

Spherical and deformed nuclei within the EDF framework

Danilo Gambacurta
gambacurta@Ins.infn.it
INFN-LNS Catania



*WS 3x60: Multifaceted aspects of collaborative research
on nuclear structure at UNIMI and INFN-MI
17–18 October 2024, Milano, Italy*

Outline

- ...
- ...
- ...

Outline → Story-line

- ...
- ...
- ...

Outline → Storyline

- The first time I met Gianluca

Outline → Storyline

- The first time I met Gianluca
- The first paper with Gianluca (back in time or ... “*ab initio*”)

Outline → Storyline

- The first time I met Gianluca
- The first paper with Gianluca (back in time or ... “*ab initio*”)
- The most recent collaboration with Gianluca (deformed nuclei)

Outline → Storyline

- The first time I met Gianluca
- The first paper with Gianluca (back in time or ... “*ab initio*”)
- The most recent collaboration with Gianluca (deformed nuclei)
- Future work with Gianluca



*Terzo Corso
Otranto 28 Maggio - 2 Giugno 2007*

II - Exotic nuclei

G. Colò

University of Milano, and INFN



colo@mi.infn.it

*Otranto,
May 28th-30th, 2007*

I - Effective interactions for nuclear structure

G. Colò

University of Milano, and INFN



colo@mi.infn.it

*Otranto,
May 28th-30th, 2007*

III - Time-dependent theories

G. Colò

University of Milano, and INFN



colo@mi.infn.it

*Otranto,
May 28th-30th, 2007*



Terzo Corso

Otranto 28 Maggio - 2 Giugno 2007

Teachers:
P.F. Bortignon

A. Bracco

L. Coraggio

G. Colò

A. Gargano

P. Finelli

L. Fortunato

R. Fossion

E.D. Lanza

G. Pollarolo

E. Vigezzi

A. Vitturi

II - Exotic n

G. Colò

University of Milano, and INFN

colo@mi.infn.it



I - Effective interactions for nuclear structure

Otranto,

May 28th-30th, 2007

, and INFN

olo@mi.infn.it

e-dependent theories

Otranto,

May 28th-30th, 2007

, and INFN

olo@mi.infn.it

80%



*Terzo Corso
Otranto 28 Maggio - 2 Giugno 2007*

II - Exotic nuclei

G. Colò

University of Milano, and INFN



colo@mi.infn.it

*Otranto,
May 28th-30th, 2007*

I - Effective interactions for nuclear structure

G. Colò

University of Milano, and INFN



colo@mi.infn.it

*Otranto,
May 28th-30th, 2007*

III - Time-dependent theories

G. Colò

University of Milano, and INFN



colo@mi.infn.it

*Otranto,
May 28th-30th, 2007*



Terzo Corso
Otranto 28 Maggio - 2 Giugno 2007

II - Exotic nuclei

G. Colò

University of Milano, and INFN



colo@mi.infn.it

Otranto,
May 28th-30th, 2007

I - Effective interactions for nuclear structure

G. Colò

University of Milano, and INFN



colo@mi.infn.it

Otranto,
May 28th-30th, 2007

III - Time-dependent theories

G. Colò

University of Milano, and INFN



colo@mi.infn.it

Otranto,
May 28th-30th, 2007

Motivations: Effective interactions and Energy Density Functional Theory

- Phenomenological forces like Skyrme (zer-range), or Gogny (finite-range), Covariant (meson-exchange) depending on some parameters
- These parameters are obtained to reproduce some observables (binding energies and charge radii) of atomic nuclei and nuclear matter properties

Advantages

- Relatively (numerically) simple to be treated
- Quite satisfactory description of nuclei along the nuclear chart
- Link with the underlying functional and Equation of State (EoS)

Disadvantages

- We fit (a few) data to reproduce (a lot) data, not the cleanest way
- We loose connection with the underlying, more fundamental theory (QCD)
- Errors and uncertainties not under control

Current Challenge for the nuclear community

- Ab initio techniques to derive in-medium nuclear interaction from the QCD (effective field theory)

