

Gamma-ray cascades probing the structure of warm rotating nuclei

The beginning of the quest for rotating nuclei

- overall stability – shape changes
- rare earth nuclei: long cascades of rotational E2 transitions.

Energy-energy correlations, damping of rotational motion

Silvias PhD work

- quantitative study of energy-energy correlations, ridges, valleys and fluctuations

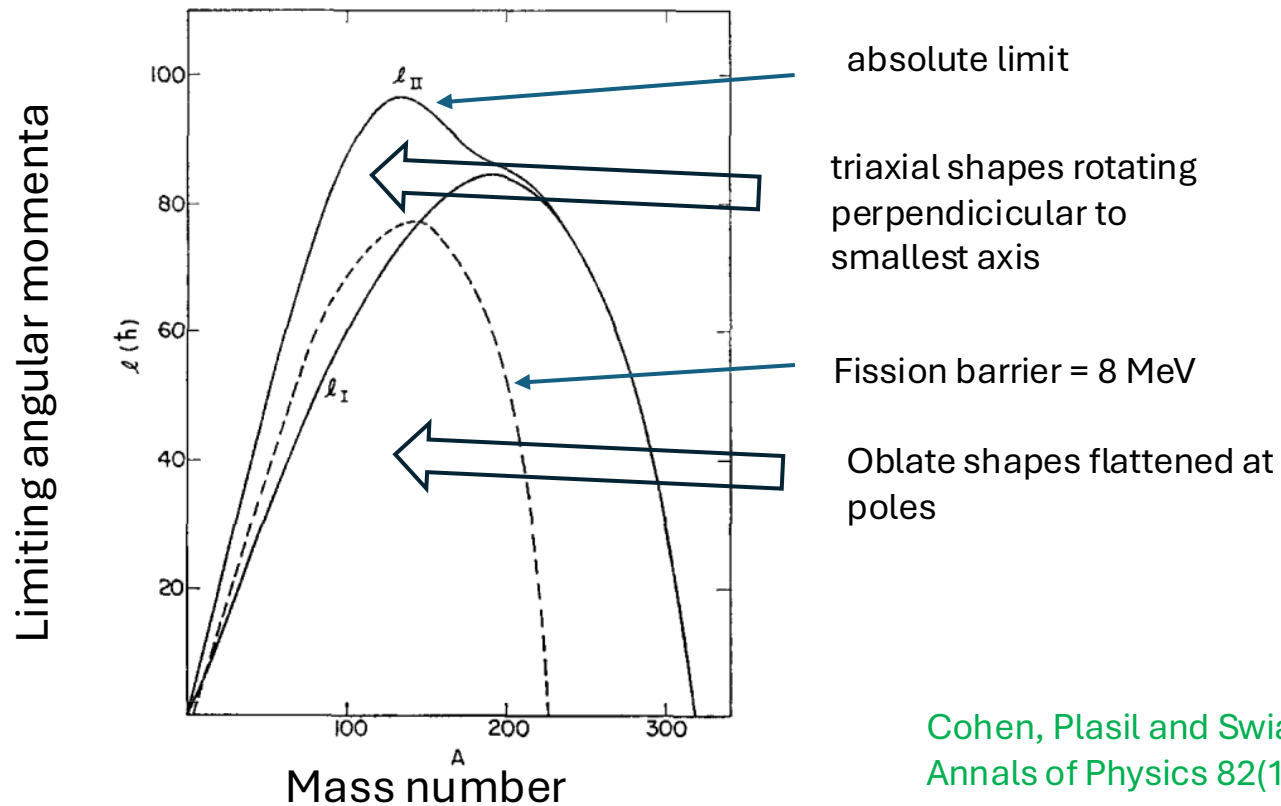
Current understanding:

- ordered and/or chaotic intrinsic and rotational motion
- realistic calculations of mixed bands
- characteristic energy –and angular momentum scales

*Milano-Kyoto-
Copenhagen*

A. Bracco
R. A. Broglia
T. Døssing
B. Herskind
S. Leoni
M. Matsuo
E. Vigezzi

Overall stability – rotating liquid drop



Cohen, Plasil and Swiatecki,
Annals of Physics 82(1974)557

Rotating liquid drop – stretching of shapes

Mass number
around $A = 170$

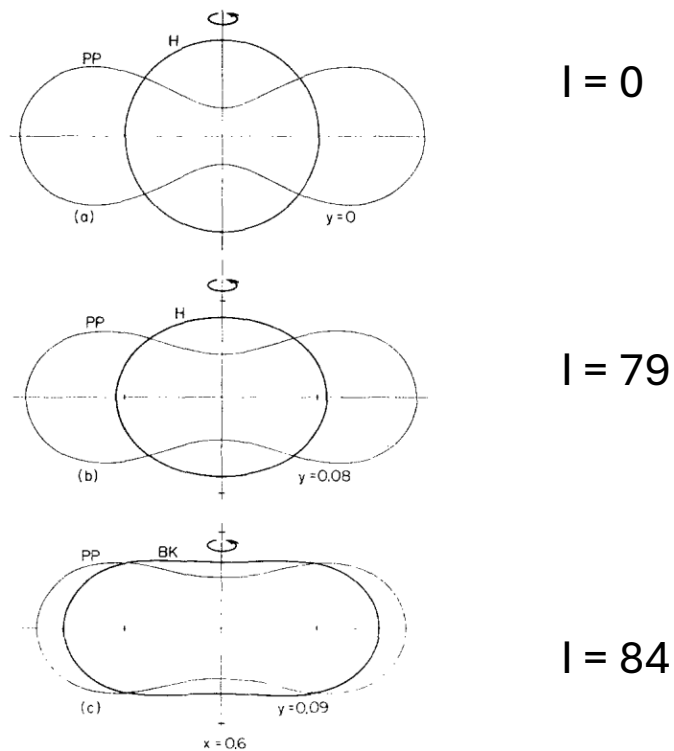


FIG. 6. Similar to Fig. 4 but for $x = 0.6$.

Influence of shell structure

Cranked mean field – development of shapes

- extensive computer program development

K. Neergaard et.al., Nuclear Physics A262(1976)61 (Dubna group)

G. Andersson et.al., Nuclear Physics A268(1976)205 (Lund group)

results in the present connection:

- mid-rare-earth nuclei around $A \sim 164$ to 178:

shell energy rather unaffected by rotation – stays at same well deformed shape up to highest angular momenta

light rare-earth nuclei – mass around $A \sim 152$: transition to oblate, then collective superdeformed

(prediction 10 years ahead of discovery of superdeformed bands)

Looking for rotational gamma-rays

GAMMA DECAY AFTER HEAVY ION REACTIONS STUDIED BY FIRST AND HIGHER ORDER MULTIPLICITY MEASUREMENTS

G. B. HAGEMANN, R. BRODA[†], B. HERSKIND, M. ISHIHARA^{††} and S. OGAZA[†]

The Niels Bohr Institute, University of Copenhagen, Denmark

and

H. RYDE

Roskilde University Center, Roskilde, Denmark

Received 23 December 1974

(Revised 7 March 1975)

quote:

.....

input which can be reached in a specific experiment. Although angular momentum input as high as $40 \hbar$ can be obtained in many laboratories, only very few states with spin higher than 18–20 have been established, and $I = 22$ is so far the maximum value.

This apparent gap between the established spin values and the expected angular momentum input is not fully understood at present. The subject is discussed by

.....

Clever setup to investigate gamma-rays

Multipolarity of Continuum γ Rays from Enhanced Angular Correlation Measurements

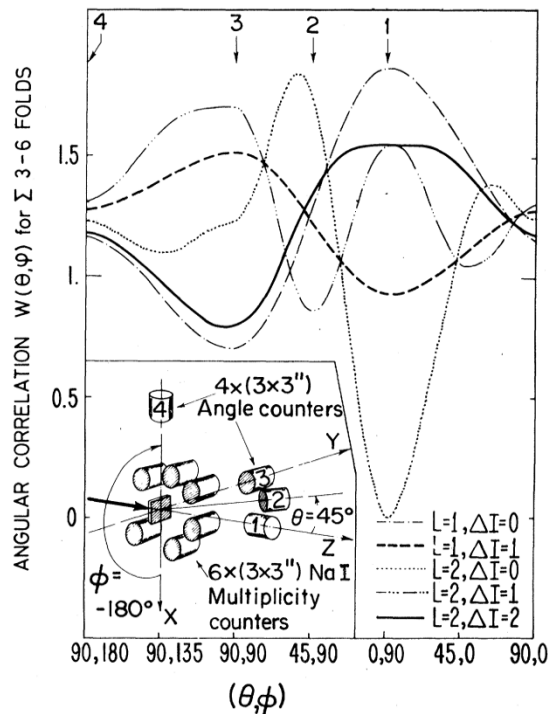
M. A. Deleplanque,^(a) Th. Byrski,^(b) R. M. Diamond, H. Hübel,^(c) and F. S. Stephens
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

and

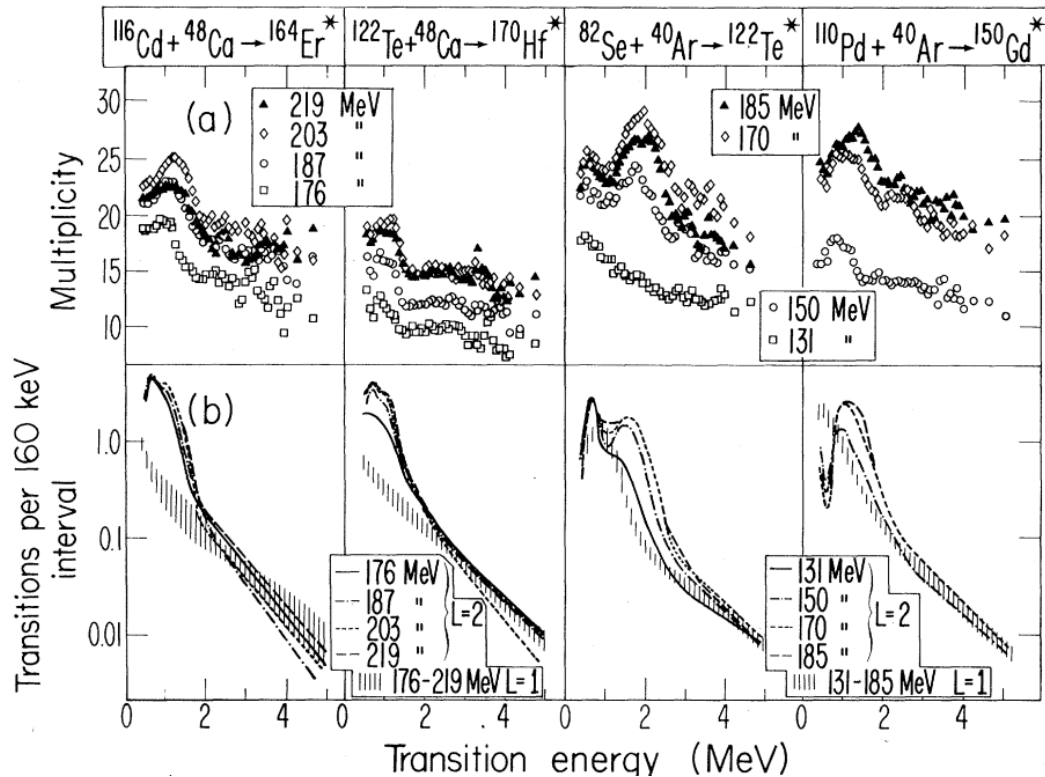
B. Herskind and R. Bauer

*The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark, and Lawrence Berkeley Laboratory,
University of California, Berkeley, California 94720*

(Received 19 May 1978)



Beautiful result from clever setup



Fusion reactions with ^{48}Ca and ^{40}Ar -> very high angular momentum

For rare earth nuclei:

Long cascades of collective transitions all the way up

Rigid body moment of inertia

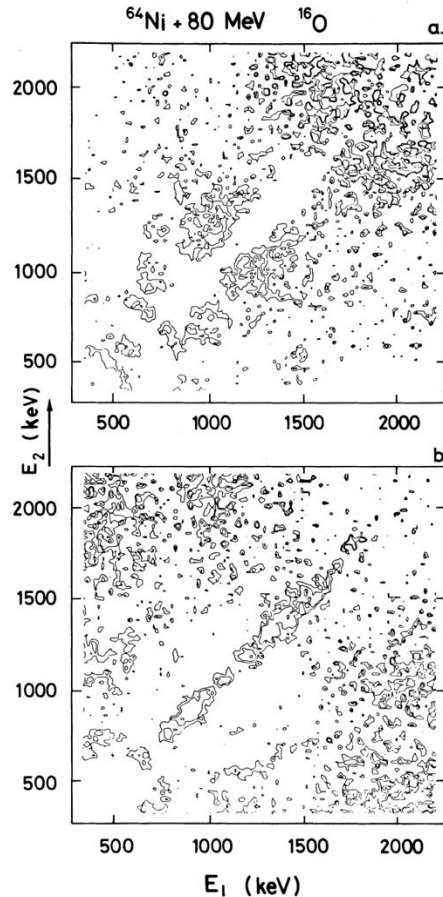
”bump spectroscopy” – pioneered by F. Stephens and R. M Diamond

Innovation and imagination

- Bent Herskind 1931-2021



Gamma-ray energy-energy correlations



nucleus: ^{72}Se

positive

COR $E_\gamma - E_\gamma$ spectrum:

$$N_{2,cor}(E_1, E_2) = N_2(E_1, E_2) - \frac{N_1(E_1) N_1(E_2)}{NORM}$$

negative

At talk by Bent Herskind:

Amand Fässler comment:

"I appreciate these foils as a piece of art, but what does it tell about the physics?"

