Eccentricity and tidal heating

Black Hole Binaries

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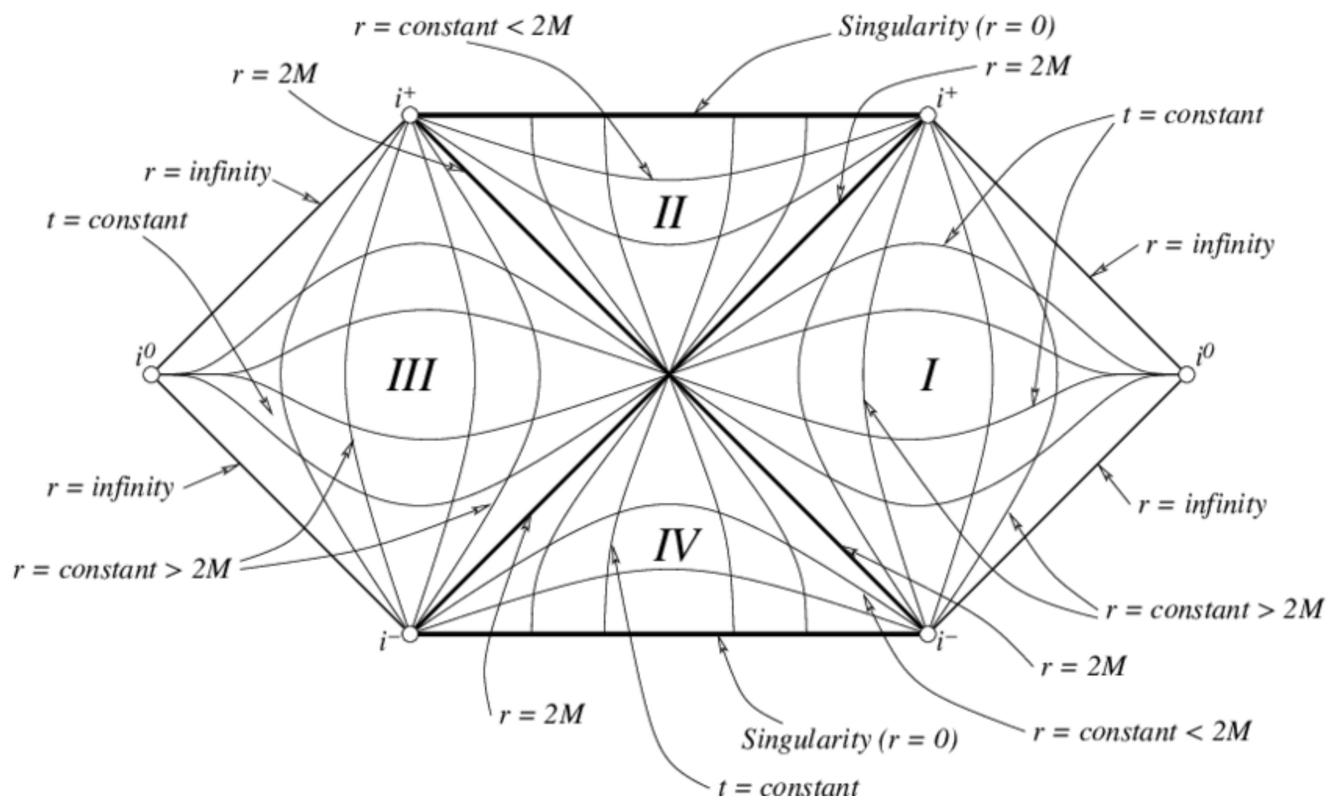
3 NOVEMBER 2024

Something about me

- Born and raised in Piedmont
- Graduated in Turin
- Went into teaching for a while

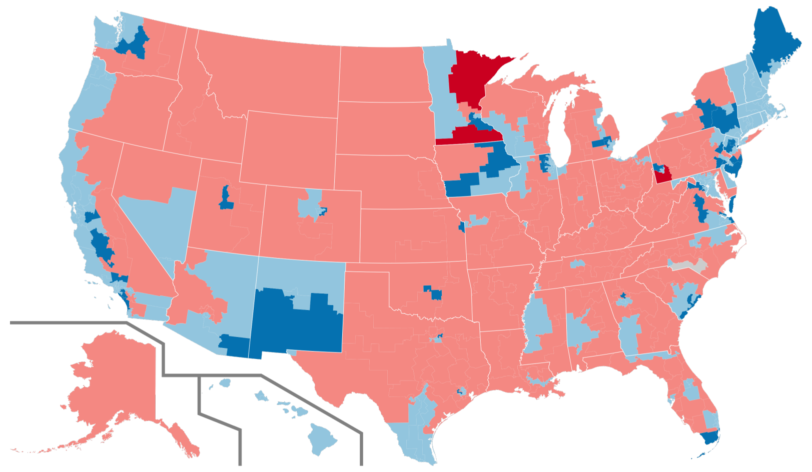


My main interests in physics: **gravity, general relativity, black holes**

Other interests include...

Baking and cooking

Politics around the world



Magic: the Gathering

General Relativity

- Einstein, 1915: new formulation of classical physics where **gravity = curvature**
- Main object: **spacetime metric**, $g_{\mu\nu}$
  - Bound by the **Einstein Field Equations** to mass-energy
  - 2nd order PDEs with gauge freedom, **very hard to solve**
- Particles and waves move along **geodesics**

"Matter tells spacetime how to curve, and curvature tells matter how to move"

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Ricci tensor: curvature of spacetime

Stress-energy tensor: matter/energy content

$$\frac{d^2 x^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} = 0$$

Christoffel symbols

Approaches to GR

- **Exact** solutions are few and far between (black holes, simple cosmological models)
- **Post-Newtonian (PN) theory:** perturbative solution for low velocity and weak field:

$$\frac{v^2}{c^2} \sim \frac{GM}{c^2 r} \ll 1$$

"nPN order" = $O(c^{-2n})$

- **Black hole perturbation theory (BHPT)/Gravitational self-force:** small body orbiting a large black hole

$$\frac{\mu}{M} \ll 1$$

- **Numerical relativity (NR):** evolution of initial "snapshot" of spacetime
  - Many theoretical and technical challenges, very expensive simulations

Black holes

- Karl **Schwarzschild**, 1916: spherically symmetric vacuum solution with a central **singularity**

$$ds^2 = -\underbrace{\left(1 - \frac{2GM}{c^2 r}\right)}_{A(r)} c^2 dt^2 + \underbrace{\left(1 - \frac{2GM}{c^2 r}\right)^{-1}}_{B(r)} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

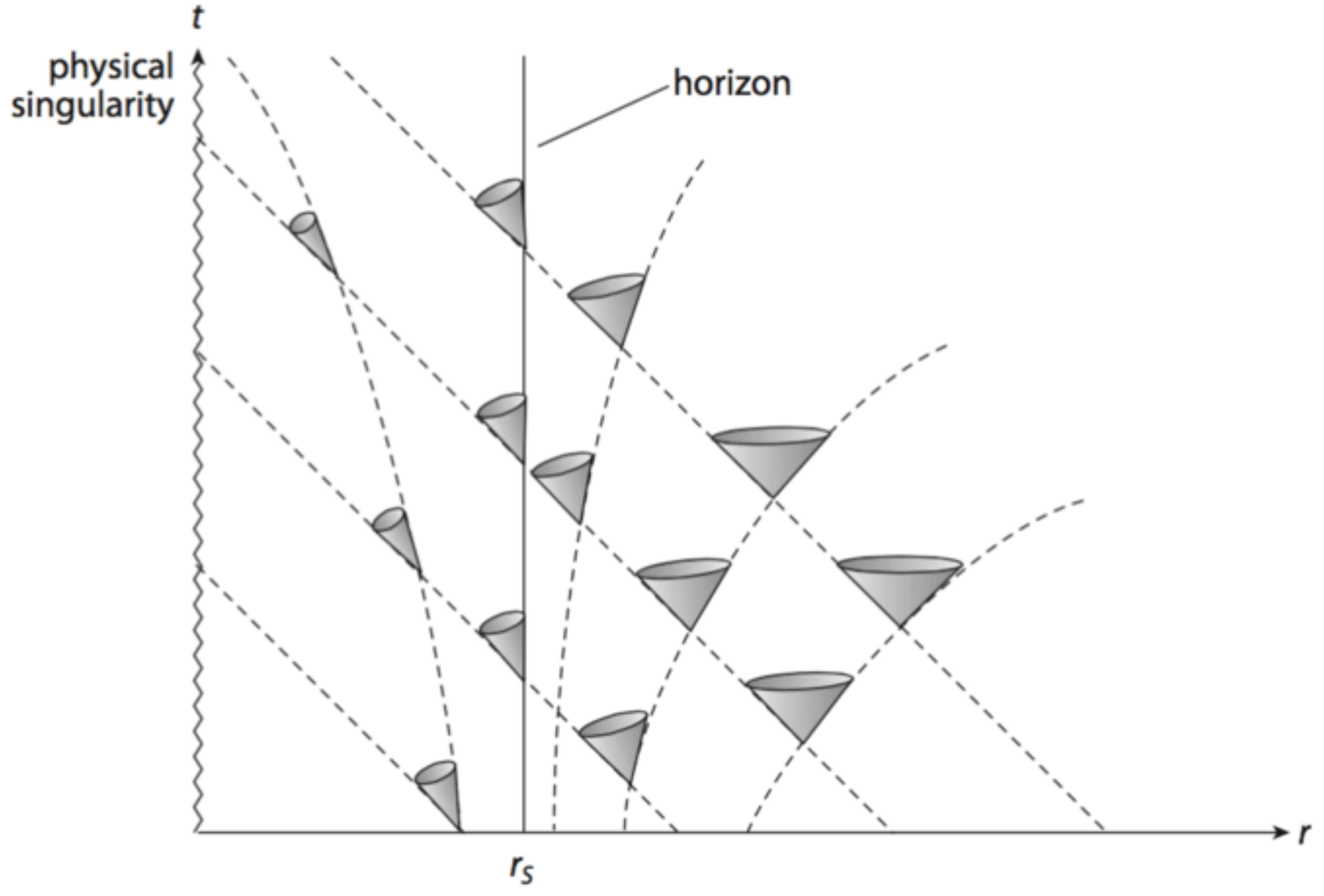
$$H_{\mathrm{Schw}} = \sqrt{A(r)\left(\mu^2 c^2 + \frac{p_\varphi^2}{r^2} + \frac{1}{B(r)} p_r^2\right)}$$

Geodesic Hamiltonian

- Roy **Kerr**, 1963: axially symmetric vacuum solution for rotating BH with spin S

- Geodesics have **Newtonian motion** as weak-field limit

- **Event horizon:** $r_{\mathrm{H}} = \dfrac{2GM}{c^2}$ (Schwarzschild)
  - Future light-cones of observers below the horizon are warped towards the center; all paths lead there
  - Matter inside the event horizon cannot affect the outside



