The Schwinger Model:
A case study in de Sitter QFT

Based on 2403.16166 w/ D. Anninos & A.R. Fukelman

Black Holes, Holography & de Sitter Spacetimes

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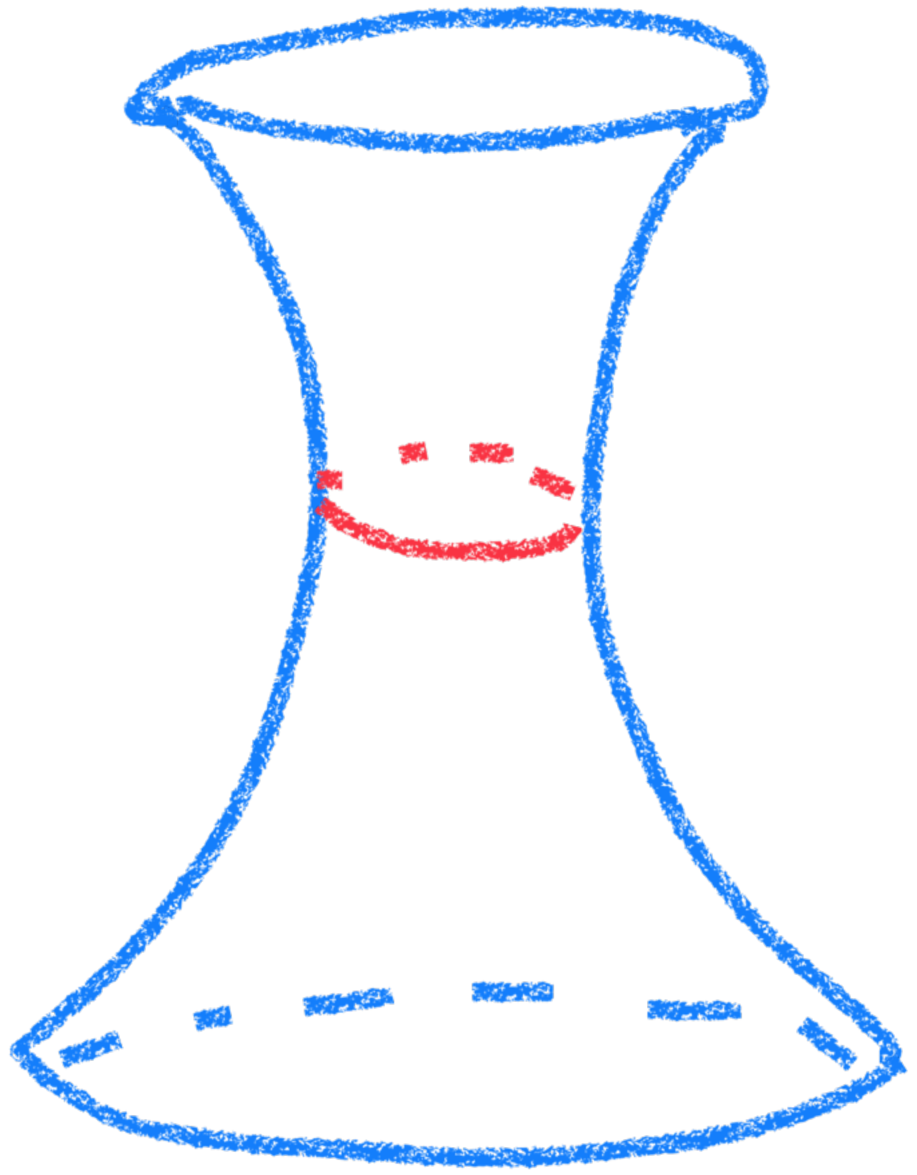
Preface: This talk will be about QFT on a rigid de Sitter spacetime

Reason: We are still confused by basic QFT notions on dS, which I will now review

As mentioned in previous talks:

$$-(X^0)^2 + (X^1)^2 + \dots + (X^d)^2 = \ell^2$$



Planar

$$\frac{ds^2}{\ell^2} = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2}$$

Global

$$\frac{ds^2}{\ell^2} = -d\tau^2 + \cosh^2\tau \, d\Omega_{d-1}^2 :$$

is time dependent

but still maximally symmetric

I will now remind you about a few asumptions that we take for granted in flat space

