New developments on asymptotically leaky boundaries

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Based on 2408.13203 with R. McNees and wip
Based also on work with M. Geiller 2205.11401 & 2401.09540

Table of contents

Asymptotic boundaries

1. Asymptotic states

2. Notion energy

Charges ~ generator of symm. → asympt. symm

Boundary → Choice BC gauge → allowed symm.

→ Charges $\left\langle \begin{array}{l} = 0 \quad \text{trivial} \\ \neq 0 \quad \text{large} \end{array} \right.$ $\quad AS = \dfrac{\text{allowed symm}}{\text{trivial symm.}}$

→ algebra ASA → AdS_3 / CFT_2

3. Holographic duality

Leaky boundaries

Symplectic potential Θ ; $\delta L = \text{EOM}\ \delta(\text{fields}) + d\Theta$

→ used for instance to compute charges

ex: $\Theta^{\mu}_{GR} = \frac{1}{2}\sqrt{g}\left(\nabla_{\nu}(\delta g)^{\nu\mu} - \nabla^{\mu}(\delta g)^{\nu}_{\nu}\right)$

$\boxed{\text{Leaky : } \Theta \neq 0}$ ⤳ Non conservation of the charges

ex: 1. 4d GR $\Lambda = 0$, $\Theta_{AS} \propto \text{News}\ \delta(\text{shear})$ $\qquad$ $\text{News} = \dot{\text{shear}}$

2. 4d GR $\Lambda \neq 0$, no Dirichlet BC
no Neumann BC
...
$\Theta_{FG} \propto T^{ij}\,\delta h^{(0)}_{ij}$

→ coupled in a unspecified way to the environment
→ compatible with radiation

3. matter falling inside a BH horizon

Challenges

1. Asympt. boundaries $\rightarrow$ ill-defined limit to the boundary

   $\rightarrow$ Renormalization

   - Compère - Marolf ['08] but based on BC
   - New prescription not tied to BC [McNees-CZ 2306.16451/2408.13203]

2. More freedom in approaching the boundary yield more charges

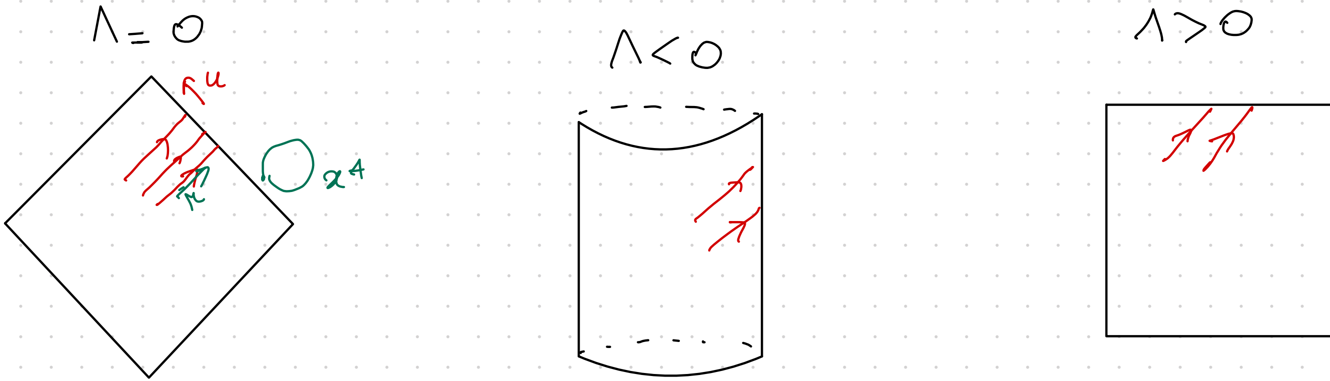
   & freedom in their definition

   (eg: change of slicing
   relaxation of the gauge, ...)

What are they ?