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“No hair” Theorem

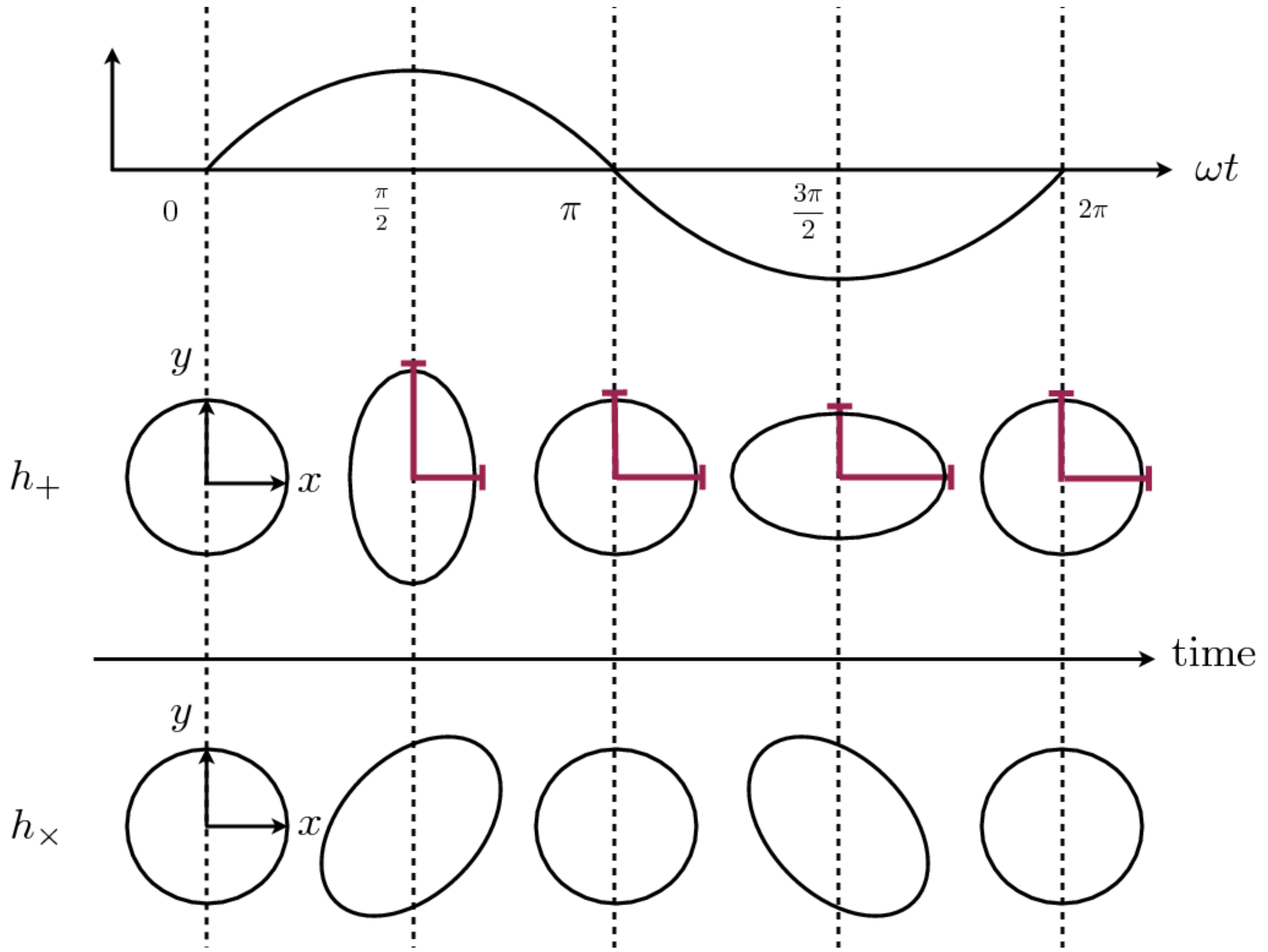
A black hole is completely characterized by three parameters:

Mass
Angular momentum
(Electric charge)

Gravitational Waves

- **Linearized theory:** $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu},\ |h| \ll 1$ ⟶ $\Box h_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$ Wave equation!
  - **GWs**: perturbations of background spacetime propagating at **speed of light**
  - Carrying **energy, momentum**
  - Transverse
  - Two polarizations $(h_+, h_\times)$

$$h^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{\mu\nu} e^{ikz}$$



- **Source?**

Sources of GWs

- Solve linearized equation by retarded Green's function: $$h_{\mu\nu}(t,\mathbf{x}) = \frac{4G}{c^3}\int d^3x' \frac{1}{|\mathbf{x}-\mathbf{x}'|} T_{\mu\nu}\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)$$
- To lowest order: **quadrupole formula**

$$h_{ij}^{\mathrm{TT}}(t,\mathbf{x}) = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}_{ij}\left(t - \frac{r}{c}\right)$$

  - Usually given as multipole expansion

Leading **(2,2)** mode ←

$$h_+ - \mathrm{i}h_\times = \sum_{\ell\geq 2}\sum_{m=-\ell}^{\ell} h_{\ell m}\ {}_{-2}Y_{\ell m}(\theta,\varphi)$$

$$\dot{E} = \frac{1}{16\pi G}\sum_{\ell\geq 2}\sum_{m=-\ell}^{\ell}\left|\dot{h}_{\ell m}\right|^2$$

Energy and angular momentum fluxes

$$\dot{J} = \frac{1}{16\pi G}\sum_{\ell\geq 2}\sum_{m=-\ell}^{\ell} \mathrm{i}m\dot{h}_{\ell m}h^*_{\ell m}$$

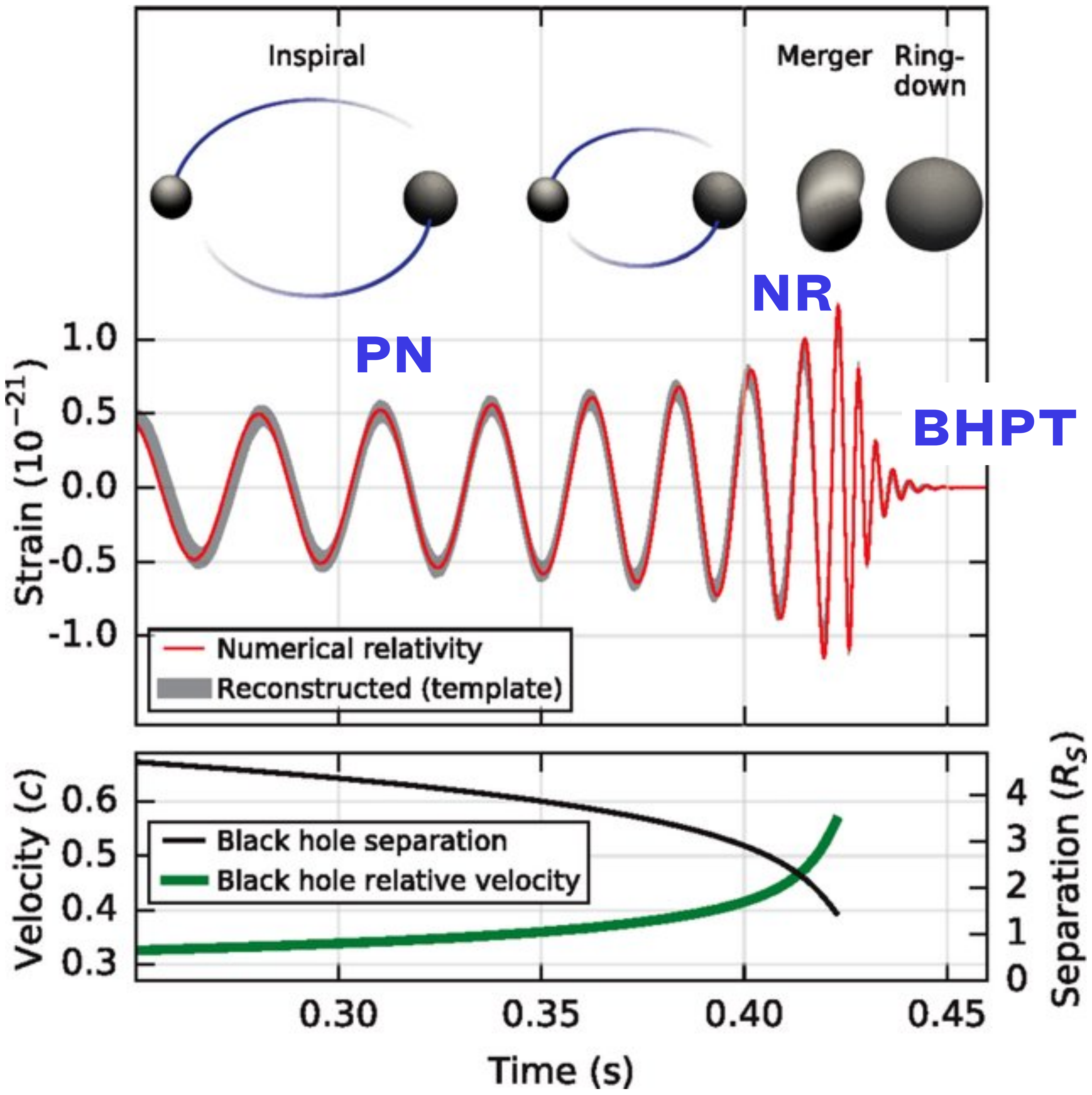
- Only the strongest sources in the universe are detectable:
  - **Compact Object Binaries (BHS, Neutron Stars)**
  - Supernovae (possibly)

Black hole binaries

- Quadrupole formula for binary system:

$$-\dot{E}_{\rm GW} = \dot{E}_{\rm system} = -\frac{32c^5}{5G}\frac{\mu^2}{M^2}\Omega^{10/3}$$

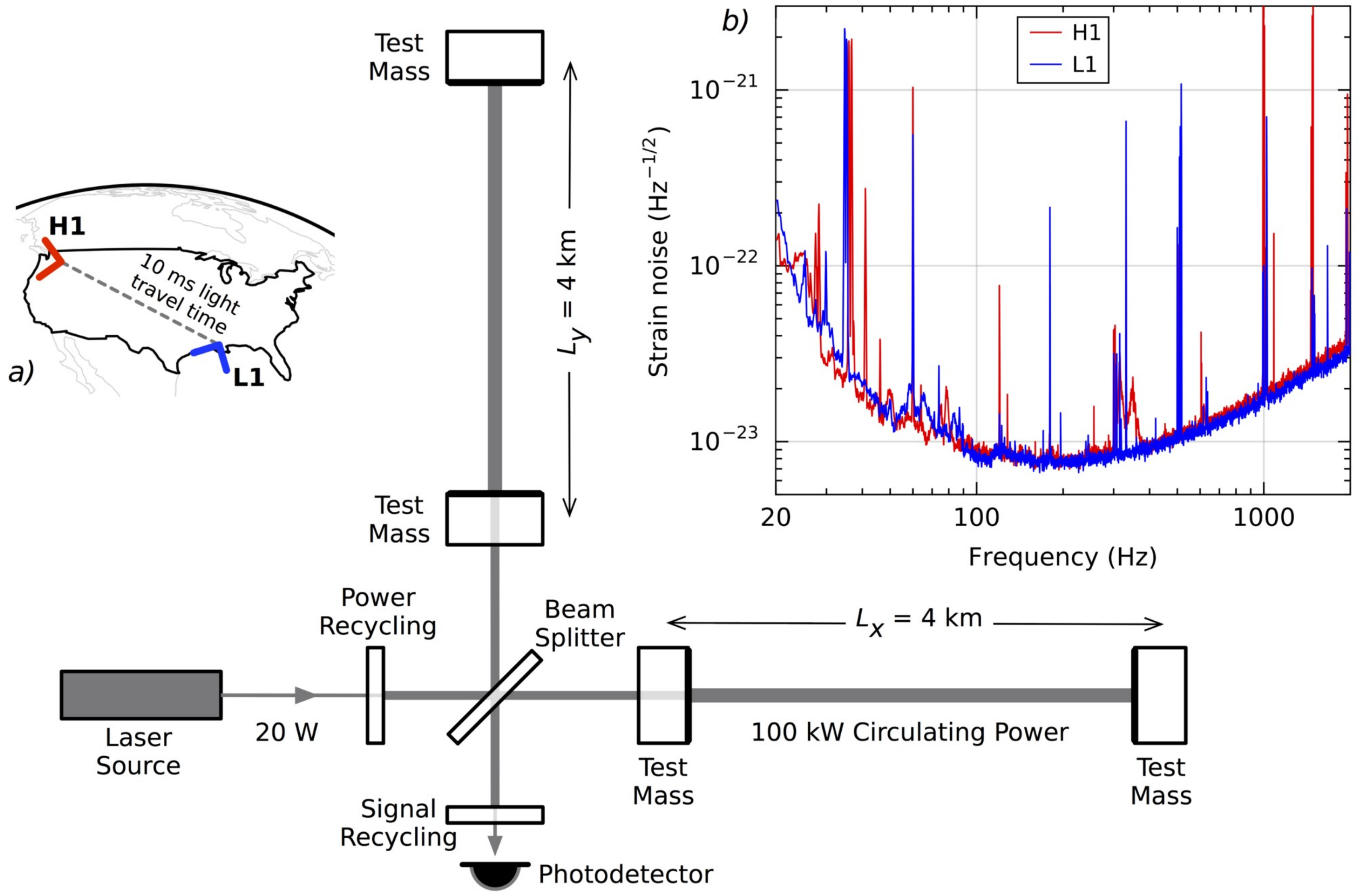
- GW frequency: $\omega_{22} = 2\Omega$
- Phases:
  - **Inspiral:** loss of energy brings BHs closer
    Velocity and frequency rise
  - **Plunge:** BHs quickly collapse onto one another
  - **Merger:** a single body forms
  - **Ringdown:** relaxation to final state (Kerr BH)

