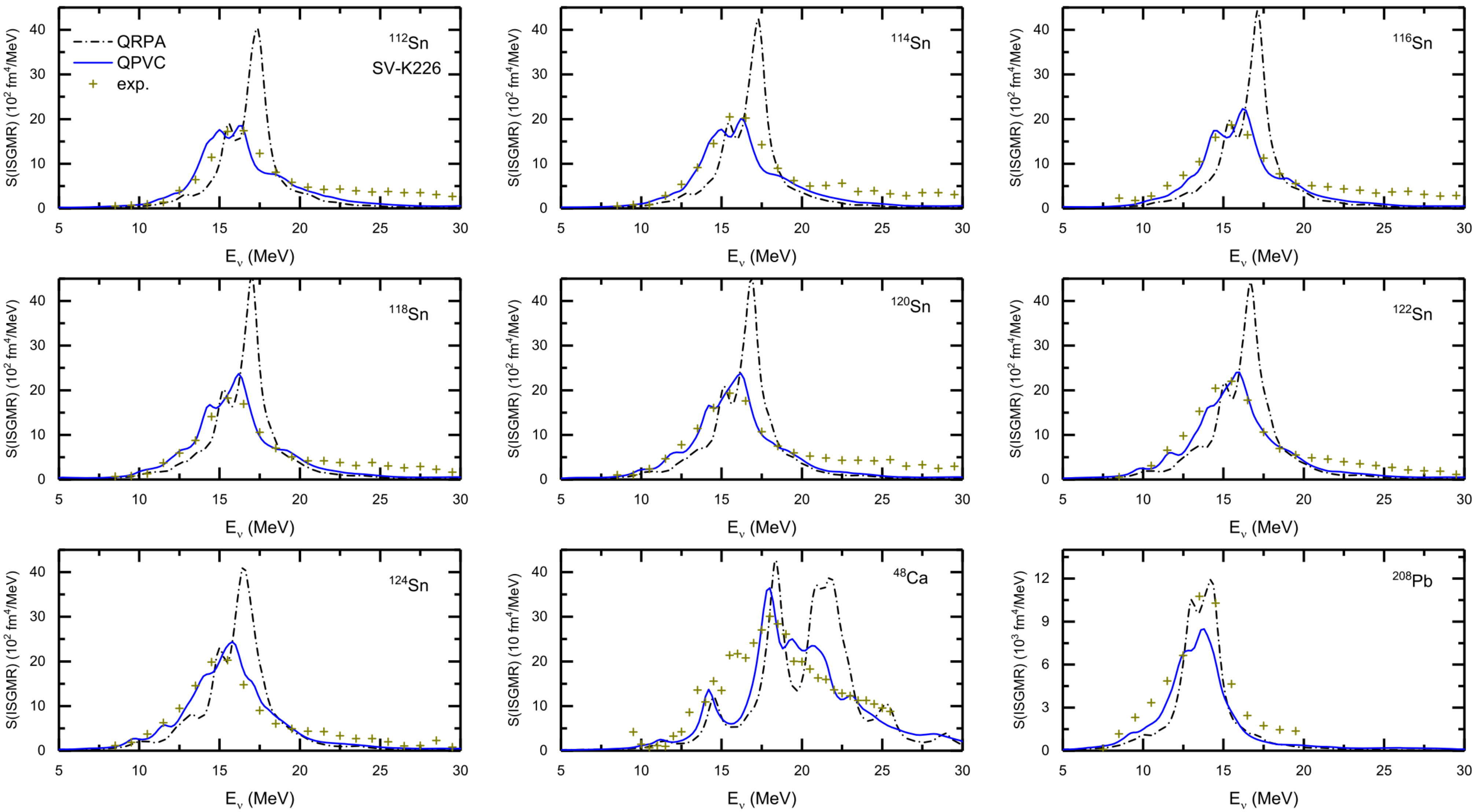
- Strength function

$$S(E) = -\frac{1}{\pi} \mathrm{Im} \sum_{\nu} \langle 0|F|\nu\rangle^2 \frac{1}{E - \hbar\Omega_\nu + i(\frac{\Gamma_\nu}{2} + \eta)} \qquad F_{00}^{\mathrm{ISGMR}} = \sum_{i=1,A} r_i^2$$



