ALESSANDRO BACCHETTA, PAVIA U. AND INFN

TMDS AND GPDS WITH EARLY DATA

A NEW ERA OF DISCOVERY

THE 2023 LONG RANGE PLAN FOR NUCLEAR SCIENCE

2023 | VERSION 1.2

https://science.osti.gov/-/media/np/nsac/pdf/202310/NSAC-LRP-2023-v12.pdf

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RECOMMENDATION 3

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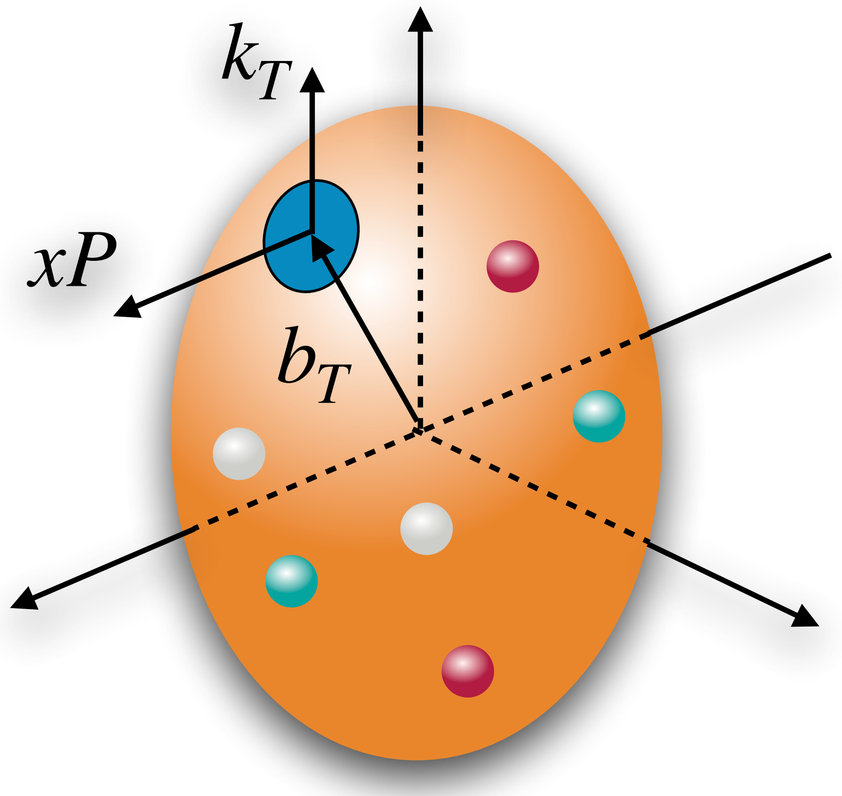
> The EIC is a powerful discovery machine, a precision microscope capable of taking three-dimensional pictures of nuclear matter at femtometer scales.

https://science.osti.gov/-/media/np/nsac/pdf/202310/NSAC-LRP-2023-v12.pdf

Wigner distributions
(Fourier transform of GTMDs =
Generalized Transverse Momentum
Distributions)



see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)

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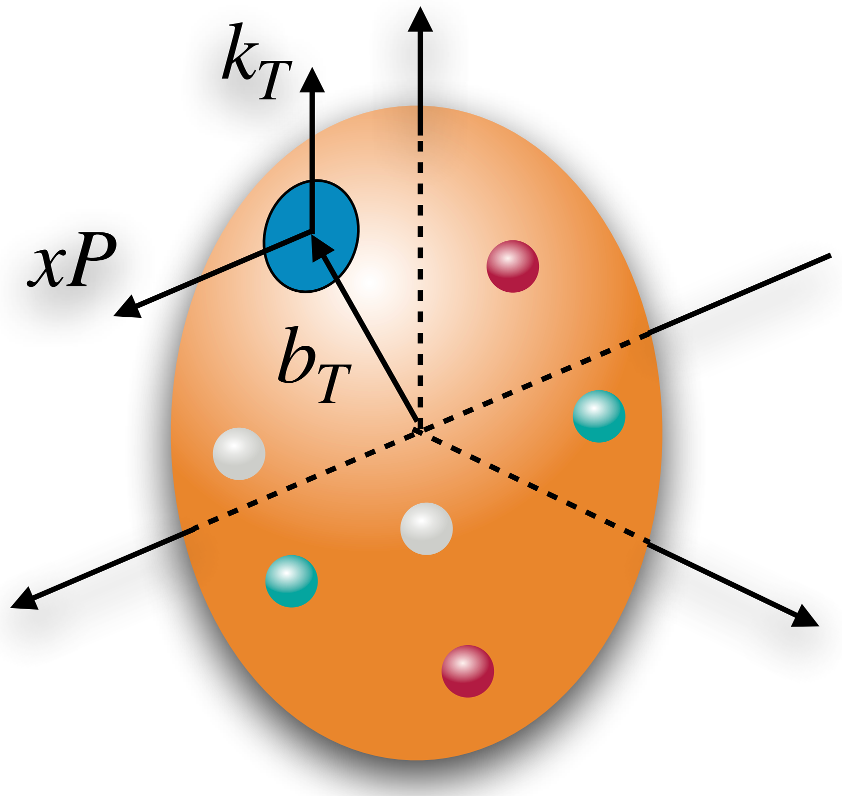

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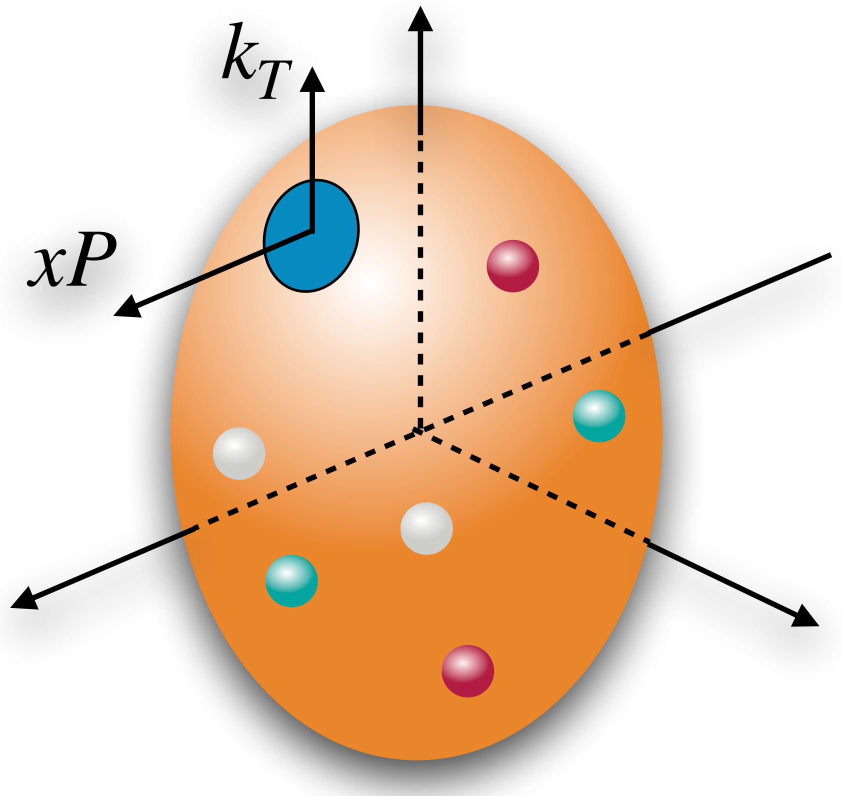

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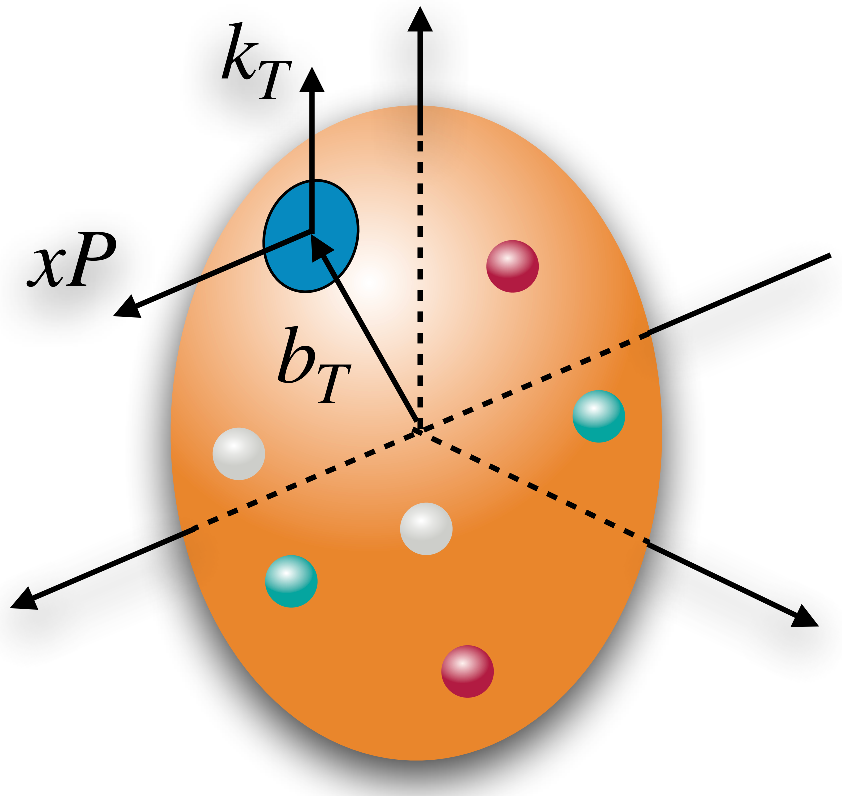

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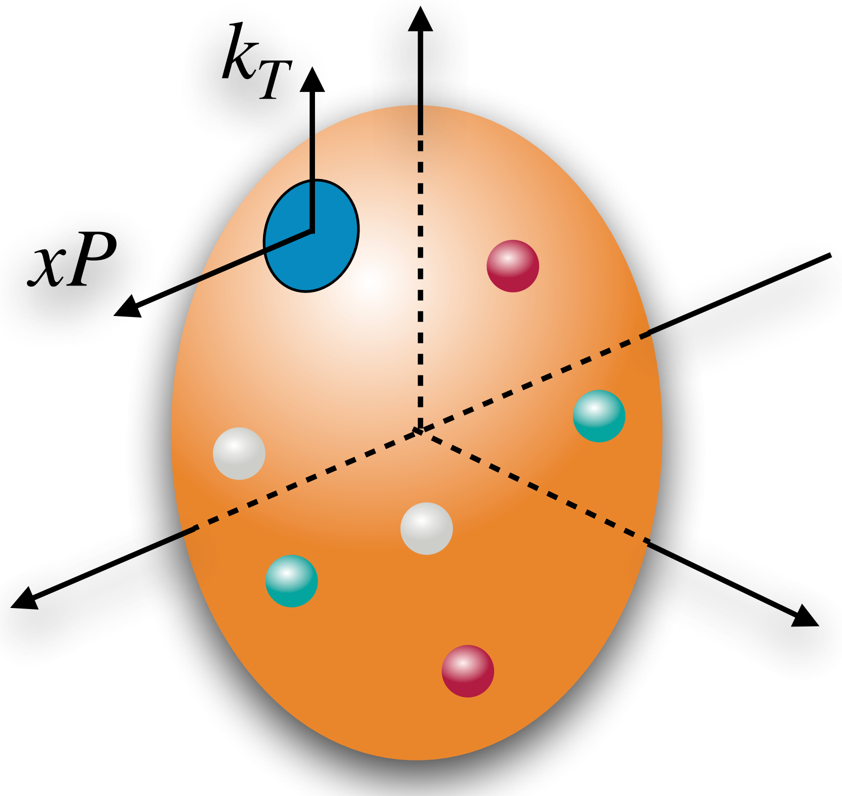

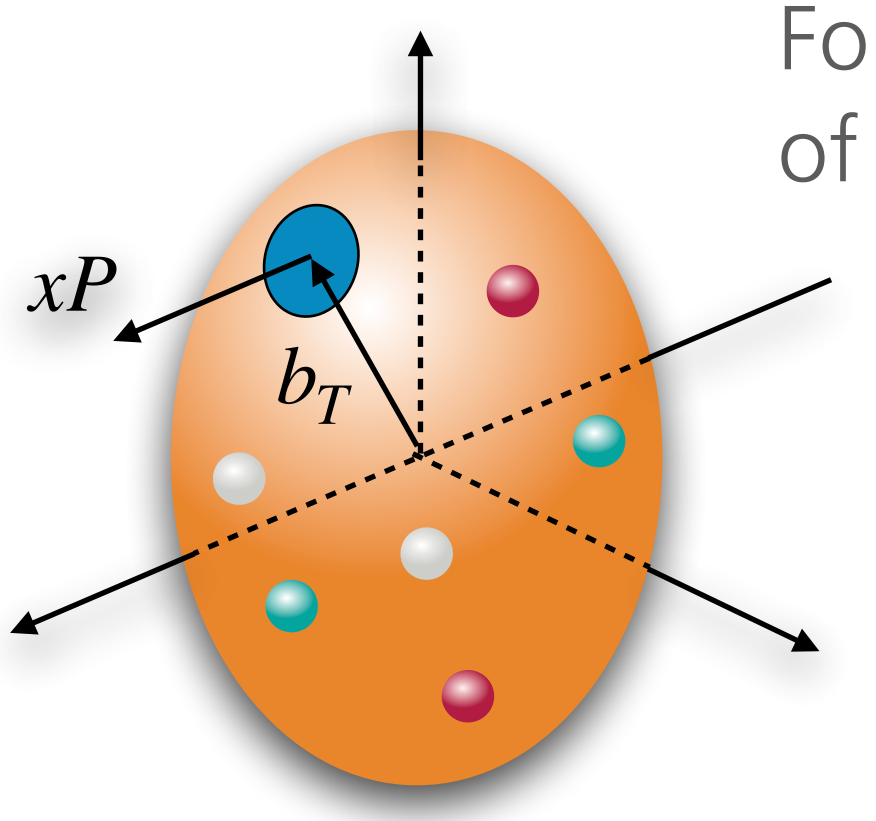

Fourier transform
of GPDs

see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)

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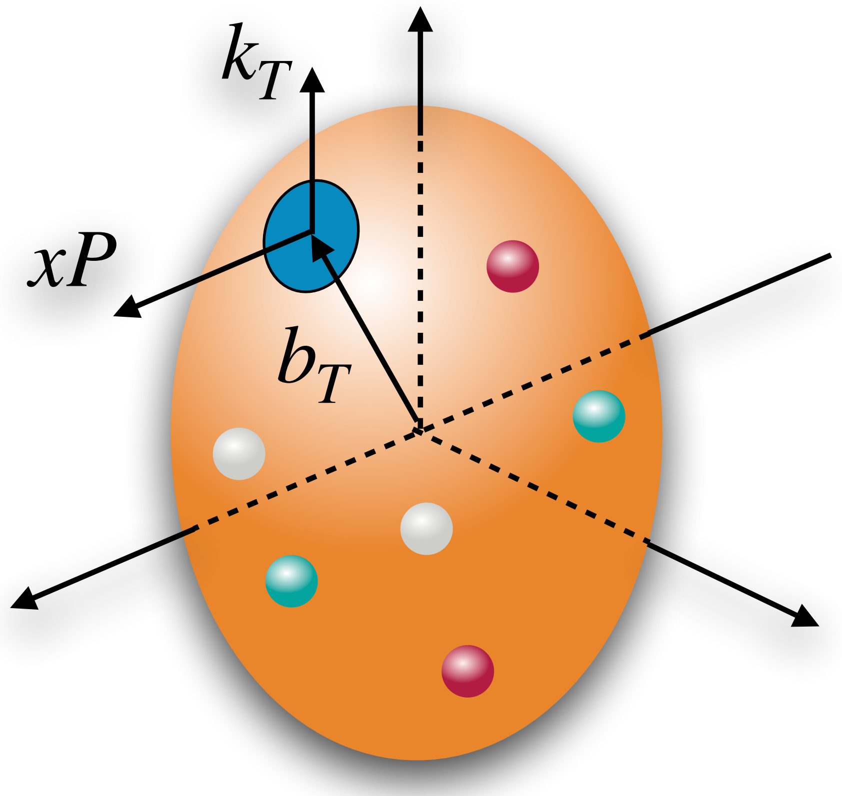

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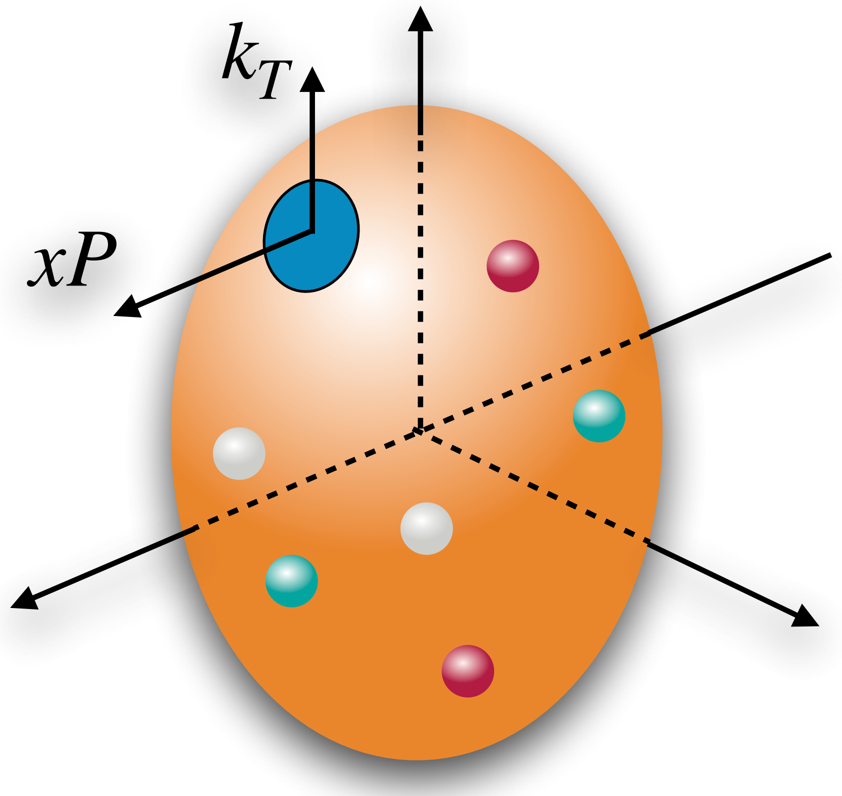

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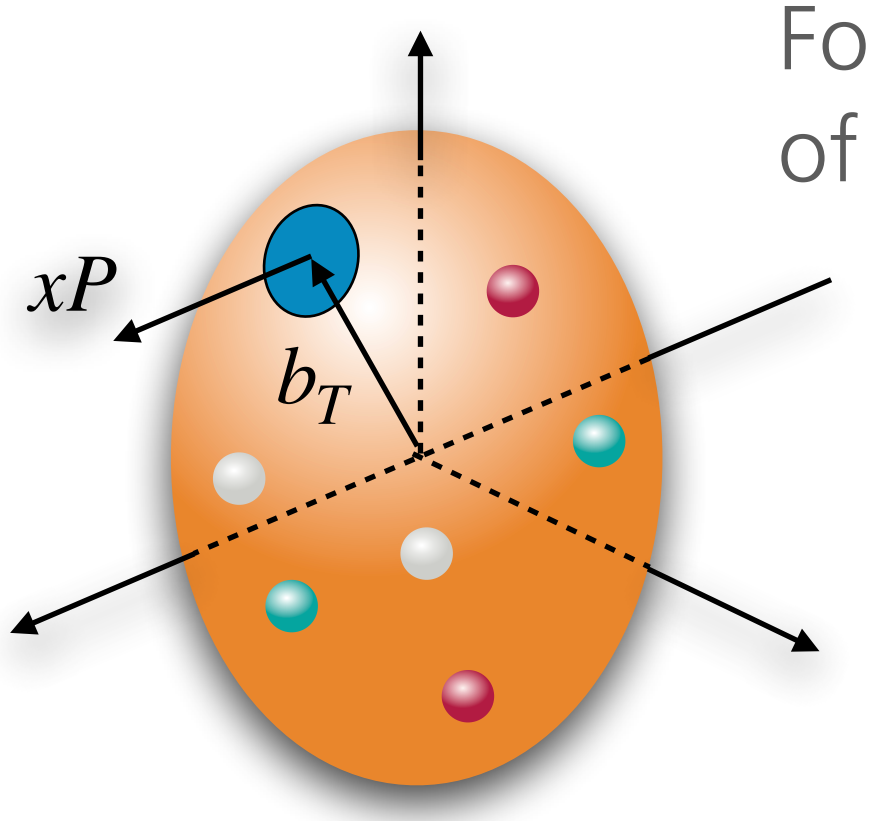
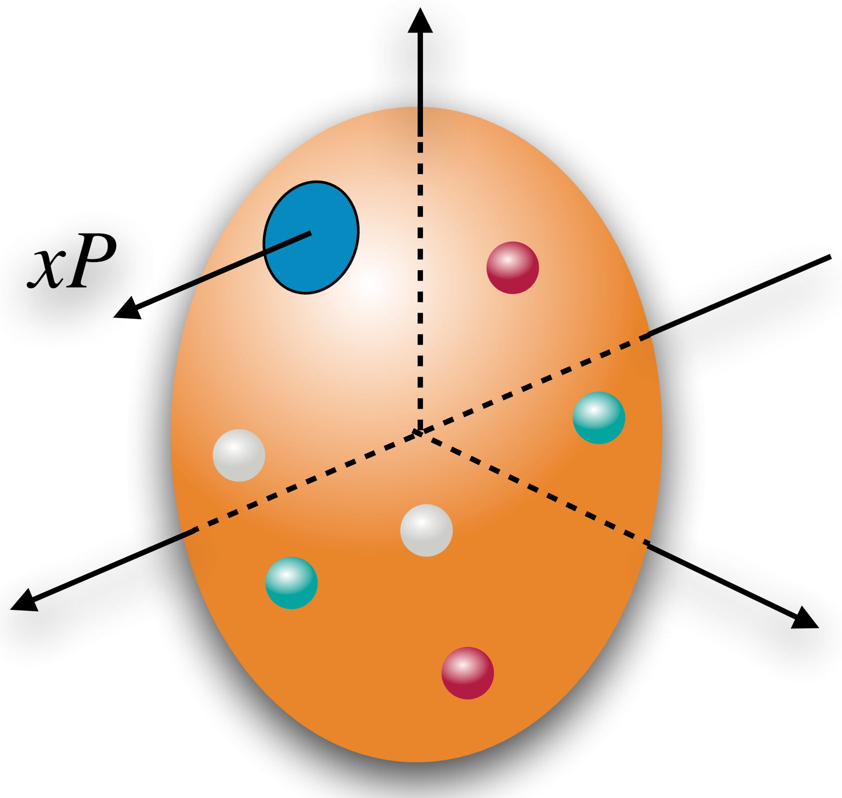
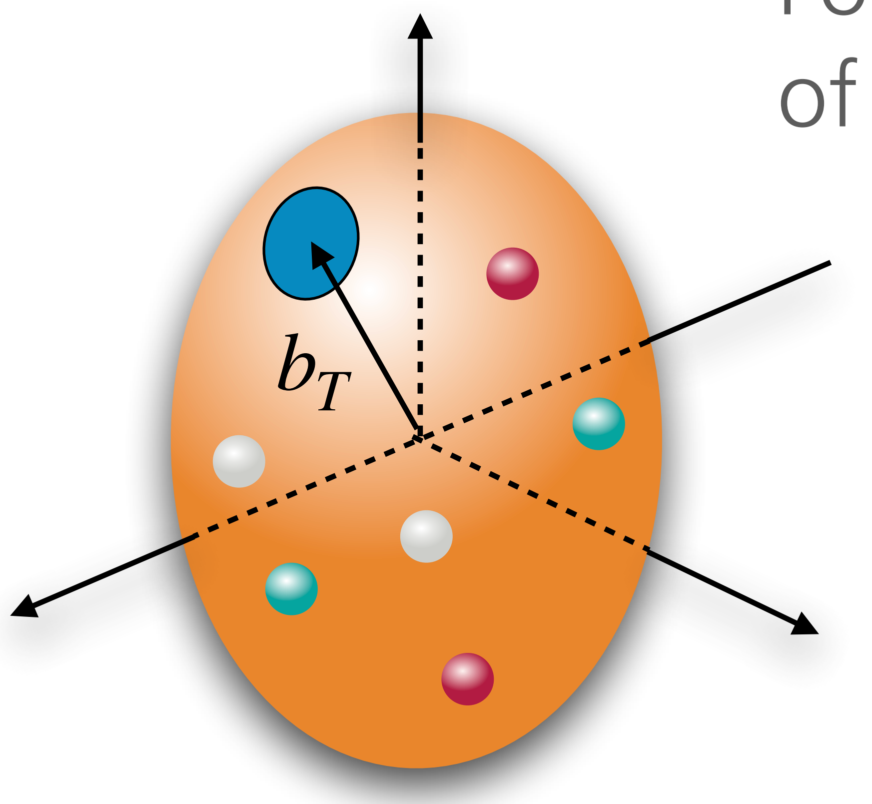


PDFs



Fourier transform
of Form Factors



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EARLY-STAGE PARAMETERS

	Species	Energy	Luminosity (fb^{-1})	e polarization	p/A polarization
Year 1	e+Ru or e+Cu	10 x 115	0.9	N/A	N/A
Year 2	e+d (21 weeks)	10 x 130	9.2	N/A	N/A
	e+p (5 weeks)	10 x 130	0.95 - 1.03	N/A	trans?
Year 3	e+p	10 x 130	4.95 - 5.33	N/A	trans & long
Year 4	e+Au (13 weeks)	10 x 100	0.42	N/A	N/A
	e+p (13 weeks)	10 x 250	3.09 - 4.59	N/A	trans & long
Year 5	e+Au (13 weeks)	10 x 100	0.42	N/A	N/A
	e+^{3}He (13 weeks)	10 x 166	4.33	N/A	trans & long

UNPOLARIZED CROSS SECTION

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, hep-ph/0611265

$$\frac{d\sigma}{dx\,dy\;dz\,d\phi_h\,dP_{h\perp}^2}$$

$$= \frac{2\pi\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Big\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\;\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\;\sin\phi_h\,F_{LU}^{\sin\phi_h}\Big\}$$

POLARIZED CROSS SECTION

without lepton polarization

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+ S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$

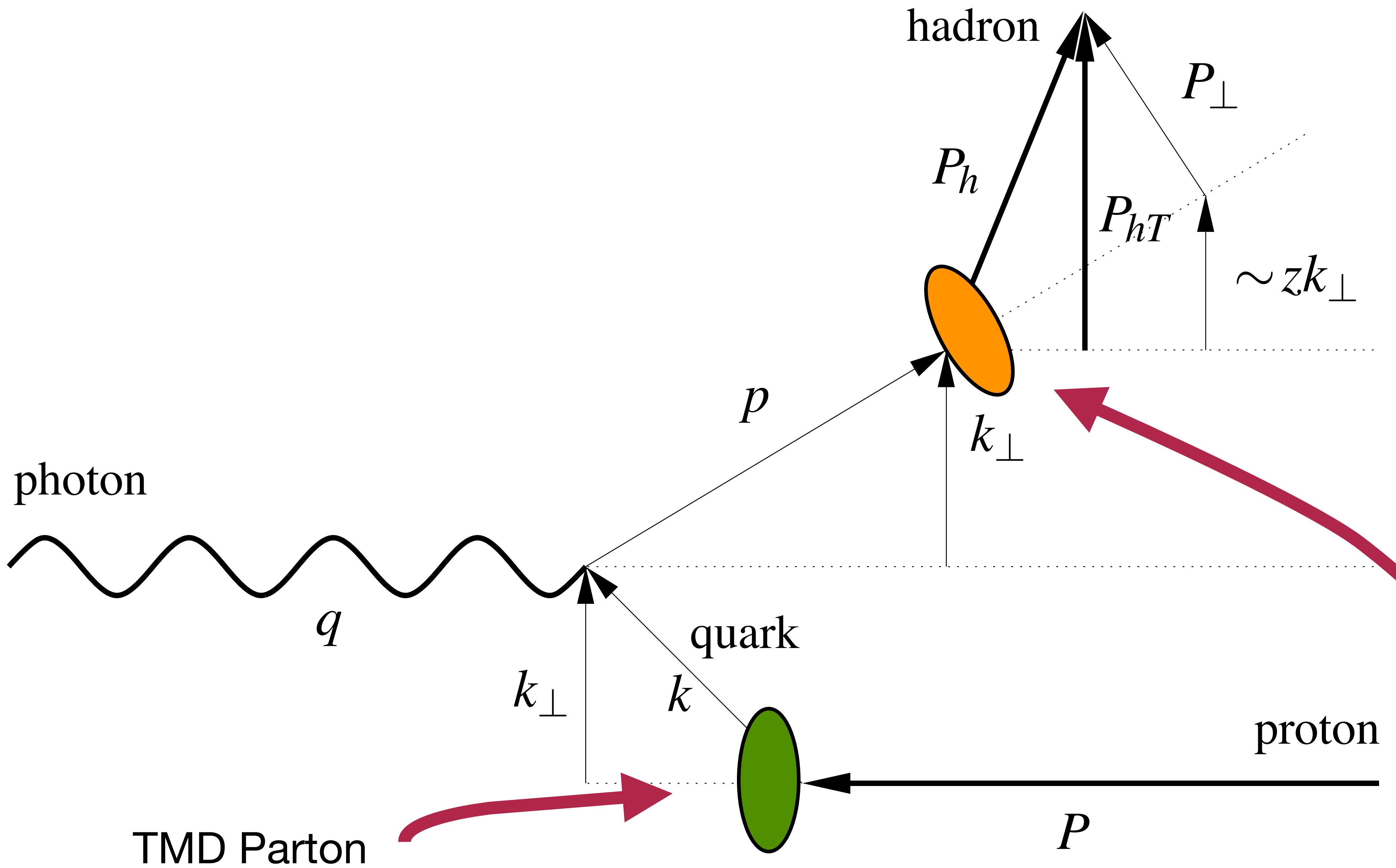
$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]$$

$$\Bigg\}$$

TMDS

TMDS IN SEMI-INCLUSIVE DIS (SIDIS)



TMD Parton
Distribution Functions

TMD Parton
Fragmentation Functions

$$
\begin{aligned}
&F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2)\\
&\quad = x\sum_q \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\,\delta(z\boldsymbol{k}_\perp-\boldsymbol{P}_{hT}+\boldsymbol{P}_\perp)\\
&\quad = x\sum_a \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|)\hat{f}_1^q(x,z^2b_\perp^2;\mu^2)\,\hat{D}_1^{a\to h}(z,b_\perp^2;\mu^2)
\end{aligned}
$$

TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

TMD STRUCTURE

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$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) \; e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} \left(\gamma_F - \gamma_K \ \ln \frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\mathrm{resum}} + g_K}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
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perturbative Sudakov
form factor

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collinear PDF

Collins-Soper kernel
(perturbative and
nonperturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

matching coefficients
(perturbative)

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
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$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

perturbative Sudakov form factor

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) \; e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} \left(\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\text{resum}} + g_K}$$

collinear PDF

Collins-Soper kernel (perturbative and nonperturbative)

matching coefficients (perturbative)

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

$$\Downarrow$$

$$\mu_{b*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

see, e.g., Collins, "Foundations of Perturbative QCD" (11)
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TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

perturbative Sudakov
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collinear PDF

matching coefficients
(perturbative)

Collins-Soper kernel
(perturbative and
nonperturbative)

nonperturbative part
of TMD

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T}$$

$$\Big\Downarrow$$

$$\mu_{b*} = \frac{2e^{-\gamma_E}}{\bar{b}_*}$$

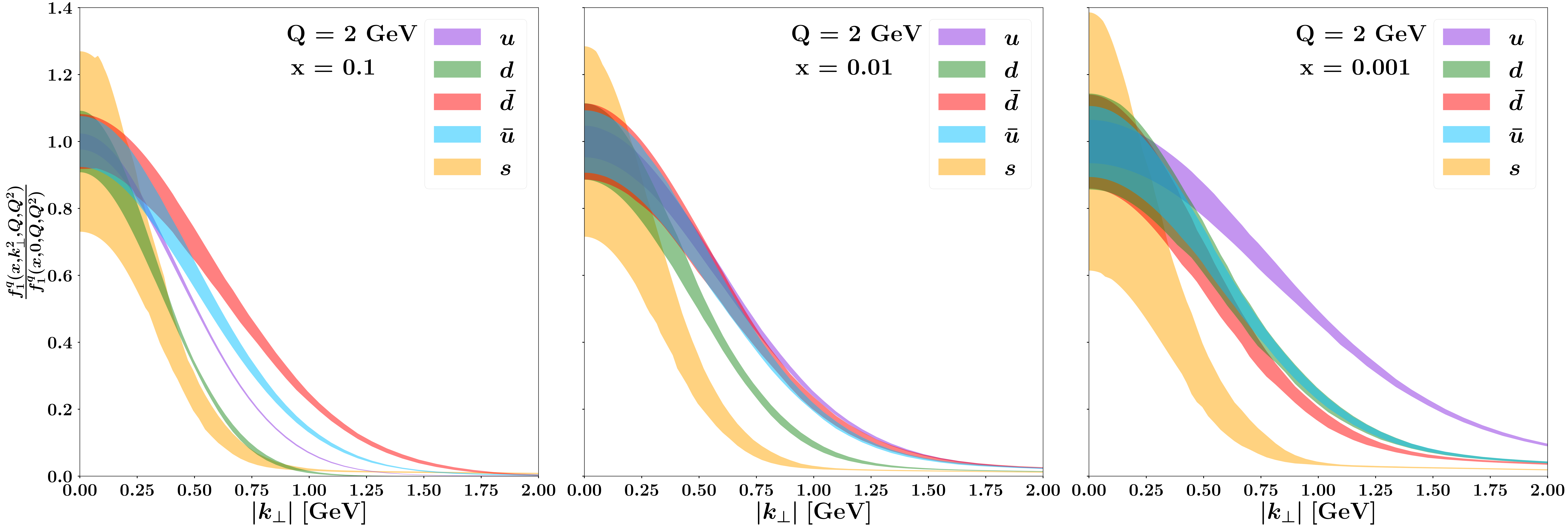
see, e.g., Collins, "Foundations of Perturbative QCD" (11)
TMD collaboration, "TMD Handbook," arXiv:2304.03302