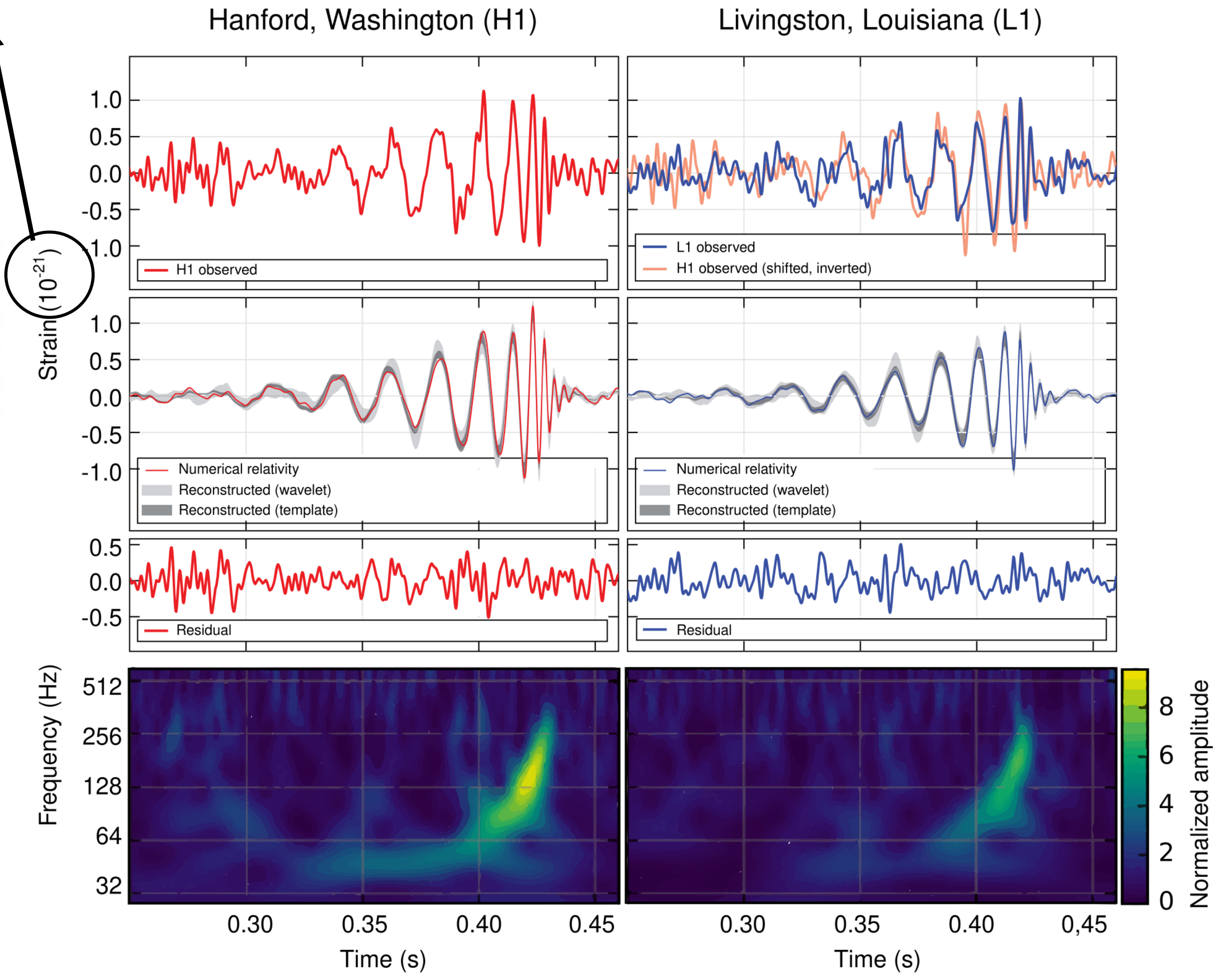


Detecting GWs

$$\text{Strain} = h \sim \frac{\delta L}{L} \sim 10^{-21}!!!$$

- **LIGO, Virgo, KAGRA**: interferometric detectors





- Technique: **matched filtering** to extract signal from noise

- Accurate models *essential* for detection and **parameter estimation**

Effective-One-Body

$!!G = c = 1!!$

- Key word: **resummation**
- PN series (dynamics, waveform) don't work in strong field

$$
\begin{aligned}
H^{\mathrm{ADM}} = & -\frac{Gm_1m_2}{2R_{12}} + \frac{P_1^2}{2m_1} \\
& + \frac{1}{c^2}\left\{-\frac{P_1^4}{8m_1^3} + \frac{G^2m_1^2m_2}{2R_{12}^2} + \frac{Gm_1m_2}{R_{12}}\left(\frac{1}{4}\frac{(N_{12}P_1)(N_{12}P_2)}{m_1m_2} - \frac{3}{2}\frac{P_1^2}{m_1^2} + \frac{7}{4}\frac{(P_1P_2)}{m_1m_2}\right)\right\} \\
& + \frac{1}{c^4}\left\{\frac{P_1^6}{16m_1^5} - \frac{G^3m_1^3m_2}{4R_{12}^3} - \frac{5G^3m_1^2m_2^2}{8R_{12}^3}\right. \\
& \quad + \frac{G^2m_1^2m_2}{R_{12}^2}\left(-\frac{3}{2}\frac{(N_{12}P_1)(N_{12}P_2)}{m_1m_2} + \frac{19}{4}\frac{P_1^2}{m_1^2} - \frac{27}{4}\frac{(P_1P_2)}{m_1m_2} + \frac{5P_2^2}{2m_2^2}\right) \\
& \quad + \frac{Gm_1m_2}{R_{12}}\left(-\frac{3}{16}\frac{(N_{12}P_1)^2(N_{12}P_2)^2}{m_1^2m_2^2} + \frac{5}{8}\frac{(N_{12}P_2)^2P_1^2}{m_1^2m_2^2}\right. \\
& \quad\quad \left.\left. + \frac{5}{8}\frac{P_1^4}{m_1^4} - \frac{3}{4}\frac{(N_{12}P_1)(N_{12}P_2)(P_1P_2)}{m_1^2m_2^2} - \frac{1}{8}\frac{(P_1P_2)^2}{m_1^2m_2^2} - \frac{11}{16}\frac{P_1^2P_2^2}{m_1^2m_2^2}\right)\right\} \\
& + \frac{1}{c^6}\left\{-\frac{5P_1^8}{128m_1^7} + \frac{G^4m_1^4m_2}{8R_{12}^4} + \frac{G^4m_1^3m_2^2}{R_{12}^4}\left(\frac{227}{24} - \frac{21}{32}\pi^2\right)\right. \\
& \quad + \frac{G^3m_1^2m_2^2}{R_{12}^3}\left(-\frac{43}{16}\frac{(N_{12}P_1)^2}{m_1^2} + \frac{119}{16}\frac{(N_{12}P_1)(N_{12}P_2)}{m_1m_2} - \frac{3}{64}\pi^2\frac{(N_{12}P_1)^2}{m_1^2}\right. \\
& \quad\quad + \frac{3}{64}\pi^2\frac{(N_{12}P_1)(N_{12}P_2)}{m_1m_2} - \frac{473}{48}\frac{P_1^2}{m_1^2} + \frac{1}{64}\pi^2\frac{P_1^2}{m_1^2} + \frac{143}{16}\frac{(P_1P_2)}{m_1m_2} \\
& \quad\quad \left. - \frac{1}{64}\pi^2\frac{(P_1P_2)}{m_1m_2}\right) \\
& \quad + \frac{G^3m_1^3m_2}{R_{12}^3}\left(\frac{5}{4}\frac{(N_{12}P_1)^2}{m_1^2} + \frac{21}{8}\frac{(N_{12}P_1)(N_{12}P_2)}{m_1m_2} - \frac{425}{48}\frac{P_1^2}{m_1^2} + \frac{77}{8}\frac{(P_1P_2)}{m_1m_2} - \frac{25P_2^2}{8m_2^2}\right) \\
& \quad + \frac{G^2m_1^2m_2}{R_{12}^2}\left(\frac{5}{12}\frac{(N_{12}P_1)^4}{m_1^4} - \frac{3}{2}\frac{(N_{12}P_1)^3(N_{12}P_2)}{m_1^3m_2} + \frac{10}{3}\frac{(N_{12}P_1)^2(N_{12}P_2)^2}{m_1^2m_2^2}\right. \\
& \quad\quad + \frac{17}{16}\frac{(N_{12}P_1)^2P_1^2}{m_1^4} - \frac{15}{8}\frac{(N_{12}P_1)(N_{12}P_2)P_1^2}{m_1^3m_2} - \frac{55}{12}\frac{(N_{12}P_2)^2P_1^2}{m_1^2m_2^2} \\
& \quad\quad + \frac{P_1^4}{16m_1^4} - \frac{11}{8}\frac{(N_{12}P_1)^2(P_1P_2)}{m_1^3m_2} + \frac{125}{12}\frac{(N_{12}P_1)(N_{12}P_2)(P_1P_2)}{m_1^2m_2^2} \\
& \quad\quad - \frac{115}{16}\frac{P_1^2(P_1P_2)}{m_1^3m_2} + \frac{25}{48}\frac{(P_1P_2)^2}{m_1^2m_2^2} - \frac{193}{48}\frac{(N_{12}P_1)^2P_2^2}{m_1^2m_2^2} + \frac{371}{48}\frac{P_1^2P_2^2}{m_1^2m_2^2} \\
& \quad\quad \left. - \frac{27}{16}\frac{P_2^4}{m_2^4}\right) \\
& \quad + \frac{Gm_1m_2}{R_{12}}\left(\frac{5}{32}\frac{(N_{12}P_1)^3(N_{12}P_2)^3}{m_1^3m_2^3} + \frac{3}{16}\frac{(N_{12}P_1)^2(N_{12}P_2)^2P_1^2}{m_1^4m_2^2}\right. \\
& \quad\quad - \frac{9}{16}\frac{(N_{12}P_1)(N_{12}P_2)^3P_1^2}{m_1^3m_2^3} - \frac{5}{16}\frac{(N_{12}P_2)^2P_1^4}{m_1^4m_2^2} - \frac{7}{16}\frac{P_1^6}{m_1^6} \\
& \quad\quad + \frac{15}{32}\frac{(N_{12}P_1)^2(N_{12}P_2)^2(P_1P_2)}{m_1^3m_2^3} + \frac{3}{4}\frac{(N_{12}P_1)(N_{12}P_2)P_1^2(P_1P_2)}{m_1^4m_2^2} \\
& \quad\quad + \frac{1}{16}\frac{(N_{12}P_2)^2P_1^2(P_1P_2)}{m_1^3m_2^3} - \frac{5}{16}\frac{(N_{12}P_1)(N_{12}P_2)(P_1P_2)^2}{m_1^3m_2^3} \\
& \quad\quad + \frac{1}{8}\frac{P_1^2(P_1P_2)^2}{m_1^4m_2^2} - \frac{1}{16}\frac{(P_1P_2)^3}{m_1^3m_2^3} - \frac{5}{16}\frac{(N_{12}P_1)^2P_1^2P_2^2}{m_1^4m_2^2} \\
& \quad\quad \left.\left. + \frac{7}{32}\frac{(N_{12}P_1)(N_{12}P_2)P_1^2P_2^2}{m_1^3m_2^3} + \frac{1}{2}\frac{P_1^4P_2^2}{m_1^4m_2^2} + \frac{1}{32}\frac{P_1^2(P_1P_2)P_2^2}{m_1^3m_2^3}\right)\right\} + 1 \leftrightarrow 2 + O\left(\frac{1}{c^7}\right)
\end{aligned}
$$

$$H^{\mathrm{ADM}} = -\frac{Gm_1m_2}{R_{12}} + \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \ldots$$

Two-body problem mapped into motion of effective particle in effective metric

Continuous deformation of Schwarzschild

$$\nu = \frac{\mu}{M} = \frac{m_1m_2}{(m_1+m_2)^2}$$

$$H_{\mathrm{EOB}} = M\sqrt{1 + 2\nu\left(\frac{H_{\mathrm{eff}}}{\mu} - 1\right)}$$

$$H_{\mathrm{eff}} = \sqrt{A(r)\left(\mu^2 + \frac{p_\varphi^2}{r^2} + \frac{1}{B(r)}p_r^2 + \nu z_3\frac{p_r^4}{r^2}\right)}$$

Non-geodesic term (3PN+)

$$A_{5\mathrm{PN}}(r) = \underbrace{1 - \frac{2M}{r}}_{\text{Schwarzschild!}} + 2\nu\left(\frac{M}{r}\right)^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu\left(\frac{M}{r}\right)^4 + \nu\left(a_5 + a_5^{\log}\log\frac{M}{r}\right)\left(\frac{M}{r}\right)^5 + \nu\left(a_6 + a_6^{\log}\log\frac{M}{r}\right)\left(\frac{M}{r}\right)^6$$

$A_{5\mathrm{PN}}(r)$ **resummed** as a (3, 3) Padé:

$$A(r) \equiv P_3^3[A_{5\mathrm{PN}}(r)] = \frac{1 + \tilde{a}_1u + \tilde{a}_2u^2 + \tilde{a}_3u^3}{1 + \tilde{a}_4u + \tilde{a}_5u^2 + \tilde{a}_6u^3} \qquad u \equiv \frac{M}{r}$$