PHYSICAL REVIEW C **84**, 024301 (2011)

Determination of local energy density functionals from Brueckner-Hartree-Fock calculations

D. Gambacurta,¹ L. Li,² G. Colò,³ U. Lombardo,¹ N. Van Giai,⁴ and W. Zuo⁵

¹*Dipartimento di Fisica and INFN-LNS, Via S. Sofia 64, I-95123 Catania, Italy*

²*State Key Lab Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China*

³*Dipartimento di Fisica, Università degli Studi, and INFN Sezione di Milano, via Celoria 16, I-20133 Milano, Italy*

⁴*Institut de Physique Nucléaire, IN2P3-CNRS and Université Paris-Sud, F-91405 Orsay, France*

⁵*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*

(Received 3 May 2011; published 8 August 2011)

Skyrme-like energy density functionals are built upon new *ab initio* calculations of nuclear matter which reproduce the empirical saturation properties. These calculations are performed in the framework of the Brueckner-Hartree-Fock approximation with consistent two- and three-body forces. The Skyrme parametrizations are obtained from a simultaneous fit of the spin-isospin components of the potential energy of symmetric nuclear matter and of the experimental energies and charge radii of a number of closed-shell and closed-subshell nuclei. These parametrizations are tested on the energies and charge radii of the Sn isotopic chain and on the ^{208}Pb giant resonances, showing a substantial improvement of the LNS functional obtained in a previous fit. The comparison with standard Skyrme forces can give some hints on how to reconcile nuclear matter and finite nuclei.

DOI: [10.1103/PhysRevC.84.024301](https://doi.org/10.1103/PhysRevC.84.024301)

PACS number(s): 21.65.Mn, 21.30.-x

PHYSICAL REVIEW C **84**, 024301 (2011)

Determination of local energy density functionals from Brueckner-Hartree-Fock calculations

D. Gambacurta,¹ L. Li,² G. Colò,³ U. Lombardo,¹ N. Van Giai,⁴ and W. Zuo⁵

¹*Dipartimento di Fisica and INFN-LNS, Via S. Sofia 64, I-95123 Catania, Italy*

²*State Key Lab Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China*

³*Dipartimento di Fisica, Università degli Studi, and INFN Sezione di Milano, via Celoria 16, I-20133 Milano, Italy*

⁴*Institut de Physique Nucléaire, IN2P3-CNRS and Université Paris-Sud, F-91405 Orsay, France*

⁵*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*

(Received 3 May 2011; published 8 August 2011)

Skyrme-like energy density functionals are built upon new *ab initio* calculations of nuclear matter which reproduce the empirical saturation properties. These calculations are performed in the framework of the Brueckner-Hartree-Fock approximation with consistent two- and three-body forces. The Skyrme parametrizations are obtained from a simultaneous fit of the spin-isospin components of the potential energy of symmetric nuclear matter and of the experimental energies and charge radii of a number of closed-shell and closed-subshell nuclei. These parametrizations are tested on the energies and charge radii of the Sn isotopic chain and on the ²⁰⁸Pb giant resonances, showing a substantial improvement of the LNS functional obtained in a previous fit. The comparison with standard Skyrme forces can give some hints on how to reconcile nuclear matter and finite nuclei.

DOI: 10.1103/PhysRevC.84.024301

PACS number(s): 21.65.Mn, 21.30.-x

Pseudo Data: BHF energy per particle

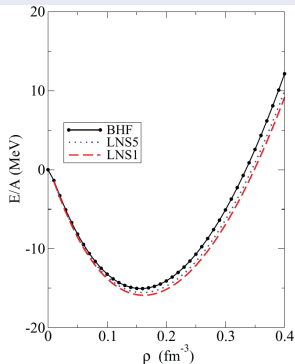


FIG. 6. (Color online) Energy per particle for symmetry nuclear matter: BHF vs. Skyrme parametrizations of Table II.