UNPOLARIZED TMD GLOBAL FITS

	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✔	✔	✔	✔	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✔	✔	✔	✔	2031	1.06
ART23 arXiv:2305.07473	N^4LL	✘	✘	✔	✔	627	0.96
MAP24 arXiv:2405.13833	N^3LL	✔	✔	✔	✔	2031	1.08

FLAVOR-DEPENDENT UNPOLARIZED TMDS

MAP Collaboration, arXiv:2405.13833

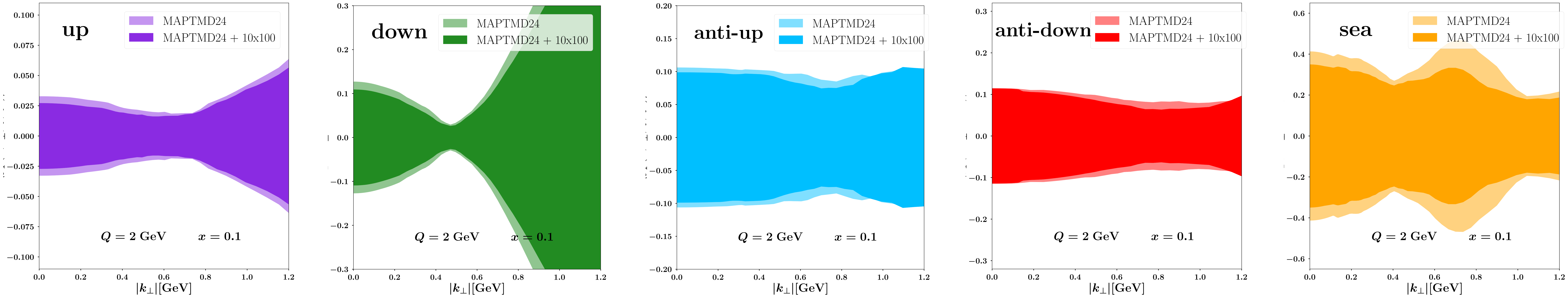


EIC IMPACT, ONE CONFIGURATION

Simulations by Lorenzo Rossi

Combined fit with existing data (2031 points) + EIC,10 x 100, 5 fb^{-1} (1611 points)

$x = 0.1$

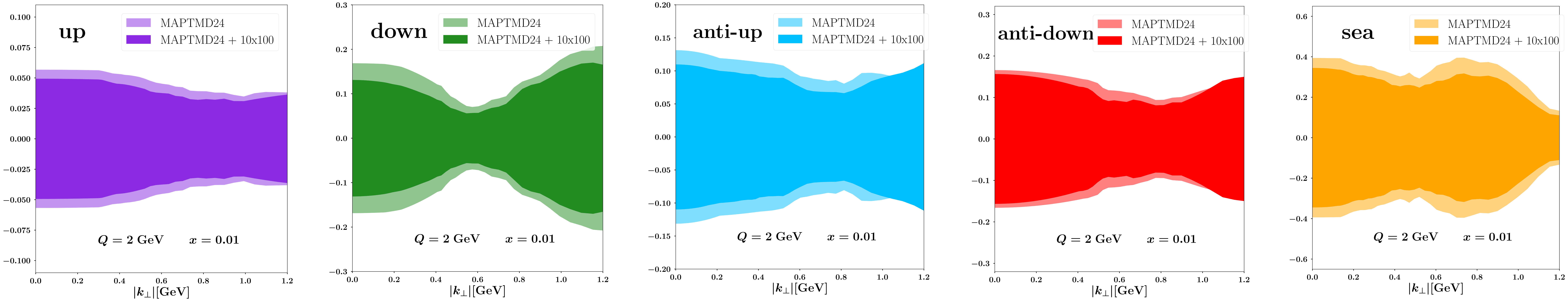


EIC IMPACT, ONE CONFIGURATION

Simulations by Lorenzo Rossi

Combined fit with existing data (2031 points) + EIC,10 x 100, 5 fb^{-1} (1611 points)

$x = 0.01$



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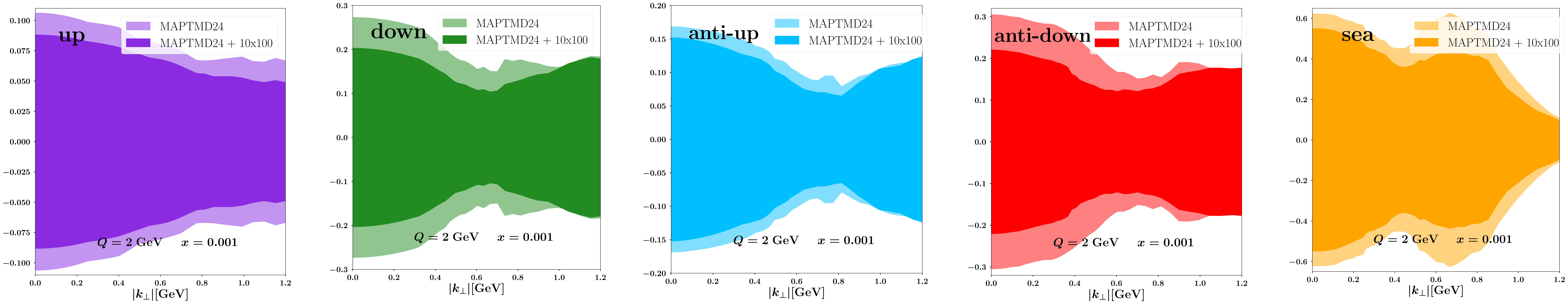
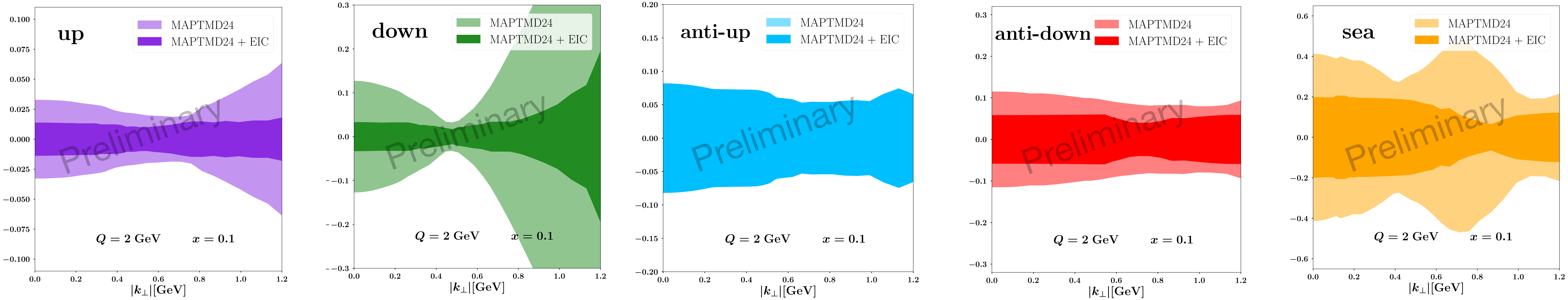


EIC IMPACT, MORE CONFIGURATIONS

Combined fit with existing data (2031 points) + EIC

Energy	Points	Lumi
5x41	1273	2.85
10x100	1611	5.13
18x275	1648	10

$$x = 0.1$$



Lorenzo Rossi's PhD Thesis

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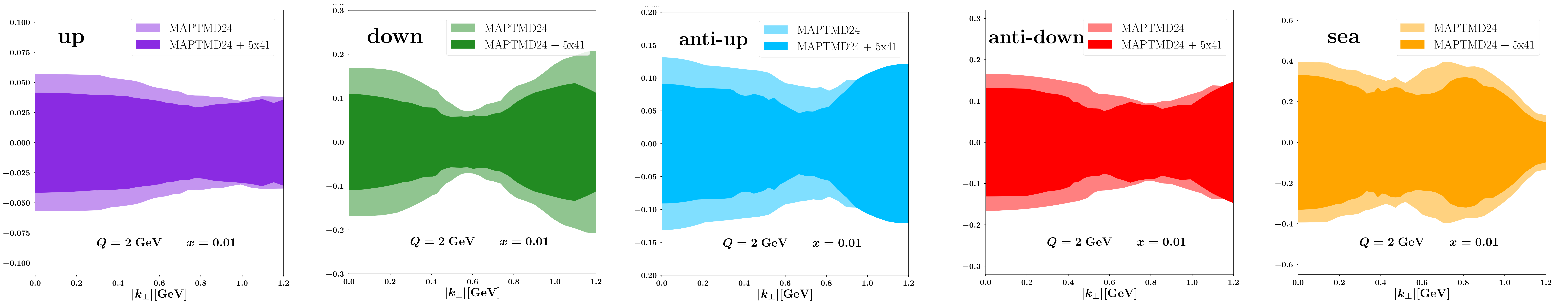
Combined fit with existing data (2031 points) + EIC

$x = 0.01$



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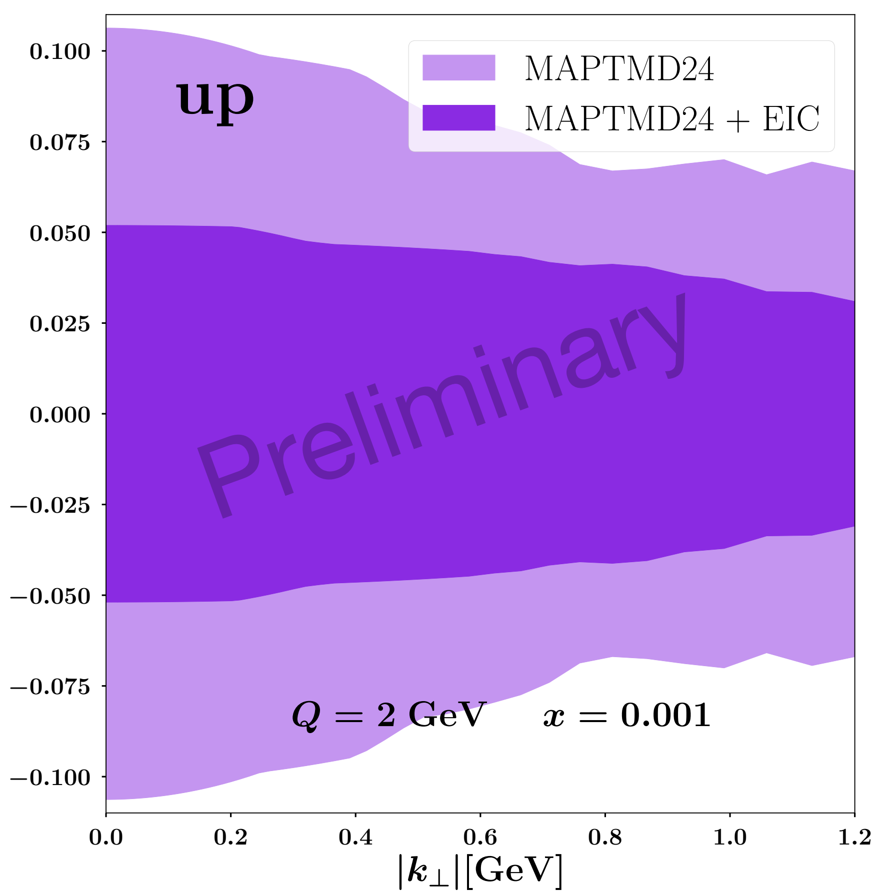
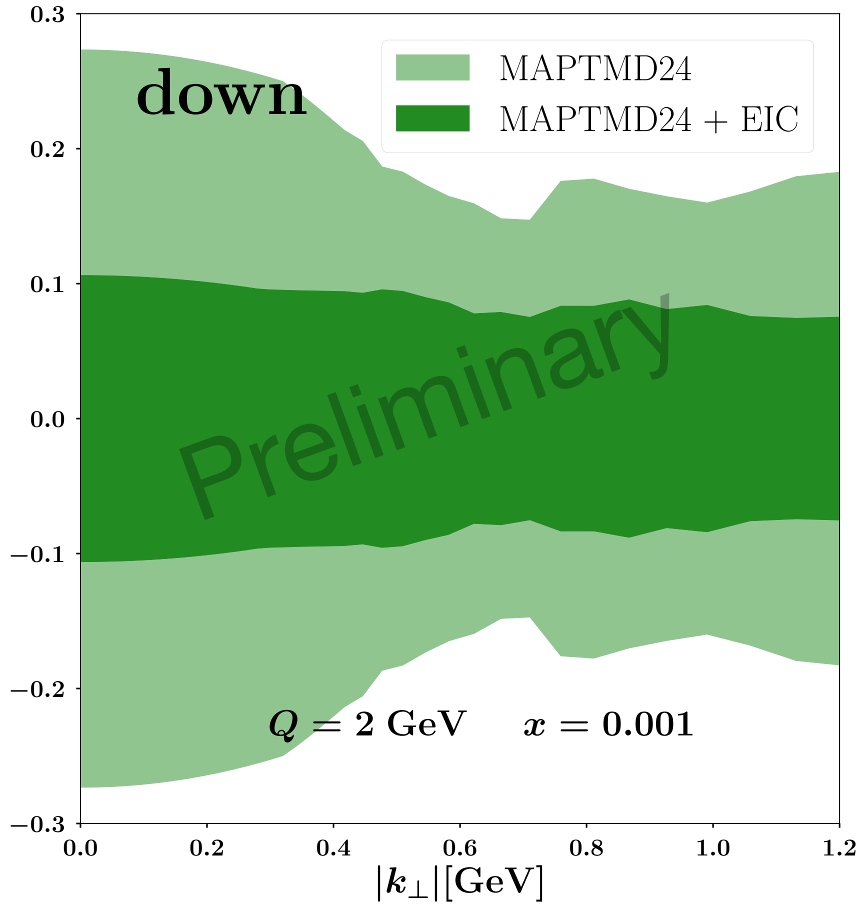
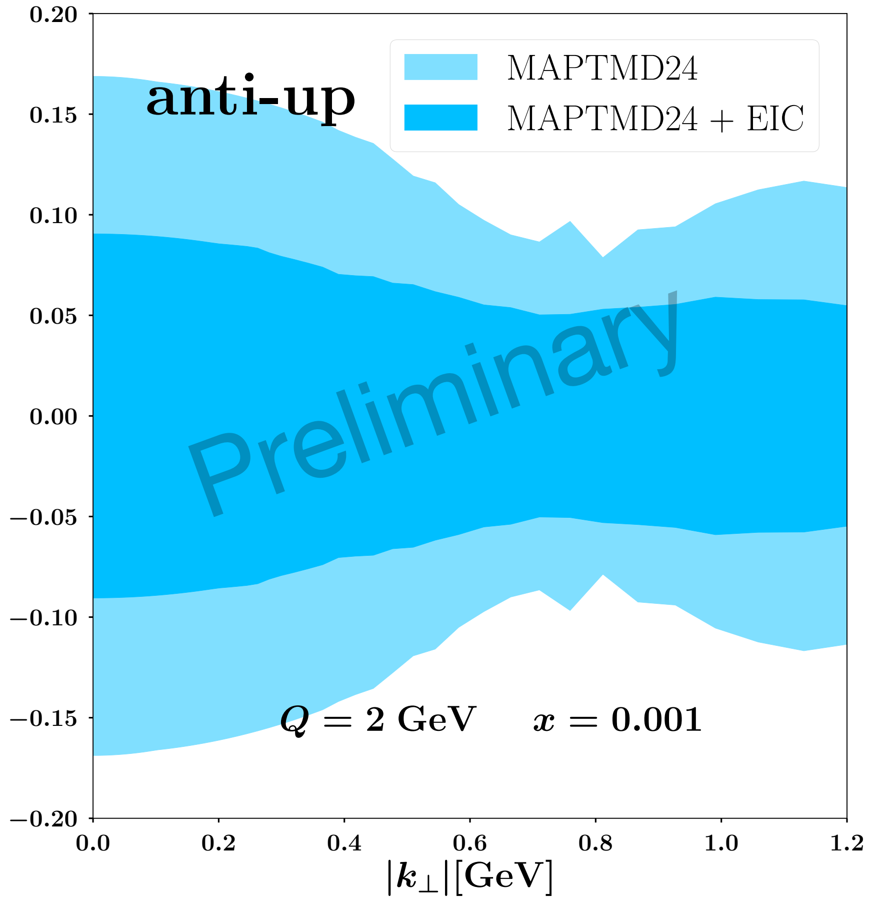
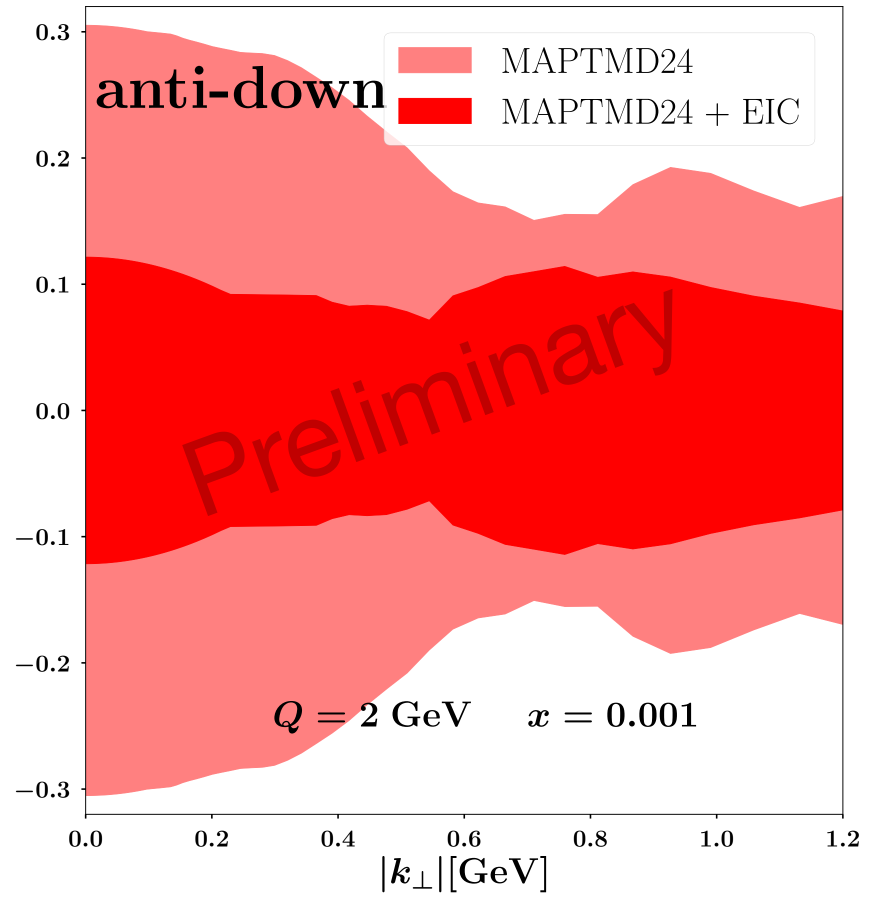
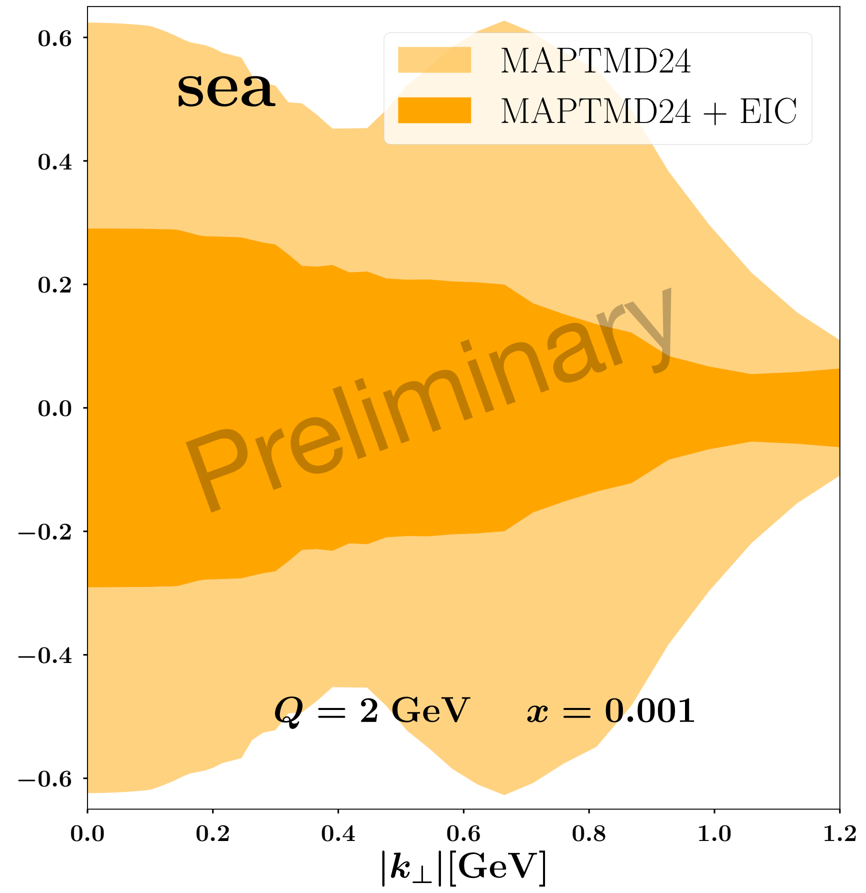






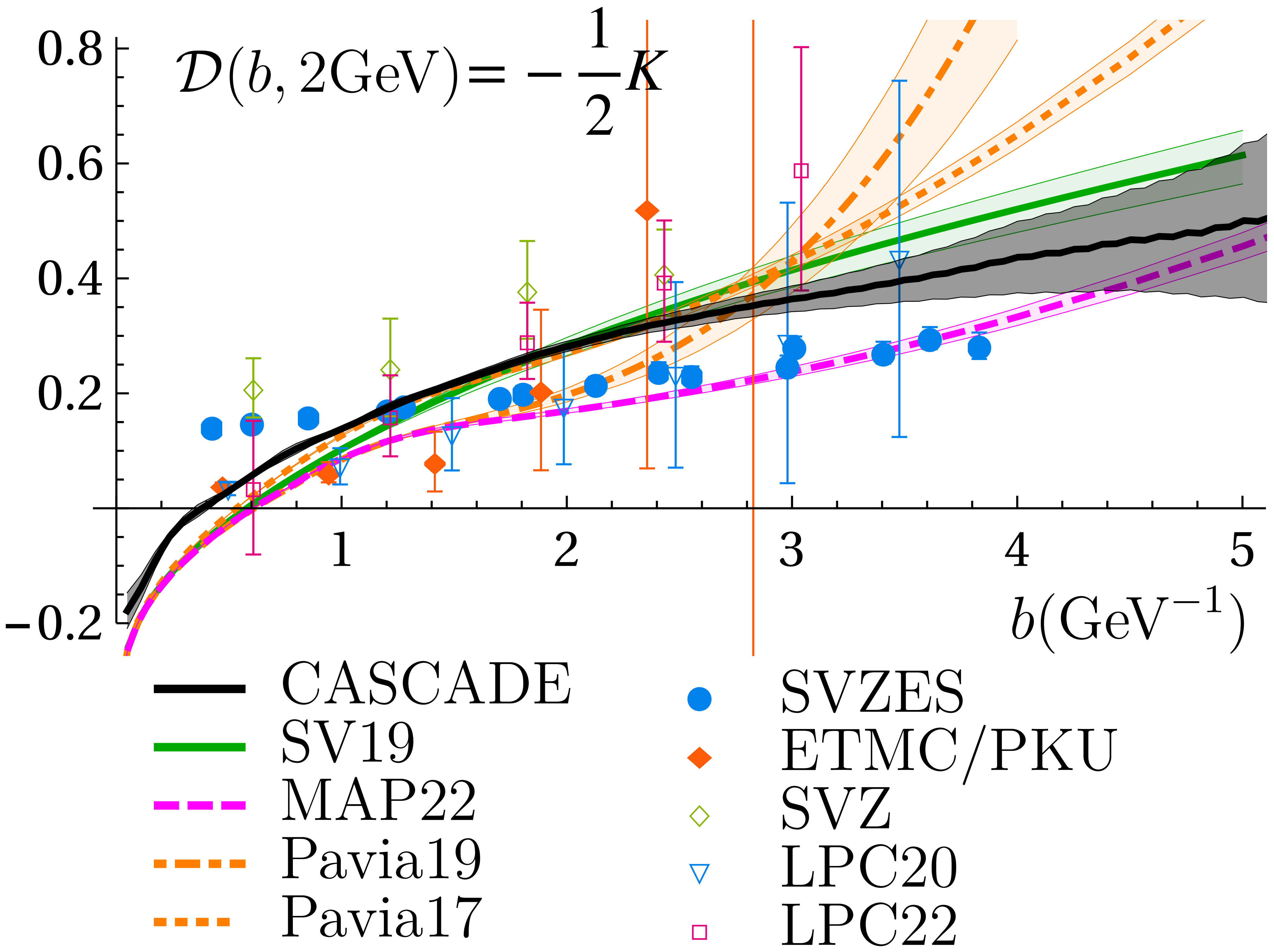




Lorenzo Rossi's PhD Thesis

COLLINS-SOPER KERNEL

Bermudez Martinez, Vladimirov, arXiv:2206.01105



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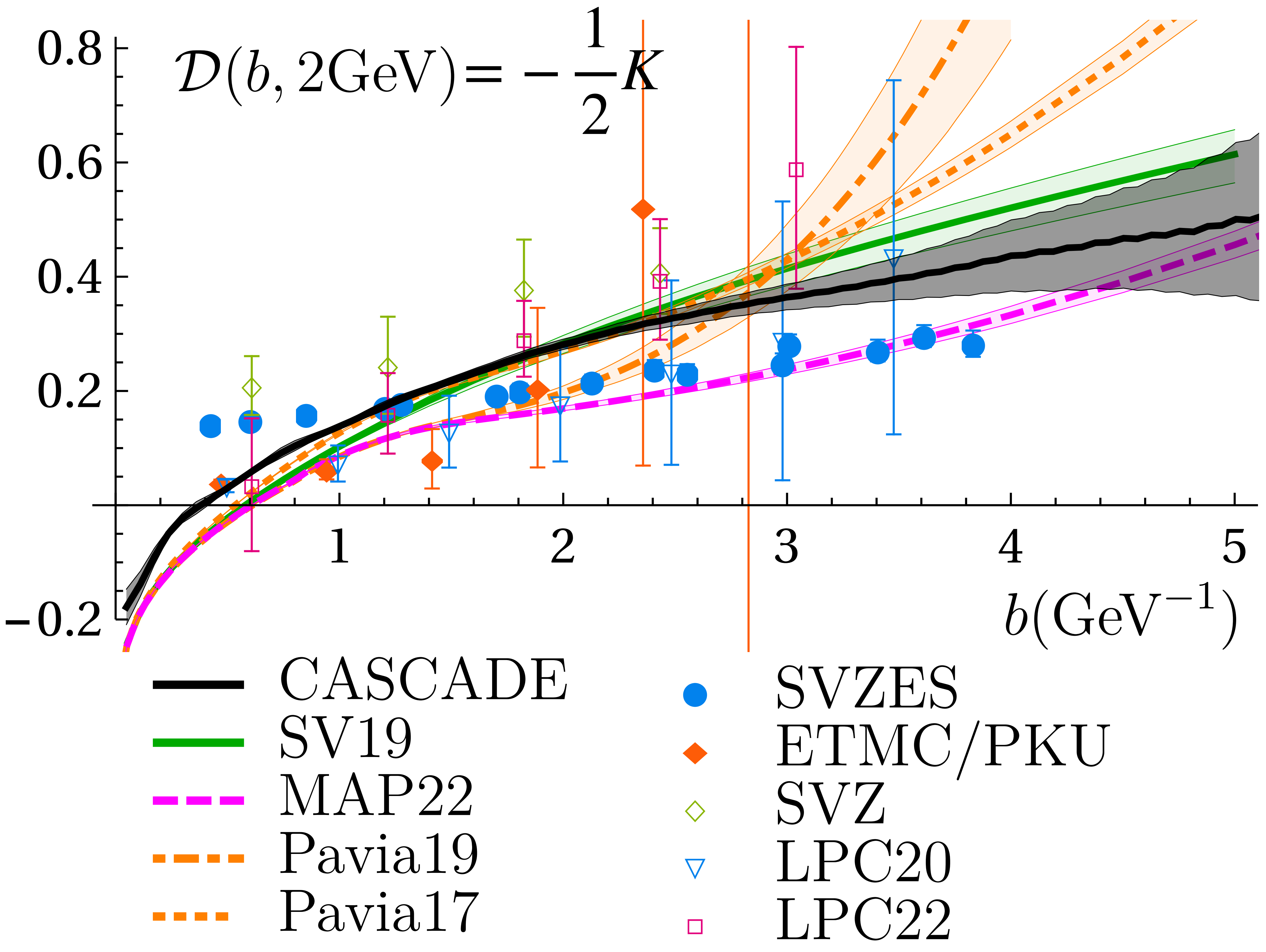

TMD phenomenology

COLLINS–SOPER KERNEL

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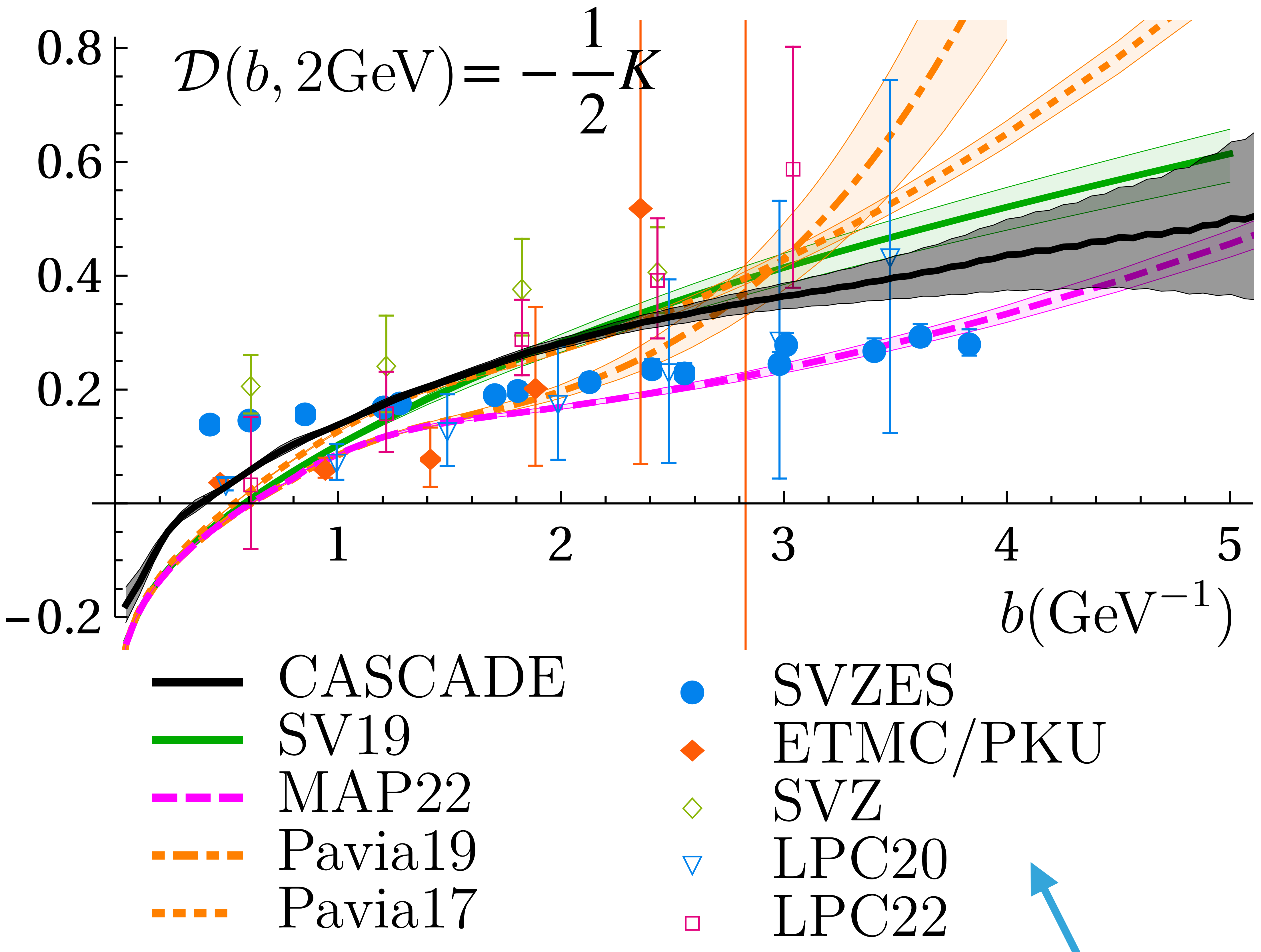


