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EXTENDED PROBES FOR THE HAWKING-PAGE TRANSITION

Based on 2403.17190 [Amariti, Glorioso, Morgante, AZ]

Idea: supersymmetric order parameter for the Hawking-Page phase transition via AdS/CFT correspondence

Main result: Computation beyond leading order of superconformal index for a 4d/2d coupled system describing holographically a D3 brane probing the BH geometry

OUTLINE

- Context and motivation
- Surface defects
- Bethe Ansatz Computation
- Comparison with Cardy approach
- Conclusions

ANTI-DE SITTER BLACK HOLES

- Black Holes and their thermodynamics central objects to study quantum properties of gravity

- AdS BH especially interesting by virtue of AdS/CFT, which provides a non-perturbative definition of QG in AdS spacetime

	Quantum Gravity **d.o.f** in AdS_{d+1}	$\longleftrightarrow$	**States** in CFT living on ∂AdS_{d+1}
In this talk:	Type IIB on $AdS_5 \times S^5$ with N unit of F_5 flux	$\longleftrightarrow$	4d $\mathcal{N} = 4$ $\mathrm{SU}(N)$ SYM on conformal boundary

- AdS/CFT provides a microscopic explanation to S_{BH}

$$S_{BH} = k_B \frac{c^3 A_{hor}}{4\hbar G_N} = k_B \log n \sim \mathcal{O}(N^2)$$

$1/G_N \sim N^2$

HAWKING–PAGE PHASE TRANSITION

- Thermodynamics of AdS BH pretty interesting: they exhibit a **first order phase transition** below a certain temperature, **HP transition**
- At finite temperature, $1/\beta$, competition of saddles contributing to euclidean gravitational path integral with same boundary geometry $S^1_\beta \times S^3$:

Semiclassical approx.

$$\mathcal{Z}[\beta] = \int \mathcal{D}g\, e^{-I[g]} \sim e^{-I_{AdS}[\beta]} + e^{-I_{BH}[\beta]}$$

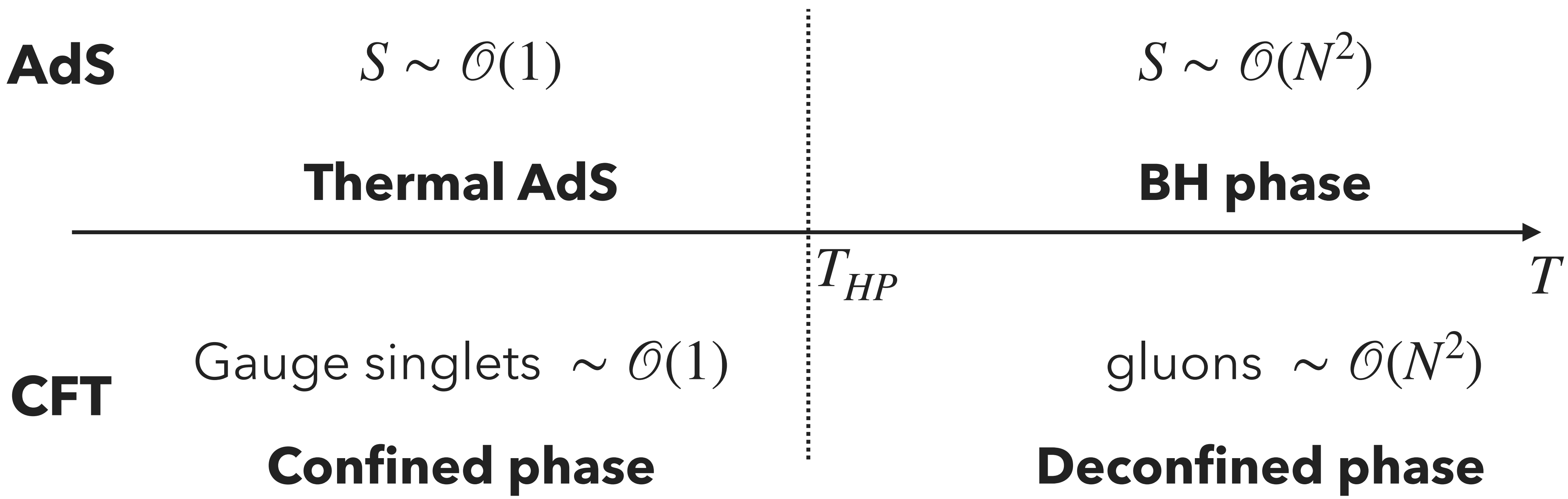
Thermal AdS phase ⟶ $T < T_{HP}$ $\quad S \sim \mathcal{O}(1)$

