



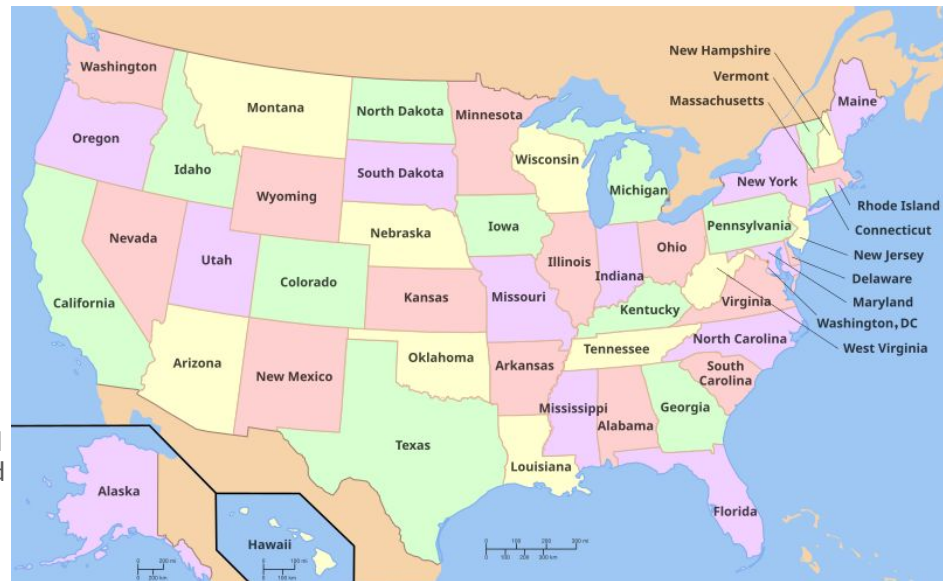
Understanding our ability to infer the properties of binary coalescences via gravitational wave radiation

Jacob Lange with many other collaborators
INFN, Sezione di Torino
November 2nd 2024, 10:30am

Outline



- Born in Tampa, Florida, USA
- Graduated with B.S. from Florida Institute of Technology in Melbourne, Florida in 2014
- Obtained M.S. (2016) and PhD (2020) from Rochester Institute of Technology in Rochester, New York
- Semester-long postdoc at Institute for Computational and Experimental Research in Mathematics, Brown University in Providence, Rhode Island
- Postdoc at University of Texas at Austin in Austin, Texas



Data Analysis Flowchart

Observation



Data Analysis Flowchart

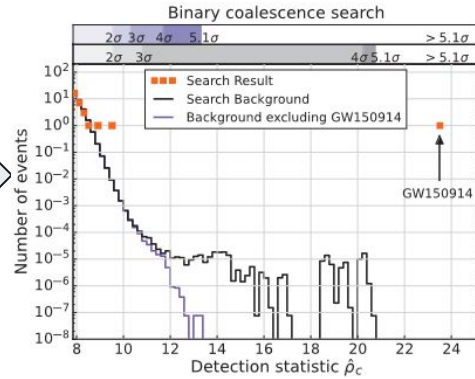


Observation



Waveforms required (for modeled searches) ✓

Searches: finding detections



Data Analysis Flowchart

Observation



Waveforms required (for modeled searches) ✓

Searches: finding detections

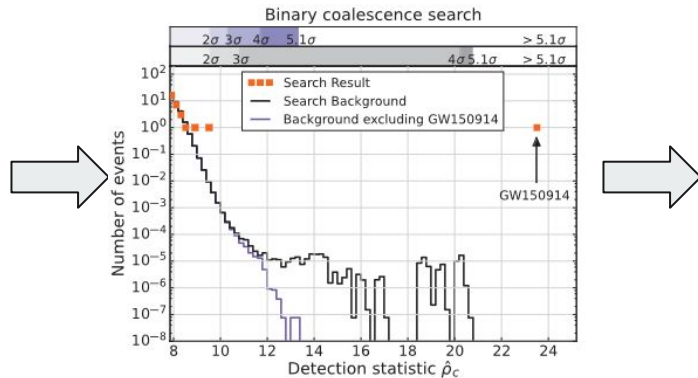


Figure from [14]

Waveforms required ✓

Parameter Estimation: inferring properties (masses, spins, etc.)

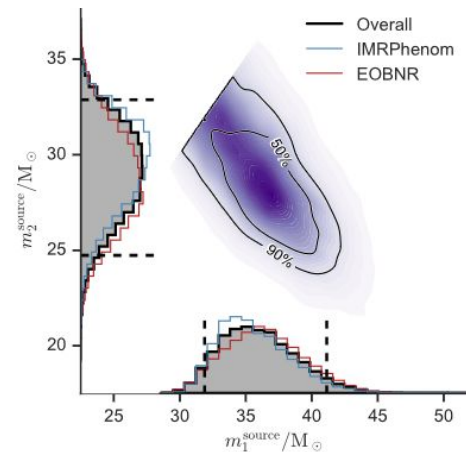


Figure from [15]

Data Analysis Flowchart

Observation



Waveforms required (for modeled searches) ✓

Searches: finding detections

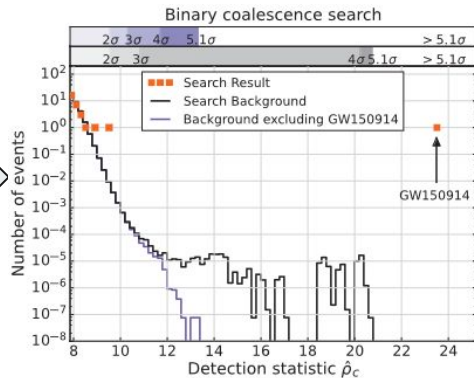


Figure from [14]

Waveforms required ✓

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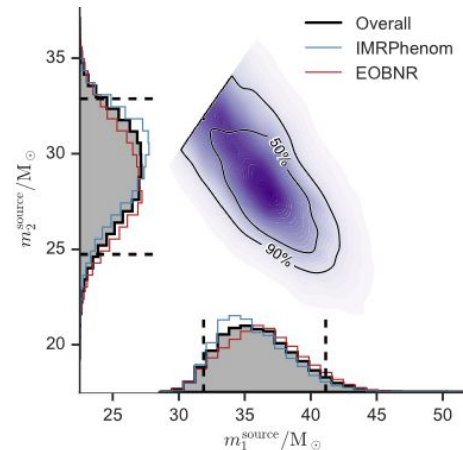


Figure from [15]

Waveforms required

Tests of General Relativity

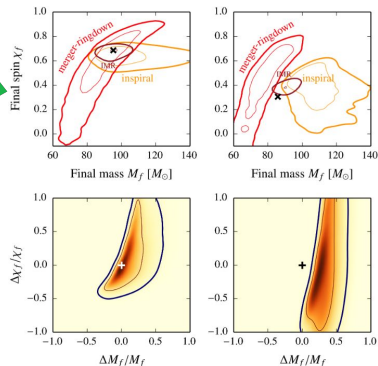


Figure from [16]

Data Analysis Flowchart

Observation



Waveforms required (for modeled searches) ✓

Searches: finding detections

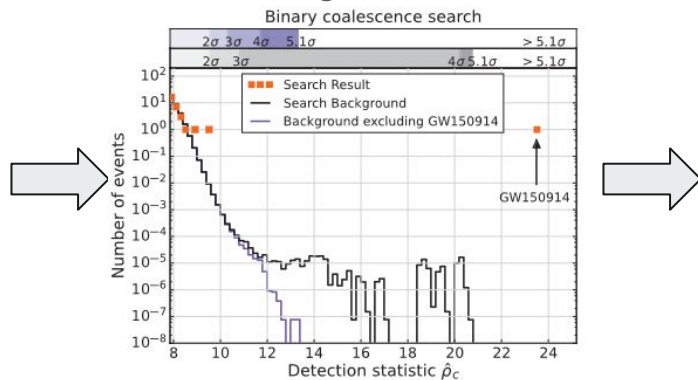


Figure from [14]

Waveforms required ✓

Parameter Estimation: inferring properties (masses, spins, etc.)