Effective-One-Body

- Conservative dynamics (Hamiltonian) completed by **radiation reaction force** and **waveform**

$$\frac{dr}{dt} = \frac{\partial H_{\mathrm{EOB}}}{\partial p_r} \qquad \frac{dp_r}{dt} = -\frac{\partial H_{\mathrm{EOB}}}{\partial r} + \mathscr{F}_r$$

$$\frac{d\varphi}{dt} = \frac{\partial H_{\mathrm{EOB}}}{\partial p_\varphi} \qquad \frac{dp_\varphi}{dt} = \mathscr{F}_\varphi$$

Balance argument:

$$\dot{E}^{\infty}_{\mathrm{GW}} + \dot{E}^{\mathrm{H}}_{\mathrm{GW}} = -\dot{H}_{\mathrm{EOB}} - \dot{E}_{\mathrm{Schott}}$$

$$\dot{J}^{\infty}_{\mathrm{GW}} + \dot{J}^{\mathrm{H}}_{\mathrm{GW}} = -\dot{p}_\varphi - \dot{J}_{\mathrm{Schott}}$$

Flux at infinity		Loss by system
+	=	+
Flux at horizon		Schott term

↓

$\mathscr{F}_r, \mathscr{F}_\varphi$ →

$$\mathscr{F}_\varphi = -\frac{32}{5}\nu^2 r_\omega^4 \Omega^5 \sum_{\ell,m} \frac{F^{\mathrm{N}}_{\ell m}}{F^{\mathrm{N}}_{22}} |\hat{h}_{\ell m}|^2 + \mathscr{F}^{\mathrm{H}}_\varphi$$

Waveform model

(2,2) mode at 3.5PN

$$h_{22} = \frac{16M\nu x}{R_L}\sqrt{\frac{\pi}{5}} H_{22} e^{-2i\varphi}$$

$$x = (M\Omega)^{2/3}$$

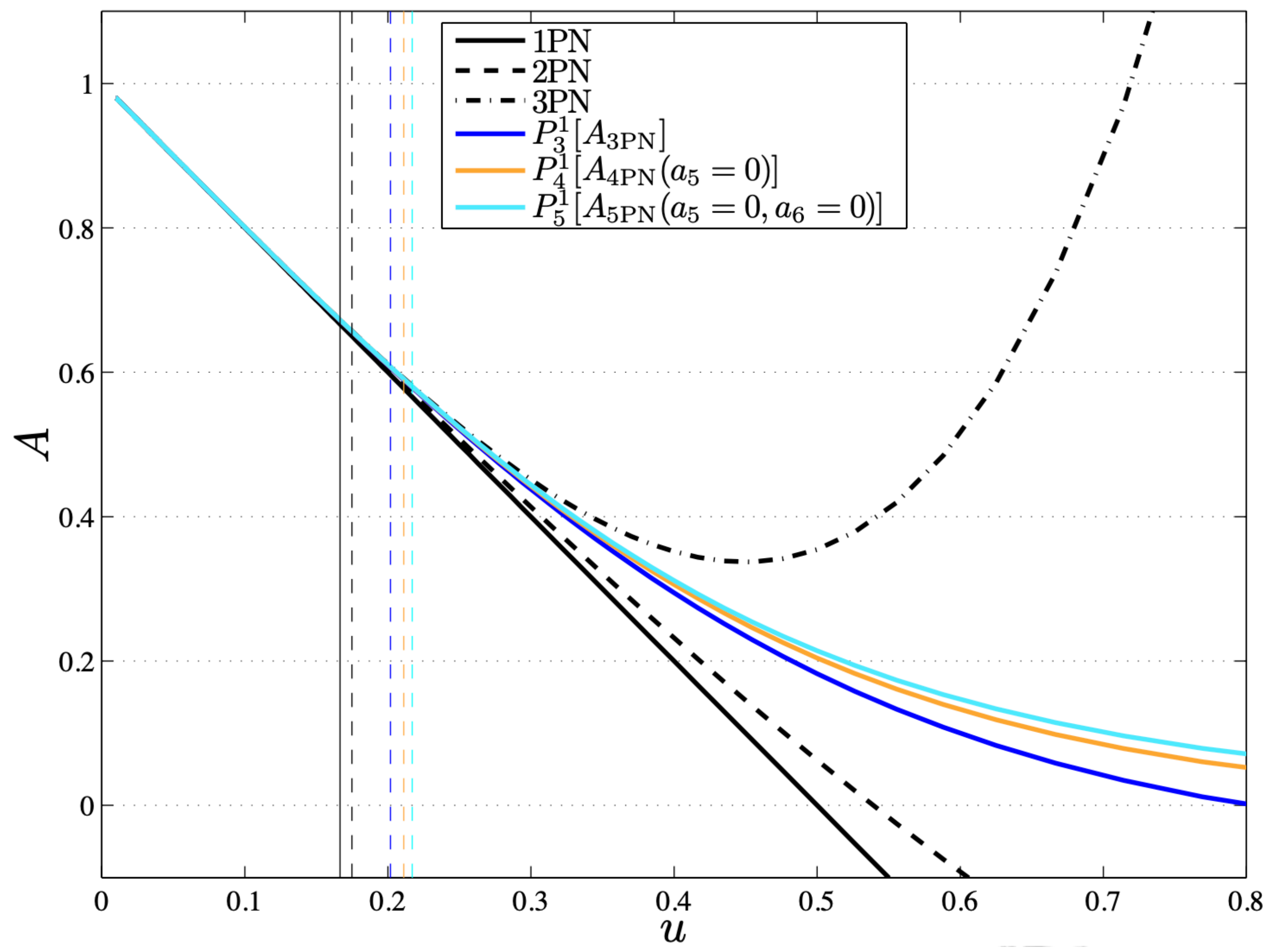
$$H_{22} = 1 + \left(-\frac{107}{42} + \frac{55}{42}\nu\right)x + 2\pi x^{3/2} + \left(-\frac{2173}{1512} - \frac{1069}{216}\nu + \frac{2047}{1512}\nu^2\right)x^2 + \left[-\frac{107\pi}{21} + \left(\frac{34\pi}{21} - 24\,\mathrm{i}\right)\nu\right]x^{5/2}$$
$$+ \left[\frac{27027409}{646800} - \frac{856}{105}\gamma_{\mathrm{E}} + \frac{428\,\mathrm{i}\,\pi}{105} + \frac{2\pi^2}{3} + \left(-\frac{278185}{33264} + \frac{41\pi^2}{96}\right)\nu - \frac{20261}{2772}\nu^2 + \frac{114635}{99792}\nu^3 - \frac{428}{105}\ln(16x)\right]x^3$$
$$+ \left[-\frac{2173\pi}{756} + \left(-\frac{2495\pi}{378} + \frac{14333\,\mathrm{i}}{162}\right)\nu + \left(\frac{40\pi}{27} - \frac{4066\,\mathrm{i}}{945}\right)\nu^2\right]x^{7/2}$$

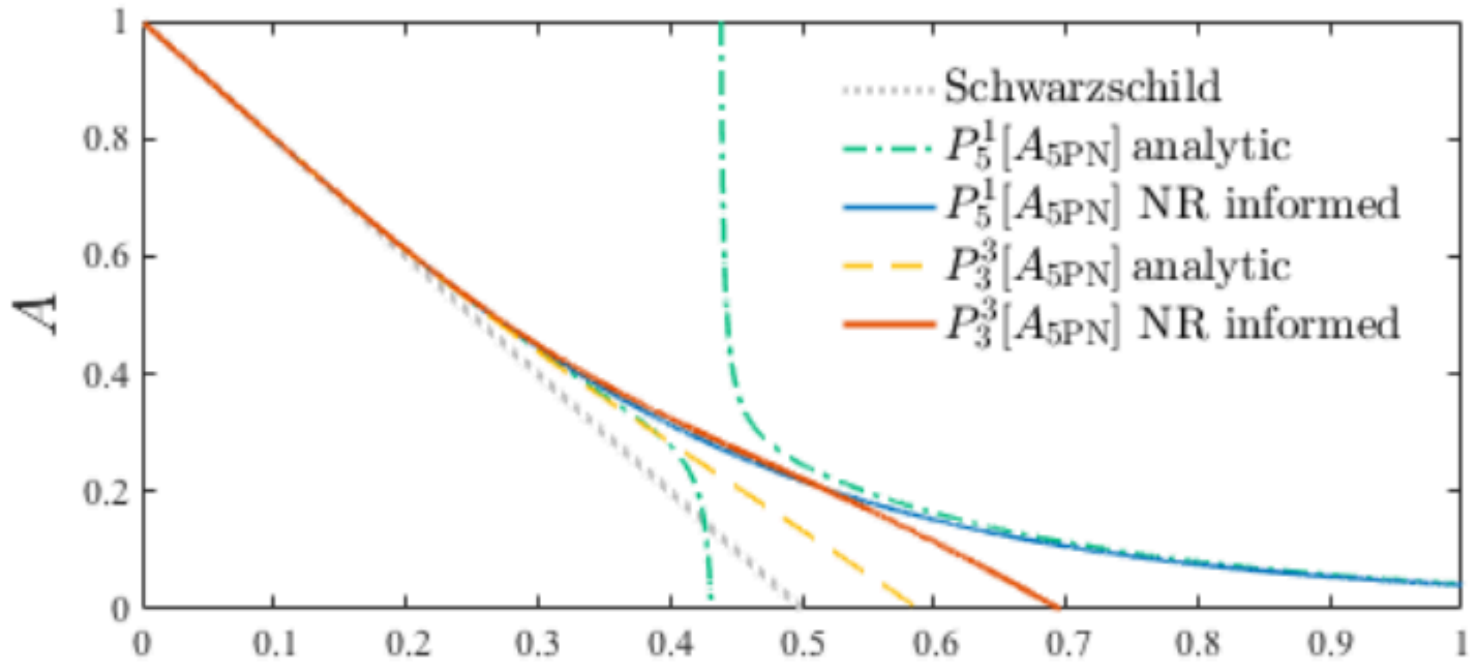
Factorize ↓ Resum

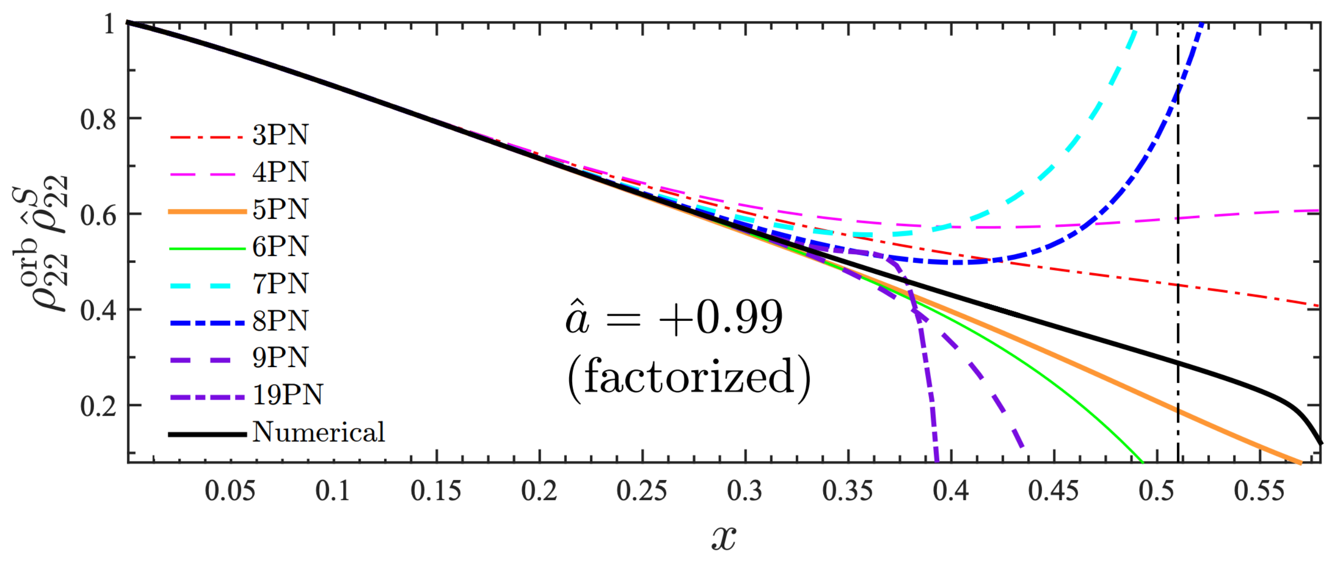
$$h_{\ell m} = h^{(N,\epsilon)}_{\ell m}\hat{h}^{(\epsilon)}_{\ell m} = h^{(N,\epsilon)}_{\ell m}\hat{S}^{(\epsilon)}_{\mathrm{eff}}\hat{h}^{\mathrm{tail}}_{\ell m}\rho^{\ell}_{\ell m}\hat{h}^{\mathrm{NQC}}_{\ell m}$$

