The Effective Field Theory of Fluctuating Defects

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PHYSICS WELCOME DAY

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Academic Education:

- 2017-2021:
  - B.Sc. Physics
- 2021-2023:
  - M.Sc. Mathematical Physics
  - "Wreathed 3d N=4 Unitary and Orthosymplectic Quivers "
  - Timo Weigand, Craig Lawrie, Alessandro Minnino
- 2023-now:
  - Doctoral School in Physics
  - Marco Meineri, Marco Billò
  - ST&FI

hhu Heinrich Heine Universität Düsseldorf

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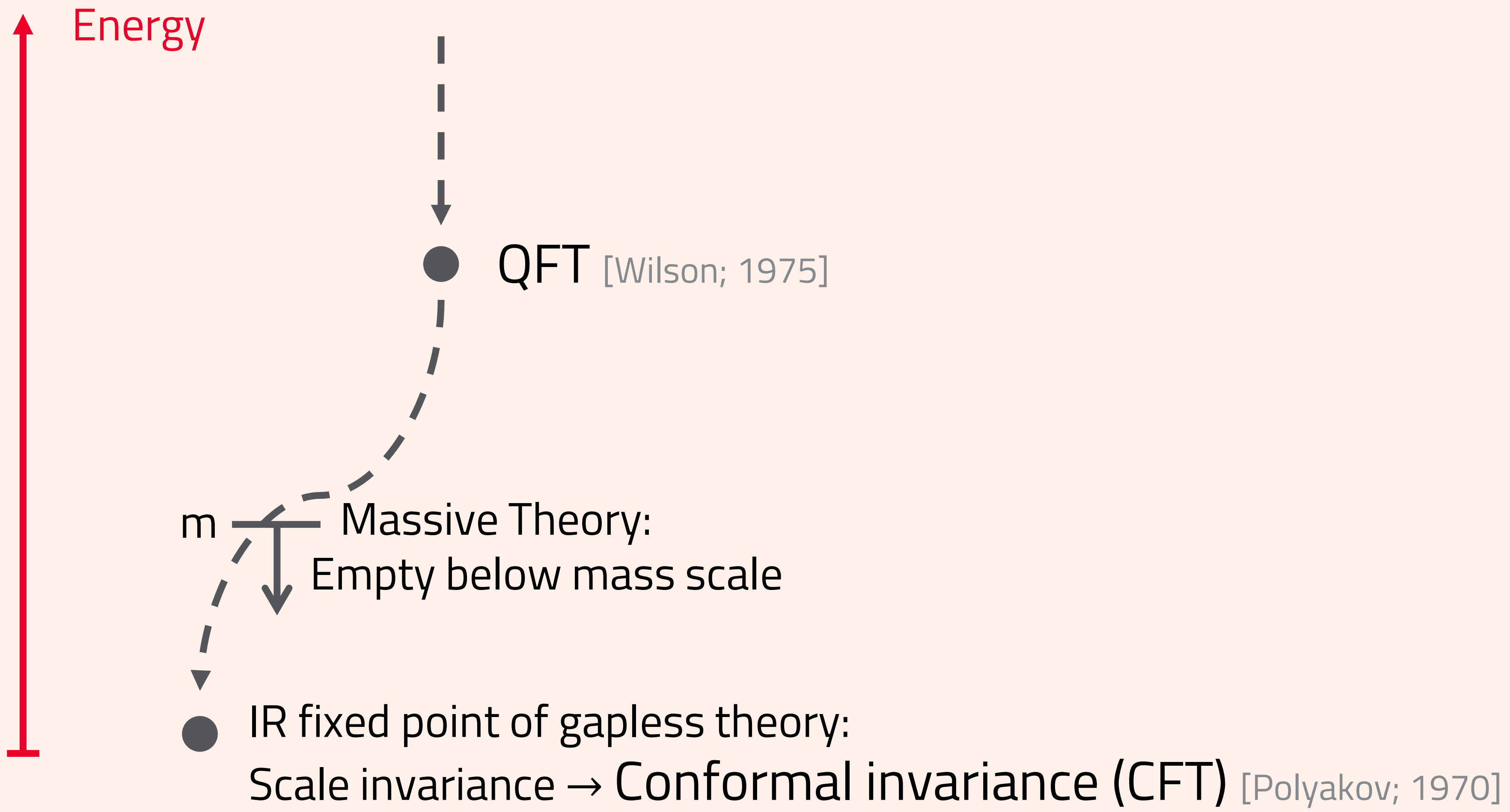
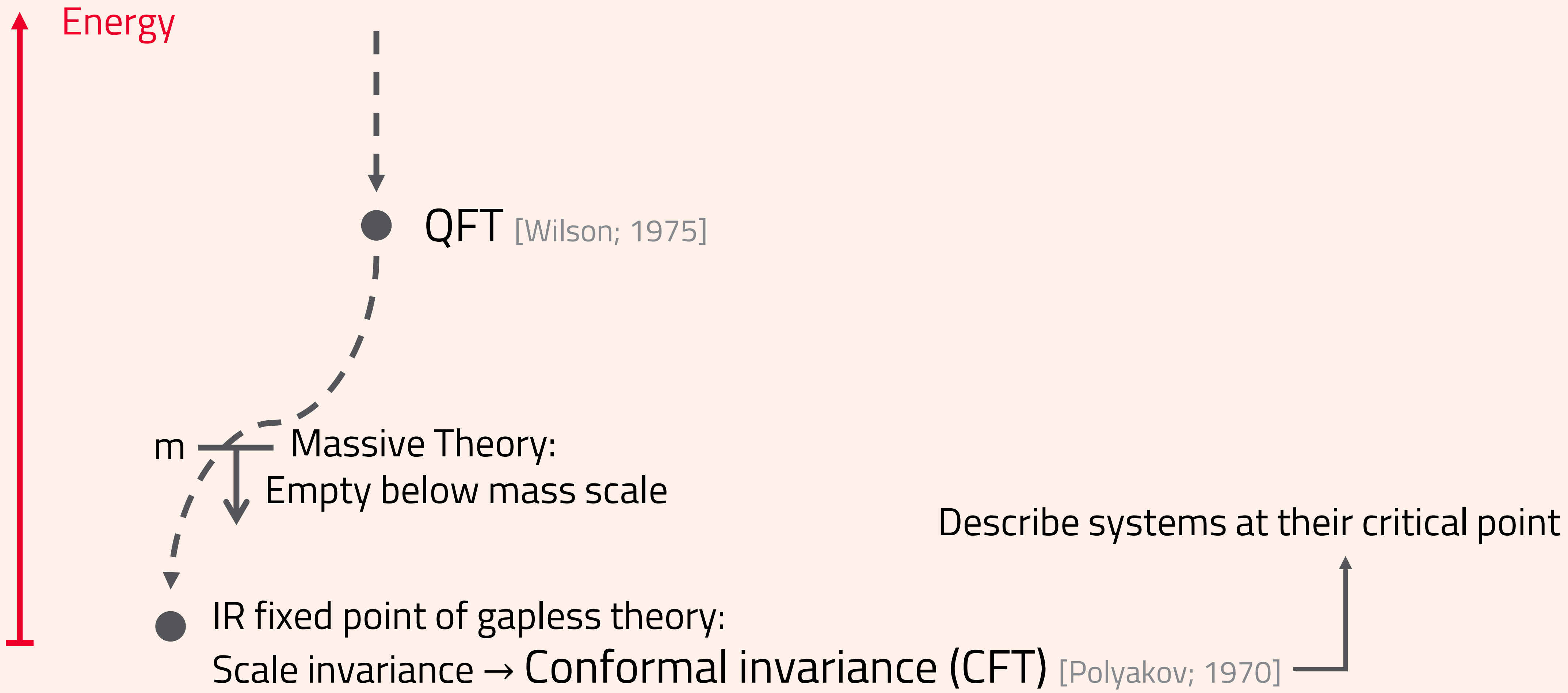
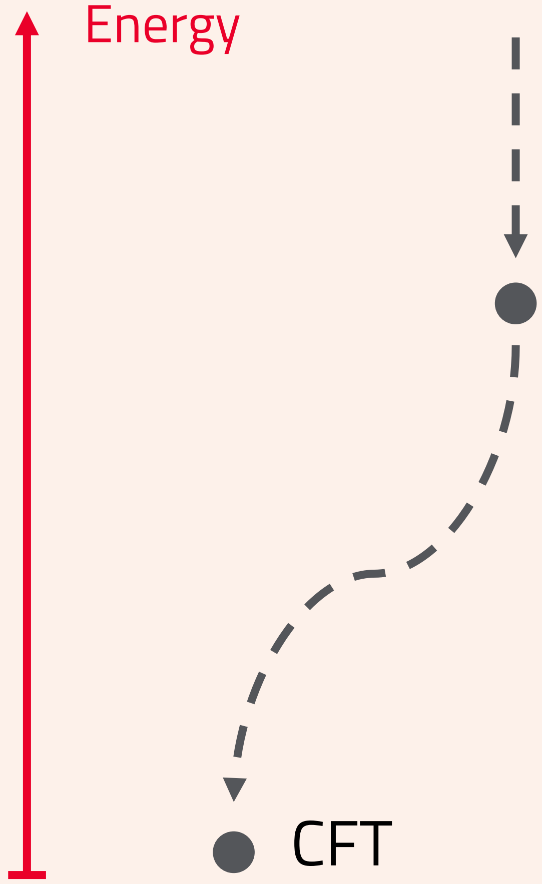
RG Flows in QFT