Bondi gauge [Bondi et al '62, Sachs '61]

- approach the asymptotic bndy along a null direction
- valid for [Poole, Skenderis, Taylor '18] [Compère-Fiorucci-Ruzziconi '19, '20]



- coordinates (u, r, x^A)

1 $g^{uu} = 0$ (∂_r is a null direct°)

2 $g^{uA} = 0$ (x^A & u are transverse)

- Condition on r? <
  - Newman-Unti condition
  - Bondi-Sachs condition

Partial Bondi gauge

[2205.11401 Geiller - CZ]

null $= \partial_u$

$g^{uu} = 0$; $g^{uA} = 0$; no specification on r more than a param. along the null geod.

$$ds^2 = e^{2\beta} V du^2 - 2e^{2\beta} du dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

+ conformal compactification

$g_{AB} = \overset{0}{\gamma}_{AB}(u, x^A) r^2 + \overset{1}{\gamma}_{AB} r + \dots$; β, V, U^A arbitrary functions of (u, r, x^A)

NU condition : r is the affine parameter of a null generator $\partial_r \to \beta = 0$

BS condition r = areal distance $\det g_{AB} = r^4 \det \underbrace{\overset{0}{q}_{AB}}_{S^2}$

EOM: $E^{\mu\nu} = -R^{\mu\nu} + \frac{1}{2} g^{\mu\nu}(R - 2\Lambda)$

Hierarchy of EOM

$$ds^2 = e^{2\beta}V\, du^2 - 2e^{2\beta}\, du\, dr + g_{AB}\,(dx^A - U^A du)(dx^B - U^B du)$$

$$g_{AB} = r^2 \overset{0}{\gamma}_{AB} + r\left(\overset{\wedge 1}{\gamma}_{AB} + \frac{1}{2}\gamma_1 \overset{0}{\gamma}_{AB}\right) + \left(\underbrace{\overset{\wedge 2}{\gamma}_{AB}}_{\overset{2}{\gamma}_{AB} + \frac{1}{4}\gamma_1 \overset{\wedge 1}{\gamma}_{AB}} + \frac{1}{2}\gamma_2 \overset{0}{\gamma}_{AB}\right) + \frac{1}{r}\left(\overset{\wedge 3}{\gamma}_{AB} + \frac{1}{2}\gamma_3 \overset{0}{\gamma}_{AB}\right) + \dots$$

$E_{rr} = 0 \Rightarrow \beta = \beta_0(u, x^A) + \frac{1}{r^2}\left(\frac{1}{32}\overset{\wedge}{\gamma}{}_1^{AB}\overset{\wedge 1}{\gamma}_{AB} + \frac{1}{64}(\gamma_1)^2 - \frac{1}{8}\gamma_2\right) + O(r^{-3})$

BS condition $\det g_{AB} \Rightarrow \gamma_1 = 0\ ,\ \gamma_2 = \frac{1}{8}\overset{\wedge}{\gamma}{}_1^{AB}\overset{\wedge 1}{\gamma}_{AB}$

NU condition $\gamma_1 = 0\ ,\ \gamma_2$ is fixed

angular momentum

$E_{rA} = 0 \Rightarrow U^A = U_0^A(u, x^A) + \frac{1}{r}(\dots) + \frac{1}{r^2}(\dots) + \frac{1}{r^3}\left(P^A(u, x^A) + \dots\right) + O\left(\frac{1}{r^4}\right)$

$E_{ur} = 0 \Rightarrow V = r^2 \frac{\Lambda}{3} e^{2\beta_0} + r(\dots) + (\dots) + \frac{1}{r}\left(M(u, x^A) + \dots\right) + \dots$

Mass

→ all the radial dependence is fixed.

Boundary metric $\lim_{r\to\infty} \frac{ds^2}{r^2} = e^{4\beta_0}\frac{\Lambda}{3}\, du^2 + \overset{0}{\gamma}_{AB}\,(dx^A - U_0^A du)(dx^B - U_0^B du)$

So we have $g_{AB}(u,r,x^A)$; $\beta_0(u,x^A)$, $\mathcal{U}^A_0(u,x^A)$, $\mathcal{P}^A(u,x^A)$, $\mathcal{M}(u,x^A)$

$$E^{TF}_{AB} = 0 \implies \underbrace{\partial_u \gamma^0_{AB} - \gamma^0_{AB}\,\partial_u \ln\sqrt{\gamma_0} + 2D_{\langle A}\mathcal{U}^0_{B\rangle}}_{\sim \text{ shear of } \partial_u} = \frac{\Lambda}{3} e^{2\beta_0} \underbrace{\hat{\gamma}^1_{AB}}_{\sim \text{ shear of } \partial_r}$$

GW - radiation

$\Lambda = 0$: $\partial_u \hat{\gamma}^{(n\geq 3)}_{AB}$ is fixed

$\Lambda \neq 0$: $\hat{\gamma}^3_{AB}$ is free
$\hat{\gamma}^{(n>3)}_{AB}$ are fixed