but that we must confront in dS

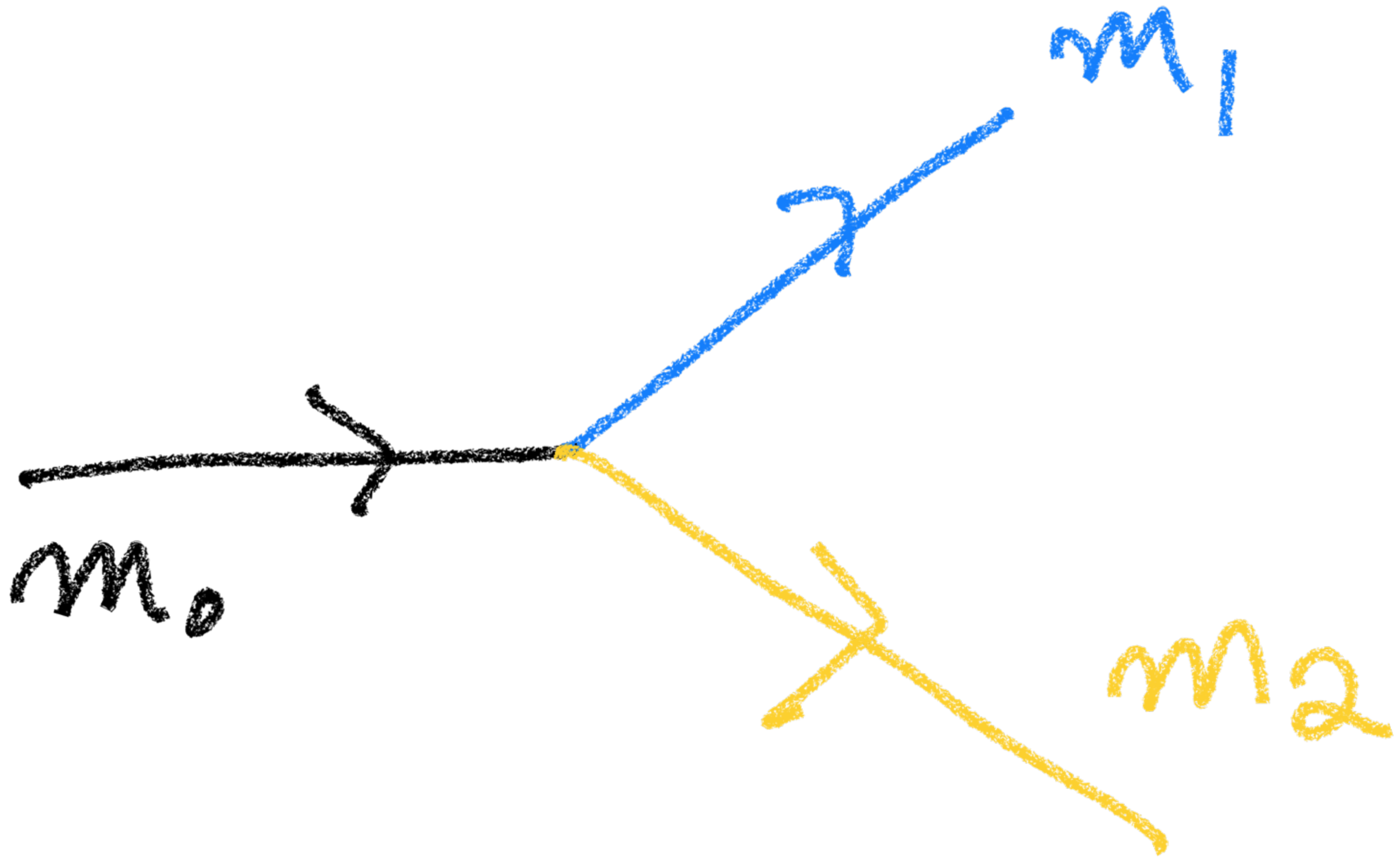
Lesson 1:

The Wilsonian paradigm on de Sitter

is dead

Simple argument

In flat space, the following process:



is forbidden if $m_0 < m_1 + m_2$

$\rightsquigarrow$ Consequence of energy conservation

In de Sitter:

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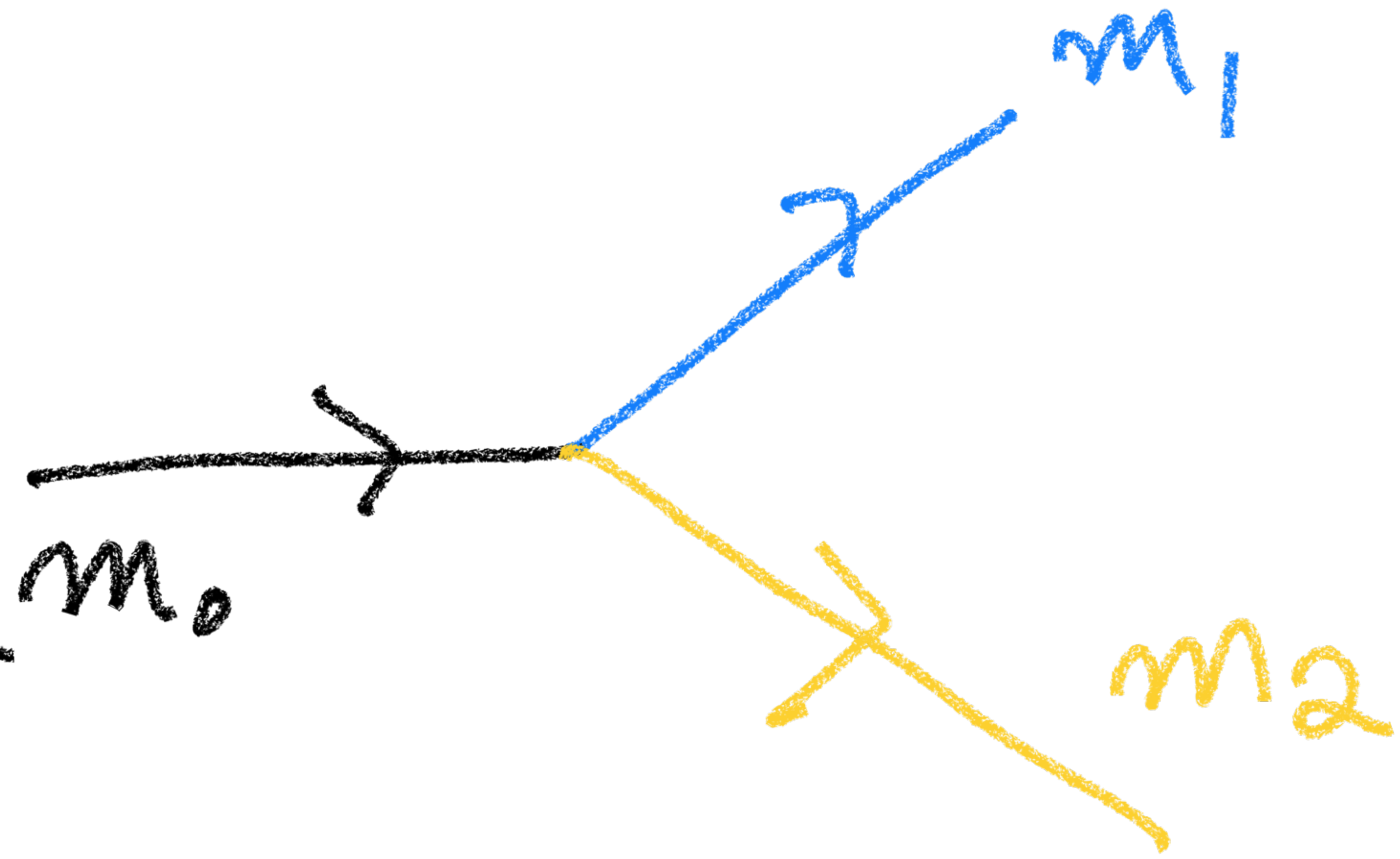


