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Hadron structure - from 1D to 3D

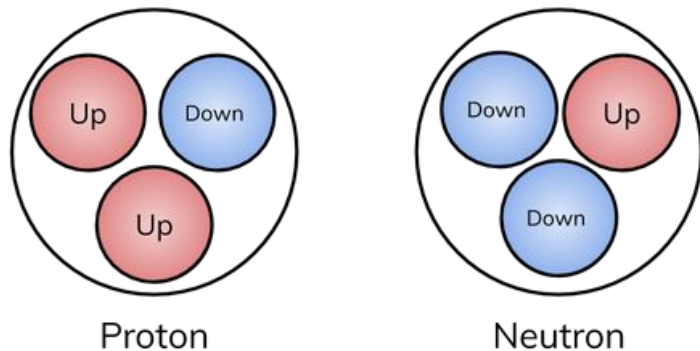
Yiyu Zhou

Department of Physics

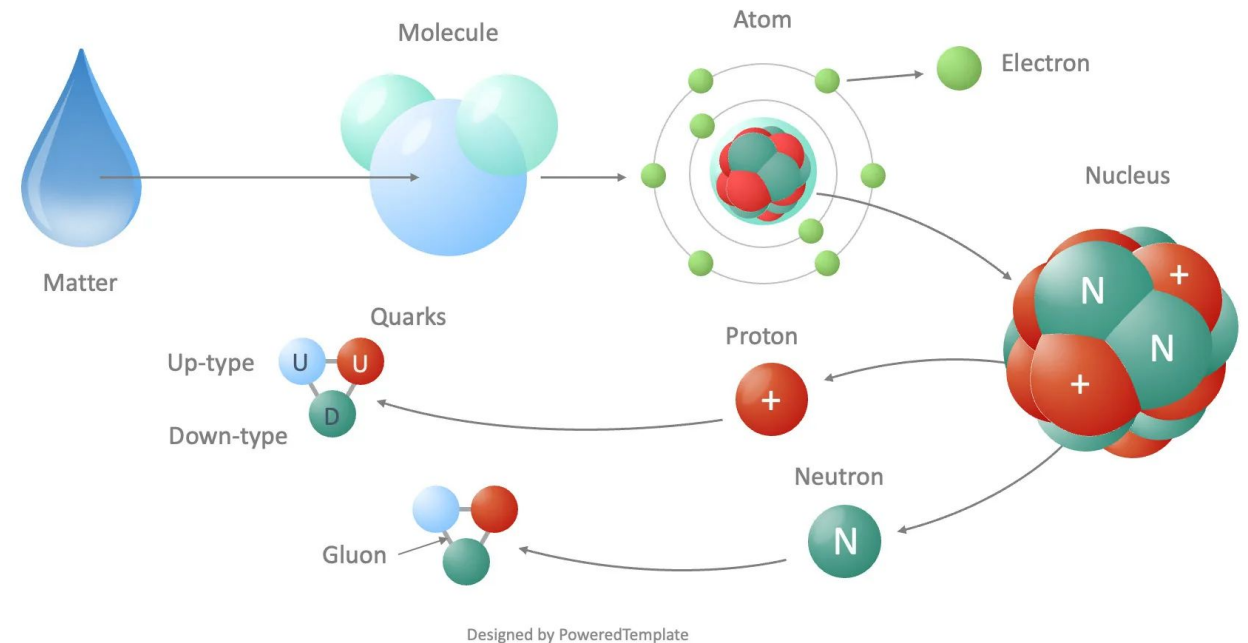
Motivation

Proton structure:

- **Mass:** how do quarks (2.3~4.8 MeV) and gluons make up the mass of proton (938 MeV)?
- **Spin:** how do quarks and gluons make up the proton spin of $1/2$?

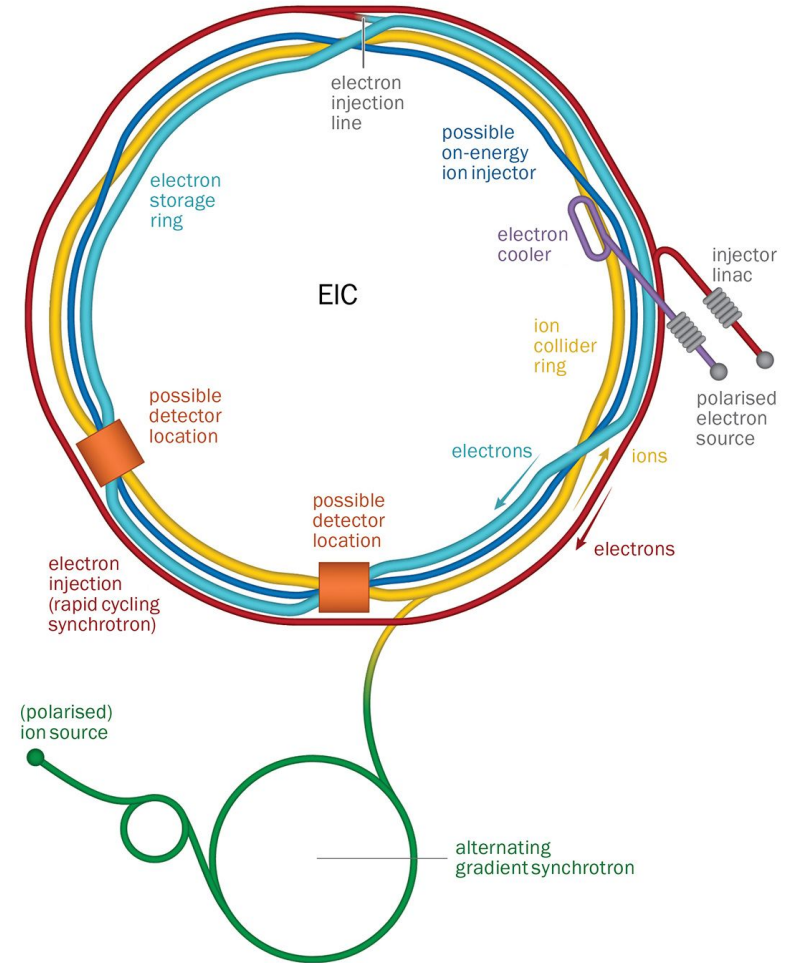
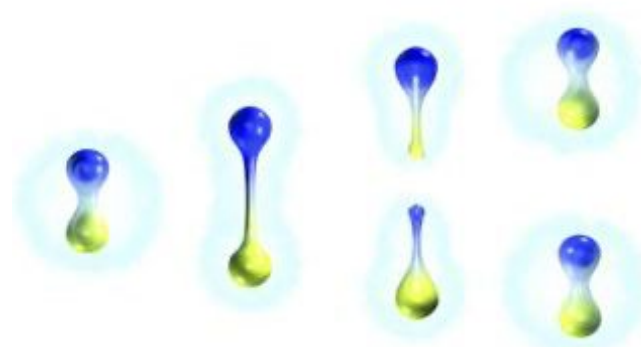
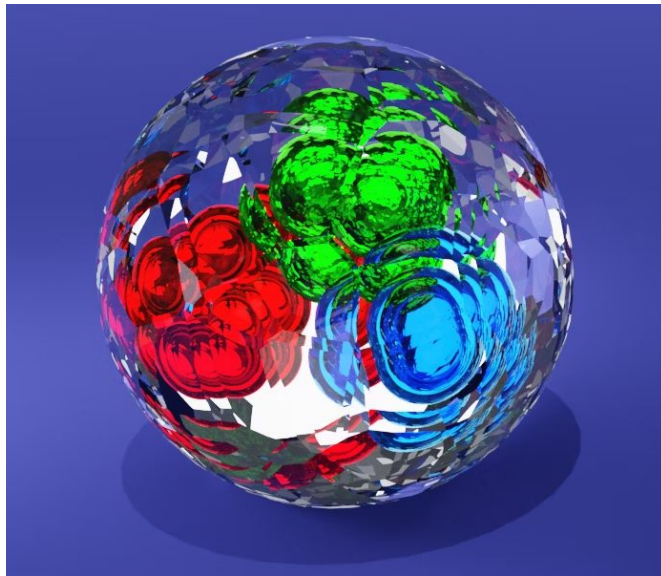


Matter from Molecule to Quark

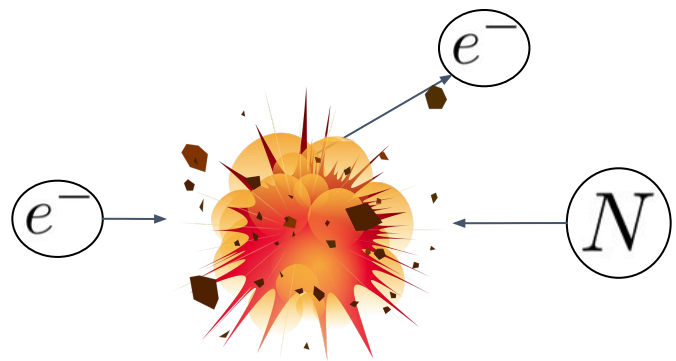


Probe proton structure

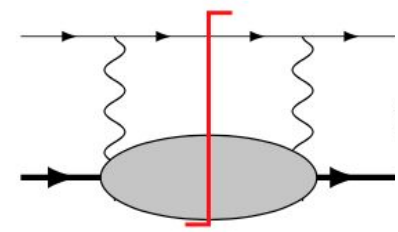
- Confinement: extraction of information is indirect
- High energy collision: send in a probe and measure the outcome