O. Andersen et. al, Phys. Rev. Lett 43 (1979) 687

Gamma energy-energy correlations – perpendicular cuts

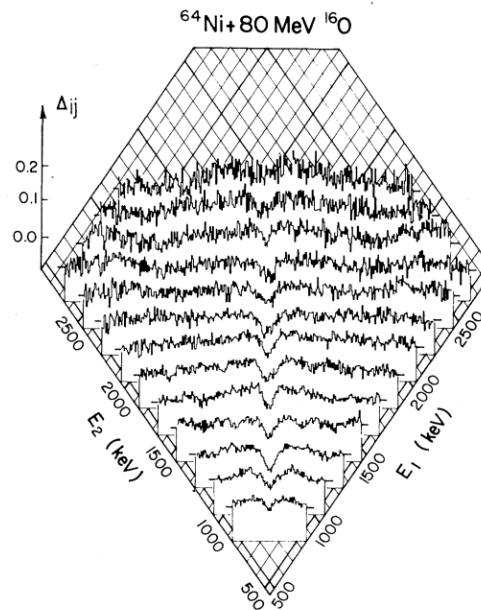
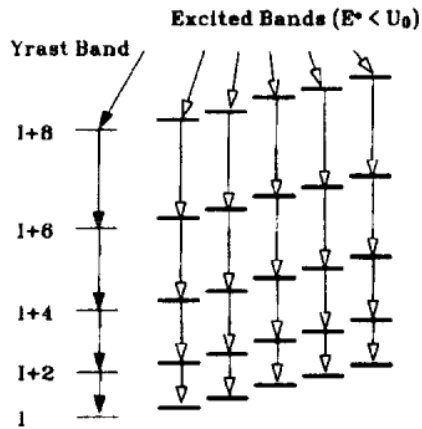


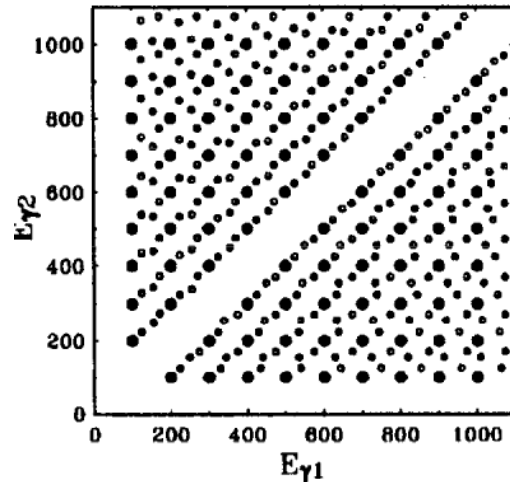
FIG. 3. Projections along the 45° diagonal of rectangular regions in the spectrum of Fig. 2 chosen such that the longest side in the rectangle is perpendicular to the 45° line. The spectra thereby show average values over intervals of 100 keV from $E_1 = E_2 = 650$ keV to $E_1 = E_2 = 1850$ keV.

Expected energy-energy correlations for decay along rotational bands

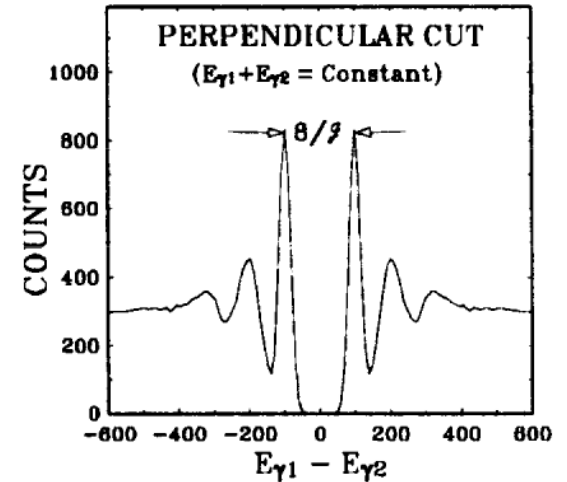
level scheme with bands



$E_\gamma - E_\gamma$ correlations

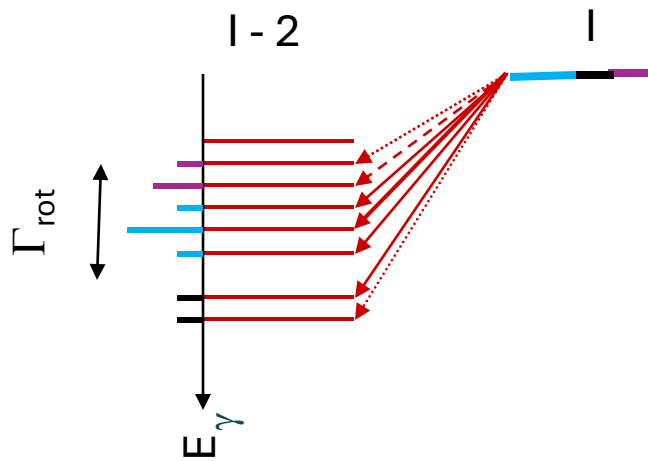


perpendicular cut

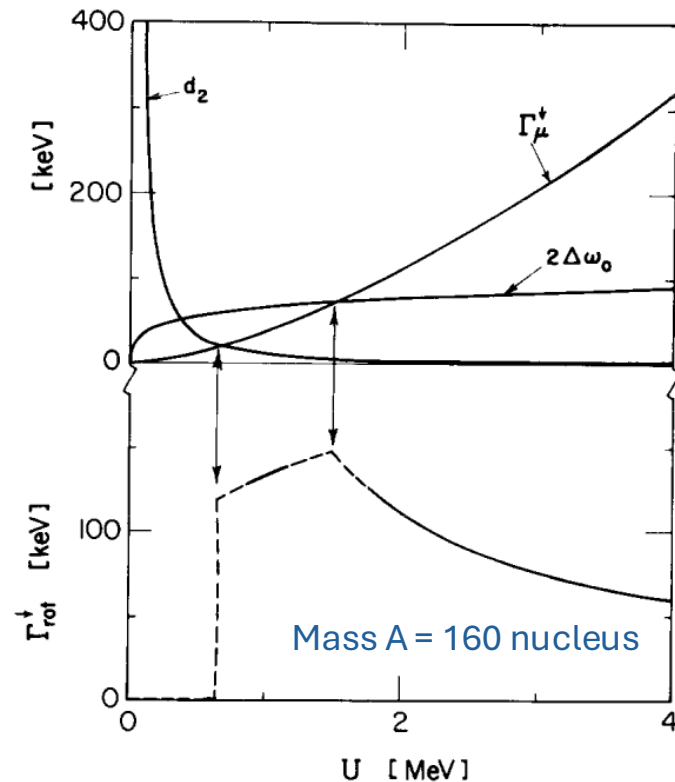


Why are energy correlations weak? Mixing of rotational bands – damping of rotational motion

rotational transitions out
of a mixed state



B. Lauritzen, T. Døssing and R.A. Broglia,
Nucl. Phys. A457(1986)61



Damping of rotational motion – analytic expressions

TABLE 1
Intervals for characteristic behavior of $\Gamma_{\text{rot}}^{\downarrow}$

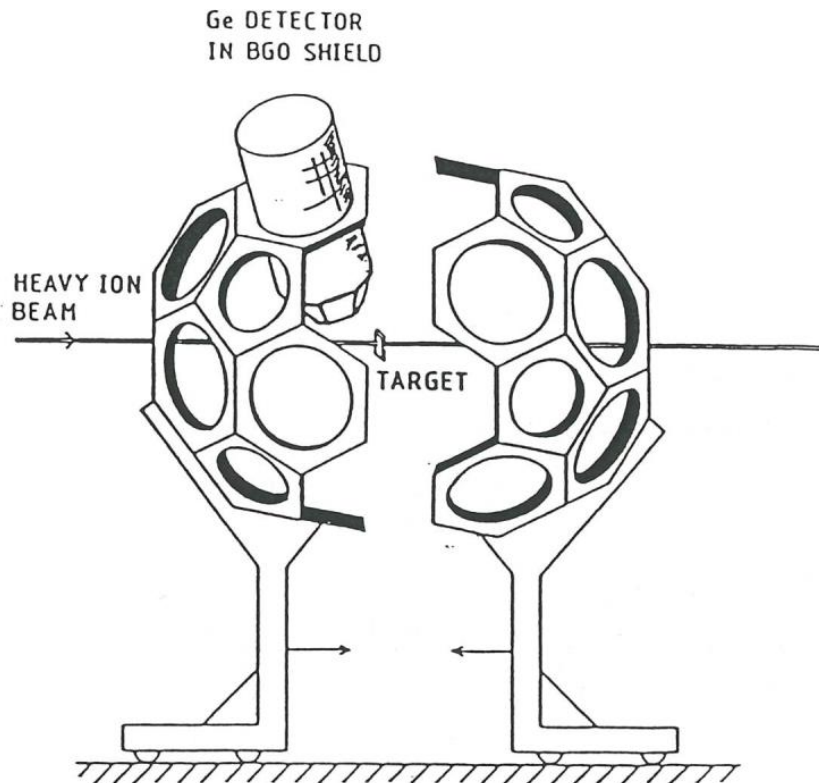
Lower limit to interval	Upper limit to interval	$\Gamma_{\text{rot}}^{\downarrow}$ within interval
0	$0.7 \left(\frac{A}{160} \right)^{-2/3}$	0
$0.7 \left(\frac{A}{160} \right)^{-2/3}$	$1.5 \frac{I}{40} \left(\frac{A}{160} \right)^{-3/2} \left(\frac{\delta}{0.3} \right)^{-2/3}$	$0.13 \frac{I}{40} \left(\frac{A}{160} \right)^{-5/2} \left(\frac{\delta}{0.3} \right)^{-1} U^{1/4}$
$1.5 \frac{I}{40} \left(\frac{A}{160} \right)^{-3/2} \left(\frac{\delta}{0.3} \right)^{-2/3}$	-	$0.22 \left(\frac{I}{40} \right)^2 \left(\frac{A}{160} \right)^{-13/3} \left(\frac{\delta}{0.3} \right)^{-2} U^{-1}$ $+ 4.4 \times 10^{-4} \left(\frac{A}{160} \right)^{-5/2} \left(\frac{\delta}{0.3} \right)^{-2} U^{3/2}$

Turning point in understanding

Silvias Thesis - december 1992 – 225 pages

- Landscapes of E_γ - E_γ correlation spectra
- Fluctuations of E_γ - E_γ correlation spectra

Silvias thesis: NORDBALL detector frame



Silvias thesis: crystals mounted in NORDBALL

- 30 days heroic experiment at the NORDBALL

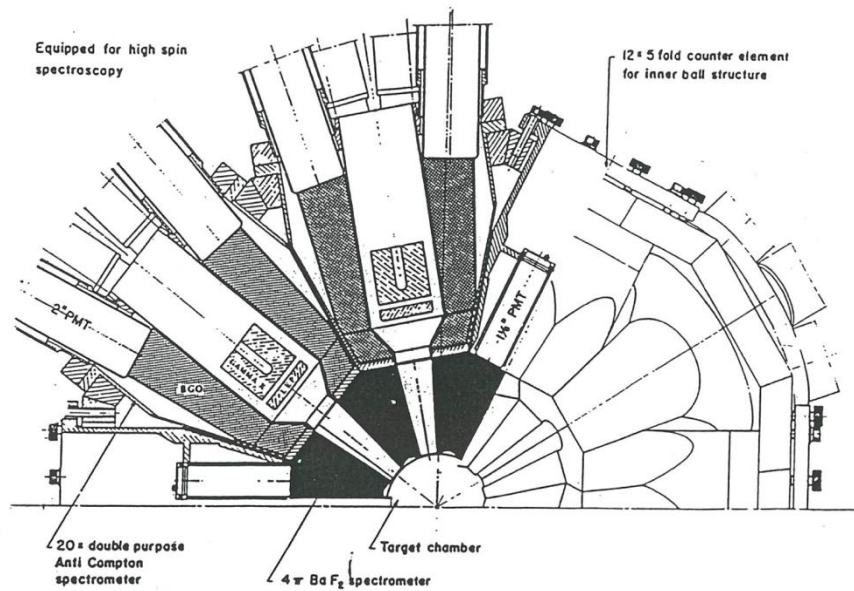
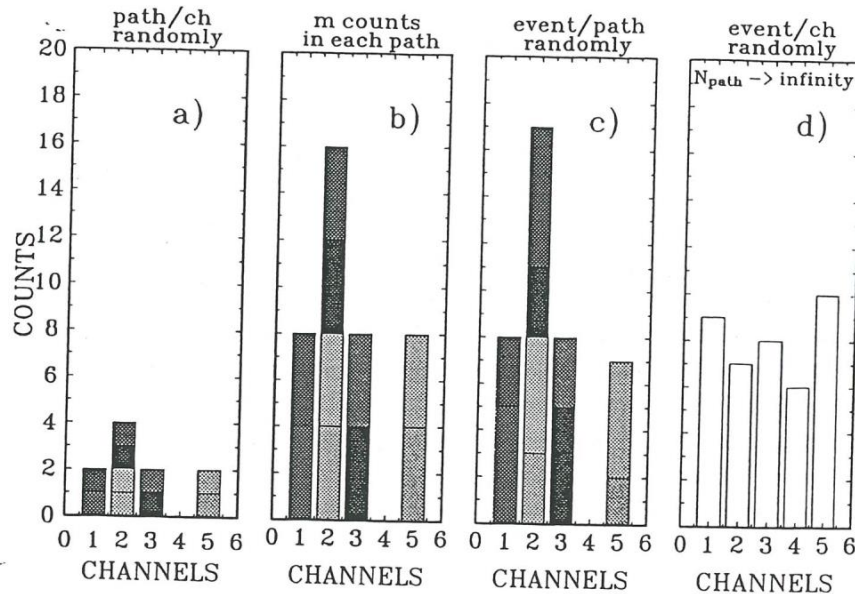


Figure 4.3: Cross section of NORDBALL.