$\Big) \neq 0$

even if $m_0 < m_1 + m_2$, because dS is time dependent $\longrightarrow$ no conservation of energy

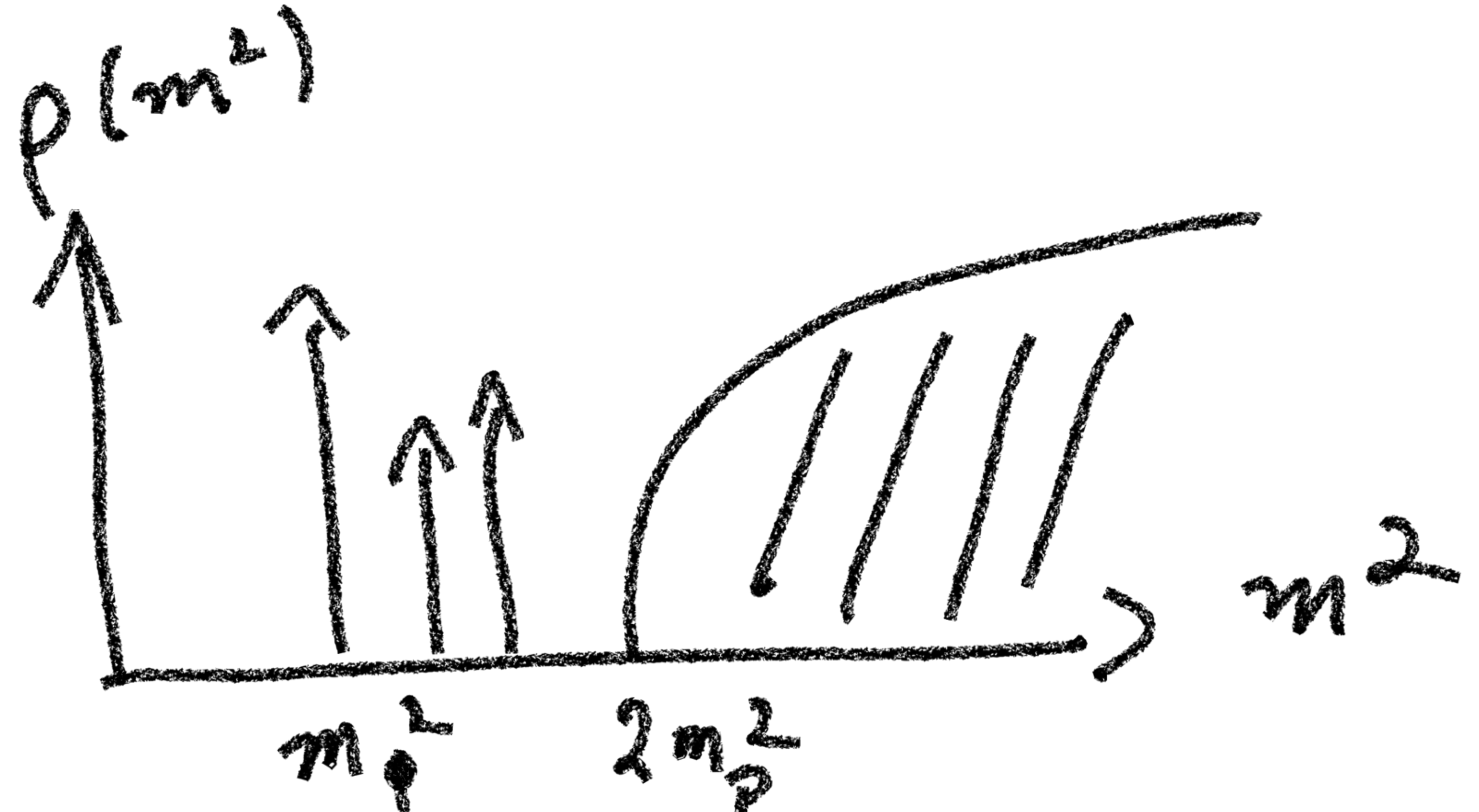
Not possible to "integrate out" heavy states consistently