Black Hole phase ⟶ $T > T_{HP}$ $\quad S \sim \mathcal{O}(N^2)$

Transition at T_{HP} where $\log \mathcal{Z}_{BH} = 0$

HAWKING–PAGE PHASE TRANSITION

- Via AdS/CFT HP transition mapped into large N confinement/ deconfinement transition in dual $\mathcal{N} = 4$ SYM on $S^1_\beta \times S^3$ [Witten]



THE SUPERCONFORMAL INDEX

$(q_1, q_2, q_3) \subset SU(4)_R$

$(j_1, j_2) \subset SO(4,2)$

- We consider 1/16-BPS Gutowski-Reall BHs:
- Entropy encoded in dual CFT by ensemble of 1/16-BPS states. A natural supersymmetric partition function is the **superconformal index** as it receives contributions only by 1/16-BPS states [Kinney, Maldacena, Minwalla, Raju; Romelsberger]:

$$I(\tau, \sigma, \xi) = \mathrm{Tr}(-1)^F e^{-\beta\{\mathcal{Q}, \bar{\mathcal{Q}}\}} e^{2\pi i \tau\left(j_1 + \frac{r}{2}\right)} e^{2\pi i \sigma\left(j_2 + \frac{r}{2}\right)} \prod_a e^{2\pi i \xi_a q_a}$$

- Analytic continuation to complex chemical potentials crucial to **obstruct boson-fermions cancellations** due to $(-1)^F$ and recover BH entropy [Benini, Zaffaroni; Hosseini, Hristov, Zaffaroni; Benini, Milan; Choi, Kim, Kim, Nahmgoong; Cabo-Bizet, Cassani, Martelli, Murthy]
- Related to Euclidean supersymmetric partition function on $S^1_\beta \times S^3$ [Gadde, Yan; Closset, Dumitrescu, Festuccia, Komargodski; Cabo-Bizet, Cassani, Martelli, Murthy]

$$\mathcal{Z}_{S^1_\beta \times S^3} = e^{-\beta \mathcal{F}} I$$

Supersymmetric Casimir Energy

- Globally defined Killing spinor on $S^1_\beta \times S^3$ **requires** a constraint on chemical potentials

$$\sum_a \xi_a - \tau - \sigma = n_0 \longrightarrow n_0 \in \mathbb{Z}$$

Cannot be **purely imaginary** for $n_0 \neq 0$!

- Reflected in supergravity in corresponding constraint on chemical potentials conjugated to BH charges [Cabo-Bizet, Cassani, Martelli, Murthy]

$n_0 = 0$: $\mathcal{Z}_{S^1_\beta \times S^3}$ with periodic conditions for fermions along S^1_β $\chi(t+\beta) = \chi(t)$ $\Longleftrightarrow$ 'First sheet' $\log \mathcal{Z}_{S^1_\beta \times S^3} \overset{N\to\infty}{\sim} \mathcal{O}(1)$

$n_0 = \pm 1$: $\mathcal{Z}_{S^1_\beta \times S^3}$ anti-periodic conditions for fermions along S^1_β $\chi(t+\beta) = -\chi(t)$, **thermal partition function** $\Longleftrightarrow$ 'Second sheet' $\log \mathcal{Z}_{S^1_\beta \times S^3} \overset{N\to\infty}{\sim} \mathcal{O}(N^2)$

SUPERSYMMETRIC PROBES

- Evaluating $\mathcal{Z}_{S^1_\beta \times S^3}$ in the BH phase at large N:

$$\log \mathcal{Z}_{S^1_\beta \times S^3} \sim -i\pi N^2 \frac{\Delta_1(\beta)\Delta_2(\beta)\Delta_3(\beta)}{\tau(\beta)\,\sigma(\beta)}$$

$\mathcal{Z}_{S^1_\beta \times S^3}$ detects **deconfinement transition** $\Longleftrightarrow$ reproduces **BH entropy function**
[Hosseini, Hristov, Zaffaroni]

β_{HP} defined by $\log \mathcal{Z}_{S^1_\beta \times S^3} = 0$

- Can we define an order parameter for the HP transition via CFT side?