$E_{uu} = 0 \implies \partial_u \mathcal{M}$ is fixed

$E_{uA} = 0 \implies \partial_u P_A$ is fixed

for $\mathcal{U}^A_0 = 0$

$$\Big(\partial_u + \frac{3}{2}\partial_u \ln\sqrt{\gamma_0}\Big)\mathcal{M} - \frac{1}{2} D_A\Big(\underset{\sim \text{"}D_A\partial_u\hat{\gamma}_1^{AB}\text{"}}{\mathcal{J}^A} - \frac{\Lambda}{6}P^A\Big)$$
$$= \frac{1}{4}\hat{\gamma}^1_{AB}\Big(\underset{\partial_u^2\hat{\gamma}_1^{AB}}{\mathcal{N}^{AB}} + \frac{\Lambda^2}{36}\underset{\hat{\gamma}_3^{AB}}{\mathcal{E}^{AB}}\Big)$$

(definition using Newman-Penrose scalars)

Solution space

$\Lambda = 0$: • boundary data $\mathring{\gamma}_{AB}$, U_0^A s.t $\partial_u \mathring{\gamma}_{AB} - \mathring{\gamma}_{AB} \partial_u \ln \sqrt{\mathring{\gamma}} + 2 D_{\langle A} \mathring{U}_{B\rangle} = 0$

• kinematical data β_0, $\gamma_{n\geq 1}$

• free data $\hat{\gamma}'_{AB}$ shear ~ Gravitational waves

• dynamical data ~ Constrainted by EOM

M, P_A, $\hat{\gamma}_{AB}^{(n\geq 3)}$ ~ mass, angular mom. , "higher spin charges"

$\Lambda \neq 0$: • kinematical data $\overset{\circ}{\gamma}_{AB}$, β_0, $\overset{\circ}{U}_A$, $\gamma_{n\geq 1}$, $\hat{\gamma}^3_{AB}$

• dynamical data $\mathcal{M}$, P_A

• $$\hat{\gamma}^1_{AB} = \frac{3}{\Lambda} e^{-2\beta_0} \left[\partial_u \overset{\circ}{\gamma}_{AB} - \partial_u \ln\sqrt{\gamma_0}\, \overset{\circ}{\gamma}_{AB} + D_{\langle A} \overset{\circ}{U}_{B\rangle} \right]$$

$\sim$ " $D_A \partial_u \hat{\gamma}_1^{AB}$ "

$$\wr$$

$$\left(\partial_u + \frac{3}{2}\partial_u \ln\sqrt{\gamma_0}\right)\mathcal{M} - \frac{1}{2} D_A\left(\mathcal{J}^A - \frac{1}{6}P^A\right)$$

$$= \frac{1}{4}\hat{\gamma}^1_{AB}\left(\mathcal{N}^{AB} + \frac{\Lambda^2}{36}\mathcal{E}^{AB}\right)$$

$\wr$ $\partial_u^2 \hat{\gamma}_1^{AB}$ $\qquad$ $\wr$ $\hat{\gamma}_3^{AB}$

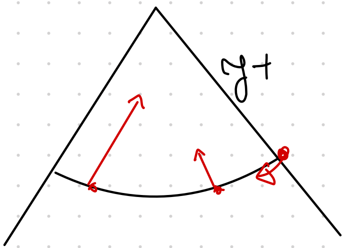
[Ashtekar, Bonga, Kesavan '15]
[Compère, Fiorucci, Ruzziconi '19, '20]
[Bonga, Bunster, Pérez '23]
[Ciambelli, Pasterski, Tabor '24]
[Compère, Hoque, Kutluk '23
[Senovilla et al]

Symmetries

ξ s.t. $\quad g + \mathcal{L}_\xi g \;\in$ Partial Bondi gauge

Preserving $\quad g_{rr} = 0 \quad \Rightarrow \quad \xi^u = f(u, x^A)$

$$g_{rA} = 0 \quad \Rightarrow \quad \xi^A = Y^A(u, x^A) - \int_r^\infty dr'\, e^{2\beta} g^{AB} \partial_B f$$
$$= Y^A + \mathcal{O}\left(\frac{1}{r}\right)$$



$$g_{AB} = \mathring{\gamma}_{AB} r^2 + \dots \quad \Rightarrow \quad \xi^r = r\, h(u, x^A) + \sum_{n=0}^{\infty} \frac{\xi^r_n}{r^n}$$

$$\xi^r = r h + \left(k - \frac{1}{2} \Delta f\right) + \frac{1}{r}\left(l - \frac{1}{2} D_A \gamma_1^{AB} \partial_B f + \frac{1}{2} D^A(\partial_A f)\right) + \mathcal{O}\left(\frac{1}{r^2}\right)$$

$$\delta \ln\sqrt{\mathring{\gamma}_0} = (2h + D_A Y^A)$$

$$\delta \gamma_1 = (\dots)\, \gamma_1 + 4k$$

$$\delta \gamma_2 = (\dots)\, \gamma_2 + 4l + k\gamma_1 - \hat{\gamma}_1^{AB} D_A \partial_B f$$

