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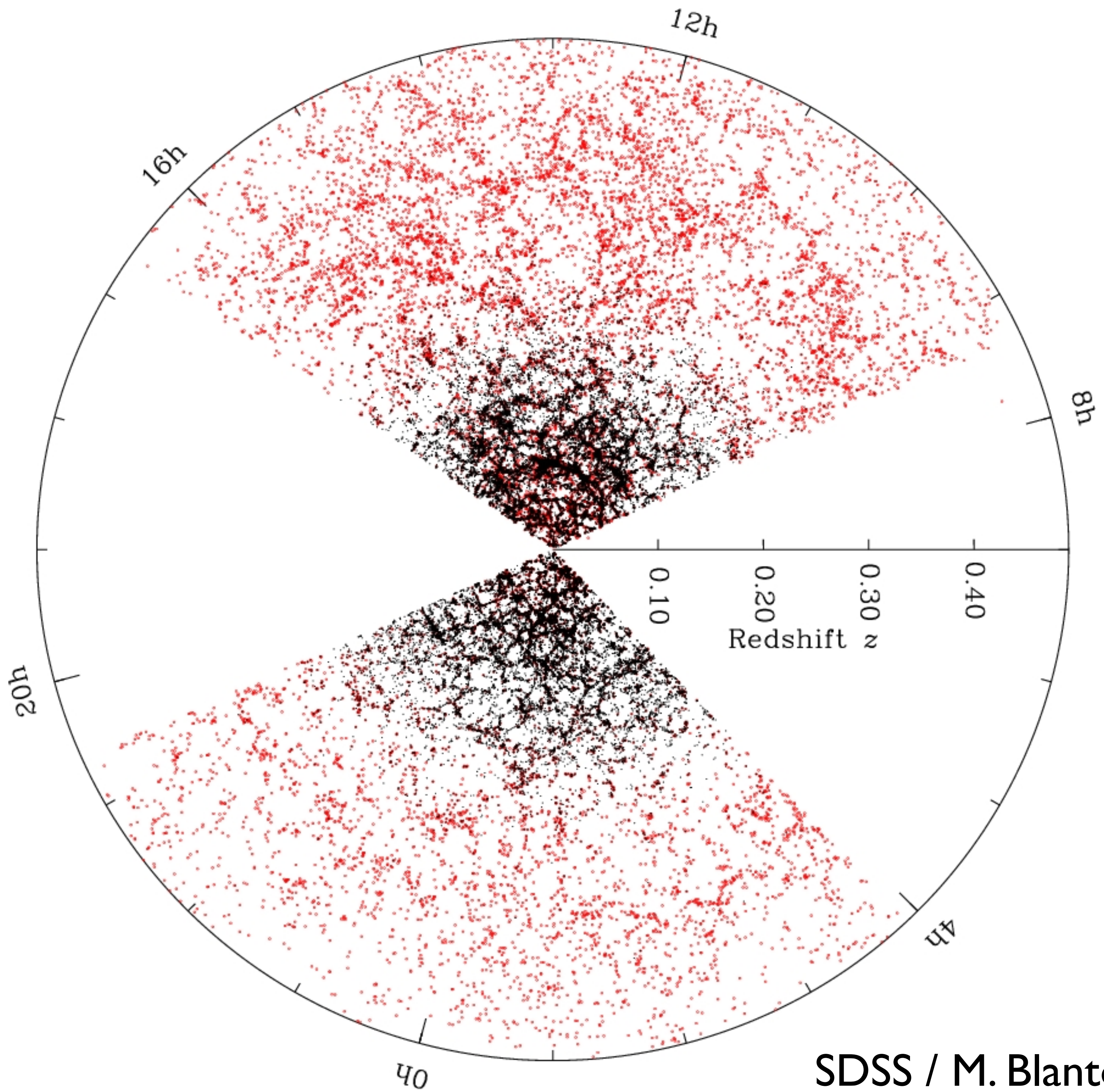
ORIGINS
Excellence Cluster

Non-Gaussianity in Large-Scale Structure

Fabian Schmidt
MPA

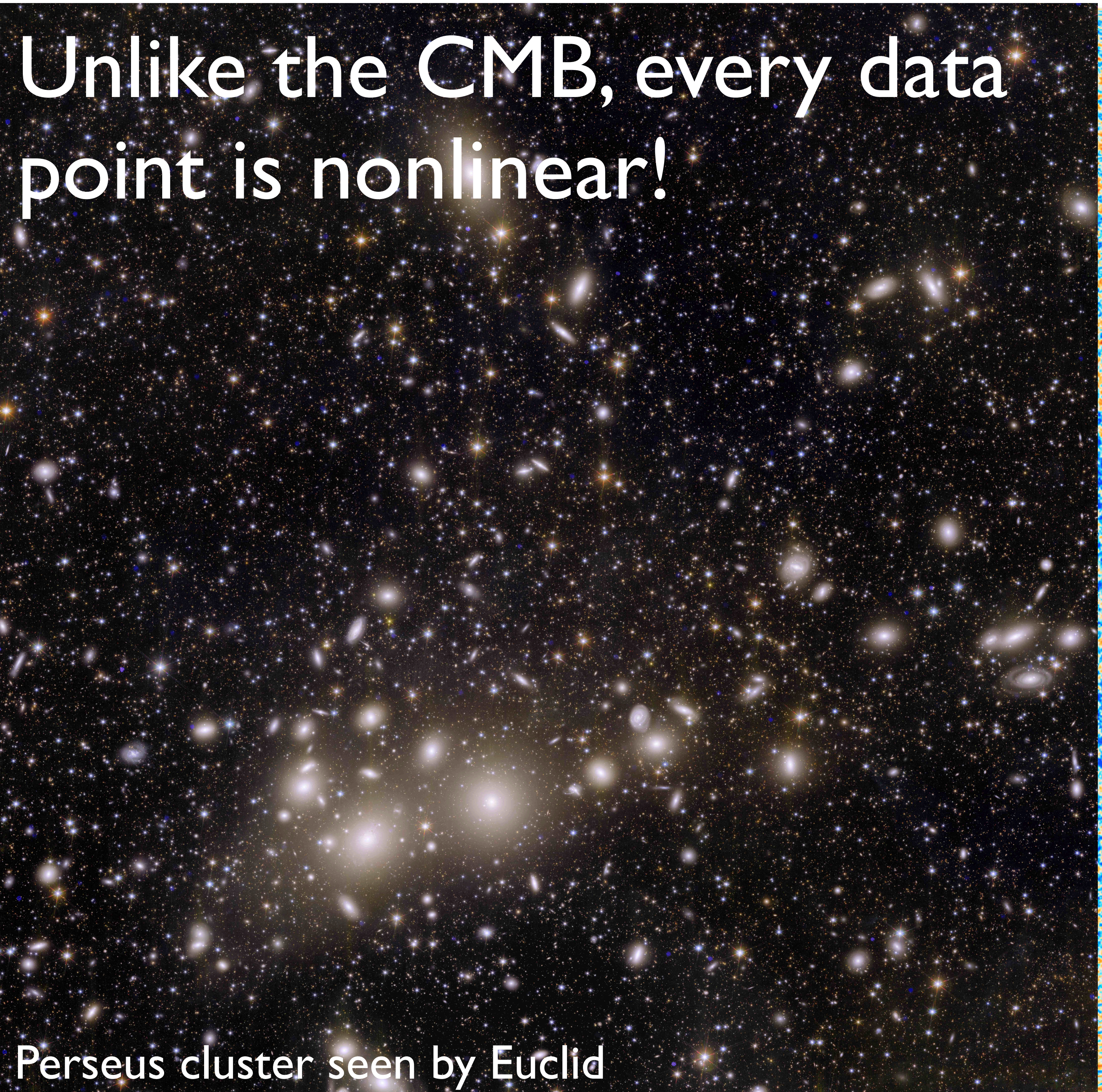
Learning the Deep Mysteries of Nature with Cosmology,
Sep 11, 2025

Non-Gaussianity in Large-Scale Structure

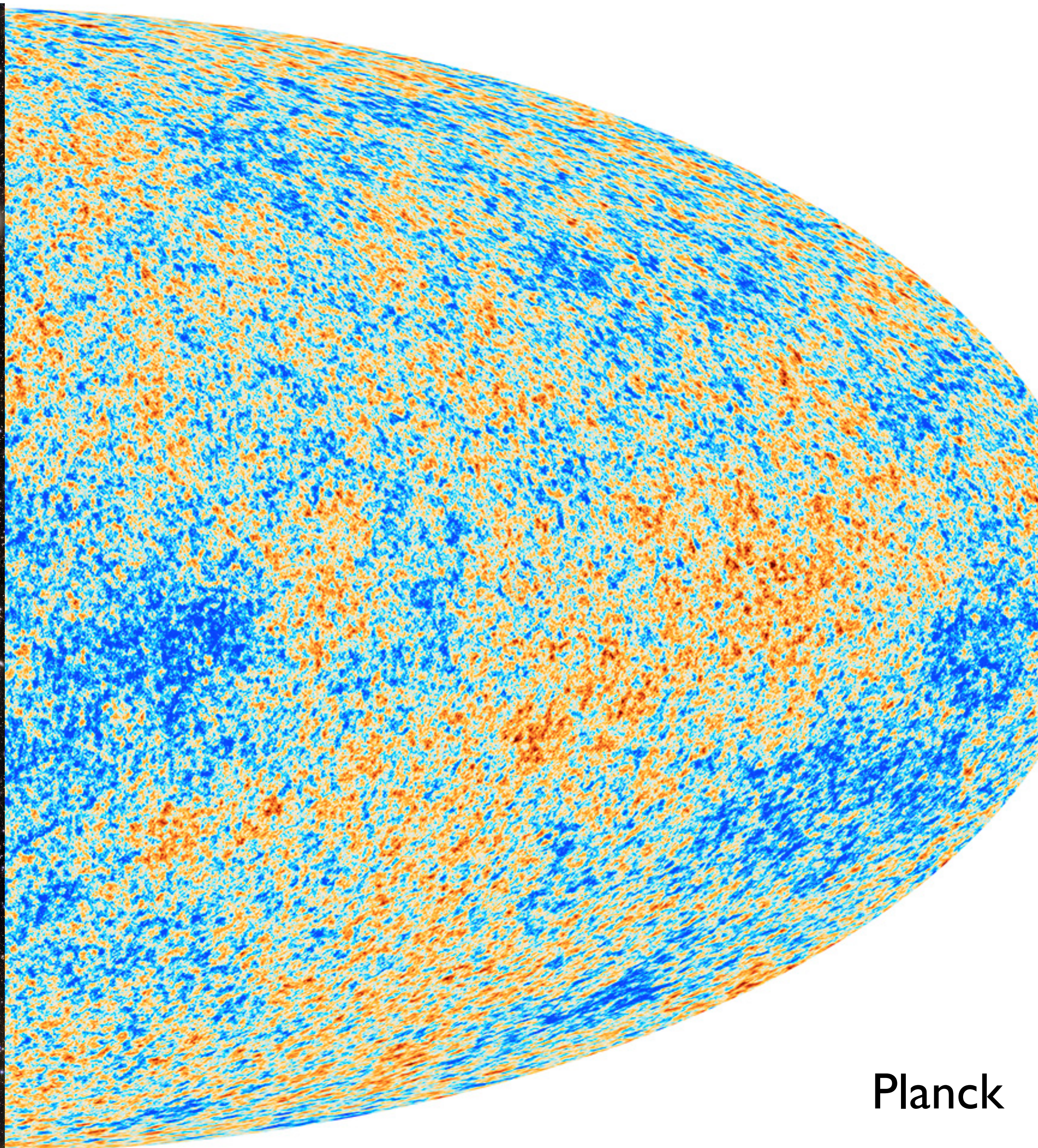


SDSS / M. Blanton

Non-Gaussianity in Large-Scale Structure

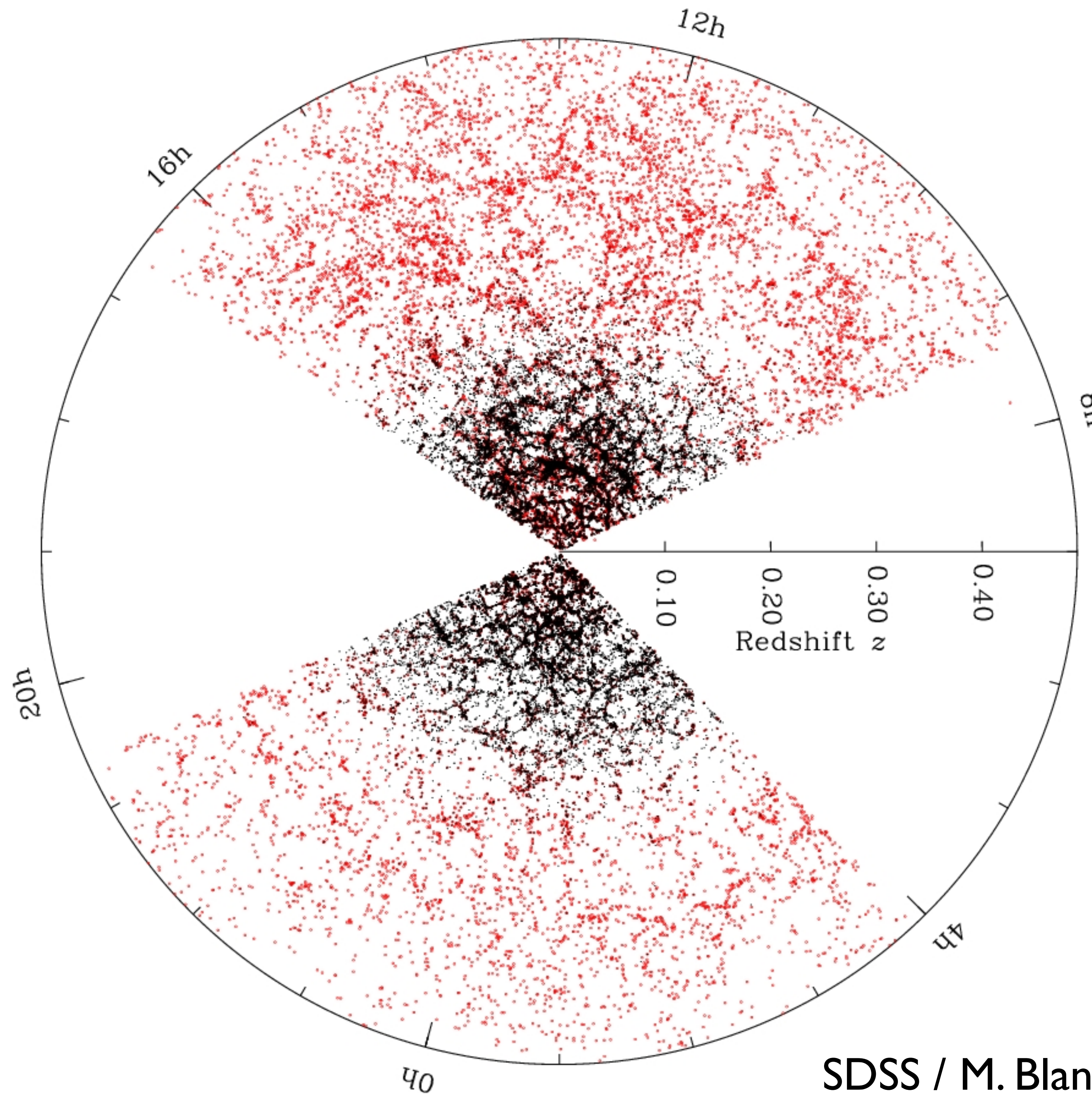


Perseus cluster seen by Euclid


Planck

Non-Gaussianity in Large-Scale Structure

- Prominent non-Gaussianity from nonlinear structure formation (part II)
- But can use this as sensitive of primordial non-Gaussianity as well (part I)