Thermal non-susy partition function
[Sundborg; Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk]
Polyakov loops insertions

Spin chain systems [Perez-Garcia, Santilli, Tierz]

Superconformal Index

Supersymmetric order parameter
Gukov-Witten surface defects
[Chen, Heydeman, Wang, Zhang]

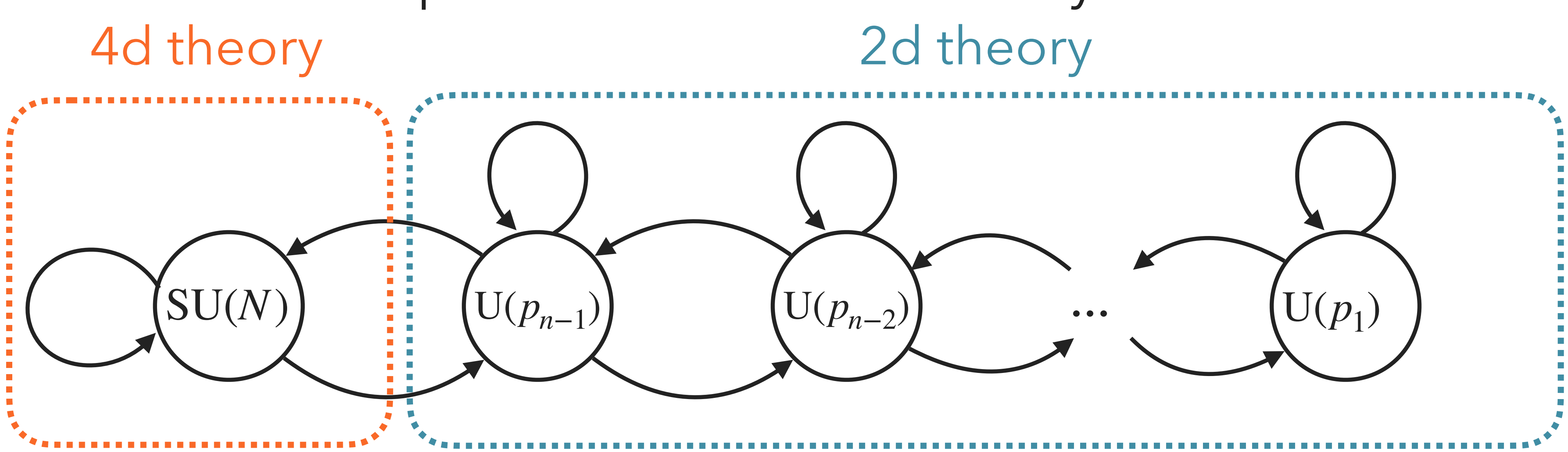
GUKOV–WITTEN DEFECTS

- We consider **half-BPS surface** operators defined by singularity for 4d fields along the surface of the defect in $\mathrm{SU}(N)$ $\mathcal{N} = 4$ SYM [Gukov, Witten]