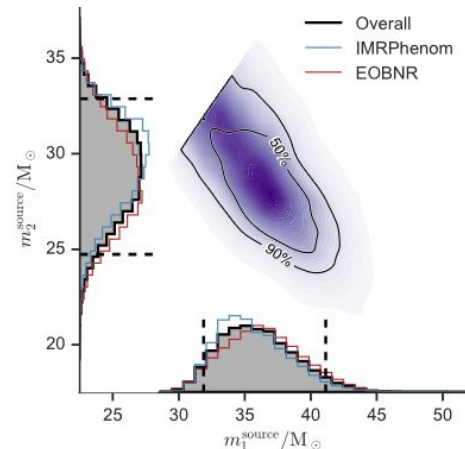


Figure from [15]

Waveforms required (for modeled searches) ✓

Tests of General Relativity

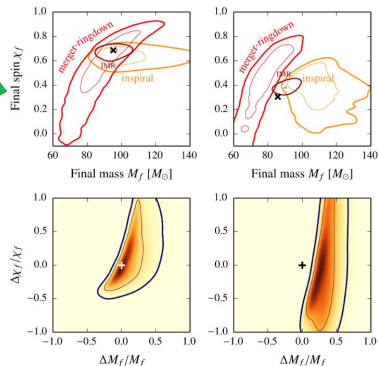


Figure from [16]

Accurate and robust waveforms (models and NR) are required for almost all GW data analyses

Data Analysis Flowchart

Observation



Waveforms required (for modeled searches) ✓

Searches: finding detections

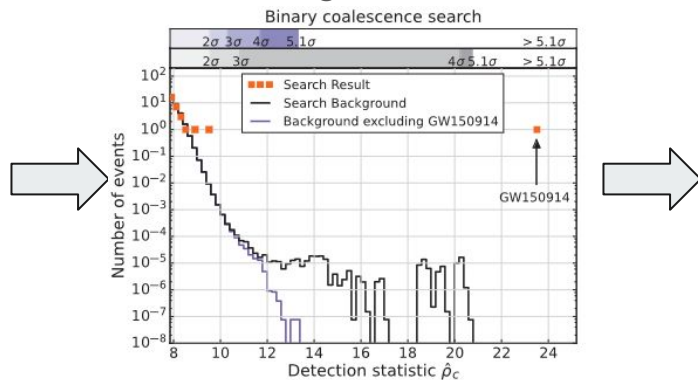


Figure from [14]

Waveforms required ✓

Parameter Estimation: inferring properties (masses, spins, etc.)

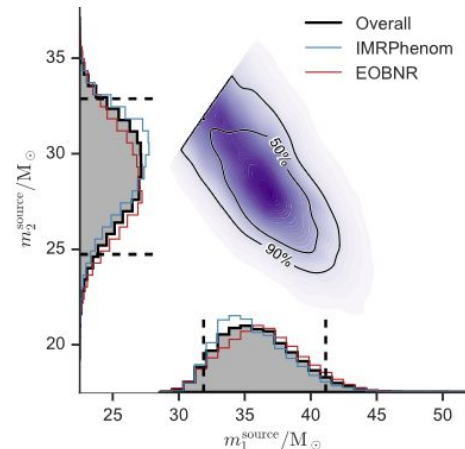


Figure from [15]

Waveforms required (for modeled searches) ✓

Tests of General Relativity

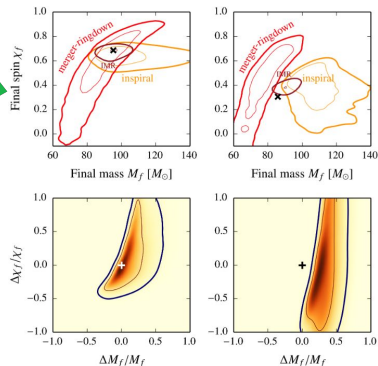


Figure from [16]

Accurate and robust waveforms (models and NR) are required for almost all GW data analyses

Parameter Estimation

$$p(\theta|d) = \frac{\mathcal{L}(d|\theta) \pi(\theta)}{\mathcal{Z}},$$

RIFT'ing the Waves: Developing and applying an algorithm to infer properties of gravitational wave sources

Jacob Lange

Initial Grid

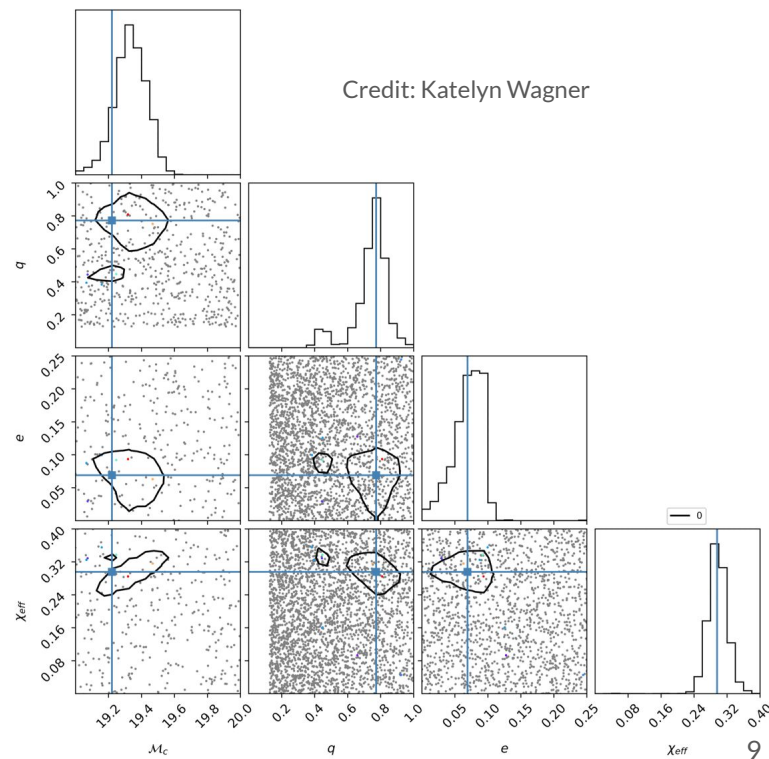
Integrate Likelihood Extrinsic (ILE)

- Compute Marginalized likelihood $\{\lambda_\alpha, L_{\text{marg},\alpha}\}$

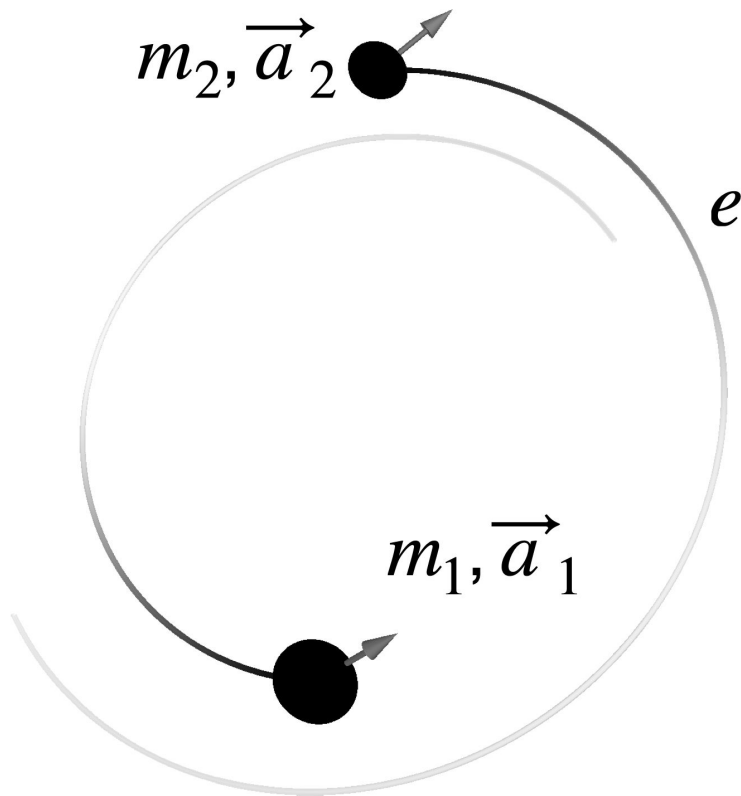
Construct Intrinsic Posterior (CIP)