Pseudo Data: spin-isospin decomposition

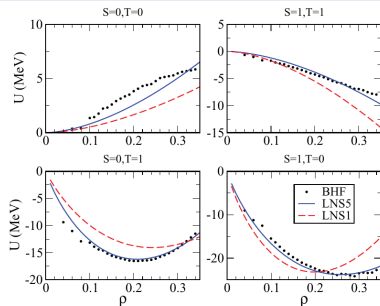
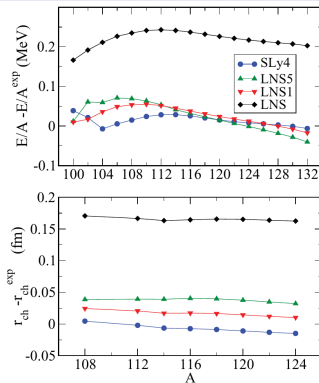


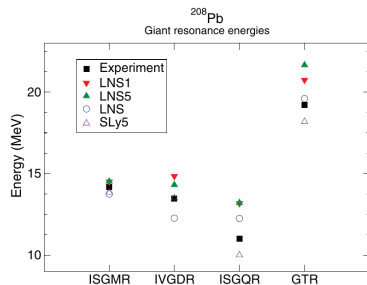
FIG. 5. (Color online) Spin-isospin components of the potential energy from the BHF calculations compared with the results from LNS5 and LNS1.

Binding energies and charge radii in tin isotopes



LNS from L. G. Cao *et al.* PRC 73, 014313 (2006).

Giant Resonances in ^{208}Pb






PHYSICAL REVIEW C **104**, 024315 (2021)

Nuclear energy density functionals grounded in *ab initio* calculations

F. Marino ^{1,2,*} C. Barbieri ^{1,2} A. Carbone,³ G. Colò ^{1,2} A. Lovato ^{4,5} F. Pederiva,^{6,5} X. Roca-Maza ^{1,2}
and E. Vigezzi ²

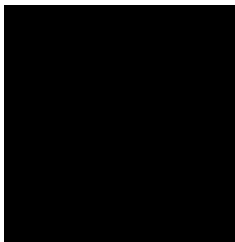
PHYSICAL REVIEW C **107**, 044311 (2023)

Perturbed nuclear matter studied within density functional theory with a finite number of particles

F. Marino ^{1,2,*} G. Colò ^{1,2} X. Roca-Maza ^{1,2} and E. Vigezzi²

¹Dipartimento di Fisica "Aldo Pontremoli", Università degli Studi di Milano, 20133 Milano, Italy

²INFN, Sezione di Milano, 20133 Milano, Italy



*Terzo Corso
Otranto 28 Maggio - 2 Giugno 2007*

II - Exotic nuclei

G. Colò

University of Milano, and INFN



colo@mi.infn.it

Otranto,
May 28th-30th, 2007

I - Effective interactions for nuclear structure

G. Colò

University of Milano, and INFN



colo@mi.infn.it

Otranto,
May 28th-30th, 2007

III - Time-dependent theories

G. Colò

University of Milano, and INFN



colo@mi.infn.it

Otranto,
May 28th-30th, 2007

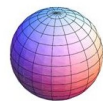
Time-dependent EDF approaches

- Study of the excited states and multipole response
- Random Phase Approximation (RPA) or TDHF, is the standard tool with the EDF framework
- Collective excitations and their impact on EoS
- Increasing number of experiments focus on deformed nuclei

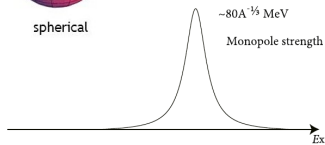
ISGMR ($L=0, S=0, T=0$): n and p in phase (isoscalar), compression (breathing) mode

$E_{ISGMR} \propto \sqrt{K_A} \sim \sqrt{K_\infty}$, $K_\infty = 240 \pm 20 \text{ MeV}$ from ^{208}Pb and from ^{90}Zr
See Garg and Colo' PNPP 101,55 (2018)