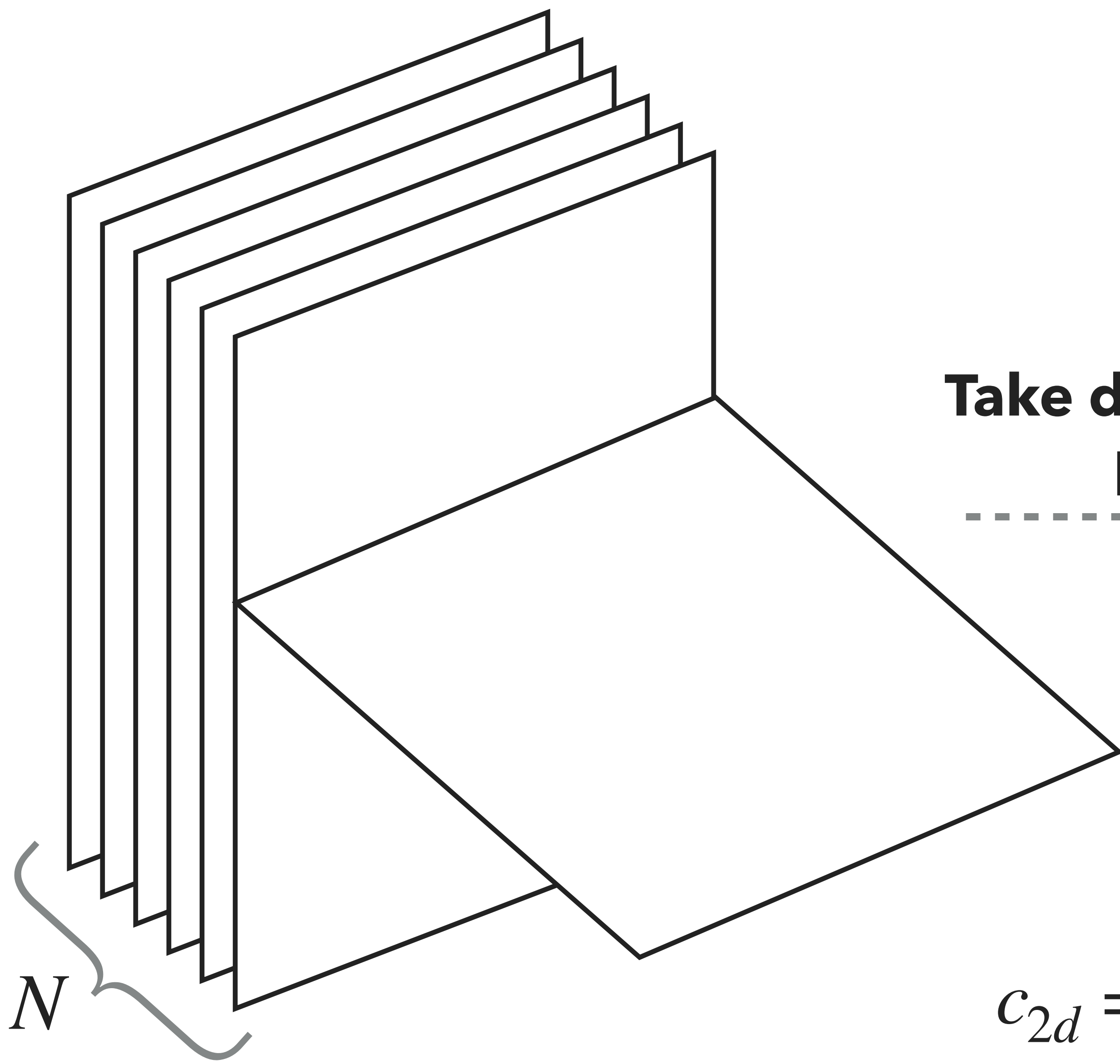
- GW defects are **uniquely** specified by choice of **Levi subgroup** $\mathbb{L} \subseteq G$ preserved on the support of the defect.

$$\mathbb{L} \subseteq \mathrm{SU}(N) \iff [\lambda_1, \ldots, \lambda_s], \quad \sum_{j=1}^{s} \lambda_j = N, \quad \lambda_j \in \mathbb{N}$$

- Equivalently, GW defects defined in terms of 2d gauge theory living on the defect surface coupled to the bulk 4d theory



- The surface defect admits a holographic interpretation in Type IIB string theory: intersecting D3 branes [Constable, Erdmenger, Guralnik, Kirsch; Koh, Yamaguchi]



Take decoupling limit

$AdS_5 \times S^5$ geometry with **probe** brane wrapping $AdS_3 \times S^1$

Probe limit

$$c_{2d} = 3\left(N^2 - \sum_i k_i^2\right) \sim \mathcal{O}(N)$$

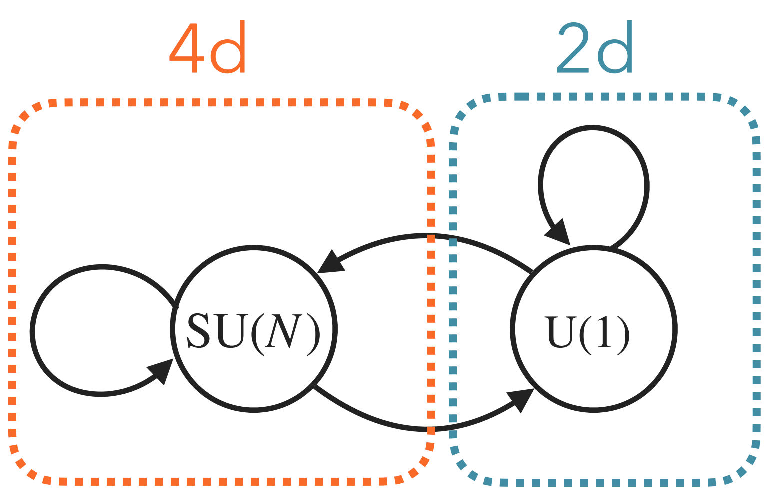
THE DEFECT INDEX

- We can define a new observable of the theory, **the defect index**, which realizes the insertion of GW defect [Gukov, Gadde]