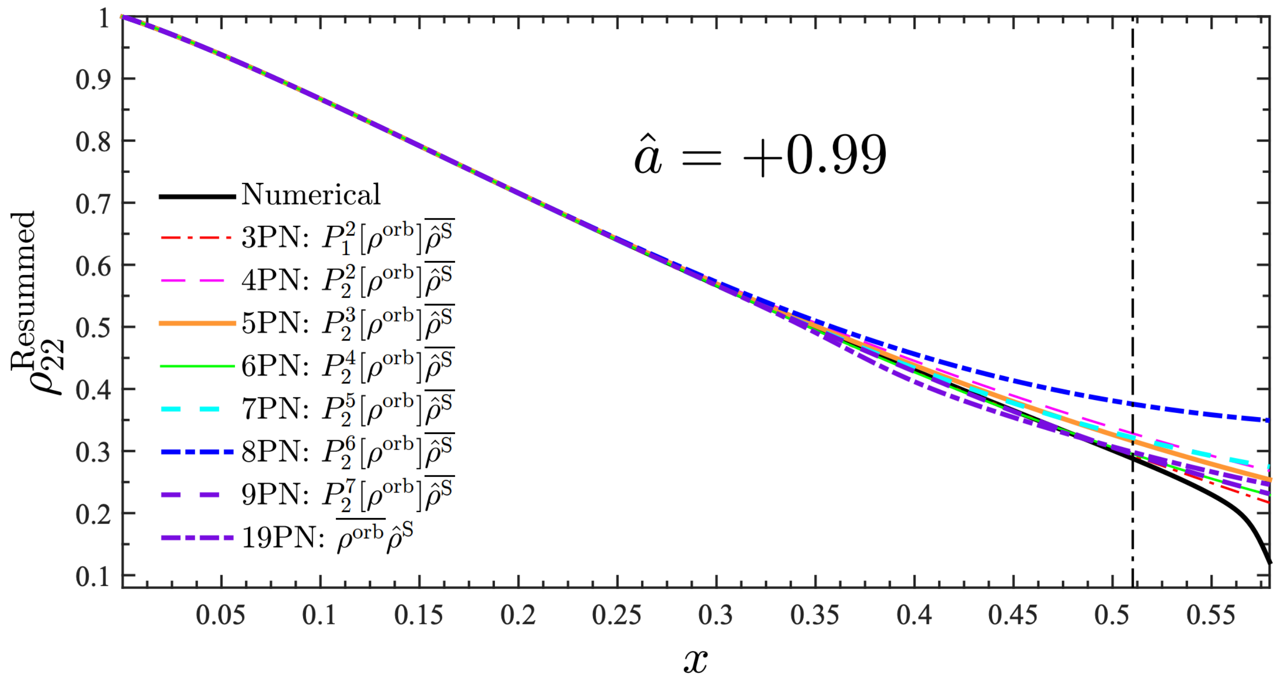
- $h^{(N,\epsilon)}_{\ell m}$: Newtonian (leading) term
- $\hat{S}^{(\epsilon)}_{\mathrm{eff}}$: Effective source (energy/ ang. mom.)
- $\hat{h}^{\mathrm{tail}}_{\ell m}$: Hereditary contributions
- $\rho^{\ell}_{\ell m}$: Residual amplitude correction — Padé-resummed!

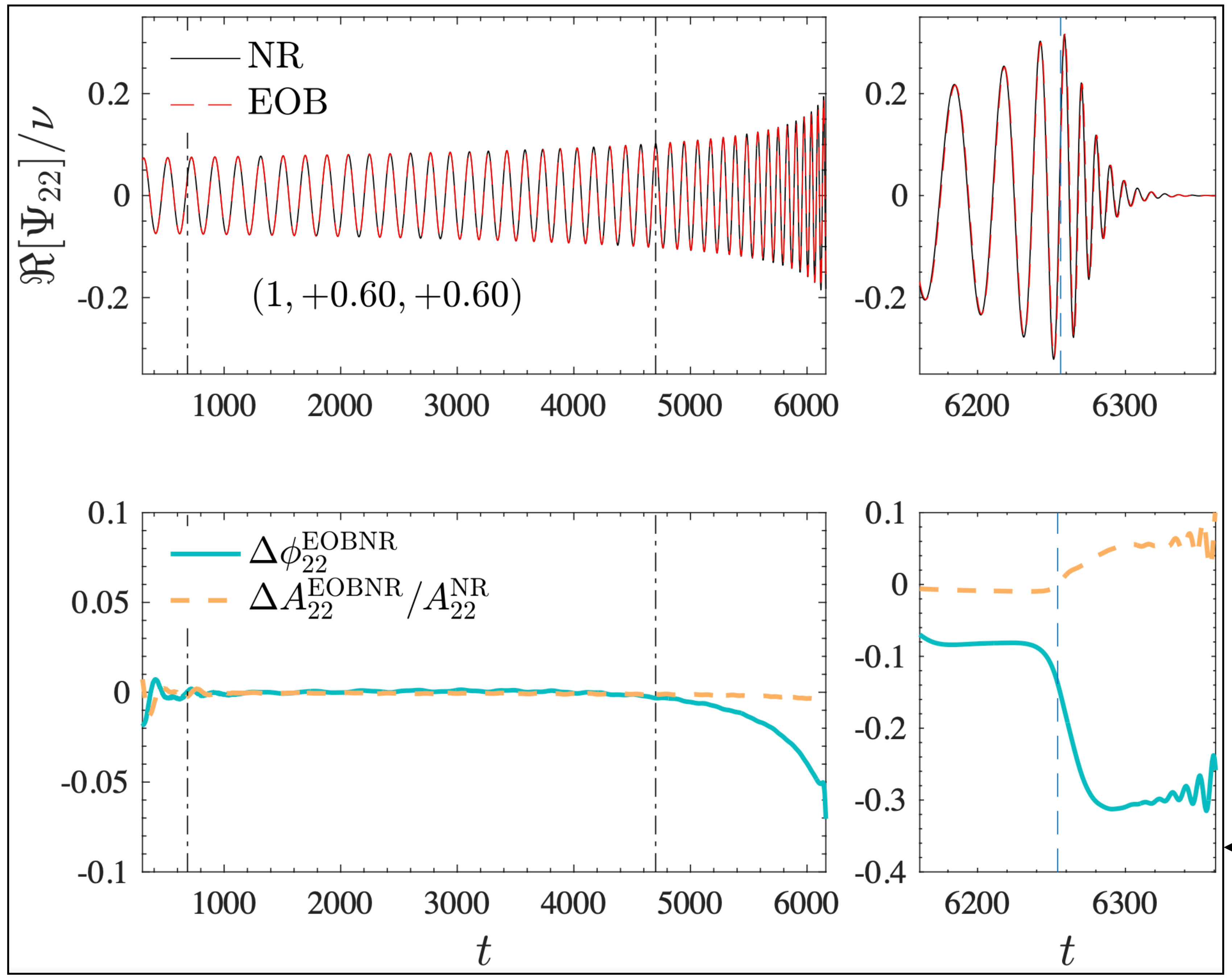
Effective-One-Body











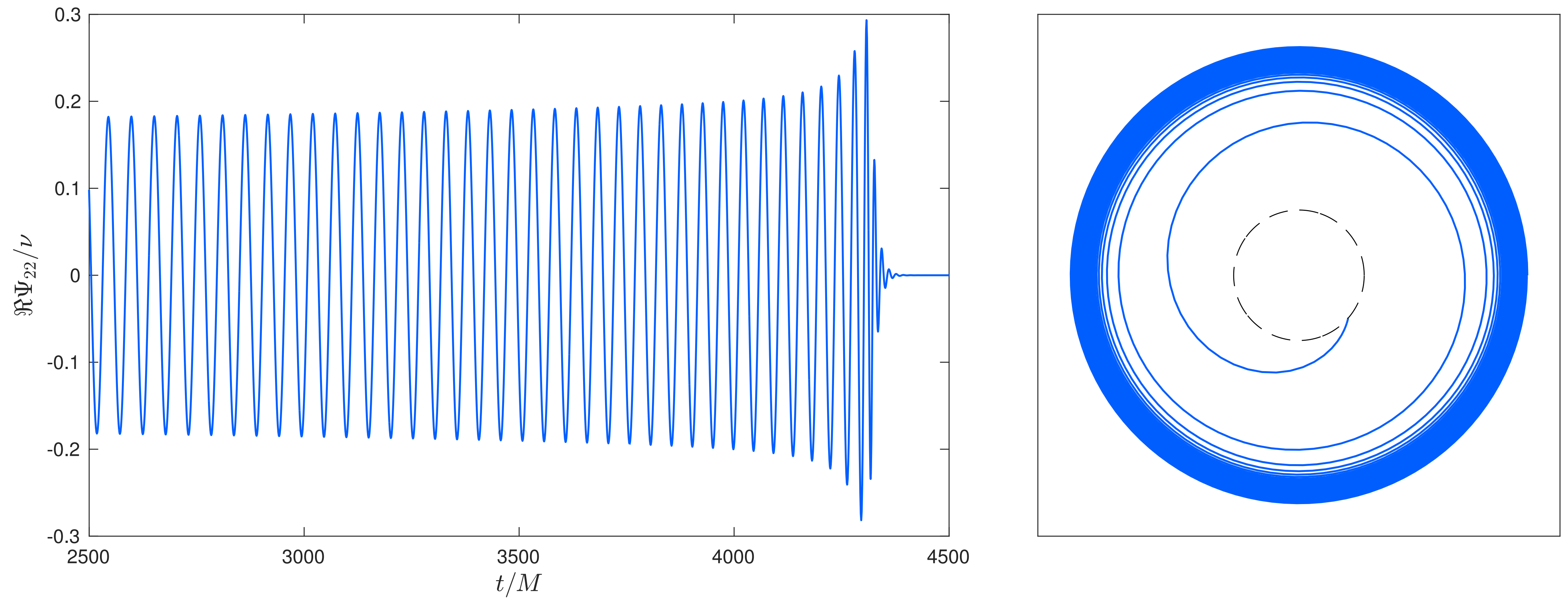
$m_1 = m_2$, spinning BHs

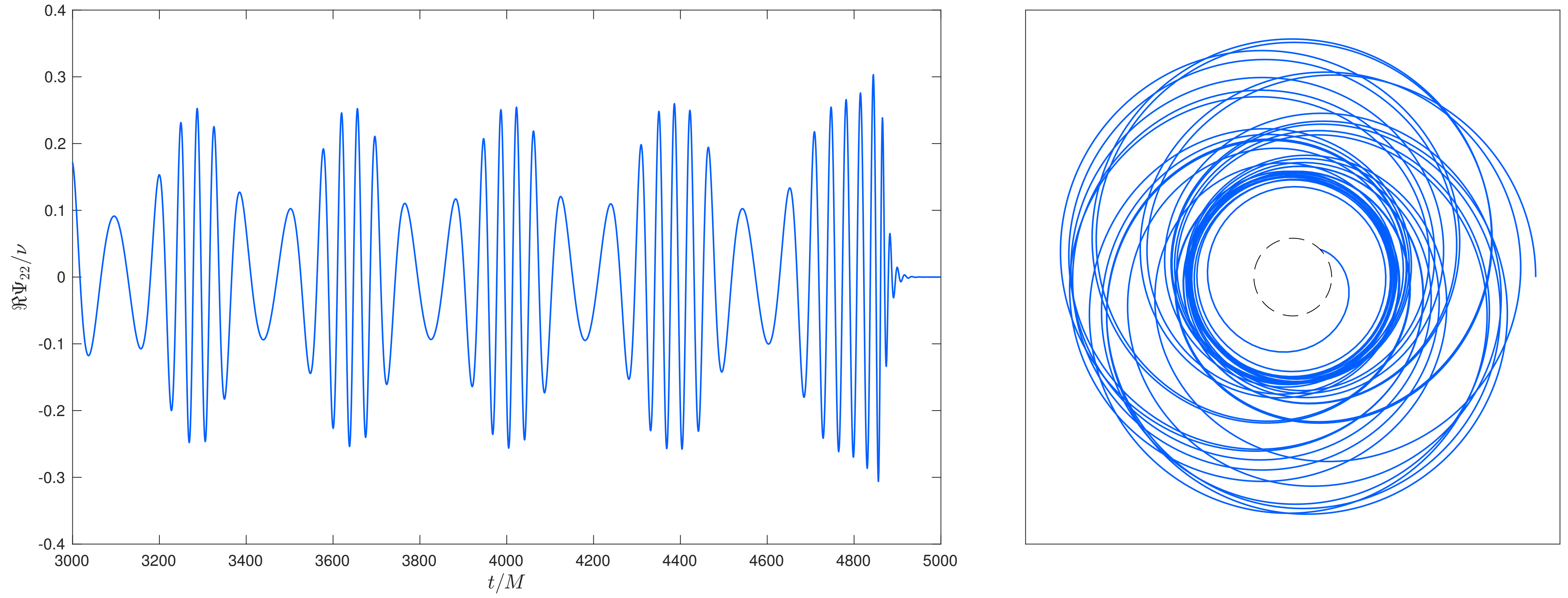
Validation:

- Comparison with **NR** (comparable-mass case)
- **BHPT** in test-mass case ($\nu \to 0$)

NR also used for **calibration**
Especially for **plunge and post-merger**

Eccentricity





Test of GR:
decay of orbit of Hulse-Taylor binary pulsar

Eccentricity-induced orbit precession:
even conservative orbits have incommensurable radial/angular periods

- **Peters-Mathews (1964):**
  Evolution of orbital period and eccentricity under radiation reaction

$$\left\langle \frac{dP}{dt} \right\rangle = -\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P}\right)^{5/3}\frac{m_1 m_2}{(m_1+m_2)^{1/3}}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}$$

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{608\pi}{15c^5}\frac{e}{P}\left(\frac{2\pi G}{P}\right)^{5/3}\frac{m_1 m_2}{(m_1+m_2)^{1/3}}\frac{1+\frac{121}{304}e^2}{(1-e^2)^{5/2}}$$

$\dot{e} < 0!$

- Most systems expected to **circularize** by the time they're detectable by LIGO-VIRGO-KAGRA
  - Models long specialized to simpler circular binaries, where $t \leftrightarrow r \leftrightarrow \Omega$
- With eccentricity: **multiple timescales** (orbital, precession, rad. reaction)
  Frequency/amplitude modulations

TEOBResumS-Dalí

State-of-the-art black hole binary model
Made in Turin

- **Simple** prescription for eccentric corrections:

$$\mathcal{F}_\varphi = -\frac{32}{5}\nu^2 r_\omega^4 \Omega^5 \sum_{\ell,m} \frac{F^{\rm N}_{\ell m}}{F^{\rm N}_{22}} |\hat{h}_{\ell m}|^2 \hat{f}^{\rm N,non-circ.}_{\ell m} + \mathcal{F}^{\rm H}_\varphi$$

$$h_{\ell m} = h^{(N,\epsilon)}_{\ell m} \hat{h}^{\rm N,non-circ.}_{\ell m} \hat{h}_{\ell m}$$

Newtonian-level correcting factors
Only for dominant (2,2) mode in $\mathcal{F}_\varphi$

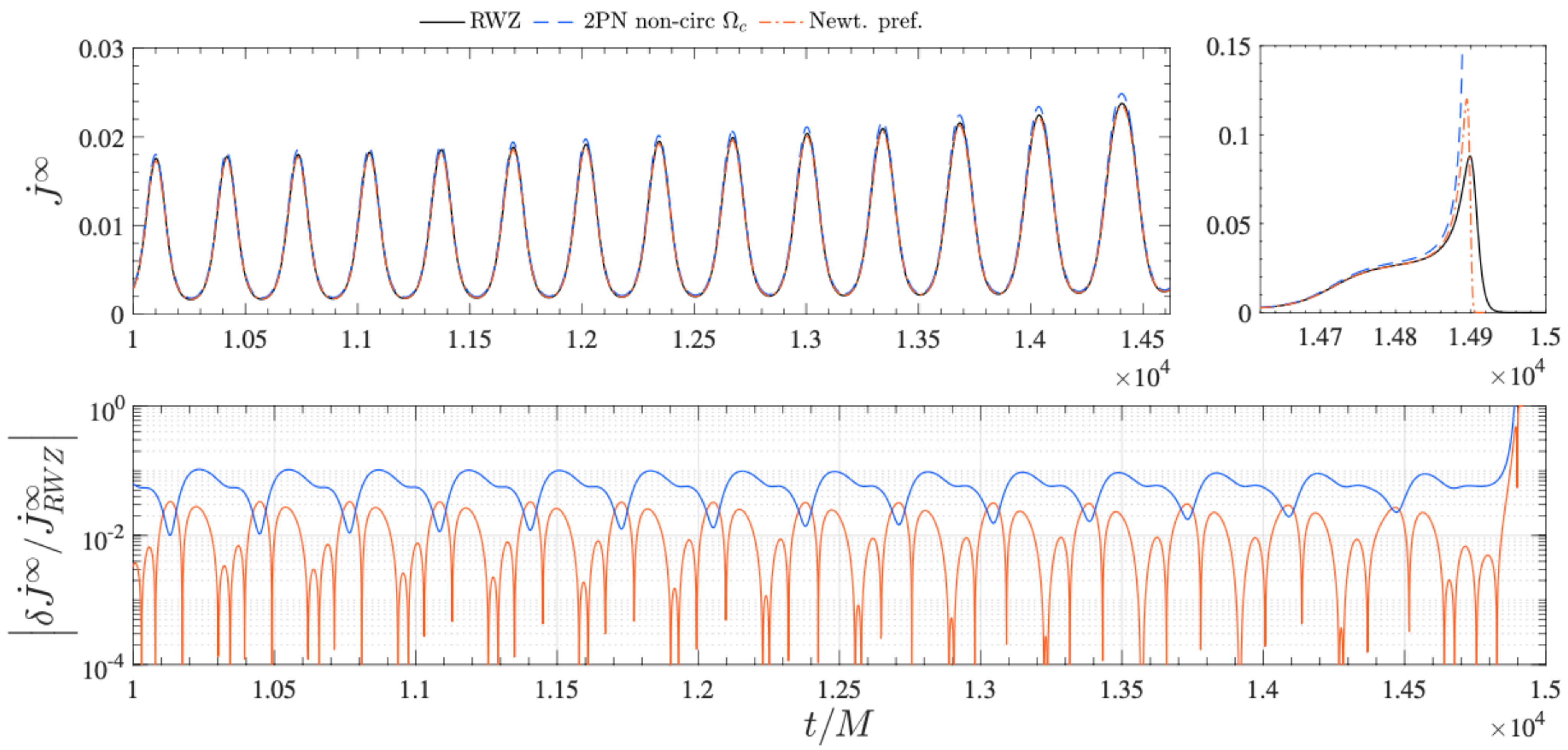
$$\hat{f}^{\rm N,non-circ.}_{22} = 1 + \frac{3}{4}\frac{\dot{r}^2}{r^2\Omega^4} - \frac{\ddot{\Omega}}{4\Omega^3} + \frac{3\dot{r}\dot{\Omega}}{r\Omega^3} + \frac{4\dot{r}^2}{r^2\Omega^2} + \frac{\ddot{\Omega}\dot{r}^2}{8r^2\Omega^5} + \frac{3}{4}\frac{\dot{r}^3\dot{\Omega}}{r^3\Omega^5} + \frac{3}{4}\frac{\dot{r}^4}{r^4\Omega^4} + \frac{3}{4}\frac{\dot{\Omega}^2}{\Omega^4} - \dddot{r}\left(\frac{\dot{r}}{2r^2\Omega^4} + \frac{\dot{\Omega}}{8r\Omega^5}\right) + \ddot{r}\left(-\frac{2}{r\Omega^2} + \frac{\ddot{\Omega}}{8r\Omega^5} + \frac{3}{8}\frac{\dot{r}\dot{\Omega}}{r^2\Omega^5}\right)$$

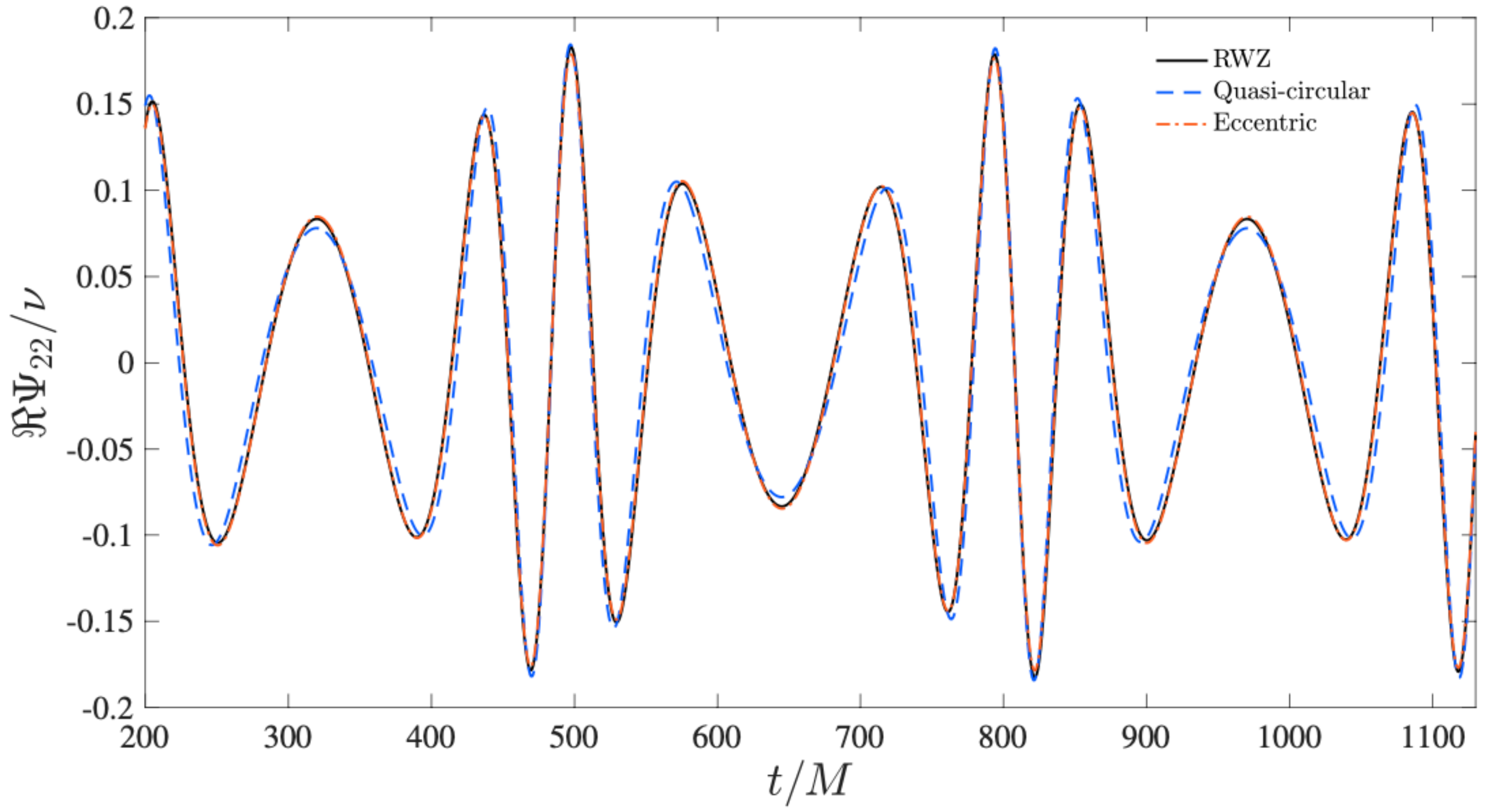
Key step: **no use of PN-expanded equations of motion** in place of **time derivatives**

Include **strong-field information** through **resummed conservative dynamics** even by using just leading correction

- **Future:**
  - Include higher order corrections
  - "Post-Adiabatic" eccentric evolution for speed
  - Review of spin-orbit effects for better agreement with high spins