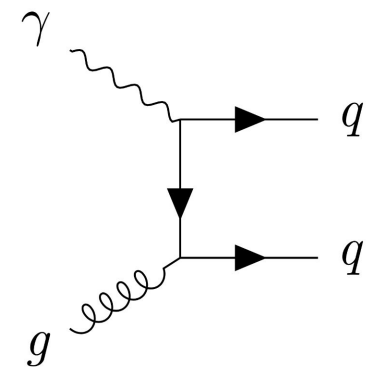
Deep-inelastic scattering (DIS)



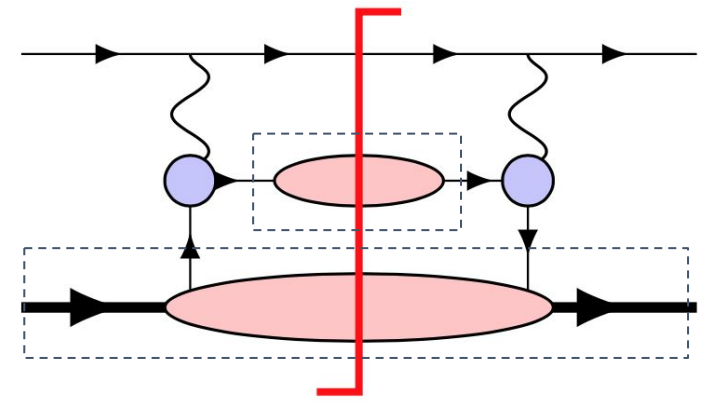
$$E' \frac{d\sigma}{d^3l'}$$



Approximations



$$Q^2 \gg m^2$$



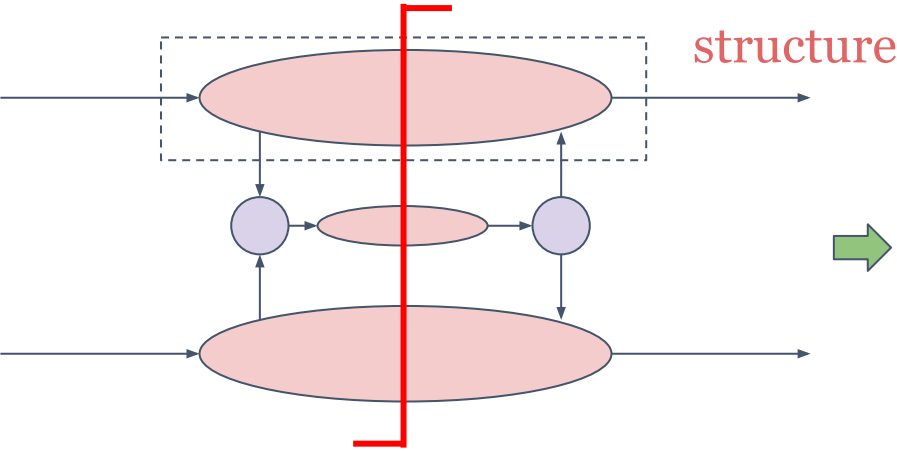
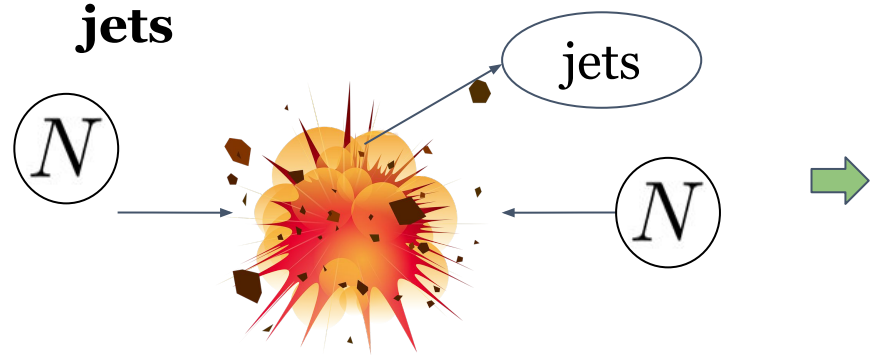
Collision dependent factor

Internal structure

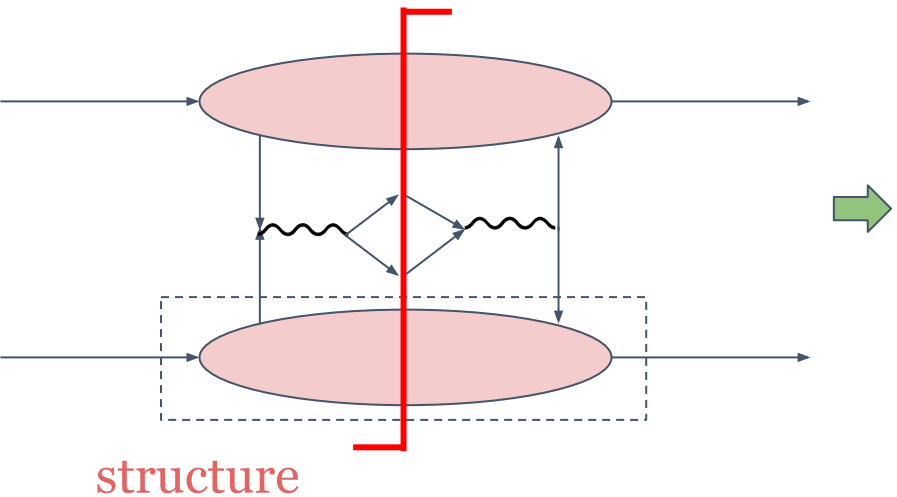
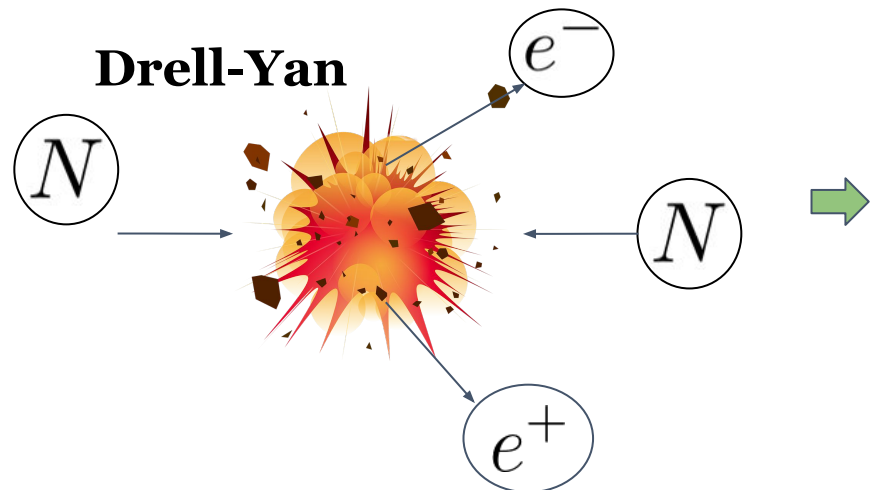
$$\sum_i \int_x^1 \frac{d\xi}{\xi} H_i(\xi) f_i\left(\frac{x}{\xi}\right) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

Error of approximations

Jets and Drell-Yan



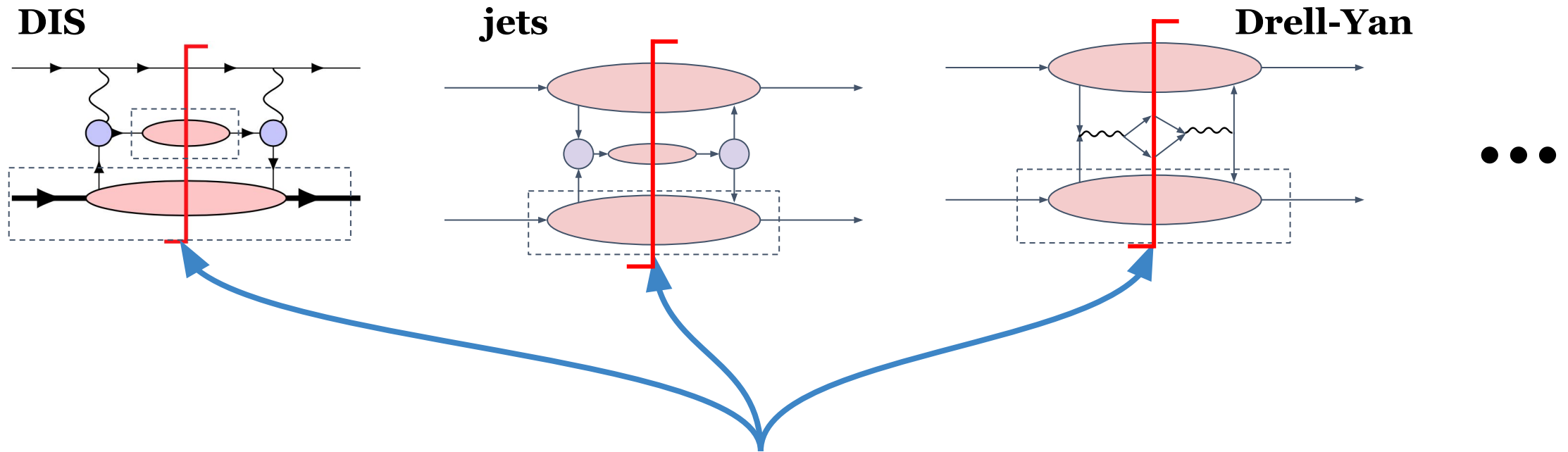
$\Rightarrow d\sigma^{\text{jets}} = \sum_{i,j} H_{ij}^{\text{jets}} \otimes f_i \otimes f_j$



$\Rightarrow d\sigma^{\text{DY}} = \sum_{i,j} H_{ij}^{\text{DY}} \otimes f_i \otimes f_j$

and many more...