- To compute index one needs to **embed** the 2d superconformal algebra in 4d $\mathfrak{u}(1)_A \ltimes \mathfrak{psu}(1,1|2) \times \mathfrak{psu}(1,1|2) \times \mathfrak{u}(1)_C \subset \mathfrak{psu}(2,2|4)$

- Focus on **maximal** GW defects $[N-1,1]$:

$$c_{2d} = 6(N-1)$$



$$I_{\mathscr{D}}(\tau,\sigma,\Delta) = \int \prod_{i=1}^{N} \mathrm{d}u_i Z_{4d}(\mathbf{u}) Z_{2d}(\mathbf{u}) = \int \prod_{i=1}^{N} \mathrm{d}u_i Z_{4d}(\mathbf{u}) \sum_{j=1}^{N} Z_{2d}^{j}(\mathbf{u})$$

$$\langle \mathscr{D} \rangle = \frac{I_D}{I_{4d}}$$

BETHE ANSATZ APPROACH

- We evaluate the index with Bethe Ansatz approach with $\tau = \sigma$
  [Closset, Kim, Willett; Benini, Milan]

- Index expanded over solution to set of auxiliary Bethe Ansatz Equations $Q_i = 1$

$$I = \sum_{u^* \in \mathfrak{M}_{\text{BAE}}} Z(u^*) H^{-1}(u^*)$$

BH entropy encoded within **'basic solution'**

$$u_i^* = \bar{u} + \frac{\tau}{N} i, \qquad \sum_{i=1}^{N} u_i^* \in \mathbb{Z} + \tau\mathbb{Z}$$

$$\log I_{4d} = -\frac{\pi i}{\tau^2} N^2 \prod_{a=1}^{3} \left(\{\Delta_a\}_\tau - n \right) + \log N + \mathcal{O}\left(N^0\right) \qquad \sum_{a=1}^{3} \{\Delta_a\}_\tau - \tau - \sigma = 1 + n$$

$n = 0, 1$

EVALUATION BETHE ANSATZ

- We want to compute I_D on 'basic solution' $\longrightarrow$ associated to BH saddle
- Does Bethe Ansatz hold for I_D? Yes, but non trivial a priori. The insertion of the defect does not modify BAE or Bethe operators

$$\log I_{\mathscr{D}} = -\frac{\pi i}{\tau^2} N^2 \prod_{a=1}^{3} \left(\{\Delta_a\}_\tau - n\right) + \frac{2\pi i}{\tau} N \prod_{a=2}^{3} \left(\{\Delta_a\}_\tau - n\right) + \log N + \mathcal{O}\left(N^0\right)$$

Controlled by **2d central charge**: consistent with probe limit, subleading effect $\mathcal{O}(N)$

Consistent and generalizes results of [Chen, Heydeman, Wang, Zhang; Cabo-Bizet, David, Lezcano]

$$\langle \mathscr{D} \rangle = \exp\left(\frac{2\pi i}{\tau} N \prod_{a=2}^{3} \left(\{\Delta_a\}_\tau - n\right) \right) \neq 0$$

COMPARISON WITH CARDY-LIKE LIMIT

- **Cardy-like limit:** Limit of small chemical potentials $|\tau|, |\sigma| \to 0$ with $\mathrm{Im}(\tau) \in \mathbb{R}^+$, $\mathrm{Im}(\sigma) \in \mathbb{R}^+$, $\frac{\tau}{\sigma}$ fixed and $\Delta \in \mathbb{C}$ fixed.