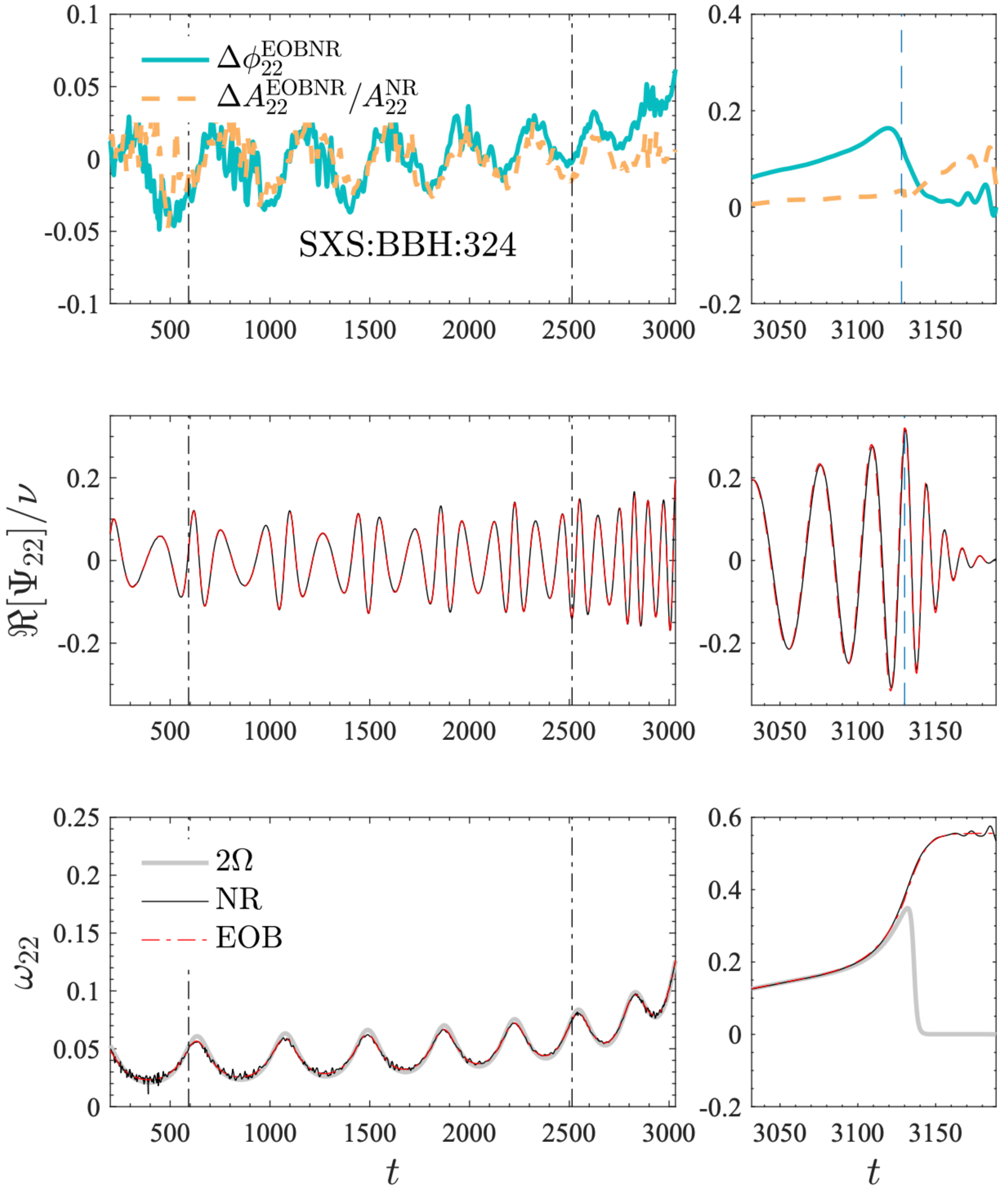
TEOBResumS-Dalí





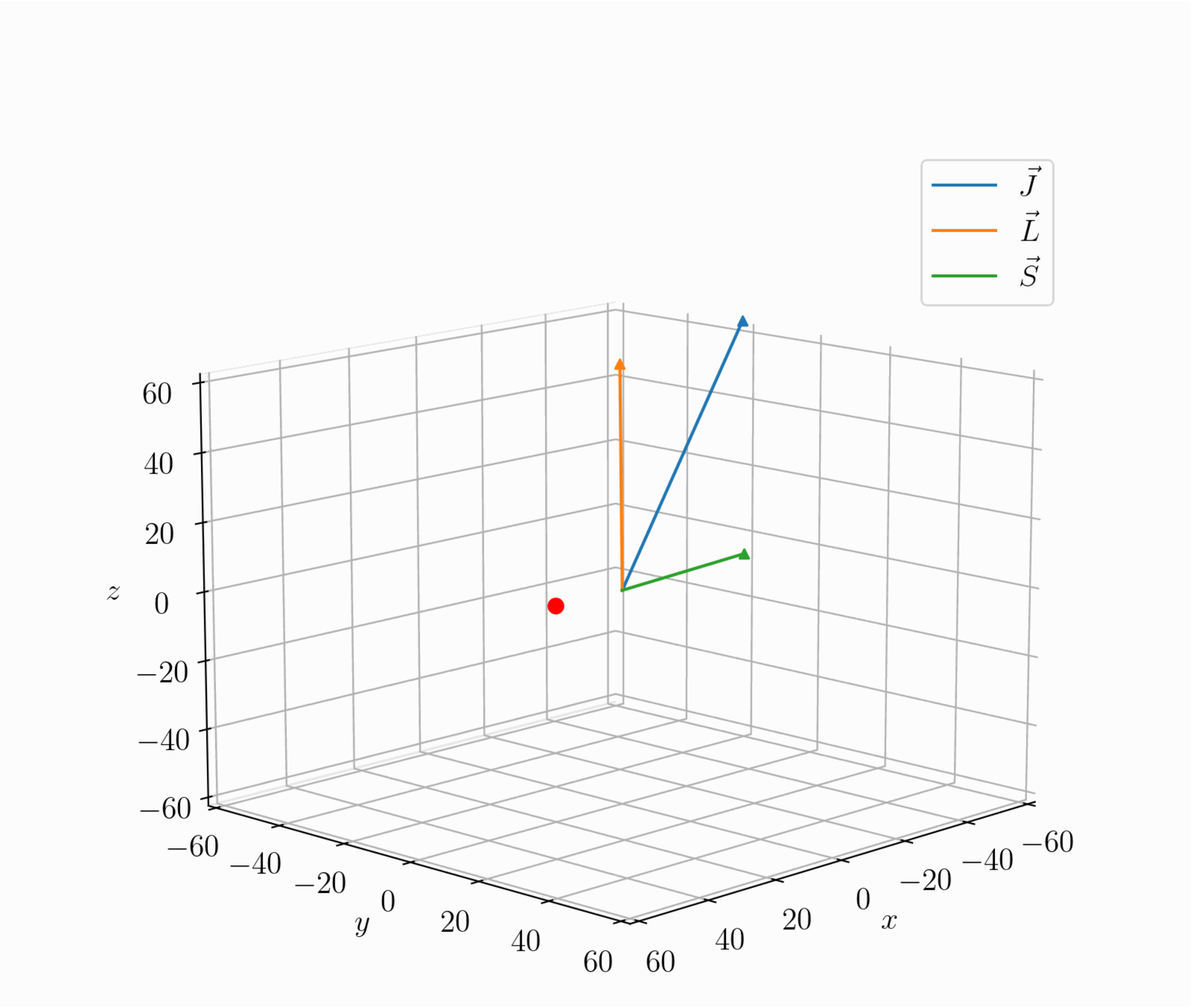
BHPT as a laboratory
to test new ideas

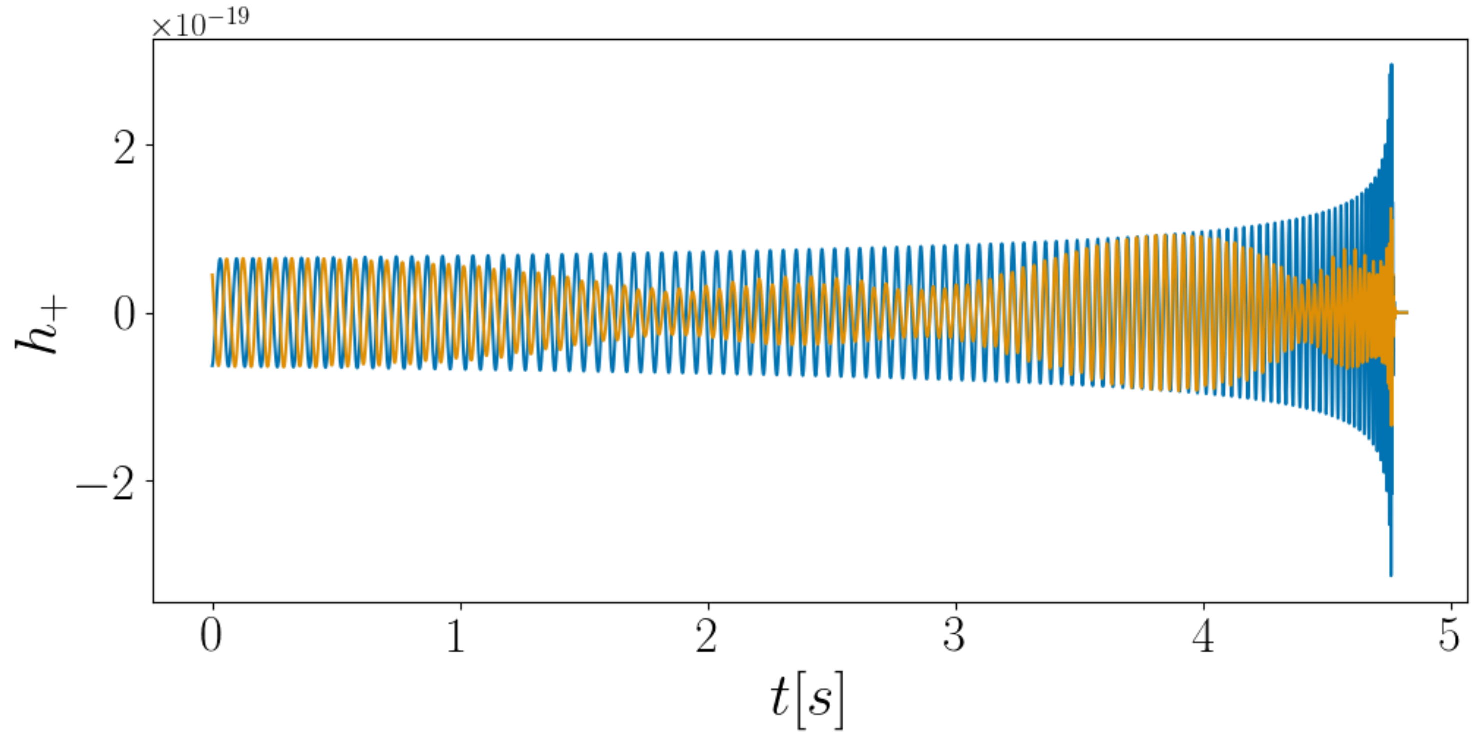
NR validation



Eccentricity and spin precession

- The **orbital plane shifts** when the BH spins and the orbital angular momentum are misaligned
  - Amplitude and frequency modulations in signal
  - Can be **degenerate with eccentricity**!
- **Twist method:** Evolve planar system → Evolve spins, $\vec{L}$ separately → Rotate wave modes accordingly
- Gamba, **D. C.**, Neogi (2024): use of PN equations for $\vec{S}, \vec{L}$ coupled with EOB Ω ~enough for moderately eccentric, precessing binaries
- **Future:** Need more testing against NR; Inclusion of in-plane spins in dynamics





Tidal heating

- Black holes in a binary **absorb energy and angular momentum** through their horizons
  - Masses, spins change during evolution
  - **Horizon fluxes** contribute to radiation reaction

"Tidal heating", "tidal torquing"
Similar to, e.g., satellites

Observed in NR!

$$\dot{m} = \Omega \dot{S} = \frac{16}{5} \frac{m^6 m_2^2}{r^6} \left(1 + \sqrt{1-\chi^2}\right)\left(1+3\chi^2\right)\left(\Omega - \Omega_H\right)\Omega$$

r^6 → Tiny effect

$(\Omega - \Omega_H)$ → **Superradiance:** If $\Omega < \Omega_H$, energy and momentum are **extracted** from BH

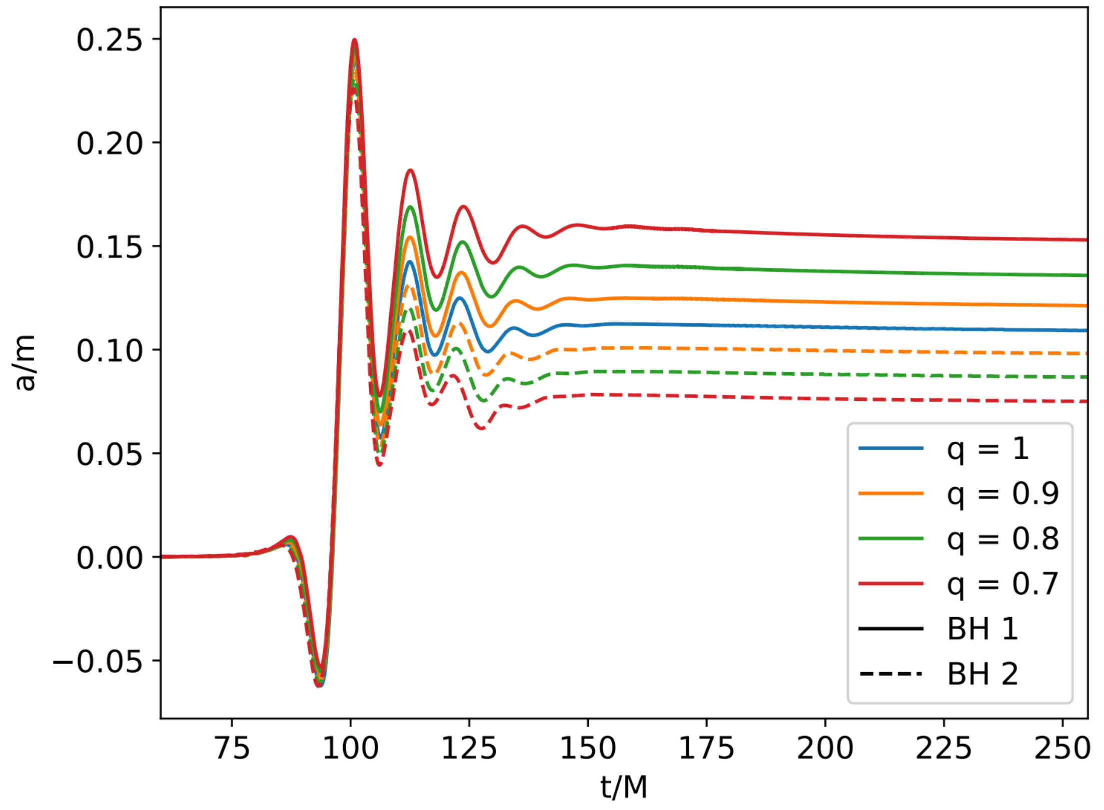
→ Leading order for circular orbits currently in TEOBResumS-Dalí
Though known up to 1.5PN ($O(c^{-3})$)

