RICCI FLOWS, MASSIVE GRAVITY AND $T\bar{T}$

TOMMASO MORONE

04 NOV 2024 — WELCOME DAY

UNIVERSITÀ DI TORINO

INFN

This is me!

This is me!


Tommaso

UNIVERSITÀ DI TORINO

2017 – 2023	BSc & MSc in physics
2023 – today	PhD in physics

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DI TORINO

2017 – 2023 BSc & MSc in physics

ysics

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Tommaso

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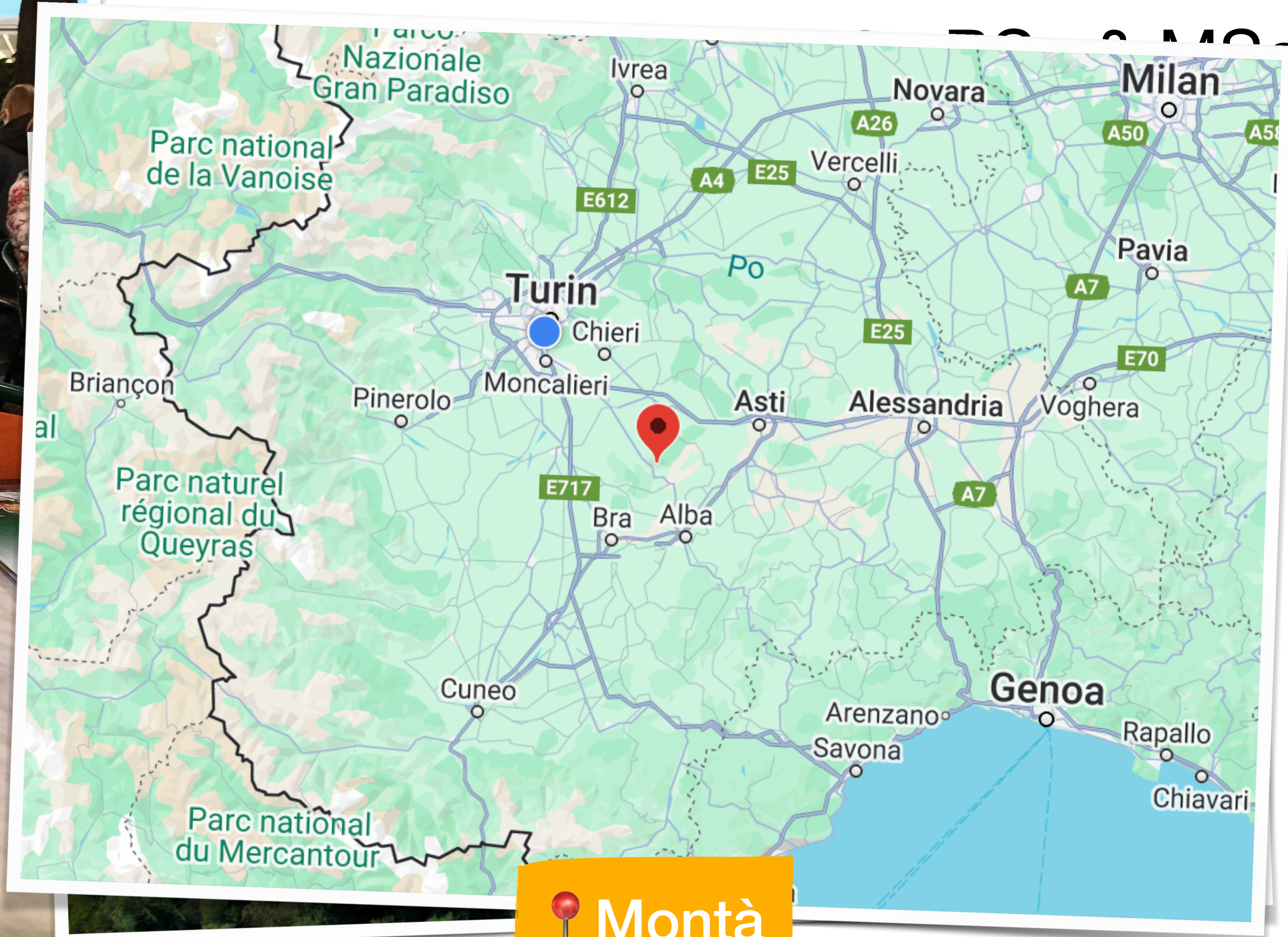

📍 Montà

... in physics

... sics

Statistical Field Theory group – Torino

Integrable models

Andrea Cavaglià
Roberto Tateo

Nicolò Brizio
Tommaso Morone
Michelangelo Preti
Nicolò Primi

Lattice field theory

Michele Caselle
Marco Panero

Andrea Bulgarelli
Tommaso Canneti
Elia Cellini
Alessandro Mariani
Alessandro Nada
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Turin for the newcomers