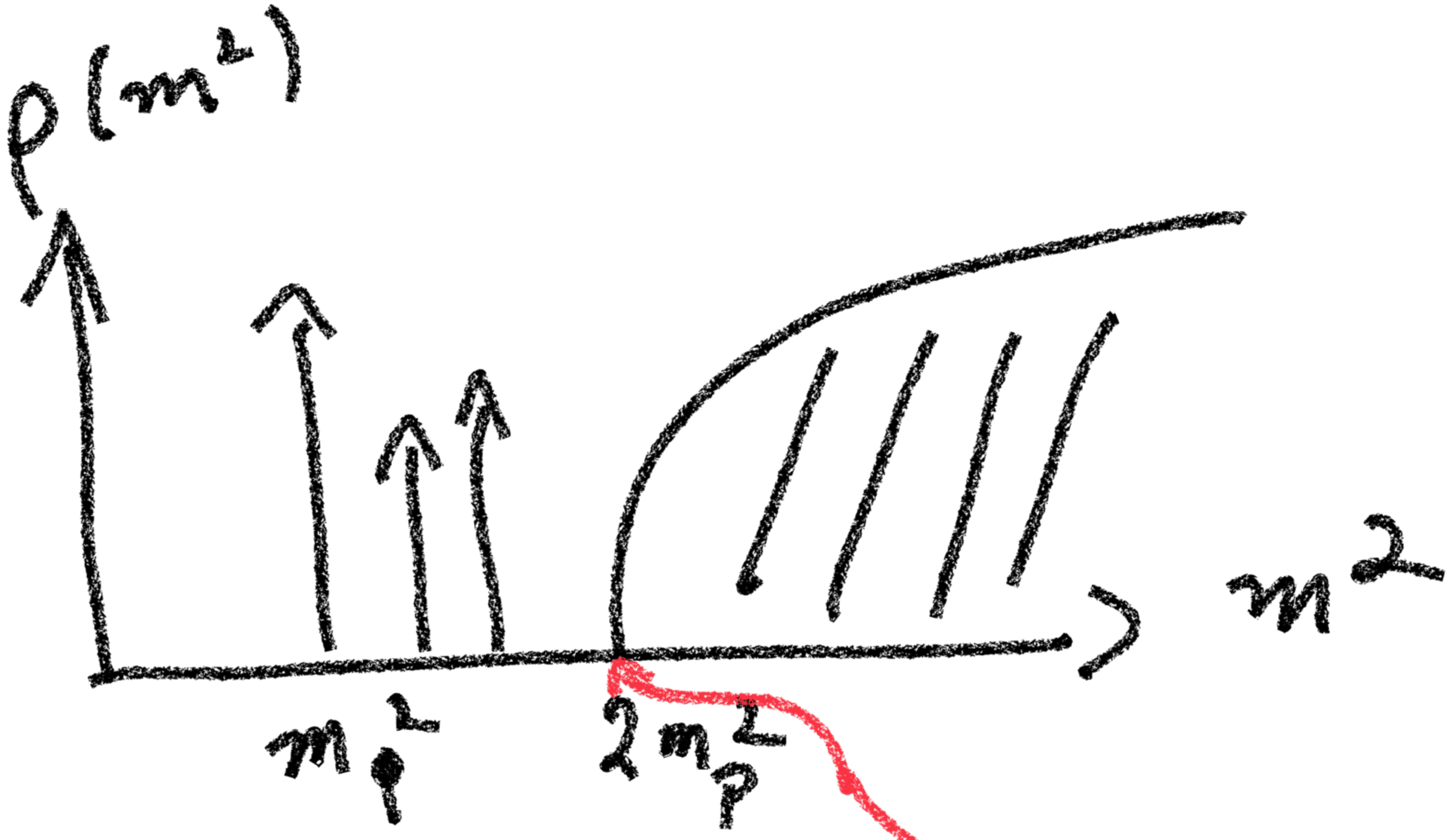
In other words, the UV never fully decouples

More precisely: Weakly interacting scalar in flat space

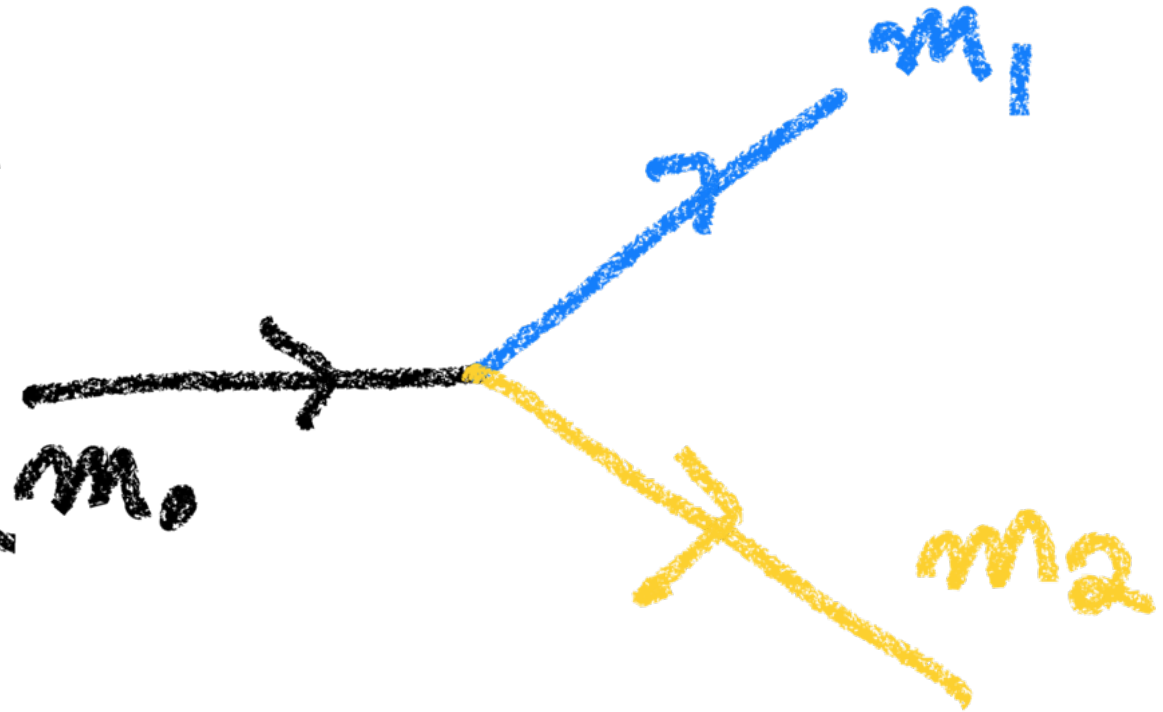
$$G_i(\vec{x},\vec{y}) = \int_0^\infty dm^2\, \rho(m^2)\, \underbrace{G_{free}(\vec{x},\vec{y};\, m^2)}_{\int \frac{d^d p}{(2\pi)^d} \frac{e^{ip\cdot(\vec{x}-\vec{y})}}{p^2+m^2}}$$