Fluctuation analysis – main idea

pioneered by F. Stephens – developed by S. Leoni et al



events -> channels randomly:

$$\mu_1 = \frac{N_{eve}}{N_{ch}} \quad \mu_2 = \frac{N_{eve}}{N_{ch}} = \mu_1$$

paths -> channels randomly

events -> paths randomly:

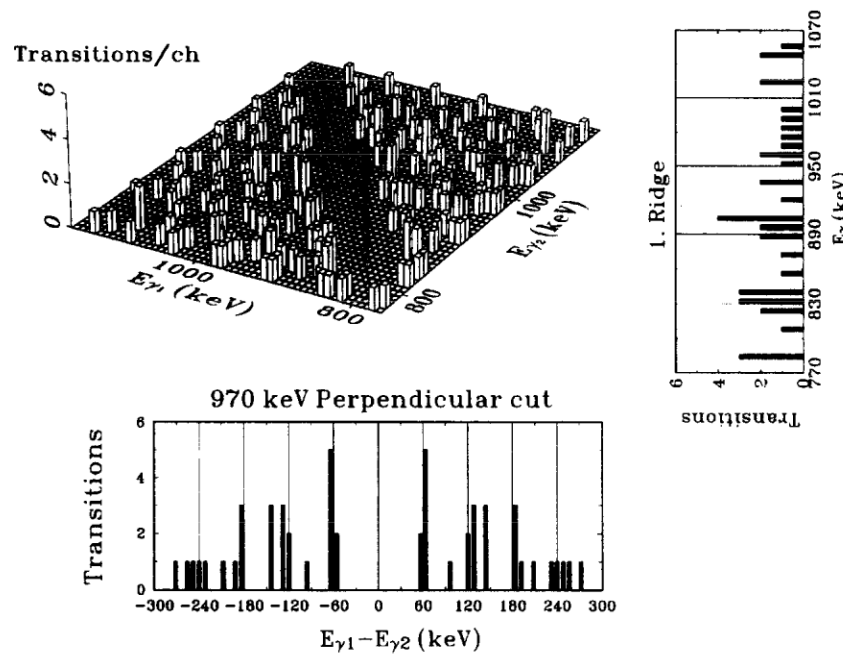
$$\mu_1 = \frac{N_{eve}}{N_{ch}} \quad \mu_2 = \mu_1 \left(\frac{N_{eve}}{N_{path}} + 1 \right)$$

number of paths in actual

cascades:

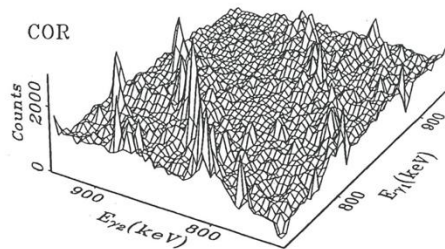
$$N_{path} = \frac{1}{\sum_i w_i^2} \quad w_i = \text{path probability}$$

Assumption of random transition energy within basic interval - illustration



Known bands
In Yb isotopes
at that time

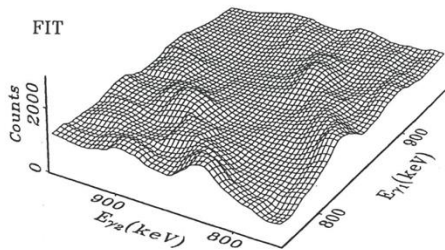
Fluctuations of spectra - illustration



a)

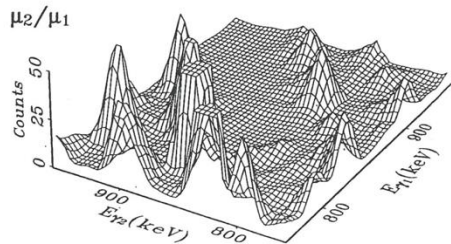
Correlation spectrum

(discrete transitions subtracted
on right hand side)



b)

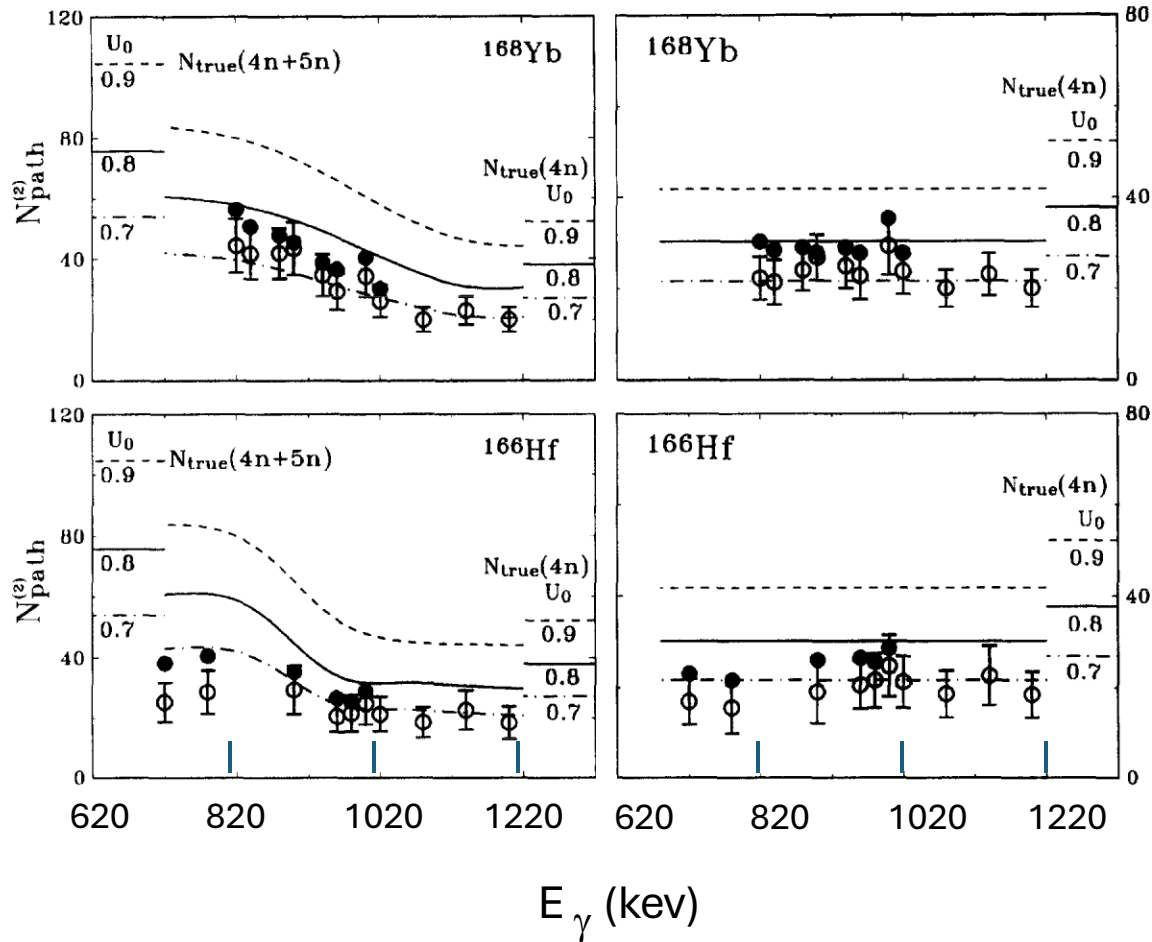
Smooth spectrum



c)

Local second moment –
that is local variance

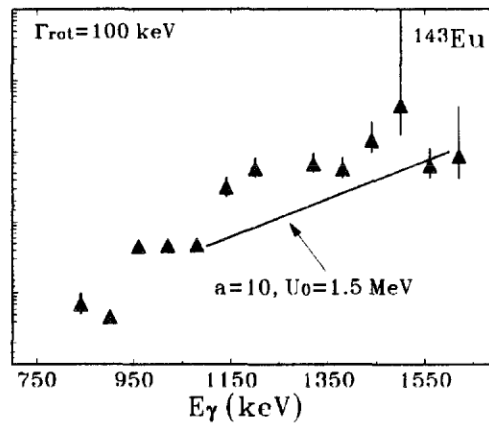
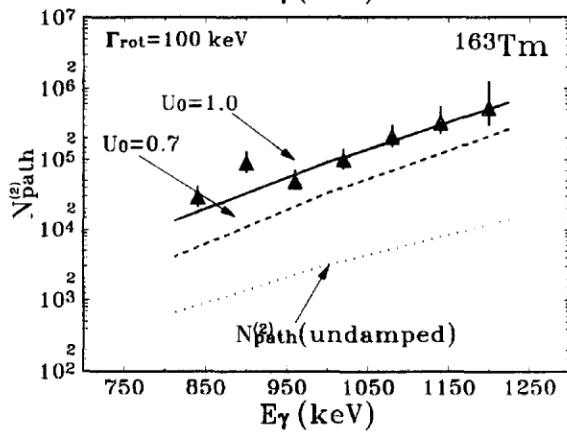
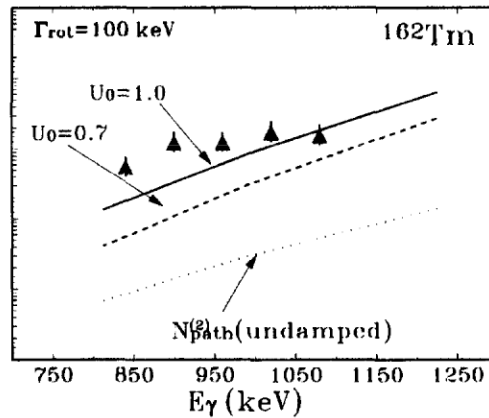
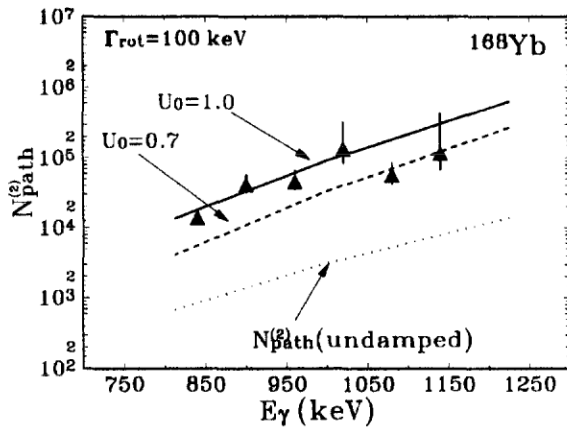
Number of paths - ridge