↳ Charges analysis will show these are pure gauge

$$\text{AKV} = \xi = f\,\partial_u + Y^A\partial_A + \Big(r\,h + k + \tfrac{1}{2}\Delta f + \tfrac{1}{r}\,\ell - \tfrac{1}{2} D_A \gamma_1^{AB}\partial_B f + \tfrac{1}{2}\partial^A\gamma_1\,\partial_A f\Big)$$

+ sublead.

Algebra:
$$\{(f_1, Y_1, h_1, k_1, \ell_1), (f_2, Y_2, h_2, k_2, \ell_2)\}_* = (f_{12}, Y_{12}, h_{12}, k_{12}, \ell_{12})$$

$$f_{12} = f_1\,\partial_u f_2 + Y_1^A\partial_A f_2 - \delta_{\xi_1} f_2 - (1 \leftrightarrow 2)$$

$$Y_{12}^A = f_1\,\partial_u Y_2^A + Y_1^B\partial_B Y_2^A - \delta_{\xi_1} Y_2^A - (1 \leftrightarrow 2)$$

$$h_{12} = f_1\,\partial_u h_2 + Y_1^A\partial_A h_2 - \delta_{\xi_1} h_2 - (1 \leftrightarrow 2)$$

$$k_{12} = f_1\,\partial_u k_2 + Y_1^A\partial_A k_2 - h_1 k_2 - \delta_{\xi_1} k_2 - (1 \leftrightarrow 2)$$

$$\ell_{12} = f_1\,\partial_u \ell_2 + Y_1^A\partial_A \ell_2 - 2h_1\ell_2 - \delta_{\xi_1}\ell_2 - (1 \leftrightarrow 2)$$

$$\boxed{(\mathrm{Diff}(\mathcal{J}^+) \oplus \mathbb{R}_h) \ltimes (\mathbb{R}_k \oplus \mathbb{R}_\ell)}$$

In [2401.09540 Geiller-CZ], for $\Lambda = 0$, $Q_k \neq 0$, $Q_\ell \neq 0$ → large!

Symplectic potential

- make it finite up δ-exact $\to$ charge will be finite

  Charge $Q_\xi = \oint_{S^{d-2}} k_\xi^{ur}\, dS^2$

  $\partial_\nu k_\xi^{\mu\nu} = \omega^\mu(\delta_\xi \phi, \delta\phi) = \delta_\xi \Theta^\mu \to \partial_r k^{ur} + \partial_A k^{uA} = \omega^u$

$\delta L = EOM\, \delta g + d\Theta \Rightarrow$ Ambiguity $\Theta \longrightarrow \Theta + dY$

$\Theta^\mu \to \Theta^\mu + \partial_\nu Y^{\mu\nu}$

Choice $Y^{ur} = -\int_r \Theta^u + o(r^{-1}) \Rightarrow \Theta^u \to o(r^{-1})$

$\Theta^r \to \Theta^r + \partial_u \int_r \Theta^u$

$\Rightarrow \partial_r \oint_{S^2} k^{ur} = o(r^{-1})$

- Rem: still finite ambiguity $\to$ we make a choice.

$$\underset{\leftarrow}{\Theta} = \frac{1}{4}\sqrt{\gamma_0}\, e^{2\beta_0}\Big[\hat{N}^{AB}\,\delta\big(e^{-2\beta_0}\hat{\gamma}^1_{AB}\big) + \hat{T}^{AB}\,\delta\gamma^0_{AB} + T\,\delta\big(\ln\sqrt{\gamma_0} - 4\beta_0\big)\Big]$$

$$+ \partial_u(\dots) + \delta(\dots)$$

$$\hat{N}_{AB} = \partial_u \hat{\gamma}^1_{AB} - \frac{1}{2}\hat{\gamma}^1_{AB}\,\partial_u \ln\sqrt{\gamma_0} - \frac{\Lambda}{6}\gamma^0_{AB}\, e^{2\beta_0}\,\hat{\gamma}_1^{CD}\hat{\gamma}^1_{CD}$$