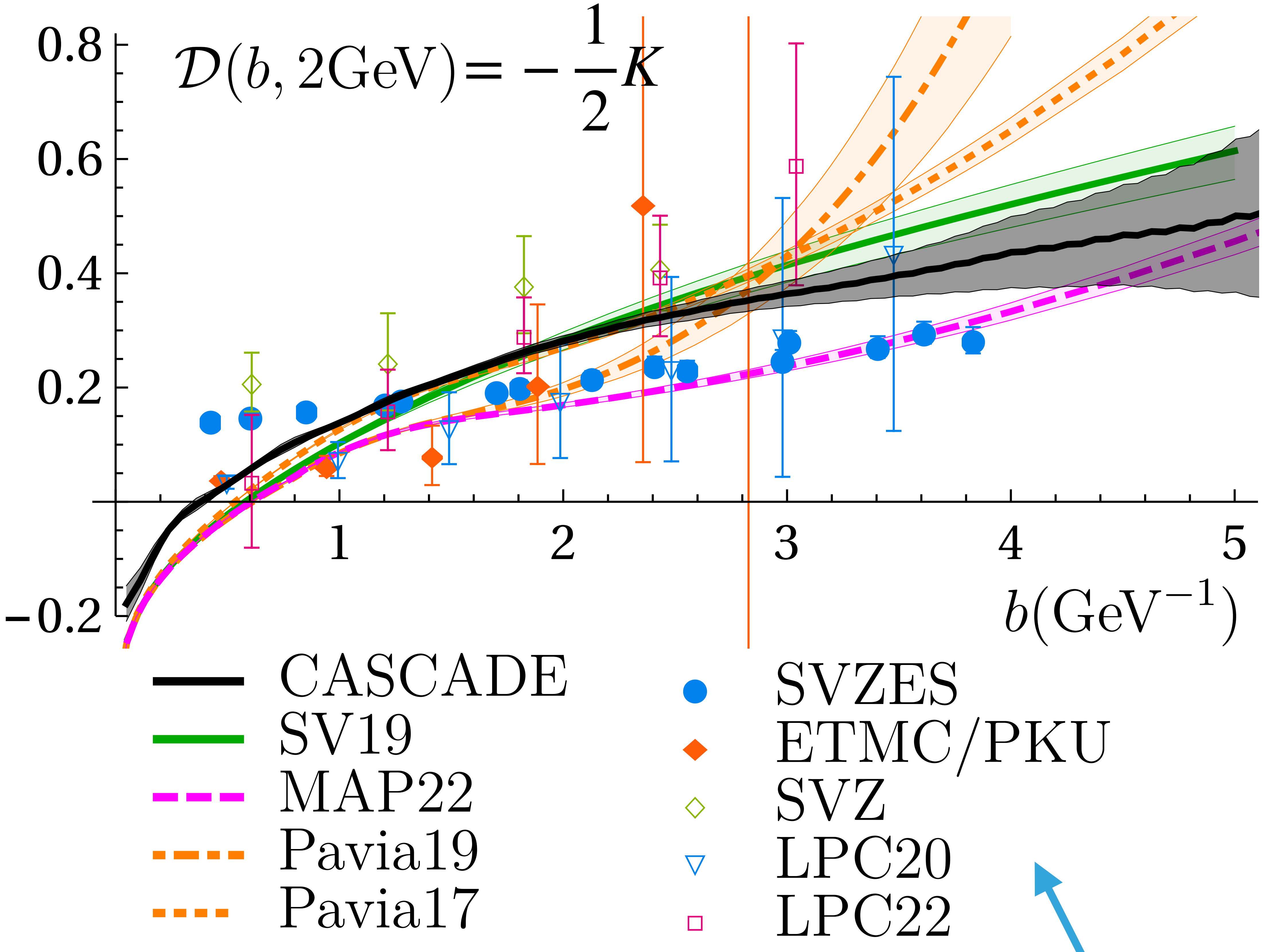
TMD phenomenology

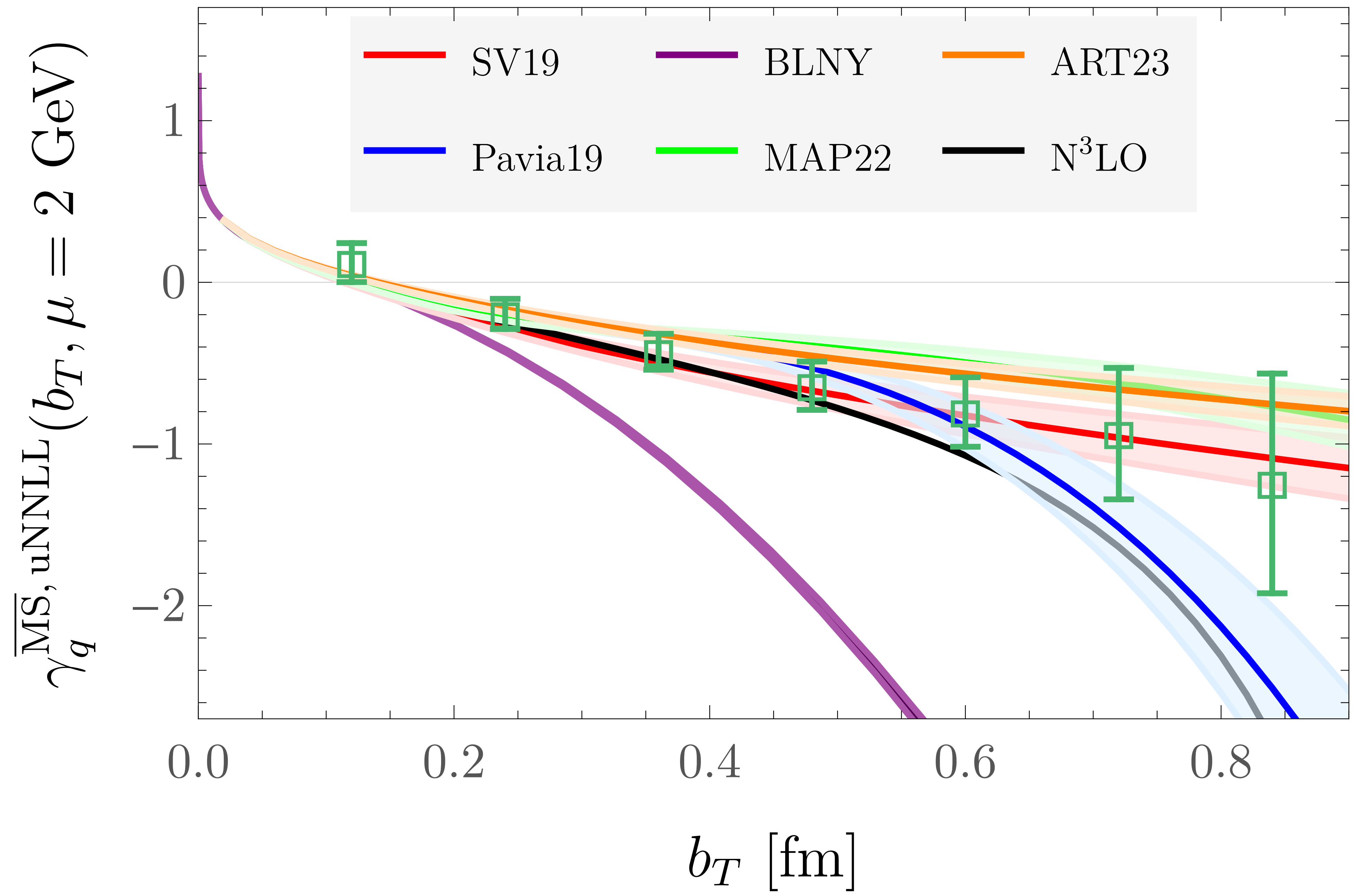
Lattice QCD

COLLINS-SOPER KERNEL

Bermudez Martinez, Vladimirov, arXiv:2206.01105

Avkhadiev, Shanahan, Wagman, Zhao, arXiv:2307.12359





TMD phenomenology

Lattice QCD

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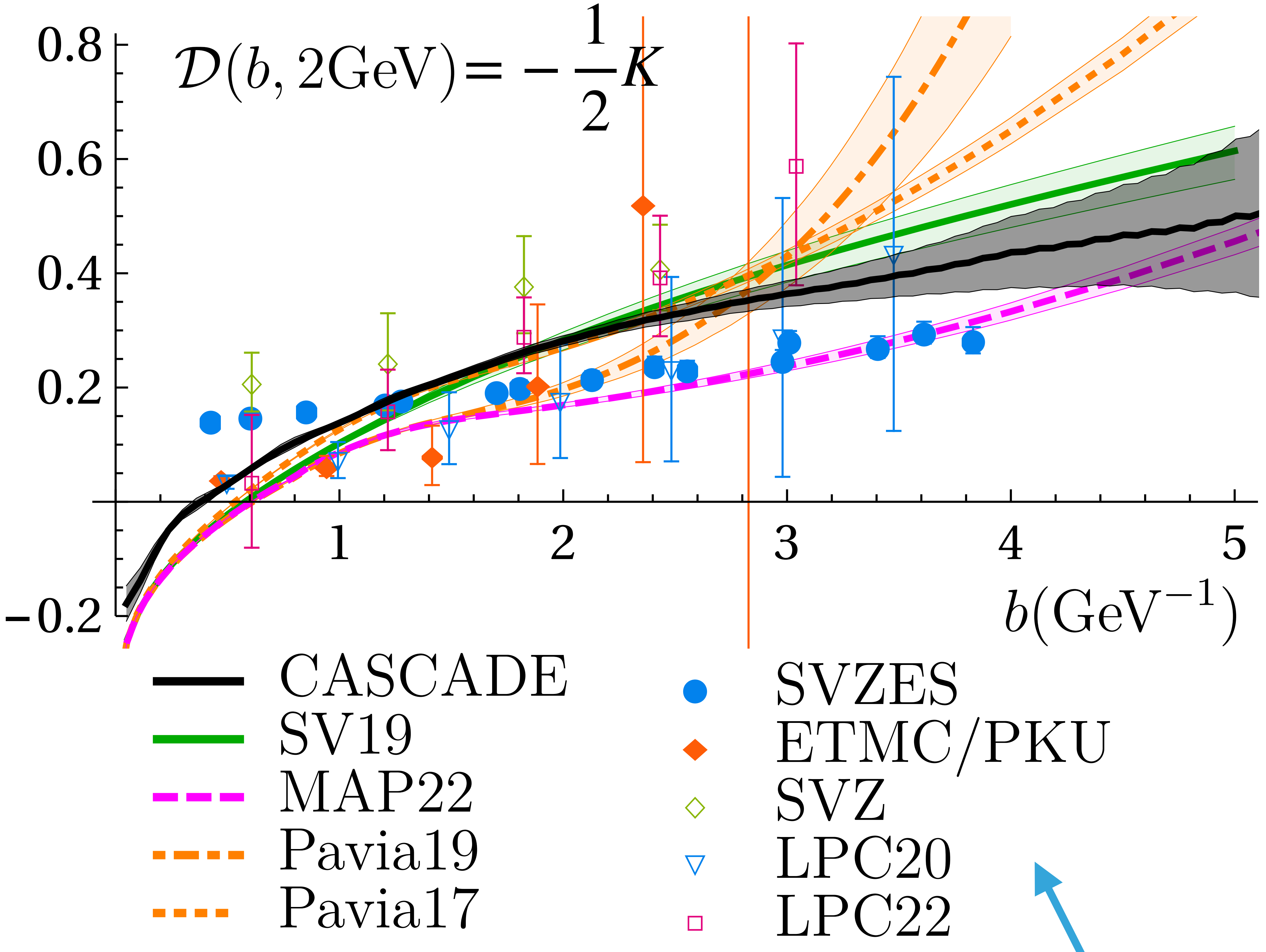
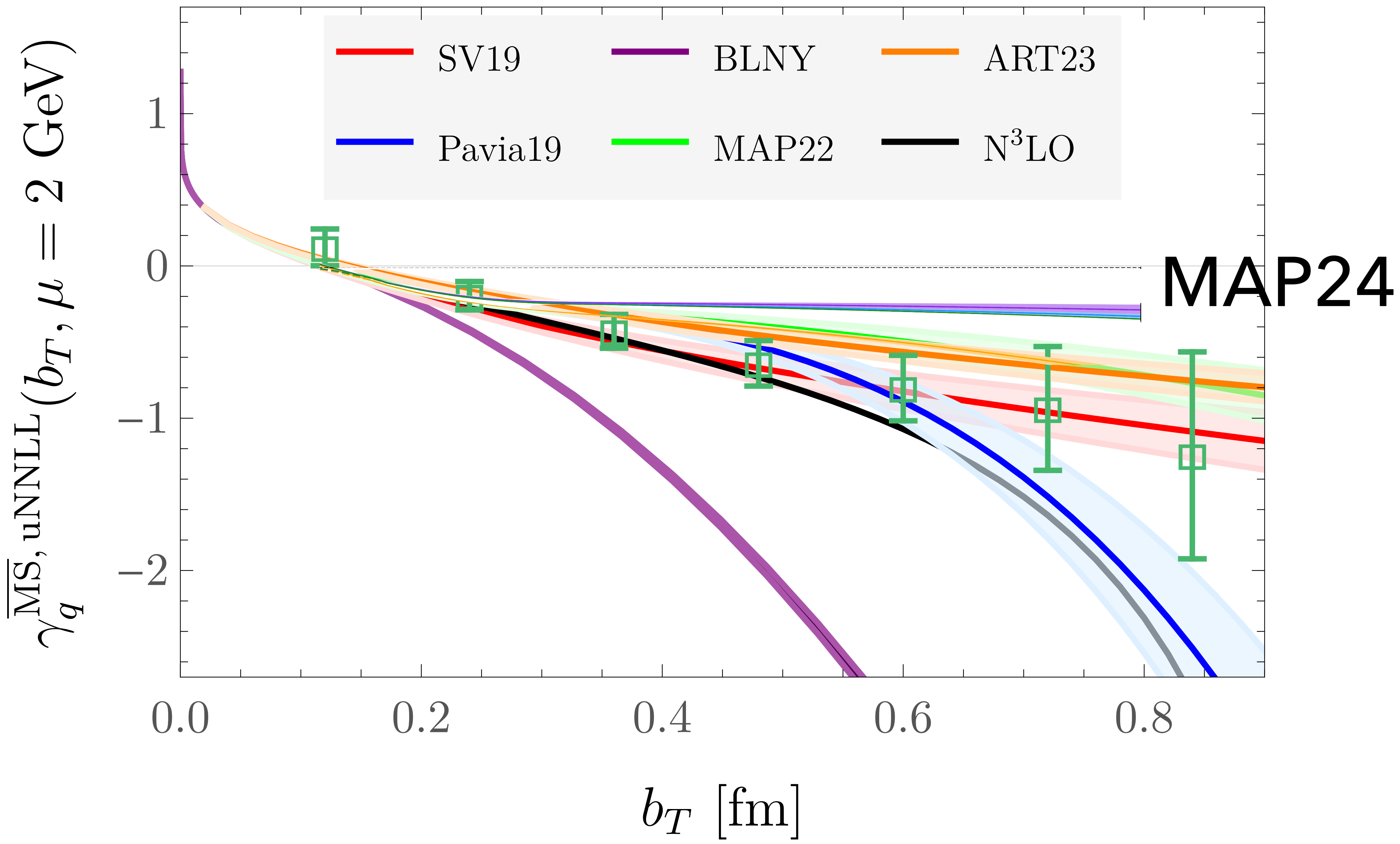


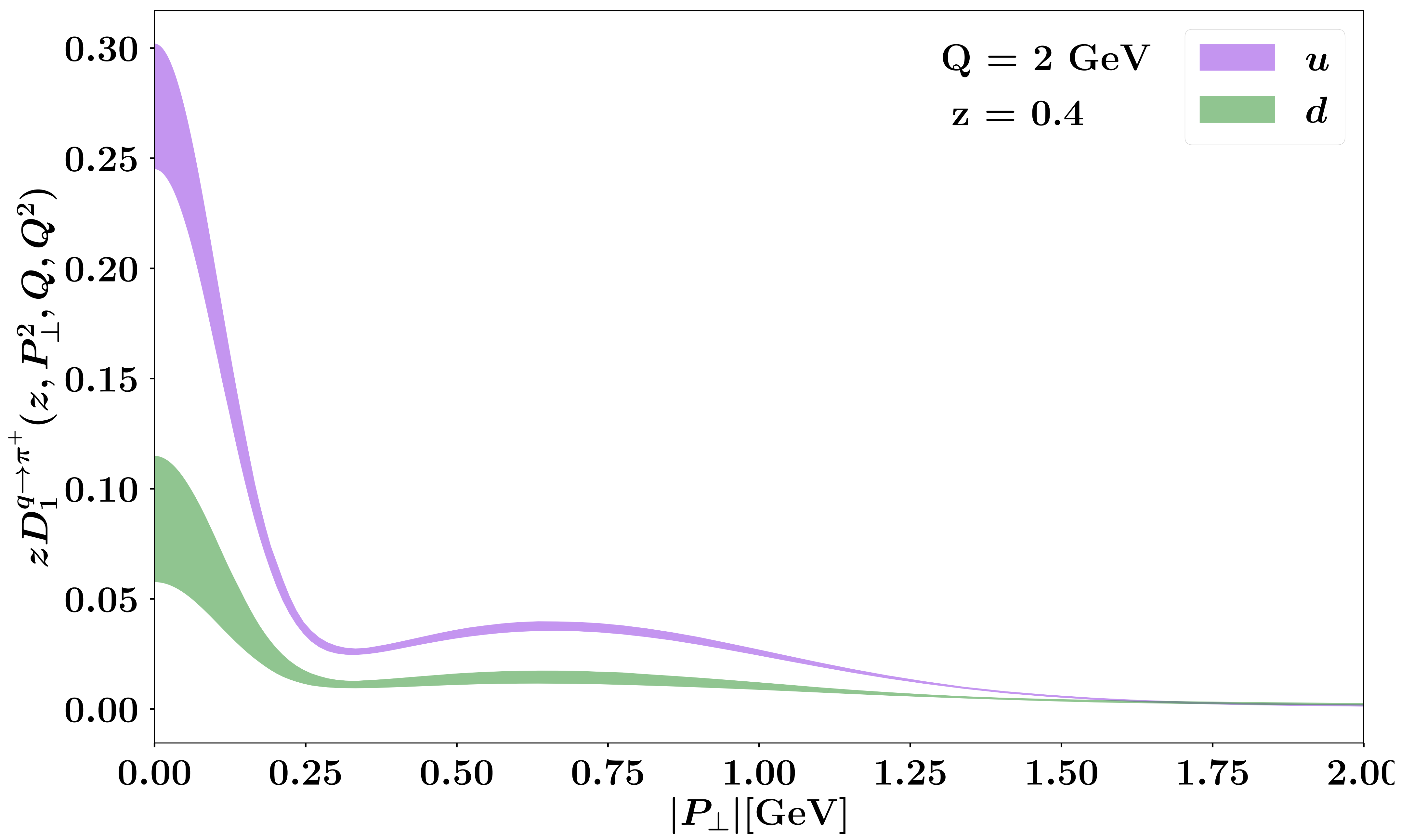


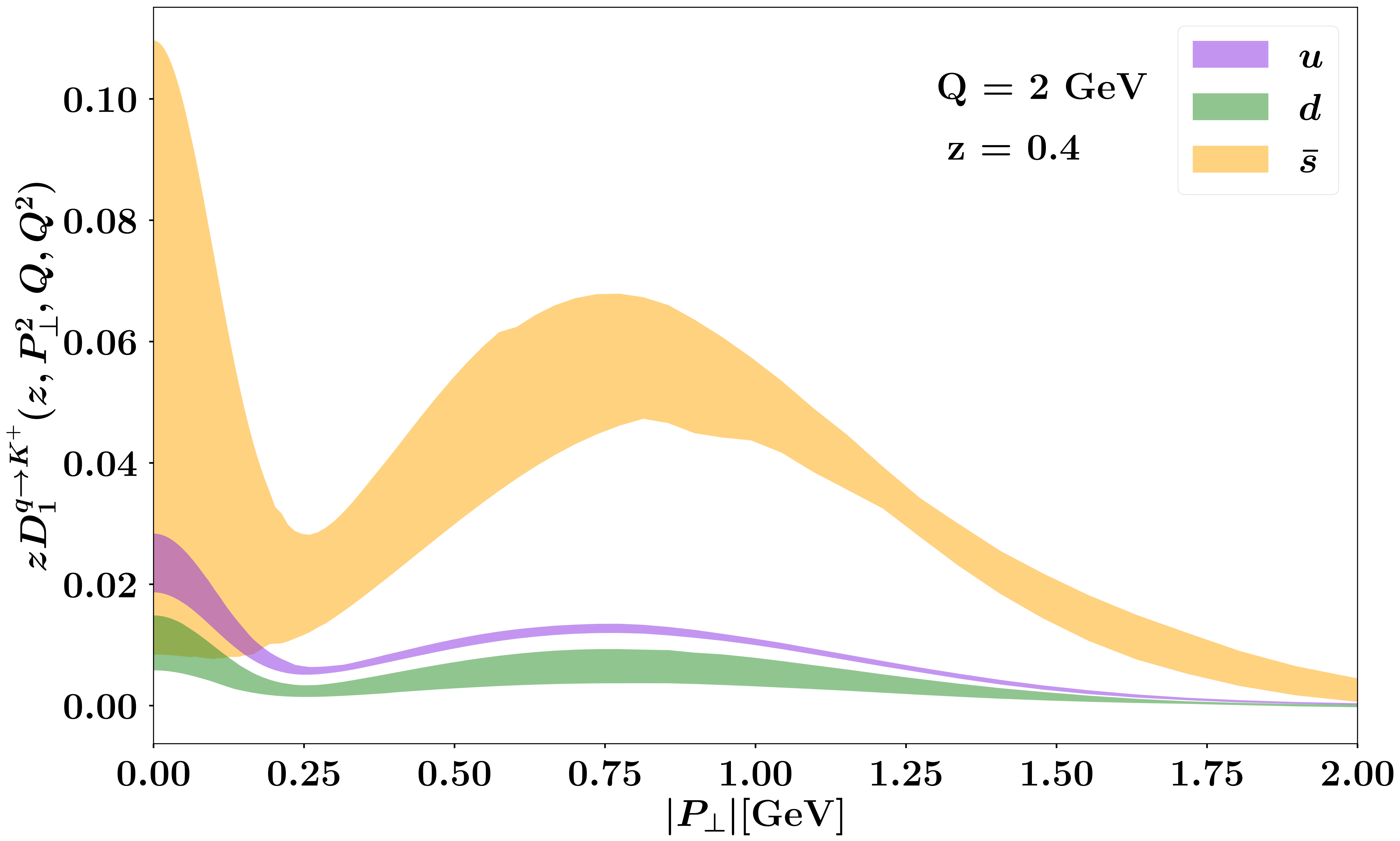
TMD phenomenology

Lattice QCD

FLAVOR-DEPENDENT TMD FRAGMENTATION FUNCTIONS

MAP Collaboration, arXiv:2405.13833





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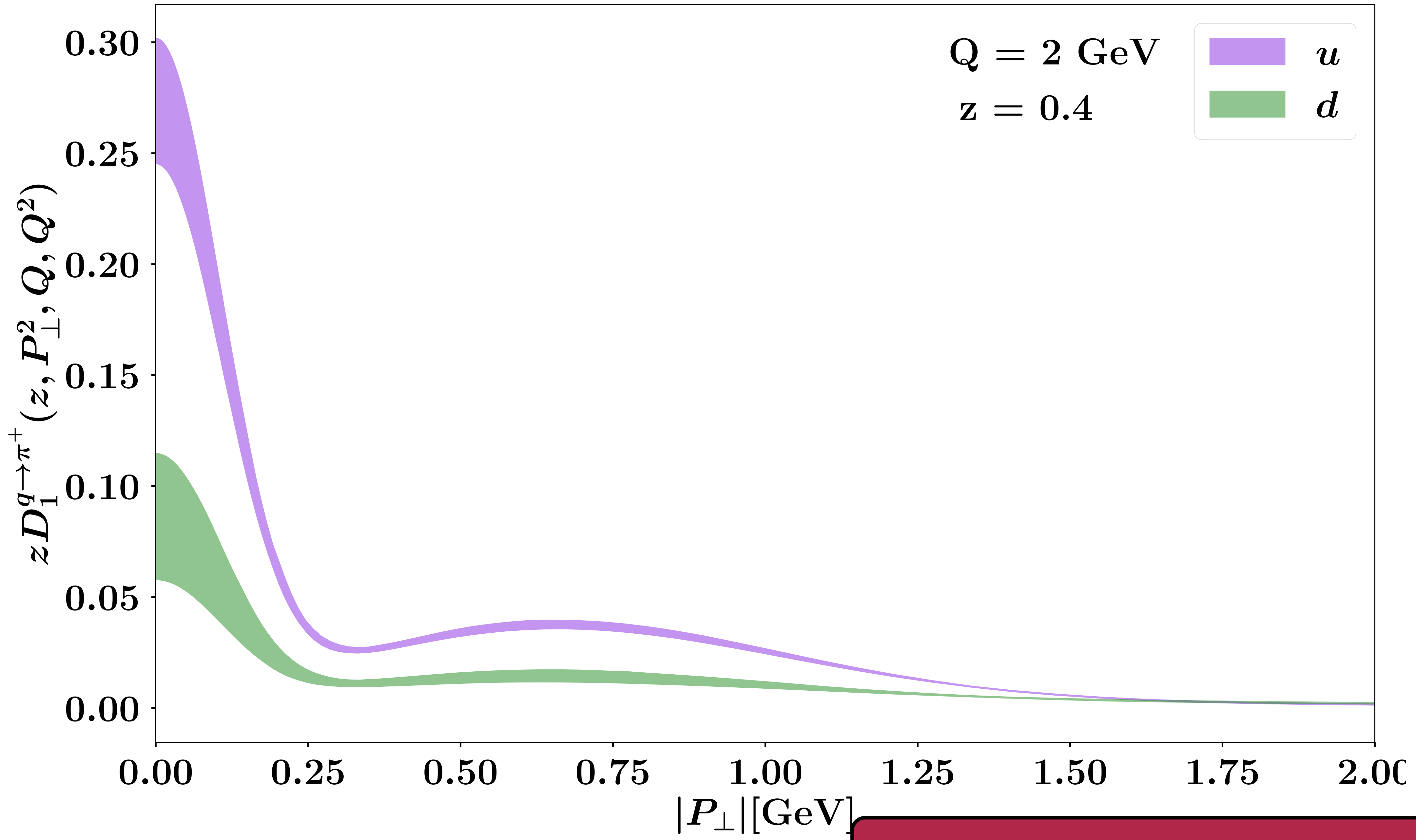
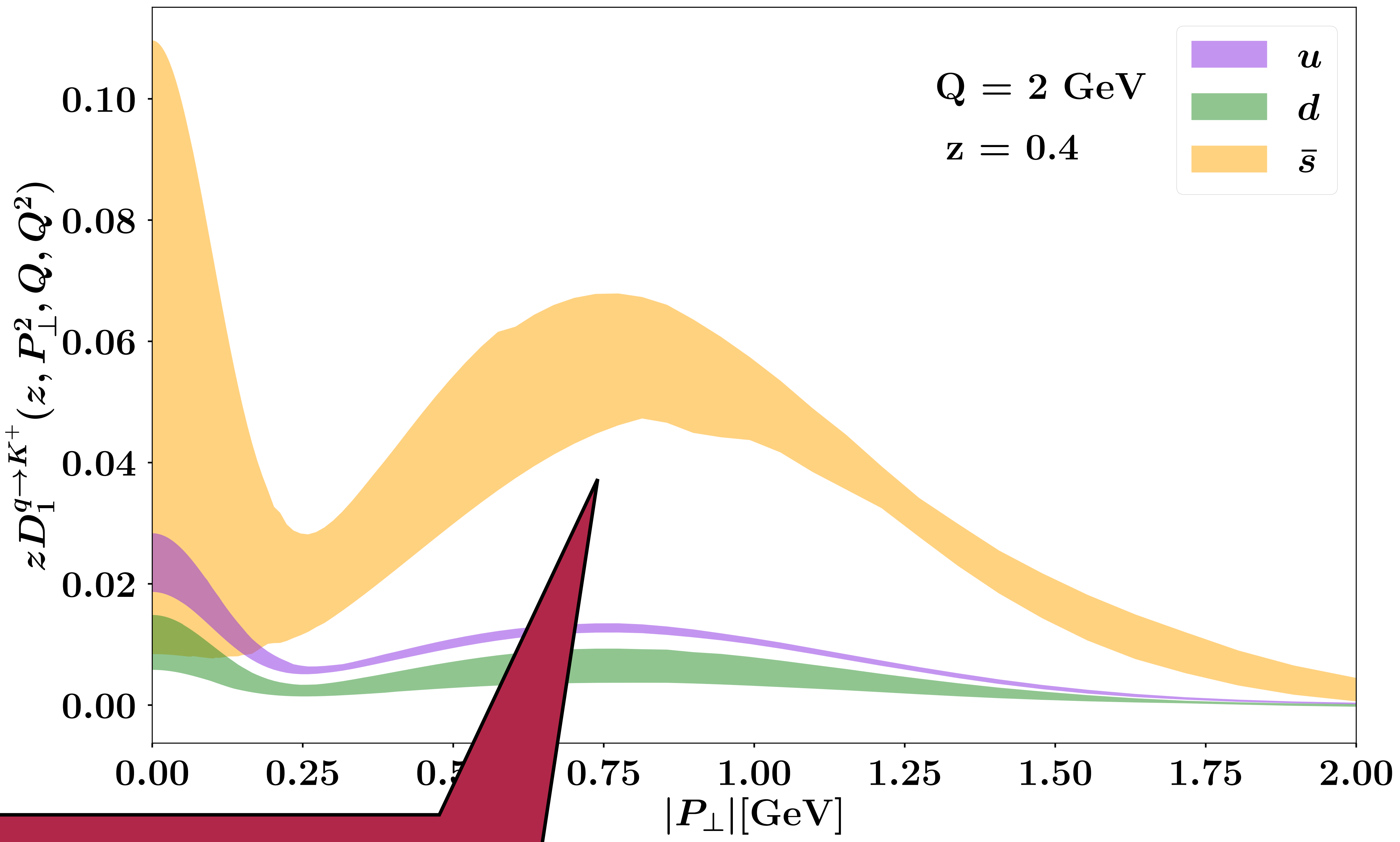
MAP Collaboration, arXiv:2405.13833

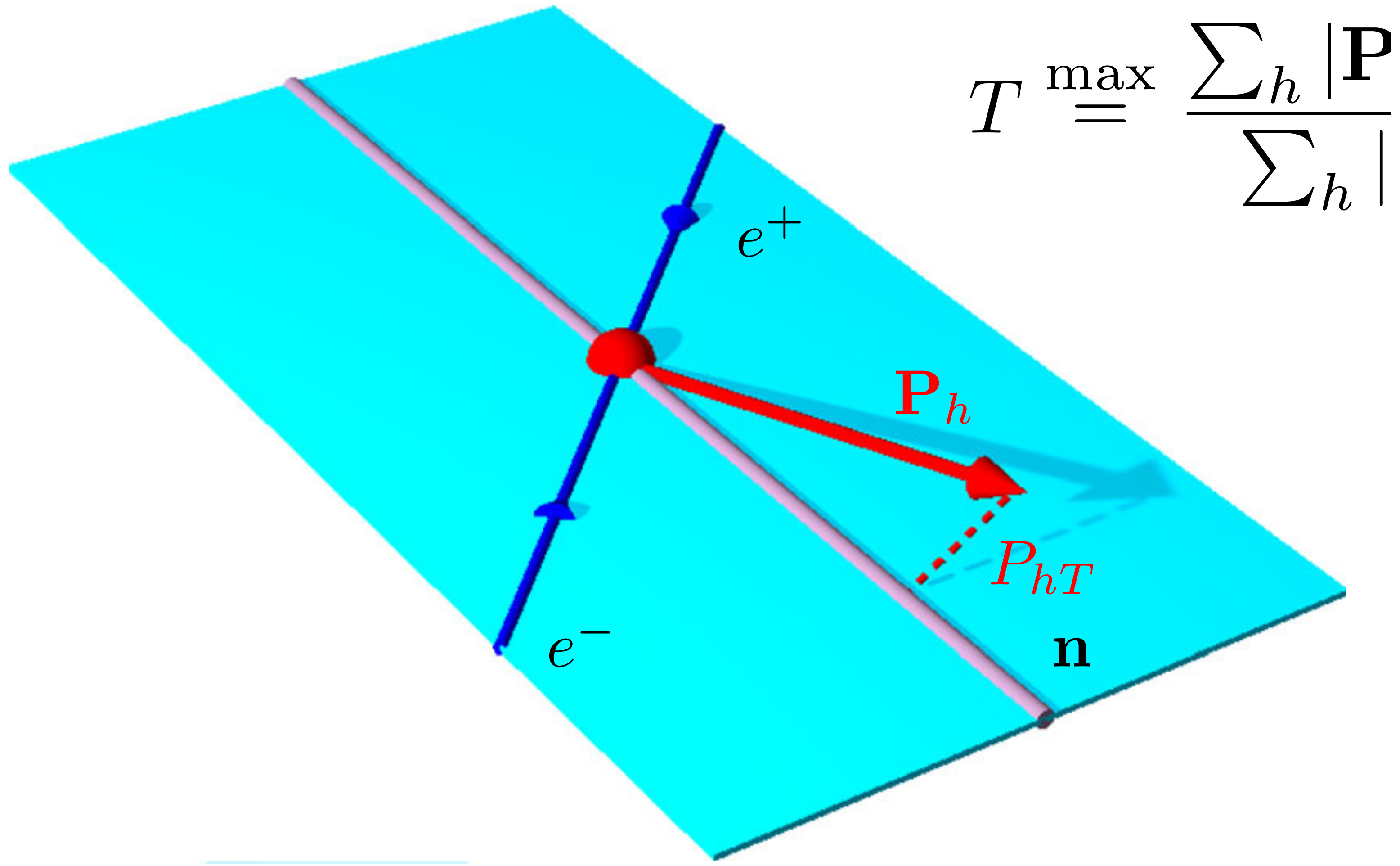


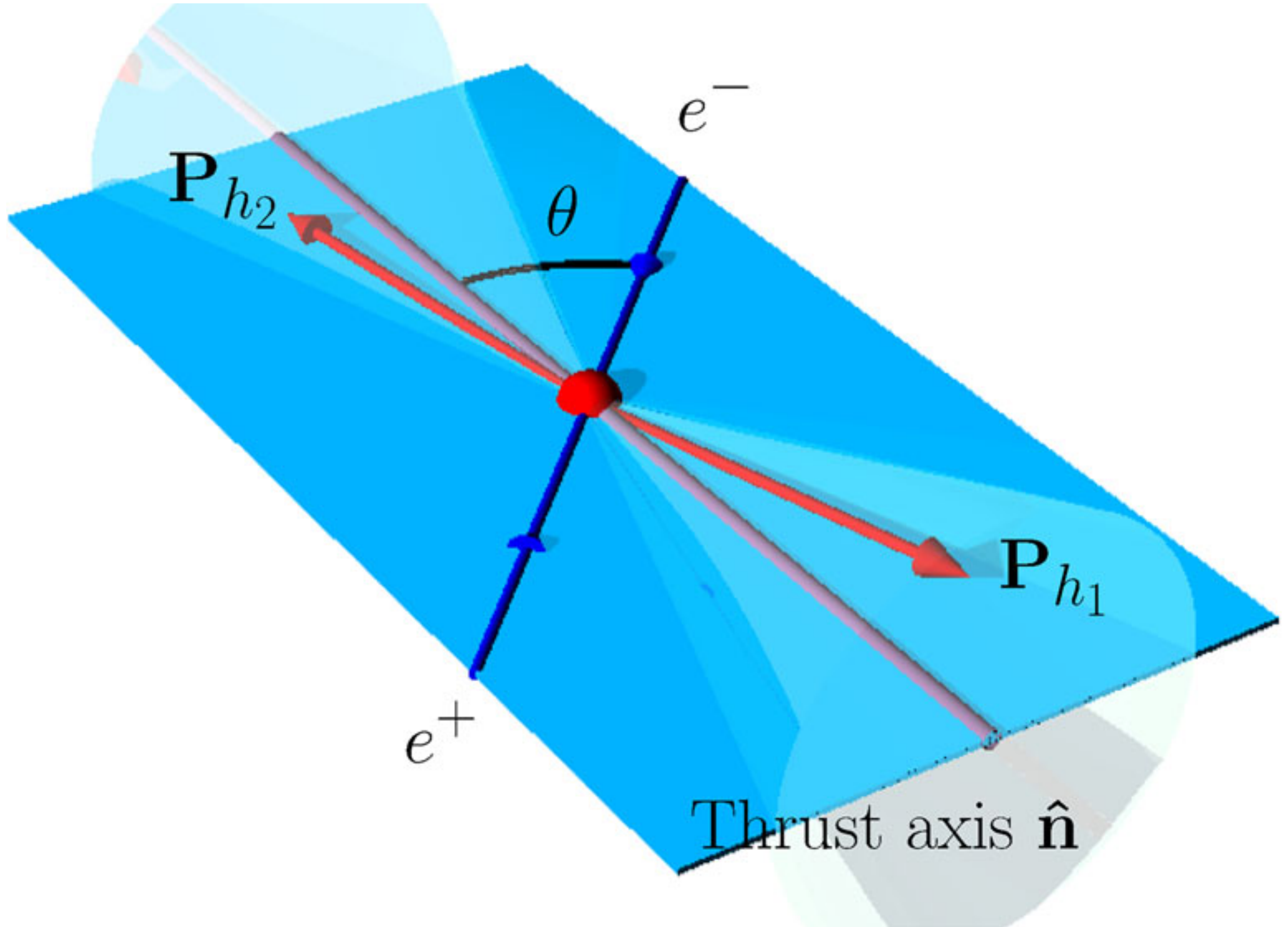


Very different from a simple Gaussian:
is it reliable?