assume
random
energies
along ridge

Number of paths - valley

Assume local
Porter-Thomas
fluctuations



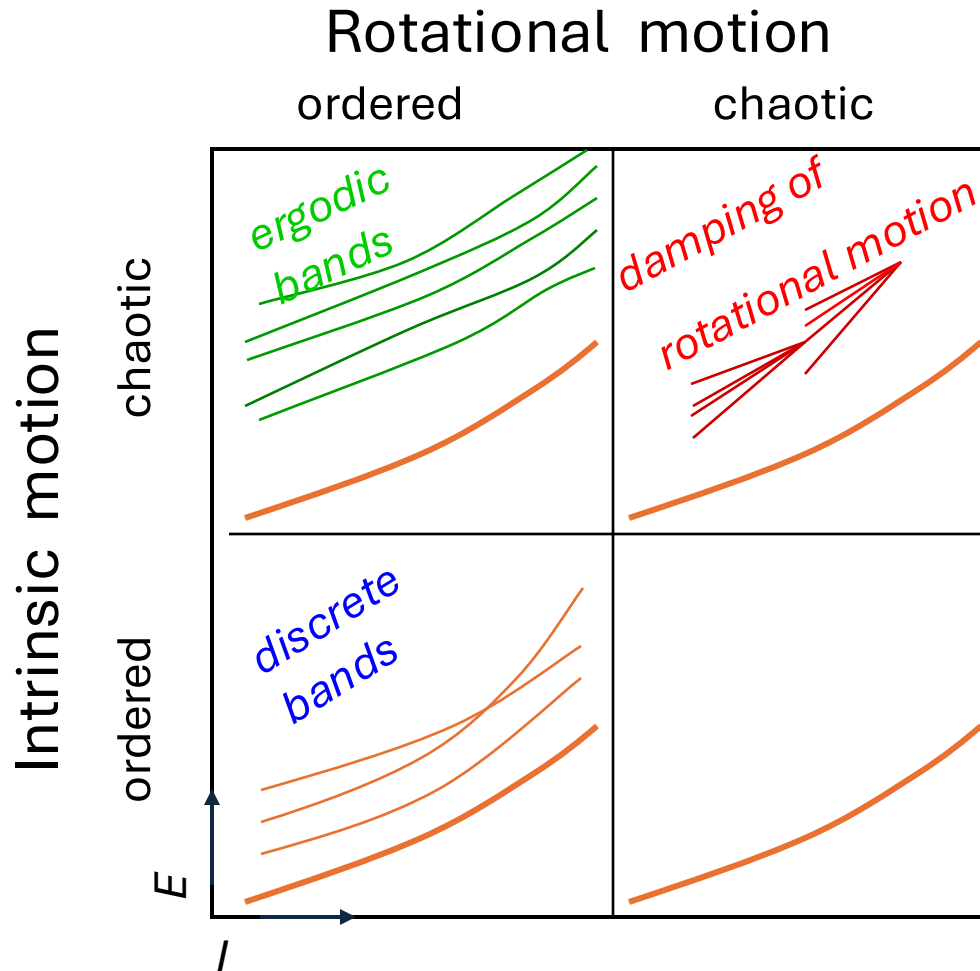
Main results:

- develop the fluctuation analysis into a well documented and reliable technique
- confirm picture of band mixing and damping of rotational motion:
discrete bands up to about 700 keV above yrast,
mixing above
- first values for the rotational damping width
 $\Gamma_{\text{rot}} \approx 100 \text{ keV}$

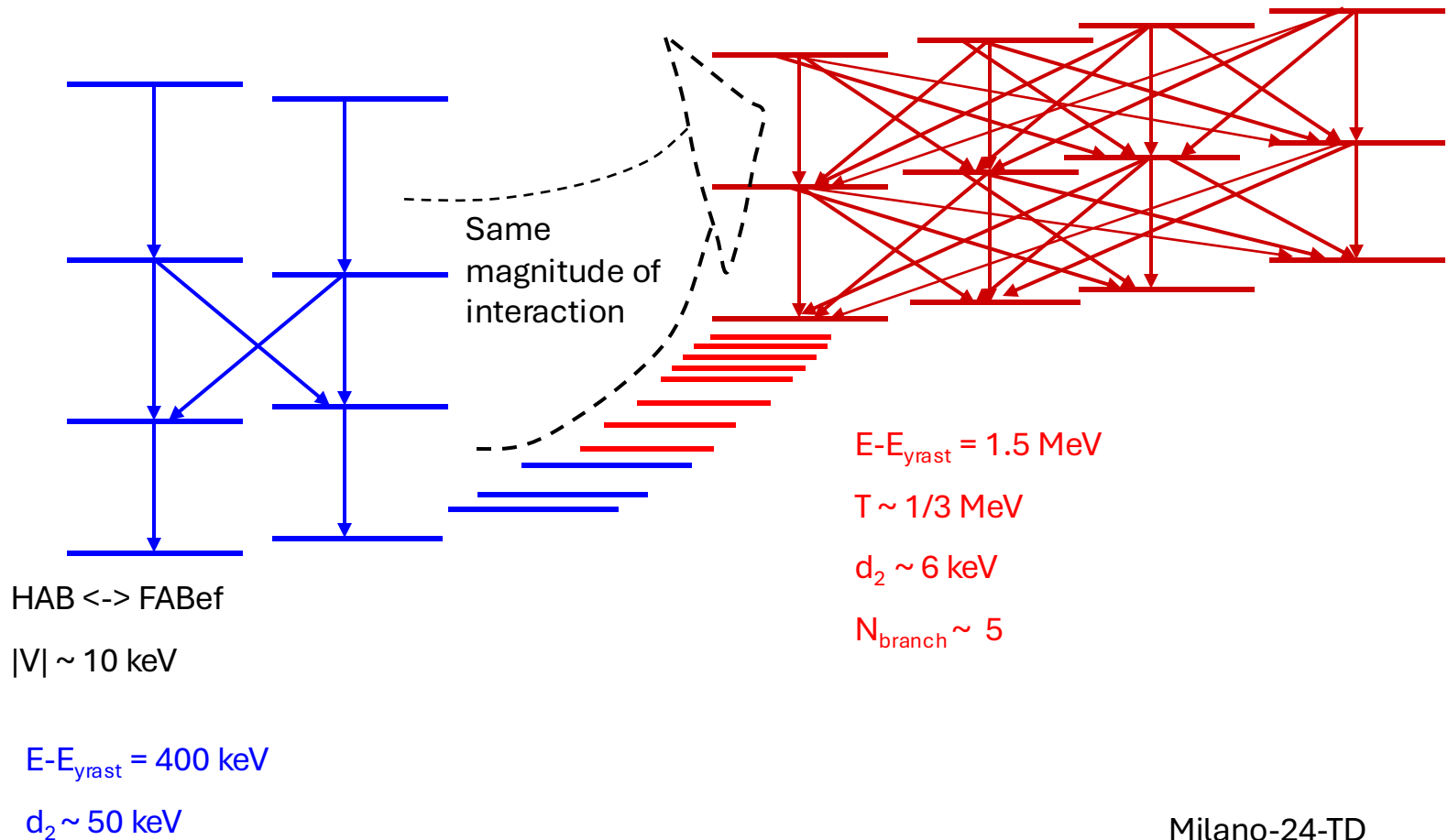
Forward to current understanding

- Results on damping since Silvias thesis:
Calculations with mixed bands
- General perspective: temperature goes together with considerations of order and chaos
- (- fruitful covariance studies)
- (- extension to SD bands)
- Unsuccessfull search for HD bands

Rotational and intrinsic motion

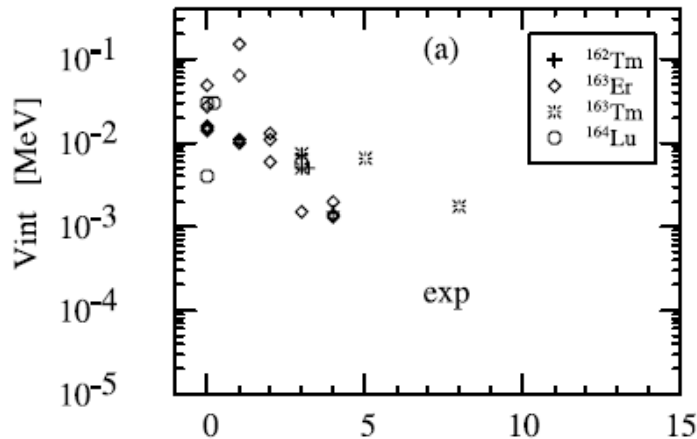


Infer: mixed bands higher up in energy

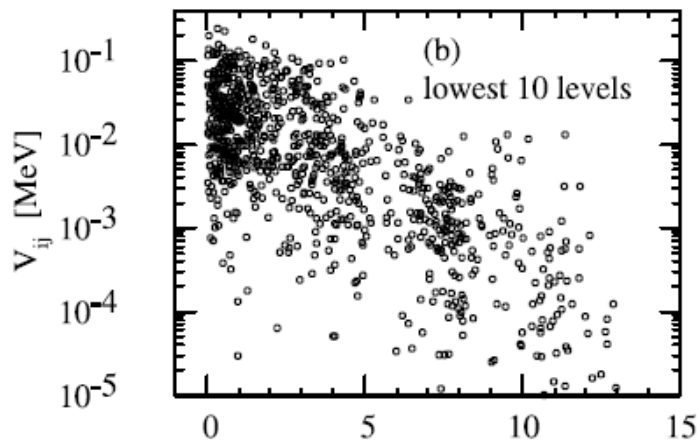


Magnitude of interactions

M. Matsuo et al. / Nuclear Physics A 736 (2004) 223–240



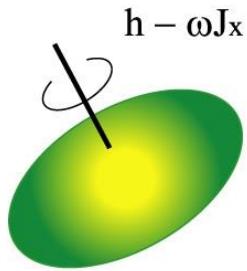
experimental, from level crossings



surface – δ interaction

ΔK

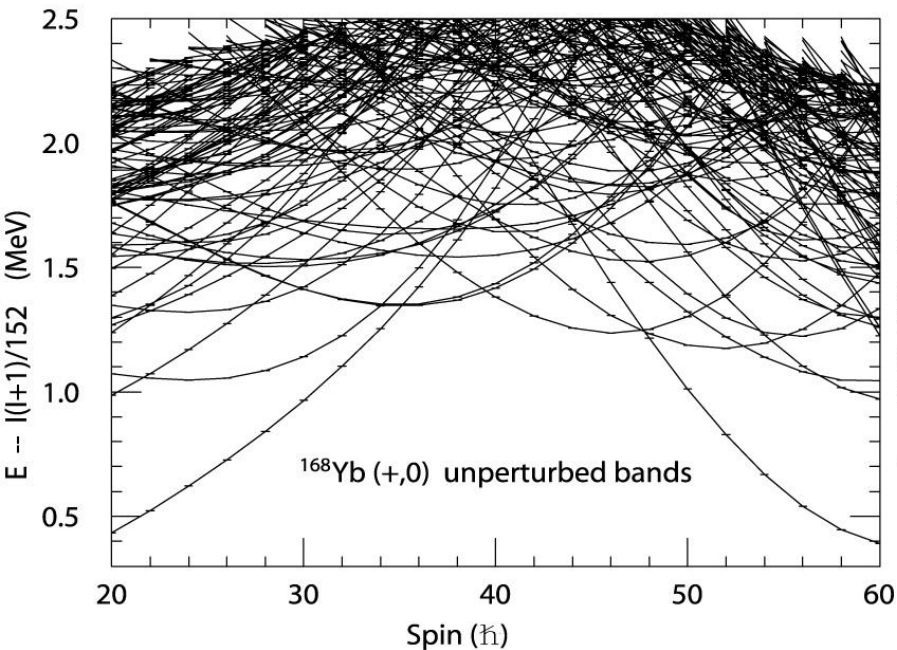
Calculations of interacting bands



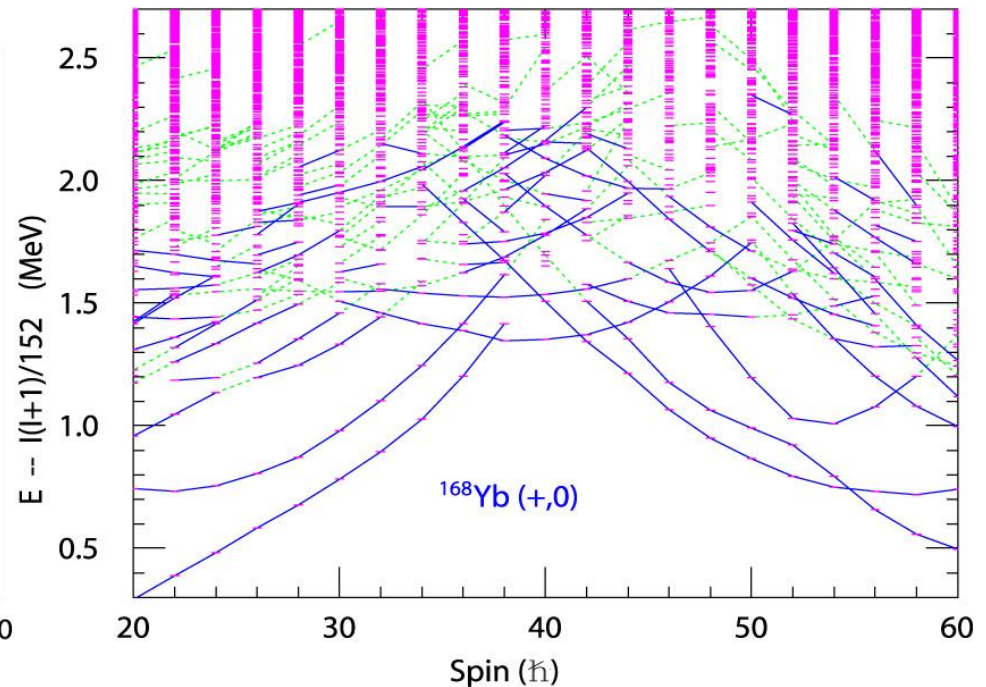
- Cranked Nilsson potential
- Surface delta interaction