The most recent collaboration with Gianluca: deformed nuclei

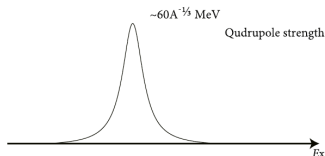


spherical



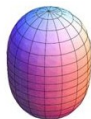
$\sim 80A^{-1/2}$ MeV

Monopole strength

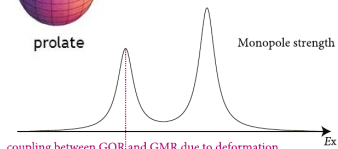


$\sim 60A^{-1/2}$ MeV

Quadrupole strength

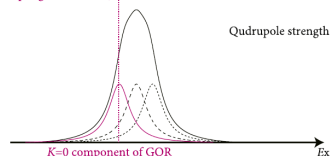


prolate



Monopole strength

coupling between GQR and GMR due to deformation



Quadrupole strength

$K=0$ component of GOR

U. Garg, G. Colò / *Progress in Particle and Nuclear Physics* 101 (2018) 55–95

PHYSICAL REVIEW C **93**, 044324 (2016)

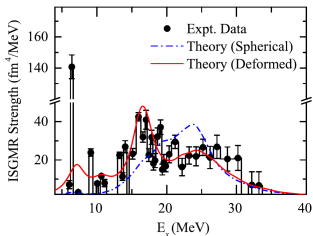
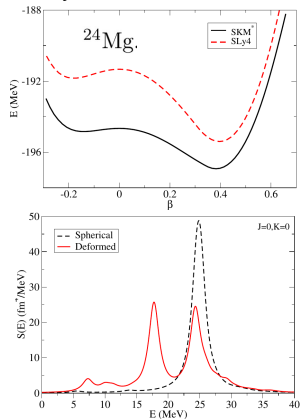


FIG. 5. ISGMR strength distributions in ^{24}Mg . The dash-dotted (blue) and solid (red) lines show microscopic calculations for spherical and prolate ground-state deformation, respectively.

Calculations by K. Yoshida were used to show that the double peak is related to deformation.

D.G. G. Colò, A. Pastore,
J. Phys.: Conf. Ser. 1643 012129, 2020





Contents lists available at [ScienceDirect](#)

Physics Letters B

www.elsevier.com/locate/physletb



Isoscalar monopole and quadrupole modes in Mo isotopes: Microscopic analysis



Gianluca Colò^{a,b,*}, Danilo Gambacurta^{c,d}, Wolfgang Kleinig^e, Jan Kvasil^f,
Valentin O. Nesterenko^{e,g,h}, Alessandro Pastoreⁱ

PHYSICAL REVIEW C **109**, 044315 (2024)

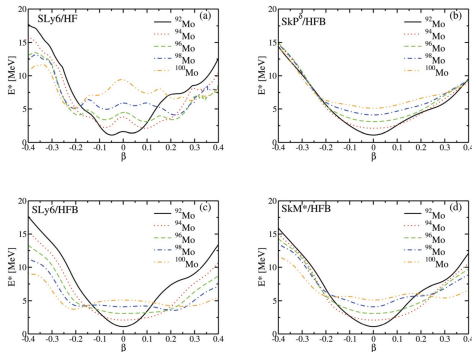
Symmetry-restored Skyrme-random-phase-approximation calculations of the monopole strength in deformed nuclei

A. Porro^{1,2,3,*}, G. Colò^{4,5,†}, T. Duguet^{6,‡}, D. Gambacurta^{6,7,§} and V. Somà^{1,||}

Physics Letters B 811 (2020) 135940

Isoscalar monopole and quadrupole modes in Mo isotopes:
Microscopic analysis

Gianluca Colò^{a,b,*}, Danilo Gambacurta^{c,d}, Wolfgang Kleinig^e, Jan Kvasil^f,
Valentin O. Nesterenko^{c,g,h}, Alessandro Pastoreⁱ