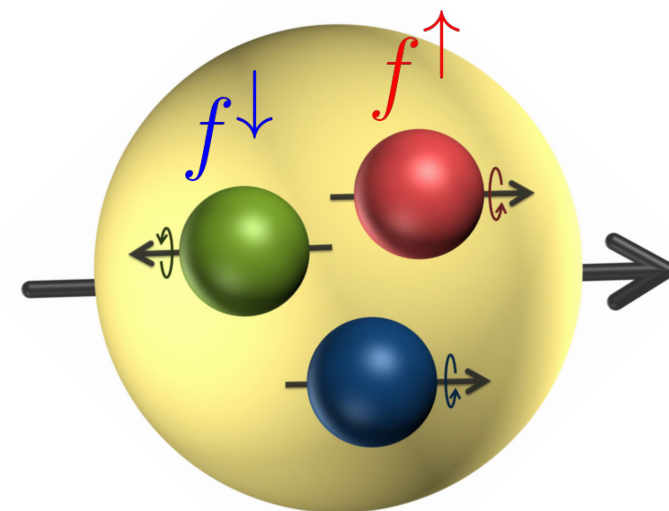
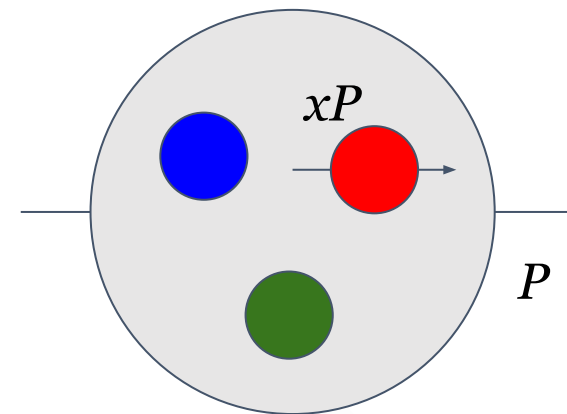
Universality



cross sections described by
universal nonperturbative
functions, *e.g.*, PDFs

Parton distribution functions - 1D

- Probability to find a quark i or a gluon g in a hadron h carrying a fraction x of the hadron's momentum.
- A function of x ($0 < x < 1$) and μ (factorization scale), flavor of partons.
- spin-averaged (unpolarized): $f = f^\uparrow + f^\downarrow$
- spin-dependent (polarized): $\Delta f = f^\uparrow - f^\downarrow$

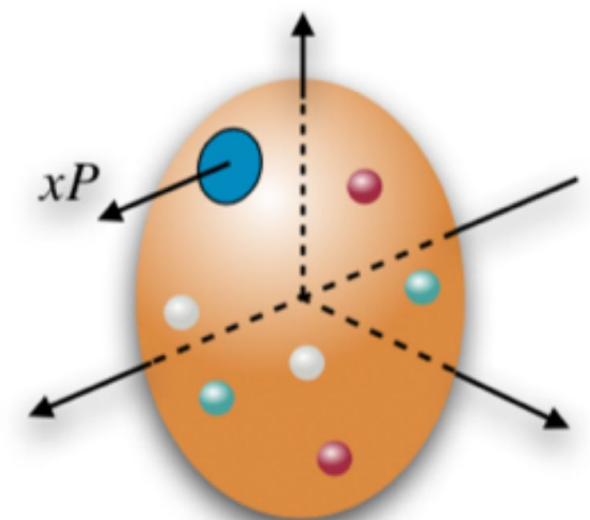
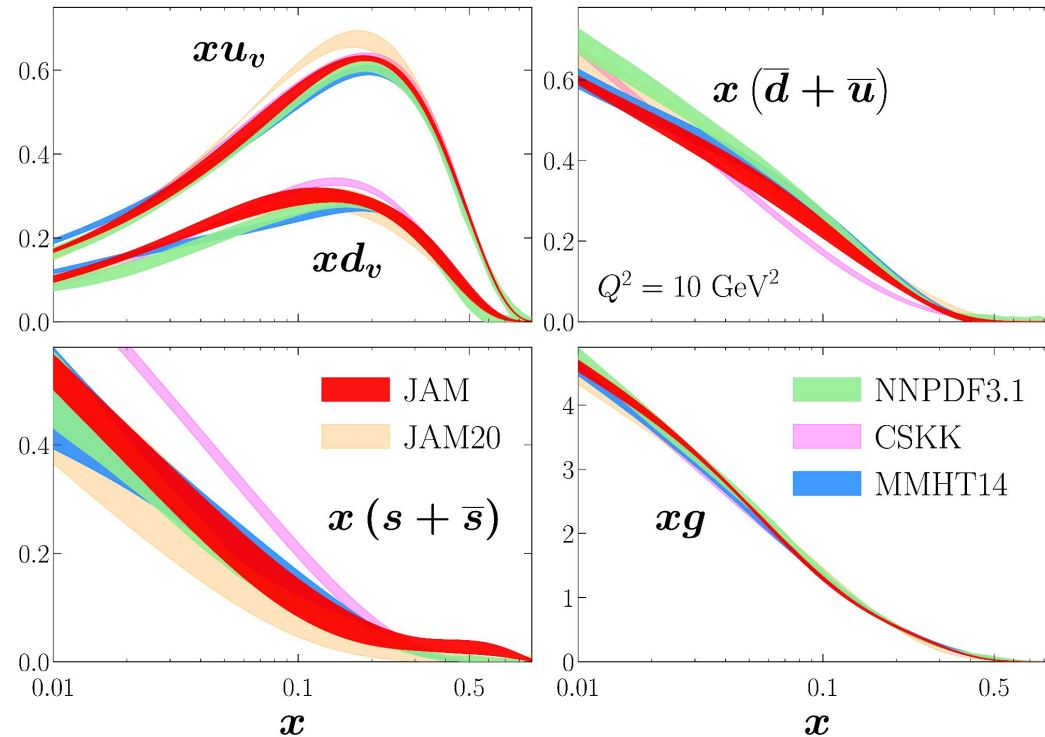


Proton 1D structure



- 1D unpolarized parton distribution functions (PDFs)
 - probability of finding a parton inside a proton with momentum fraction x

- $u_v = u - \bar{u}$
- $d_v = d - \bar{d}$
- sea: \bar{u} , \bar{d} , s , \bar{s} ...

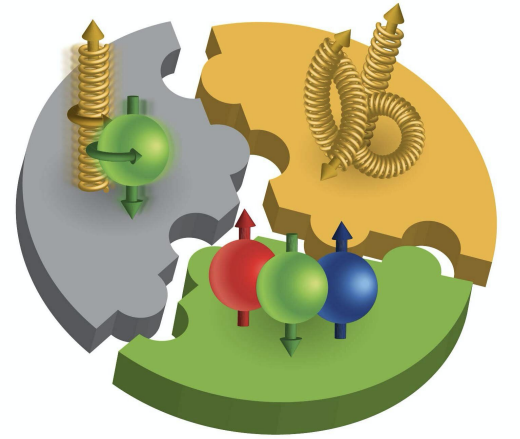


What about the spin structure?

Proton 1D structure - spin dependent

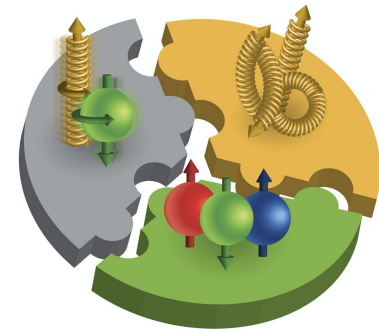
What is the decomposition of the proton spin?

- current extraction of $\Delta\Sigma$ is around 0.3 (contribution from quarks)
- spin can be extracted from parton distribution functions (PDFs)
- orbital angular momentum can be extracted from GPDs