A non-exhausting list of cool things born in Turin

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Fiat

Turin for the newcomers

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vermouth

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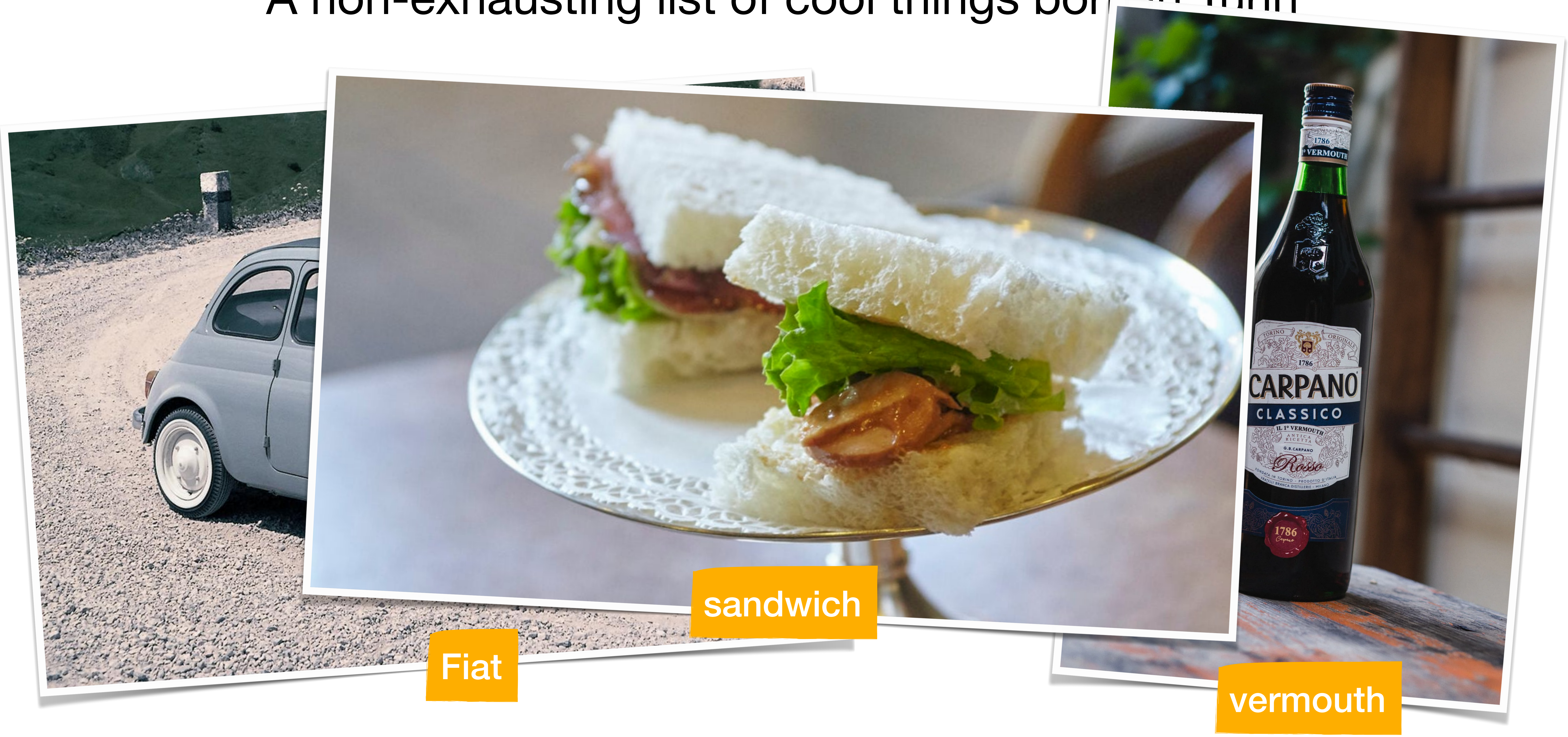

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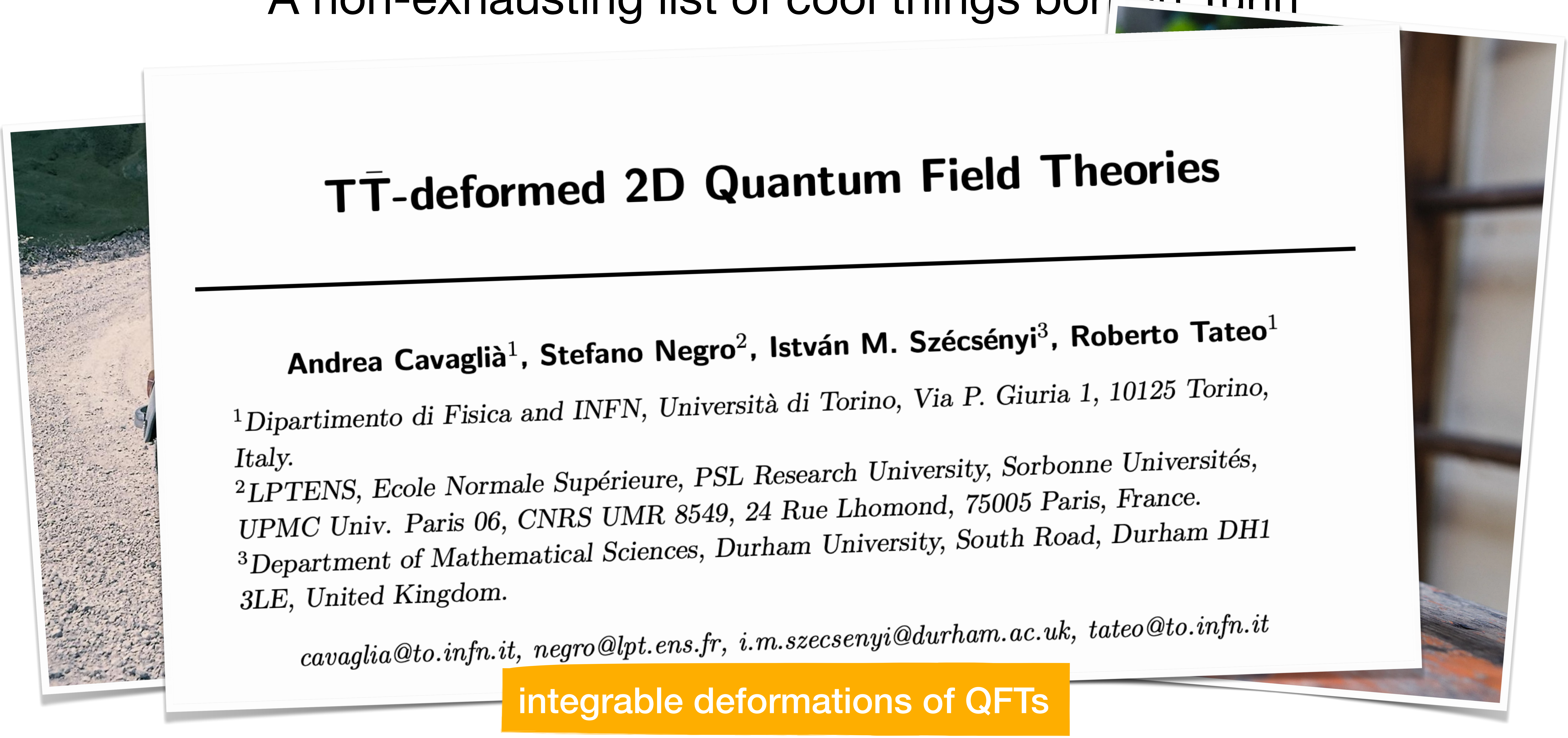