TRANSVERSE MOMENTUM IN FRAGMENTATION FUNCTIONS

Seidl et al., arXiv:1902.01552





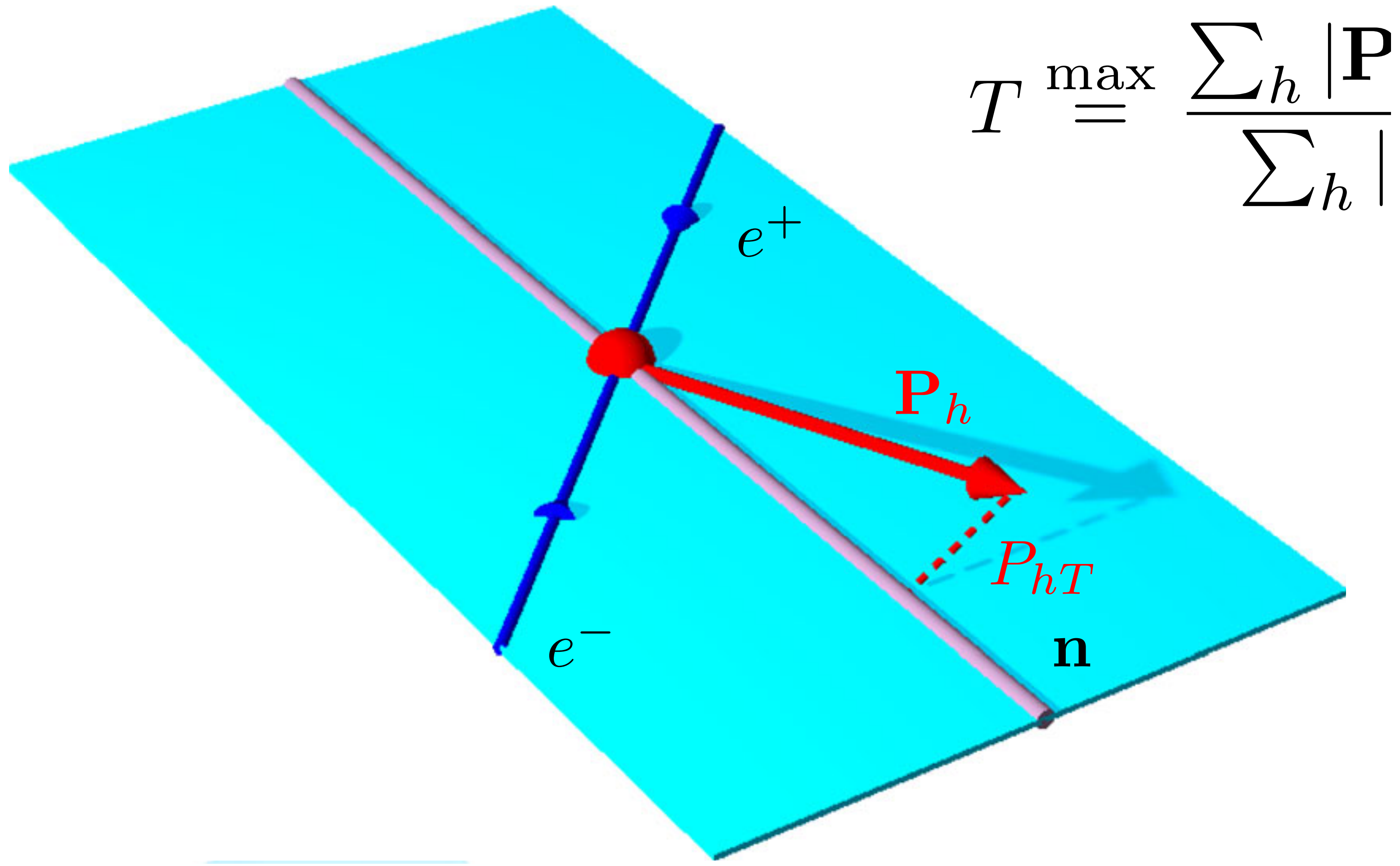
First direct measurement of TMD effects in fragmentation functions

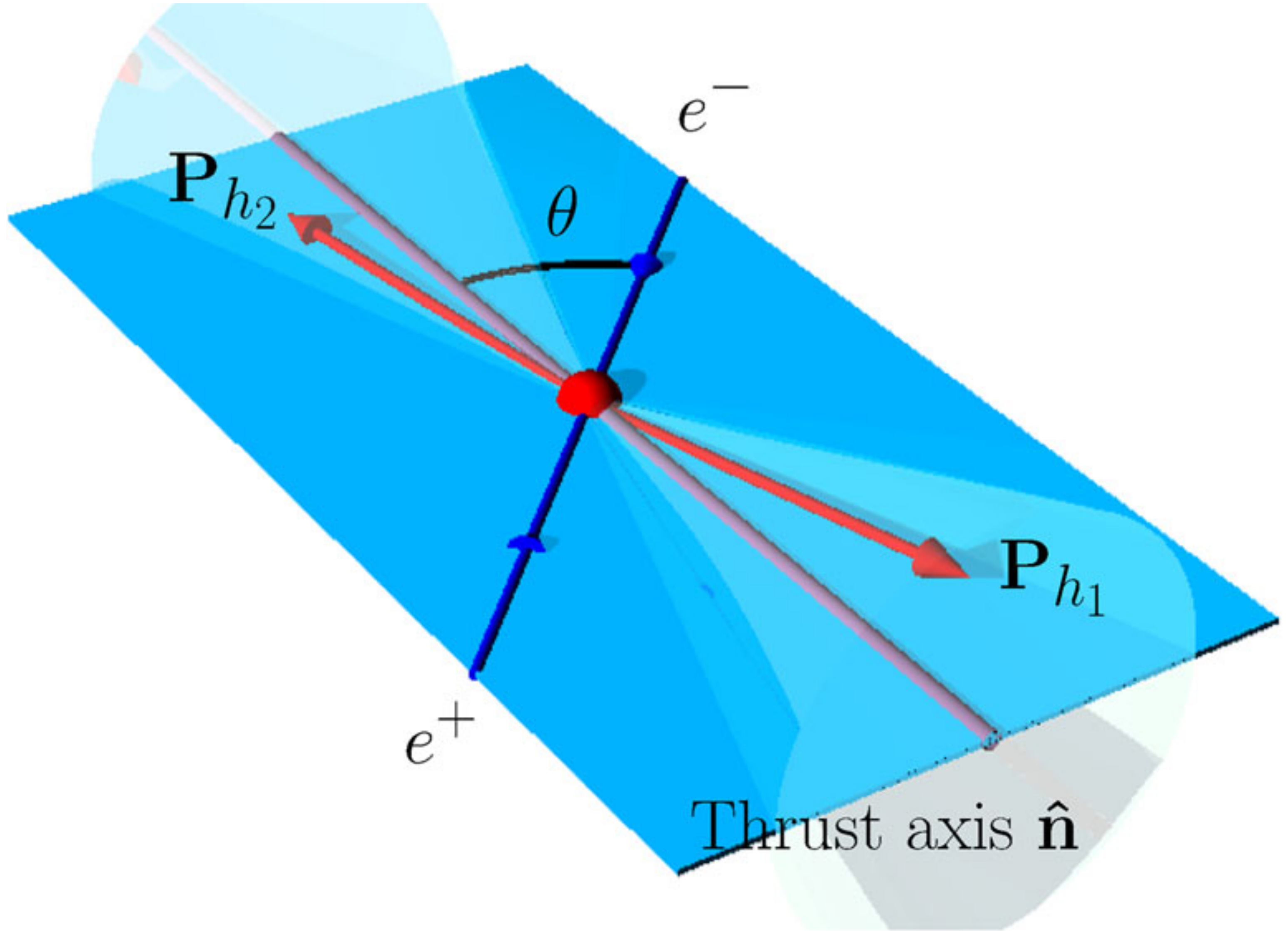
Makes use of thrust axis: the formalism should take it into account

TRANSVERSE MOMENTUM IN FRAGMENTATION FUNCTIONS

BELLE

Seidl et al., arXiv:1902.01552





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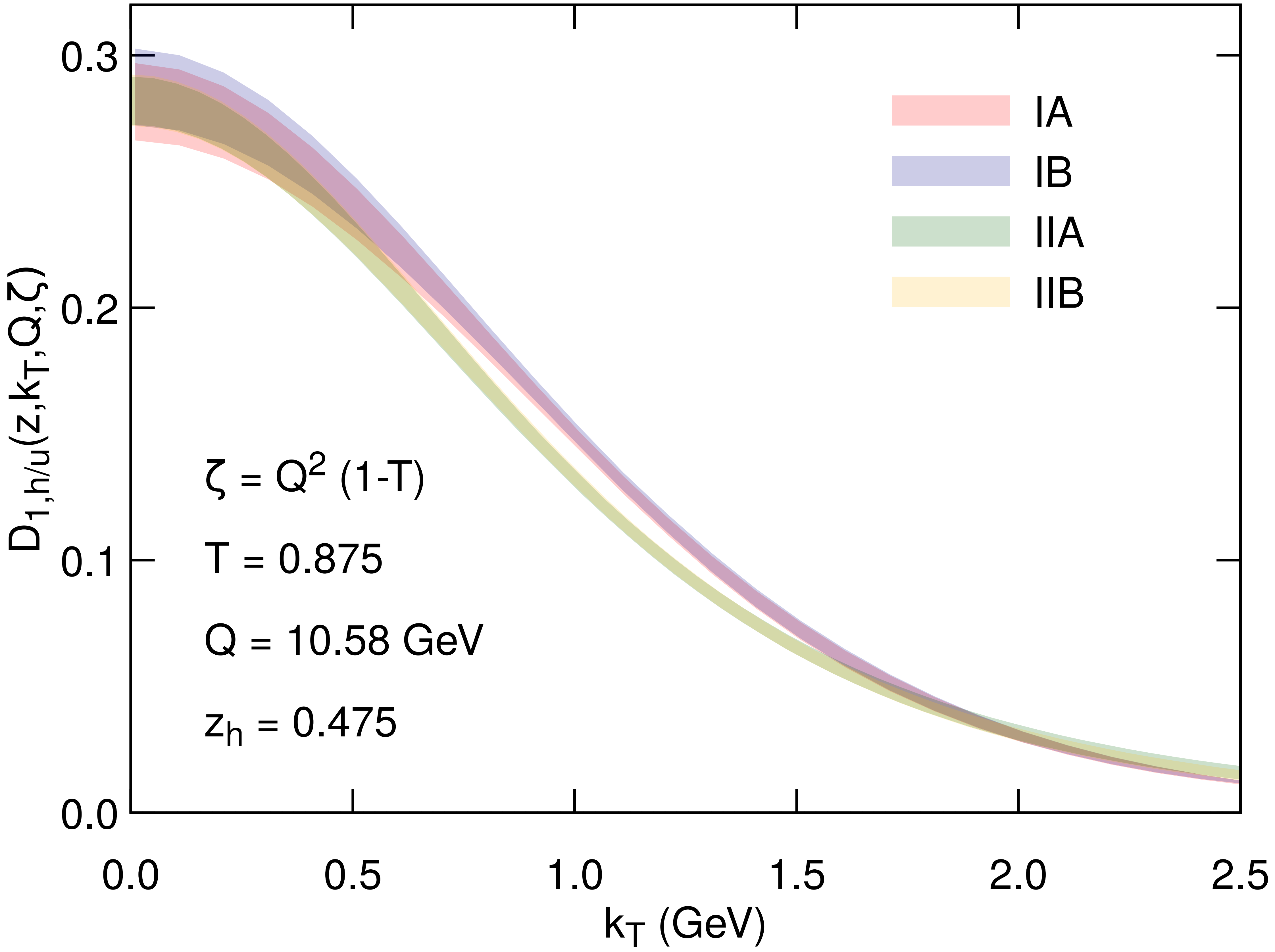

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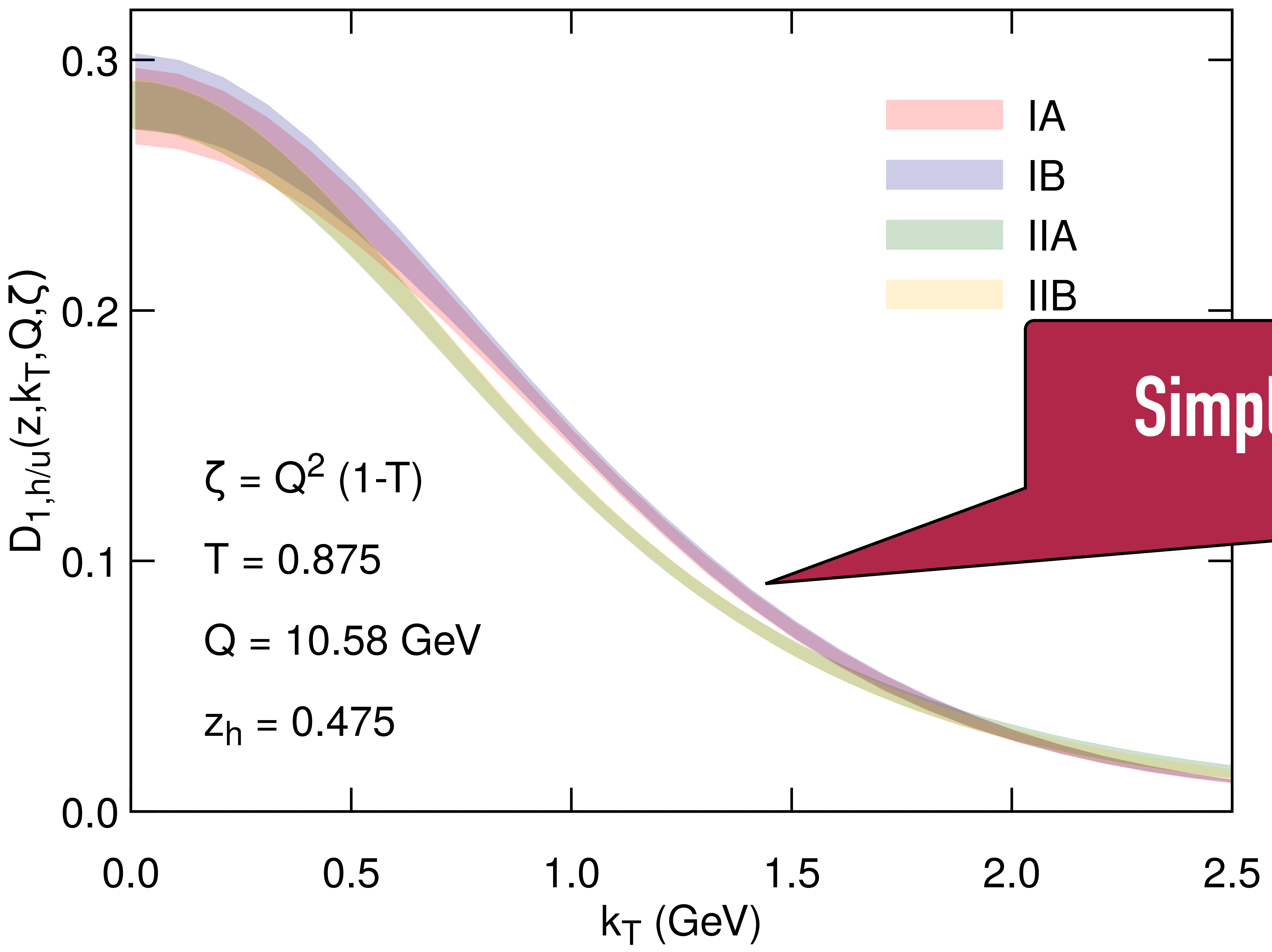
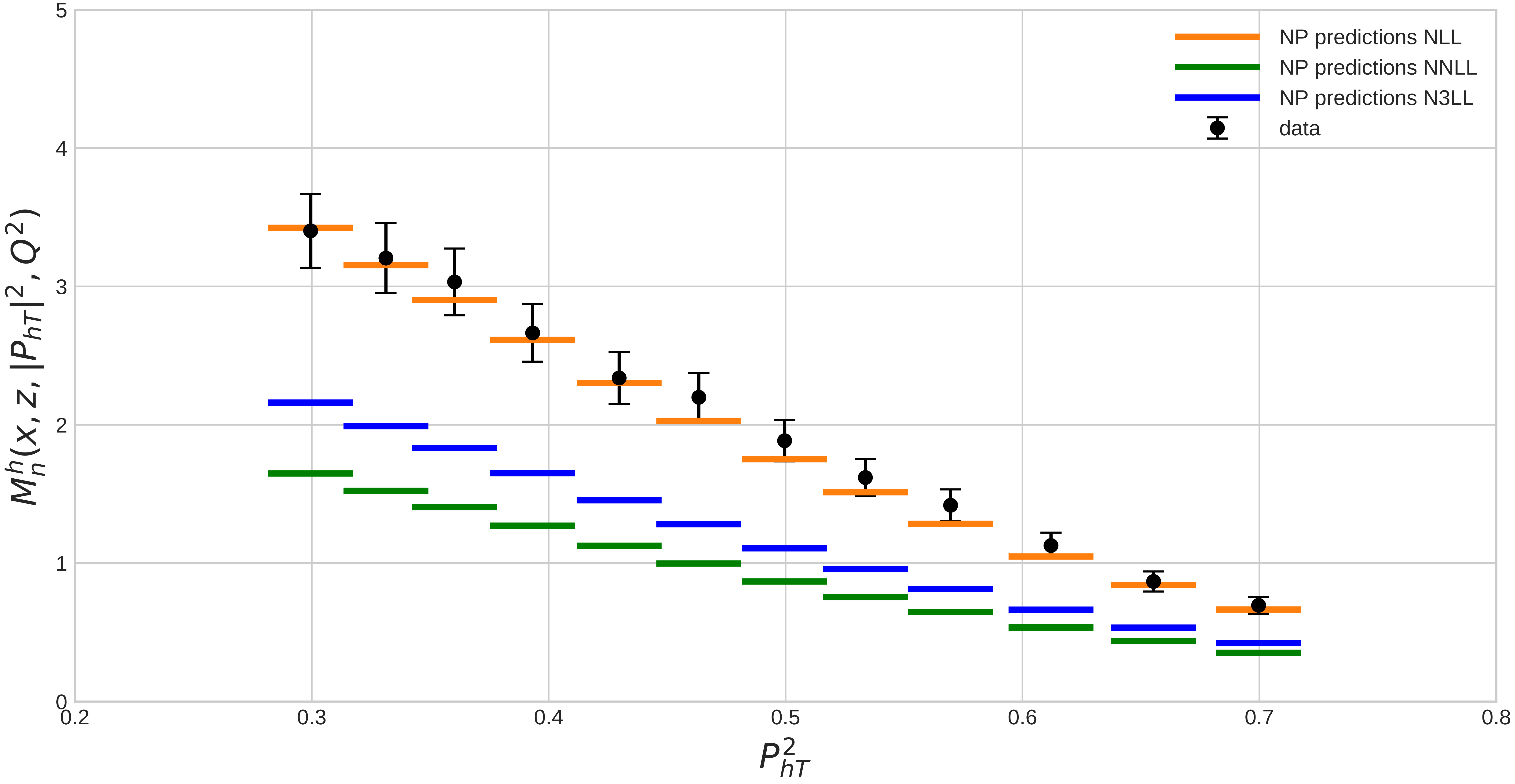


Simpler shape works well

Boglione, Gonzalez-Hernandez, Simonelli, https://arxiv.org/abs/2206.08876

IMPROVED ACCURACY AND SIDIS

COMPASS multiplicities (one of many bins)



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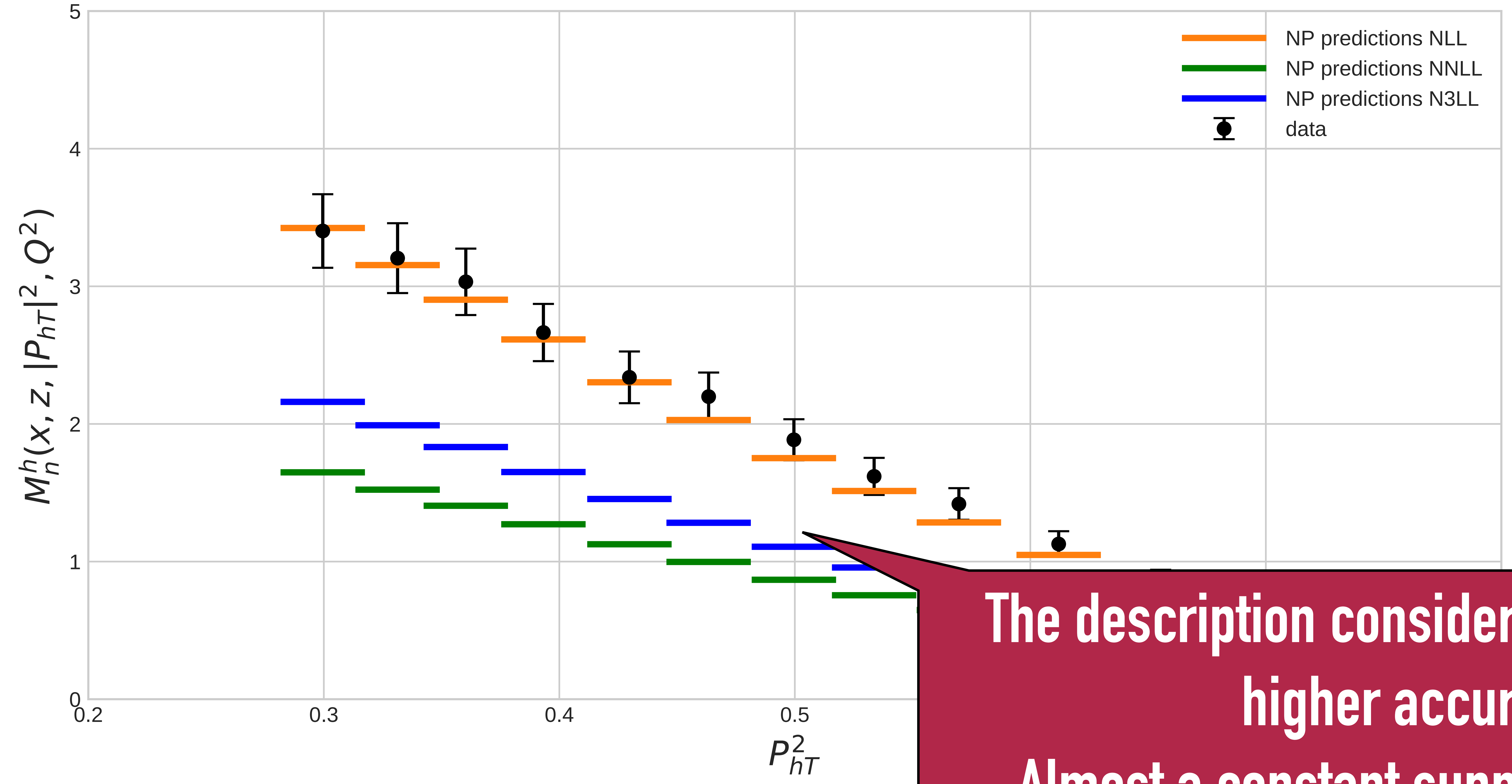
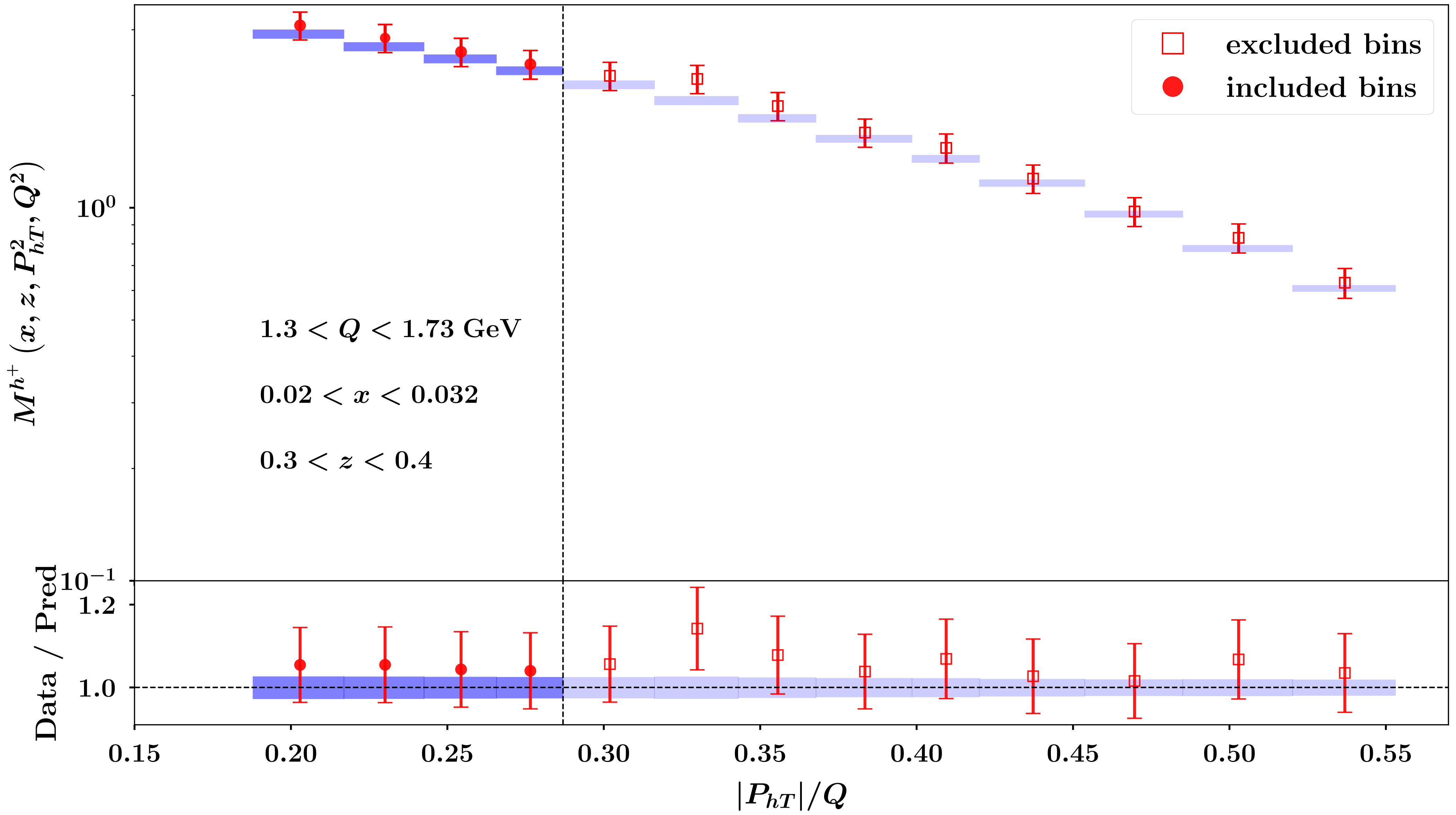


The description considerably worsens at higher accuracy.
Almost a constant suppression factor.

REGION OF VALIDITY OF TMD FORMALISM

$$|q_T| = |P_{hT}|/z \ll Q$$



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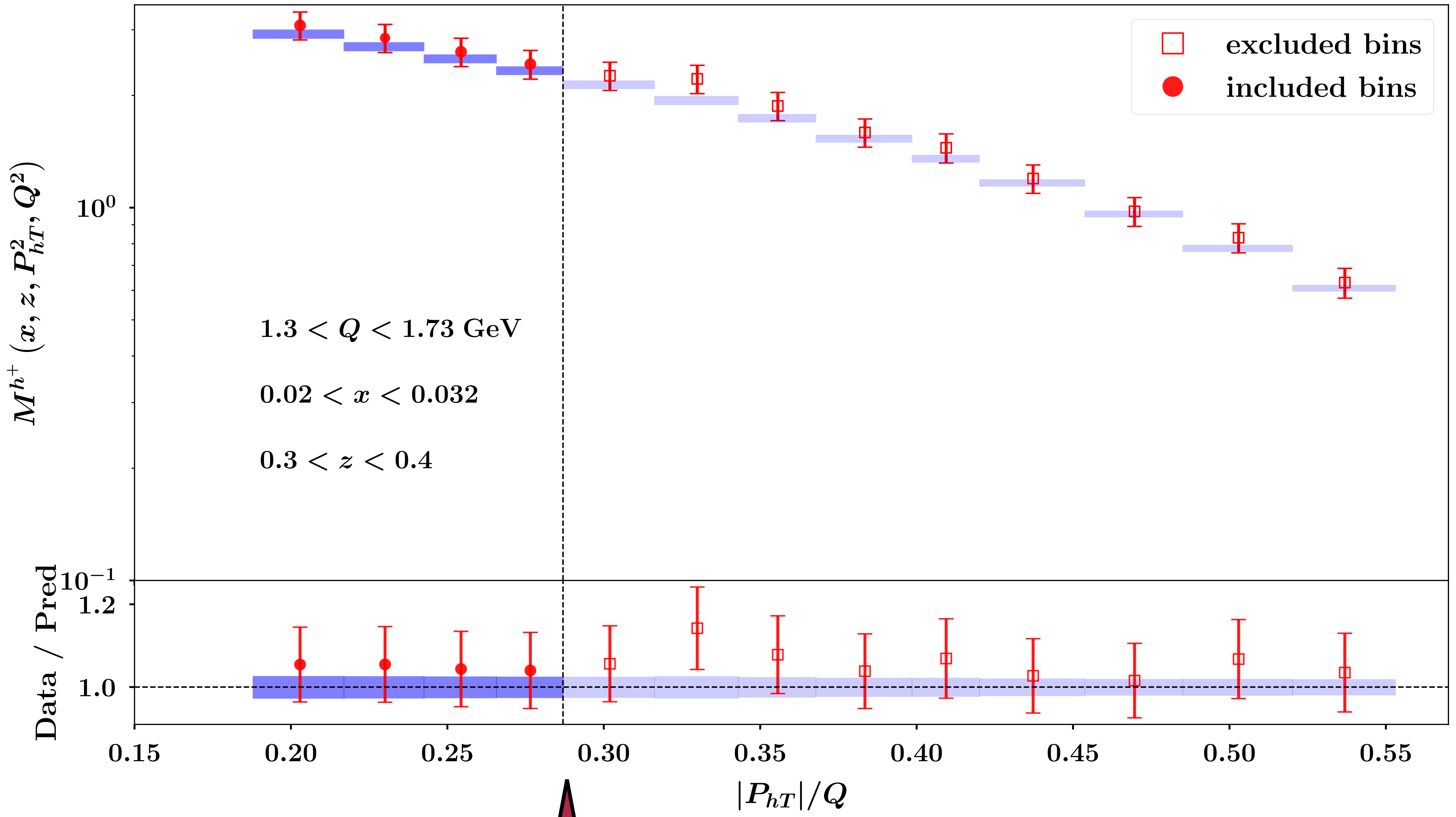

MAP22 cut

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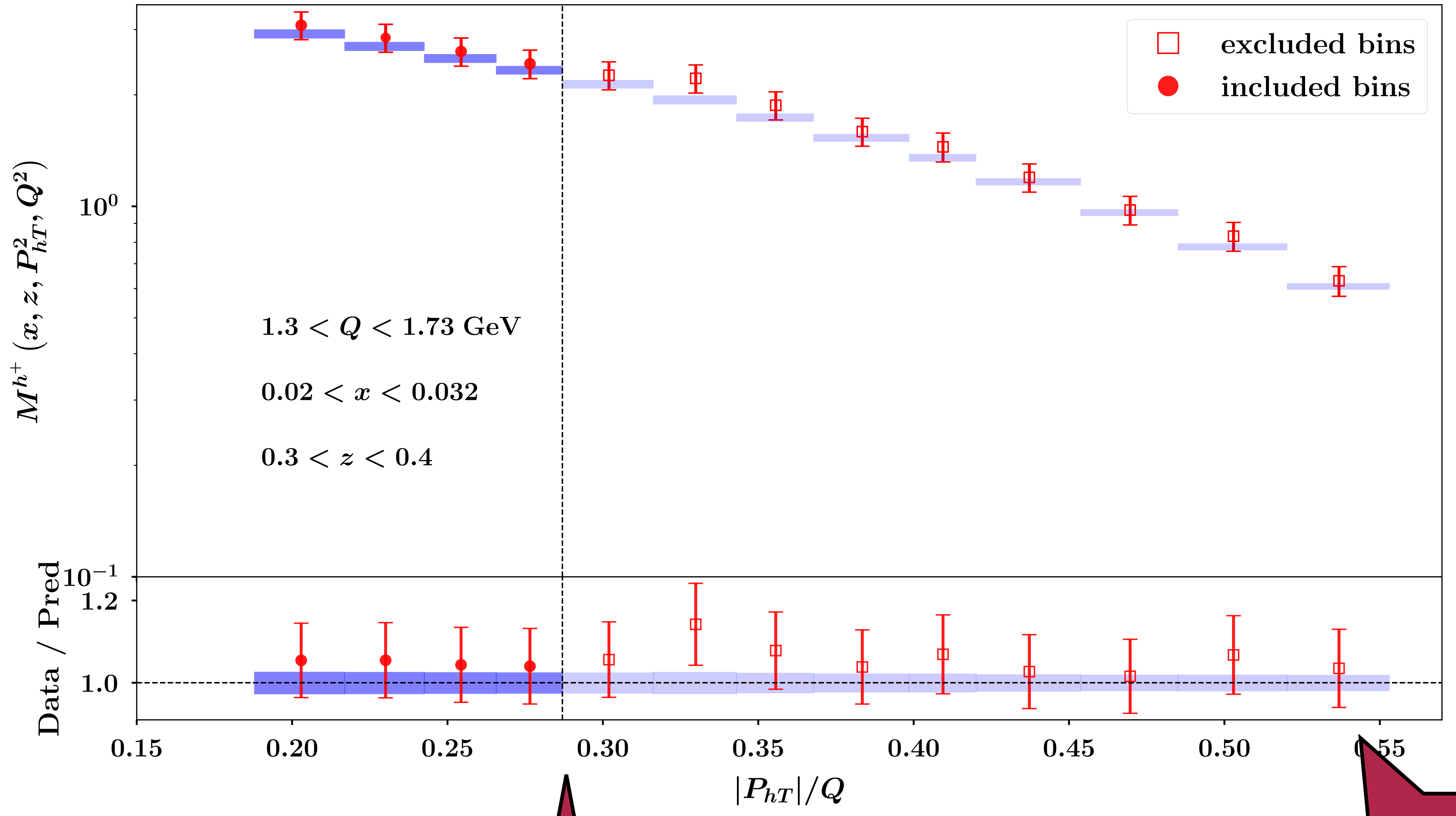