M. Matsuo et al. Nucl. Phys.
A617(1997)1

Cranking np-nh basis bands



Configuration mixing with residual interaction



Level spacings

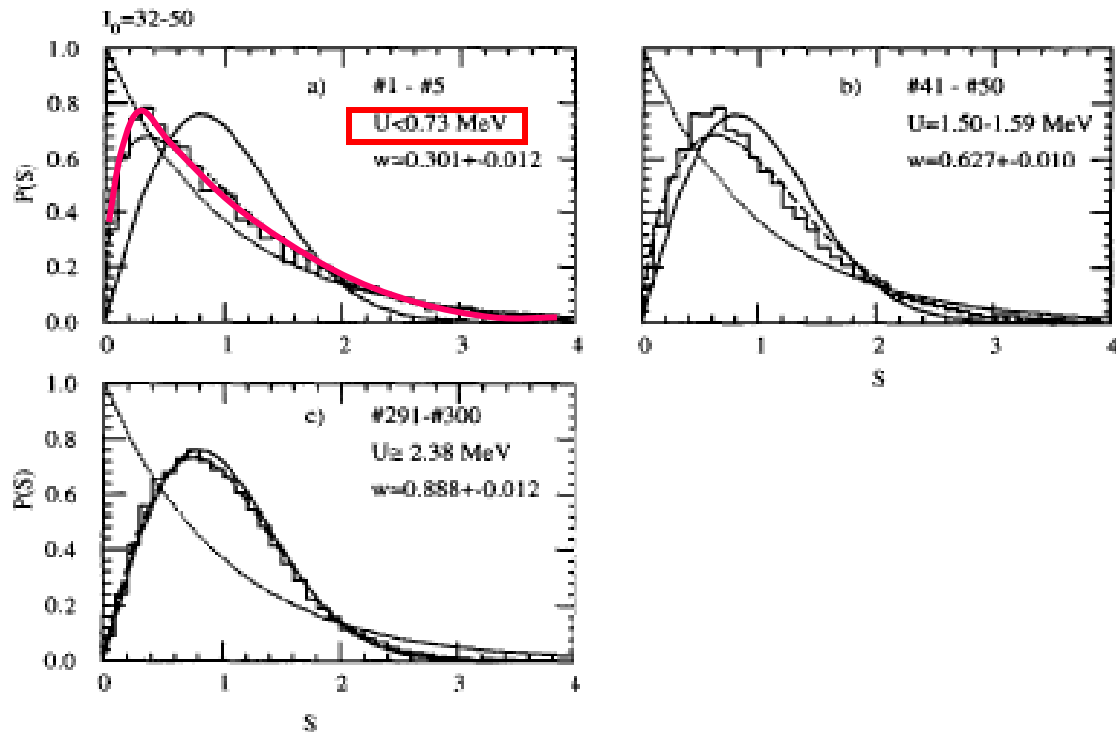
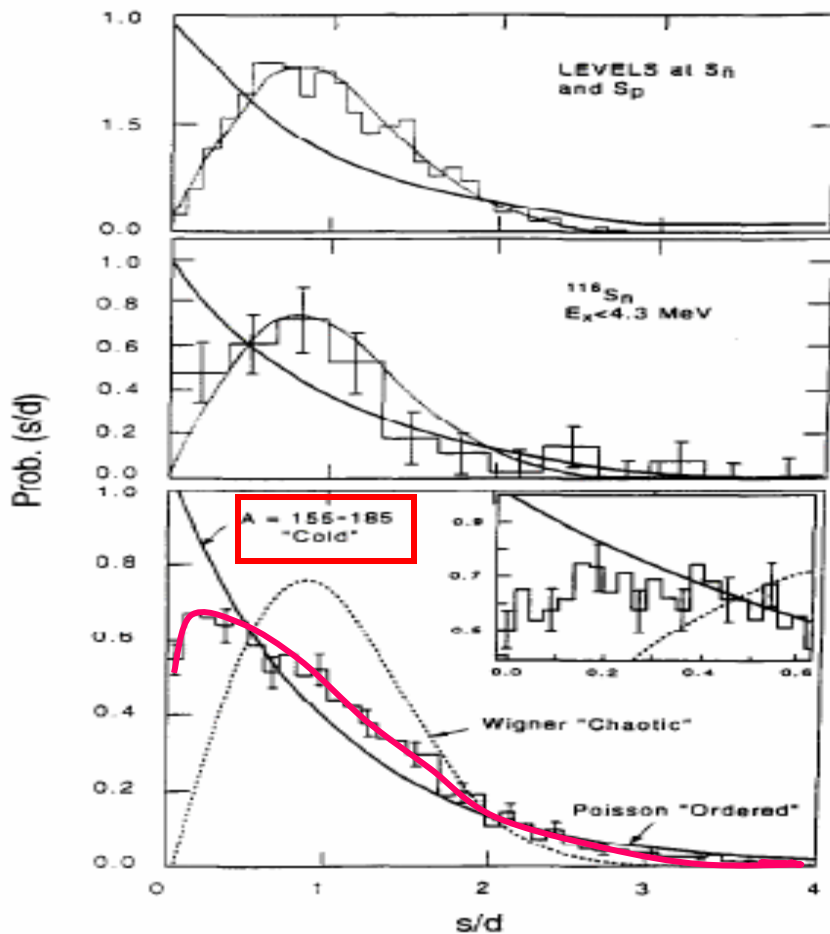


Fig. 2. The NND for energy bins containing the first to 5th, 41st to 50th, and 291st to 300th levels of each spectrum within spin interval $I_0 = 32-50$. The solid, dotted, and dashed curves represent the Wigner, the Poisson and the fitted Brody distributions, respectively. The extracted Brody parameter w and the average intrinsic excitation energy U measured from the yrast line are indicated in the figure for each bin.

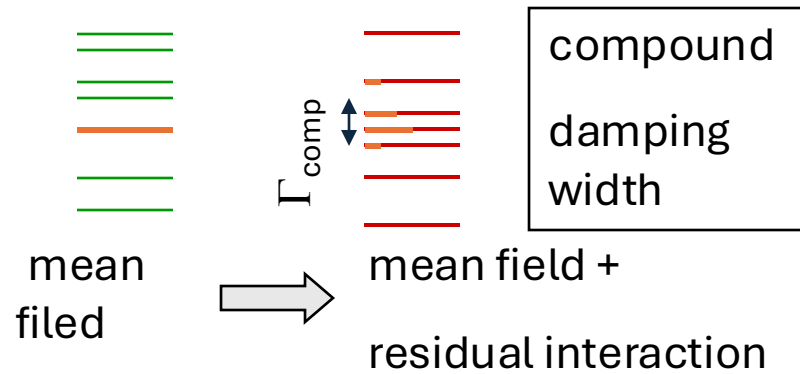
Experimental level spacings



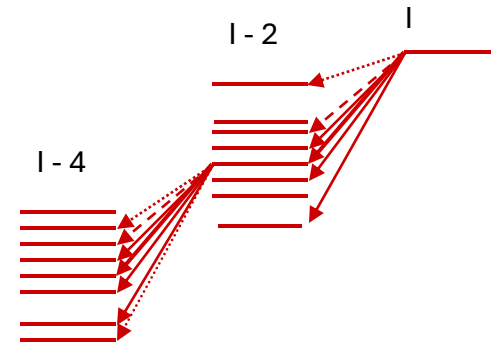
J.D Garrett et.al. Phys. Lett.
B392 (1997) 24

Mixing and Damping

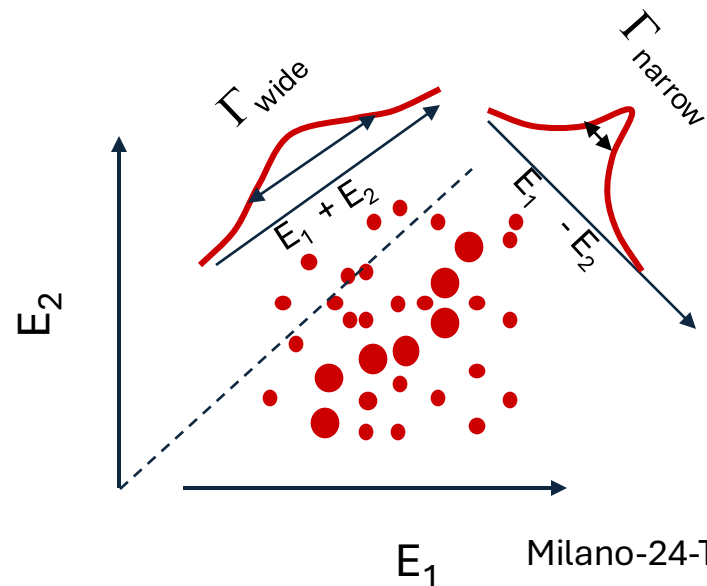
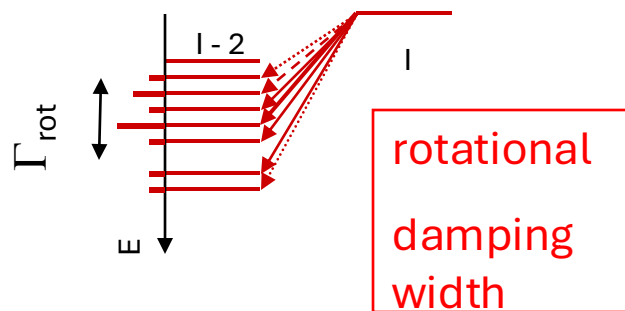
$\Delta I = 0$: spreading of basis band state over energy interval



$\Delta I = -4$: two steps in a cascade

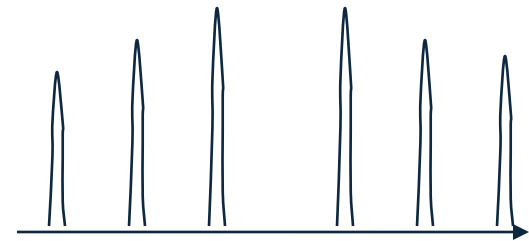
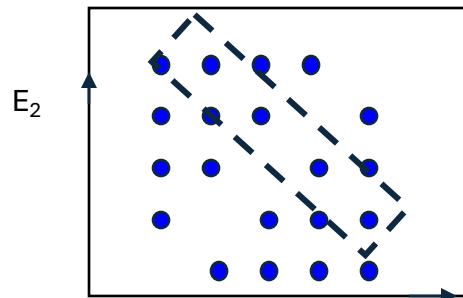


$\Delta I = -2$: one step in a cascade



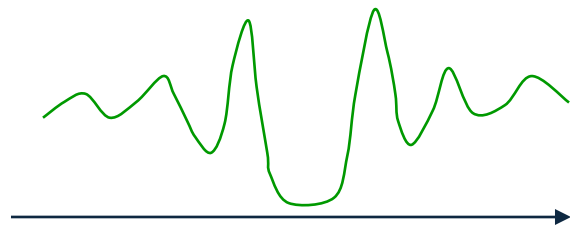
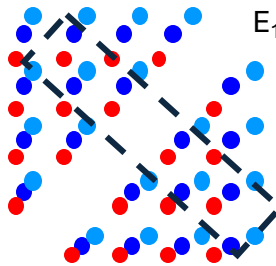
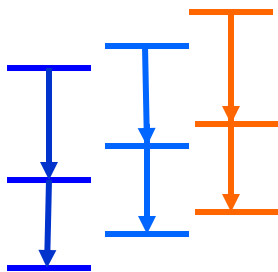
$E_\gamma - E_\gamma$ coincidence spectra

one band

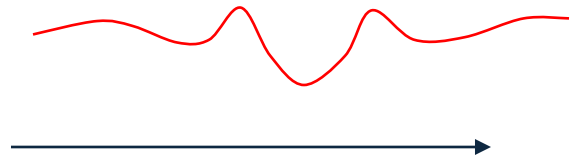
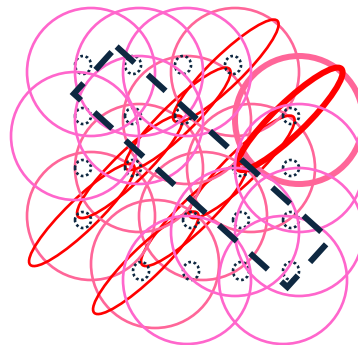
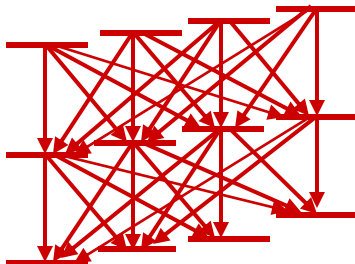