$T\bar{T}$-deformed 2D Quantum Field Theories

Andrea Cavaglià[1], Stefano Negro[2], István M. Szécsényi[3], Roberto Tateo[1]

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integrable deformations of QFTs

What is the $T\overline{T}$?

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Introduced as an irrelevant deformation of 2D QFTs [Cavaglià, Negro, Szécsényi, Tateo 2016] [Smirnov, Zamolodchikov 2016]

$$\partial_\lambda S_\lambda = \frac{1}{2} \int d^2x \sqrt{g} \left(T_{ab} T^{ab} - T^a_a \right)$$

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- Well defined at the quantum level
- Preserves integrability of the theory
- Allows exact computation of S-matrix elements and energy spectrum

What is the $T\overline{T}$?

Standard example: deforming the free boson

$$S_0 = \int d^2x \sqrt{g} (\partial \varphi)^2$$

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Nambu-Goto action in static gauge (+ vacuum energy term)

$$S_\lambda \propto \frac{1}{\lambda} \int d^2x \sqrt{g} \left(\sqrt{1 + 2\lambda(\partial\varphi)^2} - 1 \right)$$

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- Higher-dimensional extensions (at the classical level) [Taylor 2018] [Conti, Romano, Tateo 2022]

$$\partial_\lambda S_\lambda = \int d^d x \sqrt{g} f\left(T_{ab}\right)$$

- Stress-tensor deformations as couping to gravity [Dubovsky, Gorbenko, Mirbabayi 2017] [Tolley 2019] [**Morone**, Negro, Tateo 2024]

Today's goal

Look at higher-dimensional $T\overline{T}$-like deformations
from a different persective …

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Look at higher-dimensional $T\overline{T}$-like deformations
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… going back to 1904

The Poincarè conjecture

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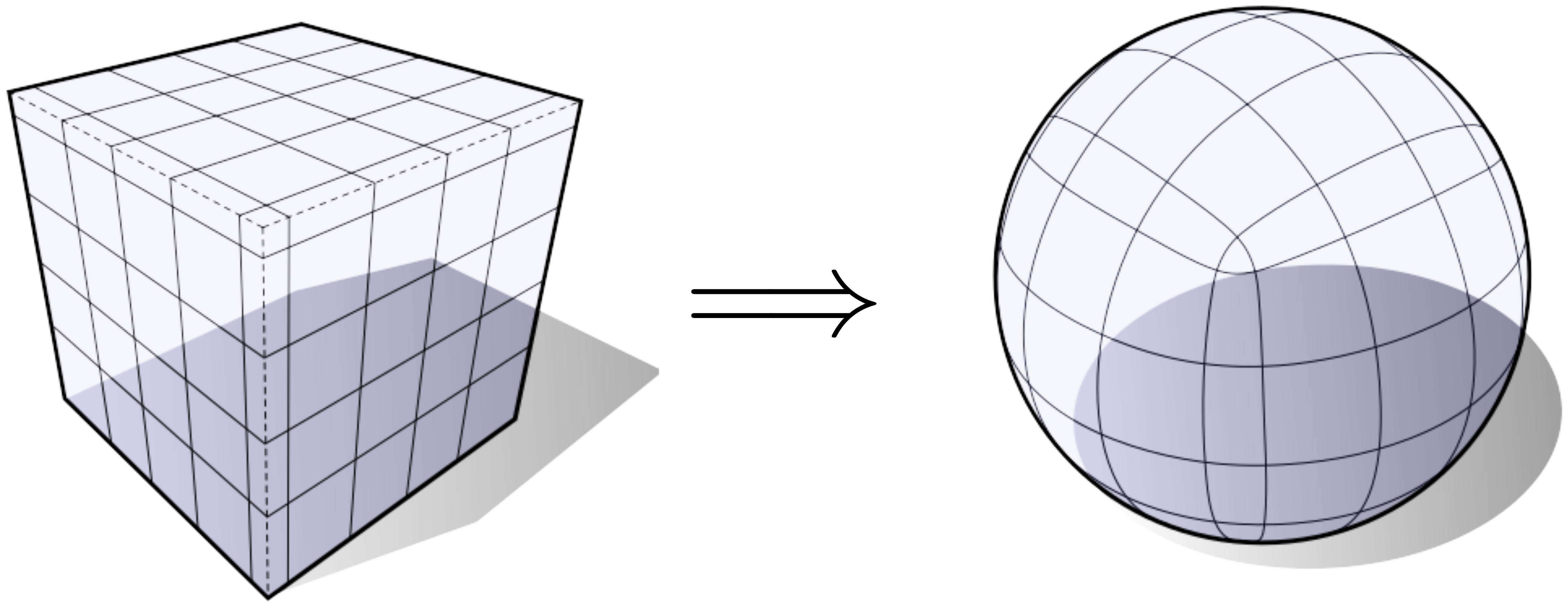
Every simply connected, closed 3-manifold
can be continuously deformed into a 3-sphere