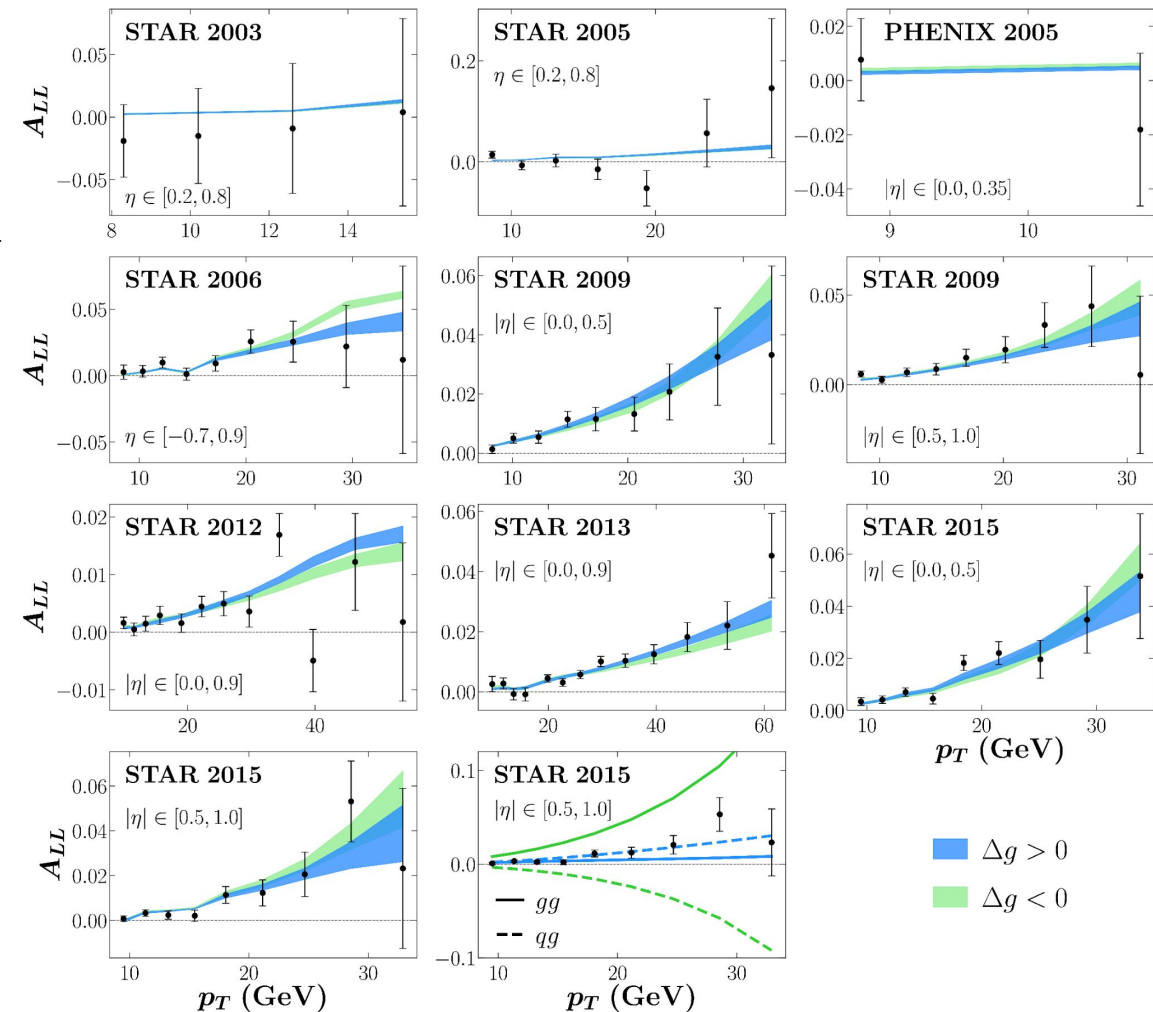
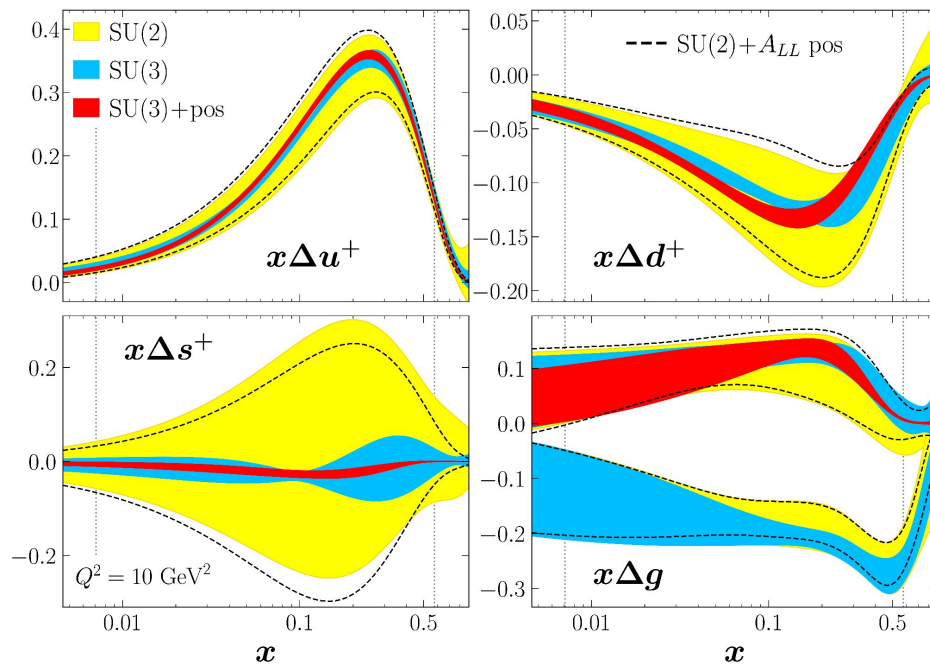
$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + L_q + \Delta G + L_g$$

Gluon polarization - sign? $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$



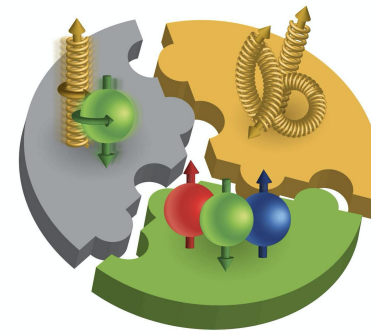
- 1D helicity distribution

- STAR experiment $pp \rightarrow \text{jet}$ is sensitive to Δg
- Δg gives the gluon contribution to proton spin

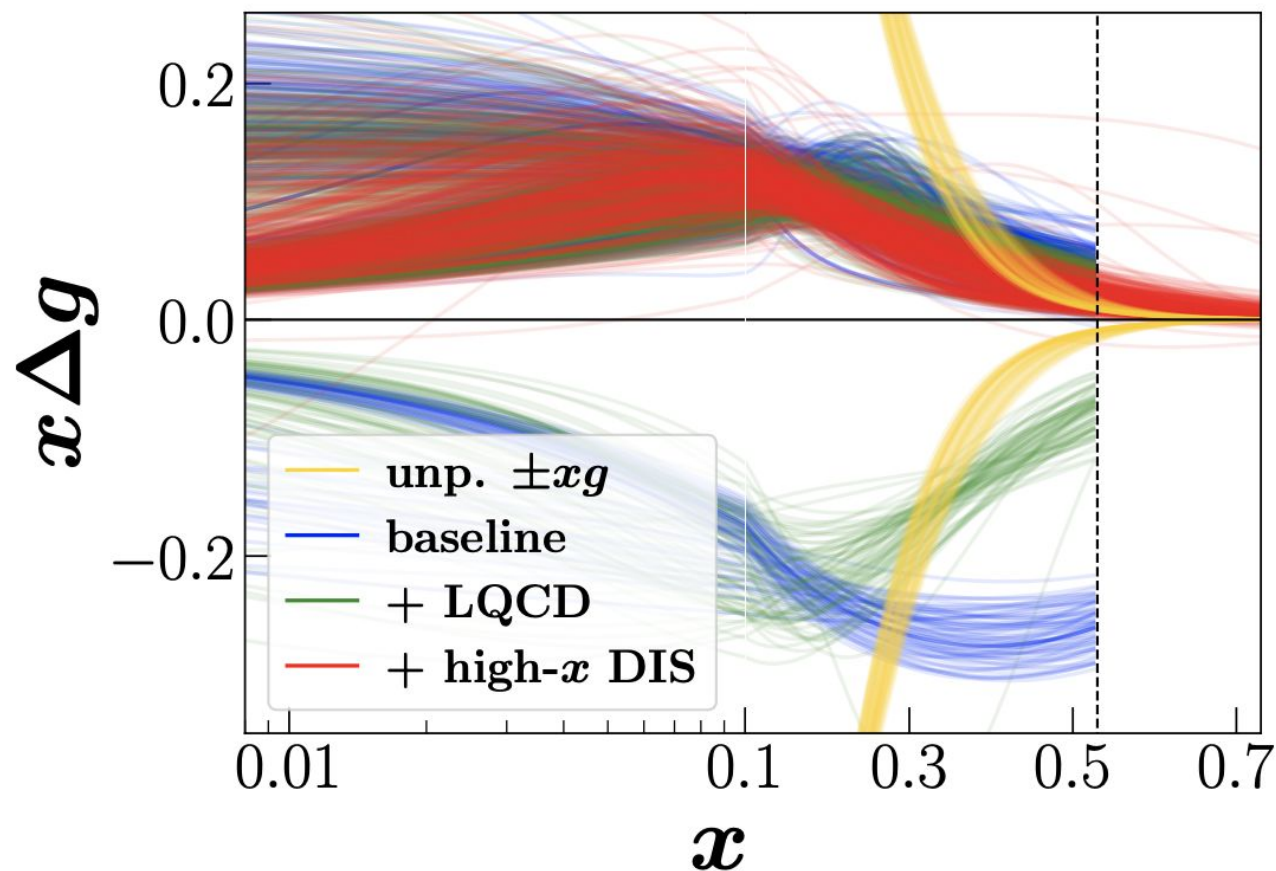
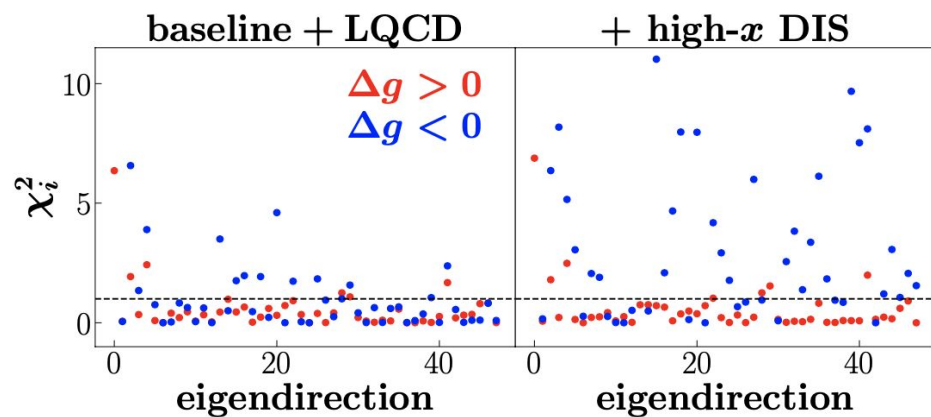


Gluon polarization - sign?