RG Flows in QFT



RG Flows in QFT



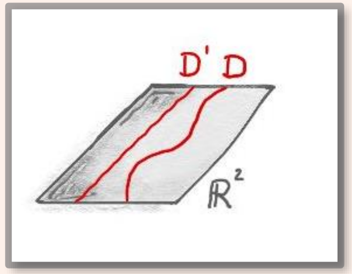
QFT is Poincaré and in particular translational invariant.

Real world is often not:

- Phase transition of liquids in a box (e.g. helium at λ point)
- Ferromagnetic samples with boundaries or impurities
- Electrons/monopoles in a gauge theory as boundary conditions for photons on a line

Defect CFT

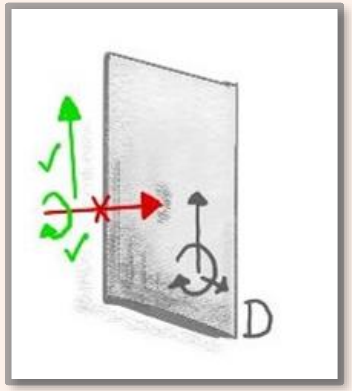
Energy



Defect CFT: [Billò, Meineri,...; 2016]

- Modification of the action on a submanifold : $S = \int_{\mathbb{R}^d} d^d x\, \mathcal{L} + \int_{\Sigma} d^p \sigma\, \hat{\mathcal{L}}$

- $\mathrm{CFT}^d \;\rightarrow\; \mathrm{CFT}^p \times \mathrm{rotations}^{d-p}$
- Translational invariance orthogonal to the defect is broken explicitly.



- Describe phase transitions in presence of, e.g., a boundary.
  [Cardy; 1984]