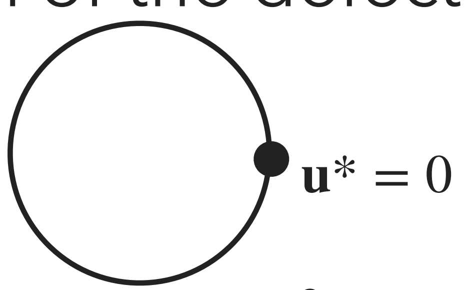
$$I \longrightarrow \sum_{\mathbf{u}^*} e^{-S_{\text{eff}}(\mathbf{u}^*)} Z_{S^3}$$

Leading contribution from saddle

Subleading corrections from perturbations around the saddle

- The insertion of the defect does not change leading order saddles

BH saddle



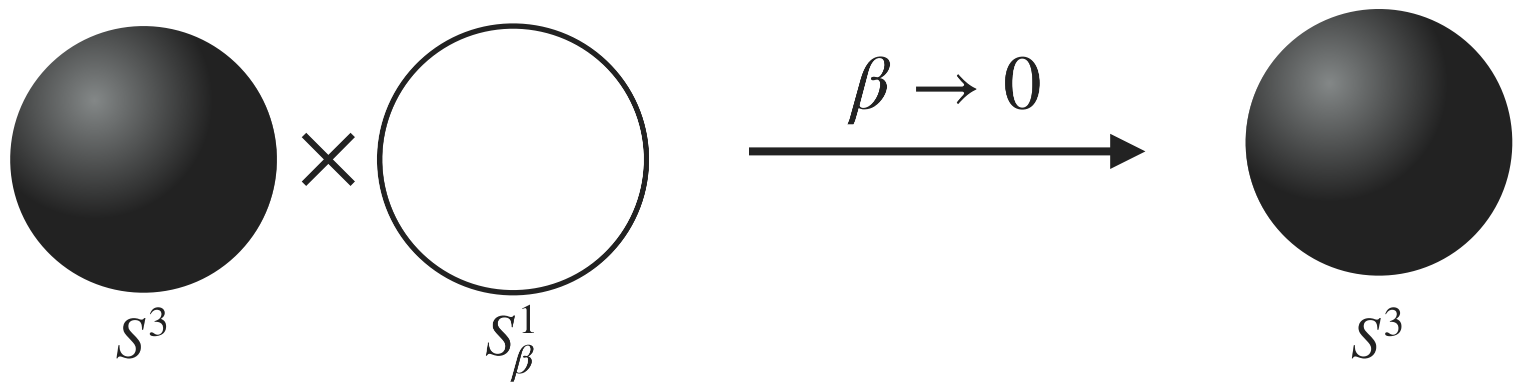
Exponentially suppressed terms in $|\tau|, |\sigma|$

$$\log I_{\mathscr{D}} = -\frac{\pi i}{\tau\sigma} N^2 \prod_{a=1}^{3} \left(\{\Delta_a\} - n\right) + \frac{2\pi i}{\sigma} N \prod_{a=2}^{3} \left(\{\Delta_a\} - n\right) + \log N + \ldots$$

Subleading corrections Z_{S^3} crucial to recover reliable result!

EFFECTIVE FIELD THEORY

- Without evaluating Z_{S^3}:
  1. $\log N^2$ correction
  2. Asymmetry between 2nd and 3rd sheets
- Z_{S^3} encodes information about underlying EFT controlling the dynamics near each saddle. **CS theories**
- Cardy-like limit is a high-temperature limit: $|\tau|, |\sigma| \propto \beta$



- In probe limit EFT unaltered by GW insertion.
- N wounded Wilson loop on one sheet from GW insertion. Contribution vanishes upon including all corrections to Z_{S^3} explicitly symmetrizing the results in the two equivalent sheets.

CONCLUSIONS

- We studied subleading corrections to superconformal index of 4d/2d coupled system associated to inserting maximal GW defect in $\mathcal{N} = 4$ SYM
- Holographically corresponds to D3-brane probing BH geometry
- Defect has non-zero expectation value in deconfinement/BH phase
- **Future directions:**
  1. $[M, N - M]$ GW defect
  2. $\mathcal{N} = 2$ cases
  3. study matrix model [Copetti, Grassi, Komargodski, Tizzano]