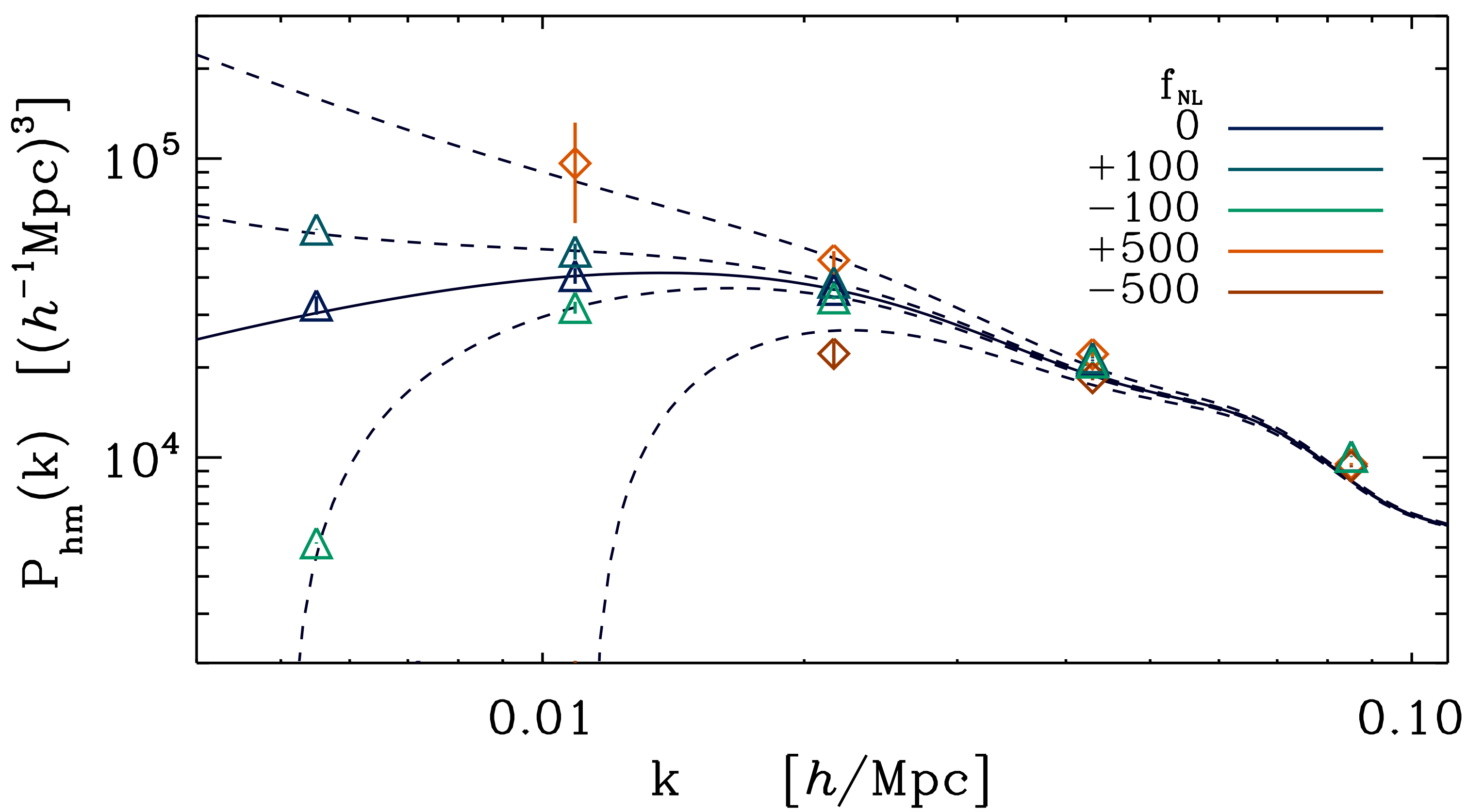
SDSS / M. Blanton

I. Early non-Gaussianities: Probing inflation

Inflation and Galaxy Clustering

- Most prominent signature of inflationary physics in LSS: scale-dependent bias induced by local-type primordial non-Gaussianity
- Discovered in simulations, but it was hiding in theory calculations all along…

$$P_g(k) = \left[b^2 + 2b\, b_{\rm NG} f_{\rm NL} \frac{A}{k^2}\right] P_m(k)$$



Dalal et al., 2008
Matarrese/Verde 2008

Inflation and Galaxy Clustering

- How does this generalize to other types of non-Gaussianity, i.e. other types of inflationary physics?
  - Different scalings in squeezed limit
  - Beyond primordial 3-point function

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A PATH-INTEGRAL APPROACH TO LARGE-SCALE MATTER DISTRIBUTION ORIGINATED BY NON-GAUSSIAN FLUCTUATIONS

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Received 1986 July 7; accepted 1986 August 1

Inflation and Galaxy Clustering

- How does this generalize to other types of non-Gaussianity, i.e. other types of inflationary physics?
  - Different scalings in squeezed limit
  - Beyond primordial 3-point function

$$\xi^{(2)}_{R_0;\,\nu,\,R}(x_1, x_2) = \langle \varepsilon_{R_0}(x_1)\,\rho_{\nu,\,R}(x_2)\rangle / \langle \rho_{\nu,\,R}\rangle$$

$$= \left\{ \sum_{L=1}^{\infty} \sum_{[m_L]} \left[\prod_{n=1}^{L} \left(w_R^{(n)}/n! \right)^{m_n} / m_n! \right] \sum_{n=1}^{L} n m_n \left(w^{(n)}_{[R_0:1;\,R:n-1]} \sigma_{R_0} / w_R^{(n)} \right) a_{L-1}(2^{-1/2}\nu) \right\} \Big/ \Pi^{(1)}_{\nu,\,R}$$

Inflation and Galaxy Clustering

- How does this generalize to other types of non-Gaussianity, i.e. other types of inflationary physics?
  - Different scalings in squeezed limit
  - Beyond primordial 3-point function
- Density peaks as a well-defined model, but we were seeking a general result for any physical LSS tracer

Inflation and Galaxy Clustering

- How does this generalize to other types of non-Gaussianity, i.e. other types of inflationary physics?
  - Different scalings in squeezed limit
  - Beyond primordial 3-point function
- Density peaks as a well-defined model, but we were seeking a general result for any physical LSS tracer
- Key ingredient: scaling of primordial correlators in squeezed limit
  - Interesting limit to constrain new particles: *cosmological collider*
- We can predict the scale-dependence of LSS statistics, but amplitude is controlled by tracer-dependent bias parameter

FS, Kamionkowski (2010)
Desjacques, Jeong, FS (2011+)

Inflation and Galaxy Clustering

- How does thi … on-Gaussianity, i.e. other types o
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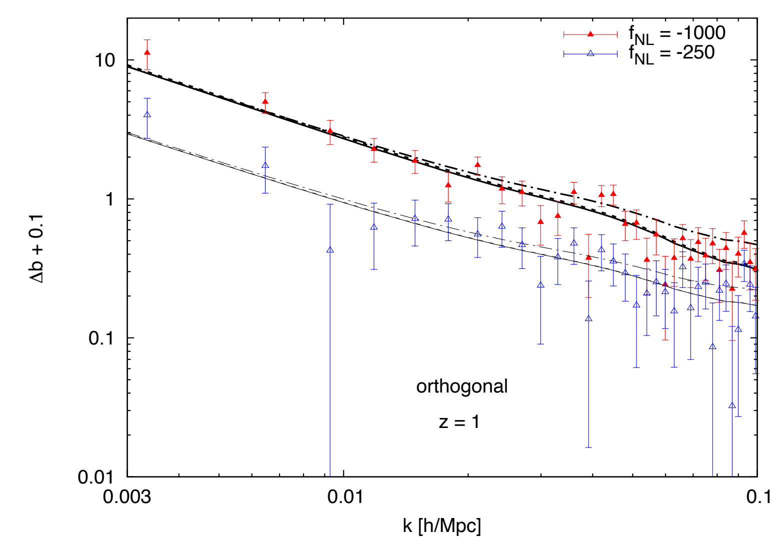


Figure 9. Same as in Fig. 8, but for the orthogonal shape of non-Gaussianity. Note, however, that here the redshift of the halos with mass 1.2×10^{14} Mpc/$h < M < 2.4 \times 10^{14}$ Mpc/h is $z = 1$.

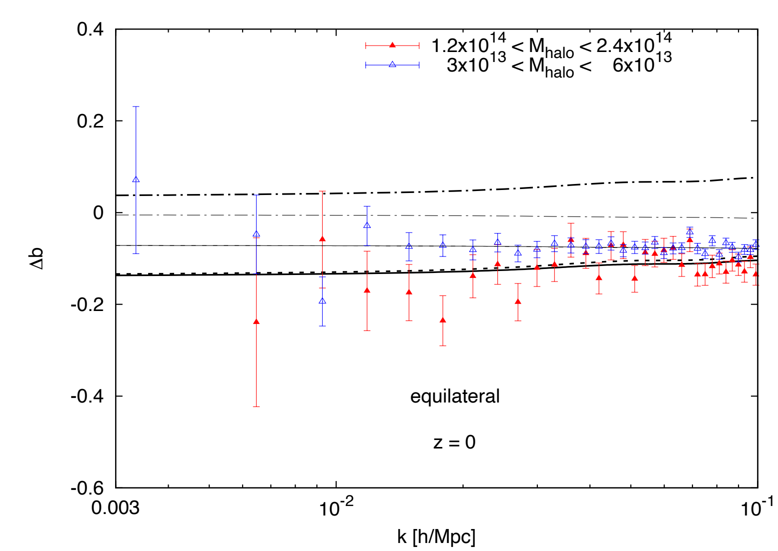


Figure 10. Same as in Fig. 8, but for the equilateral shape of non-Gaussianity. Note the linear scale of the y-axis.

Wagner & Verde (2011)

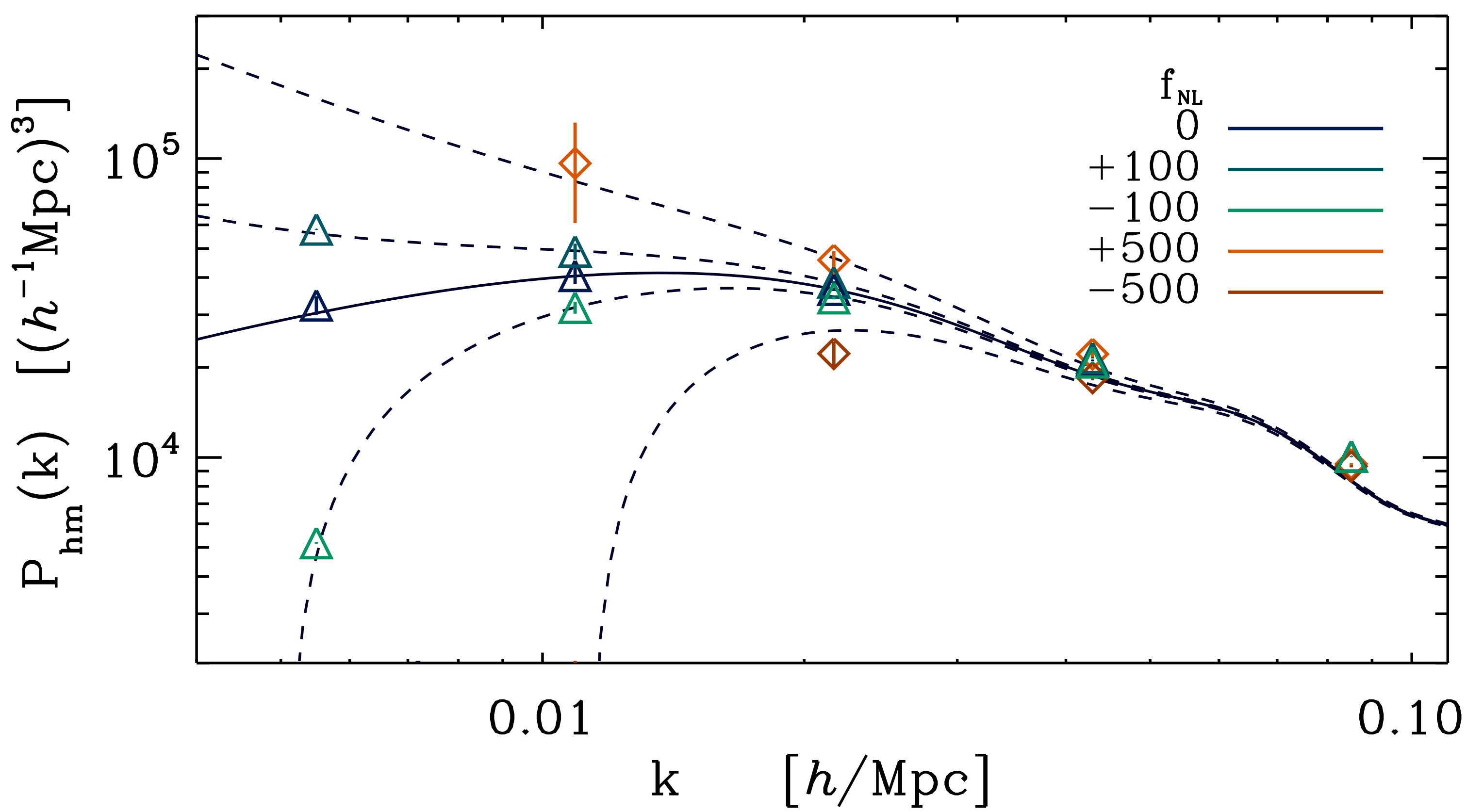
Connecting inflation with LSS in GR

- Smoking-gun signature of *multi-field inflation*
- Probing highest energy physics with galaxies on the largest scales
- Current constraints:

$$\Delta f_{\rm NL}({\rm CMB}) \sim 3$$

$$\Delta f_{\rm NL}({\rm LSS}) \sim 15$$

$$P_g(k) = \left[b^2 + 2b\, b_{\rm NG} f_{\rm NL} \frac{A}{k^2}\right] P_m(k)$$

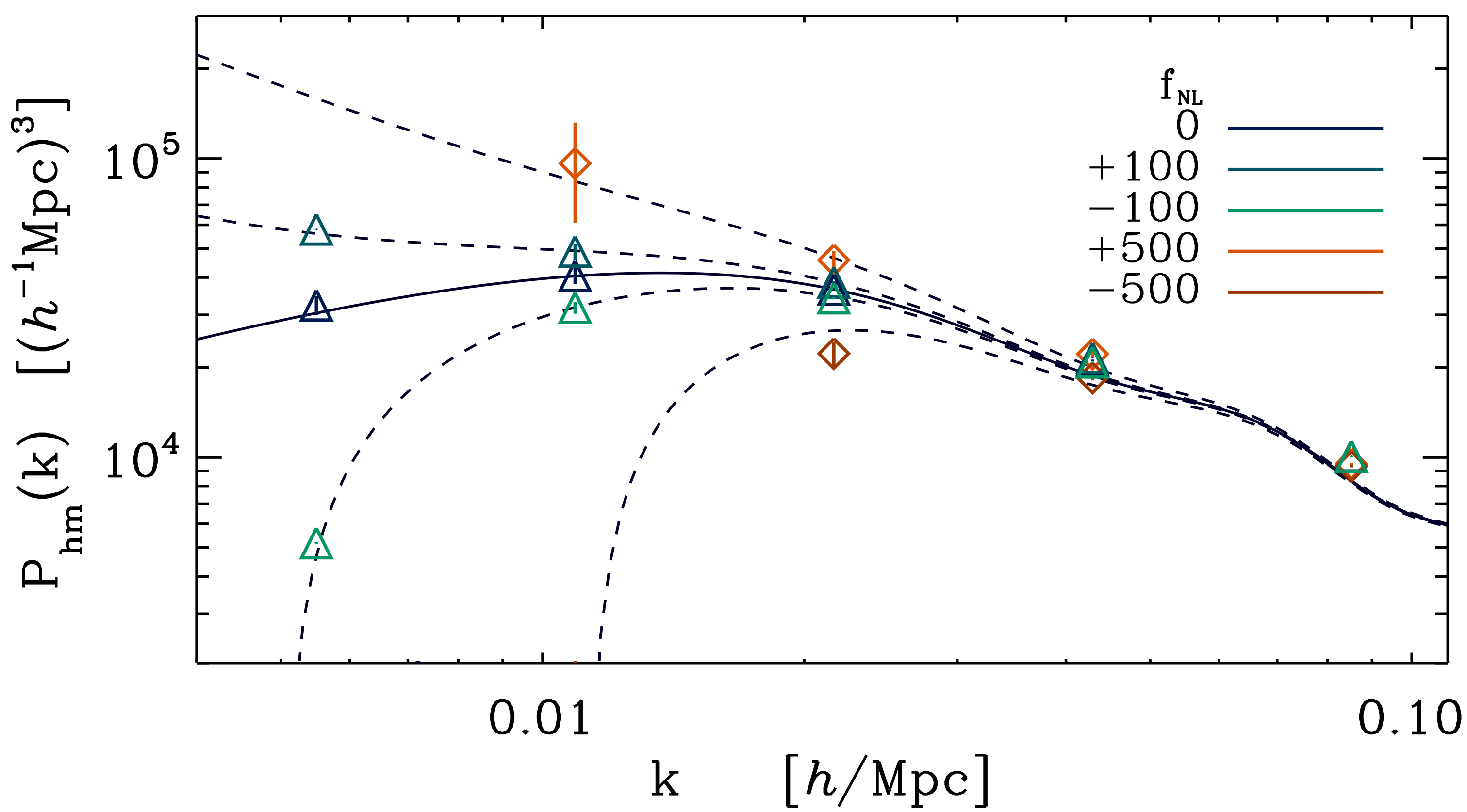


Connecting inflation with LSS in GR

$$\Delta f_{\rm NL}({\rm LSS}) \sim 15$$

- f_{NL}~1 corresponds to a contribution of order Φ to observed galaxy density
- So we need to worry about relativistic corrections to usual quasi-Newtonian treatment of galaxy clustering!

$$\Phi(\boldsymbol{k}) \sim \left(\frac{aH}{k}\right)^2 \delta(\boldsymbol{k})$$



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$$\Delta f_{\rm NL}(\rm LSS) \sim 15$$

- f_{NL}~1 corresponds to a contribution of order Φ to observed galaxy density
- So we need to worry about relativistic corrections to usual quasi-Newtonian treatment of galaxy clustering!
- In fact, since PNG is a second-order effect, it seems we need a second-order GR calculation
- Goal: get around this by focusing on squeezed limit: coupling of long- with short modes

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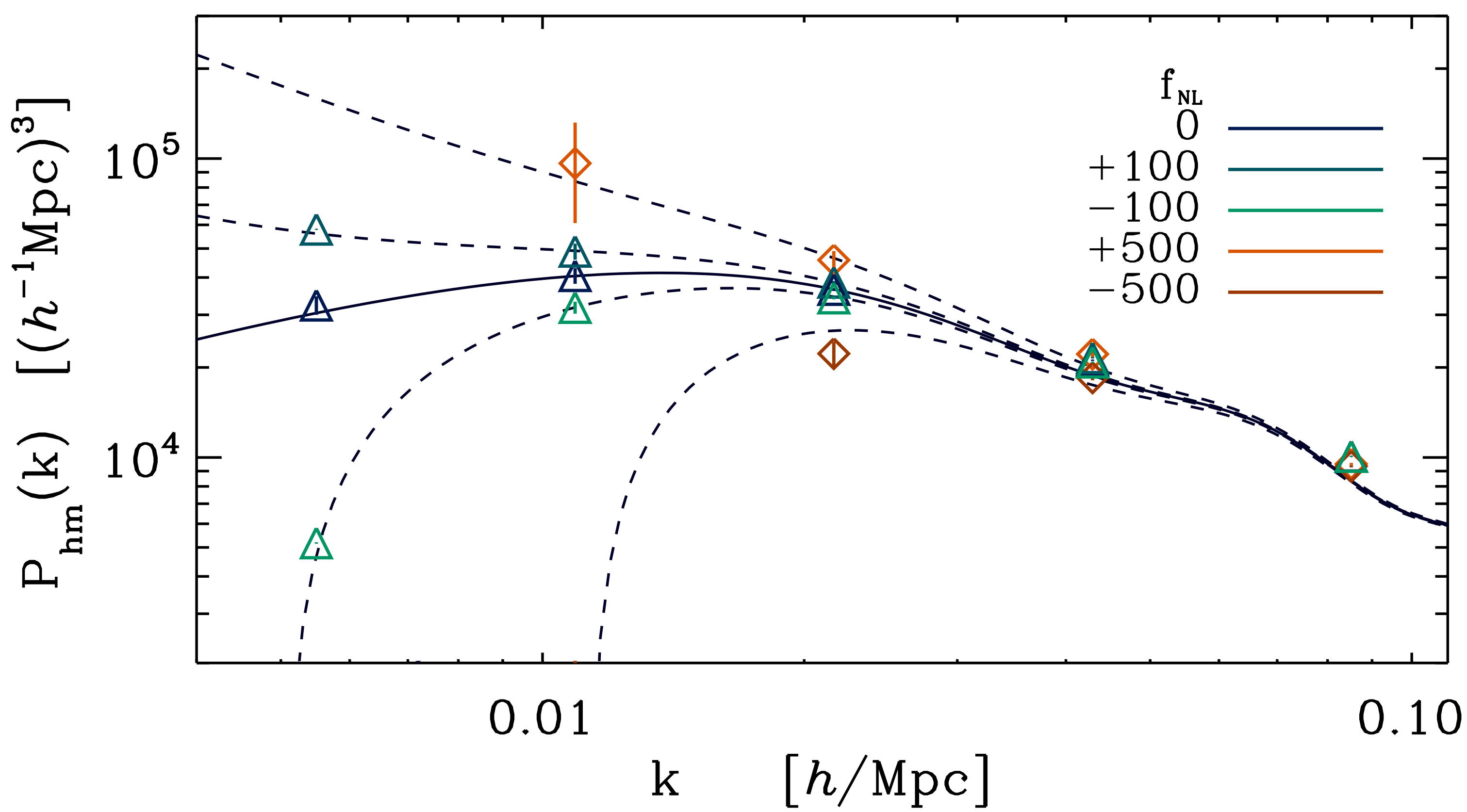
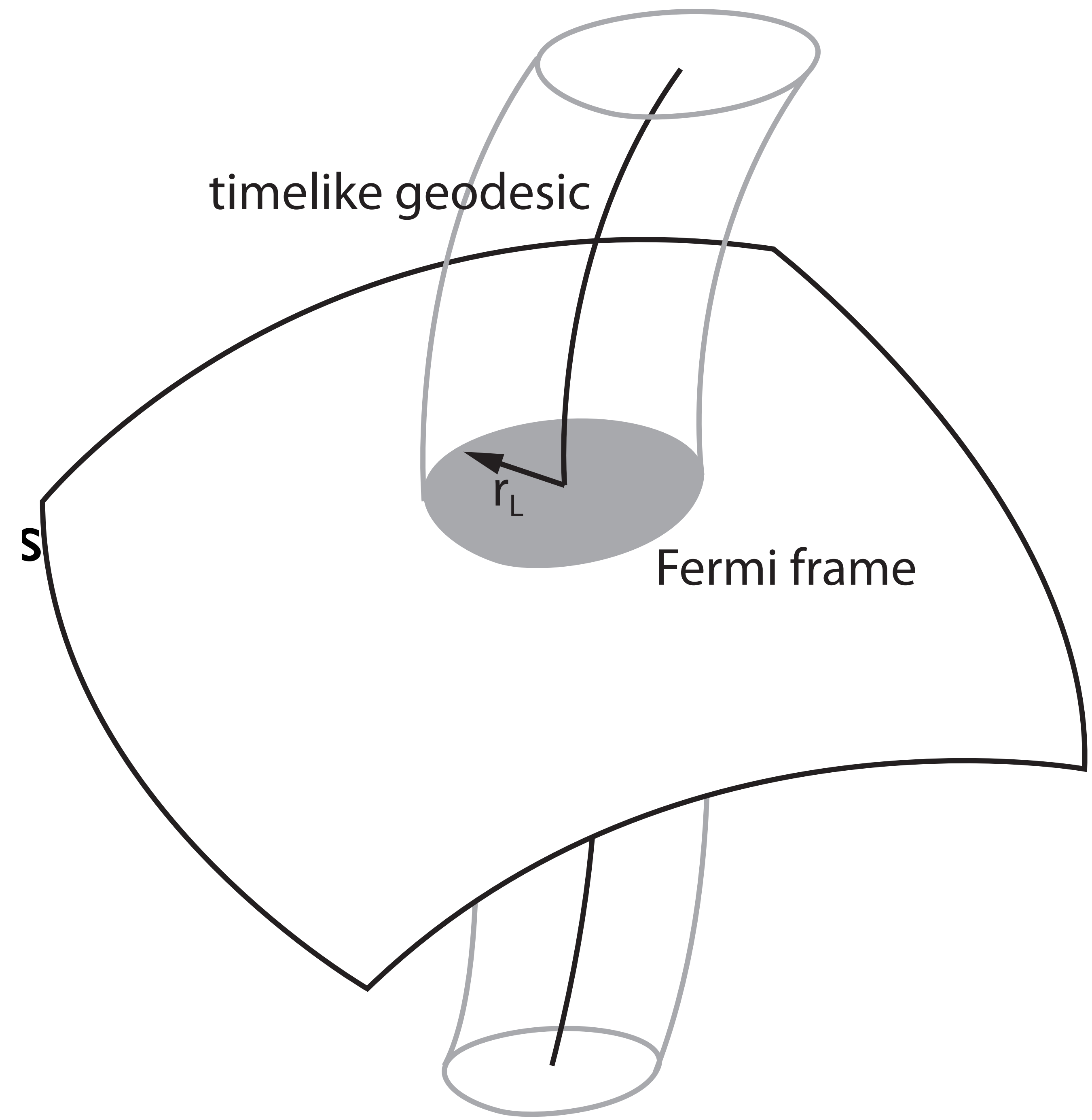


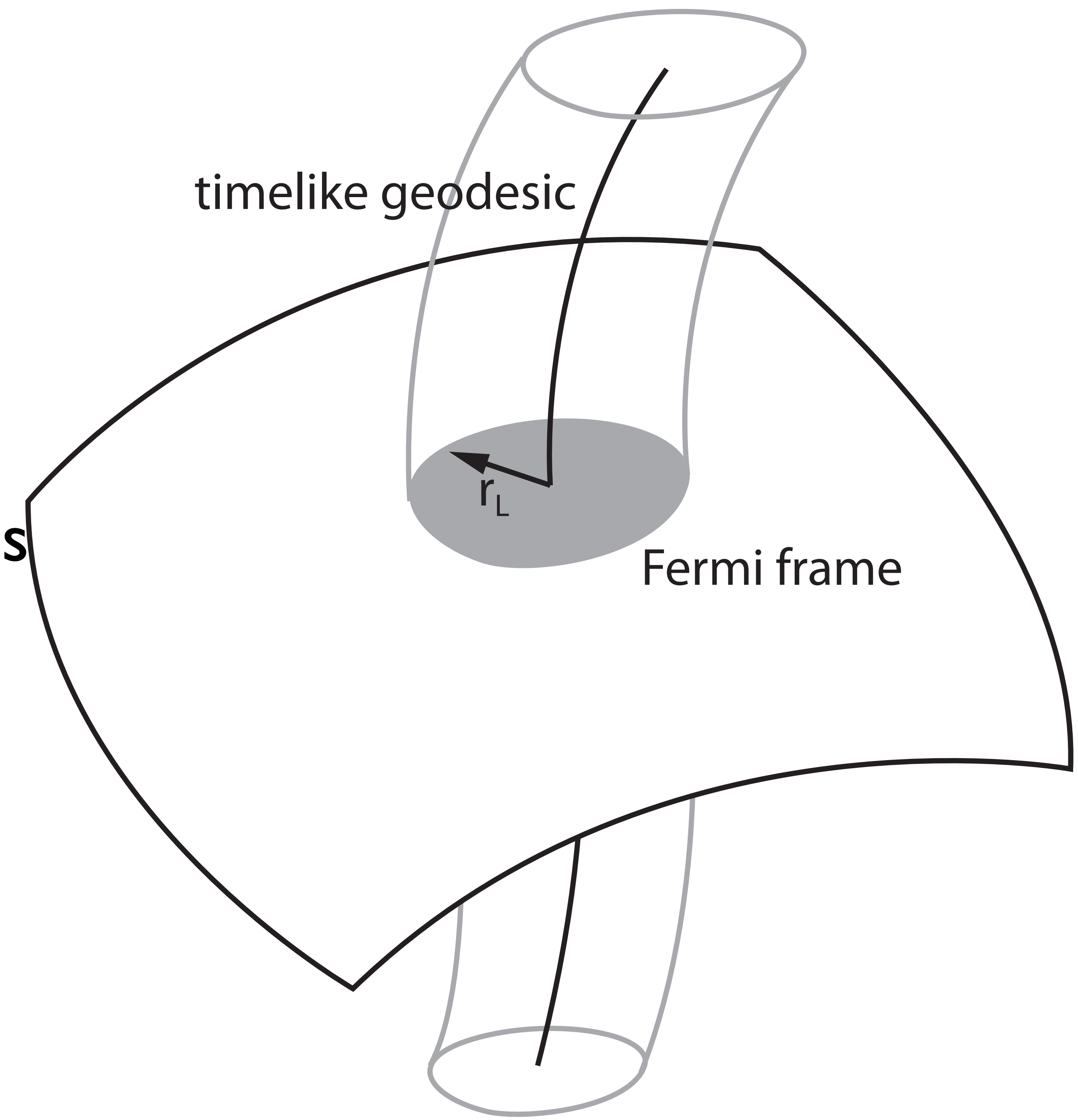
From initial conditions to galaxies

- Consider wordline of a small patch within the Universe



From initial conditions to galaxies

- Consider wordline of a small patch within the Universe
- We can go to a frame so that close to this wordline, the spacetime looks flat at all times
  - Time coordinate is proper time along geodesic
- ***Fermi frame:*** natural frame to describe local gravitational experiments