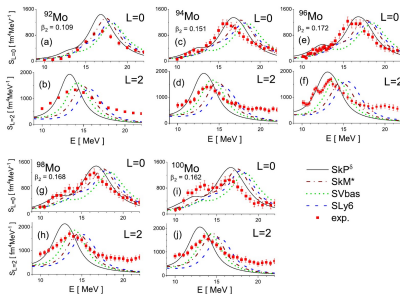


Physics Letters B 811 (2020) 135940

Isoscalar monopole and quadrupole modes in Mo isotopes:
Microscopic analysis



Gianluca Colò^{a,b,*}, Danilo Gambacurta^{c,d}, Wolfgang Kleinig^e, Jan Kvasil^f,
Valentin O. Nesterenko^{c,g,h}, Alessandro Pastoreⁱ



The “shoulder” is due to the monopole-quadrupole coupling (see for example ⁹⁸Mo)

RCNP (α, α') data

Table 1

Incompressibility K_∞ and isoscalar effective mass m^*/m for the Skyrme forces SVbas, SLy6, SkM*, and SkP^b.

	SVbas	SLy6	SkM*	SkP ^b
K_∞ [MeV]	234	230	217	202
m^*/m	0.9	0.69	0.79	1

GQR

GMR

Warning: no simultaneous description and K too low

Deformed RPA calculations break (on purpose) the **rotational symmetry**.
 Excited states **do not have a good angular momentum J**.
Restoration of J is needed!!!

$$|KM\rangle \mapsto |JM\rangle = P_{KM}^J |KM\rangle = \frac{2J+1}{8\pi^2} \int d\Omega \mathcal{D}_{MK}^{J*}(\Omega) \mathcal{R}(\Omega) |KM\rangle$$

$$|\omega\rangle = Q_\omega^\dagger |\text{RPA}\rangle \equiv \sum_{ph} \left(X_{ph}^\omega a_p^\dagger a_h |\text{RPA}\rangle - Y_{ph}^\omega a_h^\dagger a_p |\text{RPA}\rangle \right)$$

$$\langle \text{RPA} || T_\lambda || \omega \rangle = N_0 N_\omega (2J_0 + 1) (-1)^{J_0 - K_0} \sum_{ph} \sum_{\mu=-\lambda}^{+\lambda} \left\{ X_{ph}^\omega + (-1)^\mu Y_{ph}^\omega \right\} \begin{pmatrix} J_0 & \lambda & J_\omega \\ -K_0 & \mu & K_0 - \mu \end{pmatrix} \langle \text{HF} | T_{\lambda\mu} P_{K_0 - \mu, K_{ph}}^{J_\omega} c_p^\dagger c_h | \text{HF} \rangle$$

Projected
transition
amplitudes

$$N_i^{-1} = \sqrt{\langle \Phi_i | P_{K_i, K_i}^{J_i} | \Phi_i \rangle} \quad P_{MK}^J = \frac{2J+1}{2} \int_{-1}^{+1} d(\cos \beta) d_{MK}^J(\beta) e^{-i\beta J_y}$$

PHYSICAL REVIEW C **109**, 044315 (2024)

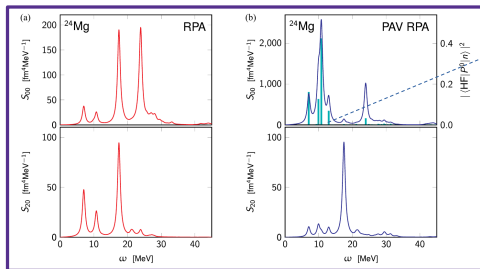
Symmetry-restored Skyrme-random-phase-approximation calculations
of the monopole strength in deformed nuclei

First ever
Projected Skyrme-RPA
calculations

A. Porro^{1,2,3,*}, G. Colò^{4,5,†}, T. Duguet^{6,1,6,‡}, D. Gambacurta^{7,8} and V. Somà^{1,1}

¹INFN-LNS, Catania; ²INFN-CA, Cagliari; ³INFN-CL, Cagliari; ⁴INFN-CL, Cagliari; ⁵INFN-CL, Cagliari; ⁶INFN-CL, Cagliari; ⁷INFN-CL, Cagliari; ⁸INFN-CL, Cagliari