[Poincaré, 1904]



100 years later

The entropy formula for the Ricci flow
and its geometric applications

Grisha Perelman*

100 years later

Mathematics genius leaves people stunned, rejects $1 million prize and awards for his work

He ref

Grigory Perelman, the maths genius who said no to $1m

Perelman cracks a century-old conundrum, refuses the reward, and barricades himself in his flat

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Grigori Perelman: The genius in hiding

By Jennifer Ouellette

Perelman’s idea:

Let geometry do (almost) all the dirty work.

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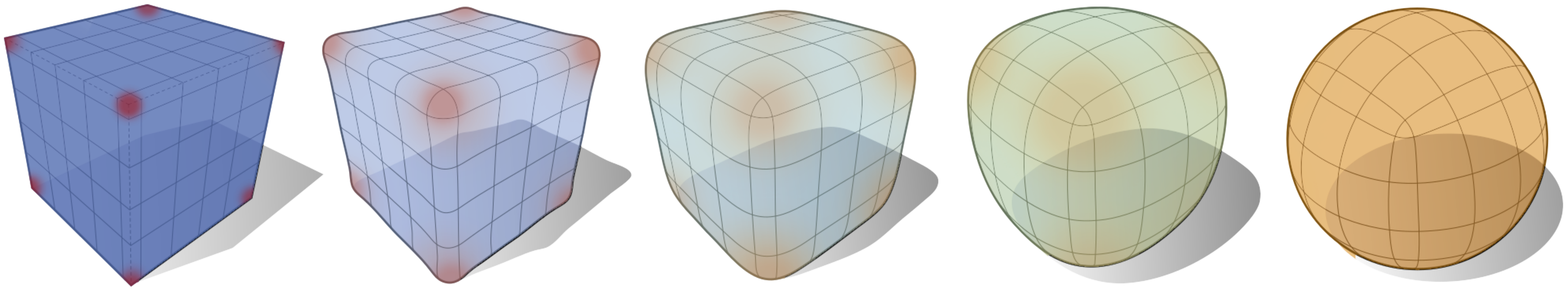
Let geometry do (almost) all the dirty work.

$$\Downarrow$$

Use geometric flows to "smooth out" high curvature regions and ultimately the manifold reduces to a sphere.



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$$\partial_\lambda g_{ab}(\lambda) \propto R_{ab}(\lambda)$$

RICCI FLOW

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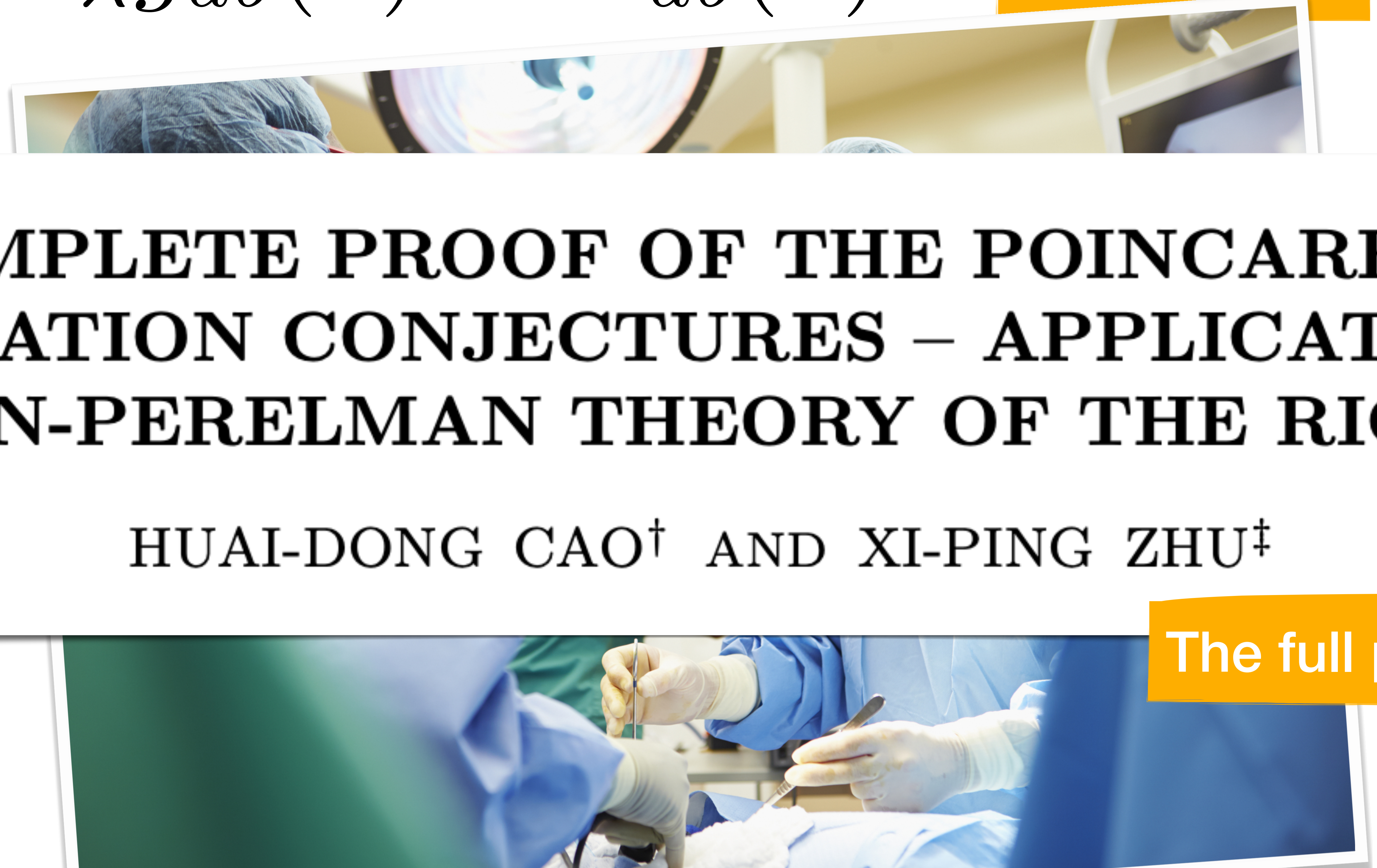