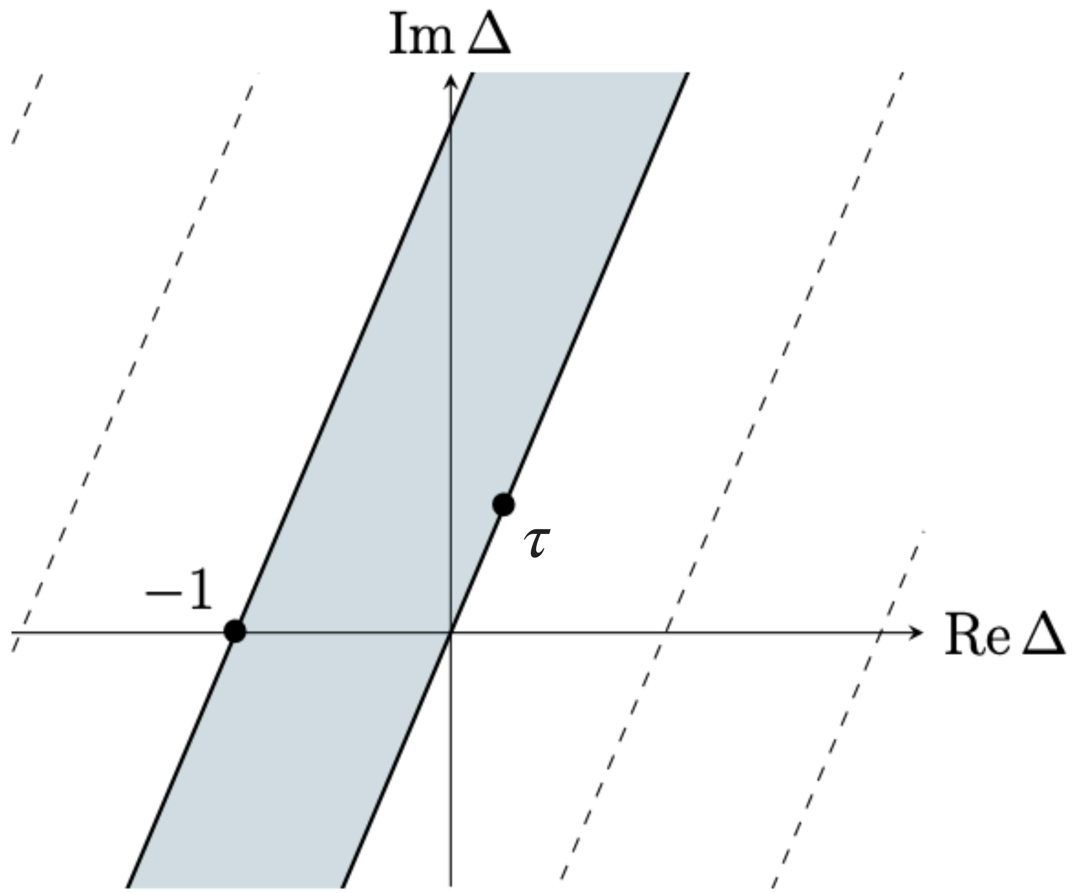
THANKS!

DEFECT INDEX

$$I_{\mathscr{D}} = \frac{(p;p)_\infty^{N-1}(q;q)^{N-1}}{N!}\prod_{a=1}^{3}\Gamma_e(\Delta_a)^{N-1}\oint d\mathbf{u}\,\frac{\prod_{a=1}^{3}\prod_{i\neq j}\tilde{\Gamma}_e(u_i-u_j+\Delta_a)}{\prod_{i\neq j}\tilde{\Gamma}_e(u_i-u_j)}\cdot$$

$$\cdot\left(\sum_{i=1}^{N}\prod_{j\neq i}\frac{\theta_0(-u_{ij}-\Delta_2+\sigma)}{\theta_0(-u_{ij}+\Delta_1-\sigma)}\frac{\theta_0(u_{ij}-\Delta_1-\Delta_2+2\sigma)}{\theta_0(u_{ij})}\right)$$

BETHE ANSATZ



EFT

$$\mathcal{I} = N \exp\Bigg(-\frac{\pi \mathrm{i}(N^2-1)}{\sigma\tau} \prod_{a=1}^{3} (\{\Delta_a\} - n) + \frac{2\pi\mathrm{i}(N-1)}{\sigma} \prod_{a=2}^{3} (\{\Delta_a\} - n) - \frac{\pi \mathrm{i} n_0 (N^2-1)(\omega_1^2 + \omega_2^2 + 3\omega_1\omega_2)}{12\omega_1\omega_2} \Bigg) .$$