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$



- 1D helicity distribution
 - negative gluon ruled out with Lattice & high- x data

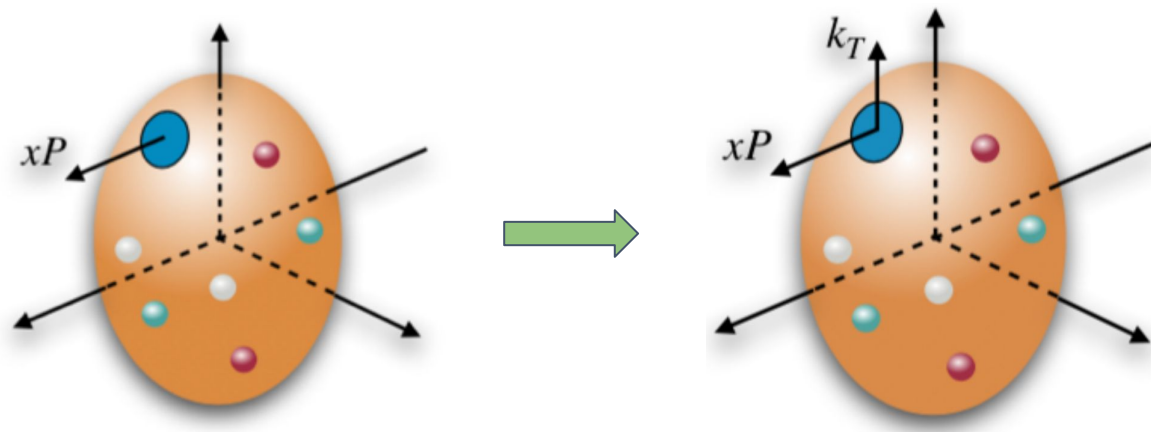


Proton structure in 3D

3D structure of hadrons

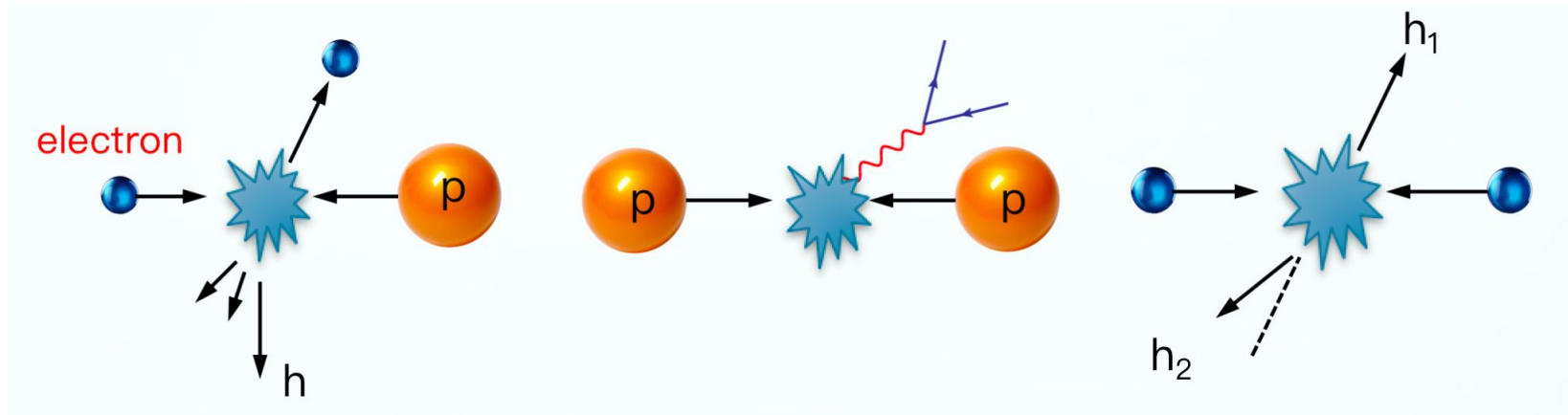
Beyond 1D:

- TMD: transverse momentum dependence (3D structure in momentum space)
- GPD: generalized parton distributions (3D structure in position space)

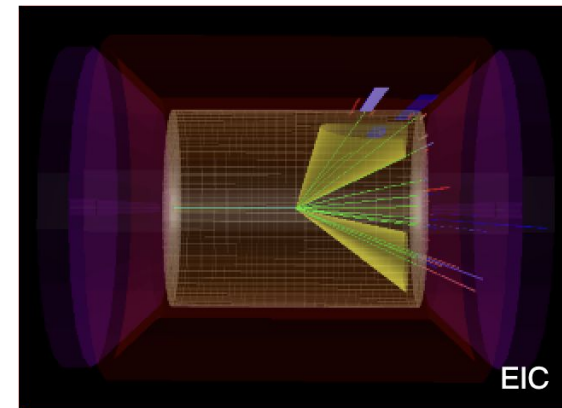
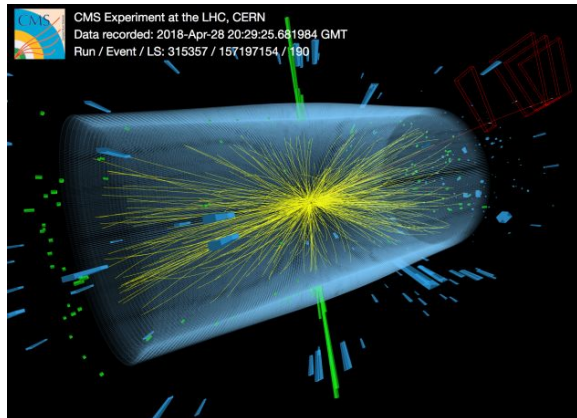


Processes to extract TMDs

- Standard processes: SIDIS, Drell-Yan, e^+e^-

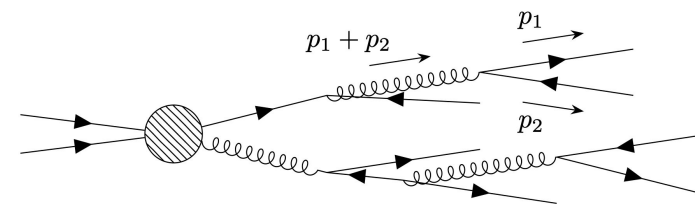


- Focus of this talk: jets for 3D imaging

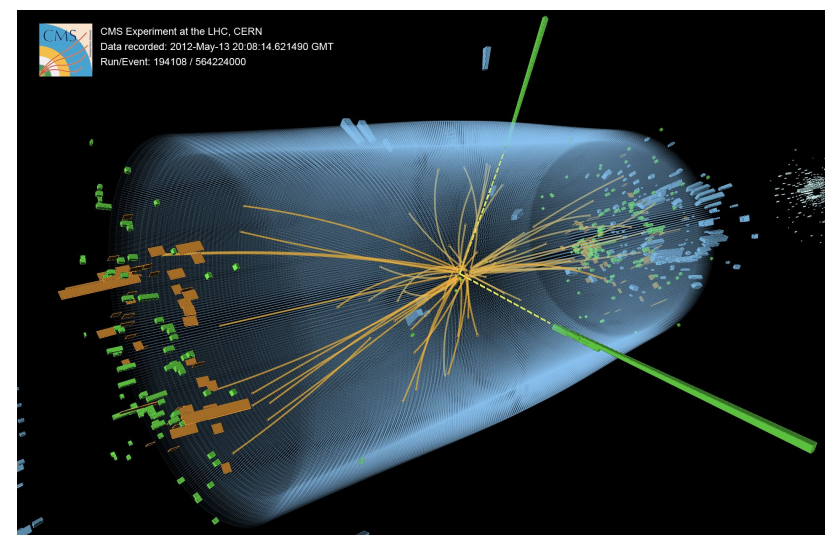


Jets as probes of hadron structure

- A jet is collimated spray of hadrons in high-energy particle reactions.
- Jets are manifestations of collinear enhancement.
- Probes hadron structure (PDFs) without needing to know about details of hadronization (fragmentation functions), in contrast to SIDIS.
- Jets are defined up to specific clustering algorithms (cone, k_T , anti- k_T ...).



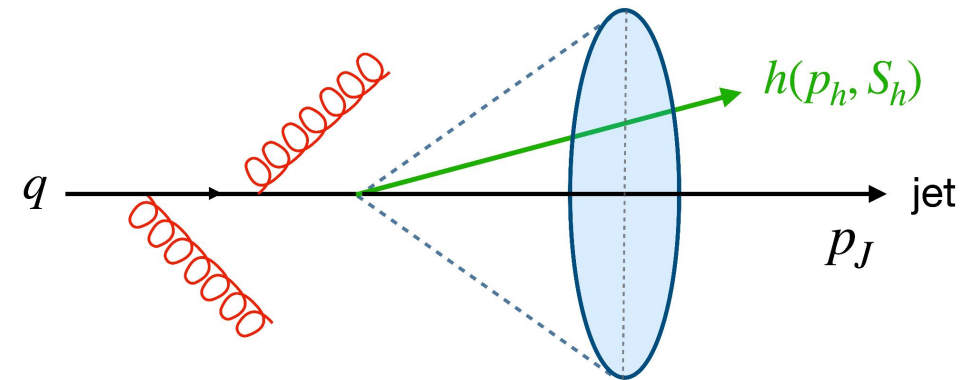
$$\frac{1}{(p_1 + p_2)^2} = \frac{1}{2E_1 E_2 (1 - \cos \theta)}$$



Jet substructure

Suppose one measures a hadron that is produced within the jet,
one can measure:

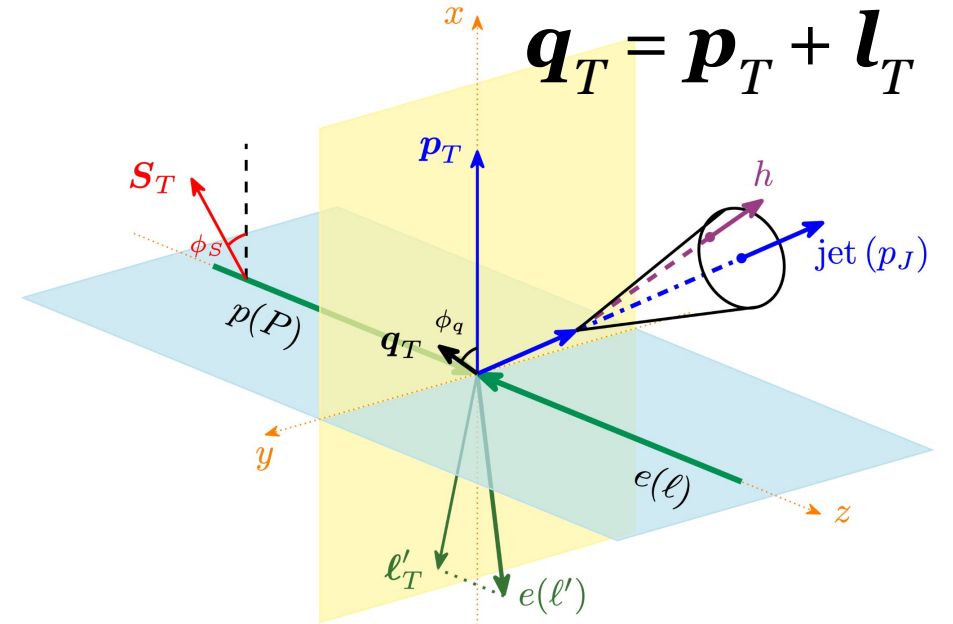
- collinear momentum fraction z_h distribution
- hadron transverse momentum \mathbf{j}_\perp with respect to jet axis
(sensitive to TMD FFs)



Exclusive jet production

- Momentum imbalance \mathbf{q}_T : sensitive to initial-state TMD distributions
- Hadron \mathbf{j}_\perp : sensitive to TMD FFs

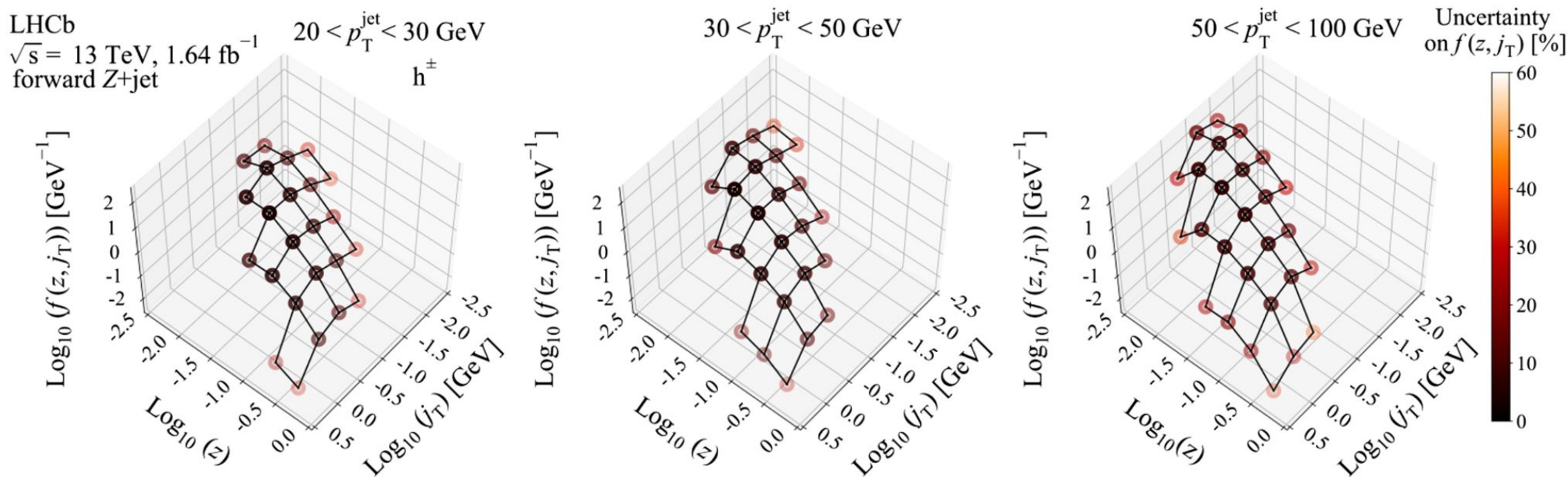
$$\frac{d\sigma_{pp}}{d\mathcal{PS}} = \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{q}_T \cdot \mathbf{b}} \tilde{f}_a^{q/p}(x_a, b) \tilde{f}_b^{q/p}(x_b, b) \\ \times \tilde{S}_{n\bar{n}n_J}(\mathbf{b}) \tilde{S}_{n_J}^{cs}(\mathbf{b}, R) H_{ab \rightarrow cZ}(p_T, m_Z) J_c(p_{JT} R)$$



- Kang, Lee, Shao & Zhao: [2106.15624](#)
- Kang, Lee, Xing, Zhao & Zhou: 2411.XXXX

Exclusive jet production: $pp \rightarrow Z + \text{jet}(h)$

- Recent measurement by LHCb ([2208.11691](#))
- First time differential in both z_h and \mathbf{j}_\perp (proposed in [1906.07187](#))



Exclusive jet production: $pp \rightarrow Z + \text{jet}(h)$

- Fragmentation functions ([2101.04664](#), [2202.03372](#))
- Data included: e^+e^- , SIDIS, polarized SIDIS

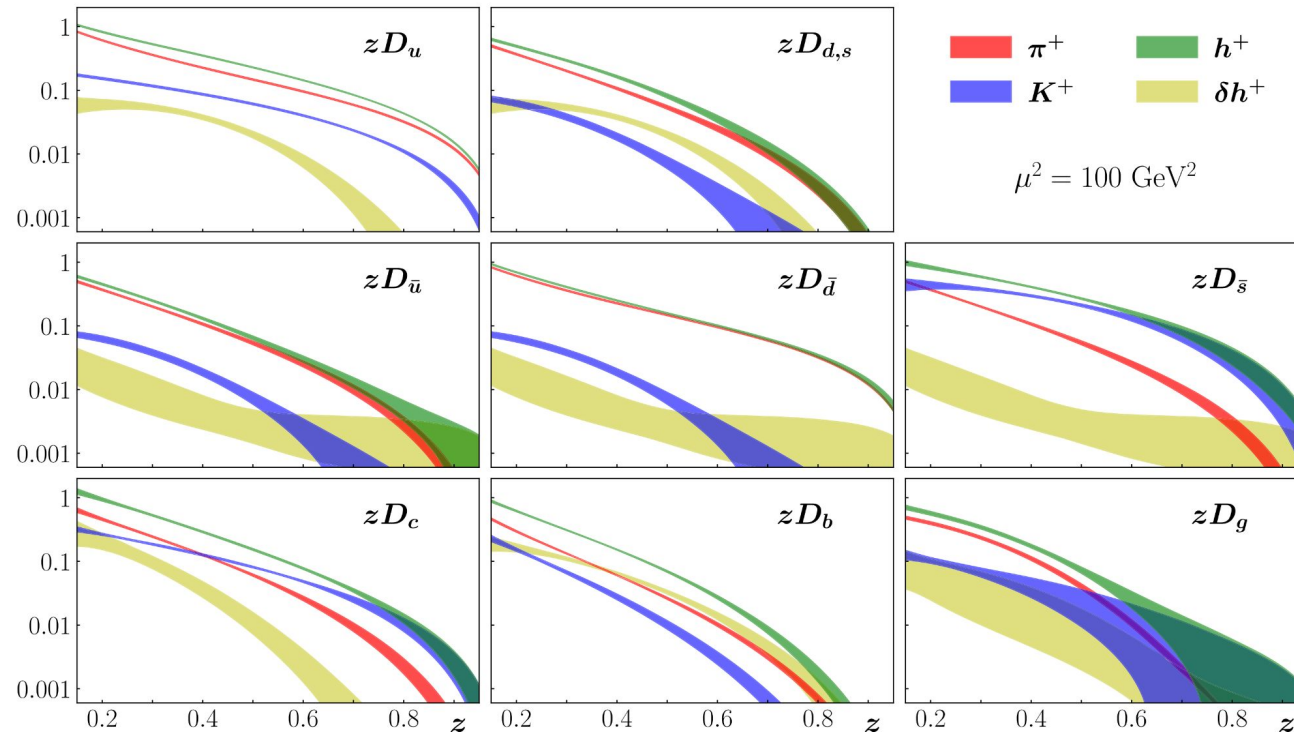
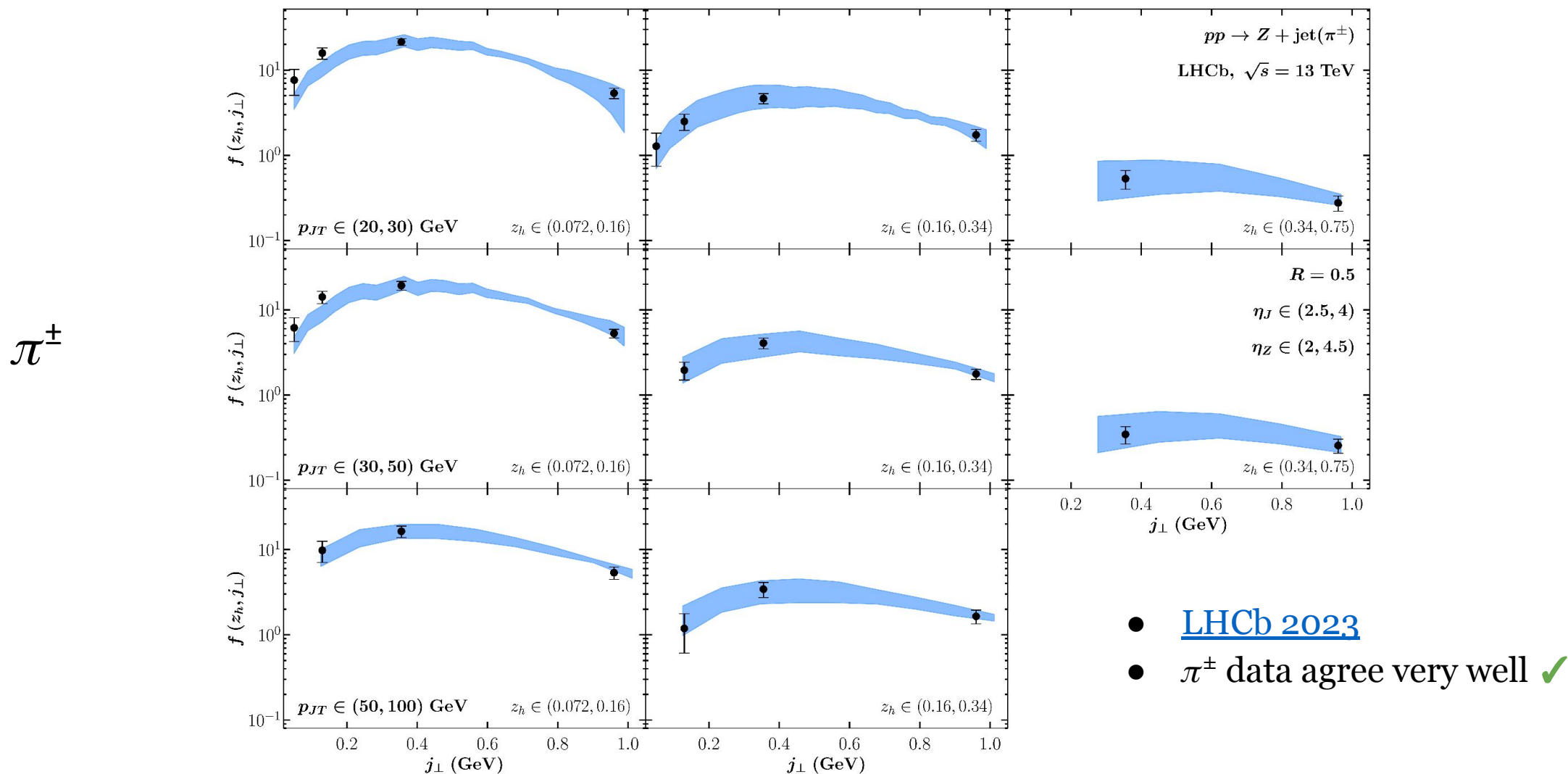