A COMPLETE PROOF OF THE POINCARÉ AND GEOMETRIZATION CONJECTURES – APPLICATION OF THE HAMILTON-PERELMAN THEORY OF THE RICCI FLOW*

HUAI-DONG CAO† AND XI-PING ZHU‡

The full proof is 320+ pages long!

100 years later

The entropy formula for the Ricci flow
and its geometric applications

Grisha Perelman*

100 years later

The entropy formula for the Ricci flow and its geometric applications

5.3* An entropy formula for the Ricci flow in dimension two was found by Chow [C]; there seems to be no relation between his formula and ours.

The interplay of statistical physics and (pseudo)-riemannian geometry occurs in the subject of Black Hole Thermodynamics, developed by Hawking et al. Unfortunately, this subject is beyond my understanding at the moment.

Question

Do Ricci flows arise in physical space-time as well?

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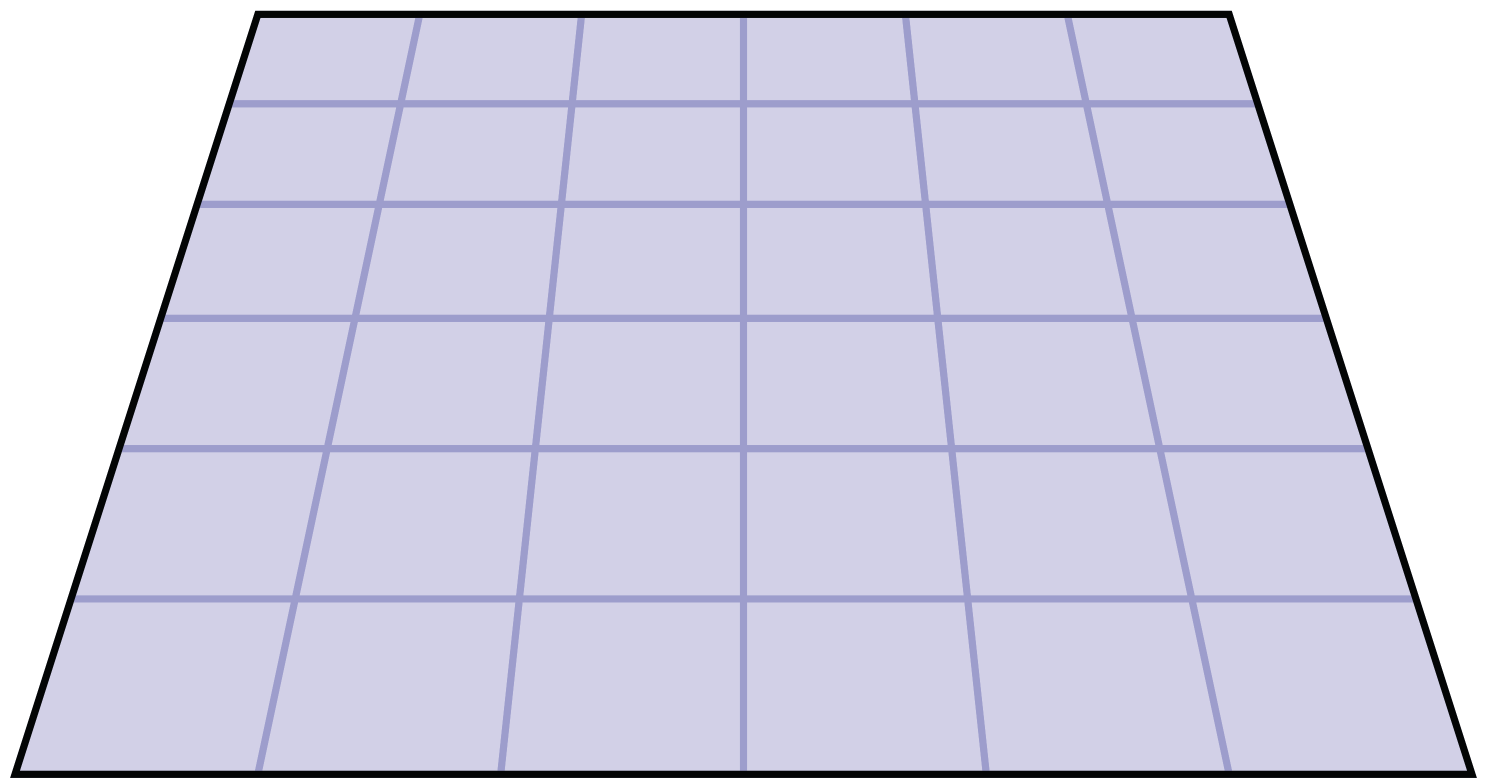
Take a lesson from General Relativity:



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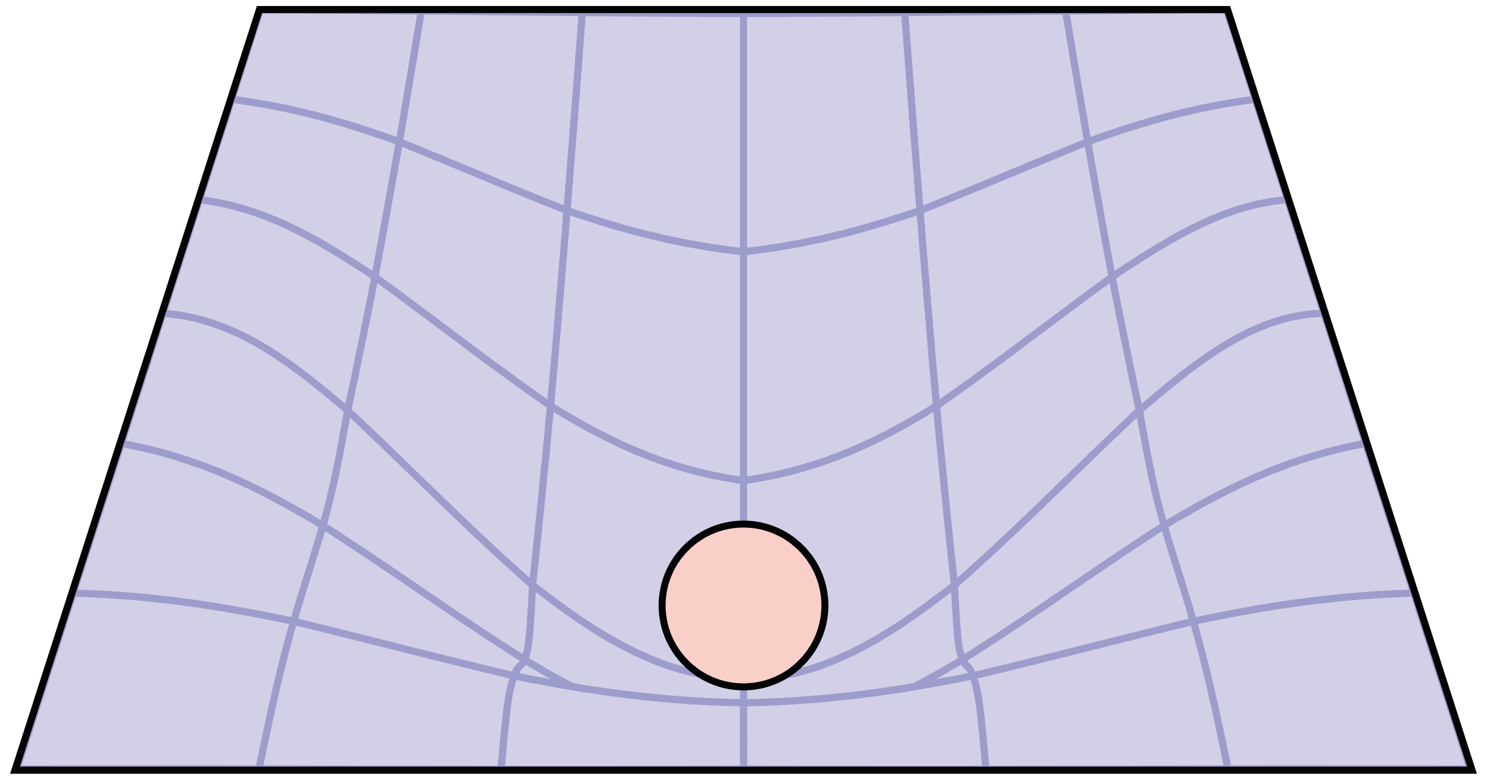

Matter fields influence space–time geometries

$$R_{ab} - \frac{1}{2}Rg_{ab} = T_{ab} \implies$$ solve Einstein field equations to see what space–time looks like

(rephrased)

Question

Are there solutions to the Einstein field equations which satisfy a Ricci flow?