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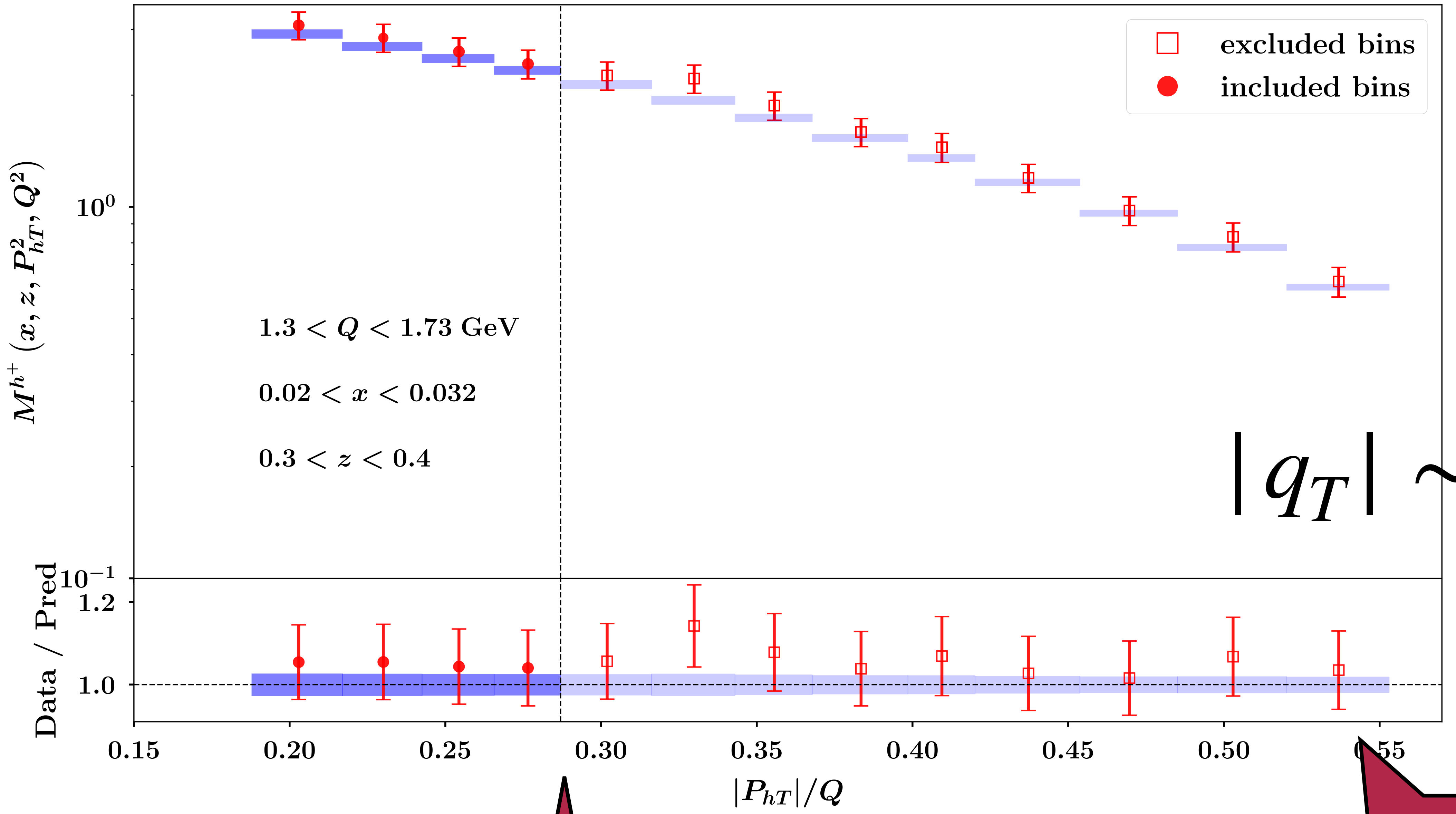

$$|q_T| \sim 1.5\ Q$$!

MAP22 cut

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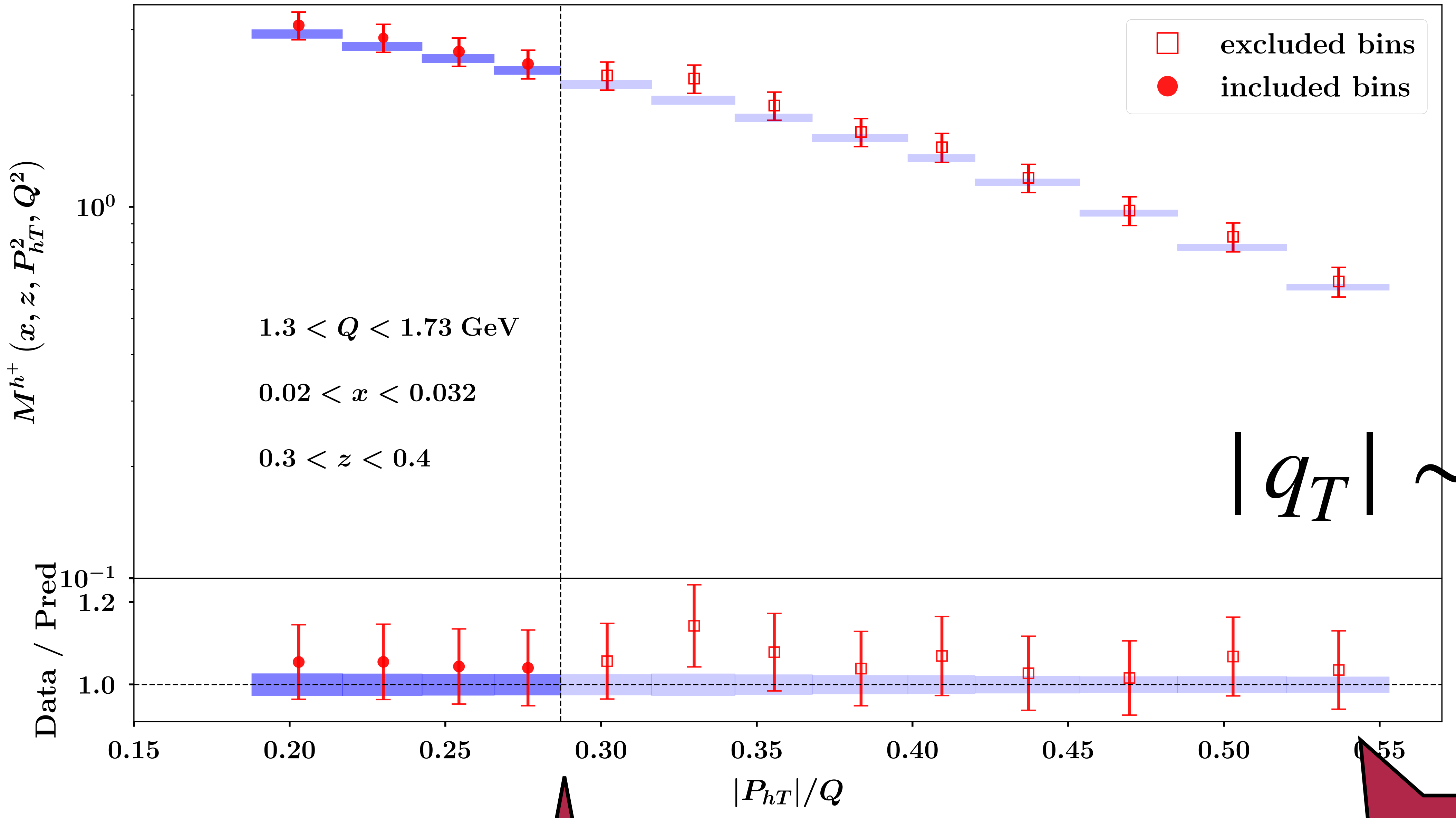

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MAP22 cut

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The MAP22 cut is already considered to be “generous”, but the physics seems to be the same for a much wider transverse momentum

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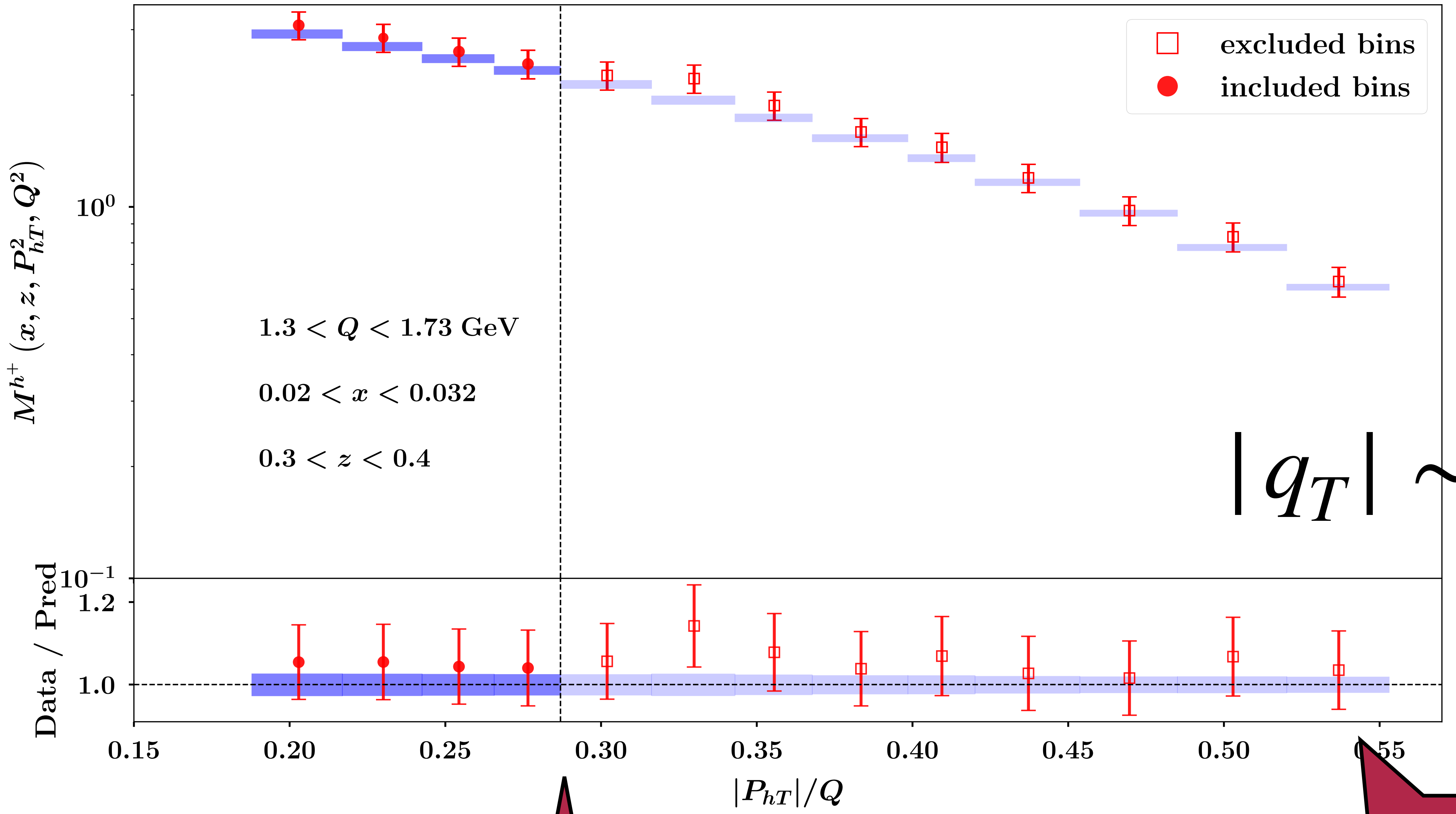
?



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TMD TABLES: QUARK, LEADING TWIST

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

TMDs in **black** survive integration over transverse momentum
TMDs in **red** are time-reversal odd

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- Good knowledge of the k_T dependence of f_1 (also for pions)
- Fair knowledge of Sivers and transversity (mainly x dependence)
- Some hints about all others

AVAILABLE EXTRACTIONS (NEWEST ONLY)

Unpol. TMD	MAP 22 arXiv:2206.07598, ART23 2305.07473, MAP24 arXiv:2405.13833
Helicity	arXiv:2409.08110, MAP24, arXiv:2409.18078
Transversity	arXiv:1505.05589, arXiv:1612.06413, arXiv:2205.00999
Sivers	MAP20 arXiv:2004.14278, arXiv:2009.10710, arXiv:2103.03270, arXiv:2205.00999, arXiv:2304.14328
Boer-Mulders	arXiv:2004.02117, arXiv:2407.06277
Worm-gear g1T	arXiv:2110.10253, arXiv:2210.07268
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Worm-gear h1L	
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Not mentioned: pion TMDs, TMD fragmentation functions, nuclear TMDs

EXAMPLE: SIVERS FUNCTION

Bacchetta, Delcarro,
Pisano, Radici, arXiv:2004.14278

$$A_{UT}^{\sin(\phi_h-\phi_S)}(x,z,P_{hT}^2,Q^2) = \frac{\int d\phi_S\, d\phi_h [d\sigma^{\uparrow} - d\sigma^{\downarrow}] \sin(\phi_h-\phi_S)}{\int d\phi_S\, d\phi_h [d\sigma^{\uparrow} + d\sigma^{\downarrow}]} \approx \frac{F_{UT,T}^{\sin(\phi_h-\phi_S)}}{F_{UU,T}}$$

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$$\begin{aligned}
F_{UU,T}(x,z,P_{hT}^2,Q^2) &= \sum_a e_a^2 x \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{P}_T\, \delta^{(2)}(z\boldsymbol{k}_T + \boldsymbol{P}_T - \boldsymbol{P}_{hT}) f_1^a(x,k_T^2;Q^2) D_1^{a\to h}(z,P_T^2;Q^2) \\
&= \frac{1}{2\pi} \sum_a e_a^2 x \int_0^\infty db_T b_T J_0(b_T P_{hT}/z) \widetilde{f}_1^a(x,b_T^2;Q^2) \widetilde{D}_1^{a\to h}(z,b_T^2;Q^2)\,,
\end{aligned}$$

$$\begin{aligned}
F_{UT,T}^{\sin(\phi_h-\phi_S)}(x,z,P_{hT}^2,Q^2) &= -\sum_a e_a^2 x \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{P}_T\, \delta^{(2)}(z\boldsymbol{k}_T + \boldsymbol{P}_T - \boldsymbol{P}_{hT}) \frac{\boldsymbol{P}_{hT}\cdot\boldsymbol{k}_T}{|\boldsymbol{P}_{hT}|M} f_{1T}^{\perp a}(x,k_T^2;Q^2) D_1^{a\to h}(z,P_T^2;Q^2) \\
&= -\frac{M}{2\pi} \sum_a e_a^2 x \int_0^\infty db_T b_T^2 J_1(b_T P_{hT}/z) \widetilde{f}_{1T}^{\perp(1)a}(x,b_T;Q^2) \widetilde{D}_1^{a\to h}(z,b_T^2;Q^2)\,,
\end{aligned}$$

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Unpolarized TMDs are needed

$$\begin{aligned} F_{UU,T}(x,z,P_{hT}^2,Q^2) &= \sum_a e_a^2 x \int d^2\boldsymbol{k}_T\, d^2\boldsymbol{P}_T\, \delta^{(2)}(z\boldsymbol{k}_T + \boldsymbol{P}_T - \boldsymbol{P}_{hT}) f_1^a(x,k_T^2;Q^2) D_1^{a\to h}(z,P_T^2;Q^2) \\ &= \frac{1}{2\pi}\sum_a e_a^2 x \int_0^\infty db_T\, b_T\, J_0(b_T P_{hT}/z)\widetilde{f}_1^a(x,b_T^2;Q^2)\widetilde{D}_1^{a\to h}(z,b_T^2;Q^2)\,, \end{aligned}$$

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SIVERS FUNCTION

$$\rho^q_{N^\uparrow}(x, k_x, k_y; Q^2) = f^q_1(x, k^2_T; Q^2) - \frac{k_x}{M} f^{\perp q}_{1T}(x, k^2_T; Q^2)$$

In a nucleon polarized in the +y direction,
the distribution of quarks can be distorted in the x direction

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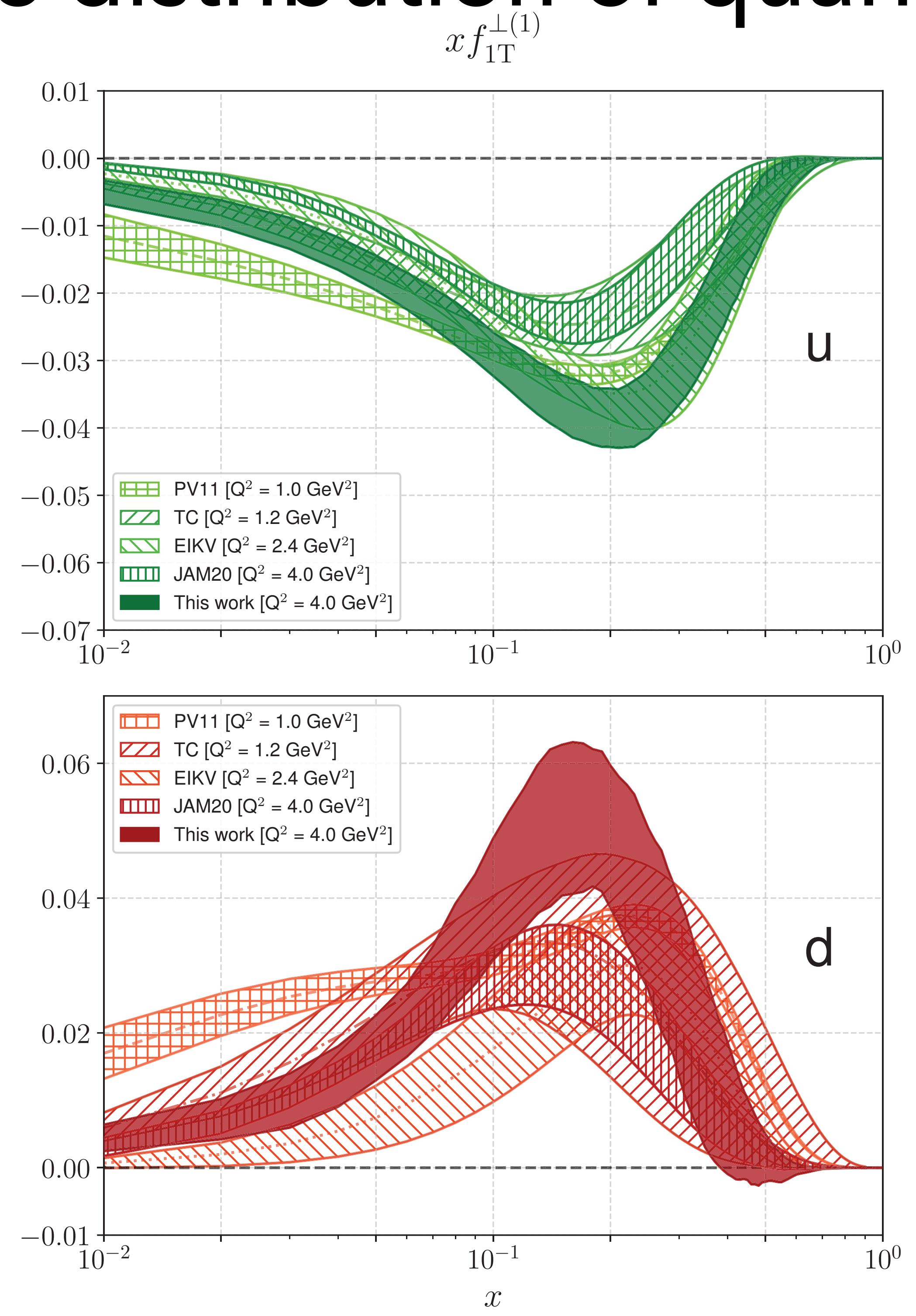
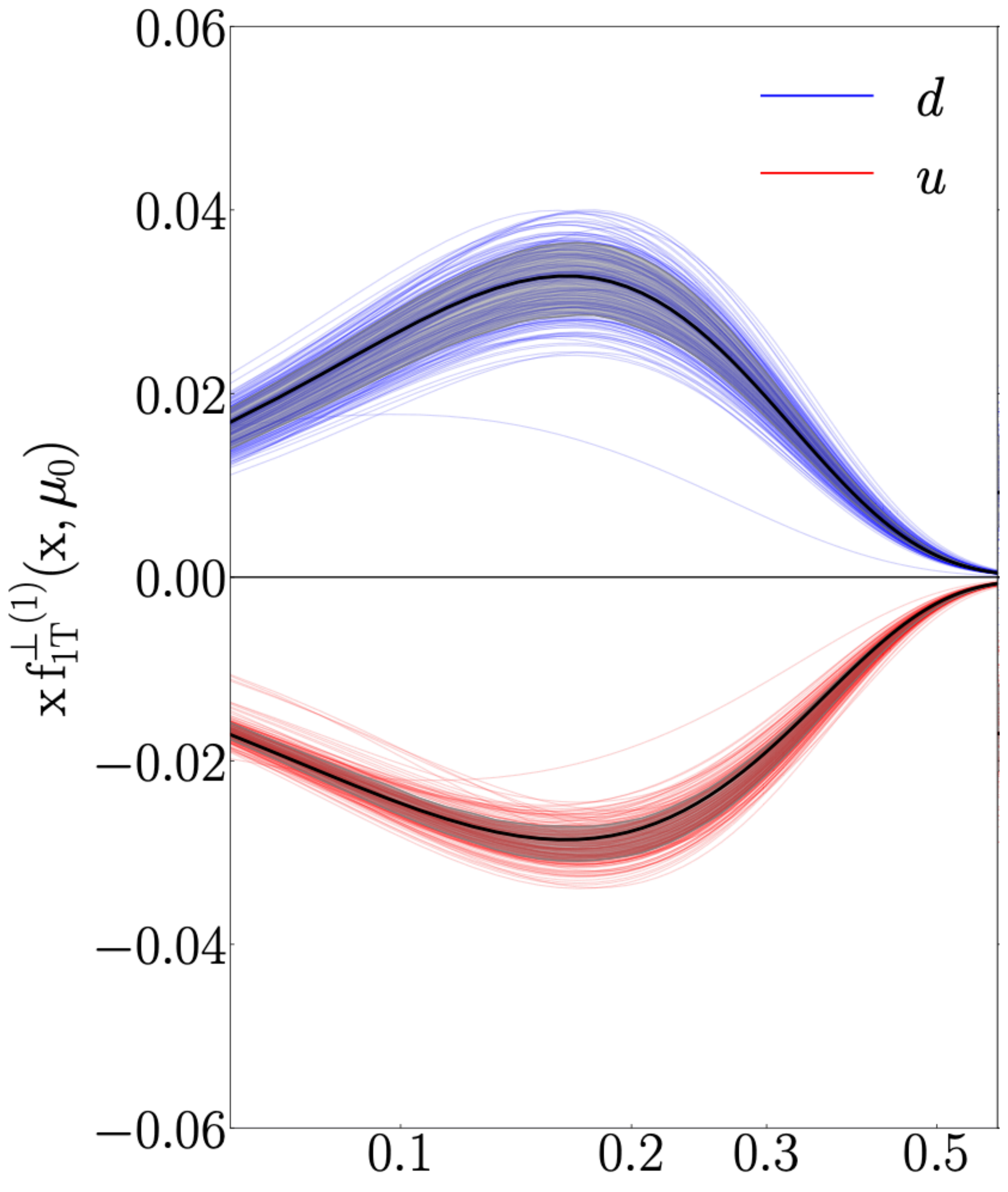
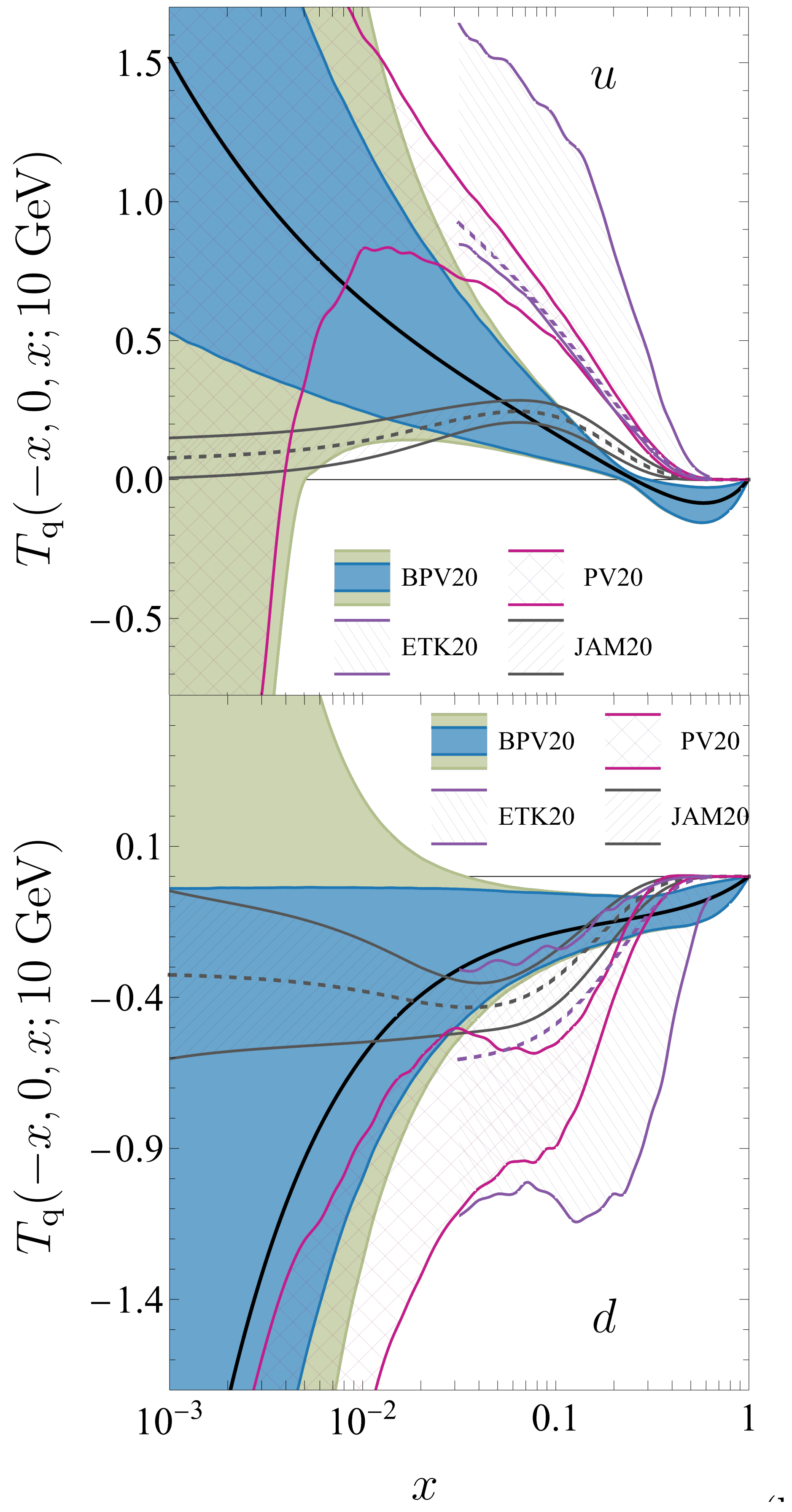


Bacchetta, Delcarro,
Pisano, Radici, arXiv:2004.14278

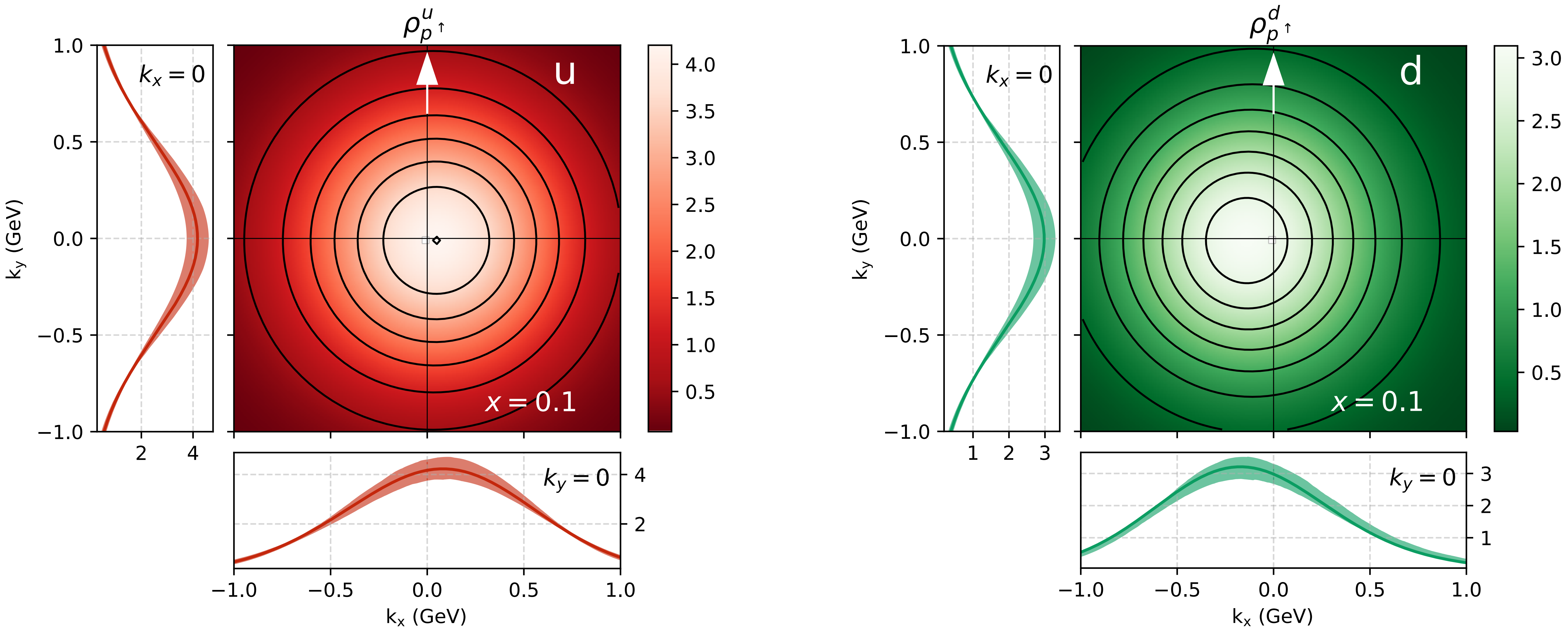


Echevarria, Kang, Terry,
arXiv:2009.10710



Bury, Prokudin, Vladimirov,
arXiv:2103.03270

3D STRUCTURE IN MOMENTUM SPACE



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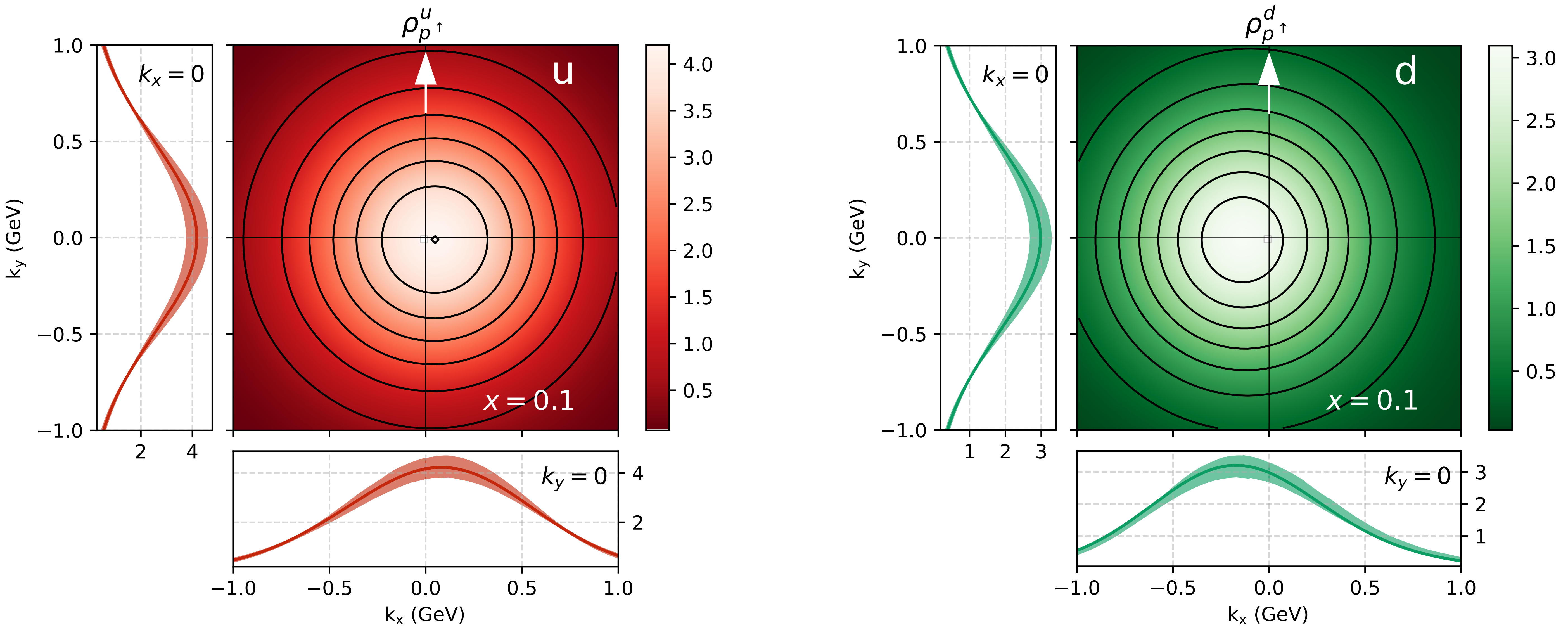