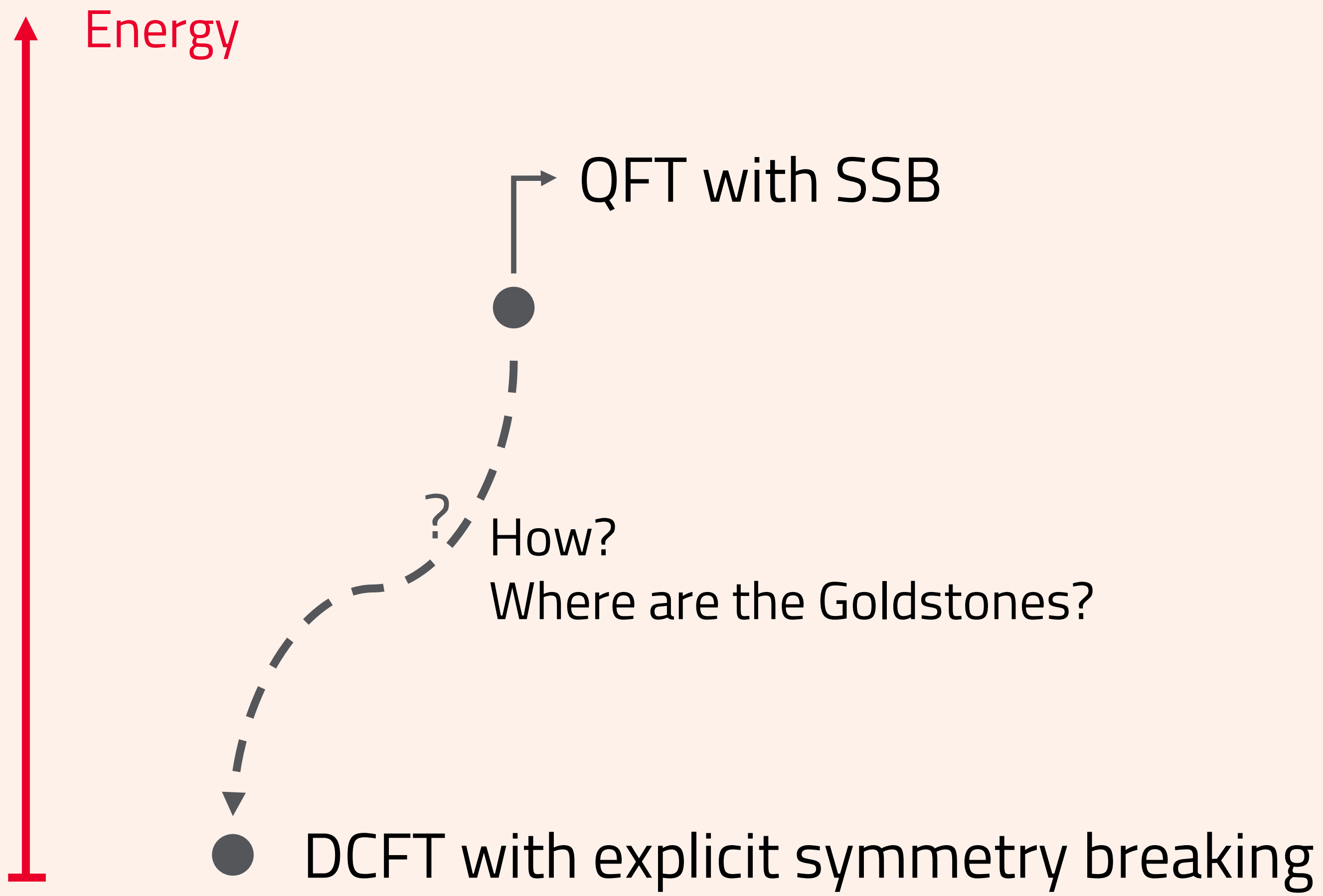
QFT with SSB

Energy

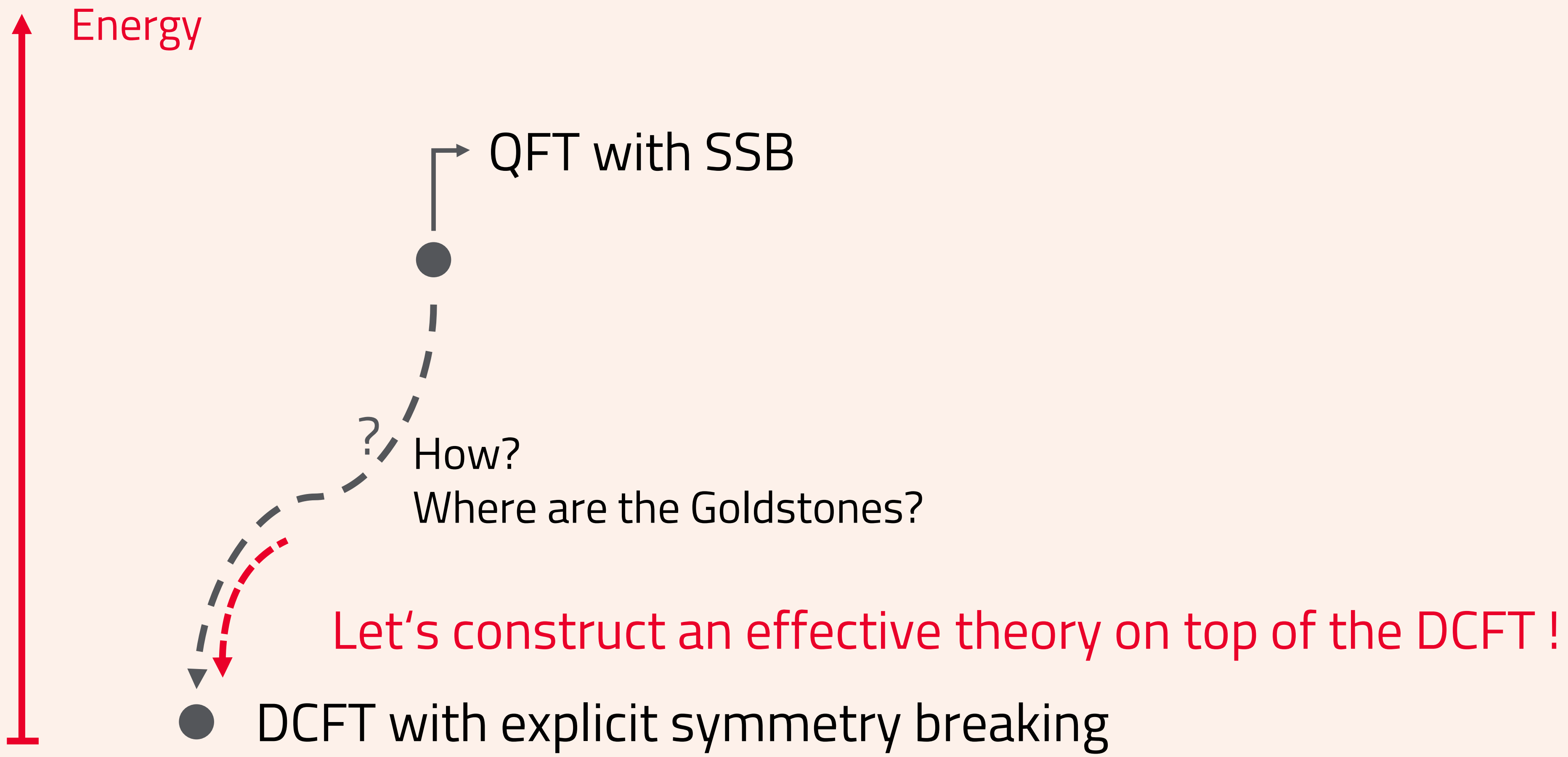
QFT :

- Action is still Poincaré invariant.
- Poincaré symmetry is broken spontaneously.
- Goldstone bosons exist. [Goldstone, Salam, Weinberg; 1962]

QFT with SSB

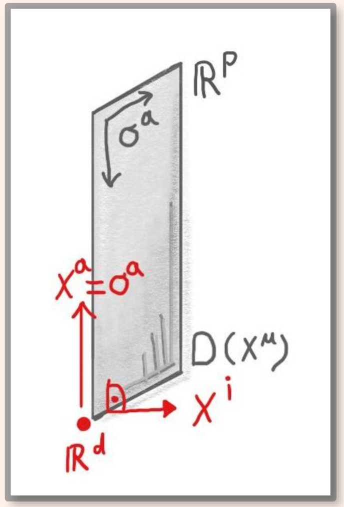

Energy
QFT with SSB
?
How?
Where are the Goldstones?
DCFT with explicit symmetry breaking