- Fit marginalized likelihood
- Construct posterior
- Fairdraw samples

Converged or Repeat

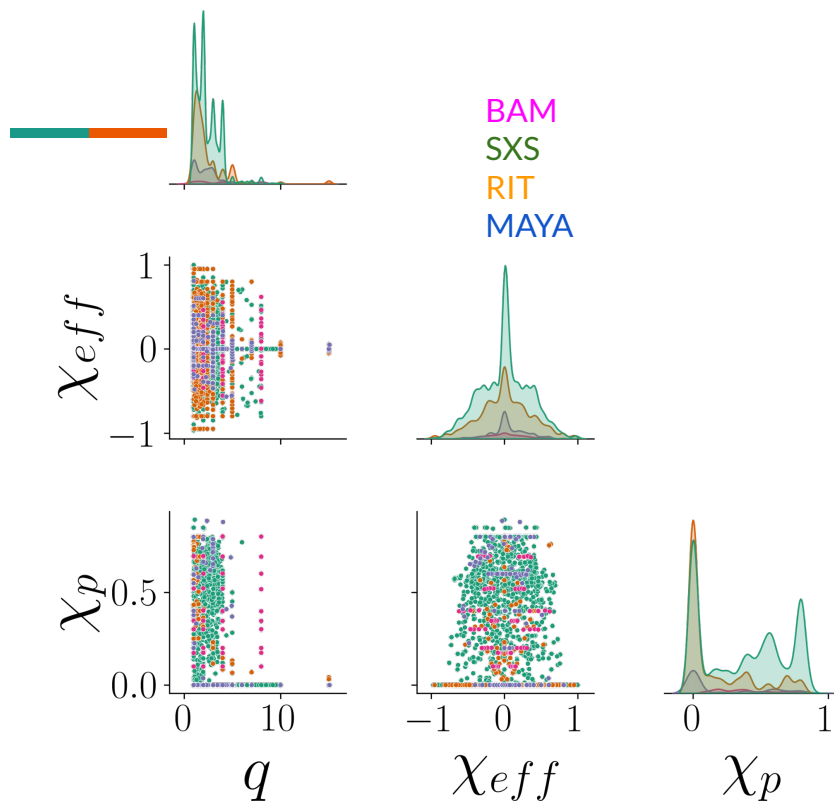


Binary Black Holes



- 7 intrinsic parameters: mass ratio and x, y, z dimensions of each object (total mass is scalable in NR) for the quasi-circular case
 - Define non-precessing systems such that only z-components are nonzero (do not evolve with time)
- +2 more parameters (e and mean anomaly) when eccentricity is allowed to be nonzero
- Numerical Relativity codes is the only method to produce for the full inspiral, merger, and ringdown waveform using all these parameters
 - Recently novel precessing, eccentric waveform SEOB_{NRE}: <https://arxiv.org/pdf/2310.04552.pdf>

Current NR Catalogs



- Up to mass ratios $q \sim 15$
 - Selected simulations at high mass ratios 128:1 [3], and more recently 1024:1 [17]
- Dense coverage up to $q \sim 4$
- Modest coverage of eccentric waveforms
- Minimal highly spinning/precessing simulations (and most are near $q=1$)

The why and how of filling the parameter space?



- If there is a dense enough NR grid, can use NR waveforms directly with PE
- Accurately training models to sufficient accuracy for current and next-generation detectors
- NR simulations are computationally expensive
- Eccentric space is extremely sparse (even more sparse with precession)
- Need to be strategic in new NR placement:
 - Machine learning [11]
 - NR-based PE influenced placement (coming up)

NR-Based PE

- NR-based PE for GW150914 in green including and excluding HOMs (dashed/solid respectively)
- Colored points represent each simulation's max InL when compared to the data over a total mass range
- Black & gray points having the highest InL
- Interpolate between the points to construct the continuous InL

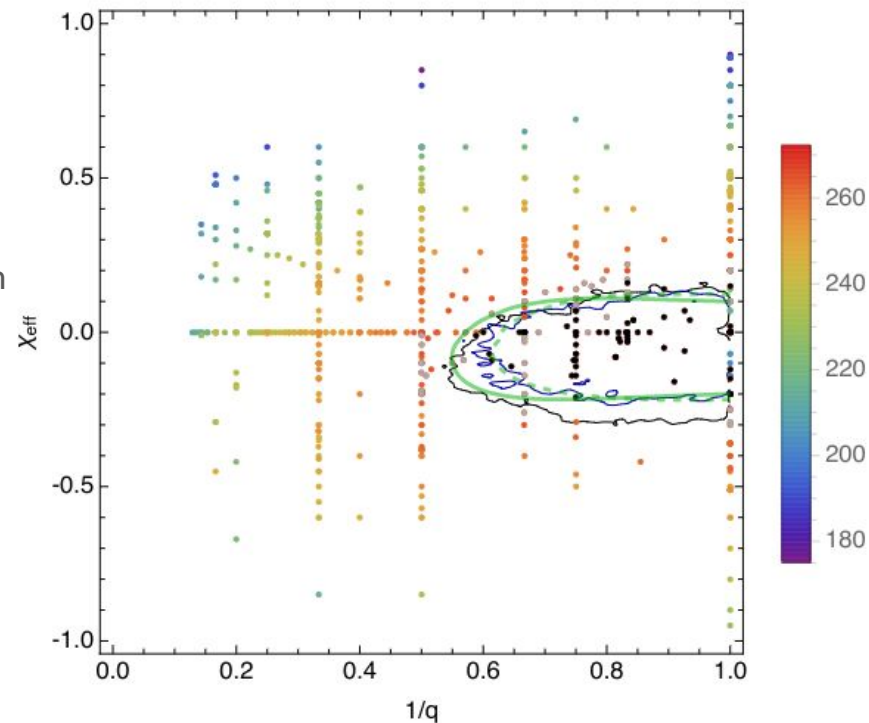
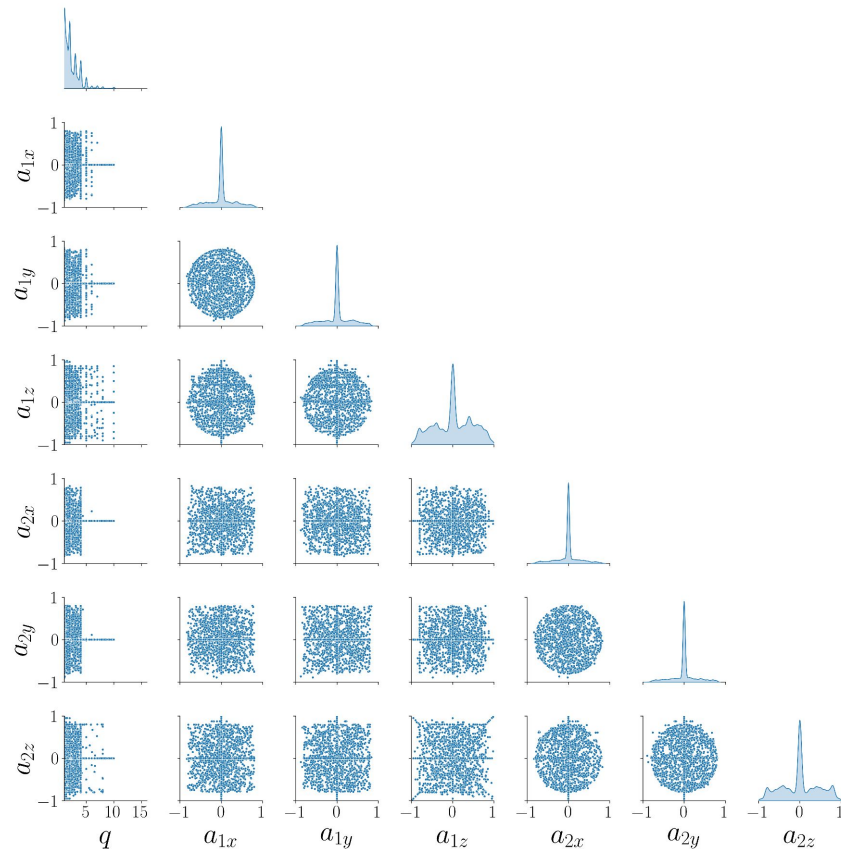


Figure 4 from [8]

Context

- Main goal of numerical relativity (NR) groups is to fully cover relevant parameter space with NR simulations. Can do this by:
 - Focusing on parts of parameter space where there are few simulations
 - Focusing on parts of parameter space that are important for existing events
- There exist previous work attempting to do both [1-4, 9]
- Would be useful to have a data-driven method to take into account the the relevant part of parameter space as well as the parse-ness of the current NR grid



Method (RIFT)

- Parameter estimation (PE) for gravitational wave (GW) sources: Compare models and data, using gaussian statistics

$$\ln \mathcal{L}(\lambda; \theta) = -\frac{1}{2} \sum_k \langle h_k(\lambda, \theta) - d_k | h_k(\lambda, \theta) - d_k \rangle_k - \langle d_k | d_k \rangle_k$$

- NR-based PE idea using RIFT: originally introduced in [7]
 - Integrate over extrinsic parameter space over a 1D grid in total mass for fixed NR intrinsic parameters

$$\mathcal{L}_{\text{marg}}(\lambda) \equiv \int \mathcal{L}(\lambda, \theta) p(\theta) d\theta$$

- Reconstruct continuous marginalized likelihood from the discrete
 - Gaussian Process Regression (GP) - returns fit as well as error of the fit
 - Choosing a free weight factor for the error, we add the error from the GP to the fit

- Construct “Error influenced” posterior via Bayes

$$p_{\text{post}}(\lambda) = \frac{\mathcal{L}_{\text{marg}}(\lambda) p(\lambda)}{\int d\lambda \mathcal{L}_{\text{marg}}(\lambda) p(\lambda)}$$