The most recent collaboration with Gianluca: deformed nuclei

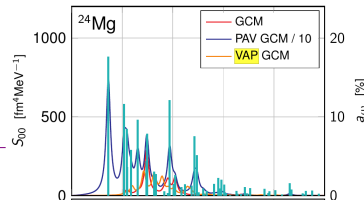


Low-lying spurious strength appears due to the coupling with (spurious) rotational motion

Similar behaviour in projected generator coordinate method (PGCM) calculations with Projection after Variation (PAV) approach

It was shown to disappear in its full variation after projection (VAP) counterpart.

Need of VAP-RPA calculations!



A. Porro et al. arXiv:2407.01325

VAP-(Q)RPA calculations

- Variation After Projection approach
- Projected (Q)RPA equations of motion
- Projecting the matrix elements of residual interaction (two-body operator)

VAP-(Q)RPA calculations

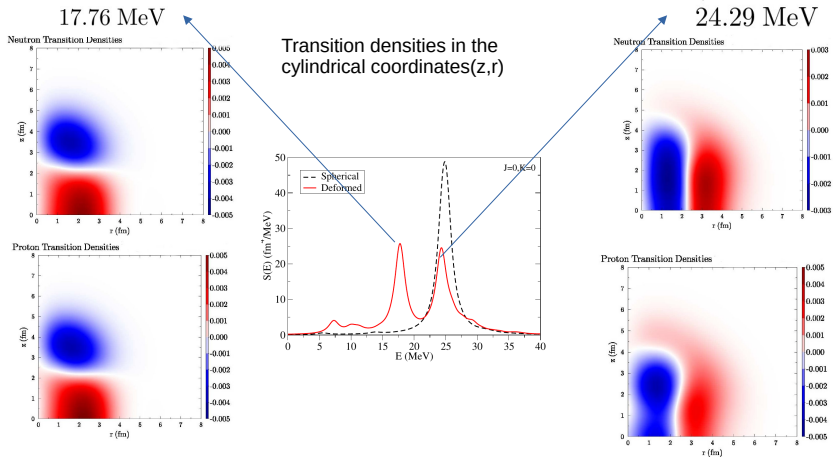
- Variation After Projection approach
- Projected (Q)RPA equations of motion
- Projecting the matrix elements of residual interaction (two-body operator)

Beyond mean-field correlations in RPA-like theories

- Particle Vibration Coupling (coupling with low-lying phonons)
- Second RPA (2 particle-2 hole configurations)
- Comparing and “merging” them

**Thanks For Your
Attention !!!**

The most recent collaboration with Gianluca: deformed nuclei



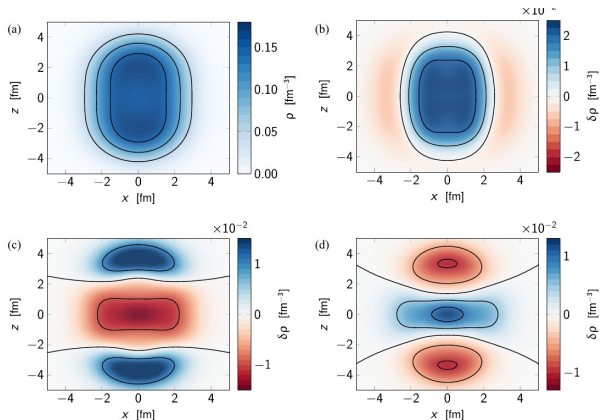


FIG. 4. ^{24}Mg (a) one-body intrinsic matter density of the HF ground state, (b) intrinsic RPA transition matter density to the ISGMR phonon at 23.9 MeV, (c) intrinsic RPA transition matter density to the ISGQR phonon at 17.5 MeV, and (d) intrinsic RPA transition matter density to the phonon at 10.8 MeV.