$E_1 - E_2$

Many bands

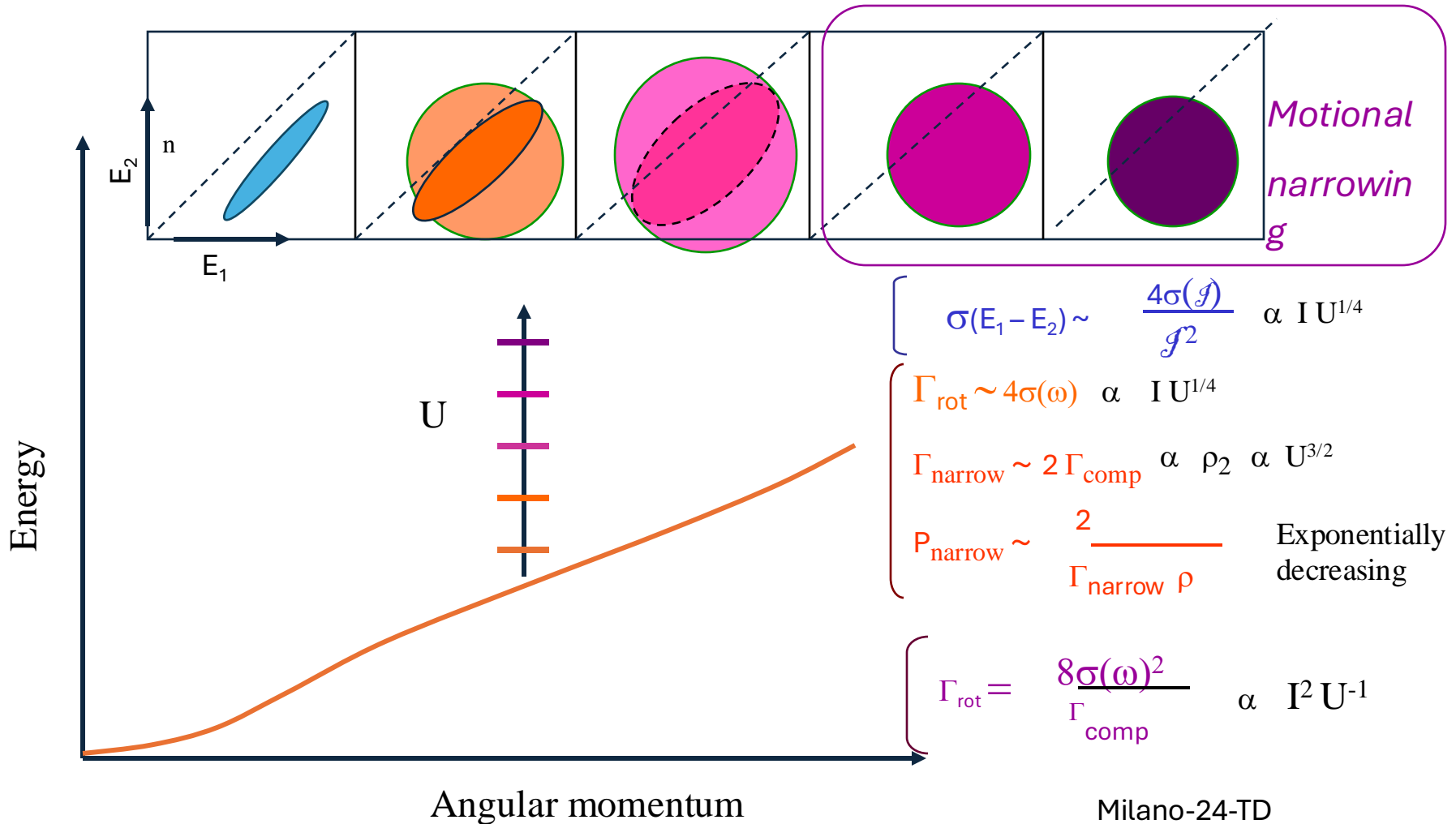


Rotational damping

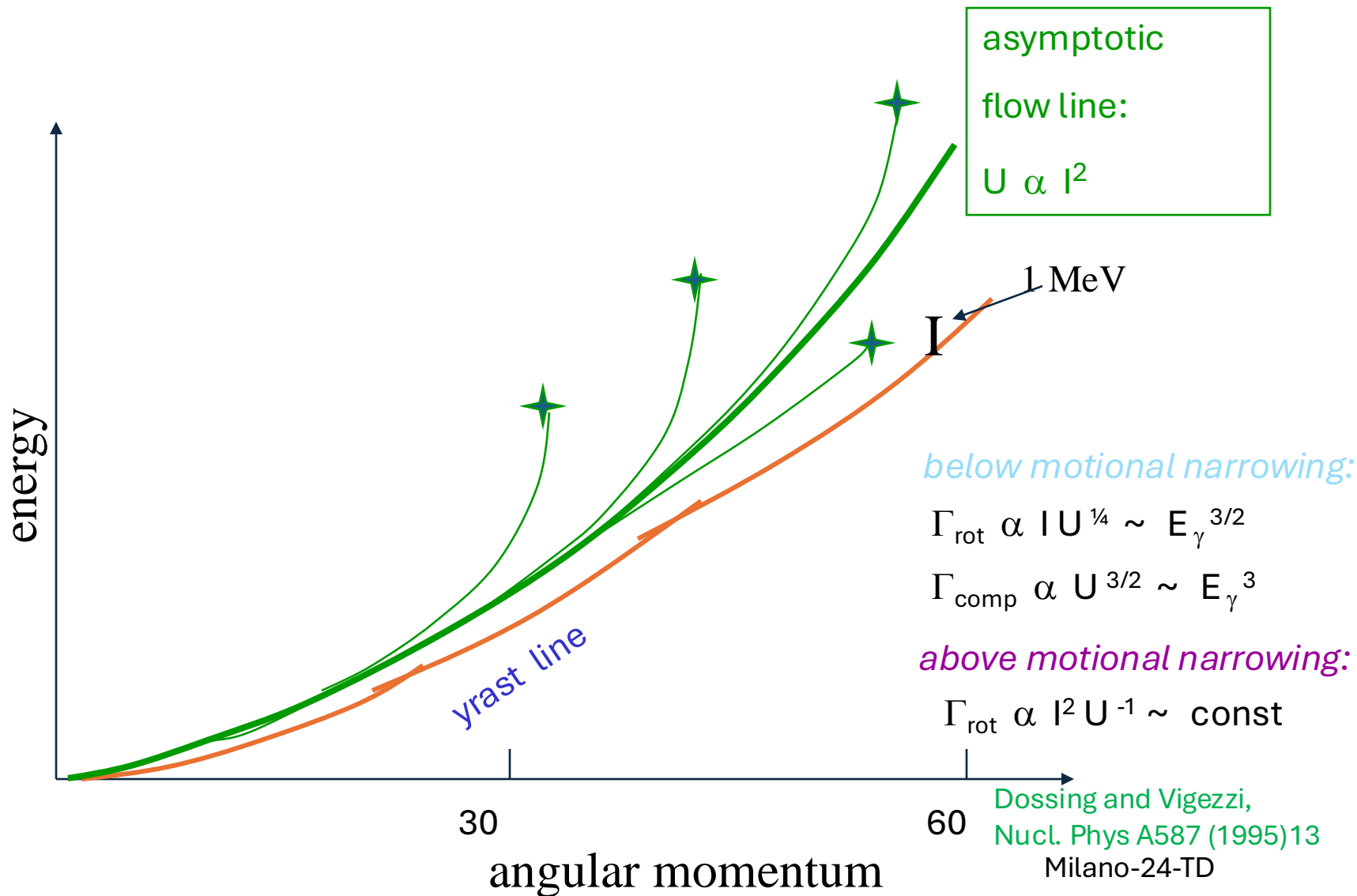


Milano-24-TD

$E_\gamma - E_\gamma$ schematic two-step strength functions



Flow in cascades



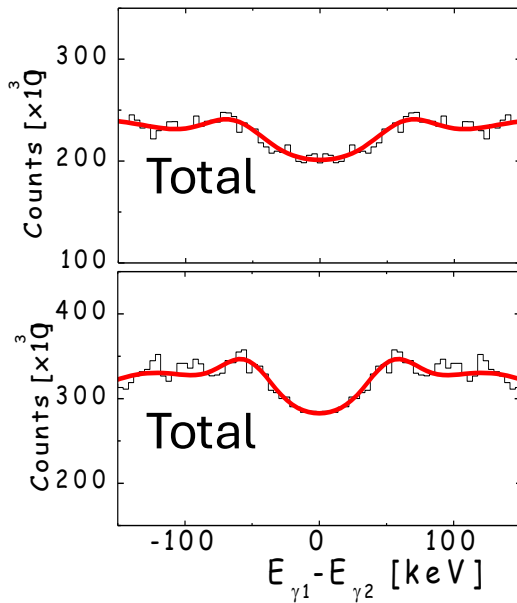
Compare data to cascade calculation with mixed bands: Perpendicular cuts => Γ_{comp} and Γ_{rot}

$^{18}\text{O} + ^{150}\text{Nd} \rightarrow ^{163}\text{Er} + 5\text{n}$

@ 87,93 MeV

$v/c = 0.96\%$

$I_{\text{max}} \approx 40\hbar$, $U_{\text{max}} \approx 4\text{ MeV}$



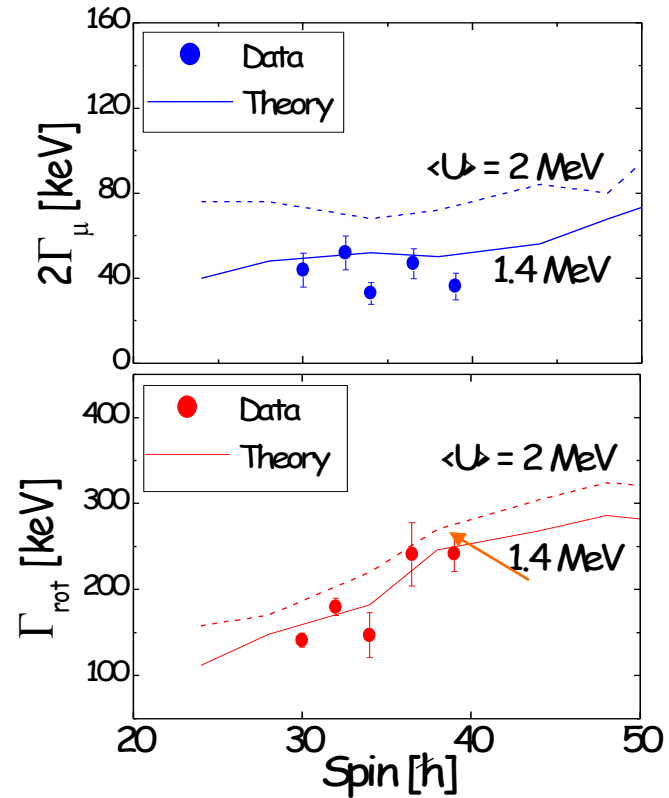
$\langle E_g \rangle = 960\text{ keV}$

$I = 32\hbar$

$\langle E_g \rangle = 900\text{ keV}$

$I = 30\hbar$

$I = 30-40:$

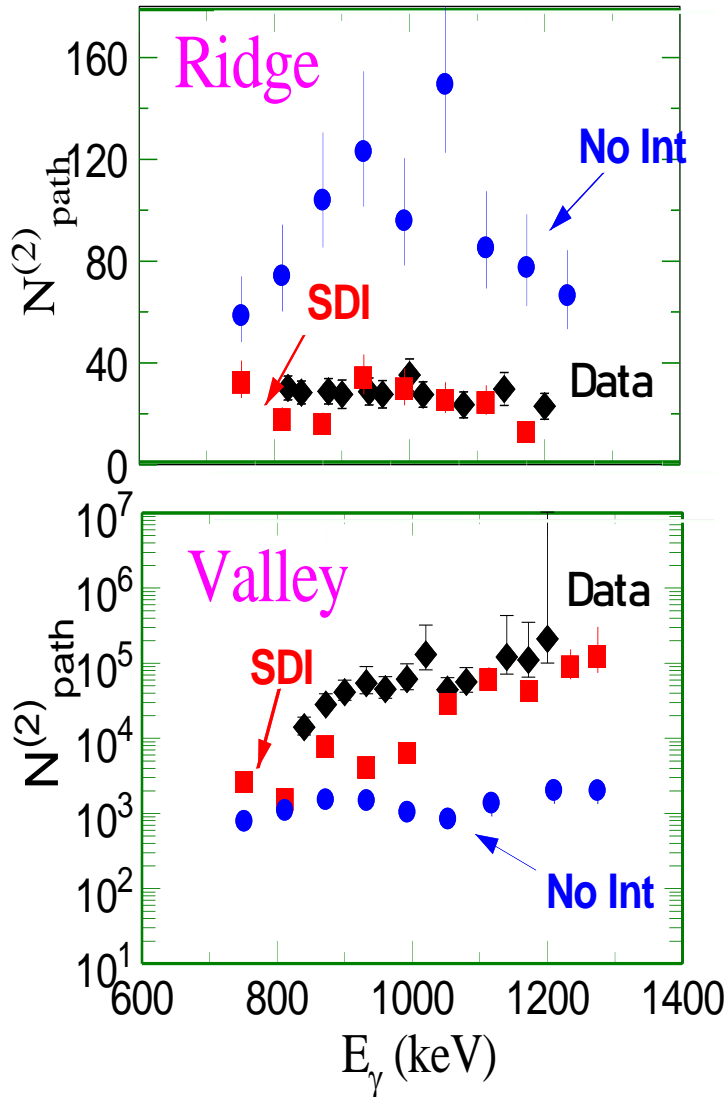


$\Gamma_{\text{comp}} \approx 20\text{ keV}$

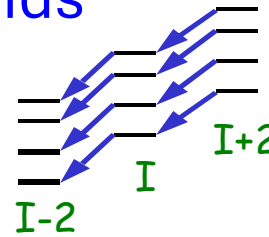
$\Gamma_{\text{rot}} \approx 150 - 200\text{ keV}$