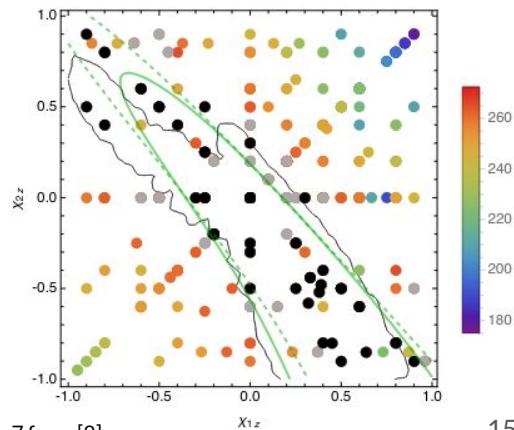


Figure 7 from [8]

Test Case: GW190521

- From [6], plot shows 4 analyses of GW190521 using SEOBNRv4PHM (green), IMRPhenomPv3HM (orange), NRSur7dq4 (blue), and directly comparing to NR (black)
- While all the results are largely consistent, there are noticeable differences between the analyses
- The NR-based PE peaks at a noticeable lower χ_p than the other analyses; could be due to only have a few highly precessing/spinning NR

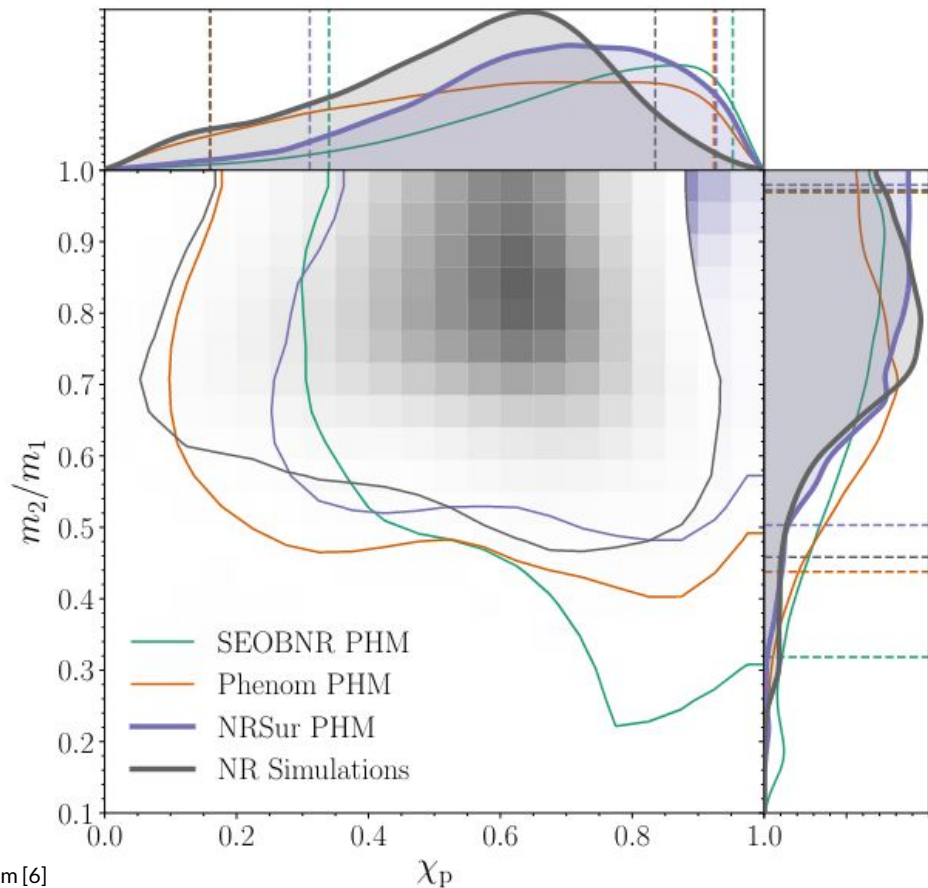
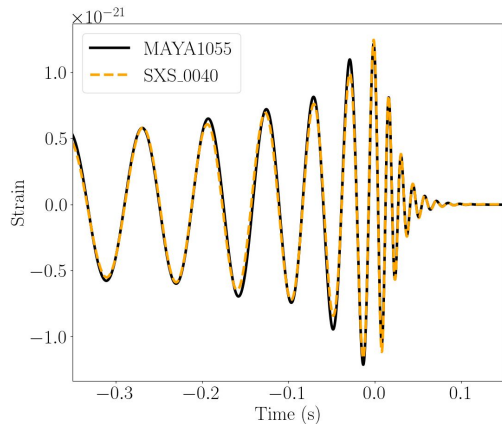


Figure 8 from [6]

The New Follow up Simulations

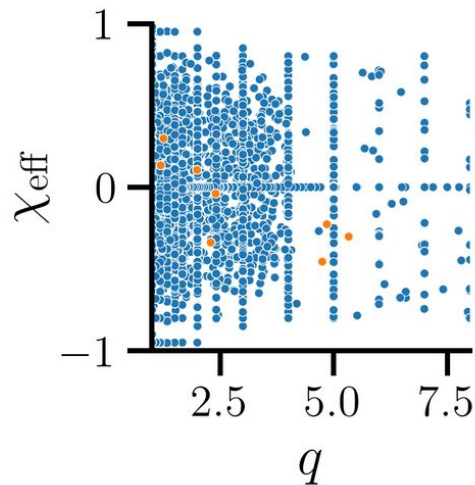
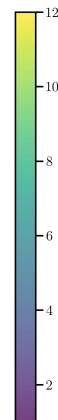
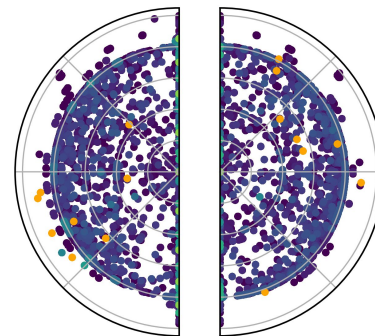
Credit:
Deborah
Ferguson



Mismatch:
0.0013

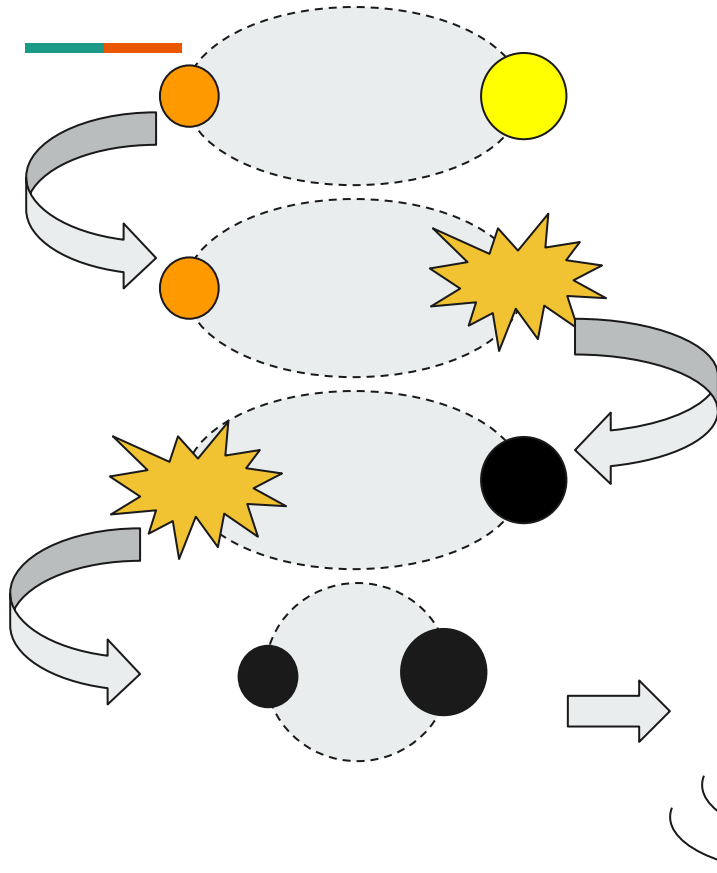
Preliminary

q	χ_{1x}	χ_{1y}	χ_{1z}	χ_{2x}	χ_{2y}	χ_{2z}	$ a_1 $	$ a_2 $
0.951	-0.642	0.042	0.129	0.832	0.375	0.340	0.657	0.974
0.729	0.915	-0.201	0.152	-0.224	0.201	-0.230	0.949	0.379
0.799	0.222	0.246	-0.040	-0.309	-0.207	0.720	0.334	0.811
0.502	0.895	-0.073	-0.164	0.082	0.348	0.648	0.913	0.740
0.188	0.117	0.453	-0.423	0.264	-0.268	0.341	0.631	0.508
0.436	0.681	-0.061	-0.542	-0.015	-0.529	0.127	0.873	0.545
0.238	0.635	0.528	-0.448	0.342	0.141	-0.233	0.940	0.437
0.210	0.542	0.604	-0.387	0.287	0.034	-0.759	0.899	0.812
0.206	0.355	0.572	-0.316	0.485	0.041	0.209	0.744	0.530
0.417	-0.737	-0.488	-0.126	-0.502	-0.555	0.177	0.893	0.769
0.842	0.047	-0.316	0.301	-0.891	0.099	-0.064	0.438	0.899