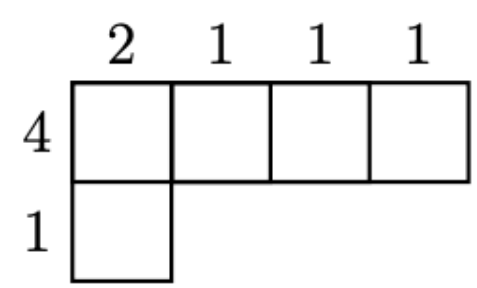
$$\cdot \frac{1}{N!} \int \mathrm{d}\Lambda \int \prod_{i=1}^{N} \frac{\mathrm{d}\lambda_i}{\sqrt{-\omega_1\omega_2}} \frac{\mathrm{e}^{-\frac{\pi \mathrm{i} n_0 N}{\omega_1\omega_2} \sum_{i=1}^{N} \lambda_i^2 + 2\pi\mathrm{i}\Lambda \sum_{j=1}^{N} \lambda_j}}{\prod_{i<j} \Gamma_h(\lambda_{ij}) \Gamma_h(-\lambda_{ij})} \sum_{i=1}^{N} \frac{\mathrm{e}^{-2\pi \mathrm{i} n N \frac{\lambda_i}{\omega_2} + \pi \mathrm{i} n (N-1)}}{\prod_{j\neq i}^{N} \left(1 - \mathrm{e}^{-2\pi\mathrm{i}\frac{\lambda_{ij}}{\omega_2}}\right)} .$$

GENERALITIES ON GW DEFECTS

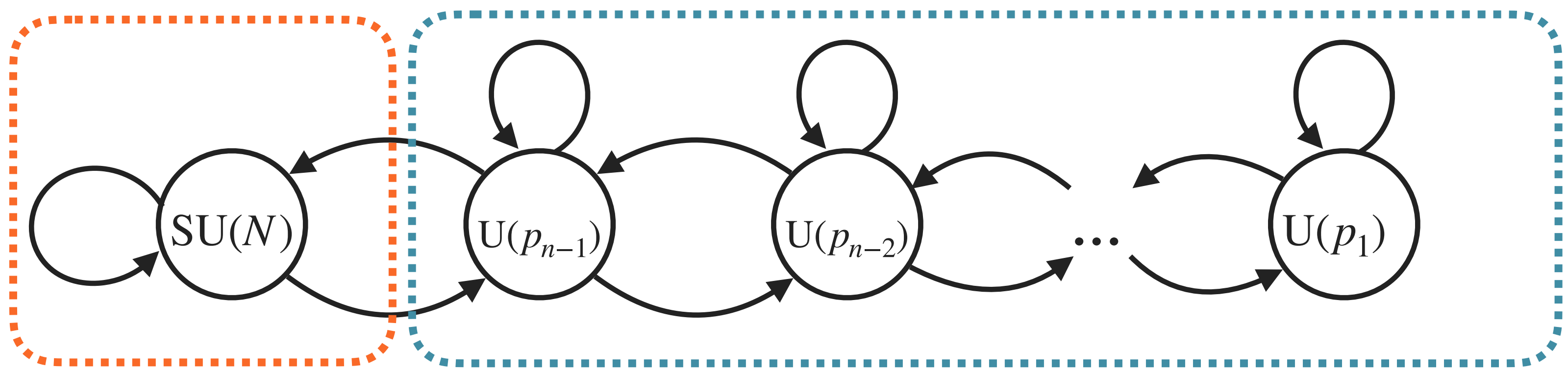
$$A = a(r)\,\mathrm{d}\theta + \cdots, \qquad \phi = b(r)\frac{\mathrm{d}r}{r} - c(r)\,\mathrm{d}\theta + \cdots$$

$G = \mathrm{SU}(5)$

$\mathbb{L} = [4, 1]$



$$p_j = \sum_{i=1}^{j} k_i$$



$$\mathcal{I}_{\mathrm{NSNS}} = \mathrm{Tr}_{\mathrm{NSNS}}(-1)^F \mathrm{e}^{2\pi\mathrm{i}\tau_{2d}L_0}\mathrm{e}^{-2\pi\mathrm{i}\bar{\tau}_{2d}(\bar{L}_0 - \frac{1}{2}\bar{J}_0)}\mathrm{e}^{2\pi\mathrm{i}z_{\mathrm{NS}}J_0}\mathrm{e}^{2\pi\mathrm{i}\chi J_A}\mathrm{e}^{2\pi\mathrm{i}uC}$$

$$\sigma = \tau_{2d}, \quad \tau = \frac{\tau_{2d}}{2} - u_C, \quad \Delta_1 = \frac{\tau_{2d}}{2} + 2\chi - u_C, \quad \Delta_2 = \frac{\tau_{2d}}{2} + z - 2\chi,$$