$P_{\text{narr}} \sim 10\%$

Fluctuations on ridge and in valley: number of paths



Discrete bands



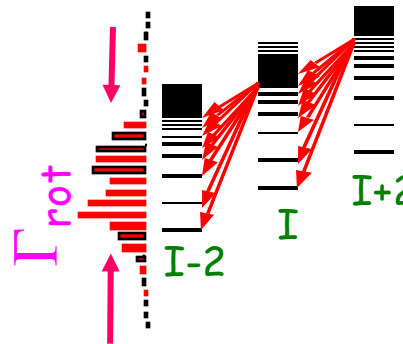
about ~ 30 discrete bands of the four π σ 's up to

$U \sim 800$ keV

cranking model level density

$\rho(U, N, Z)$

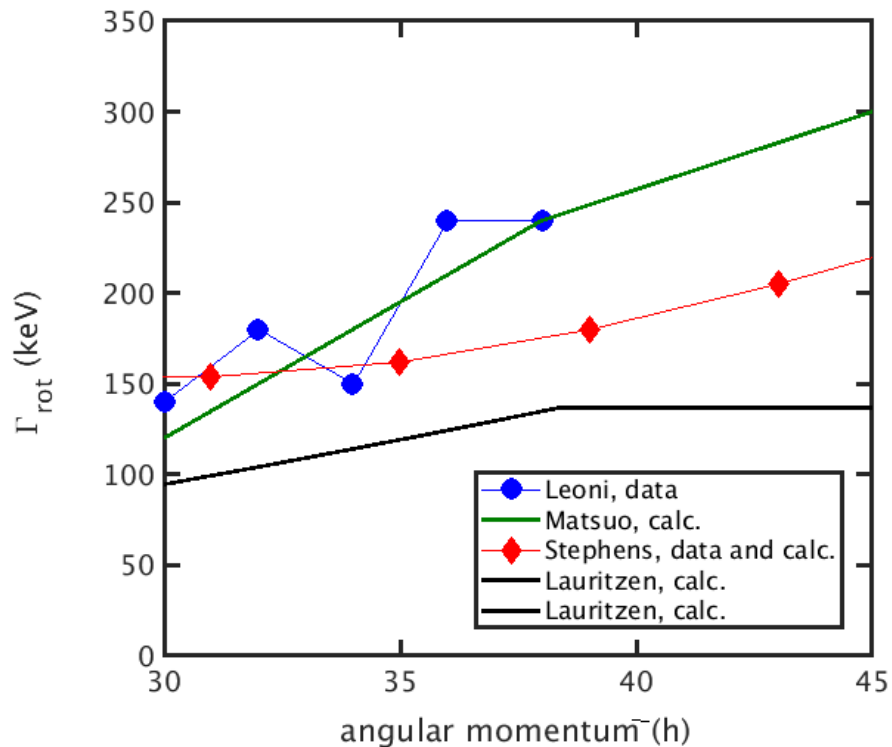
Rotational Damping



Fragmented decay of weak transitions in the valley

A. Bracco et al., PRL76(1996)4484

Compilation of Γ_{rot} results



Schematic behavior:

below motional narrowing:

$$\Gamma_{\text{rot}} \propto I U^{1/4} \sim E_{\gamma}^{3/2}$$

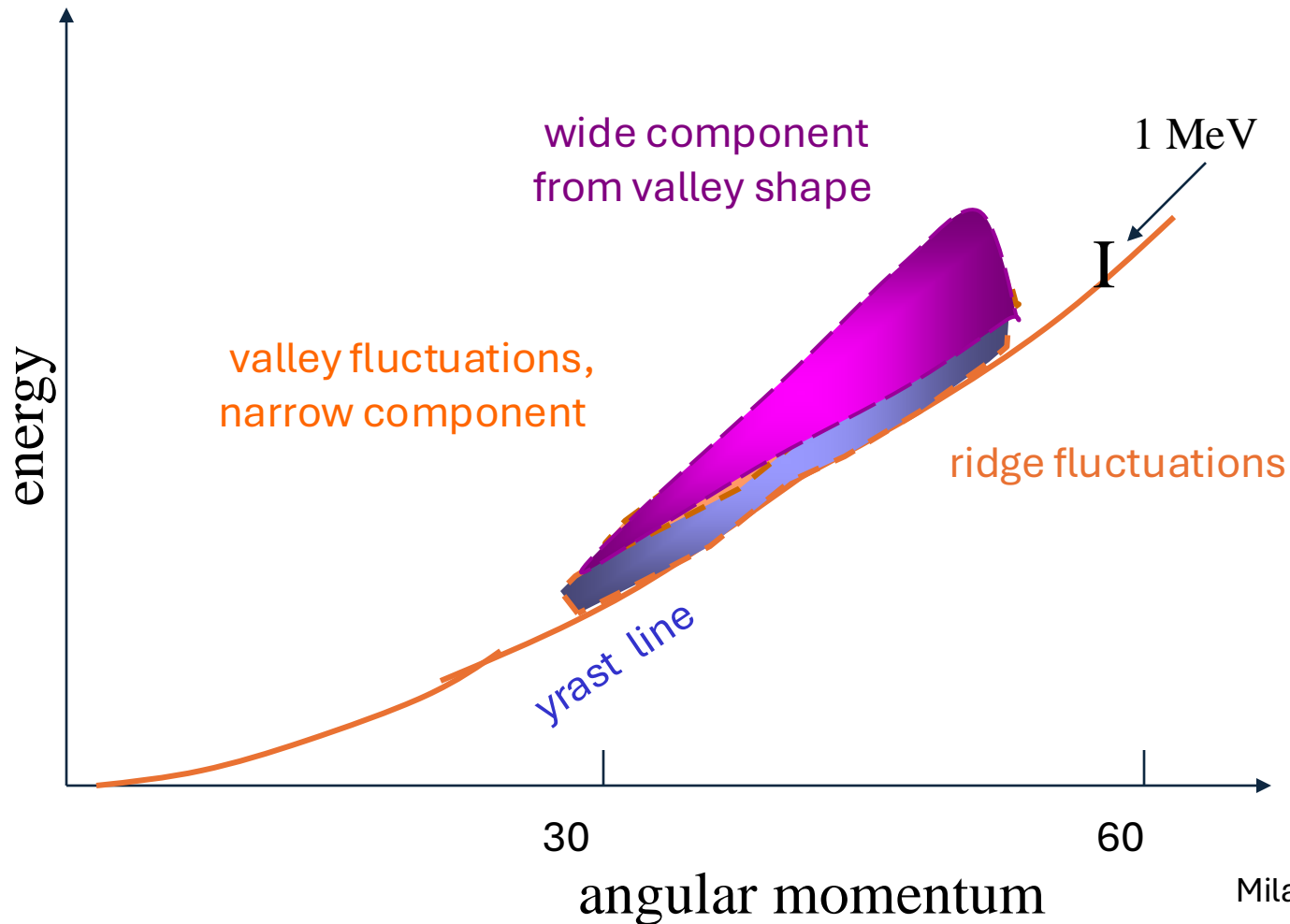
$$\Gamma_{\text{comp}} \propto U^{3/2} \sim E_{\gamma}^3$$

above motional narrowing:

$$\Gamma_{\text{rot}} \propto I^2 U^{-1} \sim \text{const}$$

F.S. Stephens et.al. Phys. Rev. C78(2008) 034303

probed regions in angular momentum and excitation energy



Conclusions

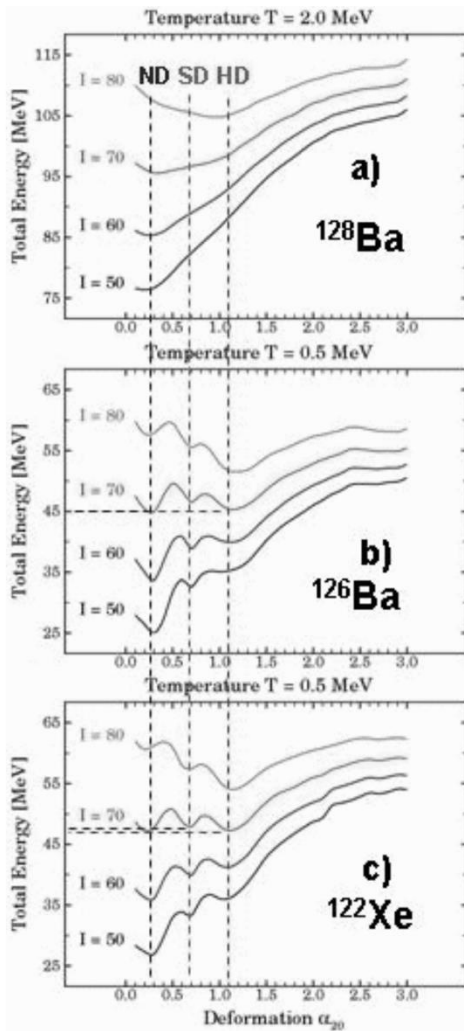
At a certain point in development of detector arrays, the notion of “complete spectroscopy” was introduced.

Comment at that time by Ben Mottelson: “I do not like this idea, it is like pinning down the nucleus, leaving no room for future surprises”.

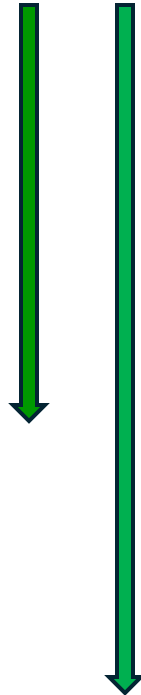
The fluctuation analysis provides insight into the coupling between thermal motion and rotation in rare-earth nuclei. This is in a way as complete as we can get, recognizing the basic statistical nature of thermal motion.

The fluctuation and covariance analysis can be used and should be used for many applications: fission fragments, transfer reactions, octupole soft nuclei, gamma-soft nuclei, nuclei with coexistence of competing shapes ...