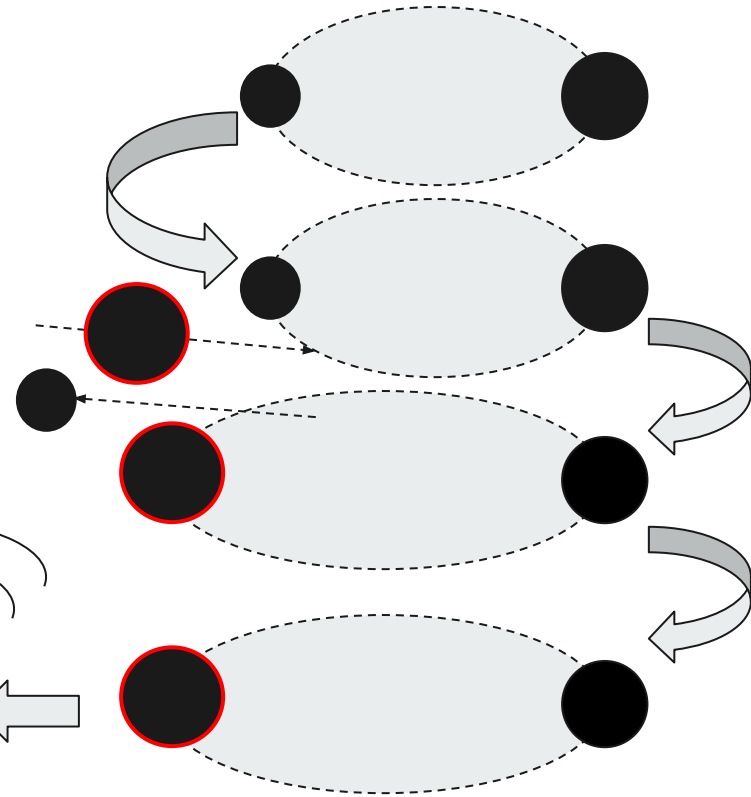


Why do we care? Formation Channels

Isolated formation

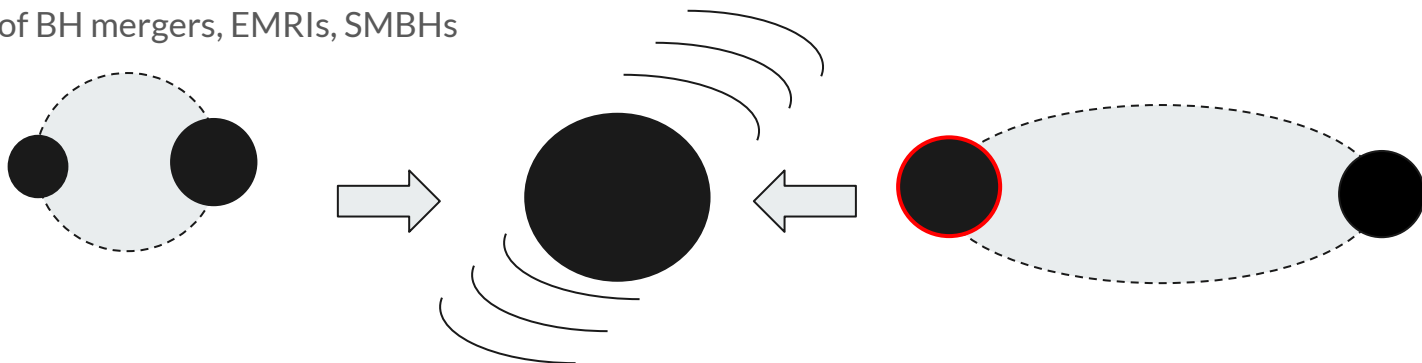


Dynamical formation



Why do we care? Formation Channels

- It is expected that eccentricity is radiated away early in a binary orbit (i.e. before aLIGO/aVirgo frequencies)
- If the signal strongly prefers quasi-circular waveforms, we can infer that the source was formed in isolation
- If the signal has any evidence of eccentricity, we can infer that the source was formed from random encounters in dense clusters
- While the recent work is focused on LIGO/Virgo frequencies, a case can also be made for expected LISA sources: early inspiral of BH mergers, EMRIs, SMBHs



Eccentric Parameter Estimation with eccentric models

TEOBResumS-DALI

SEOBNRE

SEOBNRv4EHM

Eccentricity estimation for five binary black hole mergers with higher-order gravitational wave modes SIGNS OF ECCENTRICITY IN TWO GRAVITATIONAL-WAVE SIGNALS MAY INDICATE A SUB-POPULATION OF

H. L. IGLESIAS,¹ J. LANGE,^{1,*} I. BARTOS,^{2,1} S. BHAMUK,² R. GAMBA,³ V. GAYATHRI,^{2,4} A. JAN,¹ R. NOWICKI,¹
R. O'SHAUGHNESSY,⁵ D. M. SHOEMAKER,¹ R. VENKATARAMAN,¹ AND K. WAGNER⁶

Isobel Romero-Shaw,^{1,2} Paul D. Lasky,^{1,2} and Eric Thrane^{1,2}

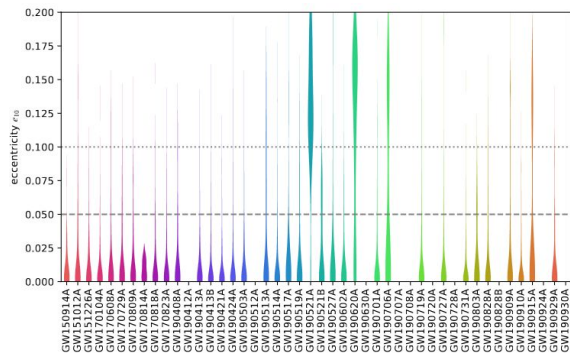
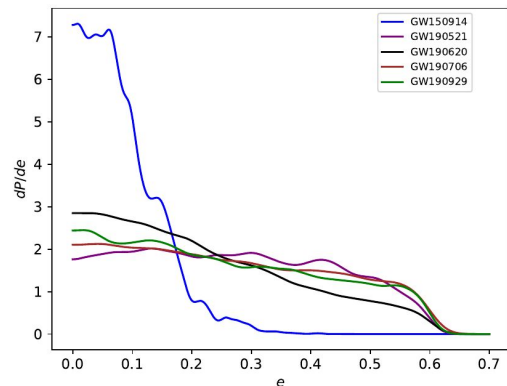
¹Center of Gravitational Physics, University of Texas at Austin, Austin, TX 78712, USA¹

²Department of Physics, University of Florida, PO Box 118440, Gainesville, FL 32611-8440, USA

³Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, 07743, Jena, Germany

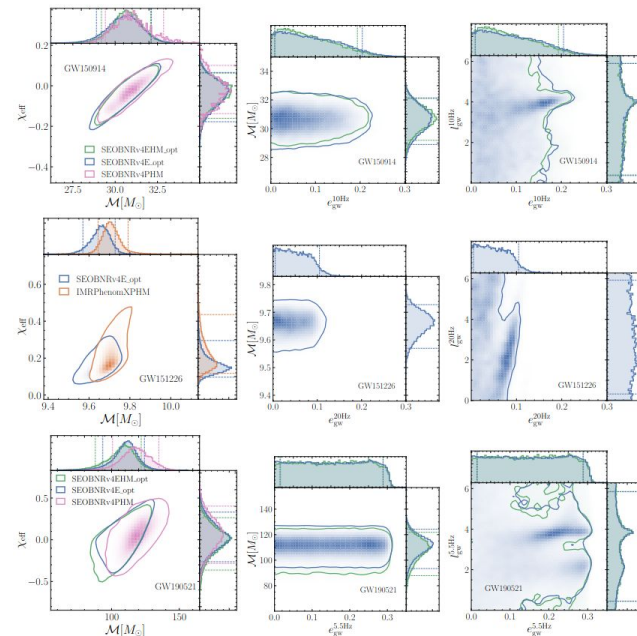
⁴Leonard E. Parker Center for Gravitation, Cosmology, and Astrophysics, University of Wisconsin-Milwaukee, Milwaukee, WI 53201, USA