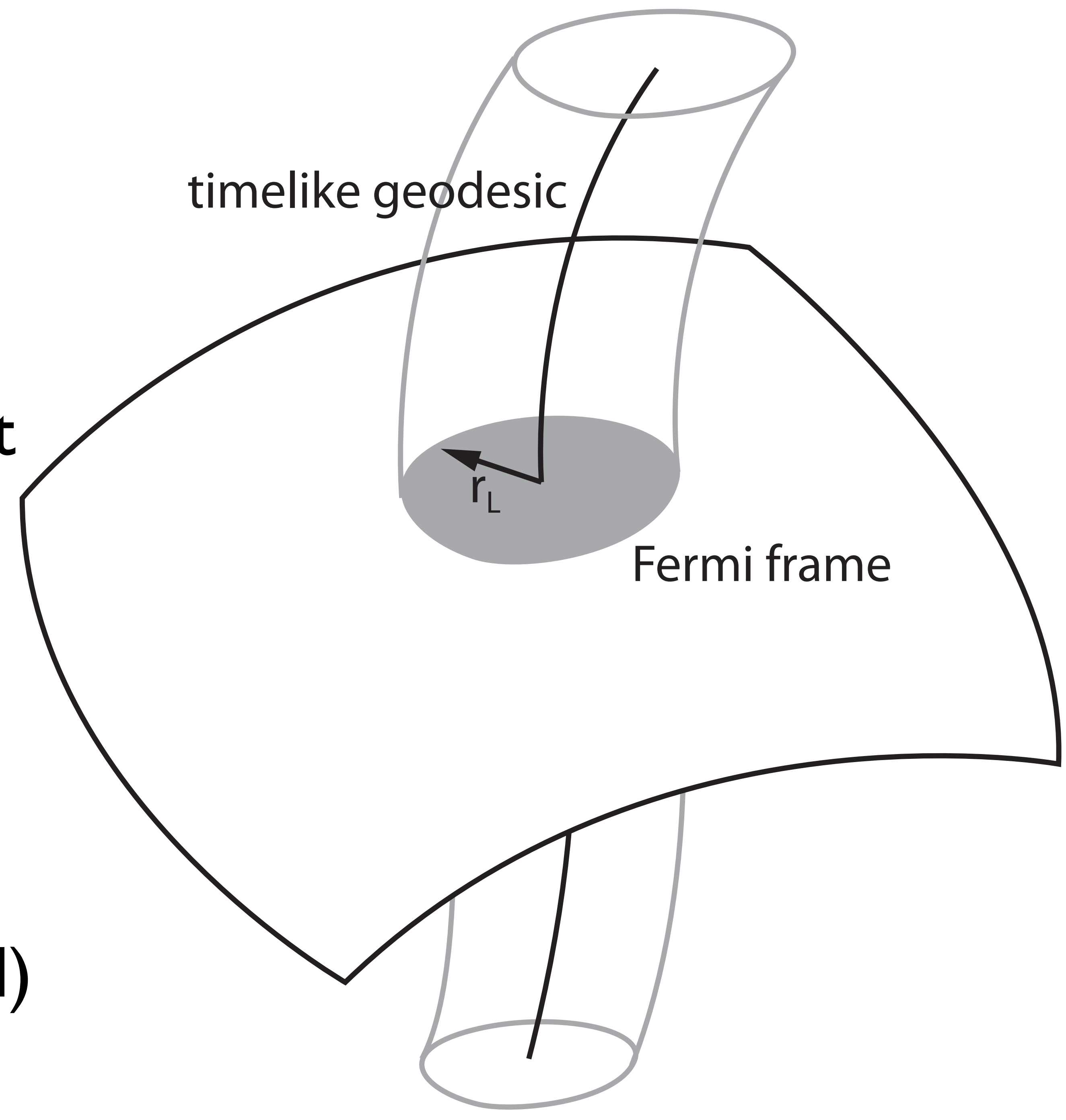


From initial conditions to galaxies

- Generalize to ***Conformal Fermi frame:***
- Valid in region (even outside horizon) around geodesic so that at all times

$$g_{\mu\nu} = a^2(\tau_F)\left[\eta_{\mu\nu} + \mathcal{O}(\mathbf{x}^2)\right]$$

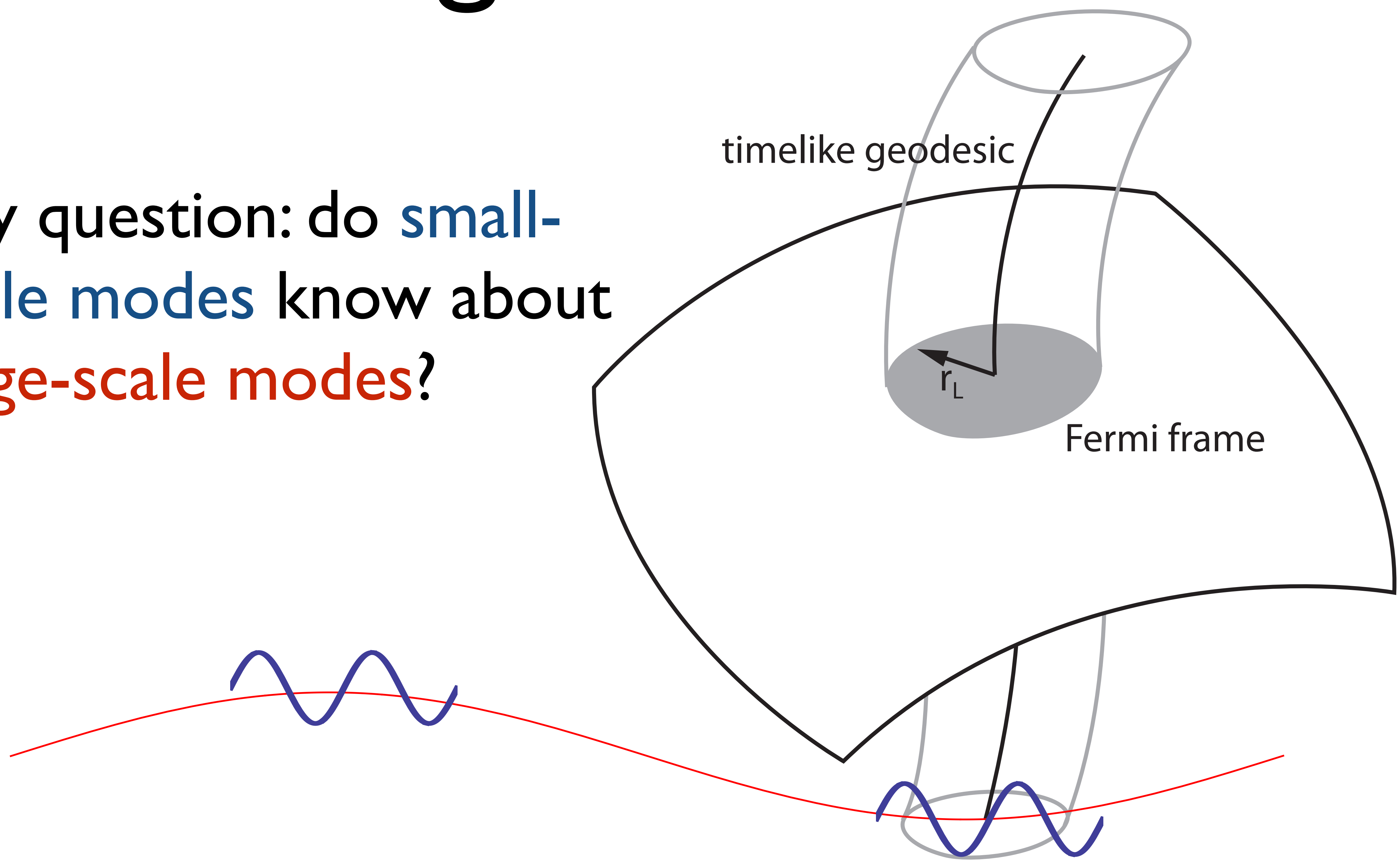
  - Spatial origin is on central geodesic
  - Time coordinate is (conformal) proper time along geodesic



Pajer, FS, Zaldarriaga (2013)

From initial conditions to galaxies

- Key question: do small-scale modes know about large-scale modes?



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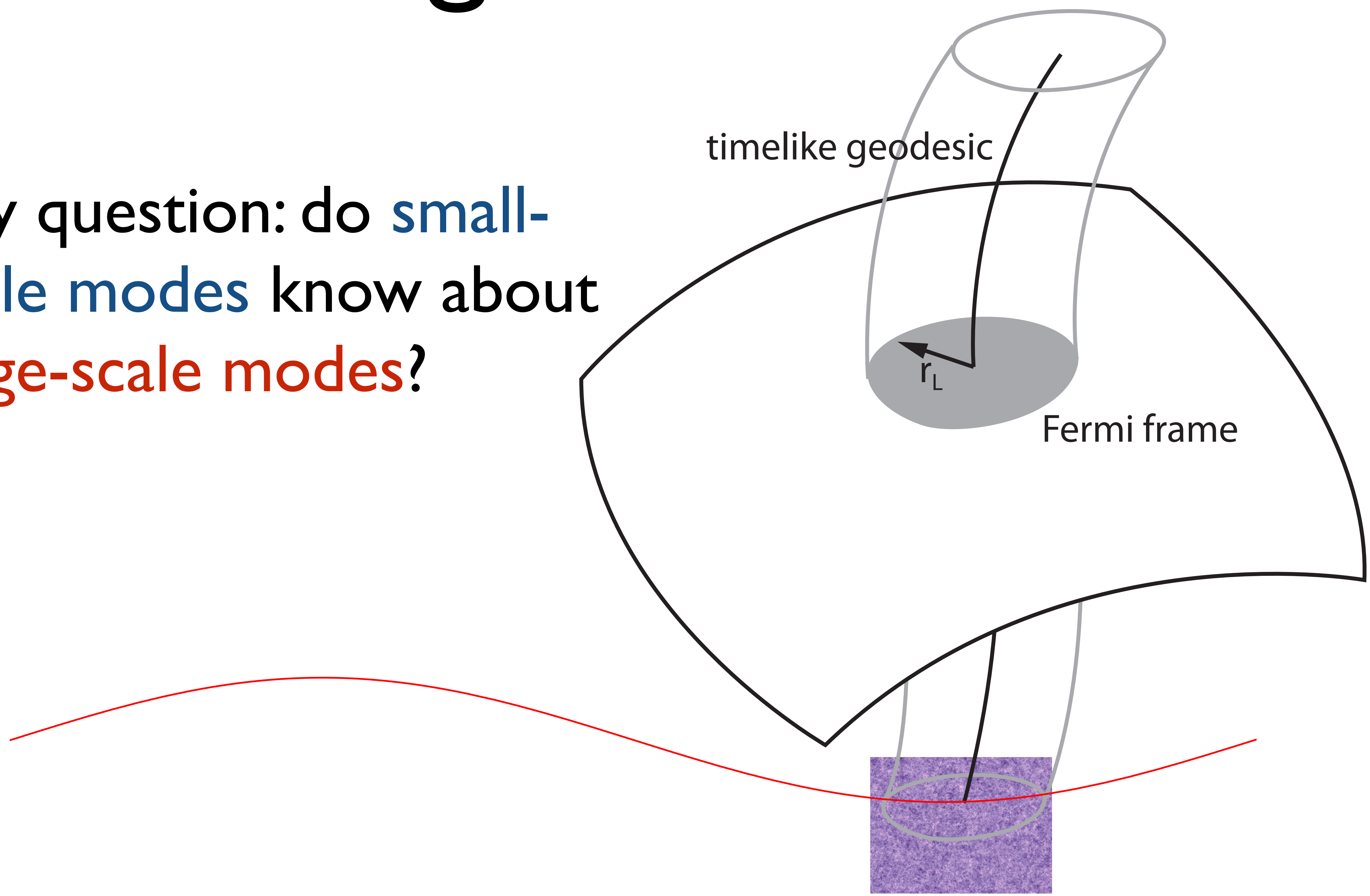
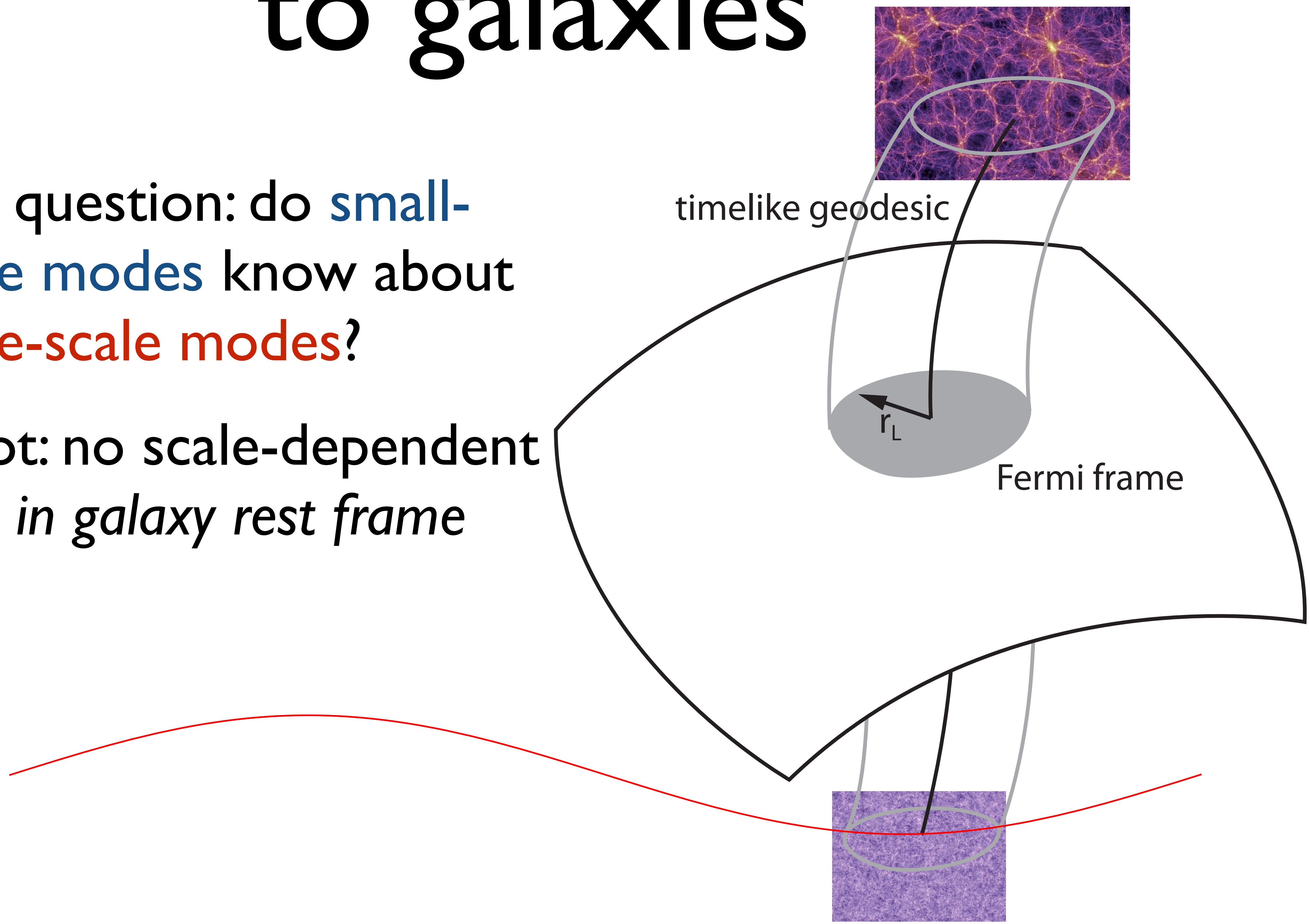