Occurrence of hyperdeformed shapes - axis ratio about 3:1



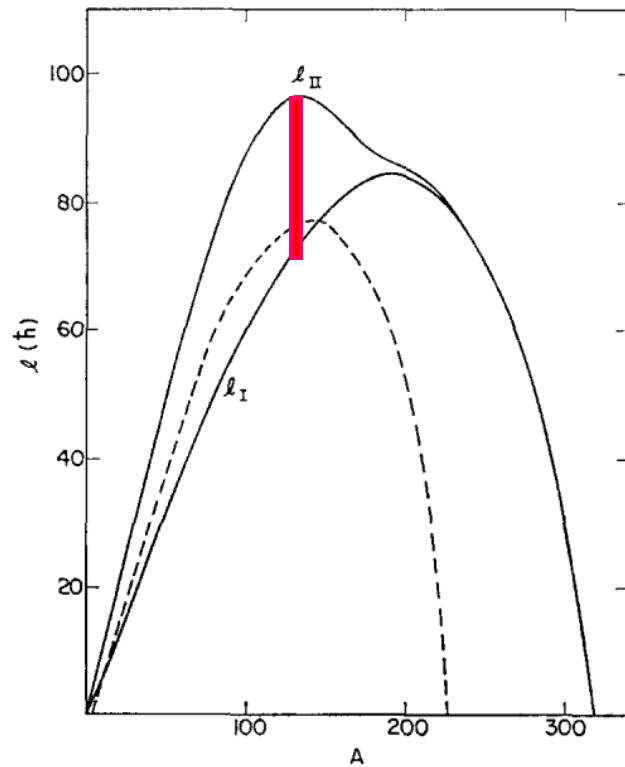
Formed
Compound nucleus



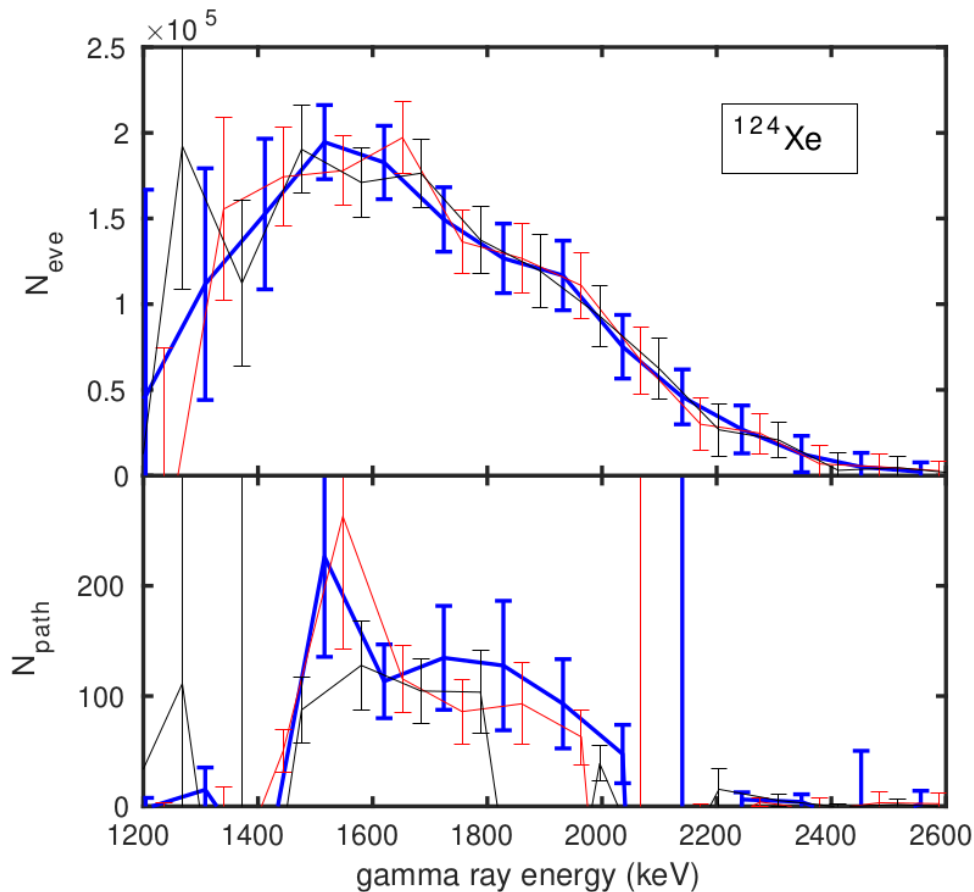
Hyperdeformed shape (axis
ratio about 3:1 favored by
both LDM and shell structure

Decay by neutrons or alpha
particles –
HD shell structure may appear

Mass around $A=130$ favored by rotating liquid drop model



Search for hyperdeformation: intense normally deformed ridges

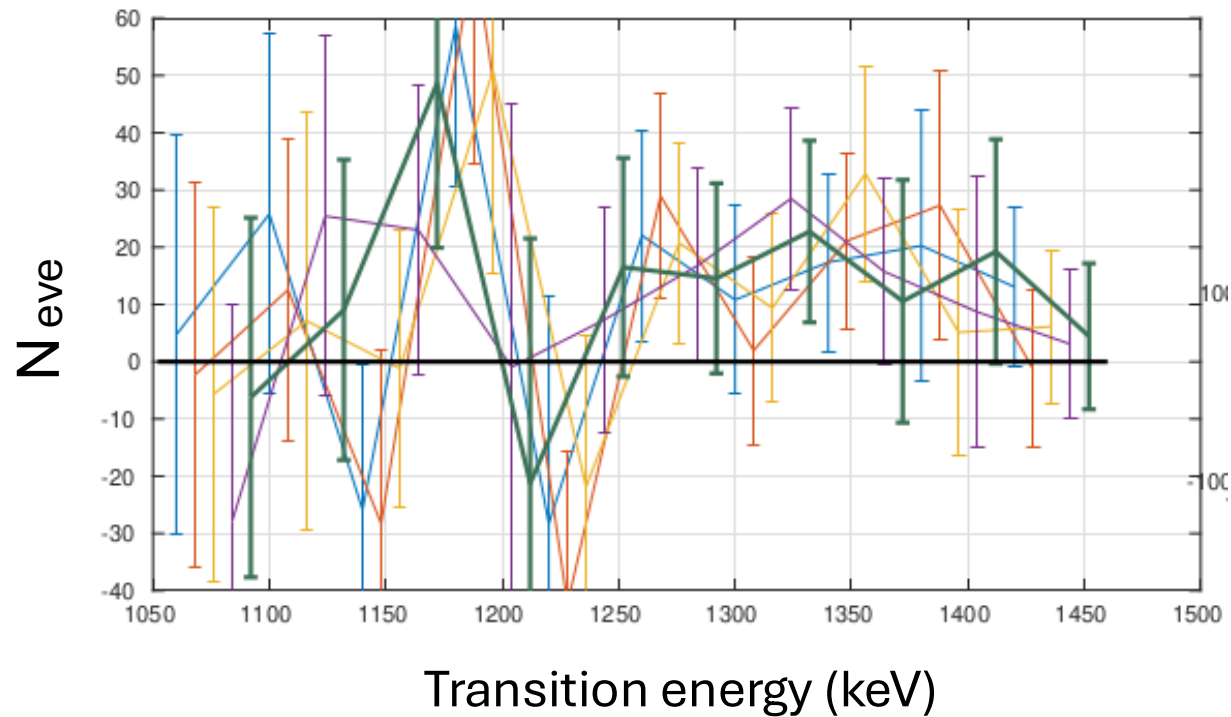


Many regular rotational bands resolved, with transition energy up to about 2400 keV

Ridge intensity about 4 times stronger than intensity in resolved bands

Analysing data from
B. Herskind et.al. ,
Physica Scripta T125 (2006) 108

Search for hyperdeformed ridge – best moment of inertia

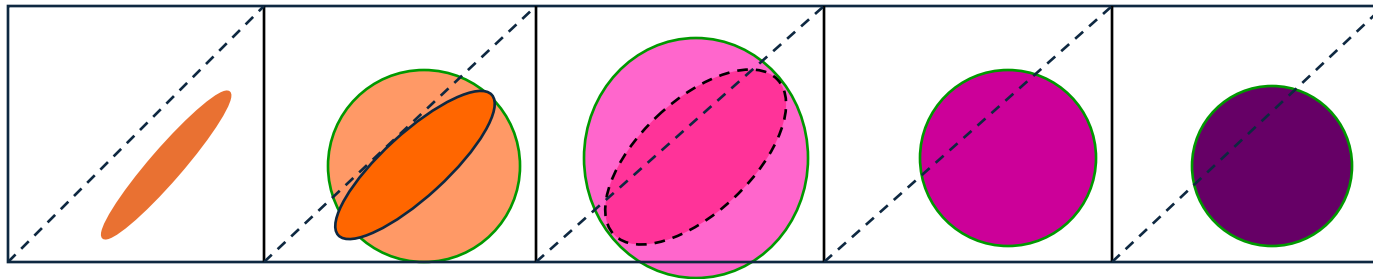


About same
result is found
searching for
ridges in
simulated
completely
random
spectra

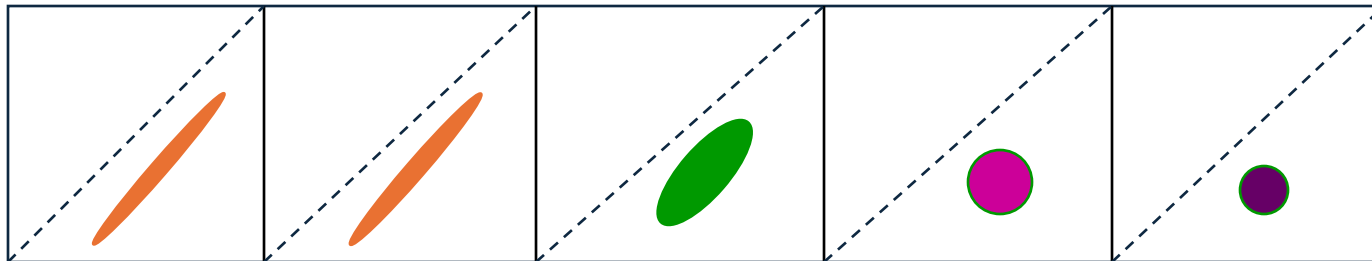
blank slide

Schematic strength functions with ergodic bands

discrete bands > rotational damping



discrete bands > ergodic bands > rotational damping



$$\sigma(E_1 - E_2) \sim \frac{4\sigma(\mathcal{J})}{\mathcal{J}^2}$$

$$\propto I U^{1/4}$$

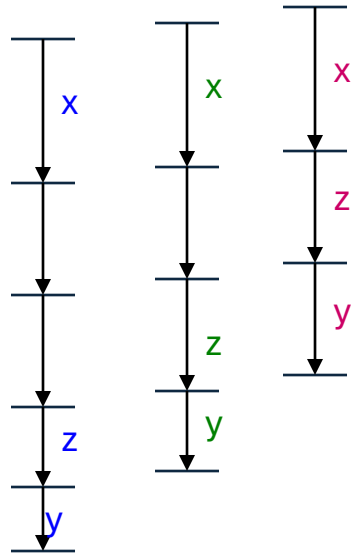
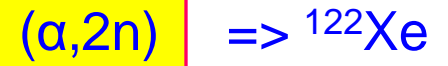
$$\Gamma_{\text{ridge}} = \frac{8\sigma(\omega)^2}{\Gamma_{\text{comp}}} \propto I^2 U^{-1}$$

$$\Gamma_{\text{long}} \sim \sqrt{\frac{2}{\Gamma_{\text{comp}} \rho}} \sigma(\omega)$$

Exponentially decreasing

$$\Gamma_{\text{rot}} = \frac{8\sigma(\omega)^2}{\Gamma_{\text{comp}}} \propto I^2 U^{-1}$$

Tilted planes in $\gamma - \gamma - \gamma$ spectra

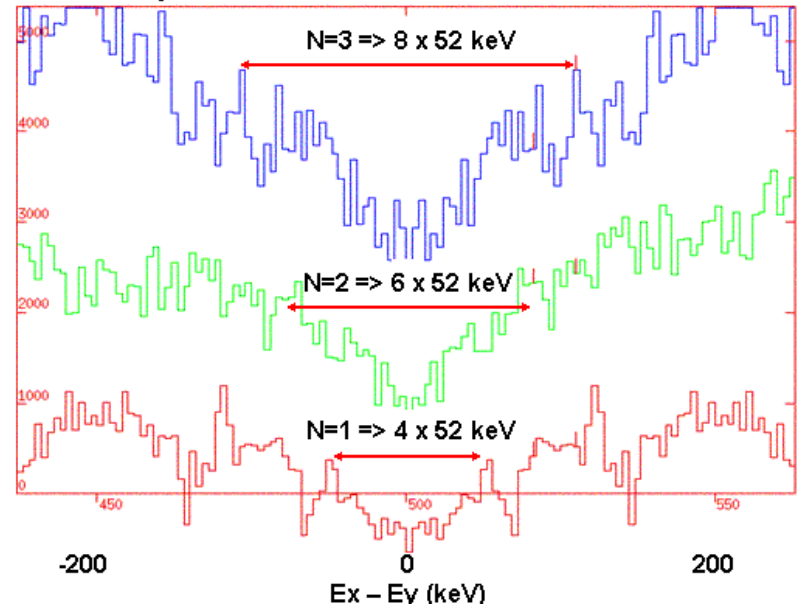


$N=3: x+3y-4z = \pm\delta$

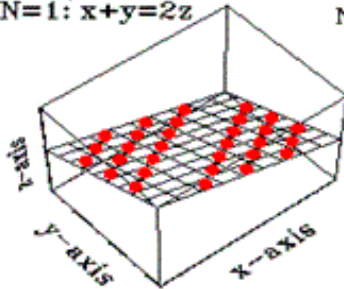
$N=2: x+2y-3z = \pm\delta$

$N=1: x+y-2z = \pm\delta$

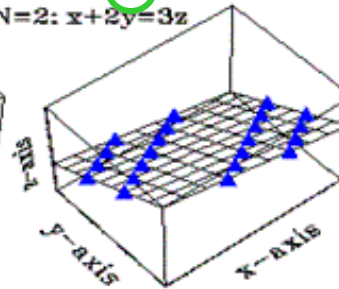
Perpendicular Cut = 1440 ± 204 keV



b) $N=1: x+y=2z$



c) $N=2: x+2y=3z$



$J(2) \sim 77 \hbar^2 / \text{MeV}$