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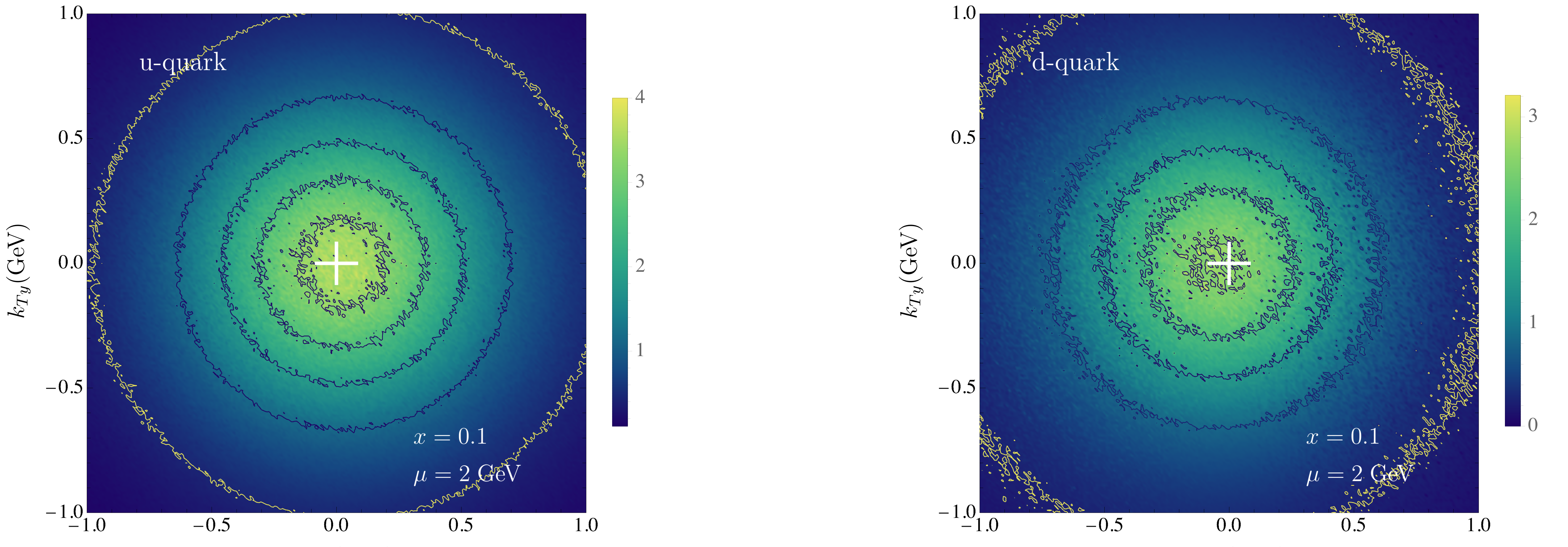


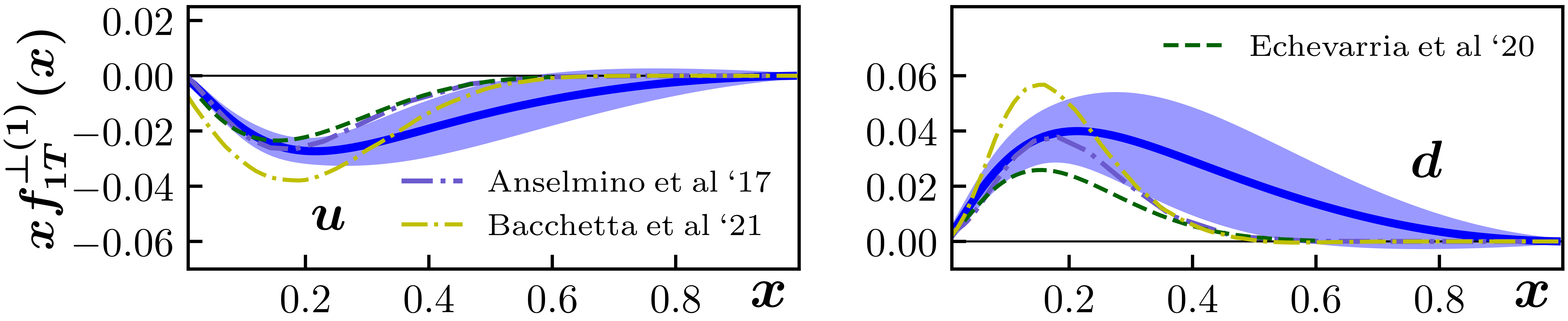
(a)

(b)

Bury, Prokudin, Vladimirov, arXiv:2103.03270

SIMULTANEOUS FIT OF SINGLE TRANSVERSE-SPIN ASY

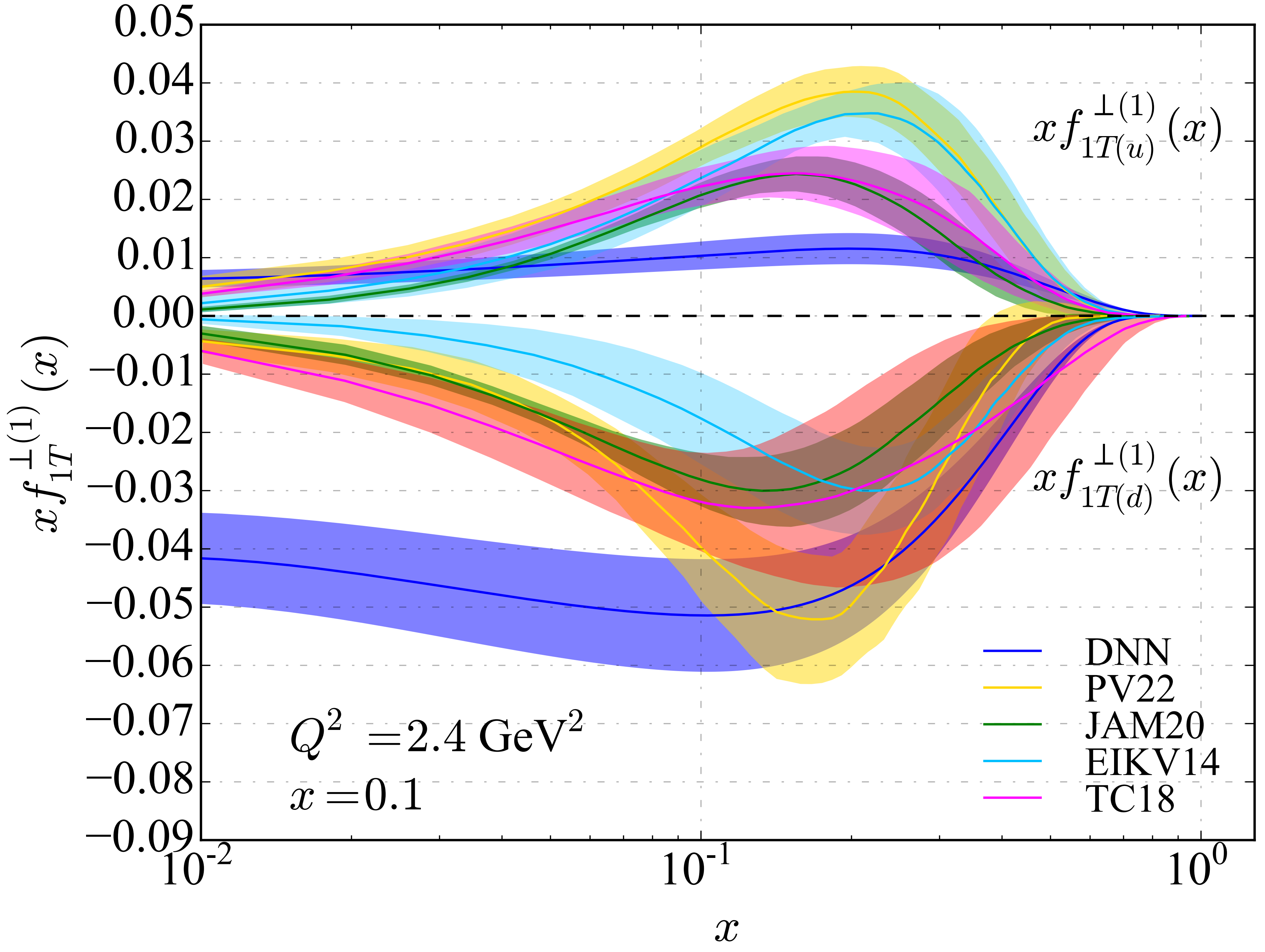
JAM, arXiv:2205.00999

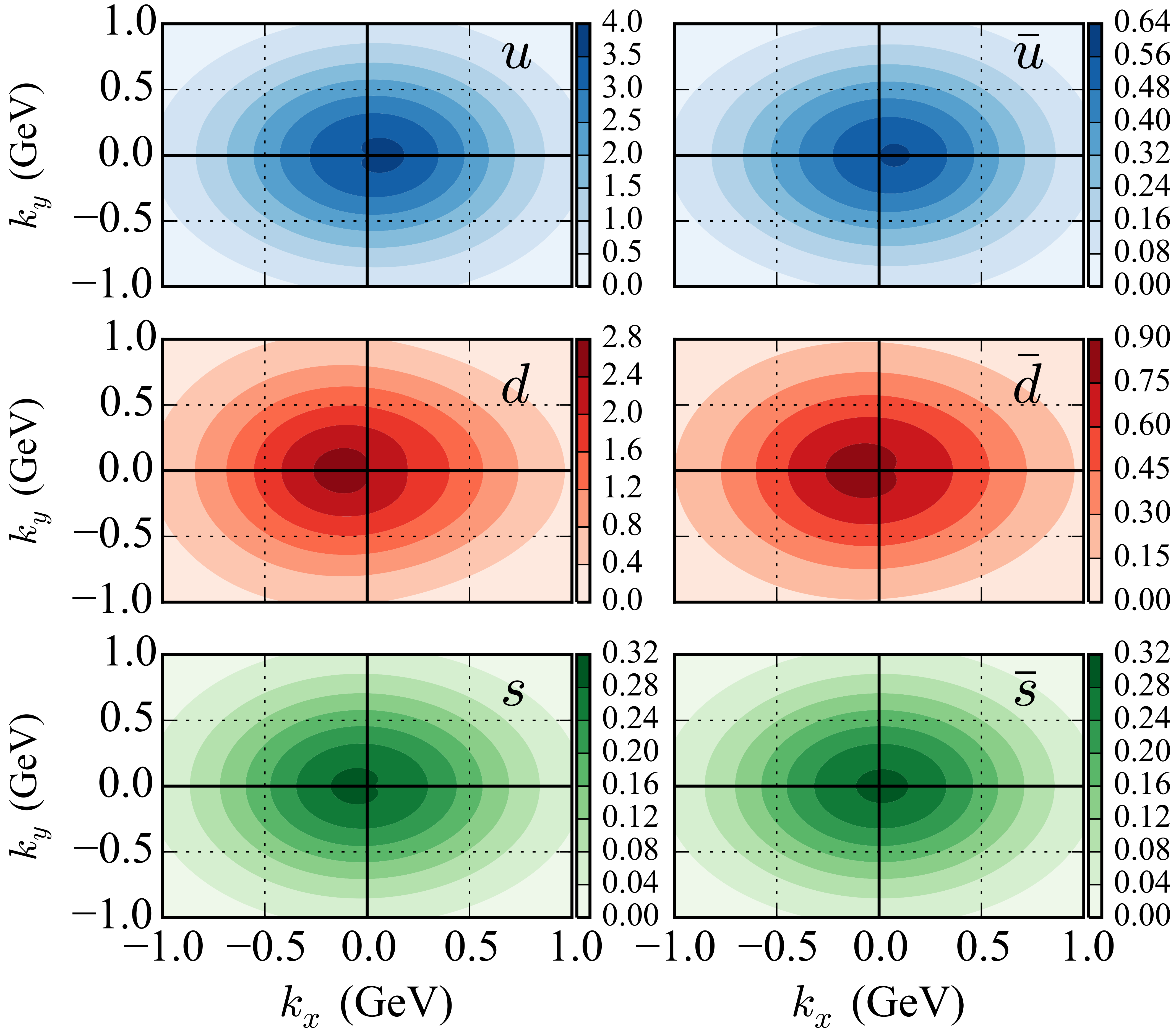


Interesting work from the point of view of simultaneous use of several measurements, but still limited from other perspectives (lack of TMD evolution and knowledge of the unpolarized function)

SIVERS FUNCTION WITH NEURAL NETWORKS

Fernando, Keller, arXiv:2304.14328





Interesting work from the point of view of the use of Neural Networks, but still limited from other perspectives (lack of TMD evolution and knowledge of the unpolarized function)

STRUCTURE FUNCTIONS

Bacchetta, Boer, Diehl, Mulders, arXiv:0803.0227

	observable	twist
“SIDIS F_T”	$F_{UU,T}$	2
“SIDIS F_L”	$F_{UU,L}$	4
“Cahn”	$F_{UU}^{\cos\phi_h}$	3
“Boer-Mulders”	$F_{UU}^{\cos 2\phi_h}$	2
	$F_{UL}^{\sin\phi_h}$	3
“Kotzinian-Mulders worm gear”	$F_{UL}^{\sin 2\phi_h}$	2
“Sivers”	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4
“Collins”	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2
“Pretzelosity”	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2
	$F_{UT}^{\sin\phi_S}$	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3

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In the absence of lepton polarization, the structure functions with long. target polarization are less interesting

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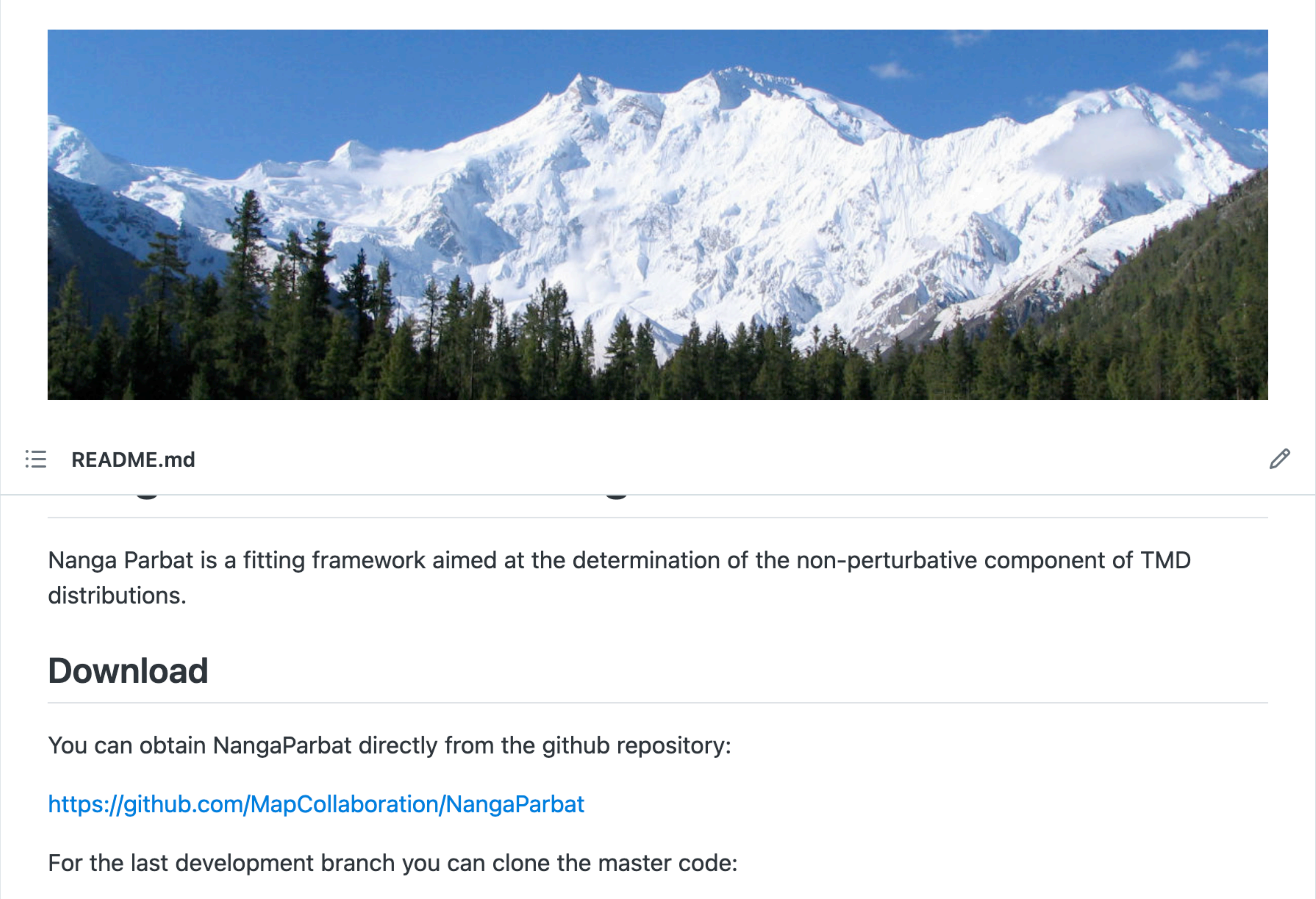
Not all of them are easy to access at EIC due to:
x-range, twist,
evolution, prefactors

AVAILABLE TOOLS: NANGA PARBAT

https://github.com/MapCollaboration/NangaParbat



Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

AVAILABLE TOOLS: ARTEMIDE

https://teorica.fis.ucm.es/artemide/

arTeMiDe

News

12 Dec 2019: Version 2.02 released (+manual update).

23 Feb 2019: Version 1.4 released (+manual update).

21 Jan 2019: Artemide now has a repository.

Archive of older links/news.

Articles, presentations & supplementary materials

Extra pictures for the paper arXiv:1902.08474

Seminar of A.Vladimirov in Pavia 2018 on TMD evolution.

Link to the text in Inspire.

Archive of older links/news.

Download

Recent version/release can be found in repository

About us & Contacts

If you have found mistakes, or have suggestions/questions, please, contact us.

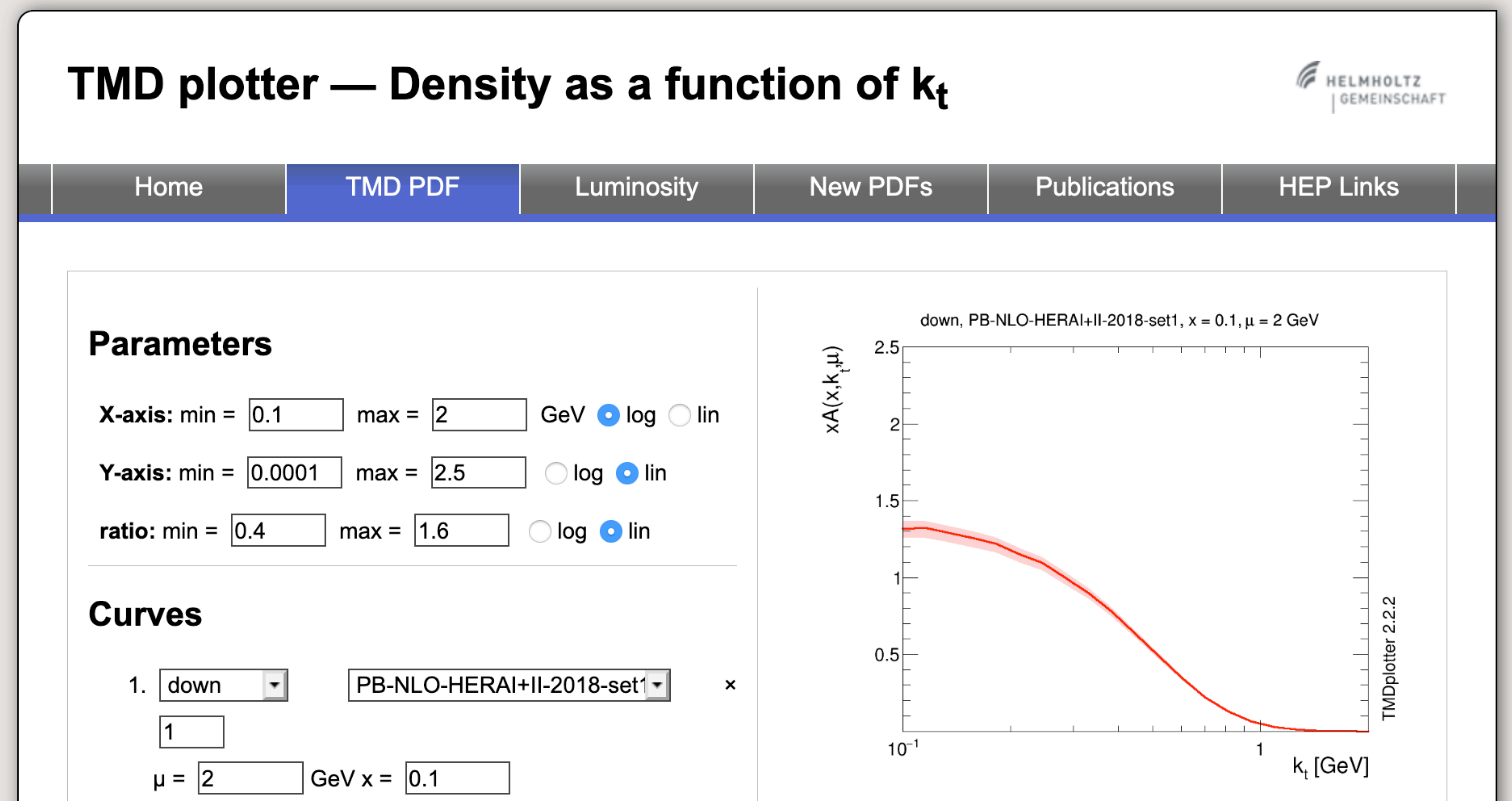
Some extra materials can be found on Alexey's web-page

Alexey Vladimirov Alexey.Vladimirov@physik.uni-regensburg.de

Ignazio Scimemi ignazios@fis.ucm.es

AVAILABLE TOOLS: TMDLIB AND TMDPLOTTER

https://tmdlib.hepforge.org/



TMD plotter — Density as a function of k_t

Parameters

X-axis: min = 0.1 max = 2 GeV ● log ○ lin

Y-axis: min = 0.0001 max = 2.5 ○ log ● lin

ratio: min = 0.4 max = 1.6 ○ log ● lin

Curves

1. down PB-NLO-HERAI+II-2018-set1 ×

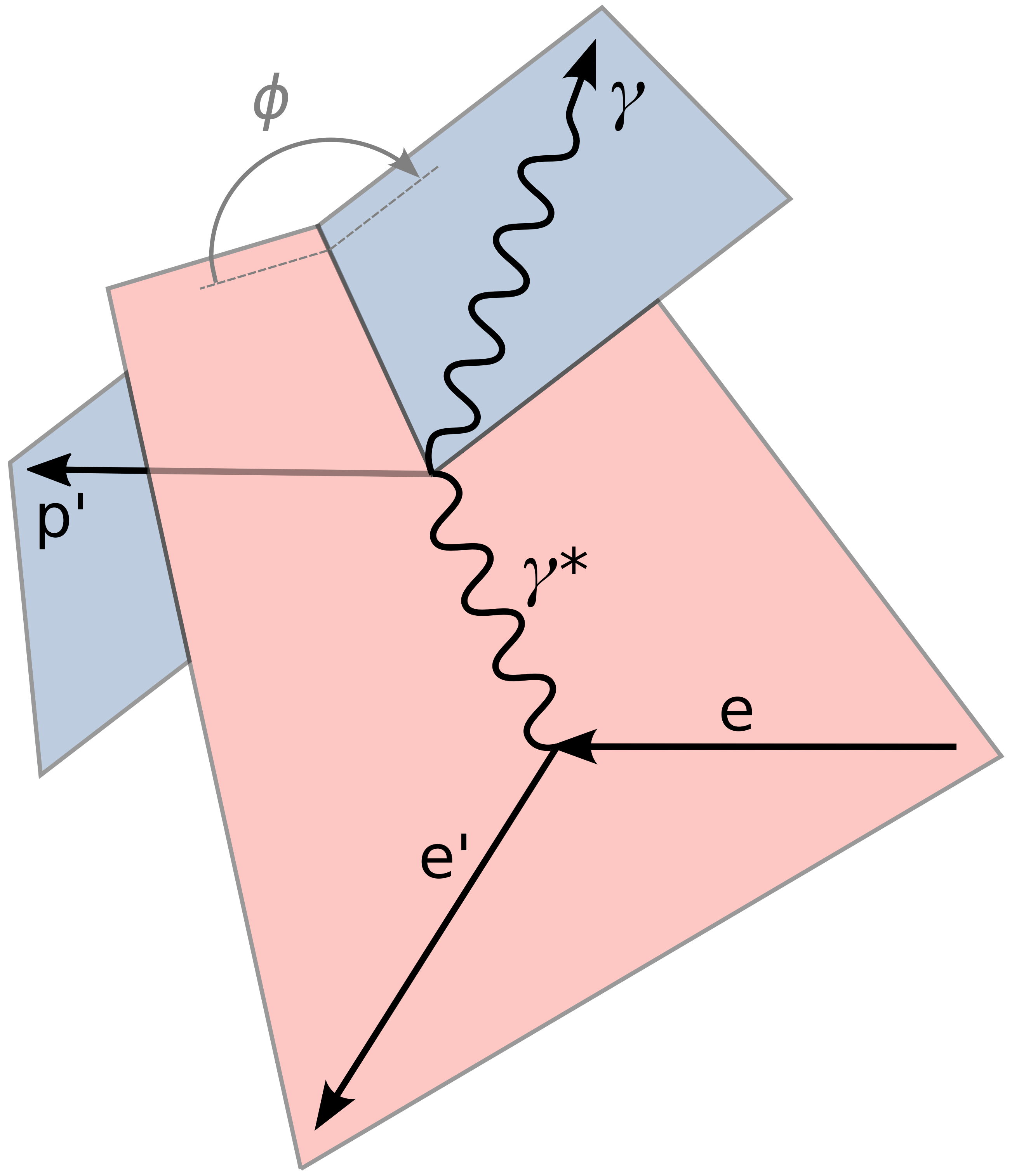
1

μ = 2 GeV x = 0.1

GPDS

DVCS CROSS SECTION

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x_B\mathrm{d}Q^2\mathrm{d}|t|\mathrm{d}\phi\mathrm{d}\phi_S} = \frac{\alpha_{\mathrm{EM}}^3 x_B y^2}{16\pi^2 Q^4\sqrt{1+\gamma^2}}\left(|\mathcal{T}_{\mathrm{DVCS}}|^2 + |\mathcal{T}_{\mathrm{BH}}|^2 + \mathcal{I}\right)$$



DVCS CROSS SECTION

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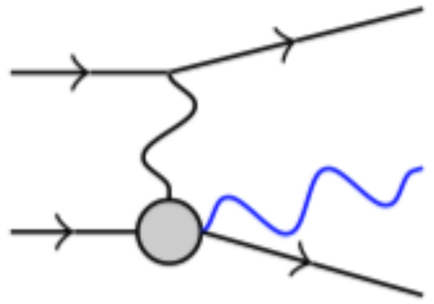
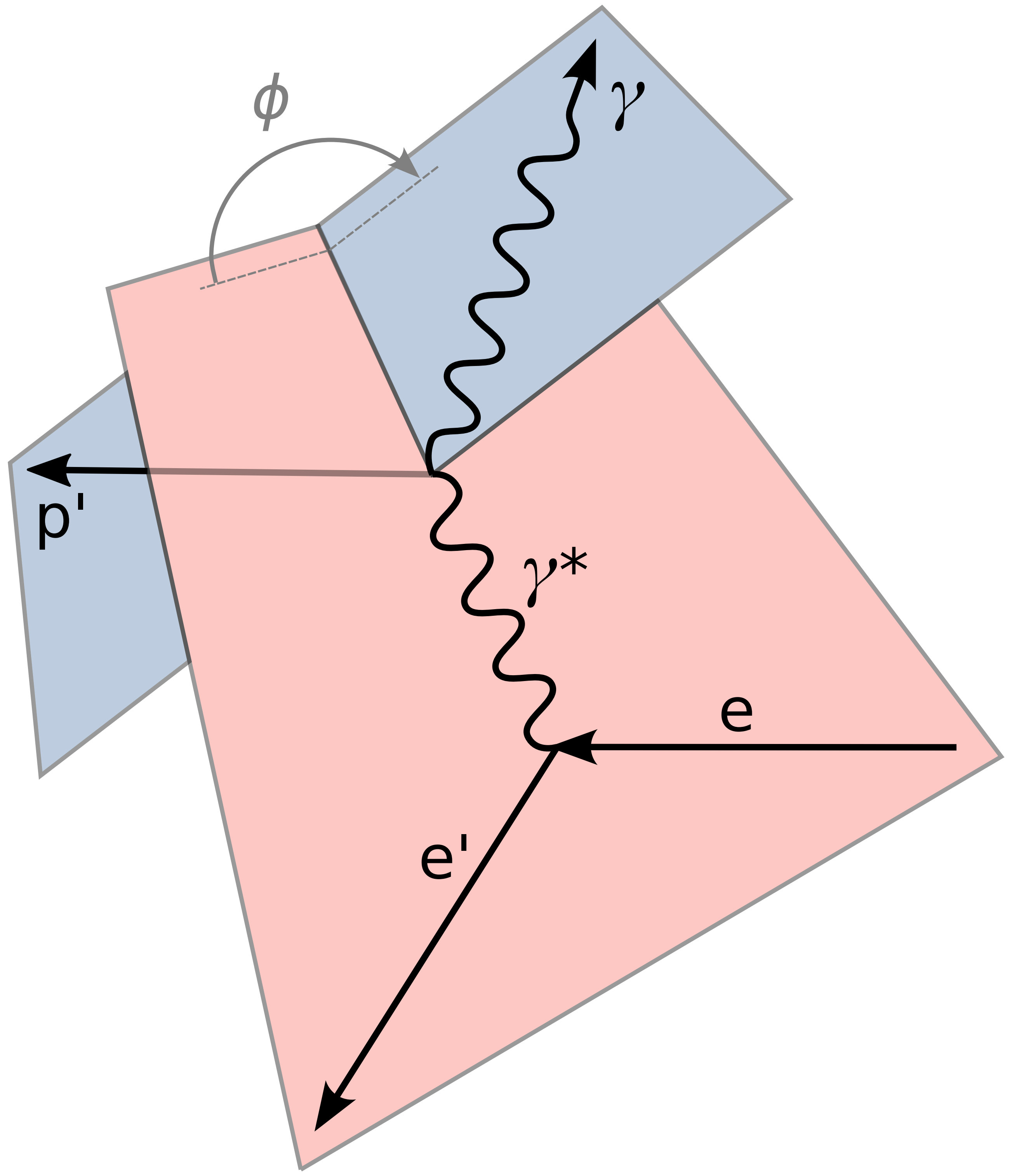
DVCS



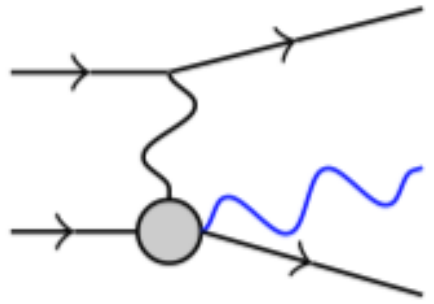
parametrised by CFFs



DVCS CROSS SECTION

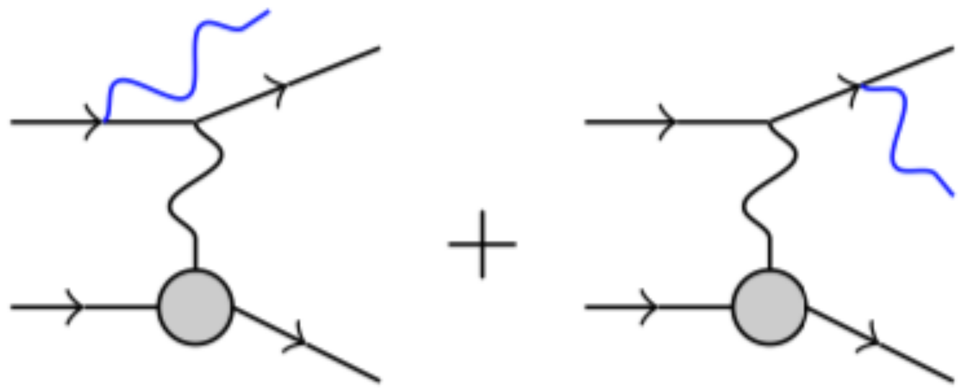
$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x_B\mathrm{d}Q^2\mathrm{d}|t|\mathrm{d}\phi\mathrm{d}\phi_S} = \frac{\alpha_{\mathrm{EM}}^3 x_B y^2}{16\pi^2 Q^4\sqrt{1+\gamma^2}}\left(|\mathcal{T}_{\mathrm{DVCS}}|^2 + |\mathcal{T}_{\mathrm{BH}}|^2 + \mathcal{I}\right)$$