QFT with SSB


Energy
QFT with SSB
?
How?
Where are the Goldstones?
Let's construct an effective theory on top of the DCFT !
DCFT with explicit symmetry breaking

Construction of the Defect EFT

- Goal:
  - Include small fluctuations of the defect shape.
  - Restore Poincaré, in particular translational invariance.
- Starting point:
  - The position of the defect is given by a vector of massless fields $X^\mu(\sigma) = \{\sigma^a, X^i\}$.
  - Translation: $X^i \to X^i + \epsilon^i$
  - The orthogonal fields are the Goldstone bosons.
- Ingredients:
  - DCFT: S_{DCFT}
  - Goldstone bosons: $S_{\text{Goldstone}}$
  - Coupling of Goldstones with defect: S_{Coupling}



Construction of the Defect EFT

- Goldstone bosons:
  - Effective theory of long strings [Lüscher; 1981][Caselle, Gliozzi; 1996]
  - Leading term is Nambu-Goto:

$$S_{\text{Goldstone}} = -M^p \int_\Sigma d^p\sigma \sqrt{-\det\left(g_{ab}\right)} = -M^p \int_\Sigma d^p\sigma \sqrt{-\det\left(\eta_{ab} + \eta_{ij}\frac{\partial X^i}{\partial\sigma^a}\frac{\partial X^j}{\partial\sigma^b}\right)}$$

  - Poincaré invariant by construction: $\delta_\epsilon S_{\text{Goldstone}} = 0$

Construction of the Defect EFT

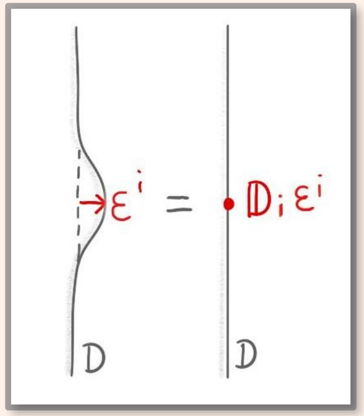
- Coupling:
  - Breaking of translational invariance induces the existence of the displacement operator

$$\delta_\epsilon S_{\text{DCFT}} = \epsilon^i \int d^p\sigma \mathbb{D}_i\,, \qquad \text{for } X^i \to X^i + \epsilon^i\,.$$

  - Displacement is a defect primary in every DCFT.
  - Displacement is a natural choice for the coupling to the Goldstones.
  - We want translational invariance: $\delta_\epsilon S = 0$

$$\Rightarrow\ \delta_\epsilon S_{\text{Coupling}} = -\delta_\epsilon S_{\text{DCFT}}$$

$$\Rightarrow\ S_{\text{Coupling}} = -\int d^p\sigma \mathbb{D}_i X^i$$



Final Defect EFT

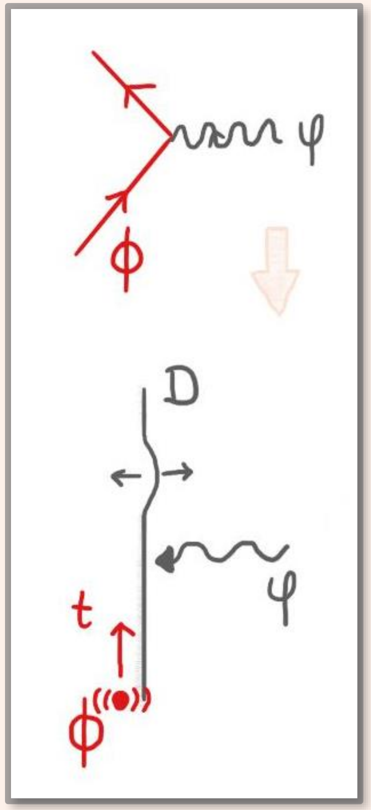
$$S = S_{\text{DCFT}} + S_{\text{Goldstones}} + S_{\text{Coupling}}$$

$$= S_{\text{DCFT}} - M^p \int_\Sigma d^p\sigma \sqrt{-\det\left(\eta_{ab} + \eta_{ij}\frac{\partial X^i}{\partial\sigma^a}\frac{\partial X^j}{\partial\sigma^b}\right)} - \int_\Sigma d^p\sigma \, \mathbb{D}_i X^i$$