(rephrased)

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Short answer

Yes! If the matter action satisfies the PDE

$$\partial_\lambda S_\lambda \propto \int d^d x \sqrt{g} \left(T_{ab} T^{ab} - \frac{1}{2} T^a_a \right)$$

[Brizio, **Morone**, Tateo 2024]

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Deformed field equations:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2\,, \quad f(r) = 1 - \frac{2}{3}\frac{\Lambda}{1+\lambda\Lambda}r^2$$

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$$\partial_\lambda g_{ab} = R_{ab}$$

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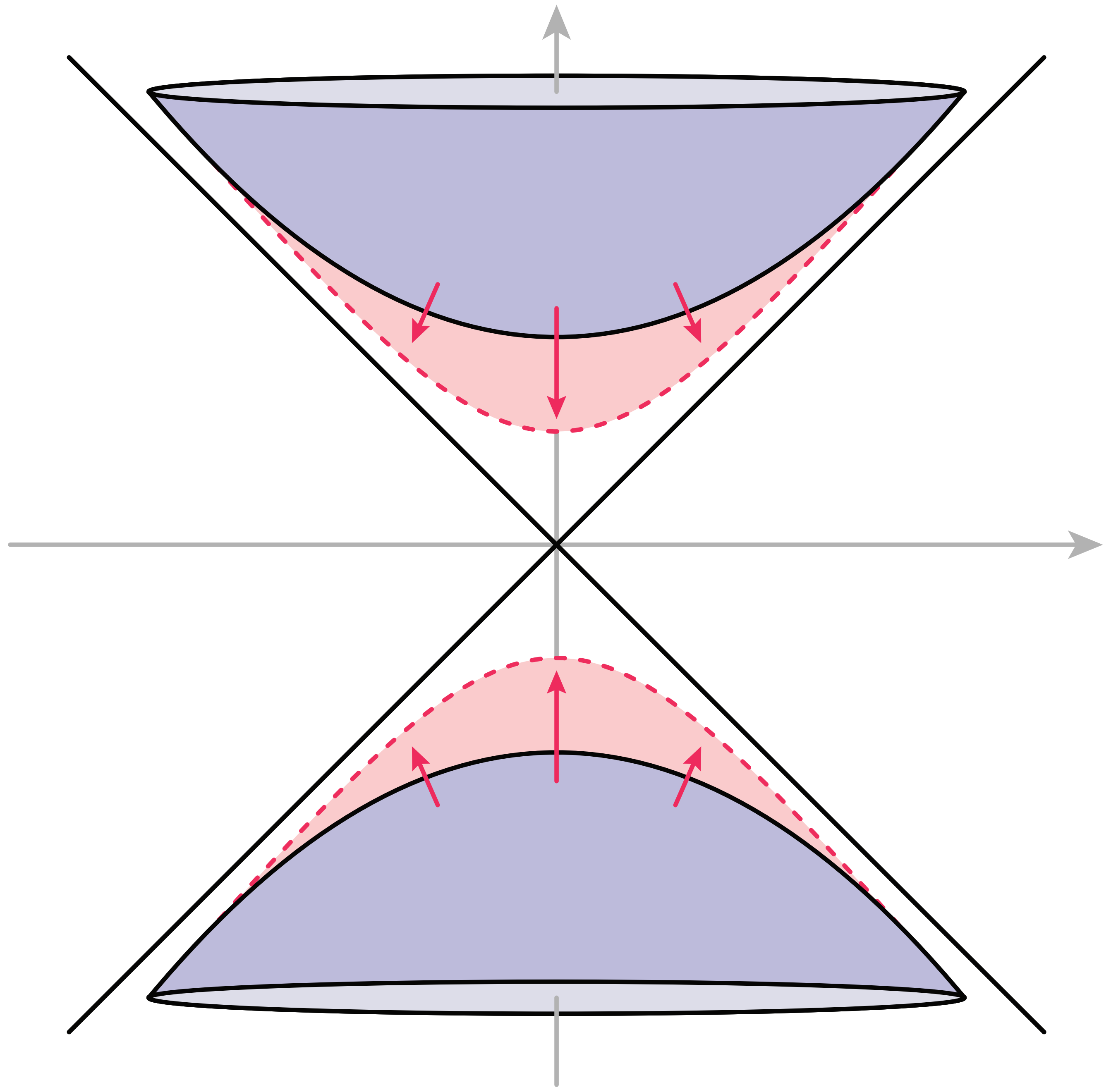