From initial conditions to galaxies

- Key question: do small-scale modes know about large-scale modes?
- If not: no scale-dependent bias *in galaxy rest frame*



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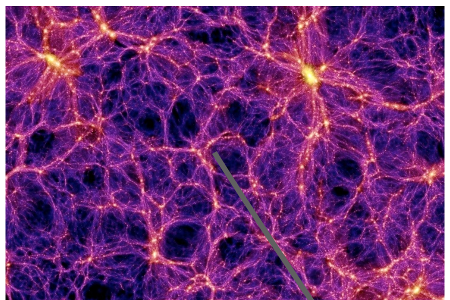
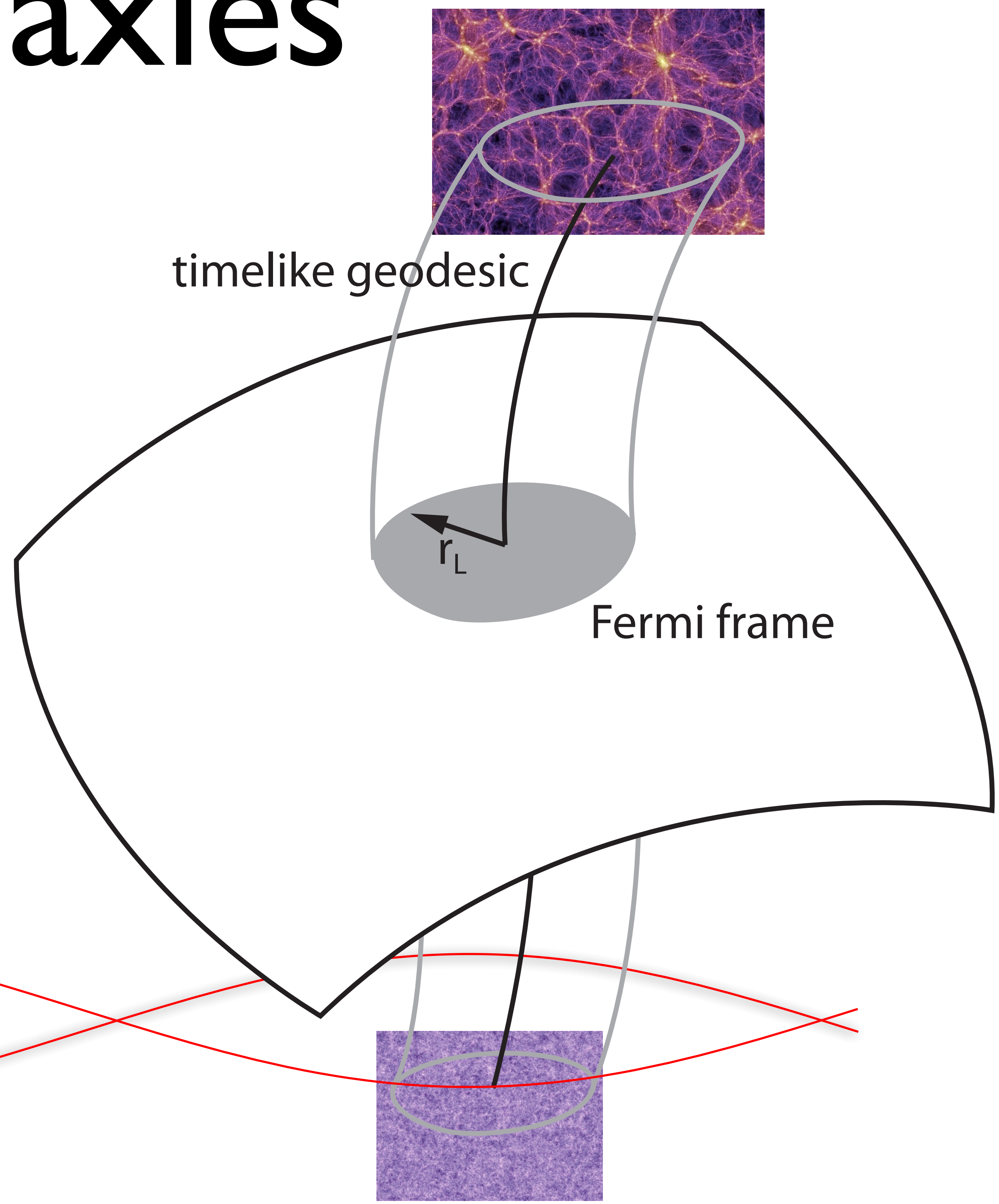
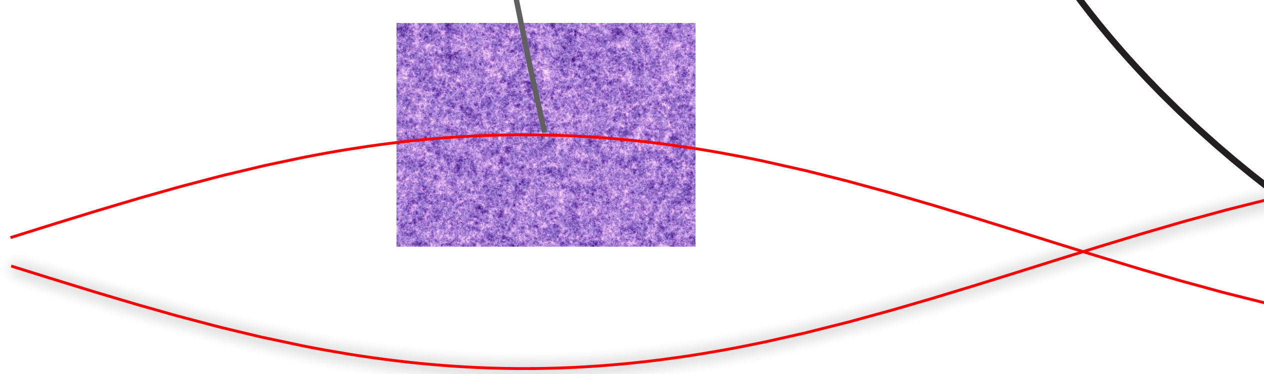


- In multi-field inflation, amplitude of initial conditions depends on large-scale ***potential***





$$\langle \delta^2 \rangle = [1 + 4 f_{\rm NL} \phi(\mathbf{x})] \sigma^2$$

Consistency relation and Fermi frame

- Metric in comoving gauge (neglecting shift and lapse): $ds^2 = a^2(\tau)[-d\tau^2 + e^{2\zeta} d\mathbf{x}^2]$
- Transform to conformal Fermi frame:

$$x'^i = (1-\zeta)x^i - \frac{1}{2}\partial_j \zeta x^j x^i$$

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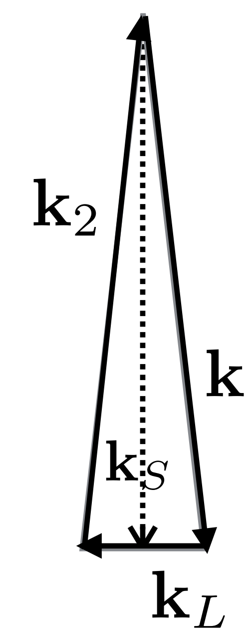

$\mathbf{k}_S = (\mathbf{k}_1 - \mathbf{k}_2)/2$

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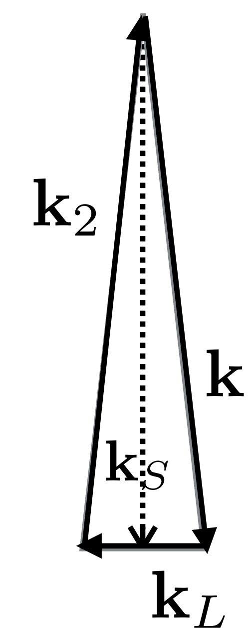

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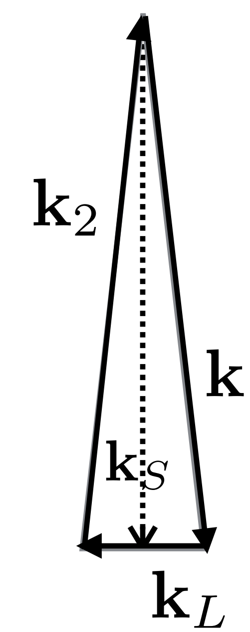
- Bispectrum in squeezed limit transforms as:

$$B'_\zeta(\mathbf{k}_L, \mathbf{k}_1, \mathbf{k}_2) = B_\zeta(\mathbf{k}_L, \mathbf{k}_1, \mathbf{k}_2) + P_\zeta(k_L)P_\zeta(k_S)\frac{d\ln k_S^3 P_\zeta(k_S)}{d\ln k_S}$$

Consistency relation and Fermi frame



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$$= \mathcal{O}\left(\frac{k_L^2}{k_S^2}\right)$$

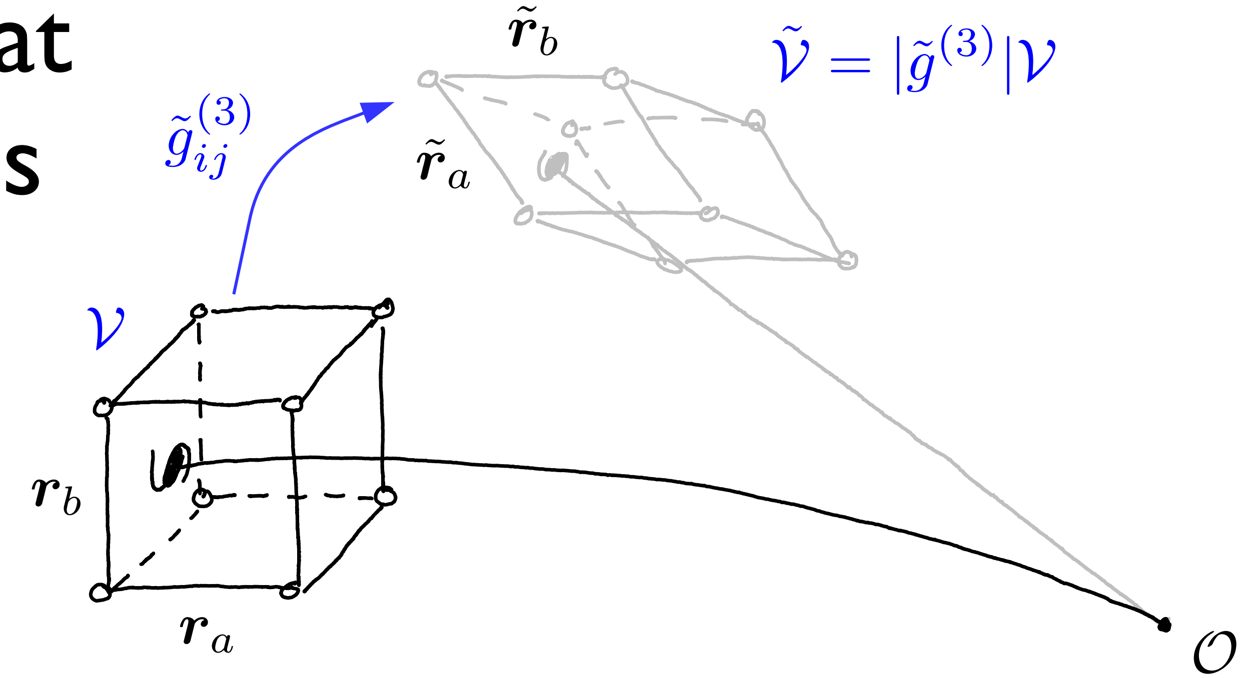
Consistency relation

Observed galaxy clustering

- No coupling of large- and small-scale fluctuations in single field inflation (attractor regime) *in local rest frame* (Fermi frame)
- But still need to map to observations of distant observer
- Nontrivial relativistic corrections at late times

Observed galaxy clustering

- Mapping local Fermi frame to distant observer's measurements understood at linear order in perturbations
  - sufficient for large-scale galaxy P(k)
- Second-order relativistic effects: still work in progress…



Bartolo, Bertacca, Bruni, Matarrese; Bonvin, Clarkson, Durrer, Maartens, Umeh; Ginat (incl. FS) …

II. Late non-Gaussianities: Probing the growth of structure

II. Late non-Gaussianities: Probing the growth of structure

Mon. Not. R. Astron. Soc. **290**, 651–662 (1997)