Unitarity: $\rho(m^2) > 0$





Flat Space

$\mathcal{A}\Big($ 

$\Big) = 0 \longleftrightarrow$ Cut starts at $2m_p^2$

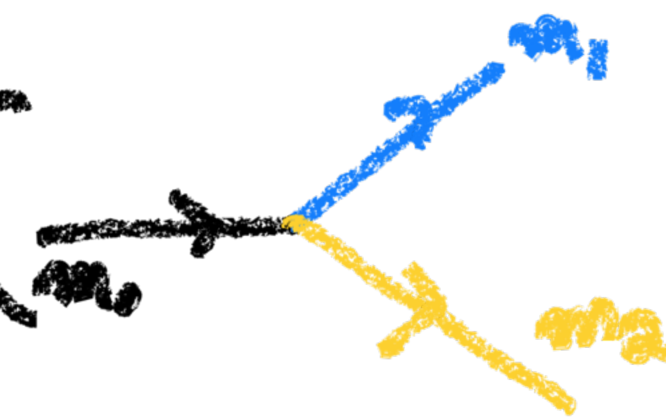
In de Sitter

$SO(1,d-1)$ casimir:

$$\Delta(\Delta-(d-1)) = -m^2\ell^2$$

$$\hookrightarrow \Delta = \frac{d-1}{2} \pm i\nu \qquad \nu = \sqrt{m^2\ell^2 - \left(\frac{d-1}{2}\right)^2}$$

$$G_i = \int_{\mathbb{R}} d\nu \, \rho(\nu) \, G_{\text{free}}(\nu, \vec{x}, \vec{y})$$