Mass-rescaled spin

$$\chi = \frac{S}{m^2} = \frac{a}{m} \leq 1$$

Horizon rotational frequency

$$\Omega_H = \frac{\chi}{2 m r_H}$$



Jaraba, Bellido (2021)

Spin-up of BHs after close scattering in NR simulations

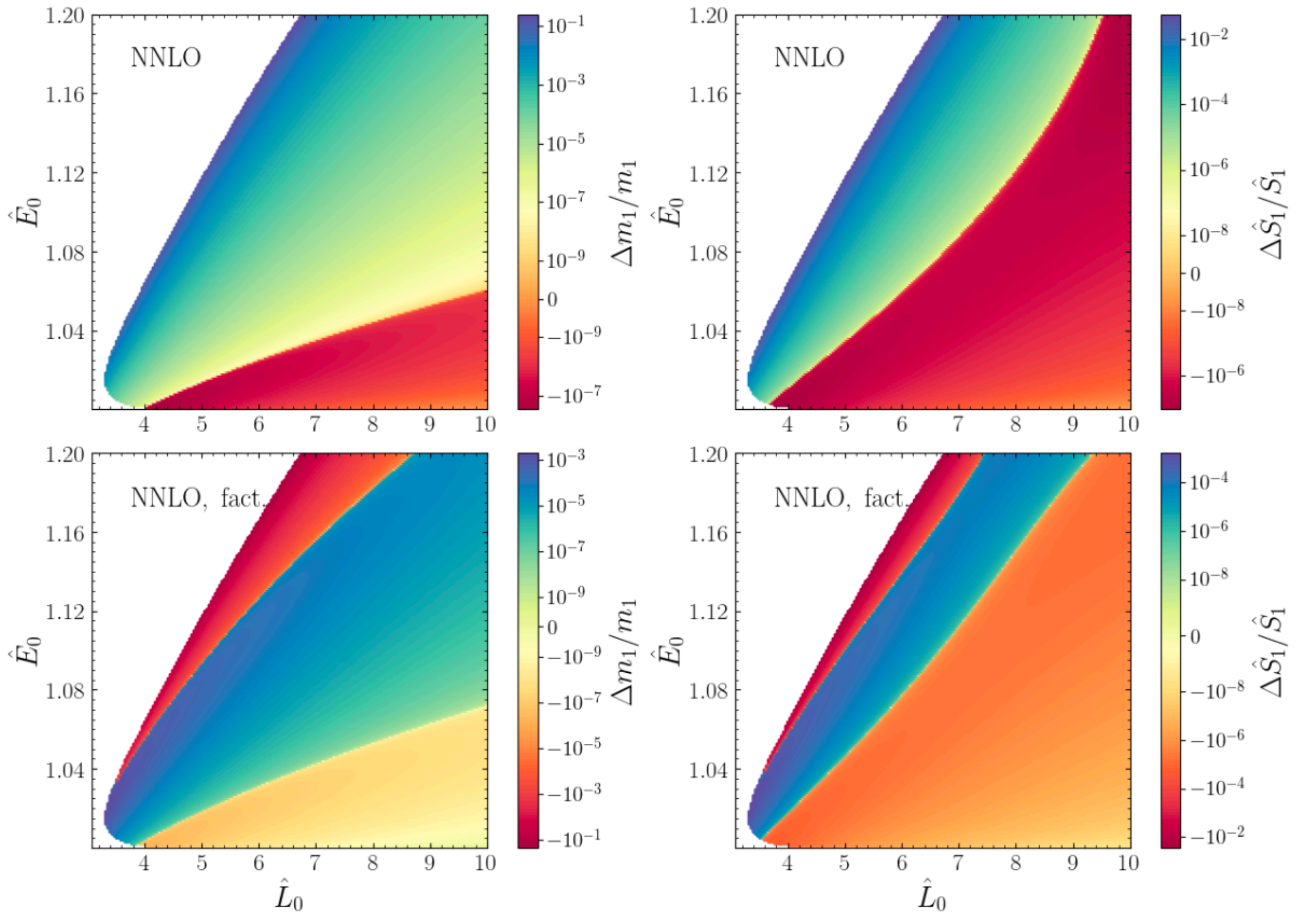
Tidal heating

- **D.C., Gamba (2024):** 1.5PN expressions for tidal heating, torquing valid on generic orbits

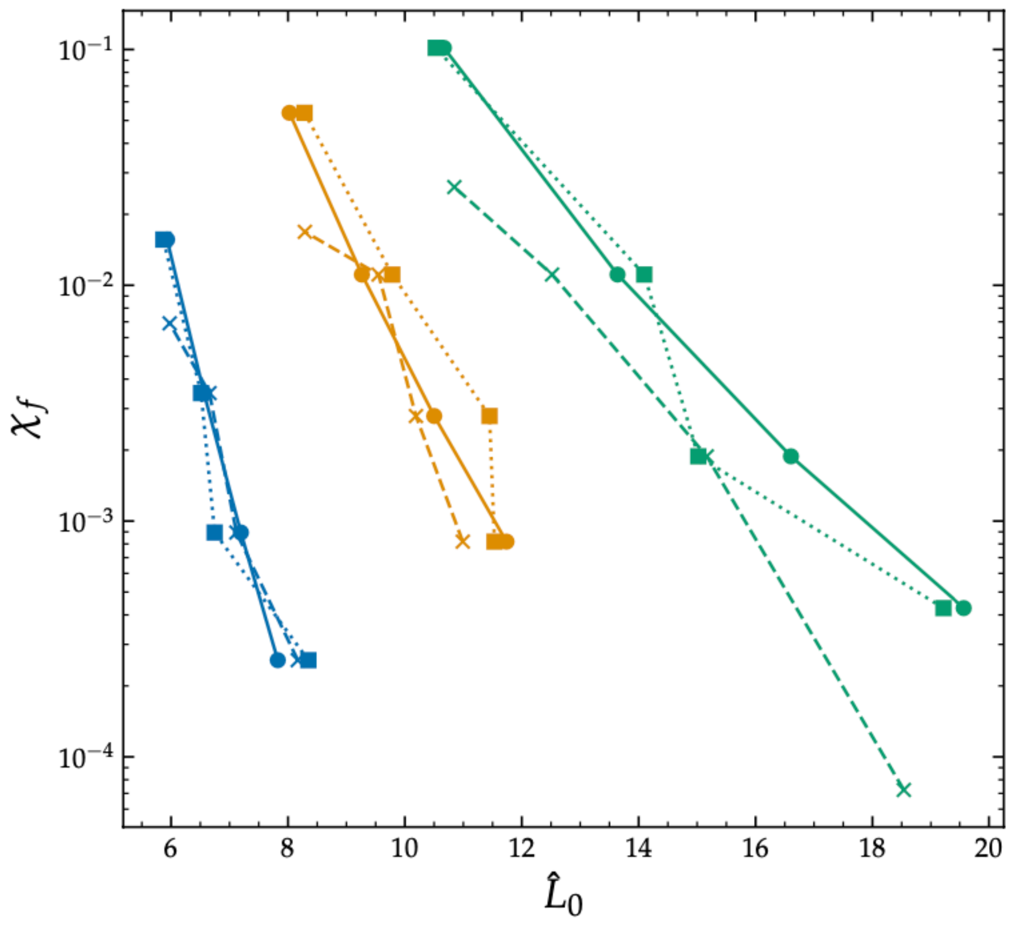
$$
\begin{aligned}
\frac{dm_1}{dt} = &-\frac{16}{5}\frac{m_1^6 m_2^2}{r^6}(1+\sigma_1)\boxed{\left[\Omega_{\rm H}^1 - \frac{1}{c^3}\left(\dot\varphi + 3\frac{\dot r^2}{r^2\dot\varphi}\right)\right]}\Bigg\{\dot\varphi\left(1+3\chi_1^2\right) + \frac{1}{c^2}\Bigg\{\left(1+3\chi_1^2\right)\Bigg[\frac{7}{4}r^2\dot\varphi^3 \\
&-\left(1-\frac{m_1}{M}+\nu\right)\frac{r\ddot r\dot\varphi + r\dot r\ddot\varphi}{2} - \left(15-\frac{5m_1}{M}+\nu\right)\frac{\dot\varphi}{2r} - \left(1+\frac{5m_1}{M}-5\nu\right)\frac{\dot\varphi\dot r^2}{2}\Bigg] + \frac{5}{4}r^2\dot\varphi^3\Bigg\} \\
&+\frac{1}{c^3}\Bigg\{-\frac{5}{6}\left(2m_1\chi_1+3m_2\chi_2\right)\dot\varphi^2 - 4m_1\chi_1\left(7\frac{\dot r^2}{r^2}+4\dot\varphi^2\right)(1+\sigma_1) \\
&+\left(1+3\chi_1^2\right)\Bigg[\left(m_1\chi_1+m_2\chi_2\right)\frac{m_2}{r^3} + \left(10m_1\chi_1 - m_2\chi_2 - 18m_1B_2(\chi_1)\right)\frac{\dot r^2}{r^2} \\
&+\left(\frac{1}{3}m_1\chi_1 - \frac{7}{2}m_2\chi_2 - 8m_1B_2(\chi_1)\right)\dot\varphi^2 + m_2\chi_2\frac{\ddot r}{r} - 16m_1\frac{\dot\varphi\dot r}{r} - m_1\chi_1\left(\frac{19}{2}\frac{\dot r^2}{r^2}+4\dot\varphi^2\right)(1+\sigma_1)\Bigg]\Bigg\}\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dS_1}{dt} = &-\frac{16}{5}\frac{m_1^6 m_2^2}{r^6}(1+\sigma_1)\boxed{\left(\Omega_{\rm H}^1 - \frac{1}{c^3}\dot\varphi\right)}\Bigg\{\left(1+3\chi_1^2\right) - \frac{1}{c^2}\Bigg\{\left(1+3\chi_1^2\right)\Bigg[\frac{7M-2m_1}{r} + \left(1+\frac{5m_1}{M}-7\nu\right)\frac{\dot r^2}{2} \\
&-\left(5+\frac{2m_1}{M}+2\nu\right)\frac{r^2\dot\varphi^2}{4}\Bigg] - \frac{5}{4}r^2\dot\varphi^2\Bigg\} + \frac{1}{c^3}\Bigg\{\dot\varphi\left(1+3\chi_1^2\right)\Bigg[-4m_1\chi_1(1+\sigma_1) + \frac{1}{3}m_1\chi_1 - \frac{7}{2}m_2\chi_2 \\
&-8m_1B_2(\chi_1)\Bigg] - 16m_1\chi_1\dot\varphi(1+\sigma_1) - \frac{5}{6}\dot\varphi\left(2m_1\chi_1+3m_2\chi_2\right) - 16m_1\left(1+3\chi_1^2\right)\frac{\dot r}{r}\Bigg\}\Bigg\}.
\end{aligned}
$$

- Factorization with $\Omega_{\rm H}$ similar to circular case
- Mass/energy and angular momentum flux decoupled ($\dot m \neq \Omega \dot S$)
- Raw PN leads to strange phenomenology on scattering dynamics
- Order of magnitude of NR data reproduced though



EOB/PN vs NR spin-up



- **Future:**

More NR comparisons

Compute horizon flux in BHPT and compare

Explore resummation for use in dynamics → Circular limit; → Generic orbits

THE END