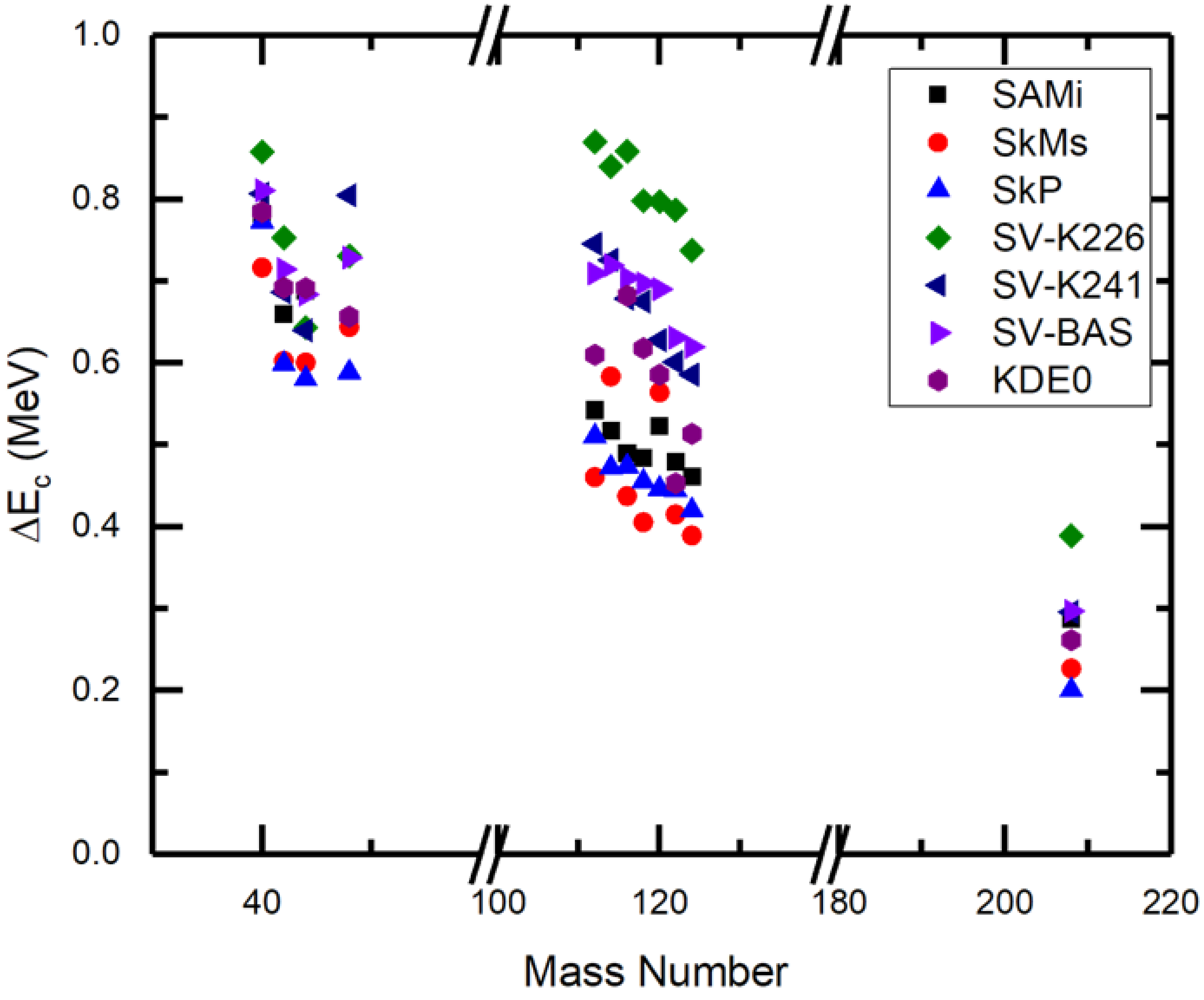
The descriptions of ISGMR

SV-K226: $K_\infty = 226$ MeV *Klupfel, et al., PRC 79, 034310 (2009).*



- QRPA: overestimate the energies in Sn, i.e., the “softness” of EOS in Sn.
- QPVC: unified descriptions of ISGMR in Sn, Pb, and Ca [both energies and widths].