$\rho(\nu)$ has support on all $\mathbb{R}$ $\longrightarrow$ $\mathcal{A}\Big($ 

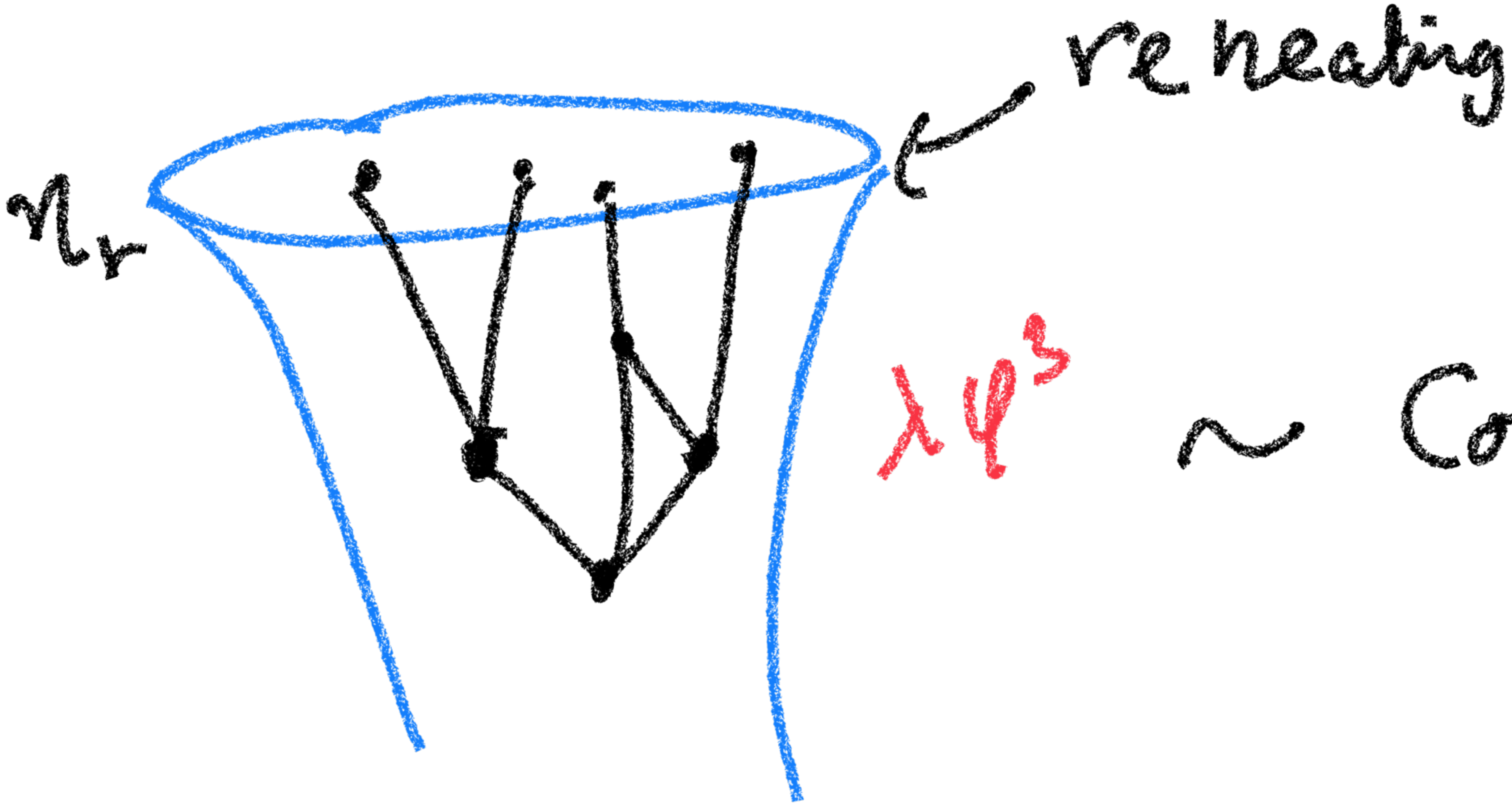
$\Big) \neq 0$

Lesson 2:

Perturbation theory in dS is not trustworthy at late times

To connect w/ the physics of inflation & the CMB we are interested in:

equal-time / late time correlators



$$\frac{ds^2}{\ell^2} = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2}$$

$\lambda\varphi^3$ ~ Computation ~ Witten diagram

Weinberg theorem 0605244

Diagram w/ N time integrals $\sim \left(\log \mu|\eta_r|\right)^N$

$\rightsquigarrow$ grows at $\eta_r \to 0$

breakdown in perturbation theory

How much can we trust a weakly coupled inflationary picture at late times?

Lesson 3

Massless fields in de Sitter lead to a violation of de Sitter invariance

Toy example: massless scalar

$$S = -\int d^d x \sqrt{g}\, (\partial\phi)^2 \qquad \frac{ds^2}{\ell^2} = -d\tau^2 + \cosh^2\tau\, d\Omega_{d-1}^2$$

dS-invariant correlator obtained from analytic continuation from S^d

$$\tau \to i(\theta - \pi/2) \qquad \frac{ds^2}{\ell^2} \to d\theta^2 + \sin^2\theta\, d\Omega_{d-1}^2$$

$$\shortparallel$$

$$S^d$$

Green's function on S^d

$$\nabla^2 G = \frac{\delta(x-y)}{\sqrt{g}}$$

integrate

$$\int_{S^d} \sqrt{g}\, \nabla^2 G = \int_{S^d} \delta(x-y)$$

$$0 = 1 \;?$$

Leads to issues quantizing massless fields on dS

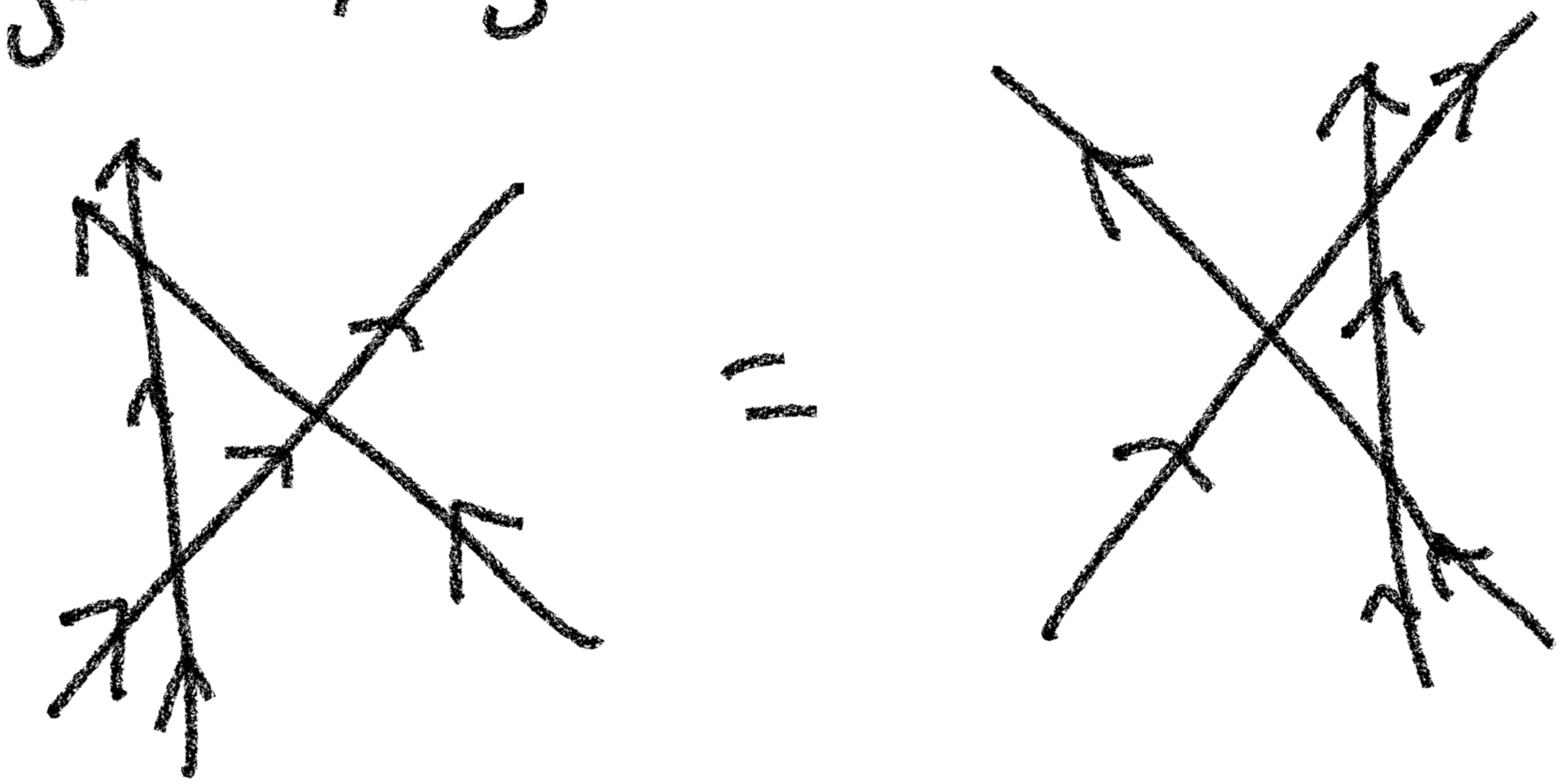
Can we build good theories w/ massless particles?