⁵Rochester Institute of Technology, Rochester, NY 14623, USA



Bayesian inference of binary black holes with inspiral-merger-ringdown waveforms using two eccentric parameters

Antoni Ramos-Buades,¹ Alessandra Buonanno,^{1,2} and Jonathan Gair¹
ck Institute for Gravitational Physics (Albert Einstein Institute), Am Mühlenberg 1, Potsdam, 14476, Germany
²Department of Physics, University of Maryland, College Park, MD 20742, USA
(Date: September 18, 2023)

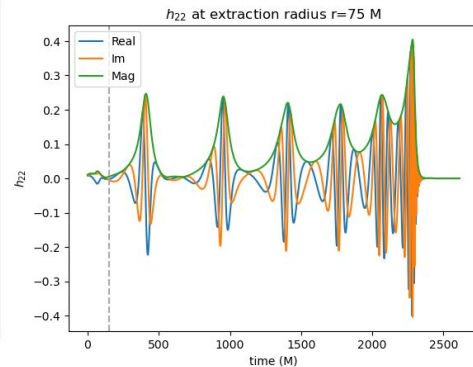
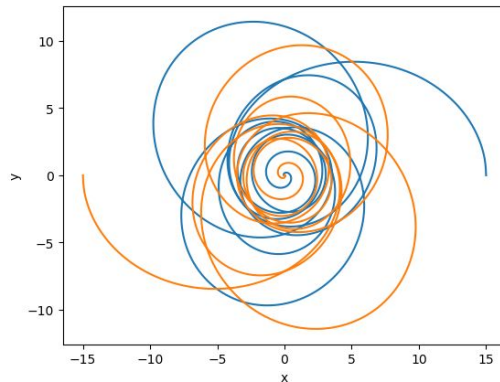


- Three selected PE studies using the 3 different eccentric models
- While using different settings and models, TEOBResumS-DALI and SEOBNRv4EHM studies did not find evidence of eccentricity and the SEOBNRE study did for a few events

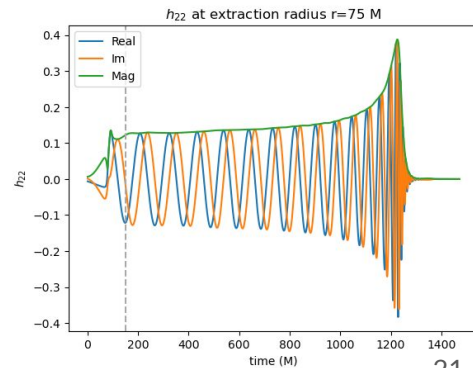
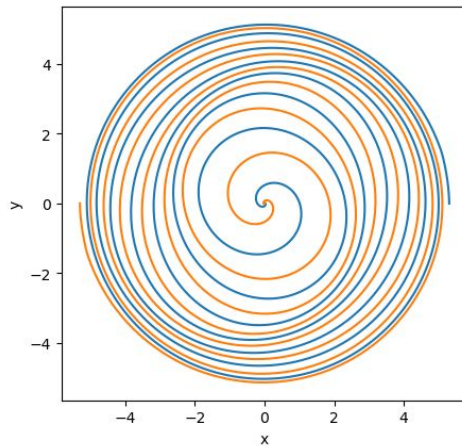
Precession vs Eccentricity

- While GW models exist that include precession or eccentricity, only numerical relativity can produce waveforms with both
- It has been theorized that precession and eccentricity can mimic each other if a model is missing some of the physics
- We want to investigate our ability to measure precession and eccentricity and to see how significant is the systematics of our current waveform models

$q=1, a_1x=a_2x=0.5, e=0.5$

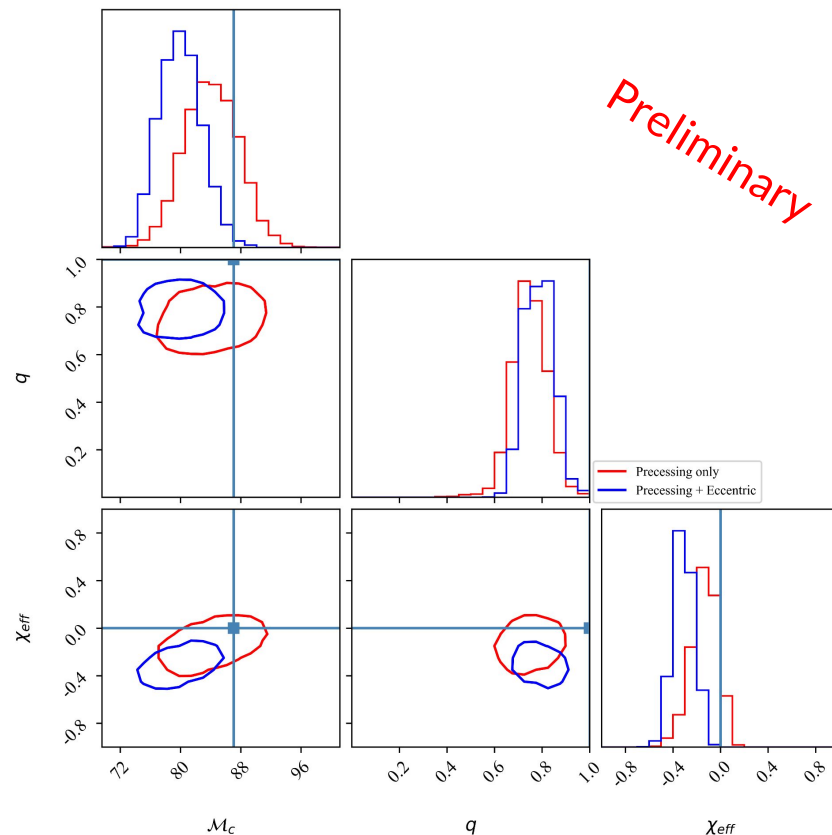


$q=1, a_1x=a_2x=0.5, e=0$



Precession vs Eccentricity

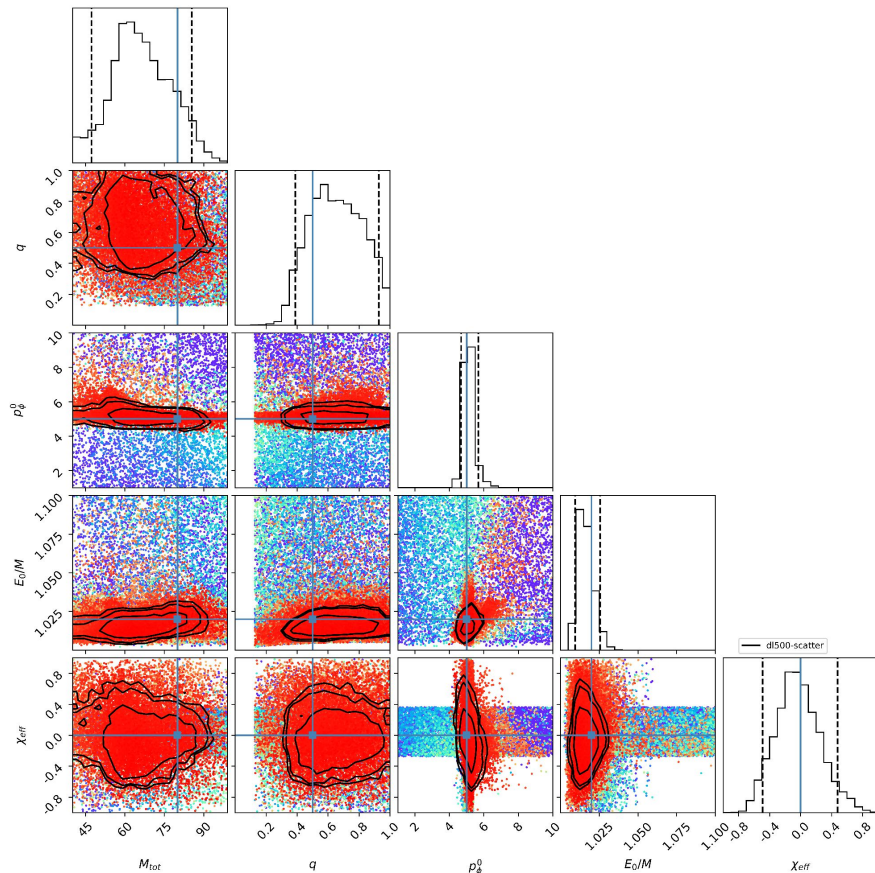
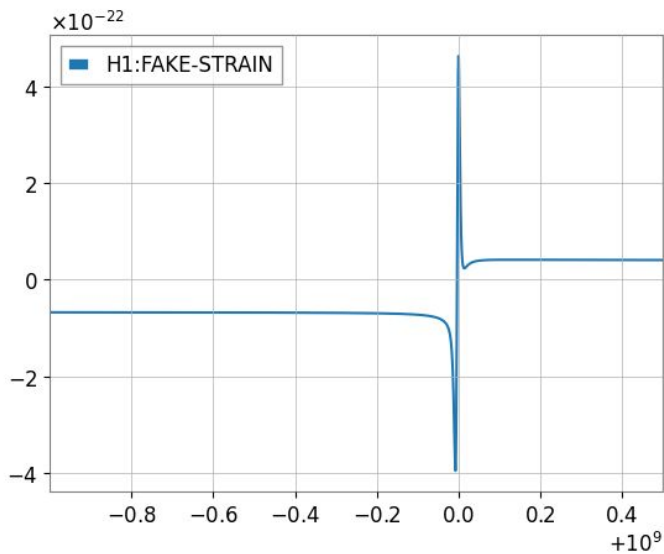
- Analysis Using precessing only model of two injections:
 - Red=Precessing only NR simulation
 - Blue=Precessing+Eccentric NR simulation
- While PE results are still converging, we already see bias in the recovery when the model does not include all the physics of the injection