$$T_{AB} = R\,\hat{\gamma}^1_{AB} - \frac{1}{2}D^2\hat{\gamma}^1_{AB} + \frac{3}{2}e^{-2\beta_0}\hat{\gamma}^1_{AB}\,D^2 e^{2\beta_0} - 6\,\partial_C\beta_0\, D^C\hat{\gamma}^1_{AB} + 8\,\partial^C\beta_0\, D_{\langle A}\hat{\gamma}^1_{B\rangle C}$$

$$+ \Lambda\left(\frac{1}{12}\hat{\gamma}^1_{AB}\,\hat{\gamma}_1^{CD}\hat{\gamma}^1_{CD} - \frac{1}{3}\mathcal{E}_{AB}\right)$$

$$T = 4\mathcal{M} - \frac{e^{-2\beta_0}}{\sqrt{\gamma_0}}\,\partial_u\left(\sqrt{\gamma_0}\,\hat{\gamma}_1^{CD}\hat{\gamma}^1_{CD}\right) + D_A D_B\,\hat{\gamma}_1^{AB} - e^{-2\beta_0}\,\hat{\gamma}_1^{AB}\, D_A\partial_B e^{2\beta_0}$$

- flat limit ok
- w/ $\sqrt{\gamma_0}\, P_A\, \delta\mathcal{U}_0^A$

$$\underset{\leftarrow}{\Theta} = \frac{1}{4}\sqrt{\gamma_0}\, e^{2\beta_0}\left[\hat{N}^{AB}\,\delta\left(e^{-2\beta_0}\,\hat{\gamma}^{1}_{AB}\right) + \hat{T}^{AB}\,\delta\gamma^{0}_{AB} + T\,\delta\left(\ln\sqrt{\gamma_0} - 4\beta_0\right)\right]$$

$$+ \partial_u\left[\frac{1}{4}\sqrt{\gamma_0}\left(\hat{\tau}^{AB}\,\delta\gamma^{0}_{AB} + \tau\,\delta\left(\ln\sqrt{\gamma_0} - 4\beta_0\right)\right)\right] + \delta\left(A_B + \partial_u A_{\gamma B}\right)$$

$$\hat{\tau}_{AB} = \frac{1}{4}\hat{\gamma}^{AB}_{1}\,\gamma_1$$

$$\tau = -\gamma_2 + \frac{1}{2}\hat{\gamma}^{AB}_{1}\,\hat{\gamma}^{1}_{AB}$$

- β_0 not indep.
- γ_1, γ_2 only @ corners

Phase space for $\Lambda \neq 0$

$\rightarrow$ use $\hat{\gamma}'_{AB} = \frac{3}{\Lambda} e^{-2\beta_0} \left(\partial_u \gamma^0_{AB} - \partial_u \ln \sqrt{\gamma_0}\, \gamma^0_{AB} \right)$

$$\underset{\leftarrow}{\Theta} = \frac{1}{4} \sqrt{\gamma_0}\, e^{2\beta_0} \left[\hat{T}_\Lambda^{AB}\, \delta \gamma^0_{AB} + T_\Lambda\, \delta\left(\ln \sqrt{\gamma_0} - 4\beta_0 \right) \right]$$

$$+ \partial_u \left[\frac{1}{4} \sqrt{\gamma_0} \left(\hat{\tau}_\Lambda^{AB}\, \delta \gamma^0_{AB} + \tau_\Lambda\, \delta\left(\ln \sqrt{\gamma_0} - 4\beta_0 \right) \right) \right] + \delta(\dots)$$

$\rightarrow$ first line is from $T^{ij}\, \delta \underbrace{h^{(0)}_{ij}}$

$$\frac{\Lambda}{3} e^{2\beta_0} du^2 + \gamma^0_{AB} \left(dx^A - U^A_0 du \right) \left(dx^B - U^B_0 du \right)$$

Next steps

→ Charges & symmetries

→ Relation to holographic renormalization

Summary & Outlook

→ Radiation ⇒ leaky ⇒ in dS_4 non conformally flat boundary metric

→ Renormalization prescription suited for leaky boundaries

→ New charges in PBG <u>but</u> no flux balance laws

→ Appear in a corner term in the symplectic potential

Summary & Outlook

→ Portion AdS → Boundary CFT

→ Flat → analysis @ spatial infinity

→ Subregion on the boundary

→ dS_4 (?)