DVCS

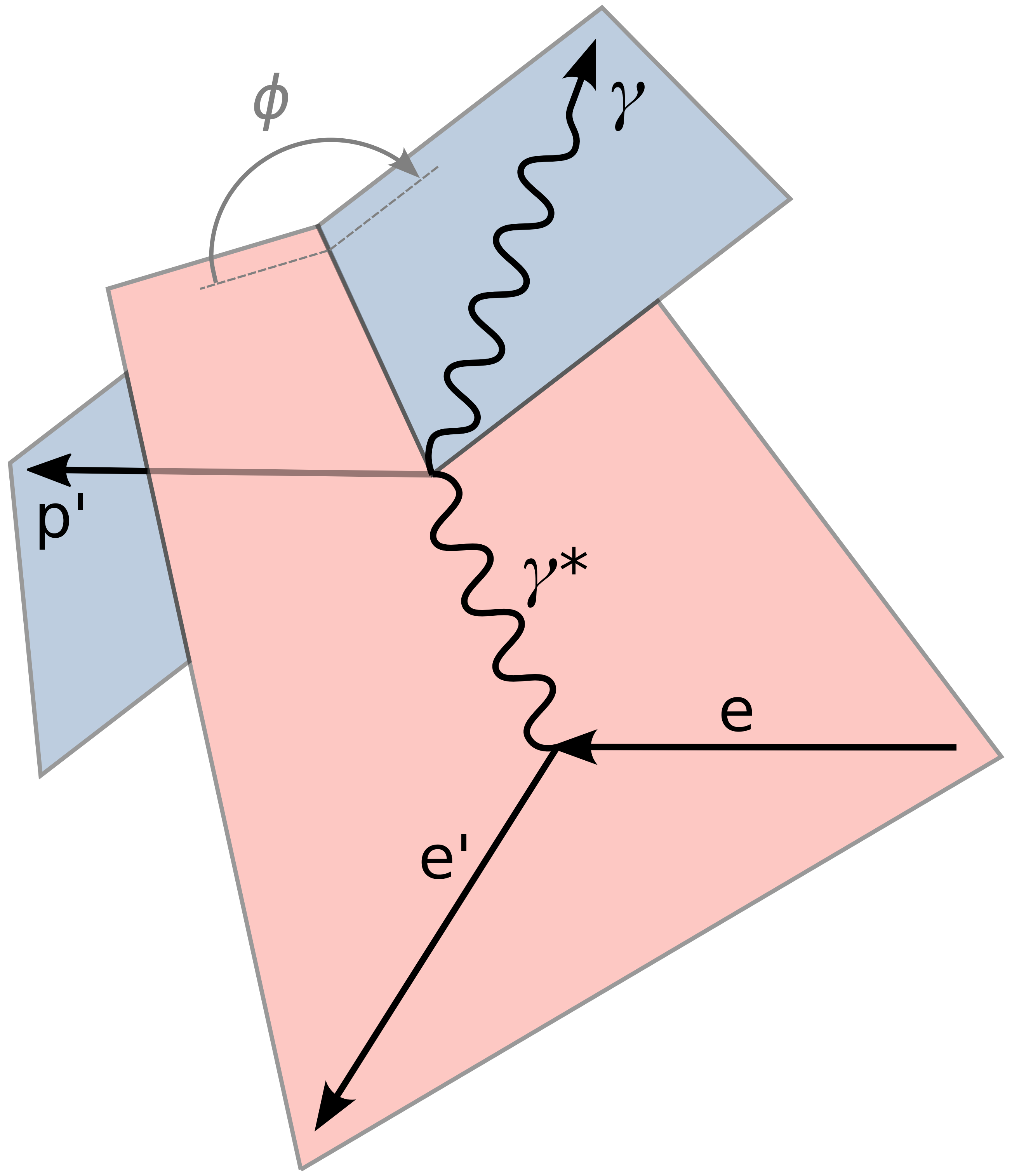


parametrised by CFFs

Bethe-Heitler process



calculable within QED



COMPTON FORM FACTORS

Shiells, Guo, Ji, arxiv:2112.15144

$$\mathrm{d}\sigma^{\mathrm{P_b P_t}} = \mathrm{d}\sigma^{\mathrm{P_b P_t}}_{\mathrm{DVCS}} + \mathrm{d}\sigma^{\mathrm{P_b P_t}}_{\mathrm{INT}} + \mathrm{d}\sigma^{\mathrm{P_b P_t}}_{\mathrm{BH}} = \sum_{i,j} A^{\mathrm{P_b P_t}}_{ij} \mathcal{F}^i \mathcal{F}^j + \sum_i B^{\mathrm{P_b P_t}}_i \mathcal{F}^i + C^{\mathrm{P_b P_t}}$$

$$\mathcal{F}_i \; = \; \{\mathrm{Re}\mathcal{H}, \mathrm{Im}\mathcal{H}, \mathrm{Re}\mathcal{E}, \mathrm{Im}\mathcal{E}, \mathrm{Re}\widetilde{\mathcal{H}}, \mathrm{Im}\widetilde{\mathcal{H}}, \mathrm{Re}\widetilde{\mathcal{E}}, \mathrm{Im}\widetilde{\mathcal{E}}\}$$

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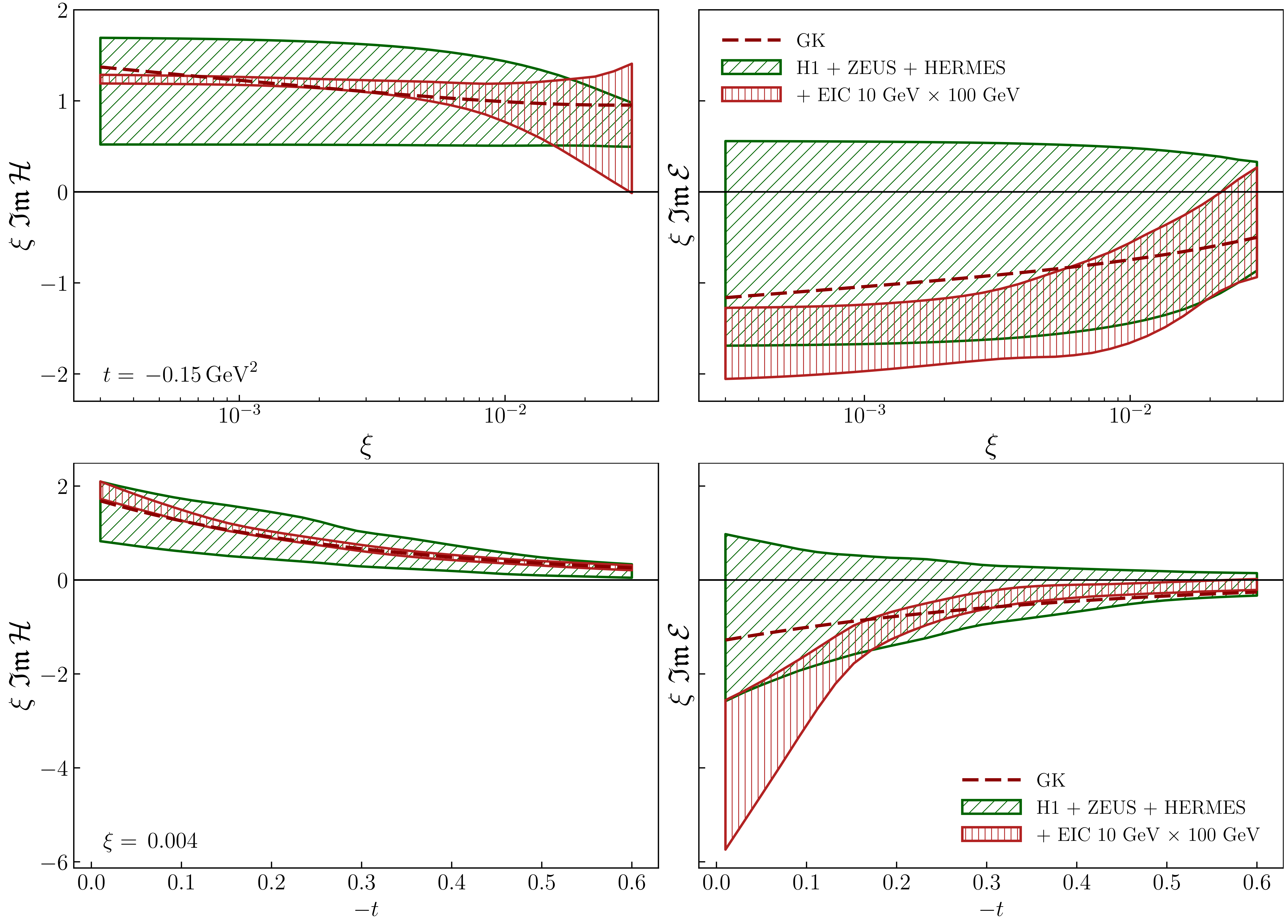
$$\mathcal{F}_i = \{\mathrm{Re}\mathcal{H}, \mathrm{Im}\mathcal{H}, \mathrm{Re}\mathcal{E}, \mathrm{Im}\mathcal{E}, \mathrm{Re}\widetilde{\mathcal{H}}, \mathrm{Im}\widetilde{\mathcal{H}}, \mathrm{Re}\widetilde{\mathcal{E}}, \mathrm{Im}\widetilde{\mathcal{E}}\}$$

Harmonic	Expressions	$\mathrm{Re}\mathcal{H}$	$\mathrm{Im}\mathcal{H}$	$\mathrm{Re}\mathcal{E}$	$\mathrm{Im}\mathcal{E}$	$\mathrm{Re}\widetilde{\mathcal{H}}$	$\mathrm{Im}\widetilde{\mathcal{H}}$	$\mathrm{Re}\widetilde{\mathcal{E}}$	$\mathrm{Im}\widetilde{\mathcal{E}}$
$\sigma^{UU}_{\cos(n\phi)}$	$\mathcal{D}^{\mathrm{DVCS}}_1, \mathcal{A}^U_{\mathrm{Re}}, \mathcal{B}^U_{\mathrm{Re}}, \mathcal{C}^U_{\mathrm{Re}}$	✔	✔	✓	✓	✔	✔	✓	✓
$\sigma^{LU}_{\sin(1\phi)}$	$\mathcal{A}^U_{\mathrm{Im}}, \mathcal{B}^U_{\mathrm{Im}}, \mathcal{C}^U_{\mathrm{Im}}$	-	✔	-	✓	-	✔	-	-
$\sigma^{UL}_{\sin(1\phi)}$	$\mathcal{A}^L_{\mathrm{Im}}, \mathcal{B}^L_{\mathrm{Im}}, \mathcal{C}^L_{\mathrm{Im}}$	-	✔	-	✓	-	✔	-	✓
$\sigma^{LL}_{\cos(n\phi)}$	$\mathcal{D}^{\mathrm{DVCS}}_2, \mathcal{A}^L_{\mathrm{Re}}, \mathcal{B}^L_{\mathrm{Re}}, \mathcal{C}^L_{\mathrm{Re}}$	✔	✔	✓	✓	✔	✔	✓	✓
$\sigma^{UT}_{\cos(n\phi)}$	$\mathcal{D}^{\mathrm{DVCS}}_3, \mathcal{A}^{\mathrm{out}}_{\mathrm{Im}}, \mathcal{B}^{\mathrm{out}}_{\mathrm{Im}}, \mathcal{C}^{\mathrm{out}}_{\mathrm{Im}}$	✓	✔	✓	✔	✓	✓	✓	✓
$\sigma^{UT}_{\sin(1\phi)}$	$\mathcal{A}^{\mathrm{in}}_{\mathrm{Im}}, \mathcal{B}^{\mathrm{in}}_{\mathrm{Im}}, \mathcal{C}^{\mathrm{in}}_{\mathrm{Im}}$	-	✔	-	✔	-	✔	-	✓
$\sigma^{LT}_{\cos(n\phi)}$	$\mathcal{D}^{\mathrm{DVCS}}_4, \mathcal{A}^{\mathrm{in}}_{\mathrm{Re}}, \mathcal{B}^{\mathrm{in}}_{\mathrm{Re}}, \mathcal{C}^{\mathrm{in}}_{\mathrm{Re}}$	✔	✓	✔	✓	✔	✓	✓	✓
$\sigma^{LT}_{\sin(1\phi)}$	$\mathcal{A}^{\mathrm{out}}_{\mathrm{Re}}, \mathcal{B}^{\mathrm{out}}_{\mathrm{Re}}, \mathcal{C}^{\mathrm{out}}_{\mathrm{Re}}$	✔	-	✔	-	✔	-	✓	-

EIC IMPACT

Aschenauer et al., in preparation, courtesy of S. Fazio

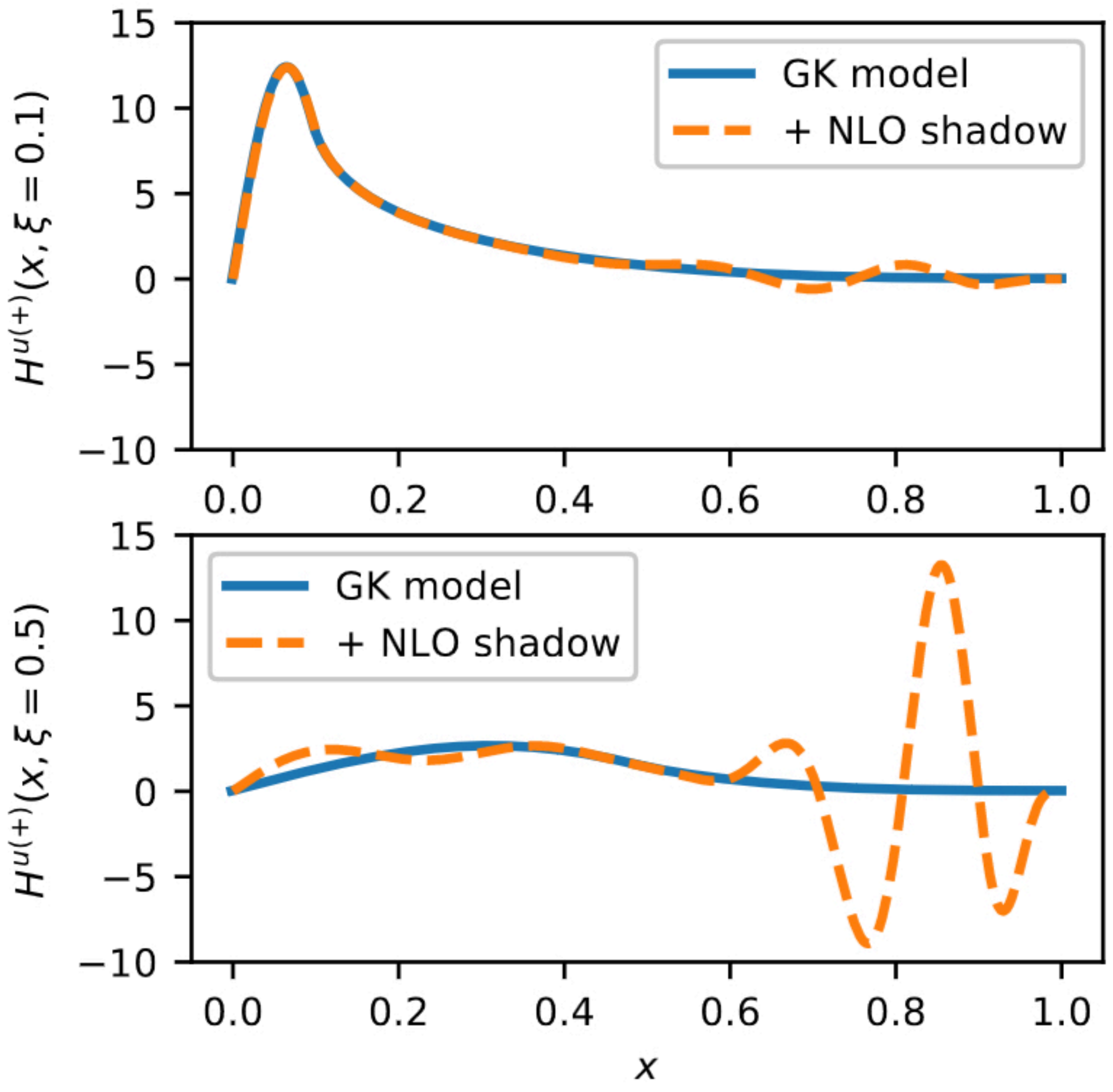


Energy	Lumi
5x41	10
10x100	10
18x275	10

FROM COMPTON FORM FACTOR TO GPDS

Bertone et al. , arxiv:2104.03836

$$\mathcal{H}(\xi,t;\mu) = \sum_q e_q^2 \int_{-1}^{1} dx\, H^q(x,\xi,t;\mu)\left(\frac{1}{\xi - x - i\varepsilon} - \frac{1}{\xi + x - i\varepsilon}\right) + \mathcal{O}(\alpha_s)$$



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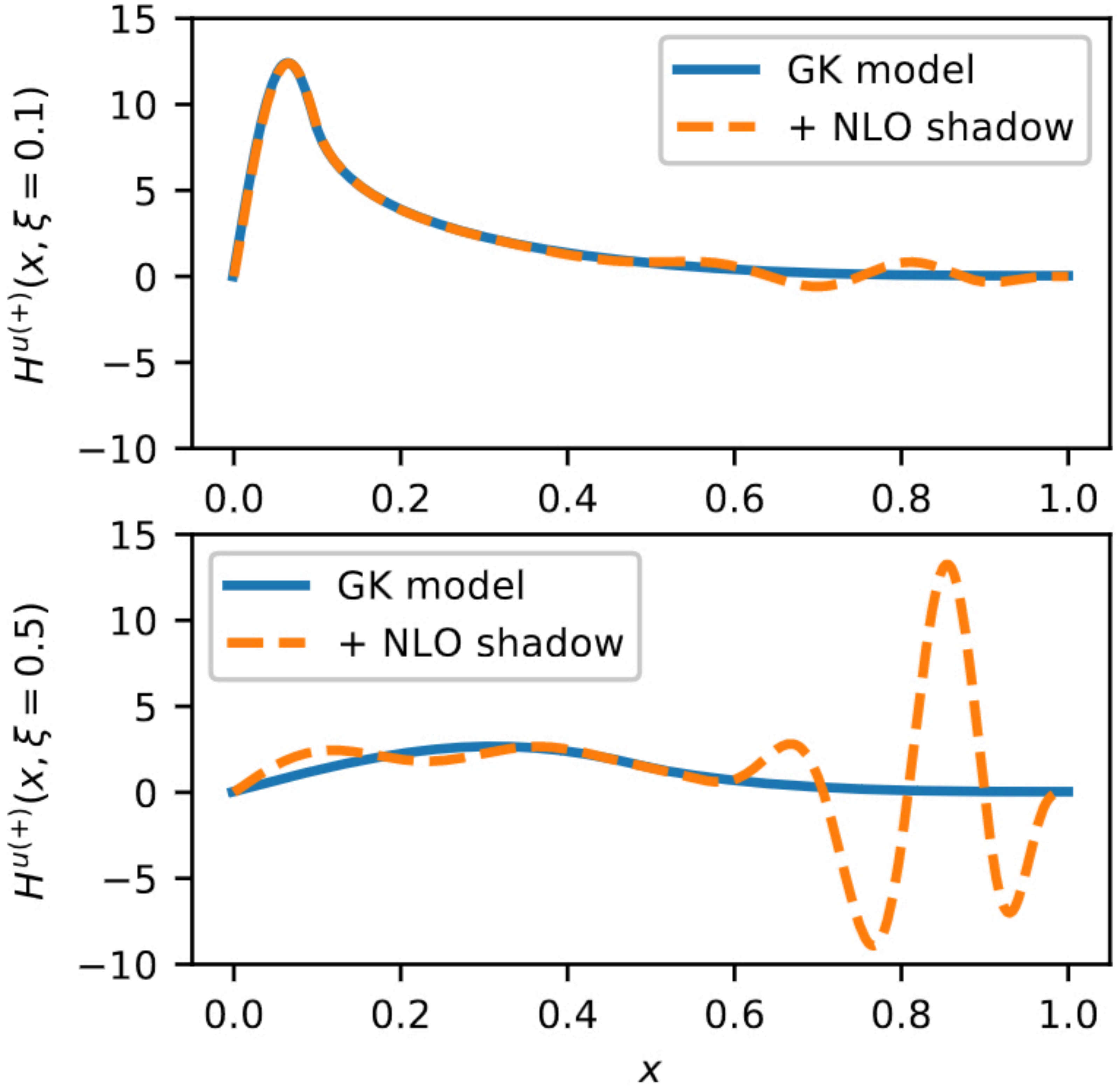
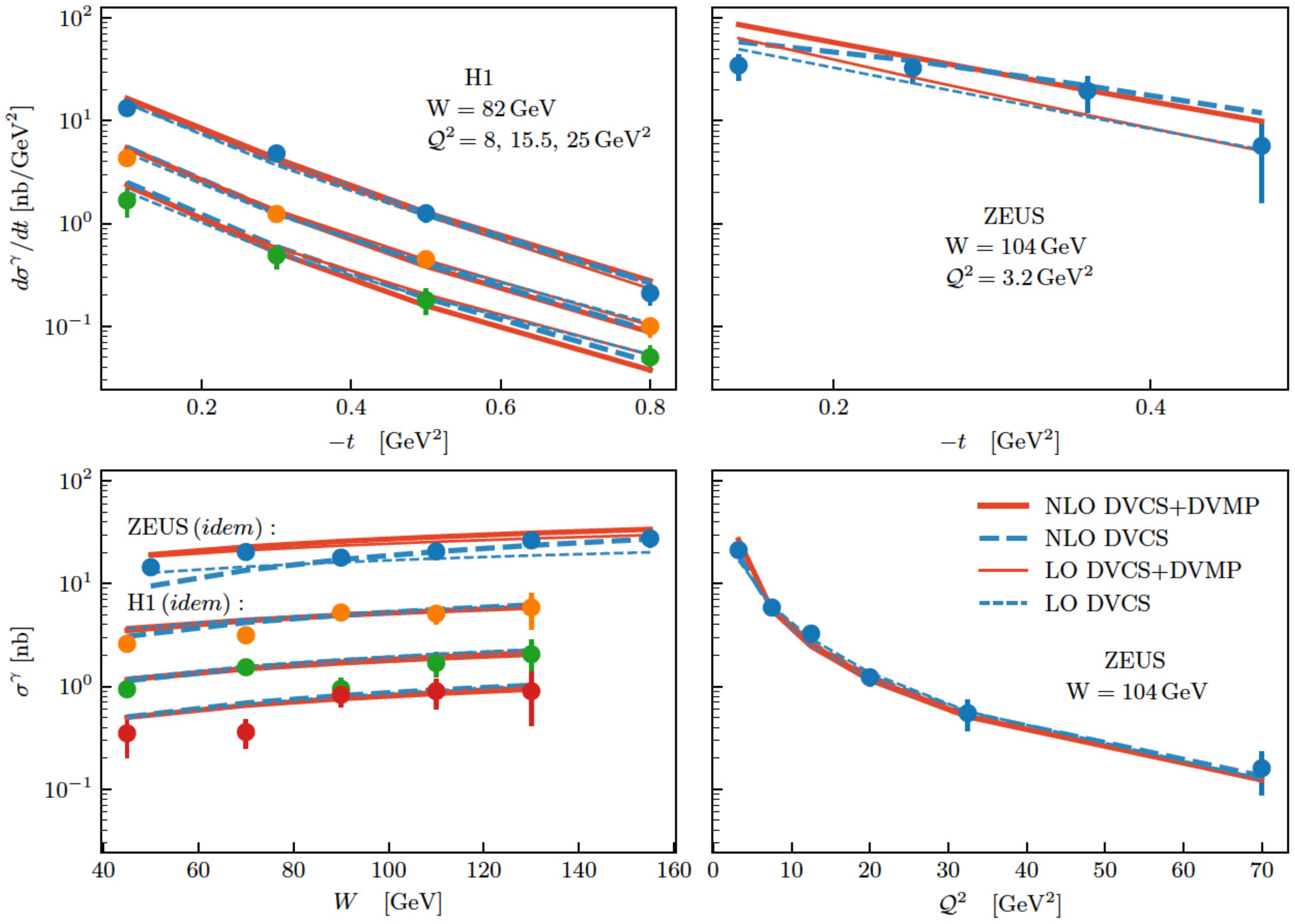


deconvolution problem

RECENT GPD FIT

Čuić, Duplančić, Kumerički, Passek-K., 2310.13837

Using DVCS and



RECENT GPD FIT

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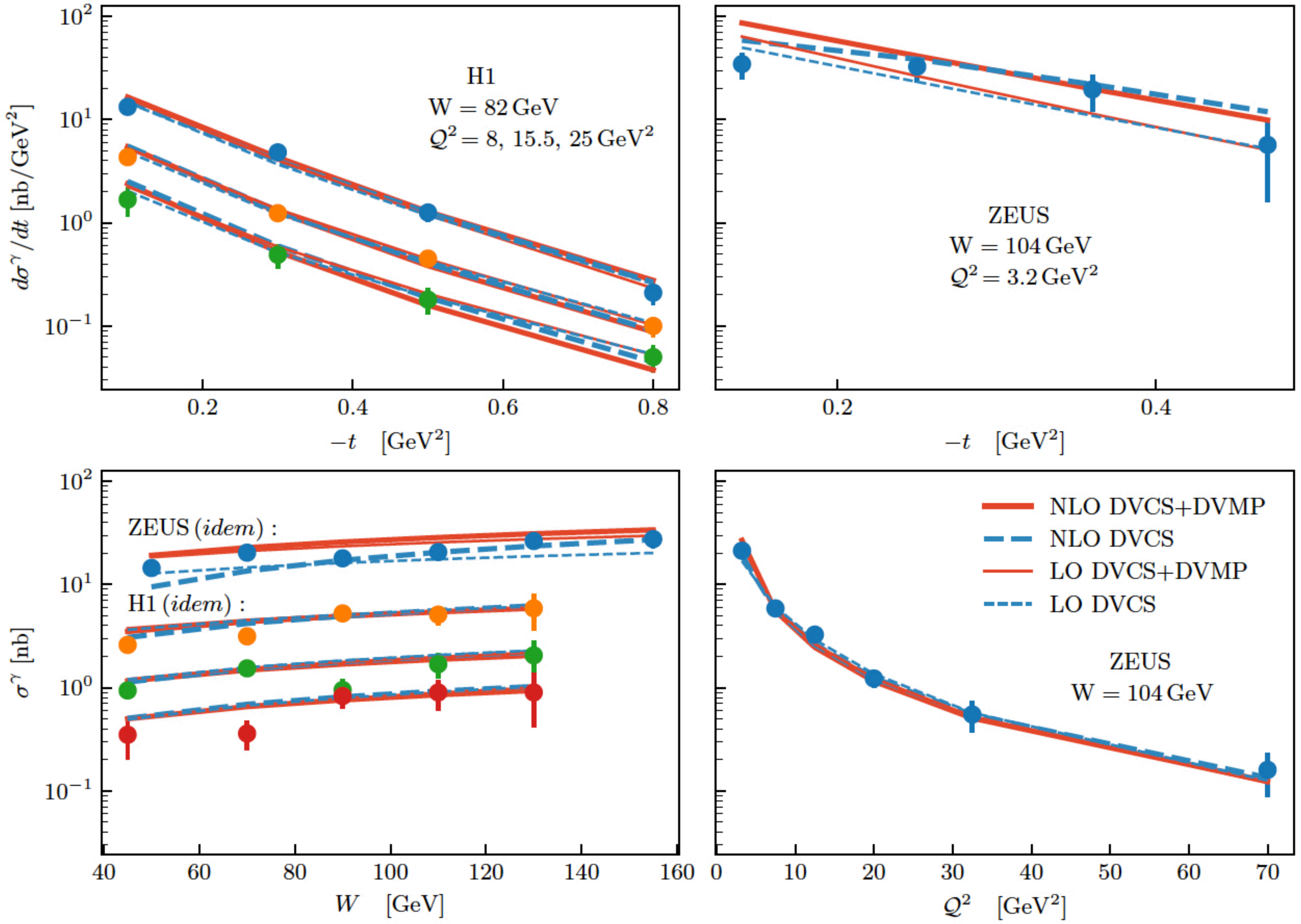
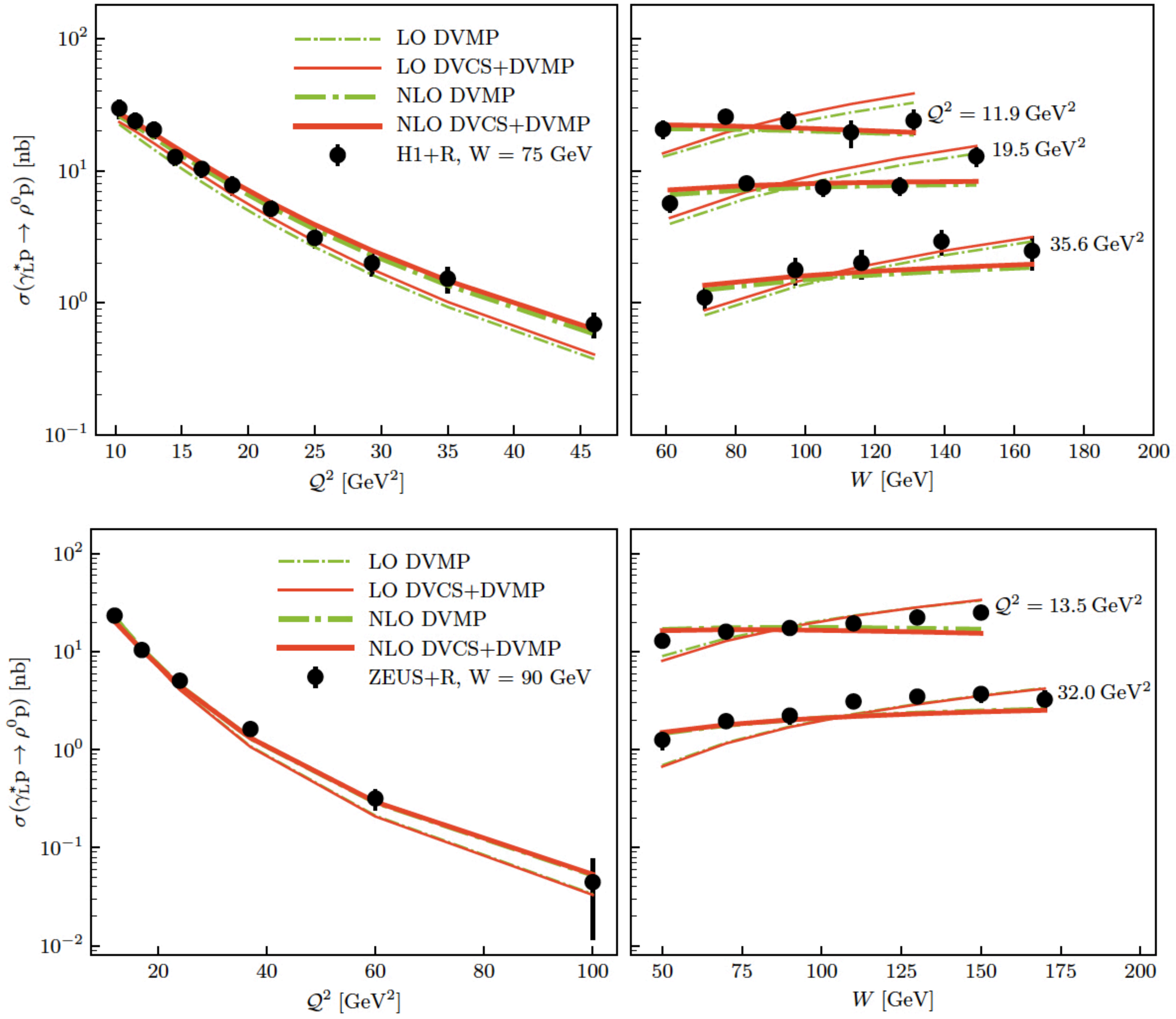
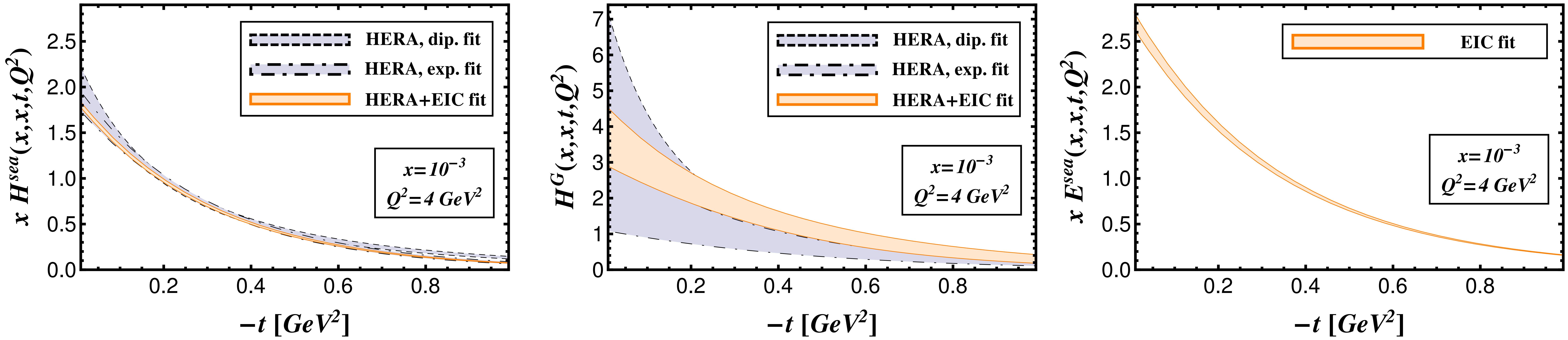