Parameter Estimation of a Hyperbolic Encounter

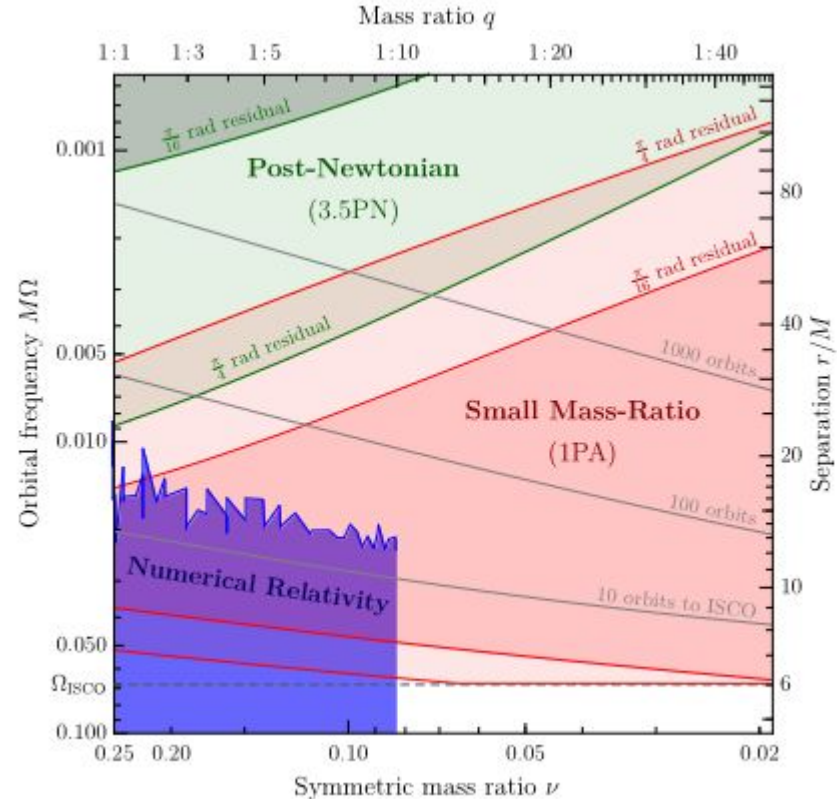
Injection Parameters:

- $M = 80M_{\text{sun}}$
- $q = 2$
- $p_{\phi} = 5$
- $E_0/M = 1.01$
- $\chi_{\text{eff}} = 0.0$
- $DL = 500 \text{ Mpc}$



Hybridization of Extreme Mass Ratio Simulations

- Self-force theory are the leading method to produce extreme mass ratio systems
- While SF can produce a long inspiral, it still breaks down near merger
- NR can be pushed to larger extreme mass ratio systems, but this requires large computing time for only a limited duration waveform
- Hybridization is the method of stitch together one waveform to another
- Allow us to run relatively limited separation simulations to use with hybrids



A Hybridized SF-NR waveform would allow for an arbitrary long IMR waveform

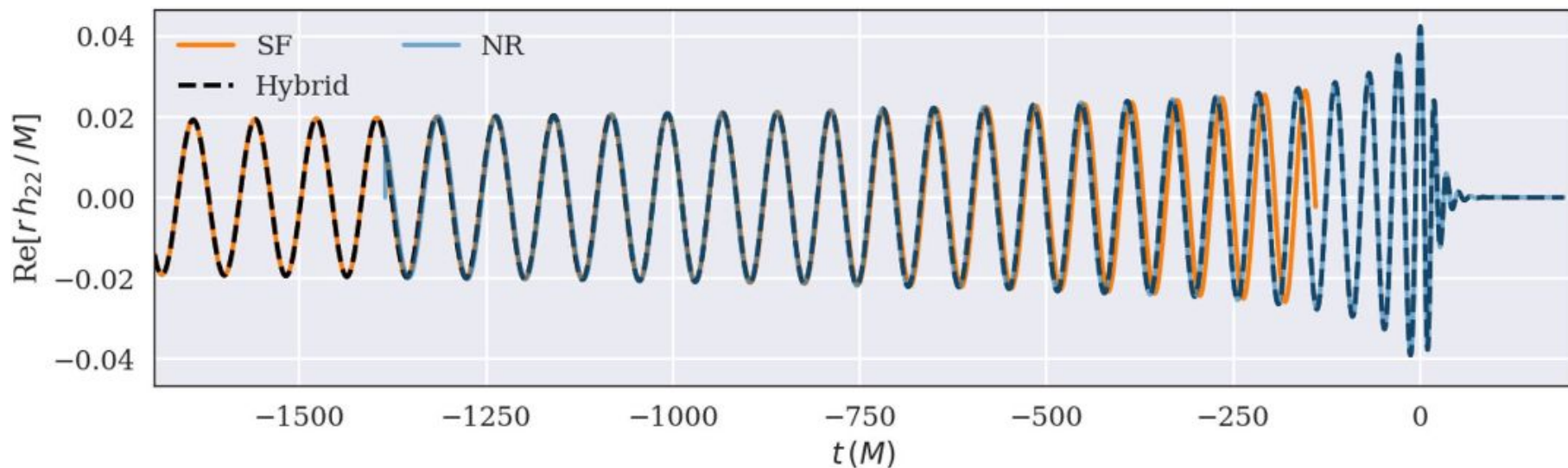
Figure 4 from [18]

Hybridization of Extreme Mass Ratio Simulations

Results: RIT

Window length = 7 GW cycles

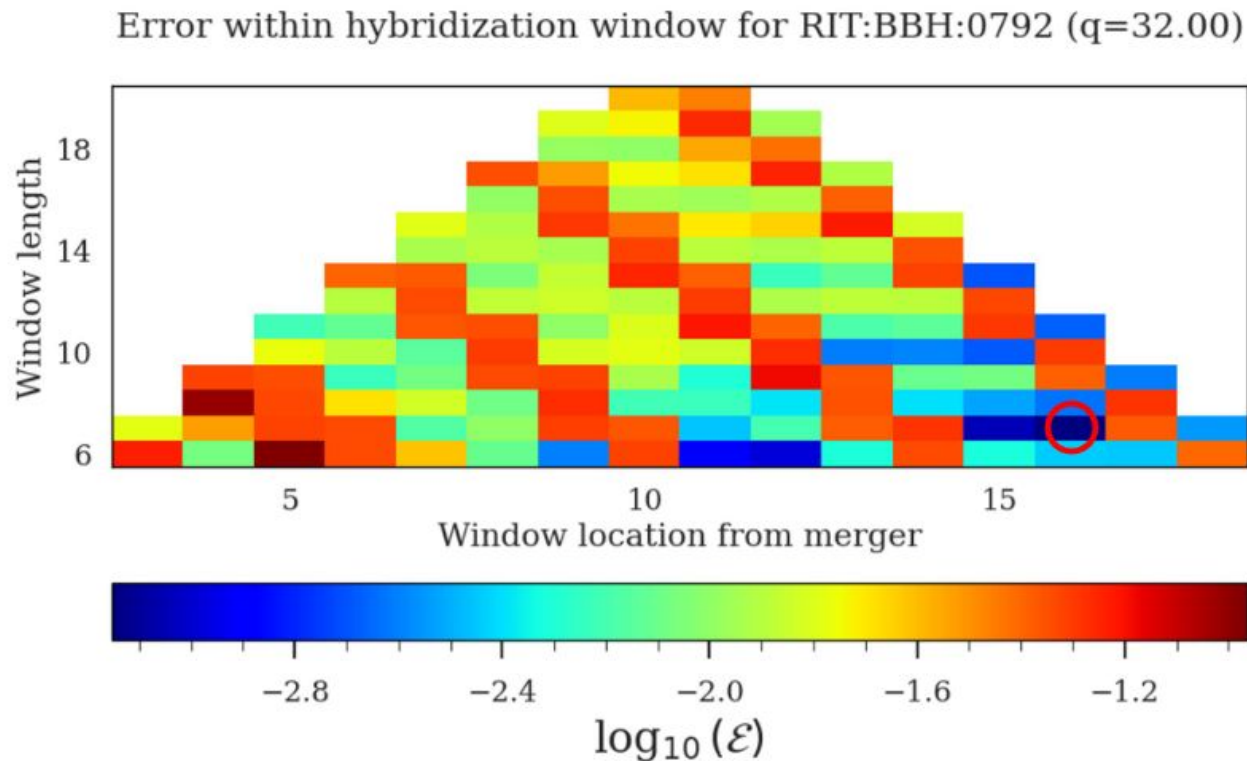
Hybridized waveform for RIT:BBH:0792 ($q=31.997$), with 16 NR cycles and 50 SF cycles