Lesson 4

Thermal nature of de Sitter ground state makes integrability seem impossible

Recall: Integrability in flat space requires
no particle production

e.g. Yang-Baxter equation presumes



In de Sitter, due to thermal occupation

particle production occurs even in free theories!

Is there a suitable notion of integrability in de Sitter?

When we have So many Questions we need to retreat to models where we can actually Compute

What Should model have?

1) Not a CFT (otherwise won't probe dS)
2) Not free
3) Have massless particles
4) Be unitary (no fishnet)

The model: Schwinger 1962

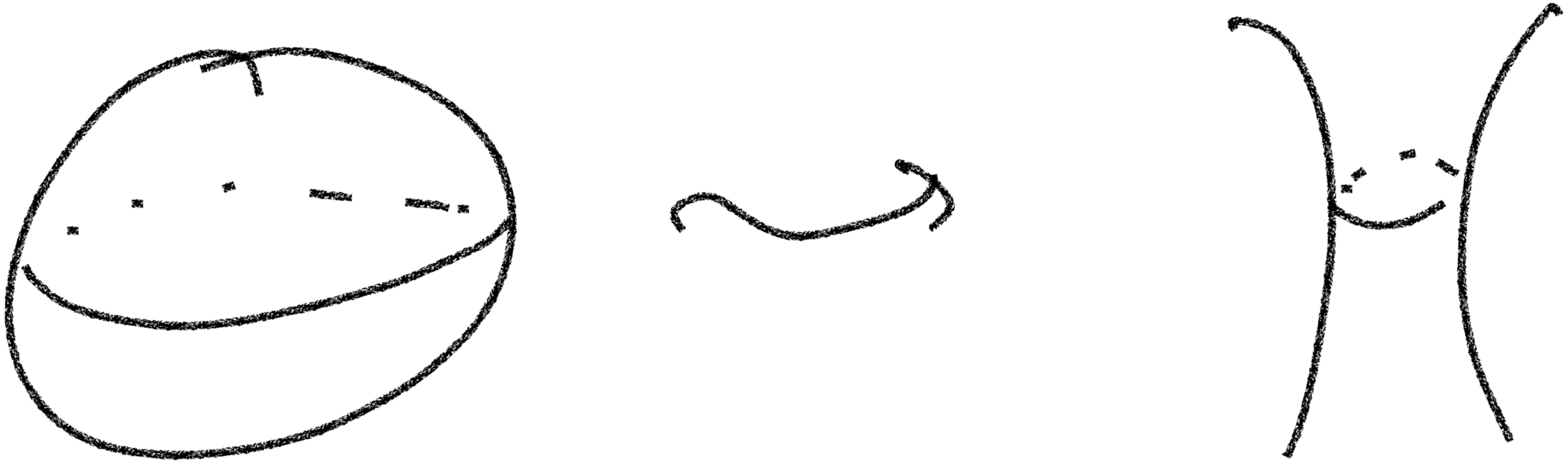
$$S = \int d^2x \sqrt{g} \left\{ \bar{\Psi} \gamma^\mu (\nabla_\mu + i q A_\mu) \Psi + \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - \frac{i\theta}{4\pi} \epsilon^{\mu\nu} F_{\mu\nu} \right\}$$

$g \equiv$ gauge coupling $[1/L]$

$q \in \mathbb{Z}$ is the fermion charge

We will consider this Euclidean theory on S^2

Correlation functions, under analytic cont., become correlators in the Bunch-Davies state on dS_2



Action exhibits

<u>Gauge invariance</u>

$$\psi \to e^{iq\alpha(x)}\psi$$

$$A_\mu \to A_\mu - \partial_\mu \alpha(x)$$

$+$

<u>Chiral Symmetry</u>

$$\underbrace{\begin{aligned}\psi &\to e^{i\beta\gamma_*}\psi \\ \bar{\psi} &\to \bar{\psi} e^{i\beta\gamma_*}\end{aligned}}_{\text{anomalous}}$$

Under a <u>local</u> chiral rotation

$\psi \to e^{i\gamma_* \beta(x)}\psi \qquad \bar{\psi} \to \bar{\psi} e^{i\beta\gamma_*} \qquad \gamma_* = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$S = \int d^2x \sqrt{g} \left\{ \bar{\psi}\gamma^\mu \left(\nabla_\mu + iqA_\mu - \epsilon_{\mu\nu}\partial^\nu \beta\right)\psi + \frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} - \frac{i}{4\pi}\left(\theta + 2q\beta\right)\epsilon^{\mu\nu}F_{\mu\nu} - \frac{1}{2\pi}\beta\nabla^2\beta \right\}$$