- Poincaré invariance fixes the prefactor to 1.
- Coupling is irrelevant and Goldstones decouple in the zero energy limit.

- [Metlitski; 2021] [Krishnan, Metlitski; 2024]

Example 1: Gauge Theory to Line Defect



- Theory with Yukawa coupling of a massive and a massless scalar:

$$S(x) = -\int d^4x \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}(\partial_\mu \varphi)^2 + e\phi^2\varphi$$

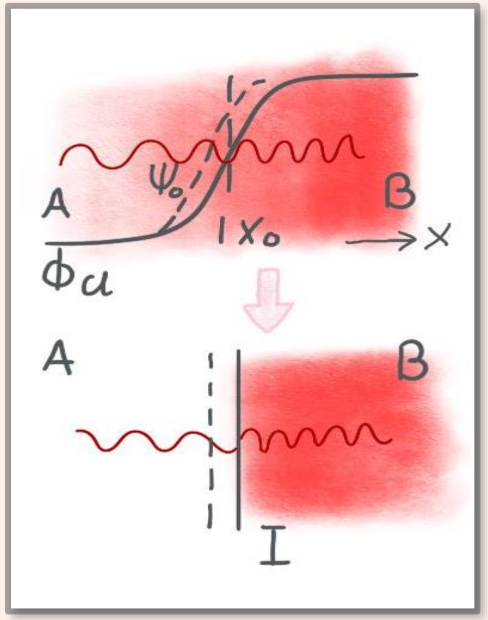
- The zero energy limit in a one-particle state is a scalar Wilson line.

- Scalar Wilson line EFT: $S = -\int dt d^3x \, \frac{1}{2}(\partial_\mu \varphi)^2 + e \int dt \, \varphi(t,0)$

$$+ m \int dt - \frac{1}{2} m \int dt \, (\partial_t X^i)^2 + \int dt \, e(\partial_i \varphi) X^i$$

- Others: QED Wilson line, N=4 SUSY Wilson line, Heavy quark EFT,...

Example 2: Soliton Theory to Interface



- Classical soliton solution in $1+1$-dimensional massive ϕ^4-theory, coupled to massless scalar:

$$S = -\int dtdx\, \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{\lambda}{4}\left(\phi^2 - a^2\right)^2 - \frac{u}{2a}(\partial_\mu \varphi)^2 \phi$$

- Kink breaks translational invariance and a zero mode ψ_0 arises upon quantization.
- At the IR fixed point is a CFT with a compact boson interface [Bachas,...; 2002].
- The low energy limit of the full theory and the EFT match exactly:

$$S = -\frac{1+u}{2}\int_{x<0} dtdx\,(\partial_\mu\varphi)^2 - \frac{1-u}{2}\int_{x>0} dtdx\,(\partial_\mu\varphi)^2 + \int dt\, M_s$$
$$-\int dt\,\frac{1}{2}M_s(\partial_t x_0)^2 - \int dt\left[\frac{1+u}{2}(\partial_\mu\varphi)^2\Big|_{0-} - \frac{1-u}{2}(\partial_\mu\varphi)^2\Big|_{0+}\right]x_0$$

Conclusion

- Embeds DCFT in real world → Universality of DCFTs
- Touches a lot of modern theory tools
- Flexible framework → Lots of fields of application

Outlook

- Include higher derivative terms from the theory of effective strings in the Goldstone action
- Include higher terms from defect perturbation theory in the coupling action
- Calculate observables
- Study RG flows

Thank You !