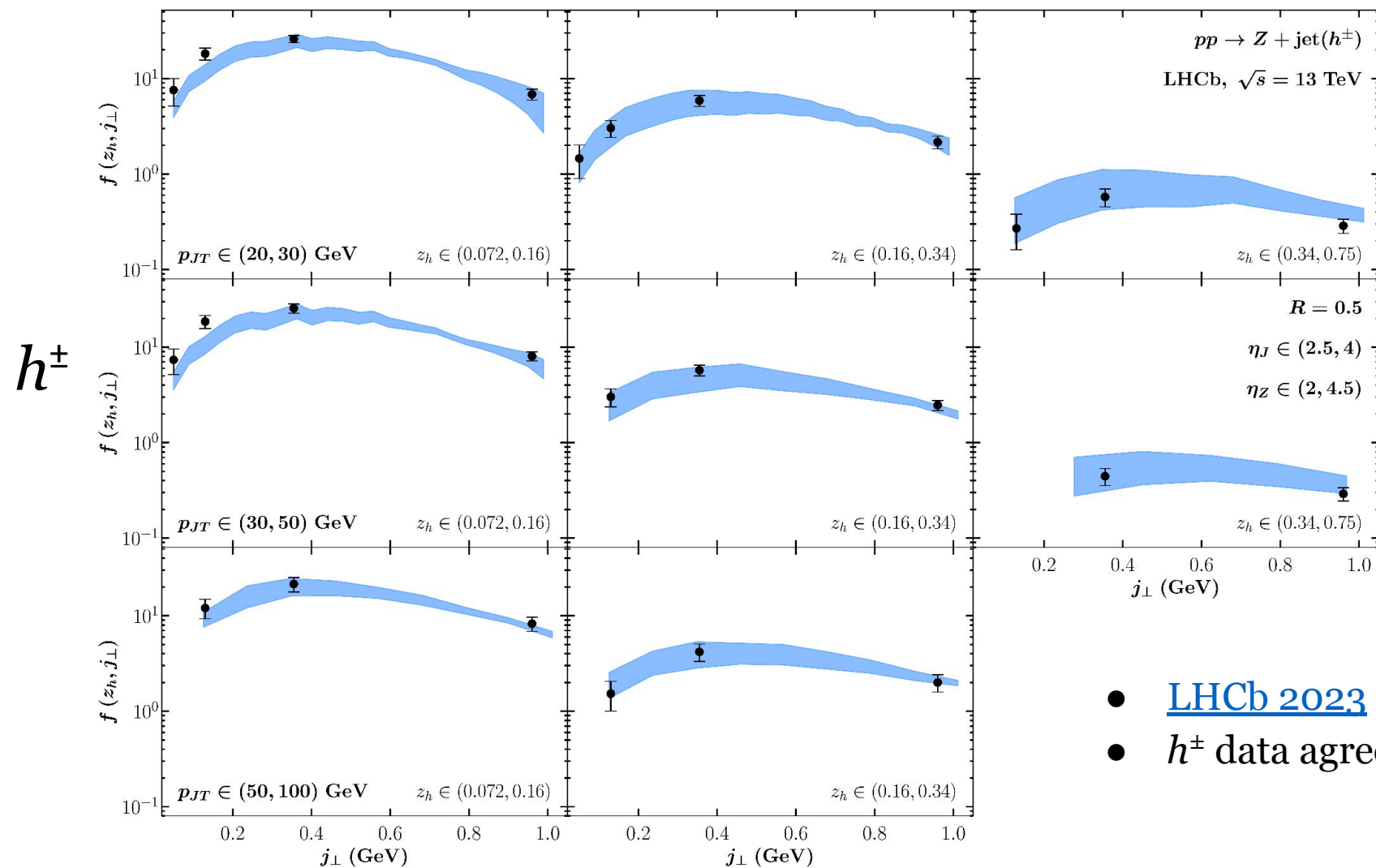
TABLE I. Summary of χ^2 values per number of points N_{dat} for the various datasets used in this analysis.

Process	N_{dat}	χ^2/N_{dat}
<i>Polarized</i>		
Inclusive DIS	365	0.95
SIDIS (π^+, π^-)	64	1.05
SIDIS (K^+, K^-)	57	0.42
SIDIS (h^+, h^-)	110	0.95
Inclusive jets	83	0.84
STAR W^\pm	12	0.65
PHENIX W^\pm/Z	6	0.50
Total	697	0.89
<i>Unpolarized</i>		
Inclusive DIS	3908	1.17
SIDIS (π^+, π^-)	498	0.94
SIDIS (K^+, K^-)	494	1.31
SIDIS (h^+, h^-)	498	0.71
Inclusive jets	198	1.28
Drell-Yan	205	1.21
W/Z production	153	1.01
Total	5954	1.12
SIA (π^\pm)	231	0.91
SIA (K^\pm)	213	0.70
SIA (h^\pm)	120	1.07
Total	7215	1.08

Exclusive jet production: $pp \rightarrow Z + \text{jet}(h)$



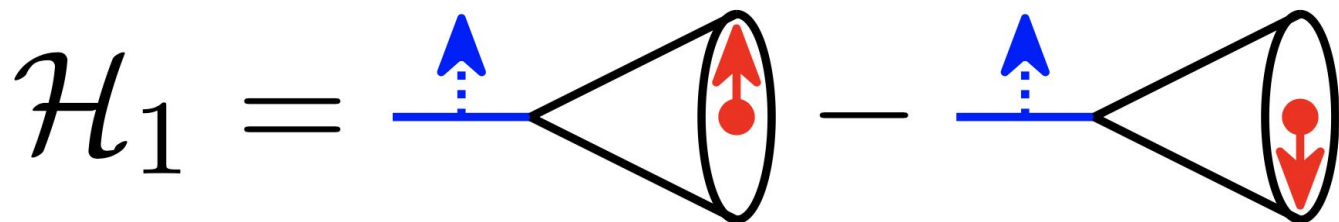
Exclusive jet production: $pp \rightarrow Z + \text{jet}(h)$