QPVC effects to energy shifts



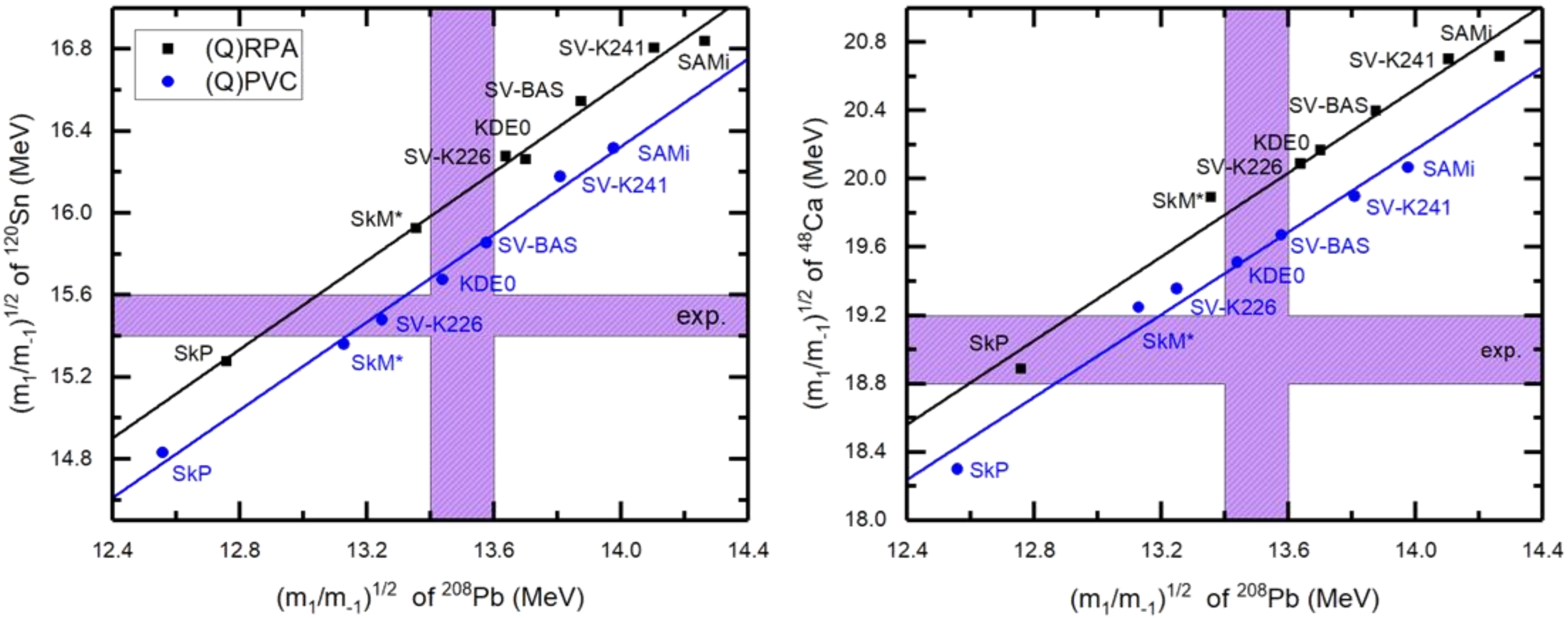
$$\Delta E_c = E_c(\text{QRPA}) - E_c(\text{QPVC})$$

$$E_c = \sqrt{m_1/m_{-1}}$$

- The energy shifts in Ca and Sn isotopes are about 0.4 MeV larger than the ones in ^{208}Pb.

ISGMR energy in ^{208}Pb vs ^{120}Sn (or ^{48}Ca)

$$E^{\mathrm{ISGMR}} = a'\sqrt{K_\infty} + b'$$



- QPVC effects are crucial for the unified description of ISGMR energy in Pb, Sn, and Ca.
- SV-K226 and KDE0 give best descriptions, with $K_\infty = 226$ and 229 MeV.

ISGMR energy deviation (MeV) from exp.

	SkP	SkM*	SV-K	KDE0	SV-bas	SV-K	SAMi
K_∞	201	217	226	229	233	241	245
(Q)RPA							
^{48}Ca	0.11	0.89	1.09	1.17	1.40	1.70	1.72
^{120}Sn	0.22	0.43	0.78	0.76	1.05	1.31	1.34
^{208}Pb	0.74	0.14	0.14	0.20	0.37	0.60	0.76
(Q)PVC							
^{48}Ca	0.70	0.25	0.36	0.51	0.67	0.90	1.07
^{120}Sn	0.67	0.14	0.02	0.18	0.36	0.68	0.82
^{208}Pb	0.94	0.37	0.25	0.06	0.08	0.31	0.48

- Best descriptions with QPVC effects:

  SV-K226 $K_\infty = 226$ MeV; KDE0 $K_\infty = 229$ MeV

✓ consistent with $K_\infty = 240 \pm 20$ MeV

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Toward a Unified Description of Isoscalar Giant Monopole Resonances in a Self-Consistent Quasiparticle-Vibration Coupling Approach

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IL NUOVO CIMENTO **47 C** (2024) 16
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COLLOQUIA: COMEX7

A consistent description of the monopole resonance in spherical nuclei

G. COLÒ(1), Z. Z. LI(2) and Y. F. NIU(2)

A self-consistent quasiparticle-vibration coupling approach for nuclear giant resonances with Skyrme interactions

Z. Z. Li (李征征),[1,2,3] Y. F. Niu (牛一斐),[2,3,*] and G. Colò[4,5,†]

Electric dipole polarizability of ^{58}Ni*

I. Brandherm,[1] F. Bonaiti,[2] P. von Neumann-Cosel,[1,†] S. Bacca,[2] G. Colò,[3,4] G. R. Jansen,[5] Z. Z. Li (李征征),[6,7,8] H. Matsubara,[9,10] Y. F. Niu (牛一斐),[7,8] P.-G. Reinhard,[11] A. Richter,[1] X. Roca-Maza,[12,13,3,4] and A. Tamii[9]

Thanks for many help from Gianluca : The man whom we can trust !

Happy 60th birthday for Gianluca, Franco, Silvia !