Hybridization of Extreme Mass Ratio Simulations

Results: RIT

- Lowest window error: 7.04×10^{-4}
- Location: 16 cycles
- Length: 7 cycles



References

- [1] D. Ferguson. Phys. Rev. D, 102, 024034 (2023)
- [2] M. Boyle, D. Hemberger, D. A. B. Iozzo, et. al. CGQ 36, 195006 (2019)
- [3] J. Healy and C. O. Lousto Phys. Rev. D, 105, 124010 (2022)
- [4] K. Jani, J. Healy, J. A. Clark, et. al. CHQ 33, 204001 (2016)
- [5] J. Lange. "RIFT'ing the Waves: Developing and applying an algorithm to infer properties of gravitational wave sources" (2020). Thesis. Rochester Institute of Technology Accessed from <https://scholarworks.rit.edu/theses/10586>
- [6] The LIGO and Virgo Collaboration. ApJL, 900, L13 (2020)
- [7] J Lange, R. O'Shaughnessy, M. Boyle, et. al. Phys, Rev D 96, 104041 (2017)
- [8] The LIGO and Virgo Collaboration. Phys. Rev. D, 94, 064035 (2016)
- [9] J. Healy, J. Lange, R. O'Shaughnessy, et. al. Phys. Rev. D, 97, 064027 (2018)
- [10] V. Varma, et. al. Phys. Rev. D, 99, 064045 (2018)
- [11] D. Ferguson, et. al. Phys Rev. D, 104, 044037 (2021)
- [12] The LIGO and Virgo Collaboration. ApJ, 883, 148 (2019)
- [13] P. C. Peters. Phys. Rev. 136, B1224 (1964)
- [14] The LIGO and Virgo Collaboration. Phys. Rev. L 116, 061102 (2016)
- [15] The LIGO and Virgo Collaboration. Phys. Rev. L 116, 241102 (2016)
- [16] A. Ghosh, et. al. Phys. Rev. D 94, 021101 (2016)
- [17] Lousti C. O., Healy, J. CQG Volume 40, Issue 9,

Hybridization of Extreme Mass Ratio Simulations

Hybridization procedure

- Procedure is based on approach used to create NRHybSur3dq8¹:

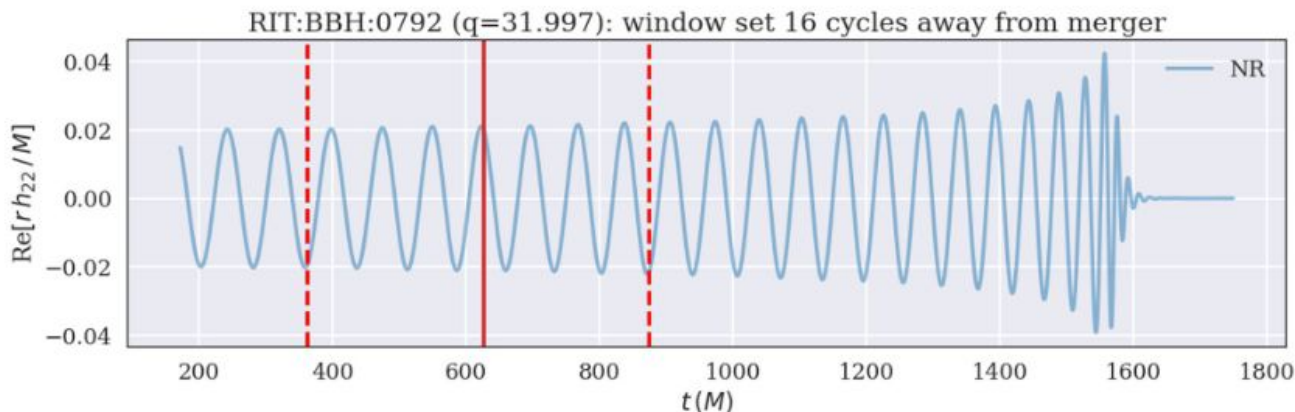
1. Match Ω_{22} at start of window

2. Find optimal time, phase shifts

3. Apply time, phase shifts and stitch waveforms together

4. Repeat 1-3 for each set of window location and length

$$\mathcal{E}[\hat{h}, \tilde{h}] = \frac{1}{2} \frac{\sum_{\ell,m} \int_{t_1}^{t_2} |\hat{h}_{\ell m}(t) - \tilde{h}_{\ell m}(t)|^2 dt}{\sum_{\ell,m} \int_{t_1}^{t_2} |\hat{h}_{\ell m}(t)|^2 dt}$$

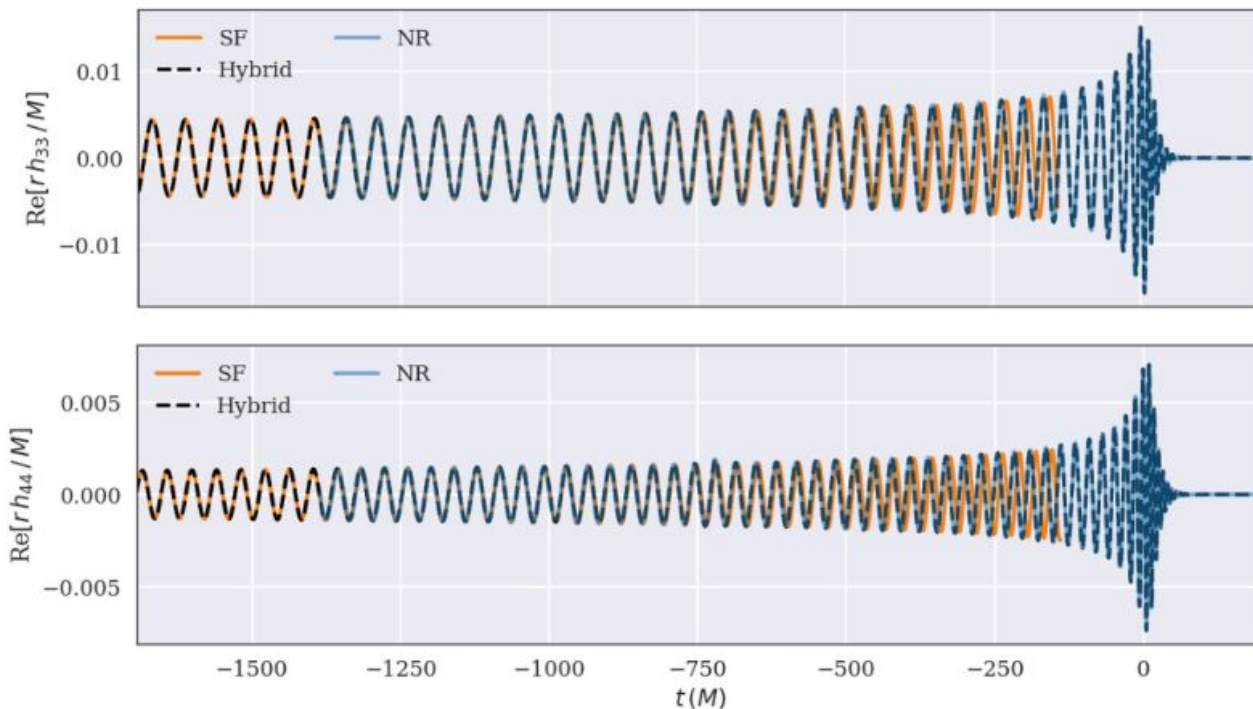


Hybridization of Extreme Mass Ratio Simulations

Appendix B: Higher-order modes

Window length = 7 GW cycles

Hybridized waveform for RIT:BBH:0792 ($q=31.997$), with 16 NR cycles and 50 SF cycles



RIT:BBH:0792