Note $\beta = \dfrac{2\pi n}{2q}$, $n = 0, \dots, 2q-1$

acts as $\theta \to \theta + 2\pi n$

Although $U(1)$-chiral is anomalous

$\mathbb{Z}_{2q}$ subgroup is preserved

$\rightsquigarrow$ not quite! for $n=q$, $e^{i\pi\gamma_*} = -1 = e^{i\pi}$

gauge

So the global symmetry is $\mathbb{Z}_q$

$$\beta = \frac{\pi n}{q} \qquad n = 0, \ldots, q-1$$

Original paper: $q=1$ $\rightsquigarrow$ no global symmetry

Why is this model solvable?

Work in Lorenz gauge: $A_\mu \equiv \epsilon_{\mu\nu}\partial^\nu \phi$

$$S' = \int d^2x \sqrt{g}\, \Big\{ \bar{\Psi}\gamma^\mu \left(\nabla_\mu + \epsilon_{\mu\nu}\left(iq\partial^\nu\phi - \partial^\nu\beta\right)\right)\Psi$$

$$+ \frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} - \frac{i}{4\pi}\left(\theta + 2q\beta\right)\epsilon^{\mu\nu}F_{\mu\nu}$$

$$- \frac{1}{2\pi}\beta\nabla^2\beta \Big\}$$

Choose $\beta = iq\phi$

Local chiral rot. gives us a frame where fermion is free

<u>Instantons</u>: Because we work on S^2 the path integral breaks up into a sum over instantons

$$Z = \sum_{k=-\infty}^{\infty} Z_k \quad \text{has } 2kq \text{ fermion}$$

Photon mass

$$S^{k\text{ inst}} = \frac{\pi k^2}{4g^2 \ell^2} + ik\theta + \frac{1}{2g^2}\int d^2x \sqrt{g}\; \phi \nabla^2\left(\nabla^2 - \frac{g^2 q^2}{\pi}\right)\phi$$

Now that we've set up the problem we can compute anything

Consider E-field (scalar): $E \equiv \frac{\epsilon^{\mu\nu} F_{\mu\nu}}{2}$

$$\langle \Omega | E(x) E(y) | \Omega \rangle = \nabla_x^2 \nabla_y^2 \langle \Omega | \phi(x) \phi(y) | \Omega \rangle$$

$$= -\frac{g^2 q^4}{\pi} \frac{\Gamma(\Delta)\Gamma(1-\Delta)}{4\pi} \, {}_2F_1\left(\Delta, 1-\Delta, 1, 1-\frac{u}{2}\right)$$

$$u = 2 \sin^2 \frac{\theta_{xy}}{2}$$

$$\Delta(\Delta - 1) = -\frac{g^2 q^2}{\pi} \ell^2$$

Exact in g!

Let us expand at small coupling:

$$\langle \Omega | E(x) E(y) | \Omega \rangle = \frac{g^2 q^4}{4\pi \ell^2} \left\{ -1 + \frac{q^2 g^2 \ell^2}{\pi} \left(1 + \log \frac{u}{2} \right) + \left(\begin{matrix} \text{Poly}\log \\ + \log 2 \end{matrix} \right) g^4 \ell^4 \right.$$

Here we see diagrammatic picture emerge w/ logs! but we also have exact answer

$\longrightarrow$ logs resum

Partition function (in some scheme)

$$Z = \# \left(\Lambda_{UV} \ell\right)^{\#} \frac{g\ell}{2\pi} \left[\frac{\Gamma(1+\bar{\Delta})}{\Gamma(1+\Delta)}\right]^{\frac{\Delta-\bar{\Delta}}{2}}$$

$$\times e^{\psi^{(-2)}(1+\Delta) + \psi^{(-2)}(1+\bar{\Delta})}$$

$$\Delta(\Delta-1) = -\frac{g^2 q^2 \ell^2}{\pi} \qquad \bar{\Delta} = 1-\Delta$$

All-loop result lets us benchmark loop calculations

Fermion 2 pt function

$$\mathrm{Tr}\,\langle \Omega |\, \bar{\Psi}(x)\, e^{-iq\int_x^y A}\, \Psi(y)\, |\Omega\rangle$$

$$= \underbrace{\#\, e^{G_\phi(0) - G_\phi(x,y)}}_{k=0} + \underbrace{\#\, e^{-\frac{\pi}{2g^2\ell^2}}\, e^{G_\phi(0) + G_\phi(x,y)}}_{\pm 1 \text{ instanton}}$$

(non-perturbative)

What happens at late times?

$$\frac{ds^2}{\ell^2} = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2} \qquad \text{as} \quad \eta \to 0$$

0-inst: $$A \left(\frac{x-y}{|\eta|}\right)^{1/2} \exp\left\{ a \left(\frac{x-y}{|\eta|}\right)^{-2\Delta} + \Delta \leftrightarrow \bar{\Delta} \right\}$$

$\pm$ 1-inst: $$A \left(\frac{x-y}{|\eta|}\right)^{1/2} \exp\left\{ -a \left(\frac{x-y}{|\eta|}\right)^{-2\Delta} + \Delta \to \bar{\Delta} \right\}$$

$\rightsquigarrow$ Relative non-perturbative suppression doesn't survive late-times

This gives further evidence that weak coupling & late-times don't play nicely

$\longrightarrow$ Need to explore more integrable models to extract general rules!