- [LHCb 2023](#)
- h^\pm data agree very well ✓

Single inclusive jet production

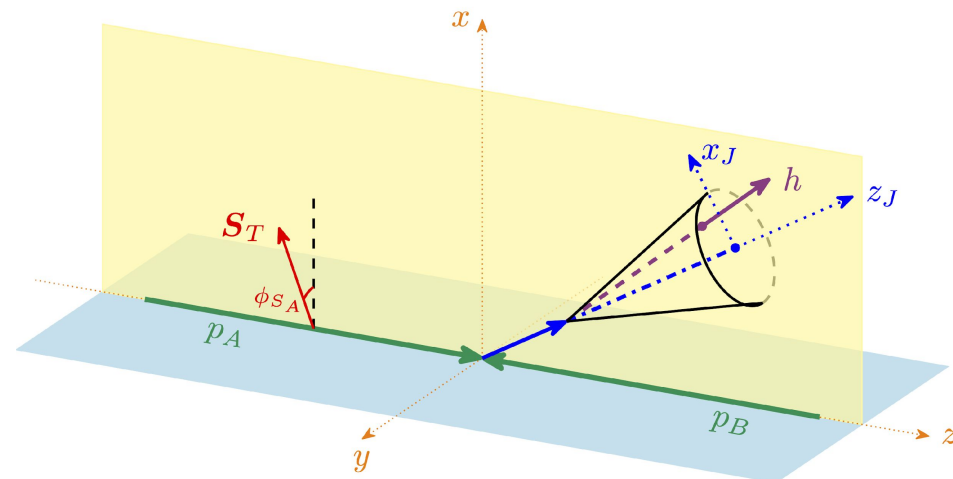
- Collinear PDFs: only one scale p_T is measured.
- TMD FFs: when hadron transverse momentum distribution is measured.



$$\frac{d\sigma^{pp \rightarrow \text{jet}(h)+X}}{dp_T d\eta dz_h} \propto f_a \otimes f_b \otimes H_{ab \rightarrow c} \otimes \mathcal{D}_1^{h/c}(z, z_h, p_T R, \mu),$$

$$\frac{d\sigma^{pp \rightarrow \text{jet}(h)+X}}{dp_T d\eta dz_h d^2\mathbf{j}_\perp} \propto f_a \otimes f_b \otimes H_{ab \rightarrow c} \otimes \mathcal{G}_1^{h/c}(z, z_h, \mathbf{j}_\perp, p_T R, \mu, \zeta_J),$$

Kang, Ringer & Vitev: 16; Dai, Kim & Leibovich: 16; Kaufmann, Mukherjee & Vogelsang: 15

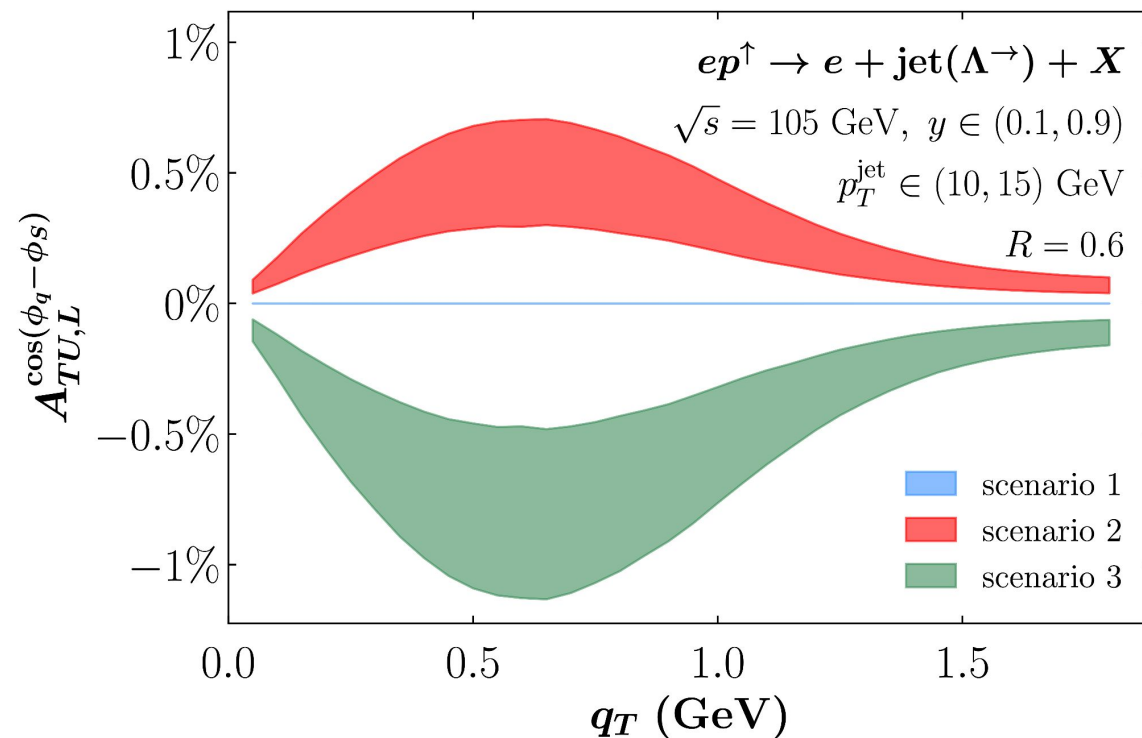
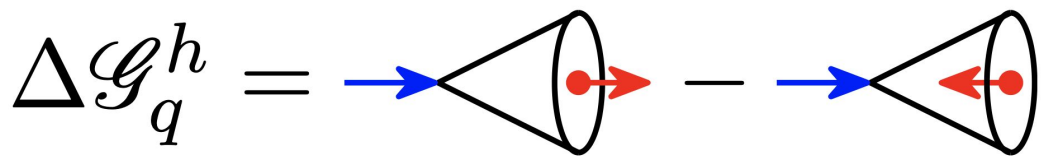
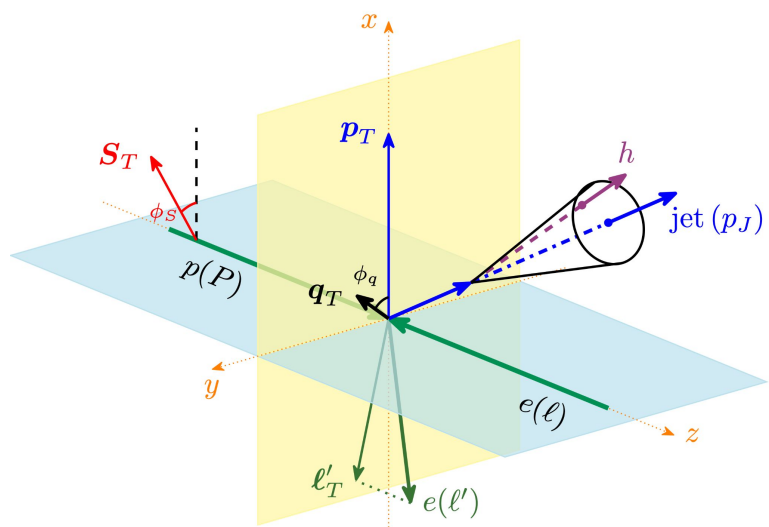


- z_h : large momentum fraction of hadron v.s. jet
- \mathbf{j}_\perp : hadron transverse momentum w.r.t. jet axis

Exclusive jet production: $ep \rightarrow e + \text{jet}(h)$

- worm-gear function
- longitudinally polarized FFs

$$A_{TU,L}(z_h, j_\perp) \equiv \frac{\hat{\sigma}_0 H \mathcal{G}_{1L} \tilde{g}_{1T} \bar{S}_{\text{global}} \bar{S}_{\text{cs}}}{\hat{\sigma}_0 H \mathcal{D}_1 \tilde{f}_1 \bar{S}_{\text{global}} \bar{S}_{\text{cs}}}$$



Relation between TMD FFs and TMD FJFs

If you measure both z_h and \mathbf{j}_\perp

Leading Quark TMDFFs  Hadron Spin  Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Polarized Hadrons	L		$G_1 = \text{Hadron Spin} \rightarrow - \text{Hadron Spin} \rightarrow$ Helicity	$H_{1L}^\perp = \text{Quark Spin} \rightarrow - \text{Quark Spin} \rightarrow$
	T	$D_{1T}^\perp = \text{Quark Spin} \uparrow - \text{Quark Spin} \downarrow$ Polarizing FF	$G_{1T}^\perp = \text{Hadron Spin} \uparrow - \text{Hadron Spin} \downarrow$	$H_{1T}^\perp = \text{Quark Spin} \uparrow - \text{Quark Spin} \downarrow$ Transversity
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{Unpolarized}$		$H_1^\perp = \text{Collins}$

[TMD handbook, 2304.03302](#)

		Quark polarization		
		U	L	T
Hadron polarization	U	$\mathcal{D}_1 = \text{Hadron Spin} \rightarrow$		$\mathcal{H}_1^\perp = \text{Quark Spin} \uparrow - \text{Quark Spin} \downarrow$
	L		$\mathcal{G}_{1L} = \text{Hadron Spin} \rightarrow - \text{Hadron Spin} \rightarrow$	$\mathcal{H}_{1L}^\perp = \text{Quark Spin} \uparrow - \text{Quark Spin} \downarrow$
	T	$\mathcal{D}_{1T}^\perp = \text{Quark Spin} \uparrow - \text{Quark Spin} \downarrow$	$\mathcal{G}_{1T} = \text{Hadron Spin} \rightarrow - \text{Hadron Spin} \rightarrow$	$\mathcal{H}_{1T}^\perp = \text{Quark Spin} \uparrow - \text{Quark Spin} \downarrow$

[Kang, Xing, Zhao and Zhou, 2311.00672](#)

How do we connect them?

Summary

- Hadron 1D structure:
 - unpolarized PDFs – proton collinear momentum distribution
 - polarized PDFs – proton spin decomposition
- Hadron 3D structure:
 - transverse momentum dependent distributions – more detailed mapping of hadron structure
 - jet substructure – polarization opens up a wide range of possibilities

Thanks for your attention!