Large-scale bias in the Universe: bispectrum method

S. Matarrese,[1] L. Verde[1,2] and A. F. Heavens[2]

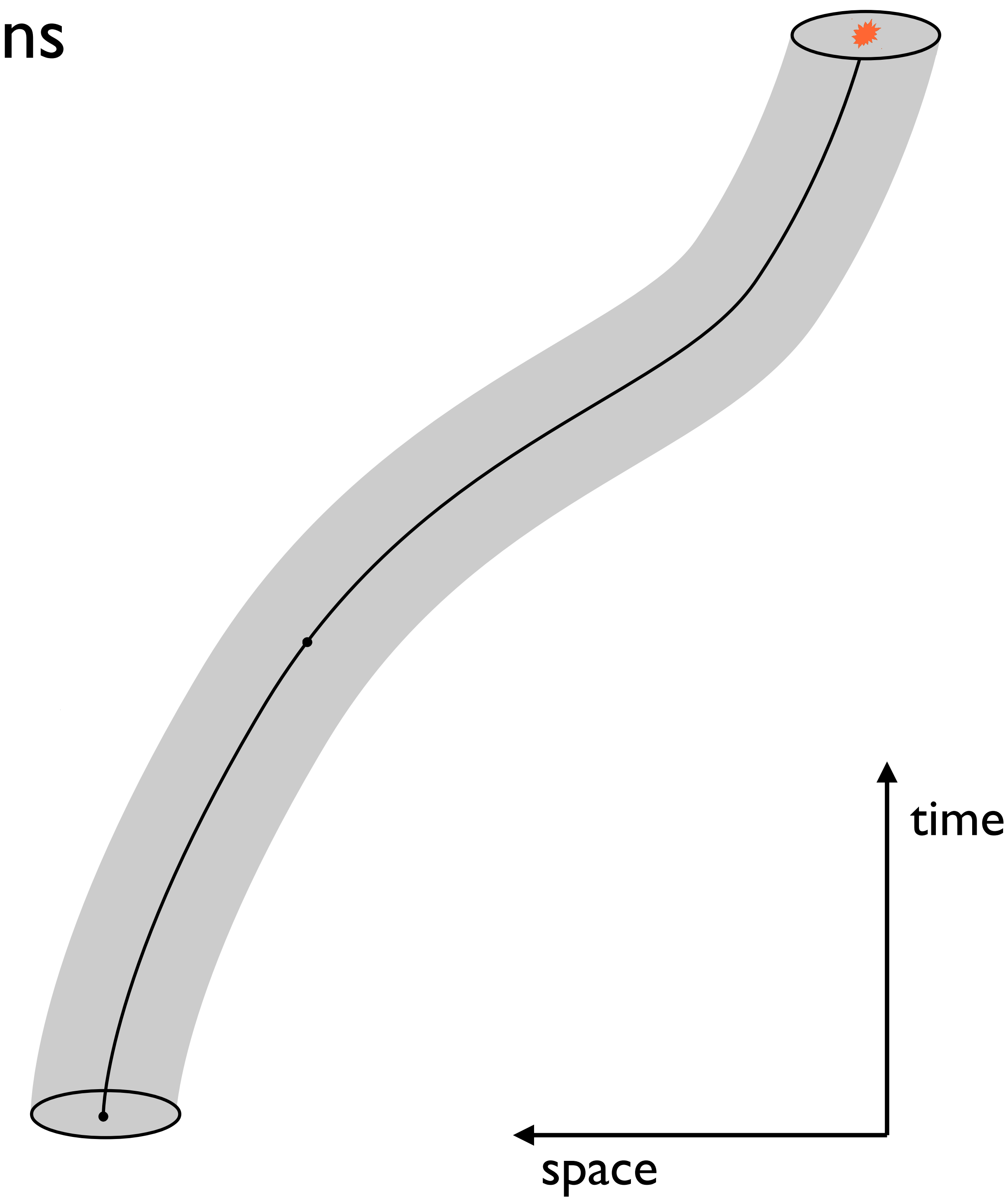
[1]*Dipartimento di Fisica Galileo Galilei, Università di Padova, via Marzolo 8, I-35131 Padova, Italy*
[2]*Institute for Astronomy, University of Edinburgh, Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ*

In this paper, we develop an idea of Fry, using second-order perturbation theory to investigate how to measure the bias parameter on large scales. The use of higher order statistics allows the degeneracy between b and Ω_0 to be lifted, and an unambiguous determination of Ω_0 then becomes possible. We apply a likelihood approach to the bispectrum, the three-point function in Fourier space. This paper is

$$\delta_g(\boldsymbol{x}) \simeq b_1 \delta^{(1)}(\boldsymbol{x}) + b_1 \delta^{(2)}(\boldsymbol{x}) + \frac{1}{2} b_2 \delta^{(1)2}(\boldsymbol{x}). \tag{9}$$

Bispectrum breaks bias degeneracies thanks to equivalence principle

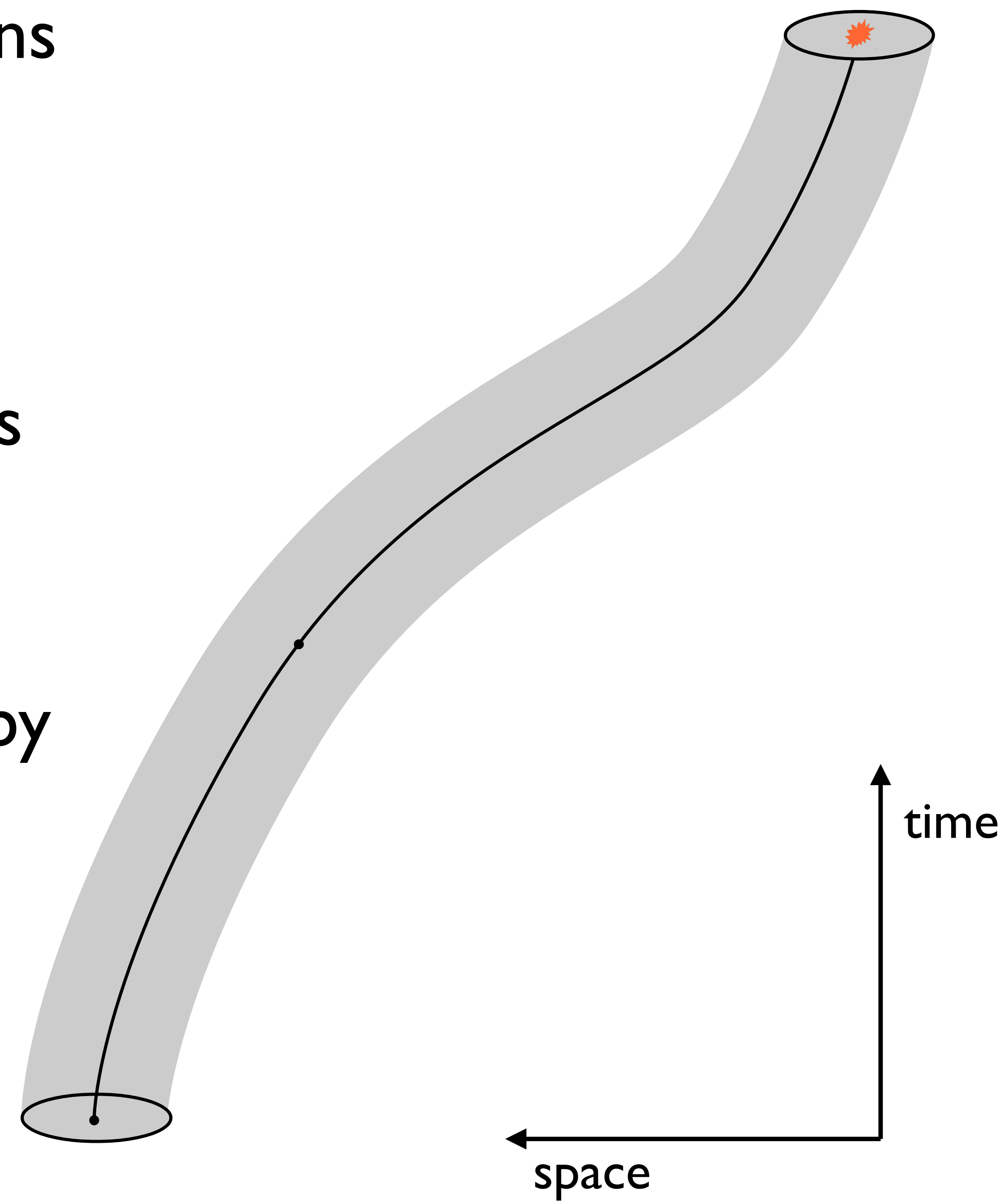
- We cannot predict galaxy positions from first principles; capture uncertainties in effective bias coefficients



Desjacques, Jeong, FS (2017)

Bispectrum breaks bias degeneracies thanks to equivalence principle

- We cannot predict galaxy positions from first principles; capture uncertainties in effective bias coefficients
- Leading gravitational observable is tidal field $\partial_i \partial_j \Phi$ which includes density $\delta \propto \nabla^2 \Psi$
- Some coefficients are protected by equivalence principle - precisely the ones that allow to break degeneracy of bias and amplitude

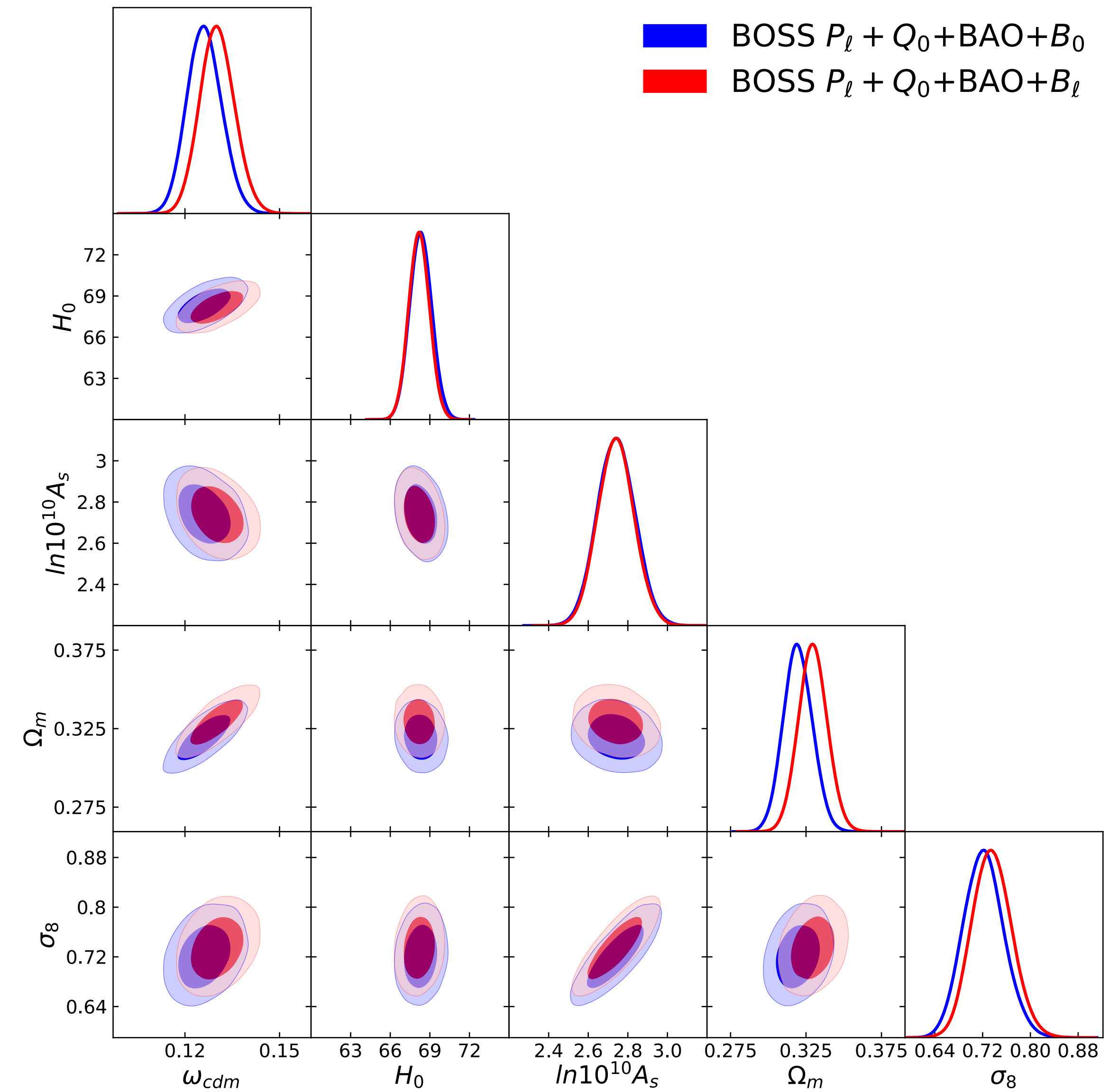


Desjacques, Jeong, FS (2017)

Current state: power spectrum + bispectrum

- Protected *displacement* terms in galaxy density start at second order
- These probe growth factor (or σ_8)
- Appear at leading order in galaxy 3-pt function = bispectrum
- Current SOTA 1-loop Pk+Bk (up to 4th order in perturbations)

$$\sigma(H_0)/H_0 \approx 1.2\%; \quad \sigma(\sigma_8)/\sigma_8 \approx 4.5\%$$



Ivanov, Philcox, et al, 2023
D'Amico et al 2023

Beyond classical n-point functions

- Much excitement in LSS about exploring information beyond 2- and 3-pt statistics, e.g.
  - *Machine-learned compressions*, coupled with simulation-based inference or emulators
  - *Field-level inference:* strictly optimal Bayesian inference, explicitly inferring initial conditions of observed universe

Field-level inference

$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in}, \theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in}, \theta\right)$$

- Scheme:
  - Discretize field on grid/lattice
  - Draw initial conditions from prior
  - Forward-evolve using gravity
  - Evaluate likelihood on data and repeat
- Results in samples from the *joint posterior of initial conditions and cosmological parameters*

Pioneered by Jasche, Kitaura, Ensslin;
Mo et al

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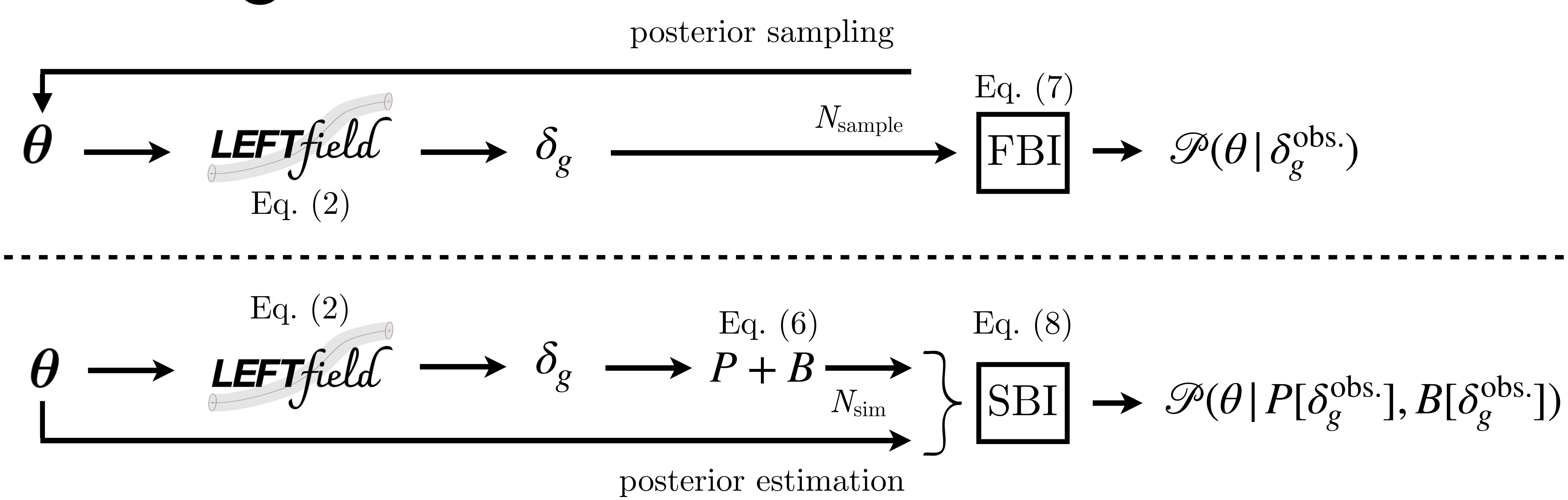
- Scheme:
  - Discretize field on grid/lattice (Nyquist frequency = cutoff Λ)
  - Draw initial conditions from prior
  - Forward-evolve using gravity
  - Evaluate likelihood on data and repeat
- Challenge: even with fairly coarse resolution, have to sample million(s) of parameters
  - Key: Hamiltonian Monte Carlo