Deeply Virtual Meson Production



EIC IMPACT

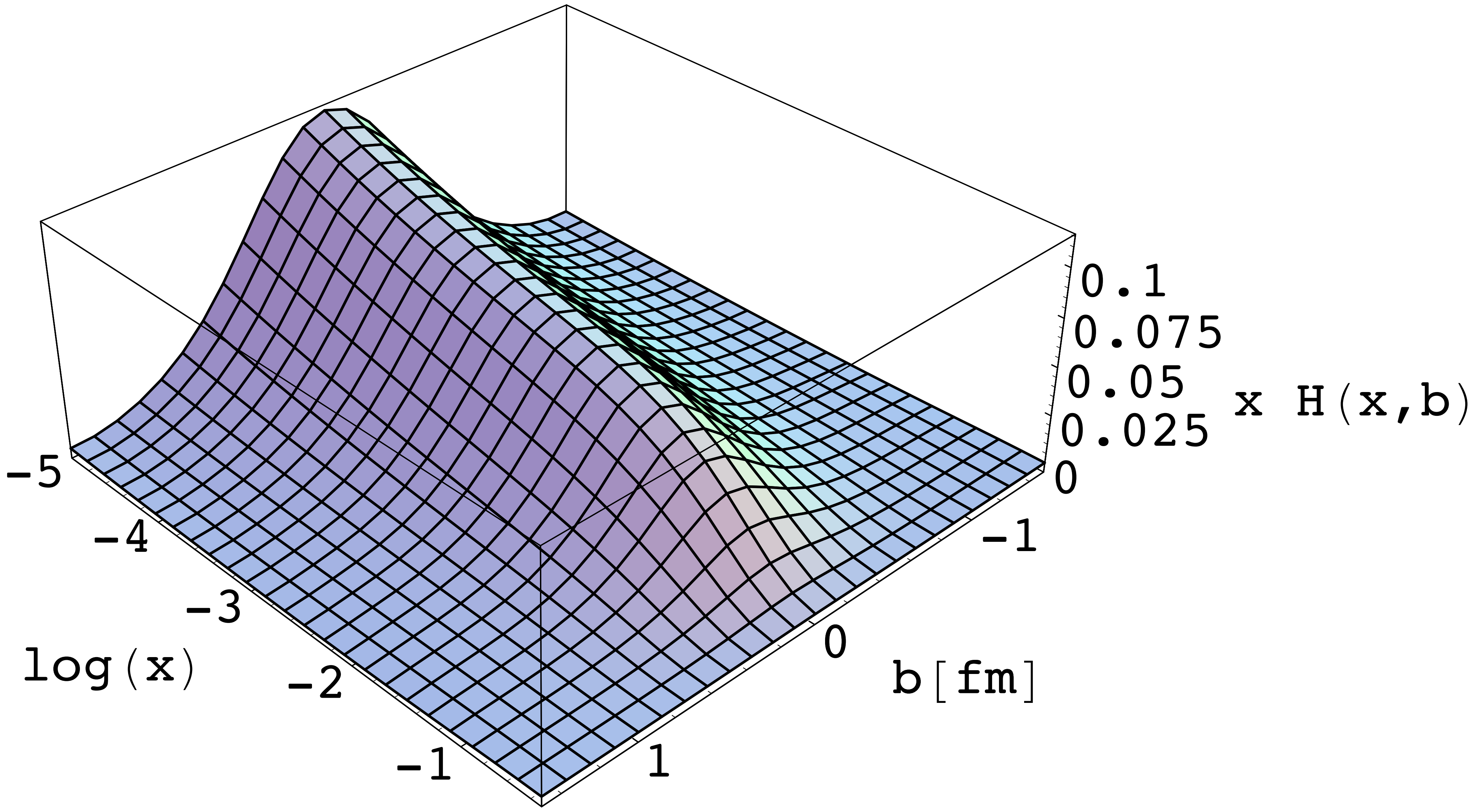
Čuić, Duplančić, Kumerički, Passek-K., 2310.13837

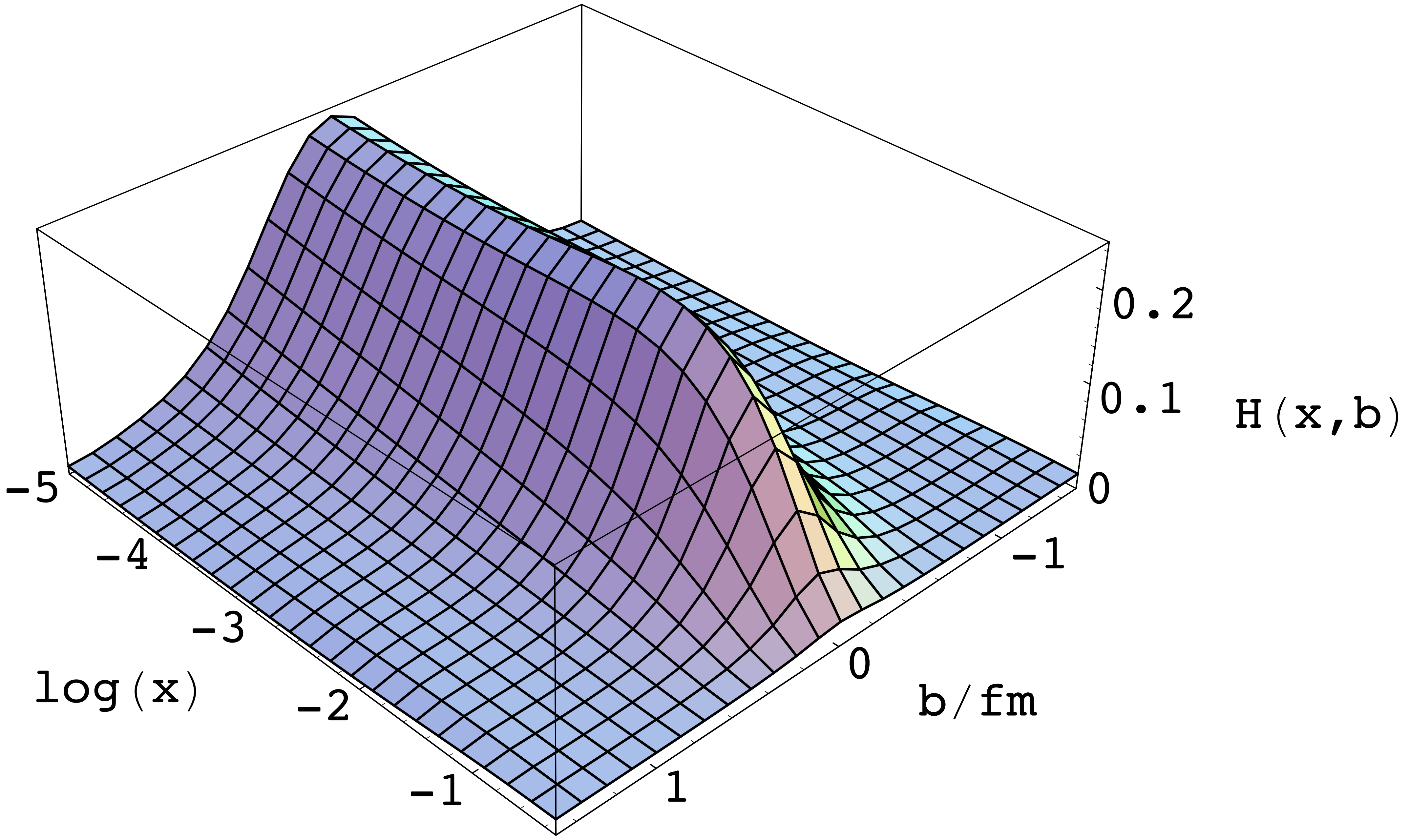


RESULTING IMPACT-PARAMETER DISTRIBUTIONS

Čuić, Duplančić, Kumerički, Passek-K., 2310.13837

$$q(x,\vec{b}_\perp) = \int \frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H^q(x,0,t=-\vec{\Delta}_\perp^2)$$





IMPACT PARAMETER DISTRIBUTIONS FROM EIC

Aschenauer et al., in preparation, courtesy of S. Fazio

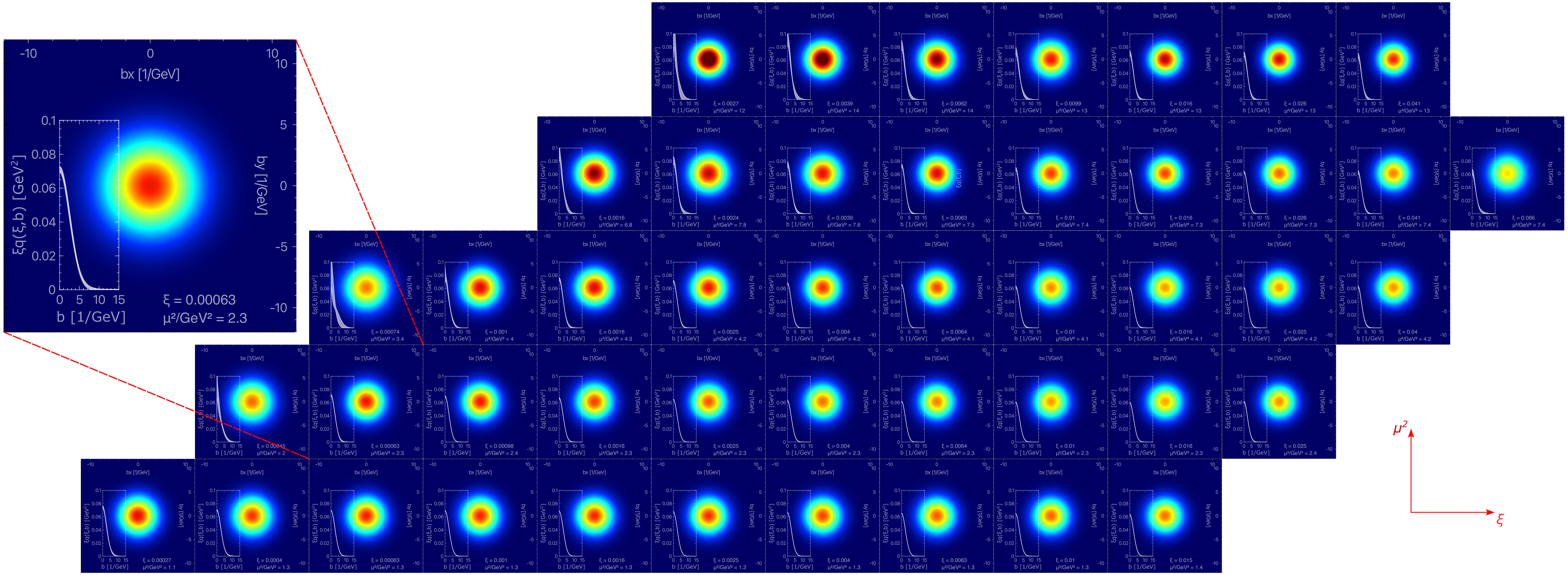
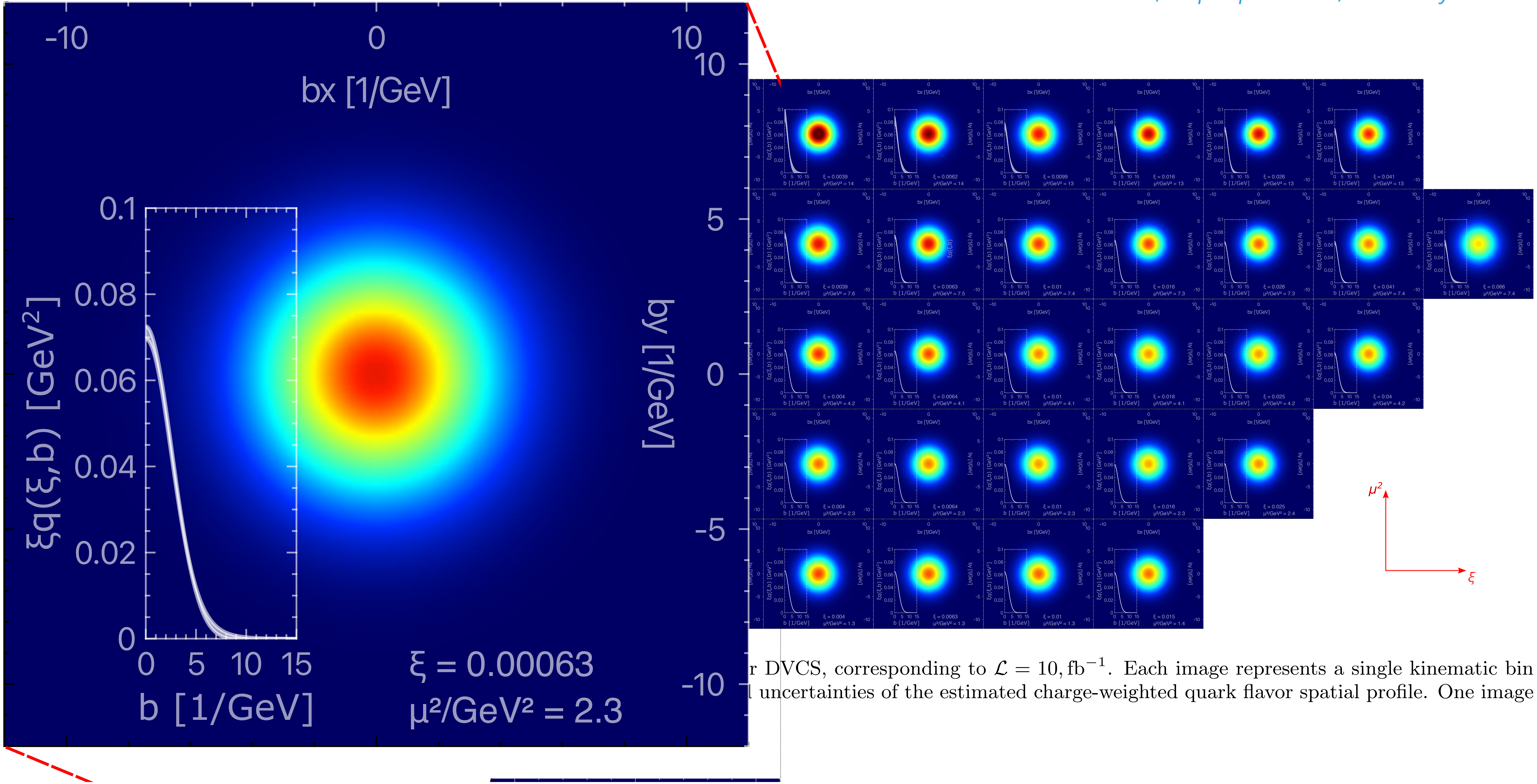


FIG. 16. 2-dimensional tomographic images obtained from EIC pseudo-data for DVCS, corresponding to $\mathcal{L} = 10, \mathrm{fb}^{-1}$. Each image represents a single kinematic bin used in this analysis and includes information about the average kinematics and uncertainties of the estimated charge-weighted quark flavor spatial profile. One image is zoomed in for better readability.

IMPACT PARAMETER DISTRIBUTIONS FROM EIC

Aschenauer et al., in preparation, courtesy of S. Fazio



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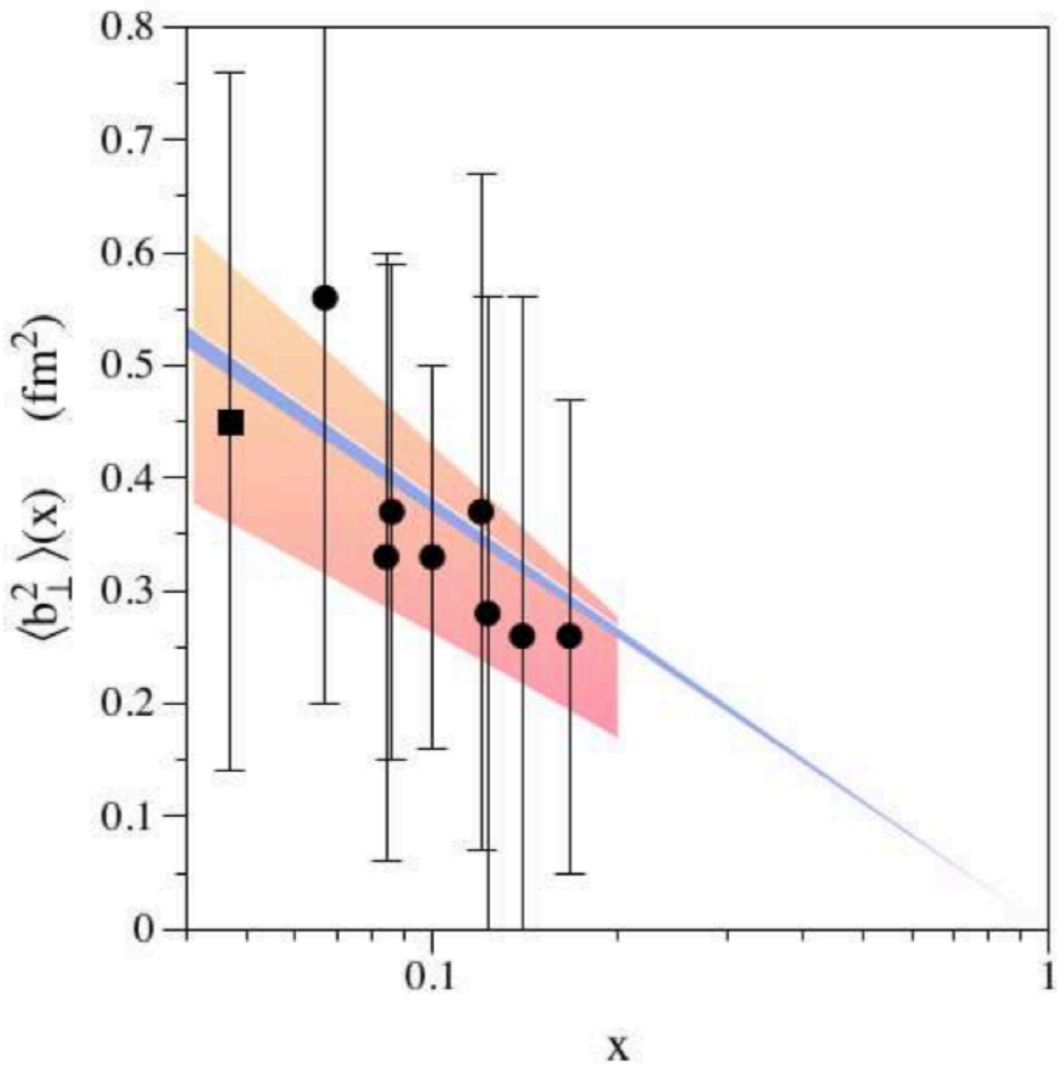
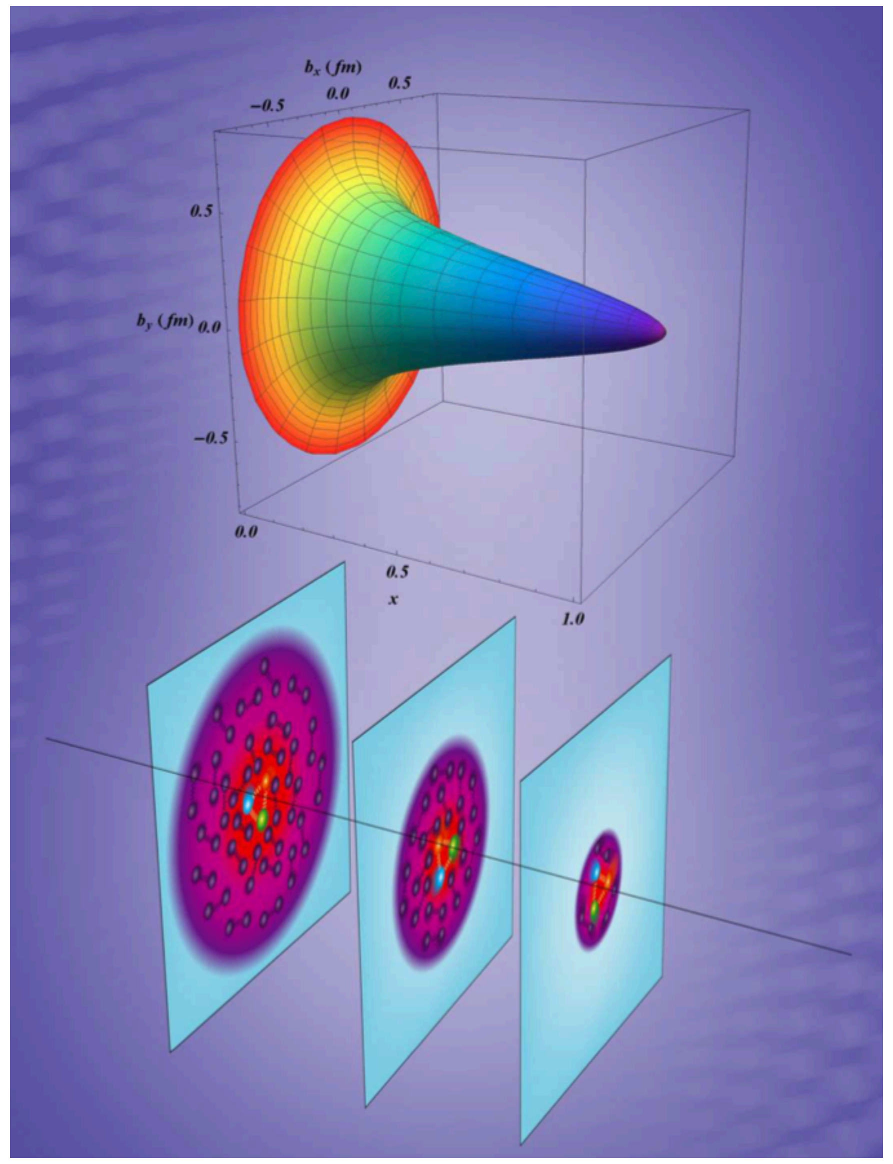
l uncertainties of the estimated charge-weighted quark flavor spatial profile. One image

IMPACT-PARAMETER DISTRIBUTION SQUARED RADIUS

$$\langle b_\perp^2\rangle^q(x) = -4\frac{\partial}{\partial \boldsymbol{\Delta}_\perp^2} \ln H_-^q(x,0,-\boldsymbol{\Delta}_\perp{}^2)\Big|_{\boldsymbol{\Delta}_\perp=0}$$





Dupré, Guidal, Niccolai, Vanderhaeghen,
arXiv:1704.07330

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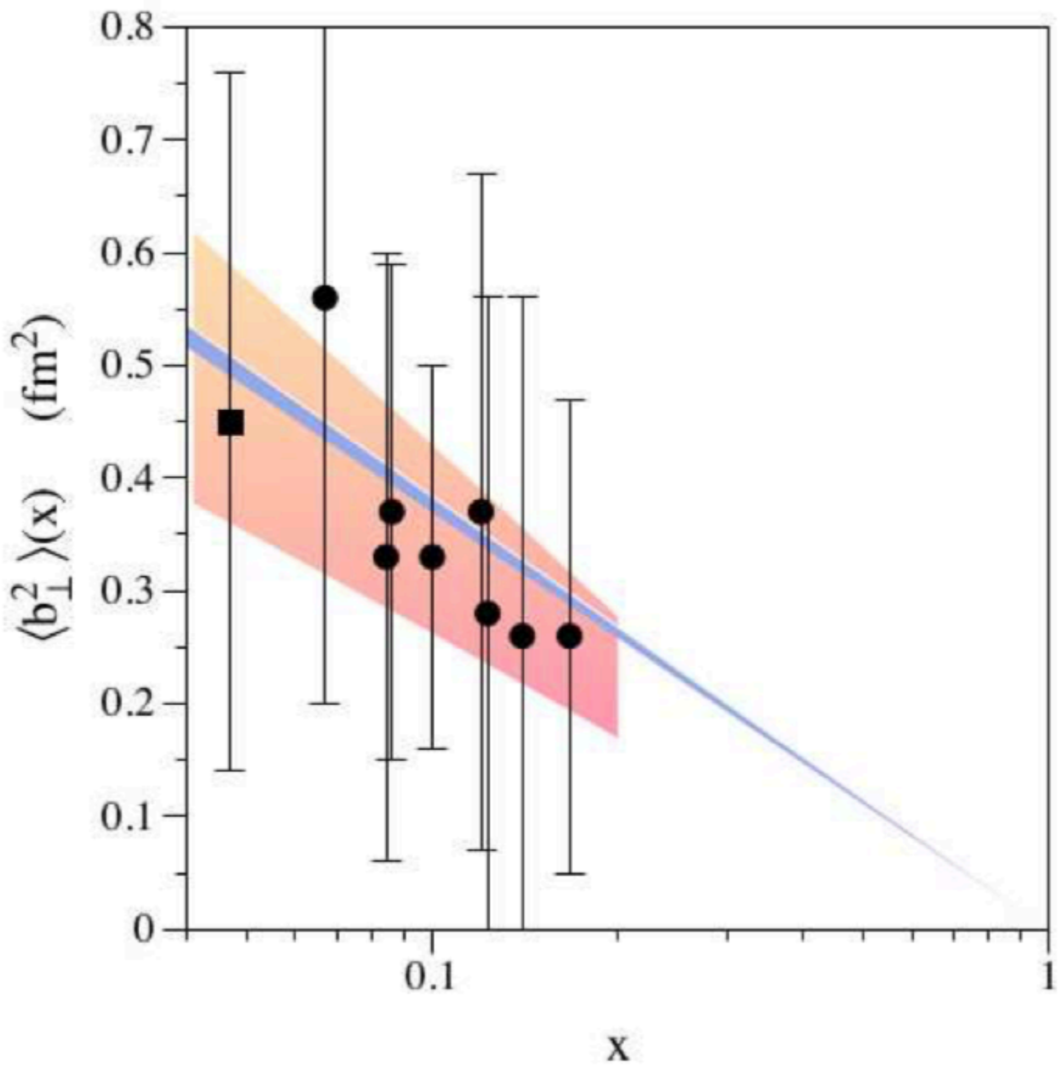
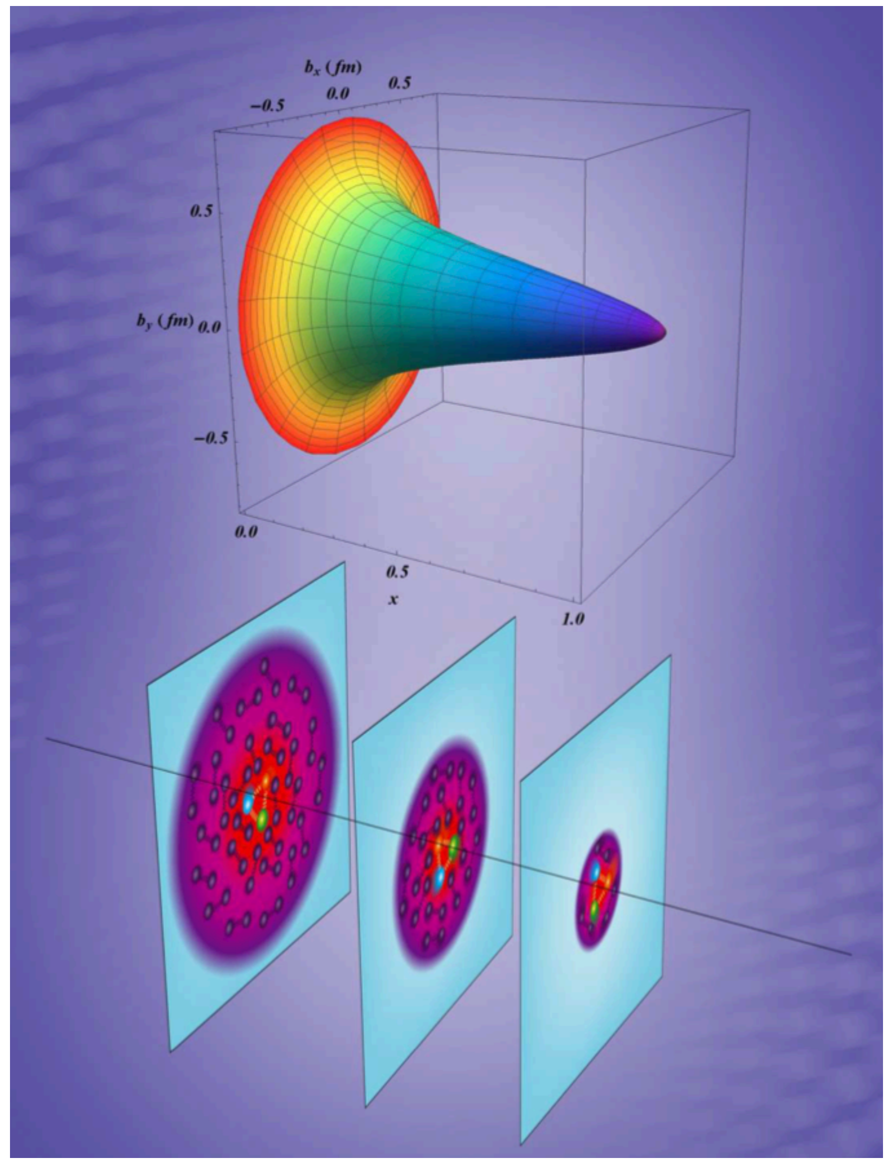
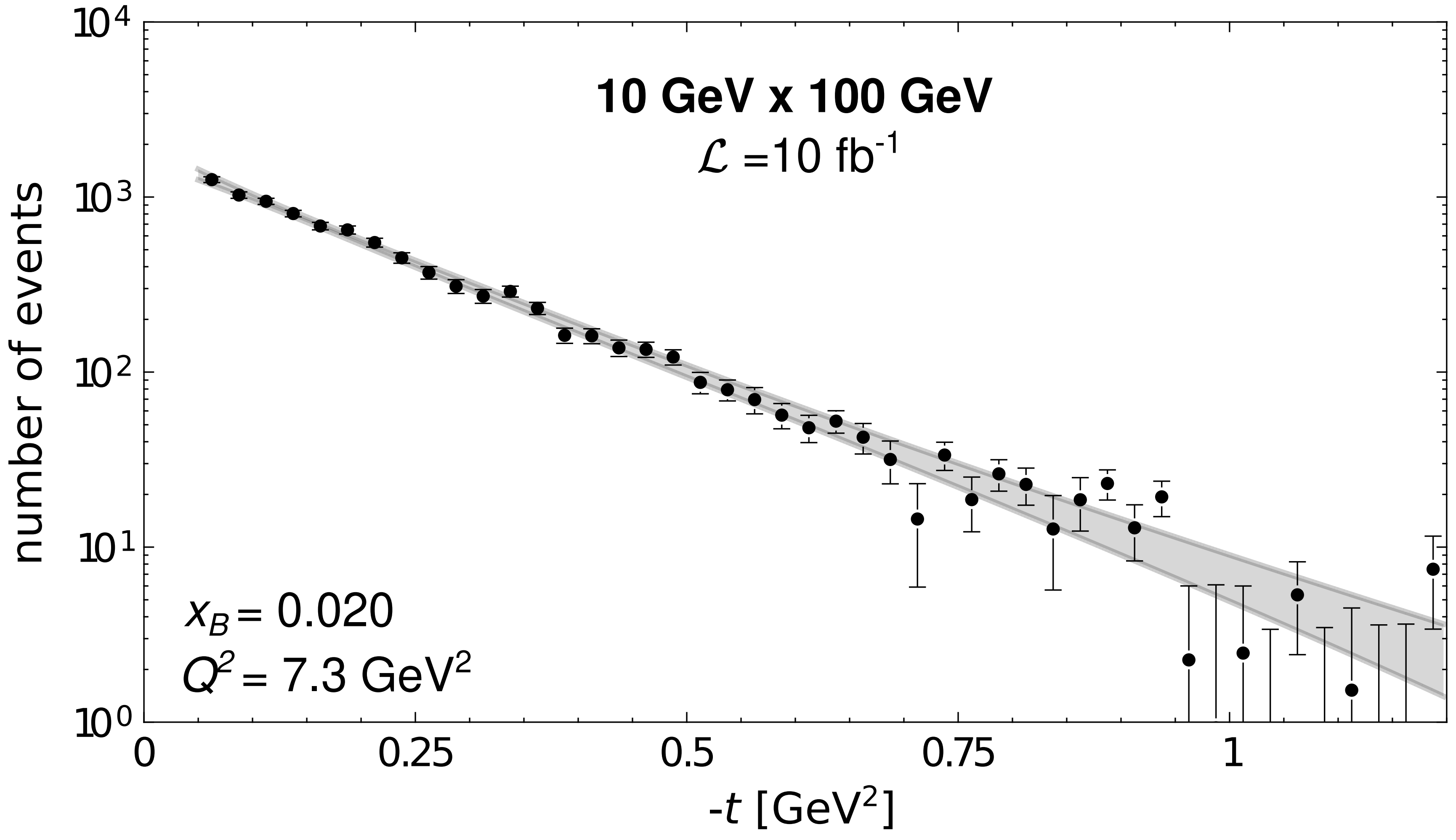
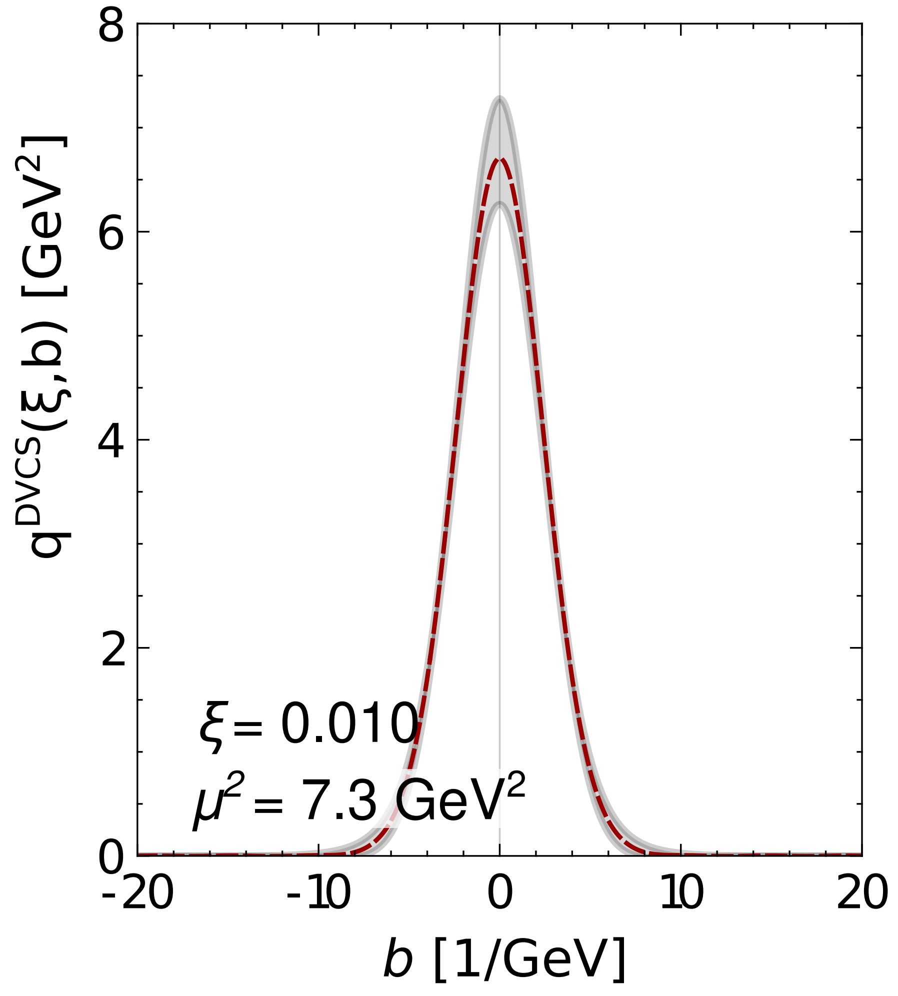




Dupré, Guidal, Niccolai, Vanderhaeghen,
arXiv:1704.07330

EIC impact





Aschenauer et al., in preparation, courtesy of S. Fazio

DEDICATED SOFTWARE

Gepard

PARTONS

https://gepard.phy.hr/

https://partons.cea.fr/

CONCLUSIONS

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- The early-stage phase will not produce dramatically different results from existing measurements in terms of number of data points and statistics

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- The machinery to extract TMDs and GPDs is in good shape
- Also, other experiments are needed (fixed target ep, electron-positron, pp)
- I did not mention many interesting topics: transversity and tensor charge, subleading-twist TMDs, dihadron production, gravitational form factors…