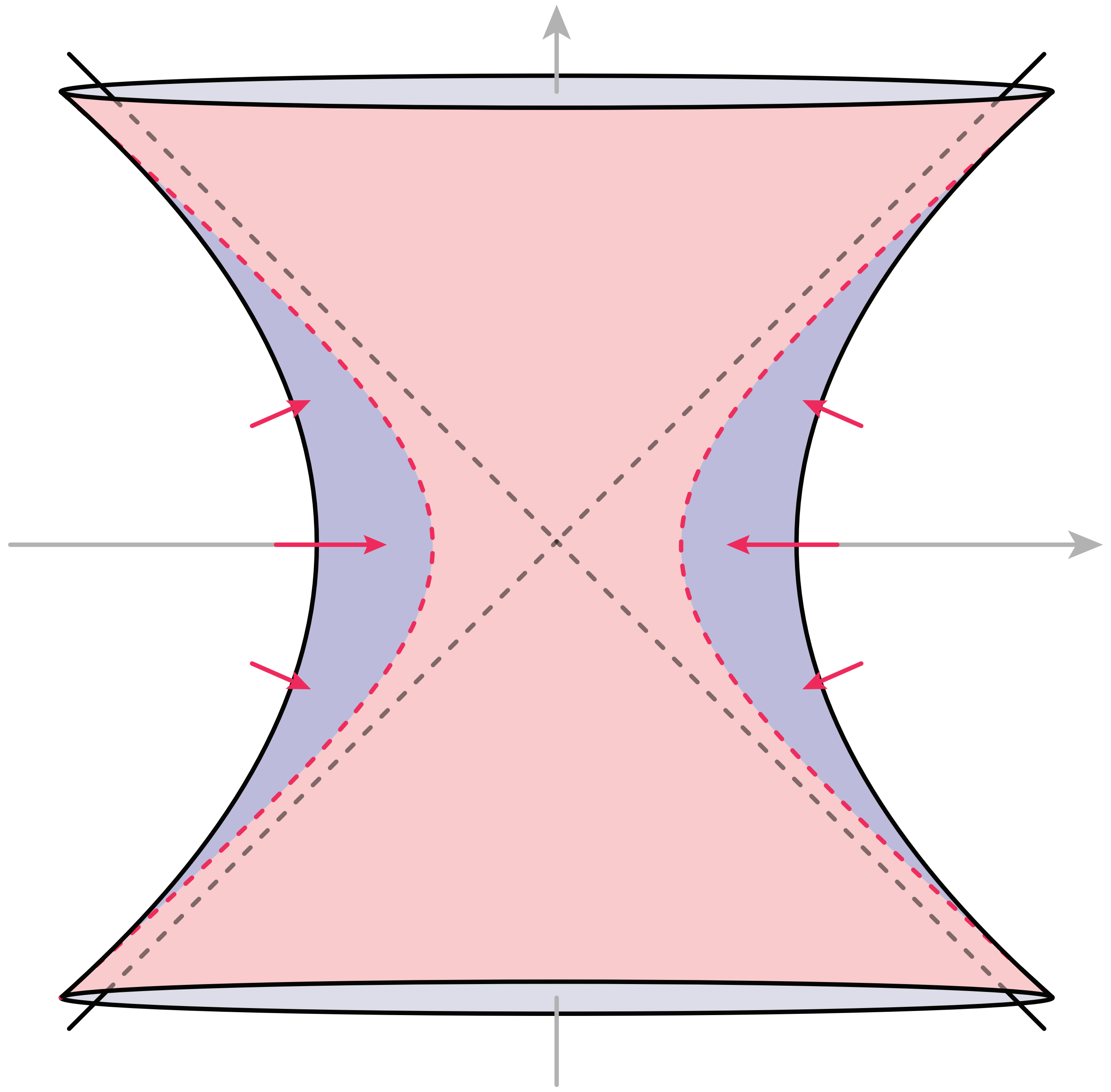

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Start from known solutions to Einstein's equations

$$\Downarrow$$

Deform them to obtain Ricci flows in space–time

A more complicated example

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Start from Reissner–Nordstrom solution
= static, spherically symmetric electromagnetic black hole

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$$f(r) = 1 - \frac{2m_0}{r} - \frac{r^2}{3\lambda}\left(1 - \sqrt{1 - \frac{2\lambda Q^2}{r^4}}\right) + \frac{4Q^2}{3r^2}\left[{}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{2\lambda Q^2}{r^4}\right)\right]$$

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(Born-Infeld black hole)

A more complicated example

$$S_0 = \frac{1}{4} \int d^4x \sqrt{g} F_{ab} F^{ab}$$

$$\Downarrow$$

$$S_\lambda = \frac{1}{\lambda} \int d^4x \left(\sqrt{\det(|g_{ab} + \lambda F_{ab})|} - \sqrt{g} \right)$$

[Conti, Iannella, Negro, Tateo 2018]

It’s all about (massive) gravity

Back to 2D:

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$T\overline{T}$-deformed matter = undeformed matter + massive gravity theory

[Tolley 2019]

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Ghost-free massive gravity theories $\Longrightarrow$ bi-metric theories

$$S_{\text{grav}} = S_{\text{grav}}[g, h]$$

[De Rham, Gabadadze, Tolley 2019]

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2D $T\overline{T}$ deformations:

$$S_\lambda[h] = S_0[g^*] + S_{\mathrm{grav}}[g^*, h]$$

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Higher dimensions: [Babaei-Aghbolagh, He, **Morone**, Ouyang, Tateo 2024]

$$\frac{1}{2}\int d^d x \sqrt{h}\, R[h] + S_\lambda[h] = S_0[g^*] + S_{\text{grav}}[g^*, h]$$

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E.g., in 4D: [**Morone**, Negro, Tateo 2024]

$$S_{\text{grav}} = \frac{1}{\lambda}\int d^4 x \left(\sqrt{|\det(g_{ab} + \lambda R_{ab}(\Gamma))|} - \sqrt{g}\right)$$

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It’s all about (massive) gravity

Generally true for any stress-energy tensor deformation.

E.g. root-$T\overline{T}$ deformations: [Babaei-Aghbolagh, He, **Morone**, Ouyang, Tateo 2024]
[**Morone**, Brizio, Tateo 2024]

$$S_{\text{grav}} = \frac{1}{2}\int d^4x\sqrt{g}\left[R(\Gamma)\cosh\frac{\gamma}{2} + \sinh\frac{\gamma}{2}\sqrt{4R^{ab}(\Gamma)R_{ab}(\Gamma) - (R(\Gamma))^2}\right]$$

Final remarks

$T\overline{T}$, Ricci flows and massive gravity theories

$$\Downarrow$$

Look like very different things, but ultimately are the same

- Quantum (or, at least, semi-classical) description?
- Connections with self-dual, non-linear electrodynamics [Aschieri, Ferrara 2013]
- $T\overline{T}$ and Bekenstein-Hawking entropy [**Morone**, Brizio, Tateo 2024]

Thanks!