Field-level inference: Inferring σ_8 from rest-frame tracers

- First results on field-level σ_8 inference from dark matter halos in real space
  - Marginalizing over bias and stochastic terms
- Idea: compare field-level result with power spectrum + bispectrum using the same forward model and modes of the data
  - Via simulation-based inference (SBI) using the same forward model as in the field-level analysis

Nguyen, FS, Tucci, Reinecke, Kostić   PRL 2024, arXiv:2403.03220

Field-level inference: Inferring σ8 from rest-frame tracers



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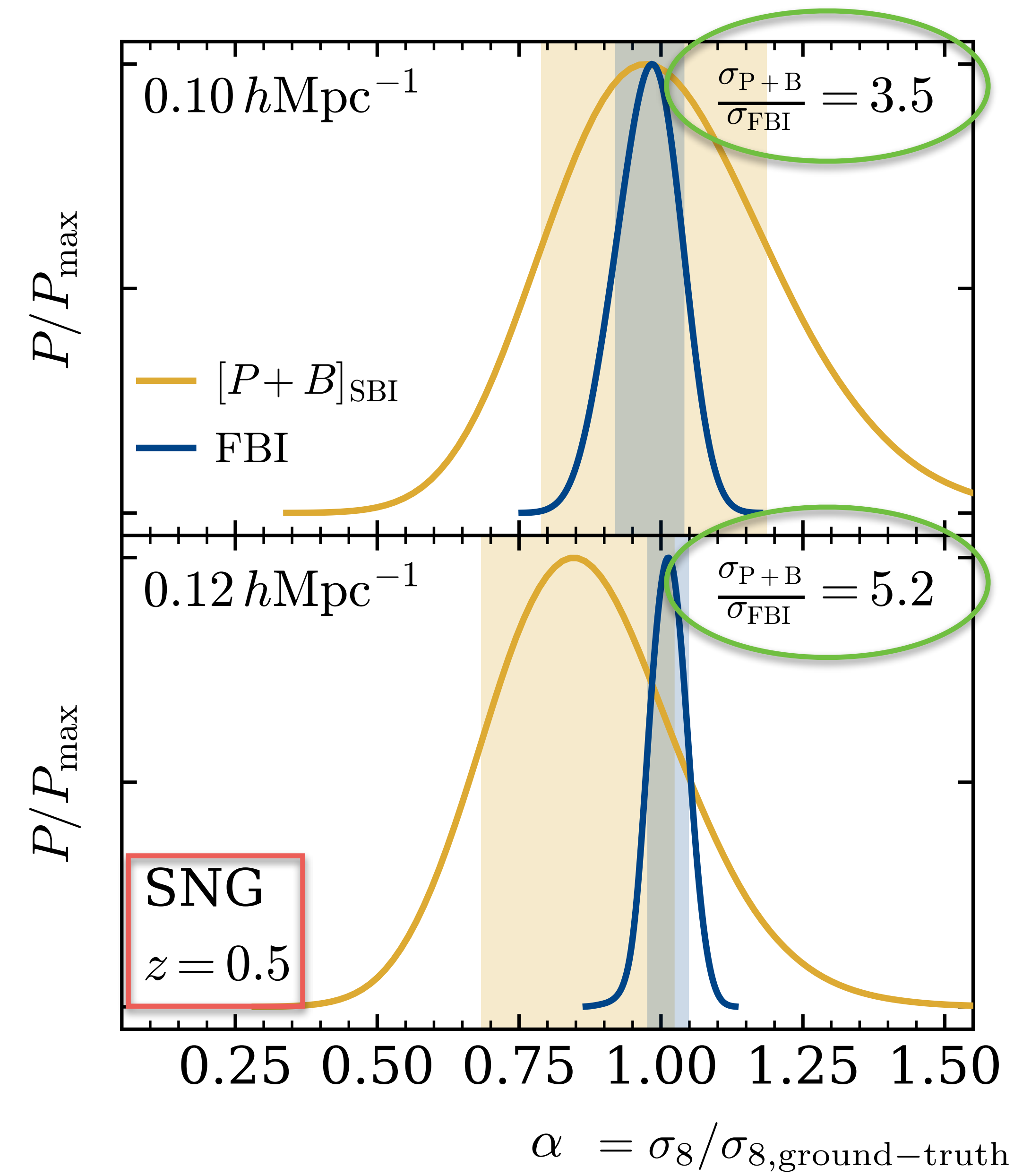
- Field-level inference vs power spectrum + bispectrum using the same forward model and modes of the data



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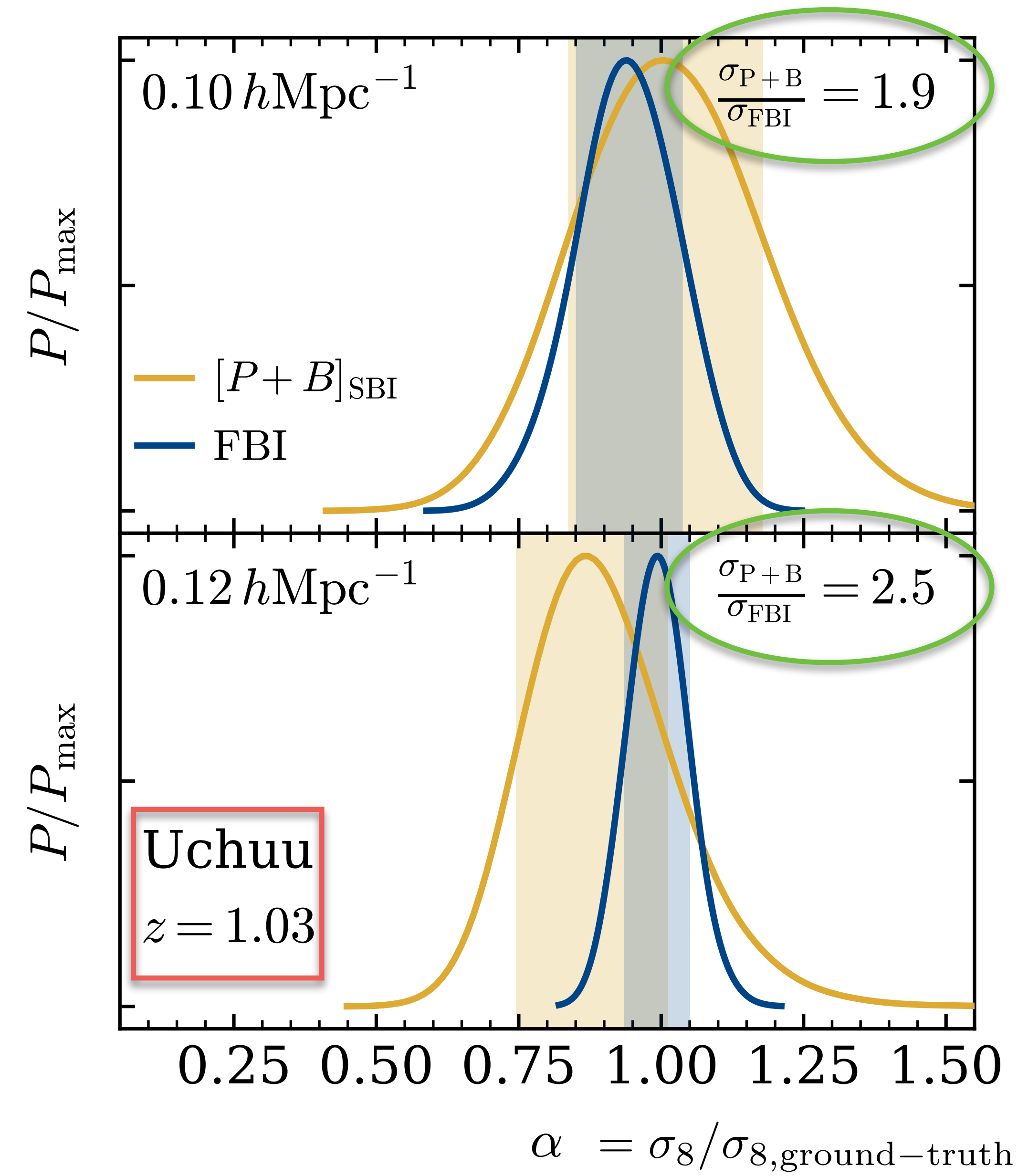

Nguyen, FS, Tucci, Reinecke, Kostić PRL 2024, arXiv:2403.03220

Where does the field-level information come from?

$$P(\theta) \propto \int \mathcal{D}\boldsymbol{\delta}_{\rm in}\, P\left(\boldsymbol{\delta}_g \middle| \boldsymbol{\delta}_{\rm fwd}[\boldsymbol{\delta}_{\rm in},\theta]\right) P_{\rm prior}\left(\boldsymbol{\delta}_{\rm in},\theta\right)$$

- Let's consider the zero-noise limit of the field-level posterior, such that likelihood becomes Dirac delta
- We can then formally perform integration over initial conditions $\delta_{\rm in}$ analytically to obtain marginalized posterior:

$$\mathcal{P}(\theta,\{b_O\}|\delta_g) \propto \mathcal{P}_{\rm prior}\left(\delta^{-1}_{\rm fwd}[\delta_g,\{b_O\}]\middle|\theta\right)\mathcal{J}[\delta_g,\{b_O\}]$$

← Jacobian $|D\delta_{\rm fwd}/D\delta_{\rm in}|^{-1}$

Cabass, Simonović, Zaldarriaga (2023); FS (2025)

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$$\propto \exp\left[-\frac{1}{2}\int_{\boldsymbol{k}} \frac{|\delta_{\rm fwd}^{-1}[\delta_g, \{b_O\}](\boldsymbol{k})|^2}{P_{\rm L}(k|\theta)}\right] \mathcal{J}[\delta_g, \{b_O\}]$$

Jacobian $|D\delta_{\rm fwd}/D\delta_{\rm in}|^{-1}$

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$$\propto \exp\left[-\frac{1}{2}\int_{\boldsymbol{k}}\frac{|\delta_{\rm fwd}^{-1}[\delta_g,\{b_O\}](\boldsymbol{k})|^2}{P_{\rm L}(k|\theta)}\right]\mathcal{J}[\delta_g,\{b_O\}]$$

- Involves inverse of forward model, evaluated on the data
- In case of linear forward model, $\delta_{\rm fwd} = b_1\delta_{\rm in}$, marginalized field-level posterior is function of the power spectrum of the data - *$P_g(k)$ is sufficient statistic*

Cabass, Simonović, Zaldarriaga (2023); FS (2025)

Where does the field-level information come from?

$$\mathcal{P}(\theta, \{b_O\}|\delta_g) \propto \mathcal{P}_{\rm prior}\left(\delta_{\rm fwd}^{-1}[\delta_g, \{b_O\}]\middle|\theta\right)\mathcal{J}[\delta_g, \{b_O\}]$$
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- If forward model is nonlinear, $\delta_{\rm fwd}^{-1}$ is a *nonlinear functional of the data* δ_g: effectively, we add higher *n*-point functions to the posterior

FS (2025), arXiv:2504.15351

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- If forward model is nonlinear, $\delta_{\rm fwd}^{-1}$ is a *nonlinear functional of the data* δ_g: effectively, we add higher *n*-point functions to the posterior
- Each term in the forward model adds a new, specific statistic to the posterior
  - Complete forward model at 2nd order: power spectrum + bispectrum
  - Complete forward model at 3d order: power spectrum + bispectrum + trispectrum …

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- Each term in the forward model adds a new, specific statistic to the posterior
- Lagrangian, LPT-based forward model as in LEFTfield: *correctly describes displacement terms at all orders, precisely those terms responsible for the degeneracy breaking*
- Impact of missing operators in forward model is proportional to scalar product of missing $O_{\rm missing}[\delta]$ with $O[\delta]$ of interest

FS (2025), arXiv:2504.15351

Cosmology results II: Field-level inference of BAO scale

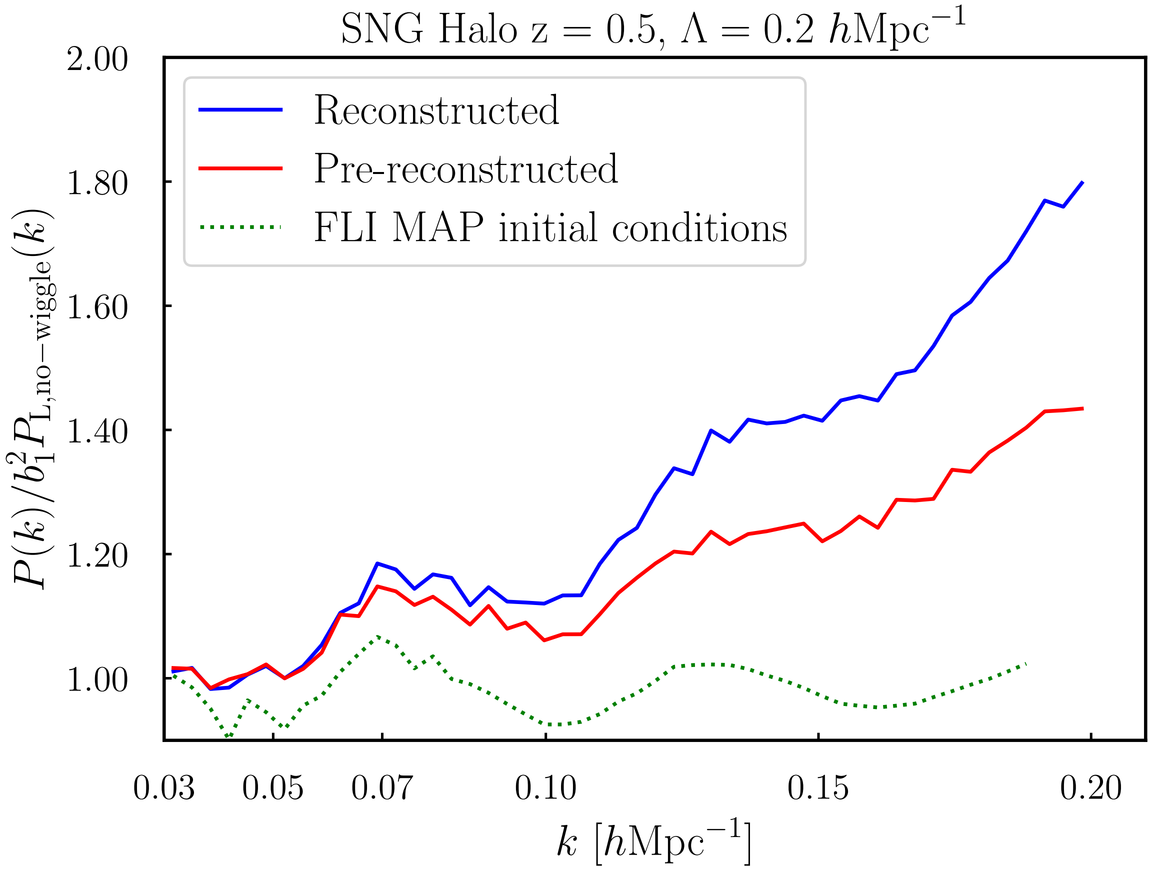
- Constraints on expansion history (dark energy) from galaxy clustering are based on the BAO standard ruler (cf. DESI results)
- These are commonly inferred by performing reconstruction procedure on galaxies, and then using the *post-reconstruction galaxy power spectrum*

Cosmology results II: Field-level inference of BAO scale

- Reconstruction idea: estimate large-scale displacements from galaxy density field, then move galaxies *back* to inferred initial positions

- Improves error bar on BAO scale by up to 50%

- Can we also do this in a *forward* approach by performing joint field-level inference of initial density field and BAO scale?



Babić, FS, Tucci (2025), arXiv:2505.13588

Cosmology results II: Field-level inference of BAO scale

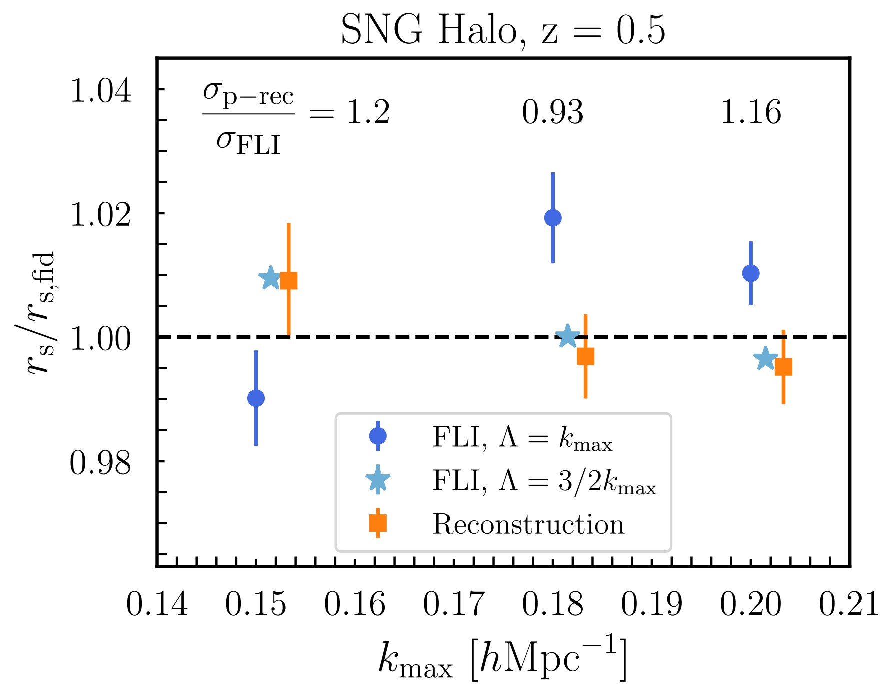
- Field-level inference of BAO scale using a trick: moving BAO feature in linear (initial) density field:

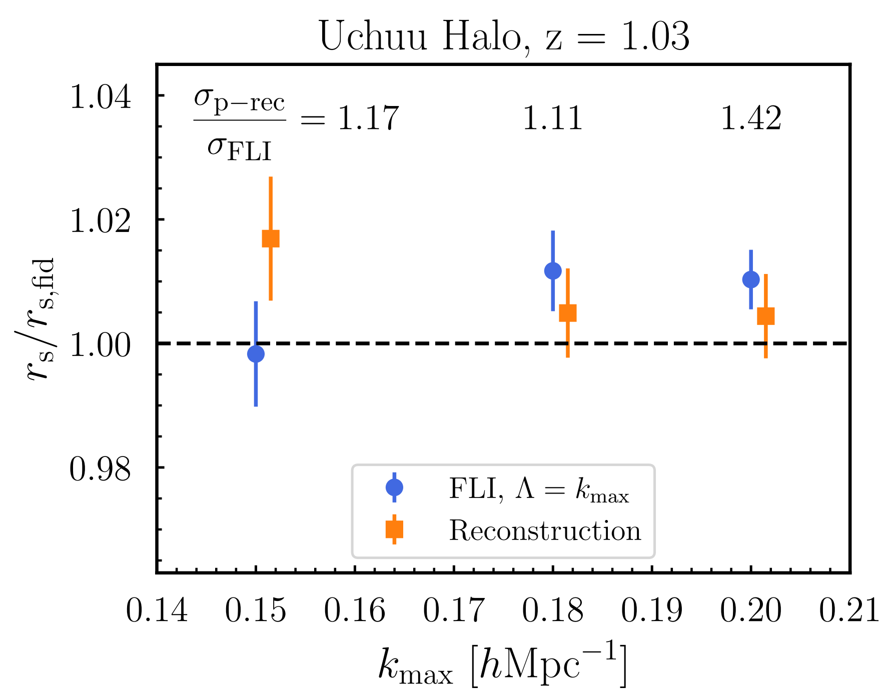
$$f(k, r_s) = \frac{T^2_{\rm BAO}(k|r_s)}{T^2_{\rm BAO}(k|r_{s,\rm fid})}\,,$$

$$T^2_{\rm BAO}(k|r_s) = 1 + A\sin(k\,r_s + \phi)\exp(-k/k_{\rm D})$$

- Compare with reconstruction analysis applied to the same scales of the data

- Note: reconstruction uses fixed linear bias, field-level inference infers all bias coefficients jointly with BAO scale





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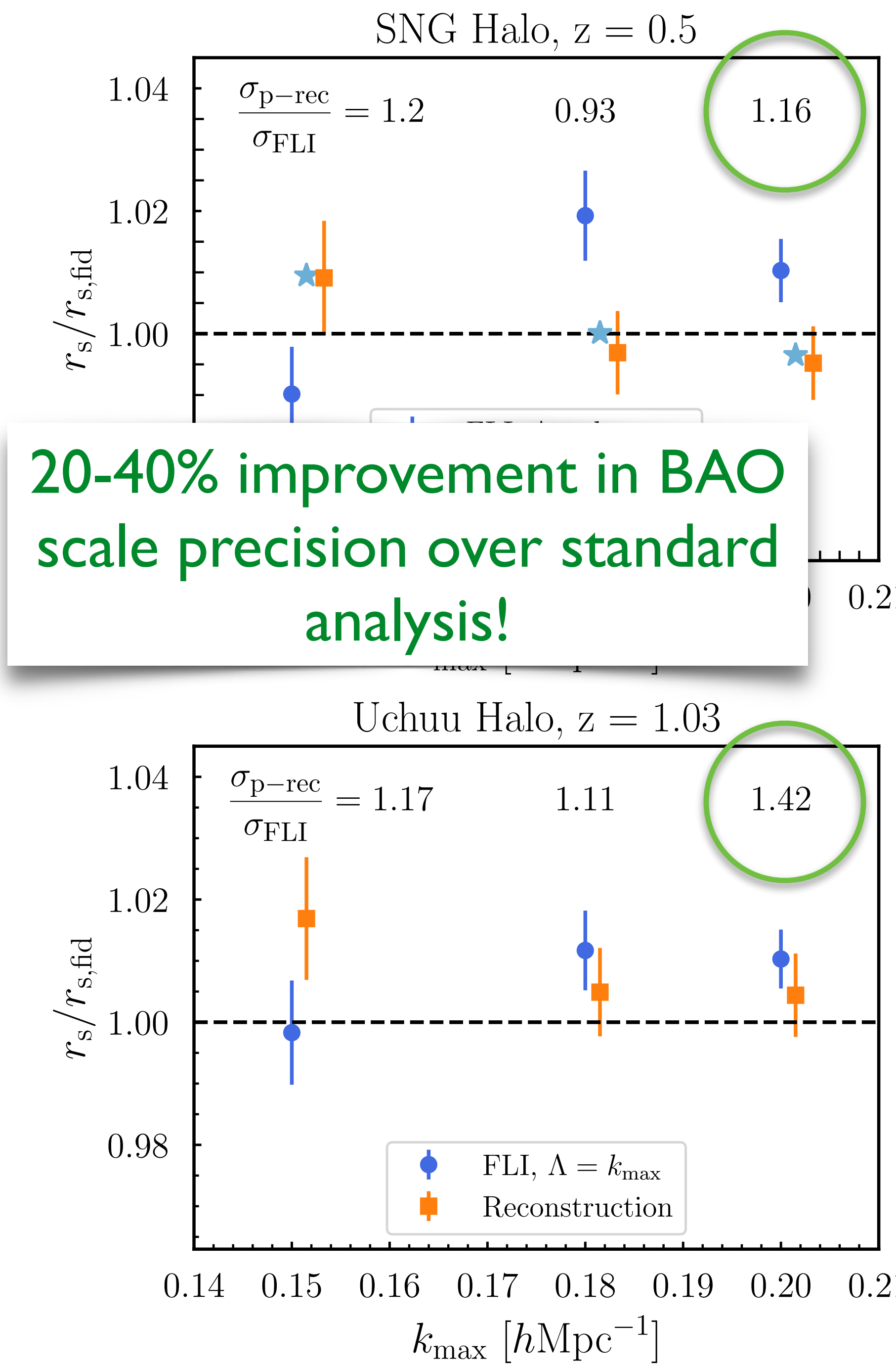

20-40% improvement in BAO scale precision over standard analysis!

Babić, FS, Tucci (2025), arXiv:2505.13588

Where does the field-level *BAO* information come from?

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- In case of *perfect* forward model, δ_{fwd}^{-1} is a sample from prior (Gaussian linear density field) - in fact, information obtained is precisely that contained in linear density field: *optimal inference*
    - Field-level inference "undoes" nonlinear evolution as well as nonlinear bias

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  - Field-level inference "undoes" nonlinear evolution as well as nonlinear bias
- On the other hand, standard BAO reconstruction leaves substantial broad-band contribution to $\delta_g^{\rm post\text{-}rec}$; this explains information gain found at field level
- Cannot easily be recuperated using higher-order n-pt functions

Babić, FS, Tucci (2025), arXiv:2505.13588

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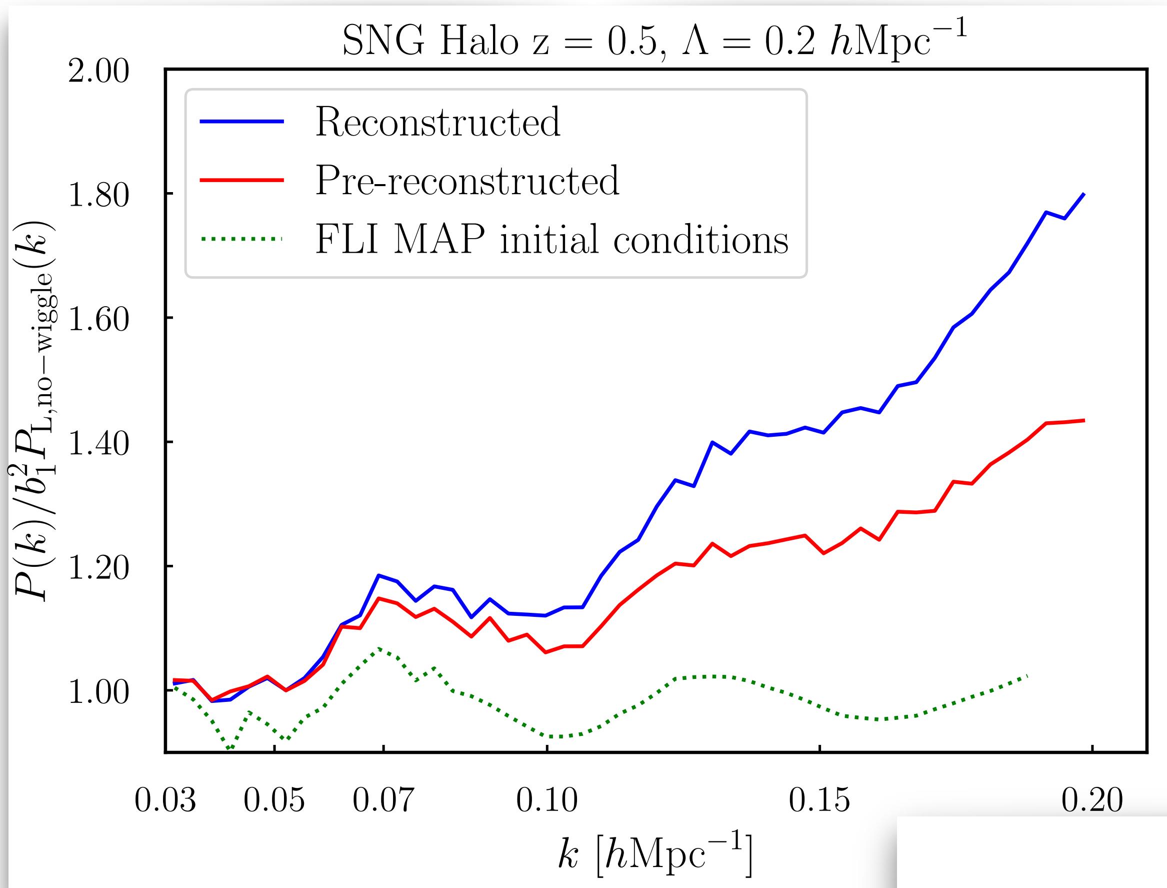

$$\ldots{}^{-1}_{\rm vd}[\delta_g, \{b_O\}]\Big|\theta\Big)\, \mathcal{J}[\delta_g, \{b_O\}]$$

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…*rence*

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$$F^{\rm FLI}_{r_s r_s} = -\left\langle \frac{\partial^2}{\partial r_s^2} \ln \mathcal{P}_{\rm FLI}[\{b_O\}, r_s|\delta_g] \right\rangle = \frac{1}{2}\sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{[P_{\rm L}(k|r_{s,\rm fid})]^2} \left(\frac{\partial P_{\rm L}(k|r_{s,\rm fid})}{\partial r_{s,\rm fid}}\right)^2$$

$$F^{\rm rec-P(k)}_{r_s r_s} = -\left\langle \frac{\partial^2}{\partial r_s^2} \ln \mathcal{P}_{\rm rec-P(k)}[r_s|\delta_g] \right\rangle = \sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{{\rm Var}[P_{\rm p\text{-}rec}(k|r_{s,\rm fid})]} \left(\frac{\partial P_{\rm p\text{-}rec}(k|r_s)}{\partial r_s}\right)^2$$
$$= \frac{1}{2}\sum_{\boldsymbol{k}}^{\Lambda} \frac{1}{[P_{\rm p\text{-}rec}(k|r_{s,\rm fid})]^2} \left(\frac{\partial P_{\rm p\text{-}rec}(k|r_s)}{\partial r_s}\right)^2 .$$

Babić, FS, Tucci (2025), arXiv:2505.13588

Why field-level?

- Evaluating the full posterior guarantees optimality in the context of the given forward model
- One can certainly hope to approach this optimum closely with suitably engineered summary statistics (i.e. *data compression*)
    - Calling that "field-level" does not seem to make sense however…
- Advantages of (actual) field-level inference apart from optimality:
    - Maximally interpretable: have access to all physically relevant variables
    - Allows for broad range of systematics checks (e.g. cross-correlating predicted mean field with systematics maps)
    - Many possibilities for ancillary science: cross-correlation with other tracers, shear, CMB lensing

Conclusions

- By now have a robust framework to predict galaxy clustering on large scales within GR
- Even after many years we are continuing to find new signals to search for in LSS
  - Light thermal relics
  - Spinning particles
  - Primordial parity violation
  - …
- Inference/analysis methods have made tremendous progress — now need to tie the two together